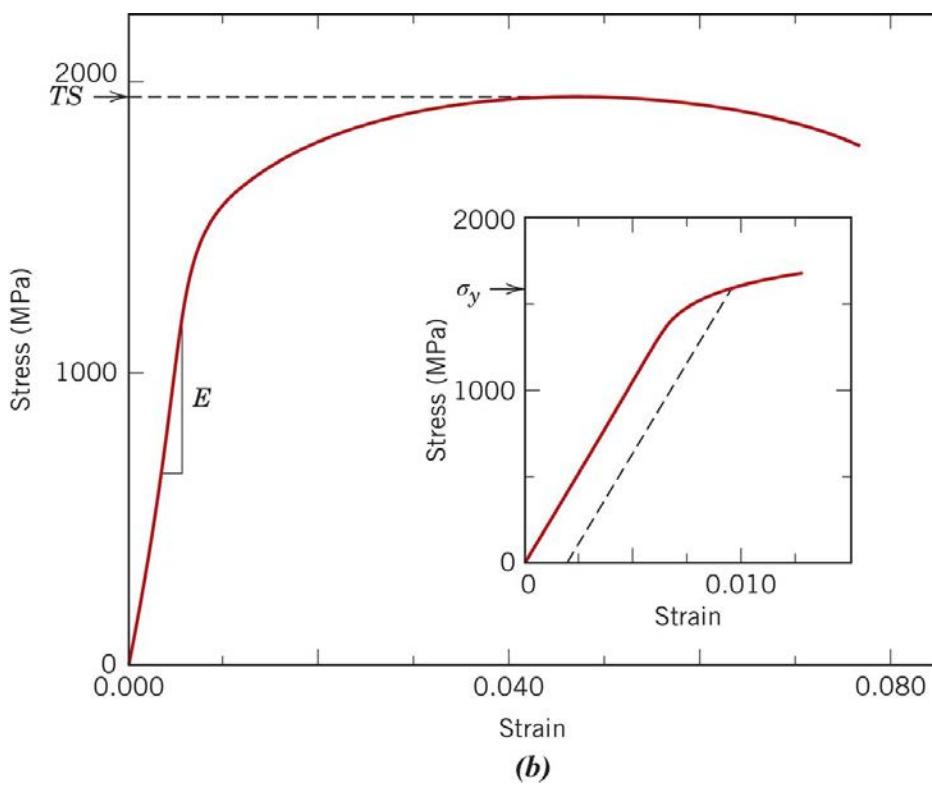


Model H300KU Universal Testing Machine by Tinius Olsen



Vertical  
suspender cables



Main suspender  
cables

# Chapter 7: Mechanical Properties

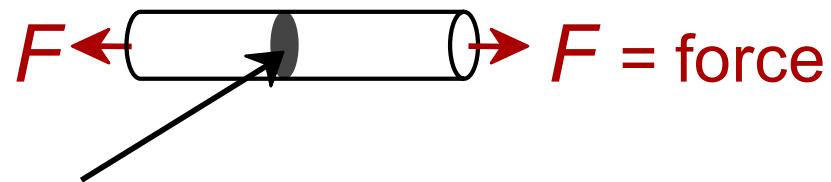
## ISSUES TO ADDRESS...

- When a metal is exposed to **mechanical forces**, what parameters are used to express force magnitude and degree of deformation?  

- What is the distinction between **elastic** and **plastic** deformations?
- How are the following mechanical characteristics of metals measured?
  - (a) **Strength**
  - (b) **Ductility** 
  - (c) **Hardness**
- What parameters are used to quantify these properties?

# Common States of Stress (應力)

- Simple tension:  
cable



$A_0$  = cross-sectional  
area of cable (with no load)

Tensile stress =  $\sigma$

拉伸應力

材料展延  $\Rightarrow$  形變



Ski lift (photo courtesy P.M. Anderson)  
滑雪電纜車

$$\sigma = \frac{F}{A_0}$$

F: 垂直試片橫截面的瞬間作用負荷

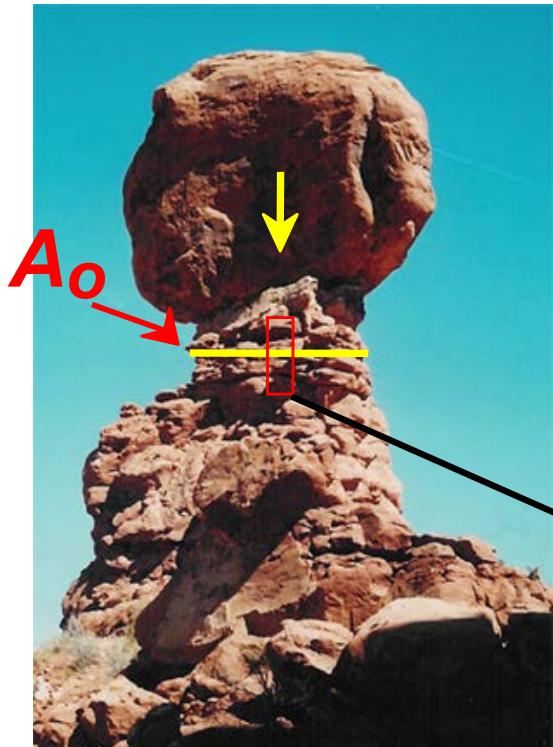
$A_0$ : 負荷作用前, 原來橫截面面積

Units for stress:

MPa =  $10^6$  Pa =  $10^6$  N/m<sup>2</sup> or lb<sub>f</sub>/in<sup>2</sup>

# OTHER COMMON STRESS STATES (i)

- Simple compression: 壓縮應力



Balanced Rock, Arches  
National Park  
(photo courtesy P.M. Anderson)



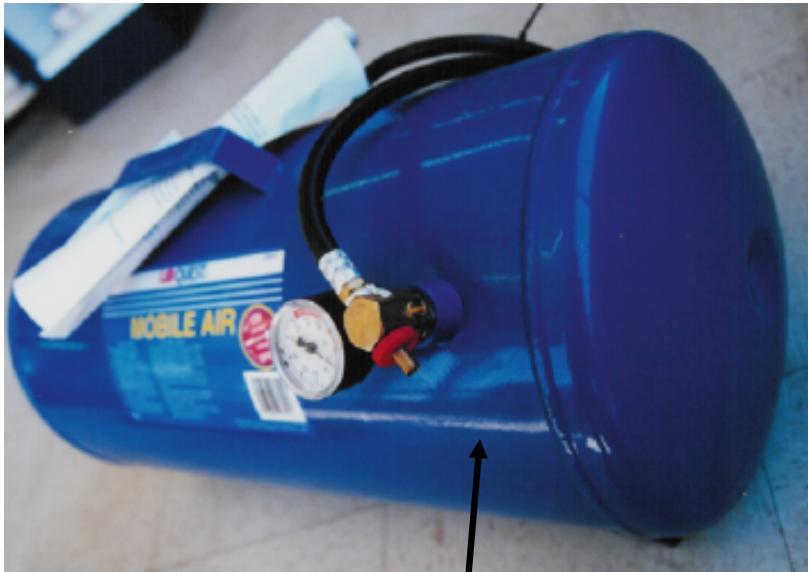
Canyon Bridge, Los Alamos, NM  
(photo courtesy P.M. Anderson)

Note: structure members  
are under compression  
( $F < 0$  and  $\sigma < 0$ ).

$$\sigma = \frac{F}{A_o}$$

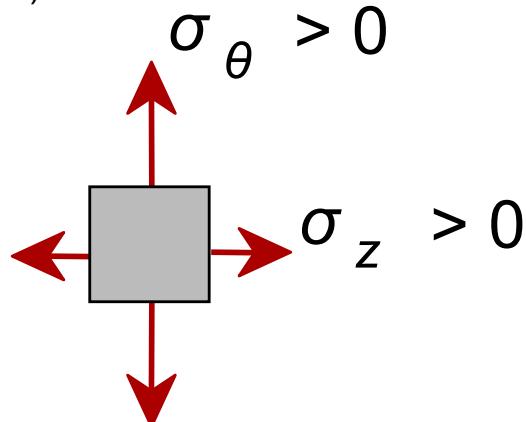
# OTHER COMMON STRESS STATES (ii)

- Bi-axial tension:



Pressurized tank

(photo courtesy  
P.M. Anderson)



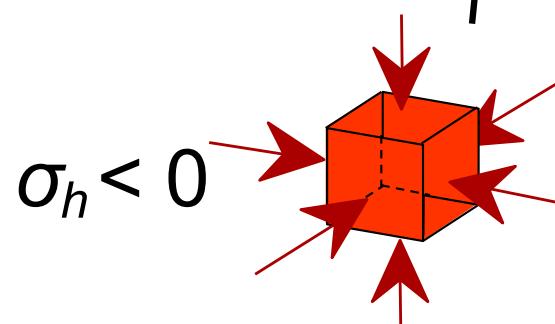
- Hydrostatic compression: 水合壓力



Fish under water

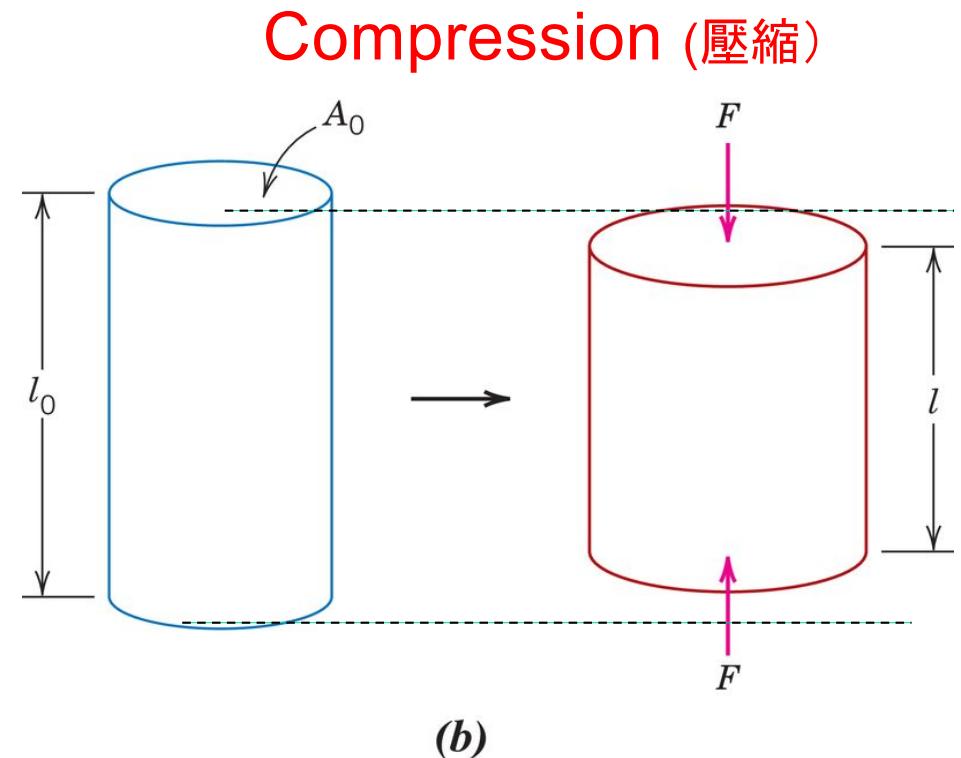
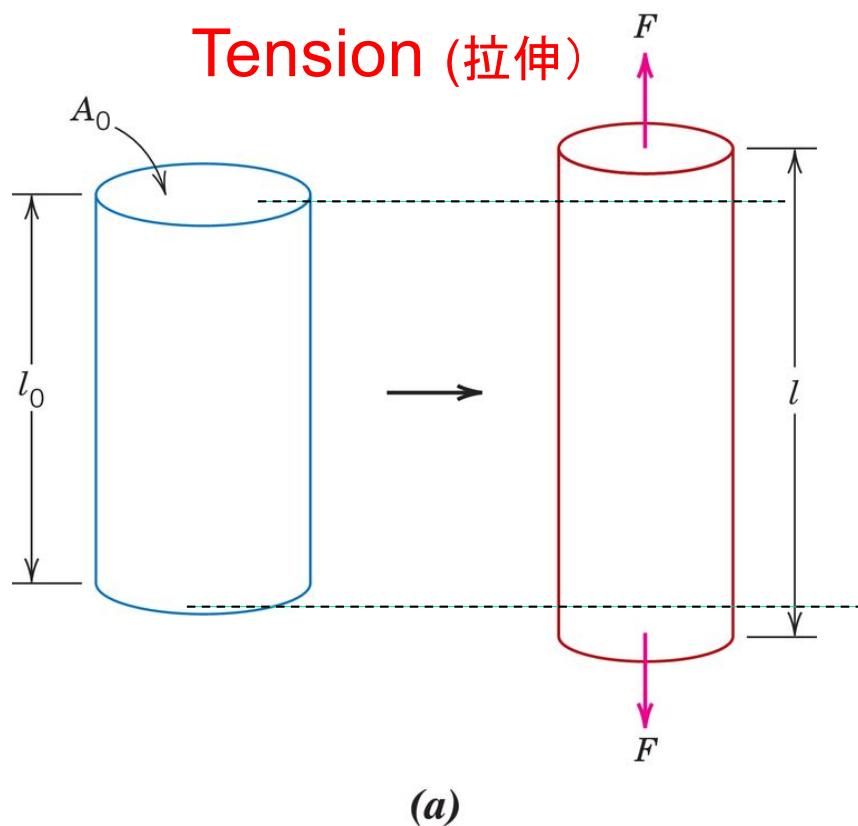
非單方向

(photo courtesy  
P.M. Anderson)



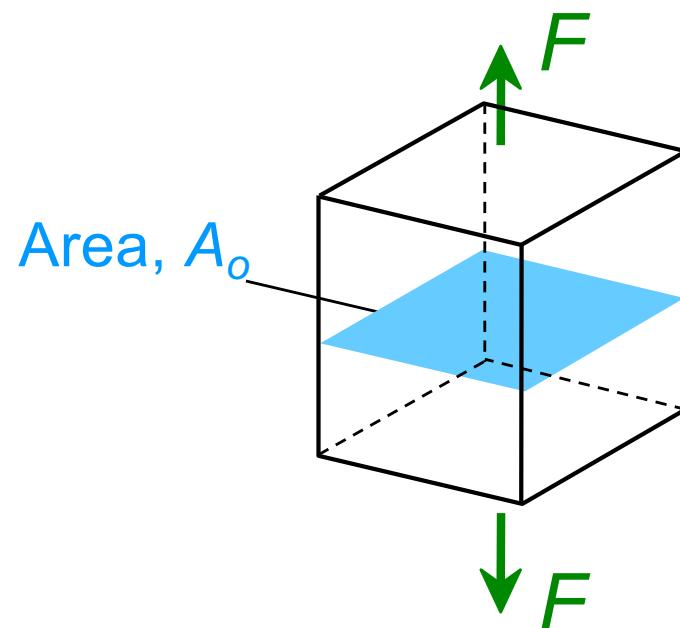
Chapter 7

# Common States of Stress (應力)



# Engineering Stress (應力)

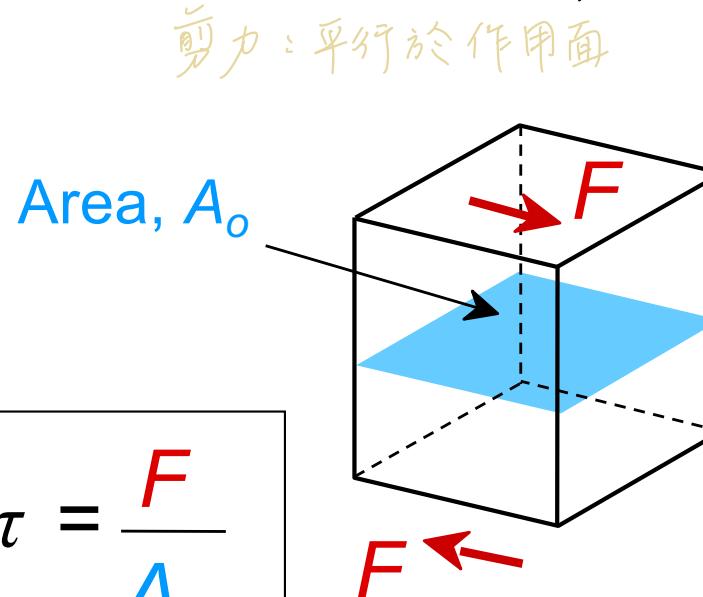
- Tensile stress,  $\sigma$ :



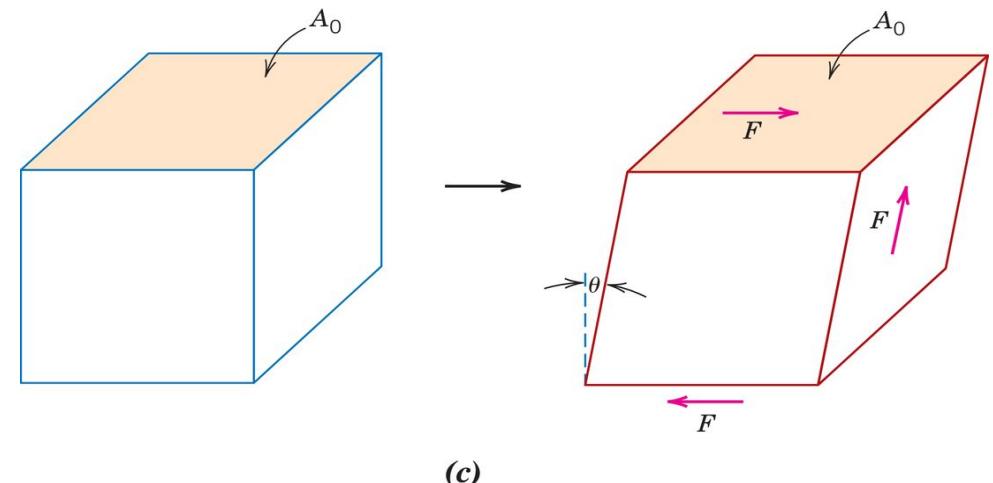
$$\sigma = \frac{F}{A_o}$$

original cross-sectional area before loading

- Shear stress,  $\tau$ :



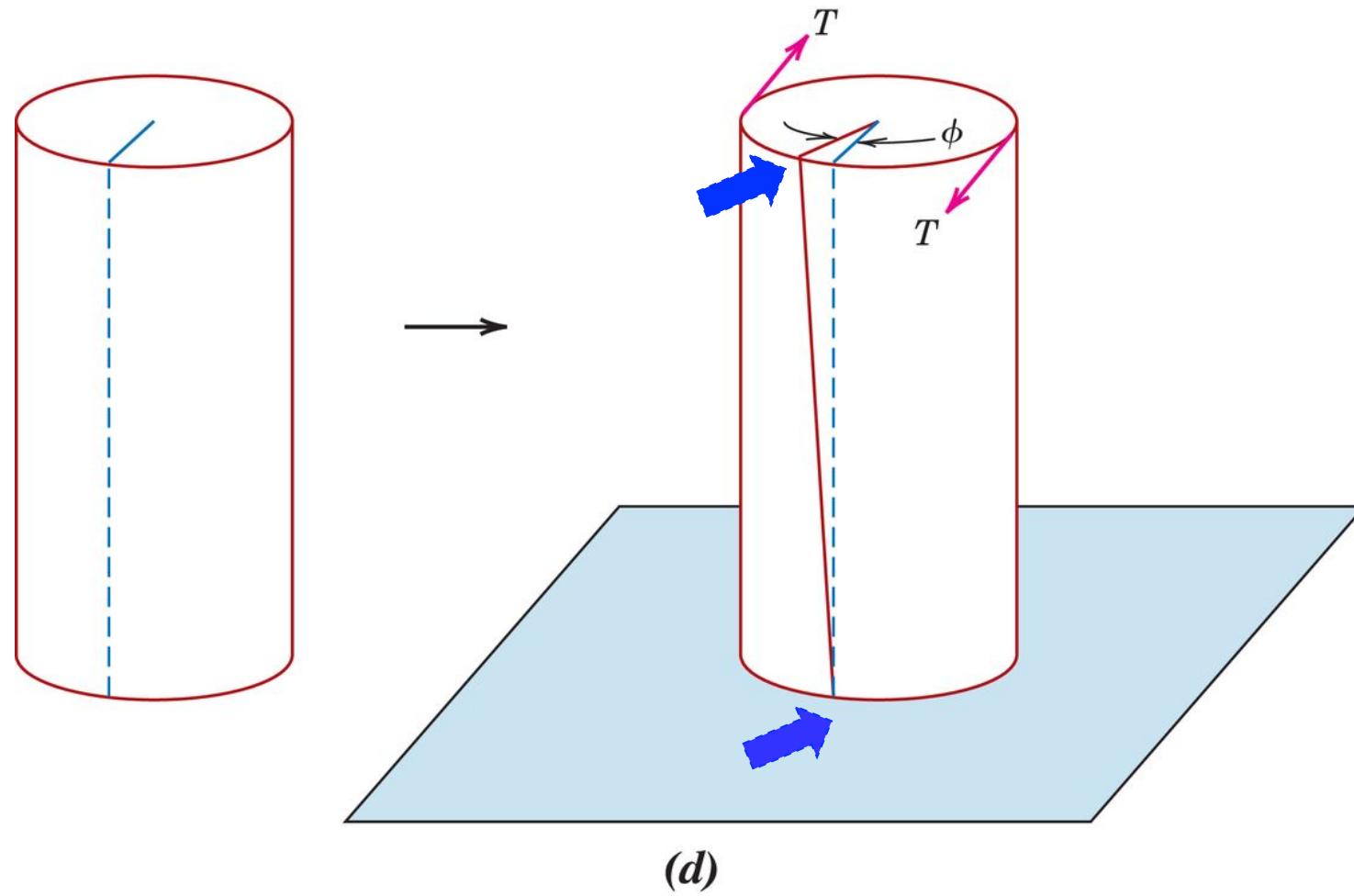
$$\tau = \frac{F}{A_o}$$



(c)

# Common States of Stress (應力)

剪切和扭轉試驗 (Shear and Torsional Tests)



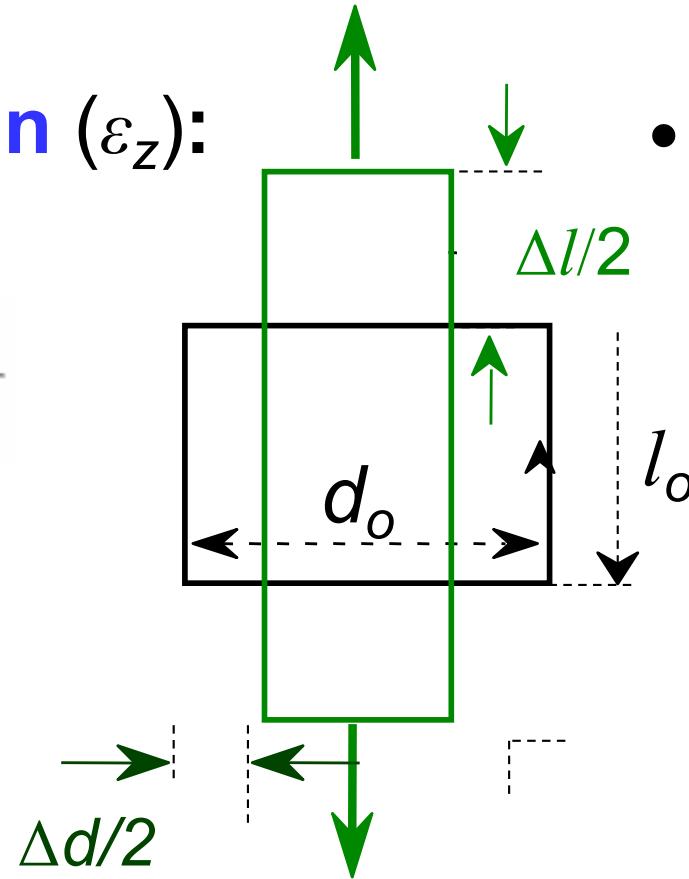
# Engineering Strain (應變)

- Tensile strain ( $\varepsilon_z$ ):

$$\varepsilon_z = \frac{\Delta l}{l_0} = \frac{l_i - l_0}{l_0}$$

- Lateral strain ( $\varepsilon_x$ ):

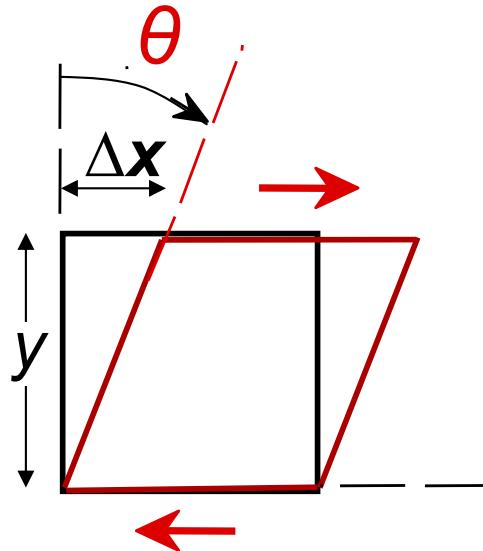
$$\varepsilon_x = -\frac{\Delta d}{d_0}$$



# Engineering Strain (應變)

- Shear strain ( $\gamma$ ):

$$\tau = \frac{F}{A_0}$$

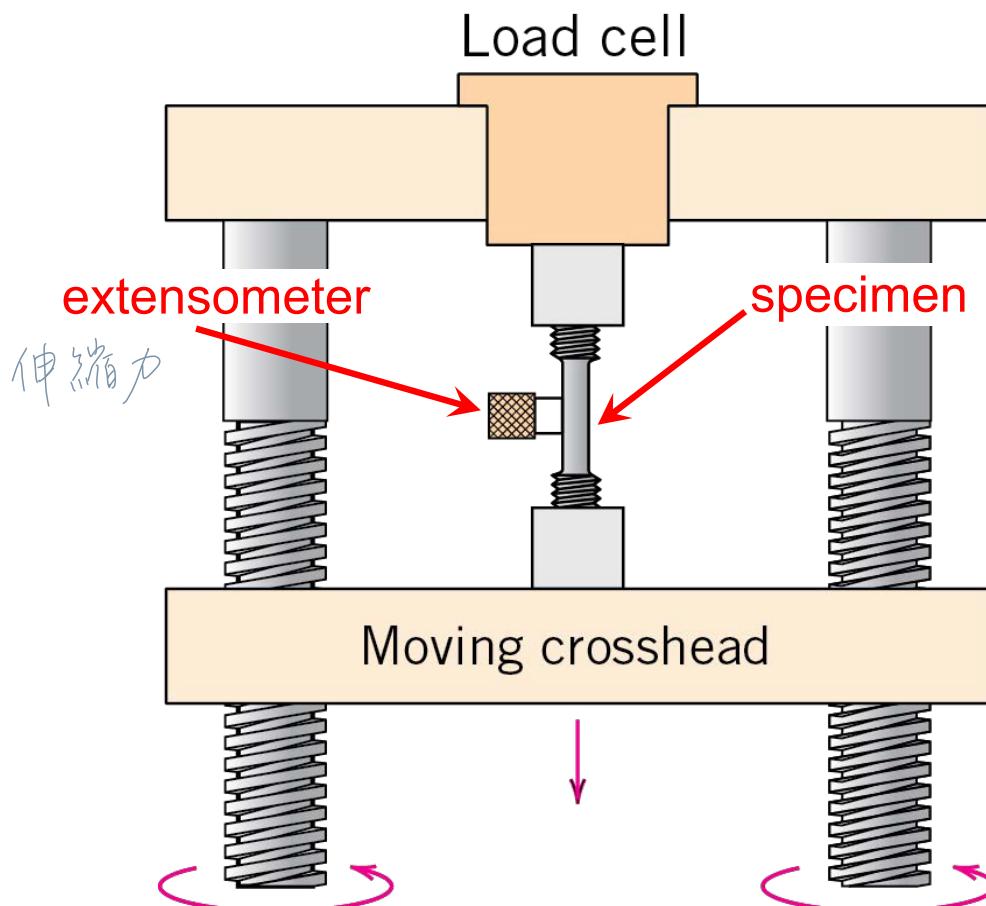


$$\gamma = \Delta x/y = \tan \theta$$

Both tensile and shear strain are dimensionless

# Stress-Strain Testing

- Typical tensile test machine



- Typical tensile specimen

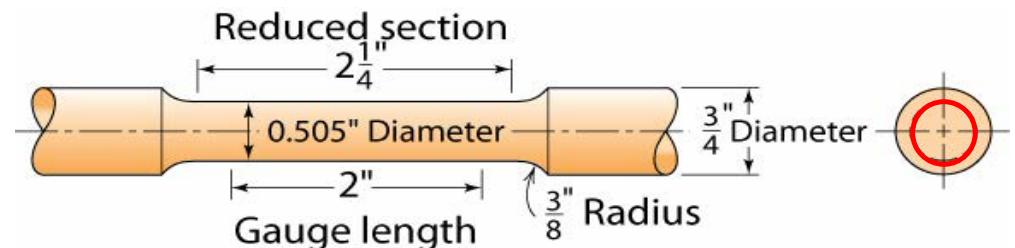
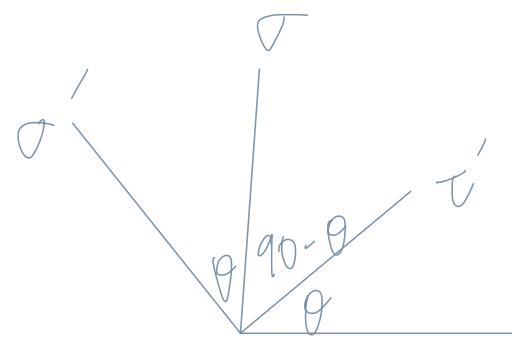
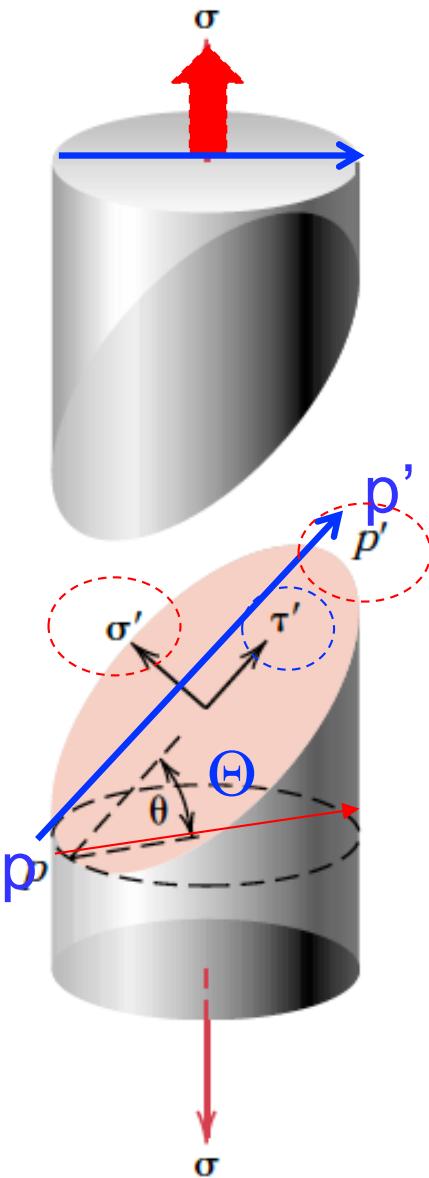


Fig. 7.3, Callister & Rethwisch 5e.

(Taken from H.W. Hayden, W.G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*, p. 2, John Wiley and Sons, New York, 1965.)

# Geometric Considerations of The Stress State

壓力狀態的幾何考量



$$\sigma' = \sigma \cos^2 \theta$$

$$\sigma' = \sigma \cos^2 \theta = \sigma \left( \frac{1 + \cos 2\theta}{2} \right) \quad (7.4a)$$

$$\tau' = \sigma \sin \theta \cos \theta = \sigma \left( \frac{\sin 2\theta}{2} \right) \quad (7.4b)$$

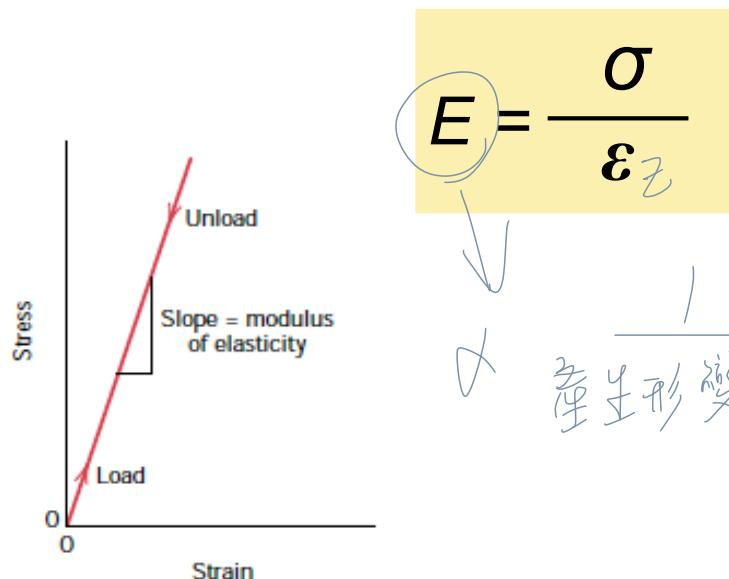
$$\sigma = \frac{F}{A} \quad \tau = \frac{F \times \cos \theta}{A \times \frac{1}{\cos \theta}}$$

# Linear Elastic Properties (線性彈性變形)

- **Elastic deformation** is **nonpermanent** and **reversible!**
  - generally valid at small deformations
  - linear stress strain curve

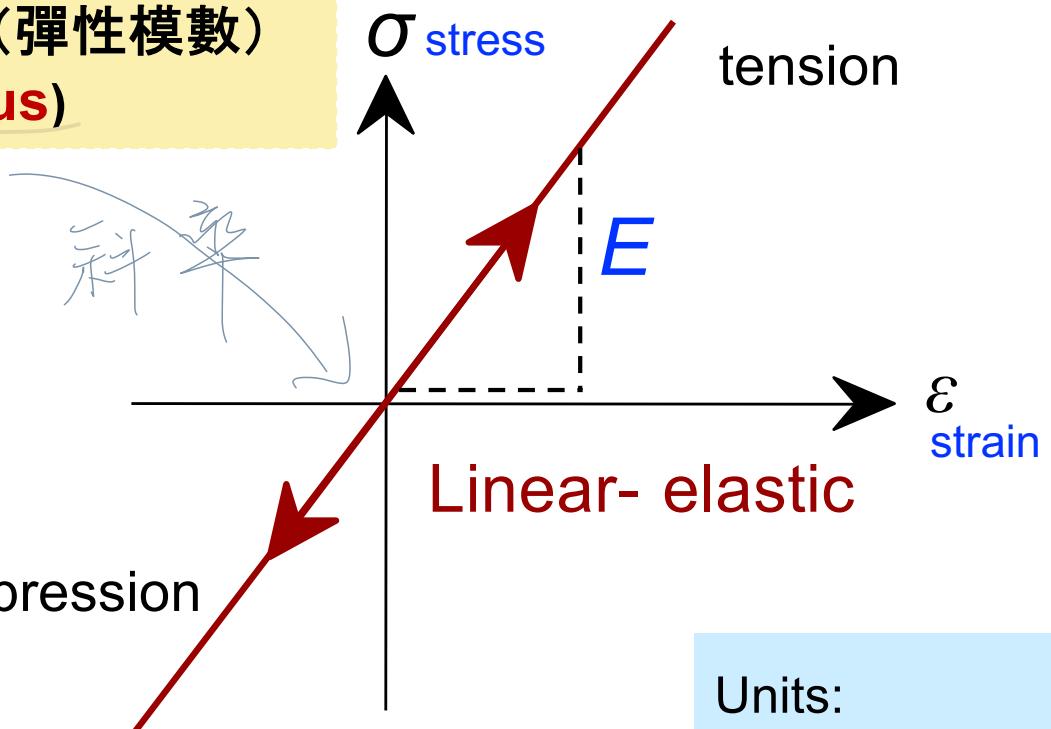
- **Modulus of Elasticity,  $E$ :** (彈性模數)  
(also known as **Young's modulus**)

- **Hooke's Law:**



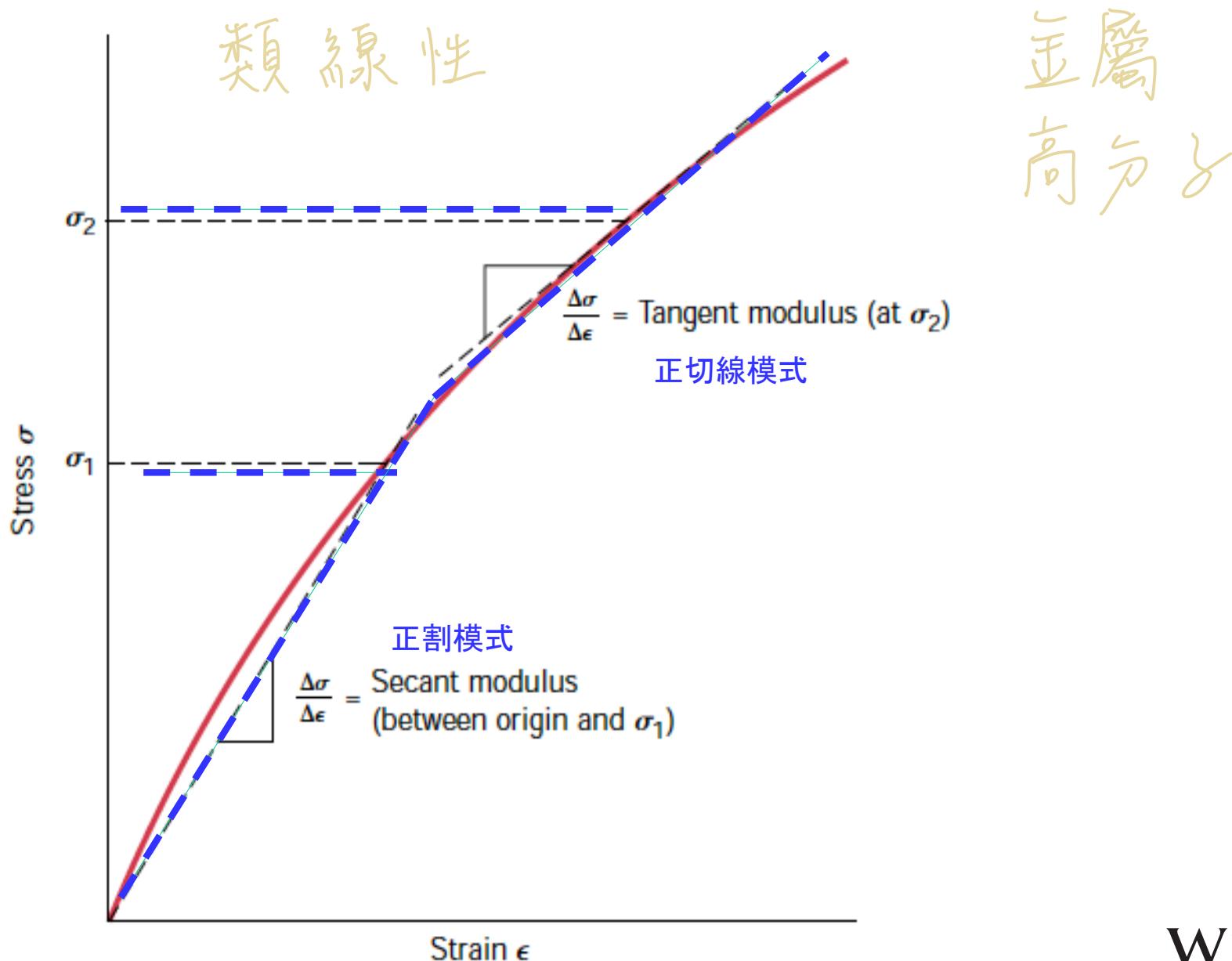
材料

產生形變的力

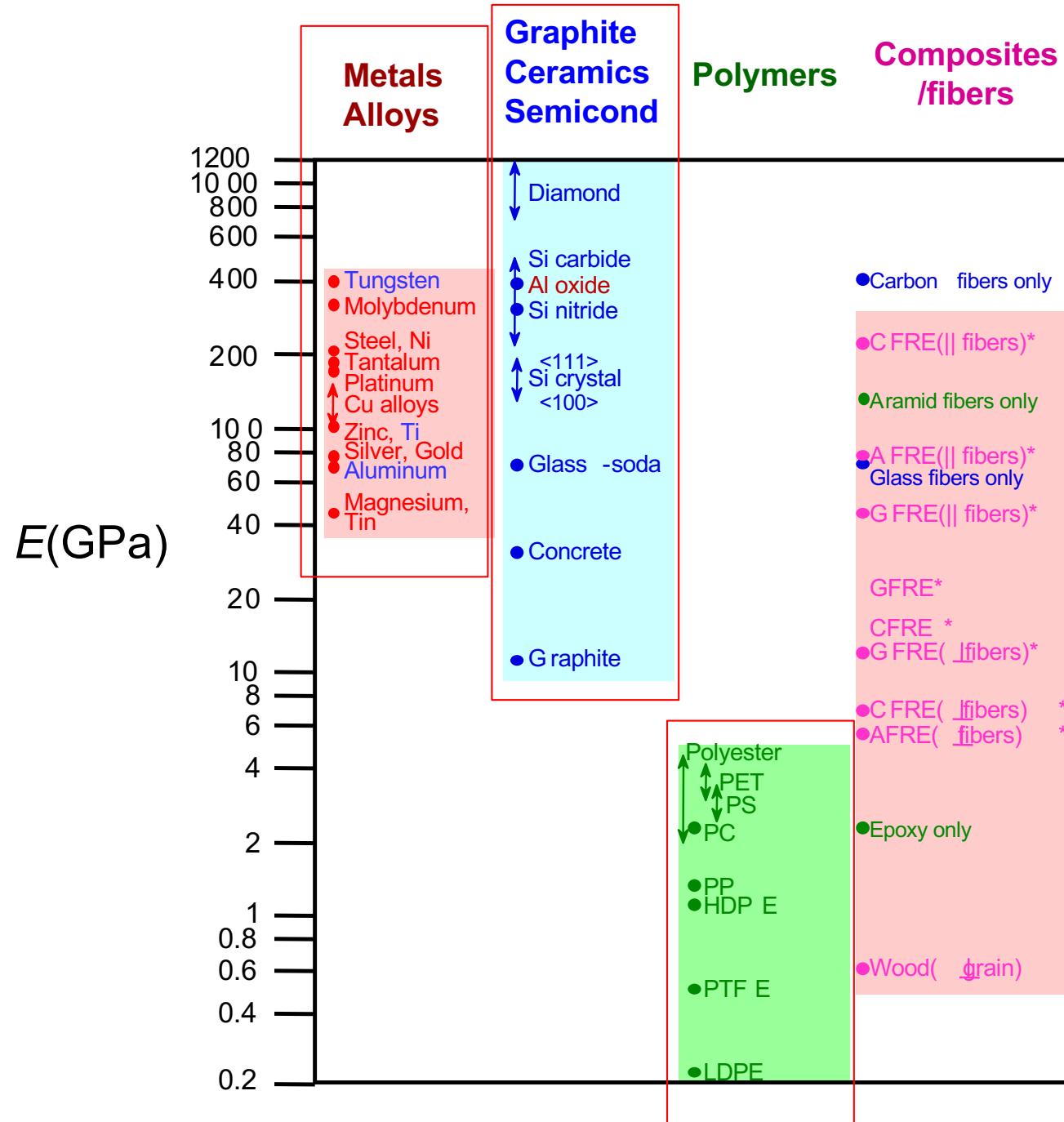


Units:  
 $E$ : [GPa] or [psi]  
1 GPa =  $10^9$  Pa

# Some material has Nonlinear elastic behavior



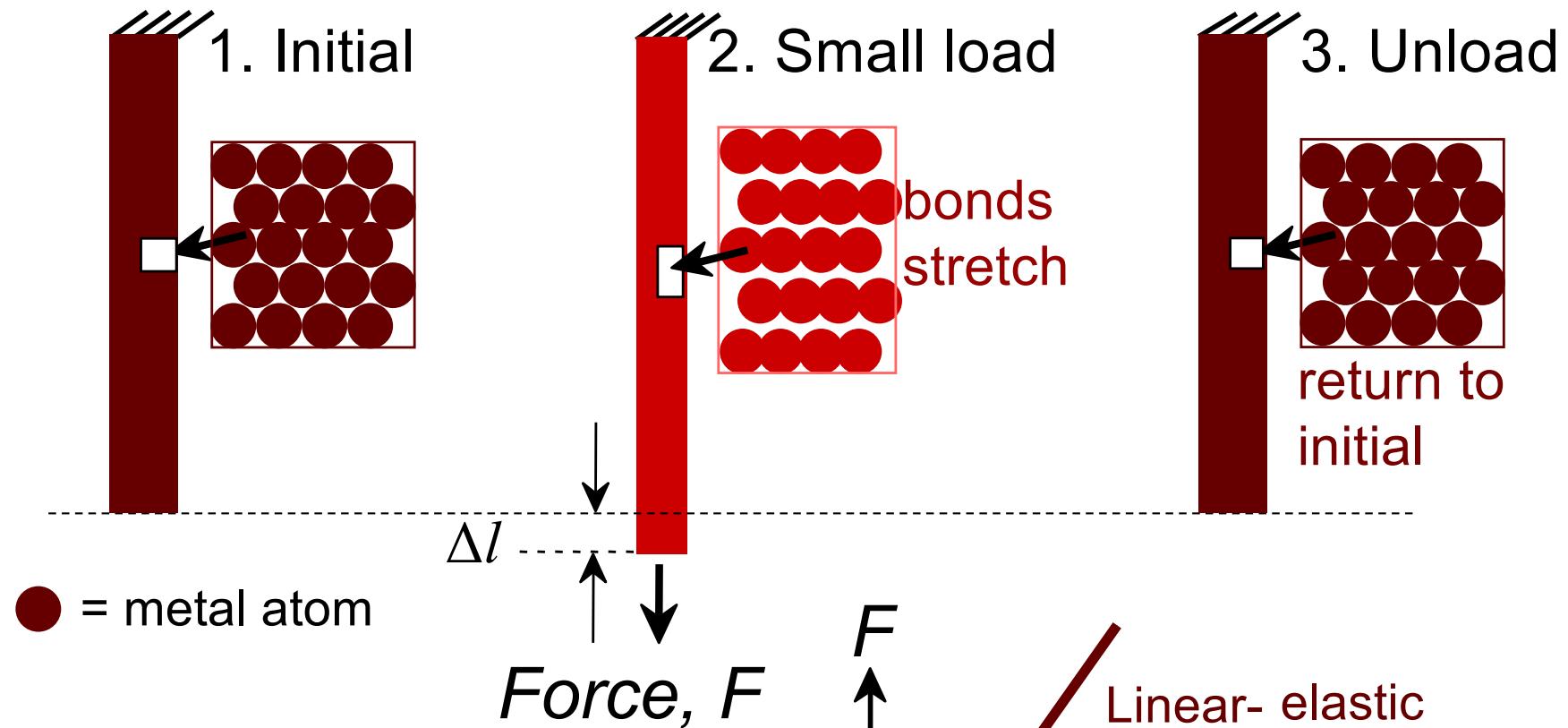
# Elastic Modulus – Comparison of Material Types



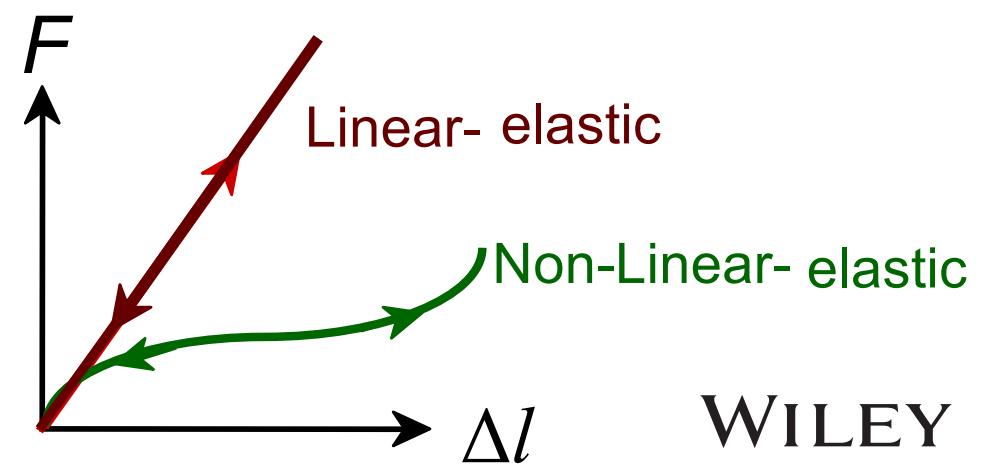
Composite data based on reinforced epoxy with 60 vol% of aligned carbon (CFRE), aramid (AFRE), or glass (GFRE) fibers.

# Elastic Deformation (彈性變形)

Atomic configurations—before, during, after load (force) application



Elastic deformation is  
nonpermanent and reversible!



# Influence of Bonding Forces

- Elastic modulus depends on interatomic bonding forces
- Modulus proportional to slope of inter-atomic force (F)-inter-atomic separation curve (r)

$$E \propto \left( \frac{dF}{dr} \right)_{r_0}$$

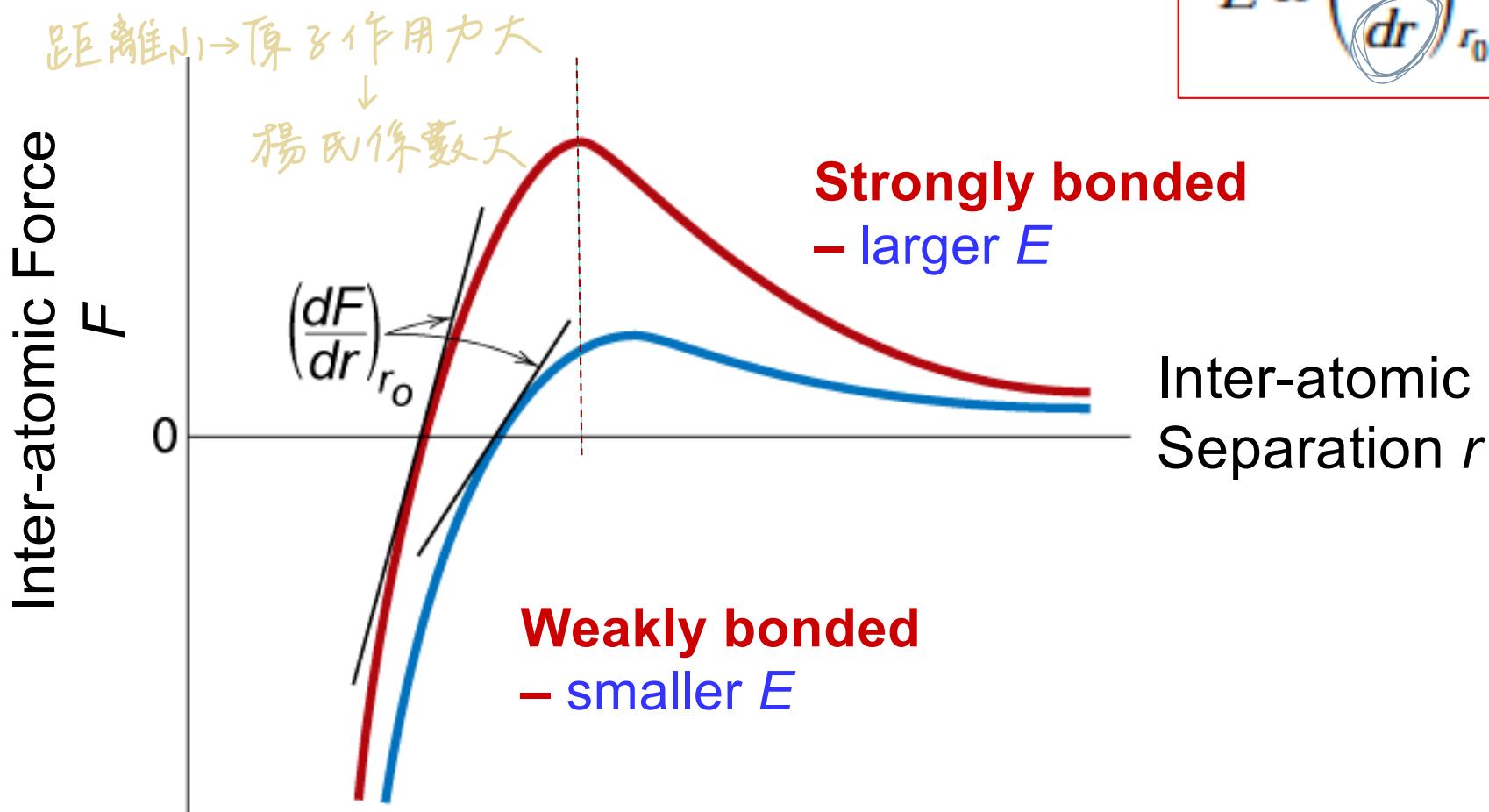
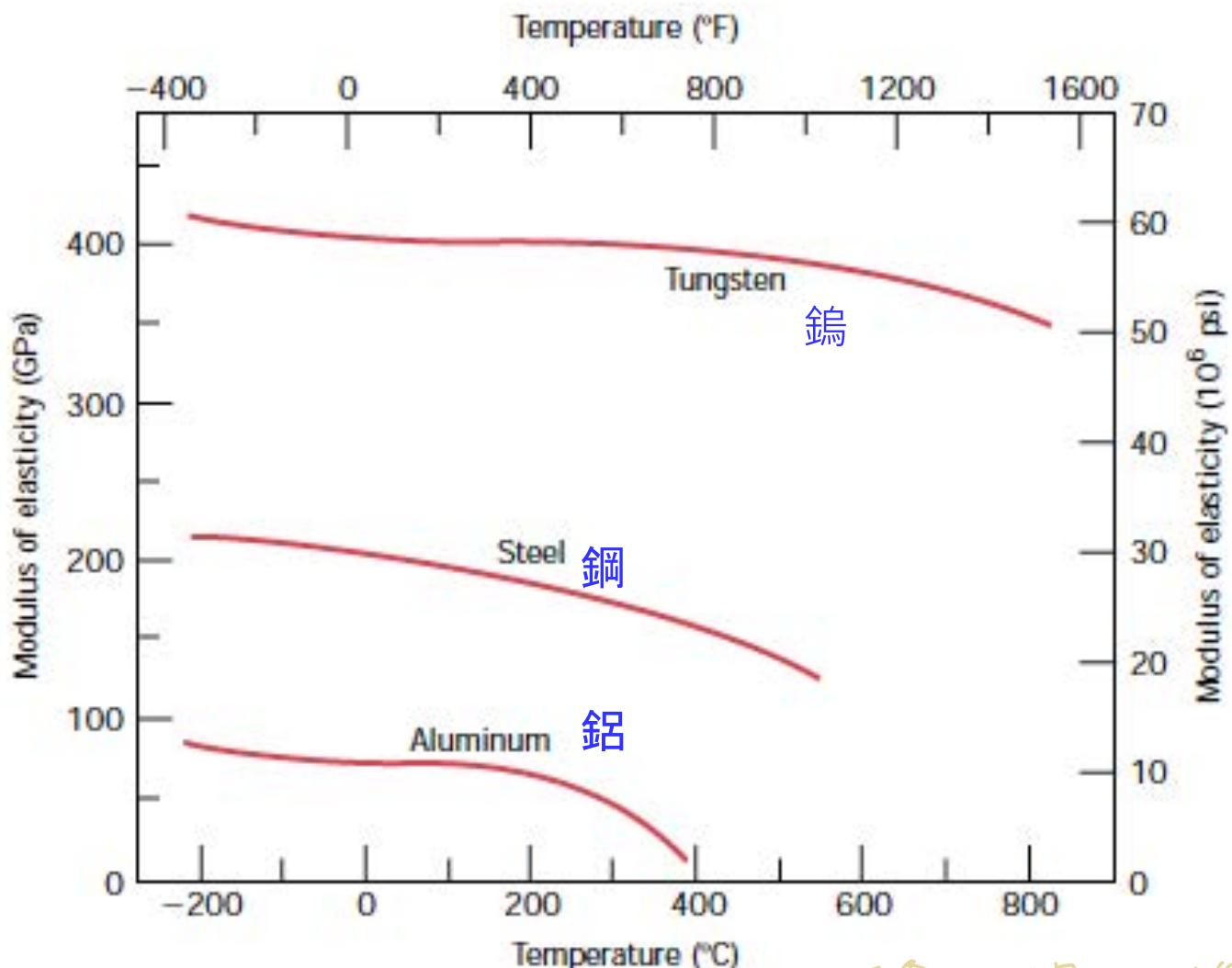


Fig. 7.7, Callister & Rethwisch 5e.

Temperature increased, modulus decreased



↑↑ ↗ 楊氏係數 ↘

# Poisson's ratio (浦松比)

- Poisson's ratio,  $\nu$ :

$$\nu = -\frac{\varepsilon_x}{\varepsilon_z}$$

x axial 和 Z axial 的 應變比值

Units  $\nu$ :  
dimensionless

無量纲

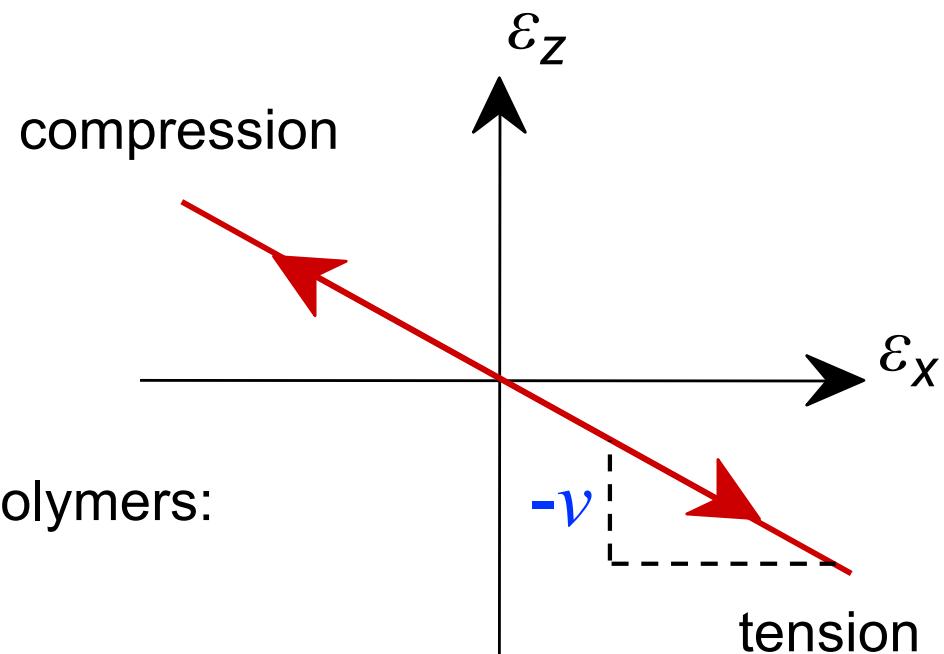
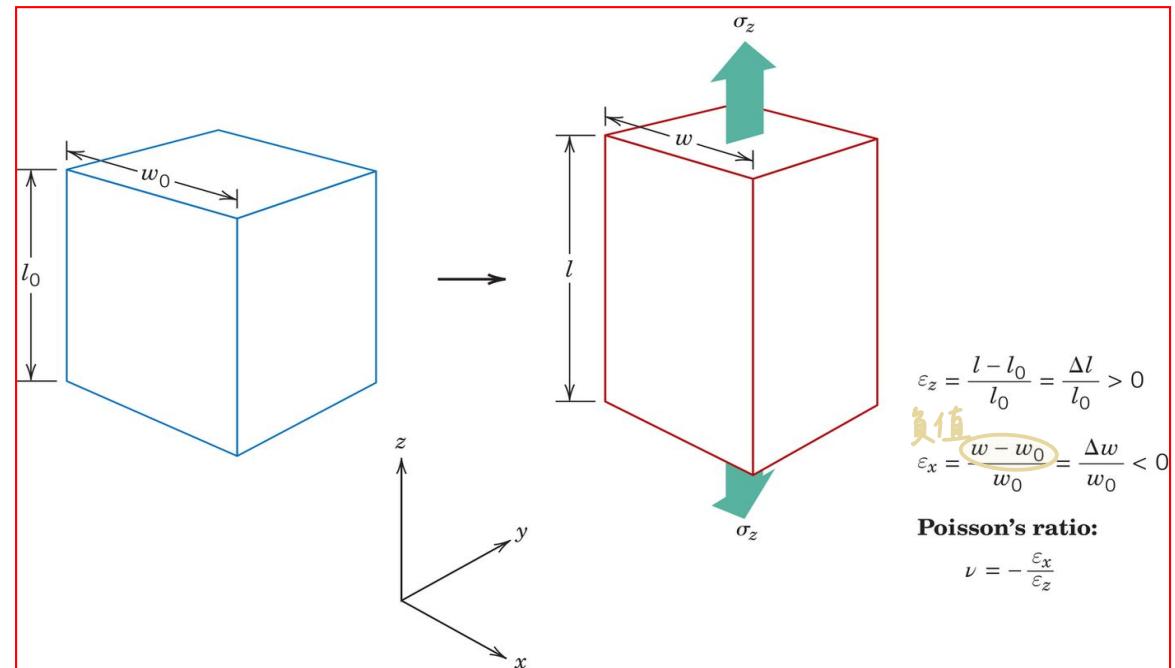
ceramics:  $\nu \sim 0.25$

metals:  $\nu \sim 0.33$

polymers:  $\nu \sim 0.40$

For most metals, ceramics and polymers:

$$0.15 < \nu \leq 0.50$$



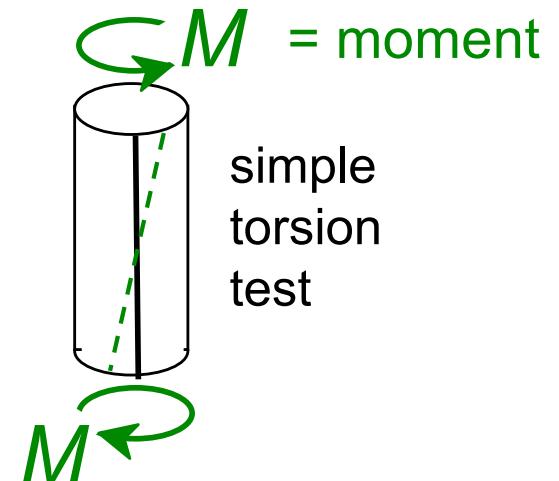
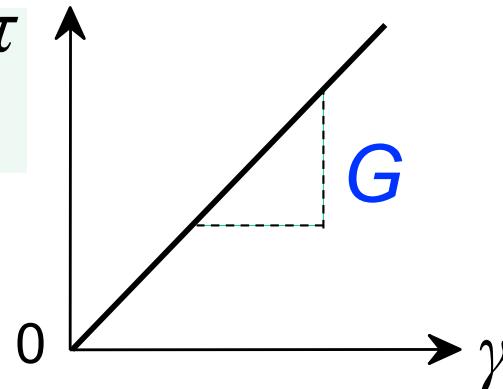
# Other Elastic Properties



- Elastic Shear modulus,  $\tau$

$G$ :

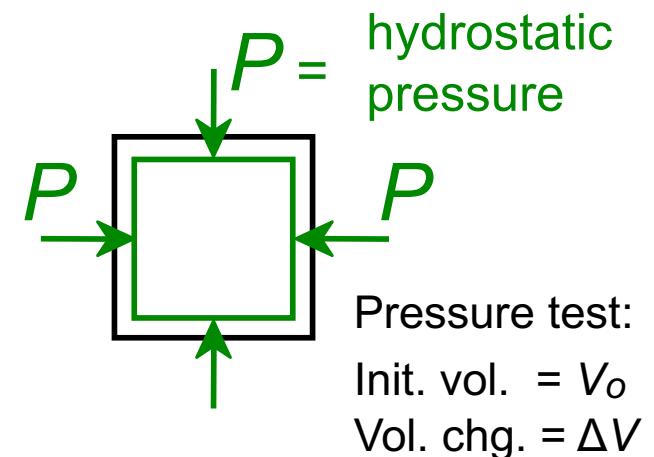
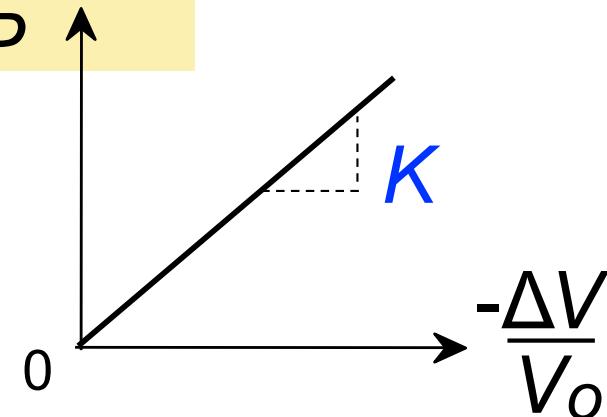
G: shear modulus  
 $\tau = G \gamma$



- Elastic Bulk modulus,  $K$

$K$ :

$$P = -K \frac{\Delta V}{V_0}$$



- Elastic constant relationships for **isotropic** materials:

$$G = \frac{E}{2(1 + \nu)}$$

$$K = \frac{E}{3(1 - 2\nu)}$$

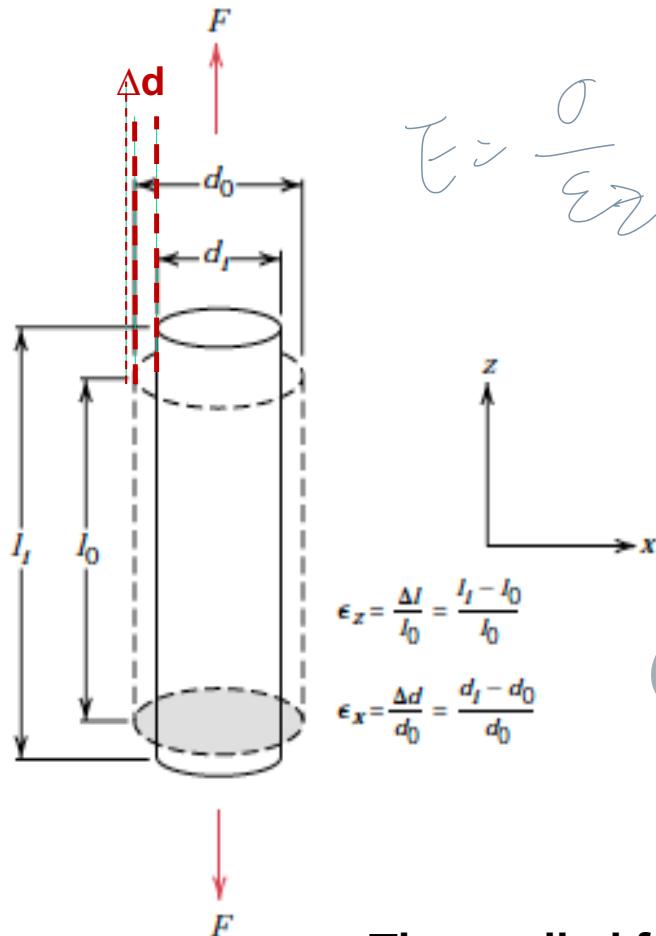
等向材料

A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm (0.4 in.). Determine the magnitude of the load required to produce a  $2.5 \times 10^{-3}$  mm ( $10^{-4}$  in.) change in diameter if the deformation is entirely elastic.

### SOLUTION

This deformation situation is represented in the accompanying drawing.

$\Delta d$ , of  $2.5 \times 10^{-3}$  mm



### For the strain in the x direction

$$\epsilon_x = \frac{\sigma}{E} \quad \text{and} \quad \epsilon_x = \frac{\Delta d}{d_0}$$

$$\epsilon_x = \frac{\Delta d}{d_0} = \frac{-2.5 \times 10^{-3} \text{ mm}}{10 \text{ mm}} = -2.5 \times 10^{-4}$$

$$F = \frac{(-\Delta d) EA_0}{d_0 \times V}$$

### For the strain in the z direction

$$\epsilon_z = -\frac{\epsilon_x}{\nu} = -\frac{(-2.5 \times 10^{-4})}{0.34} = 7.35 \times 10^{-4}$$

From Table 7-1

$$\nu = -\frac{\epsilon_x}{\epsilon_z}$$

$$\sigma = \epsilon_z E = (7.35 \times 10^{-4})(97 \times 10^3 \text{ MPa}) = 71.3 \text{ MPa}$$

$$F = \sigma A_0 = \sigma \left(\frac{d_0}{2}\right)^2 \pi$$

$$\sigma = \frac{F}{A_0}$$

$$\begin{aligned} &= (71.3 \times 10^6 \text{ N/m}^2) \left(\frac{10 \times 10^{-3} \text{ m}}{2}\right)^2 \pi = 5600 \text{ N} (1293 \text{ lb}_f) \\ &\text{原物之截面積} \end{aligned}$$

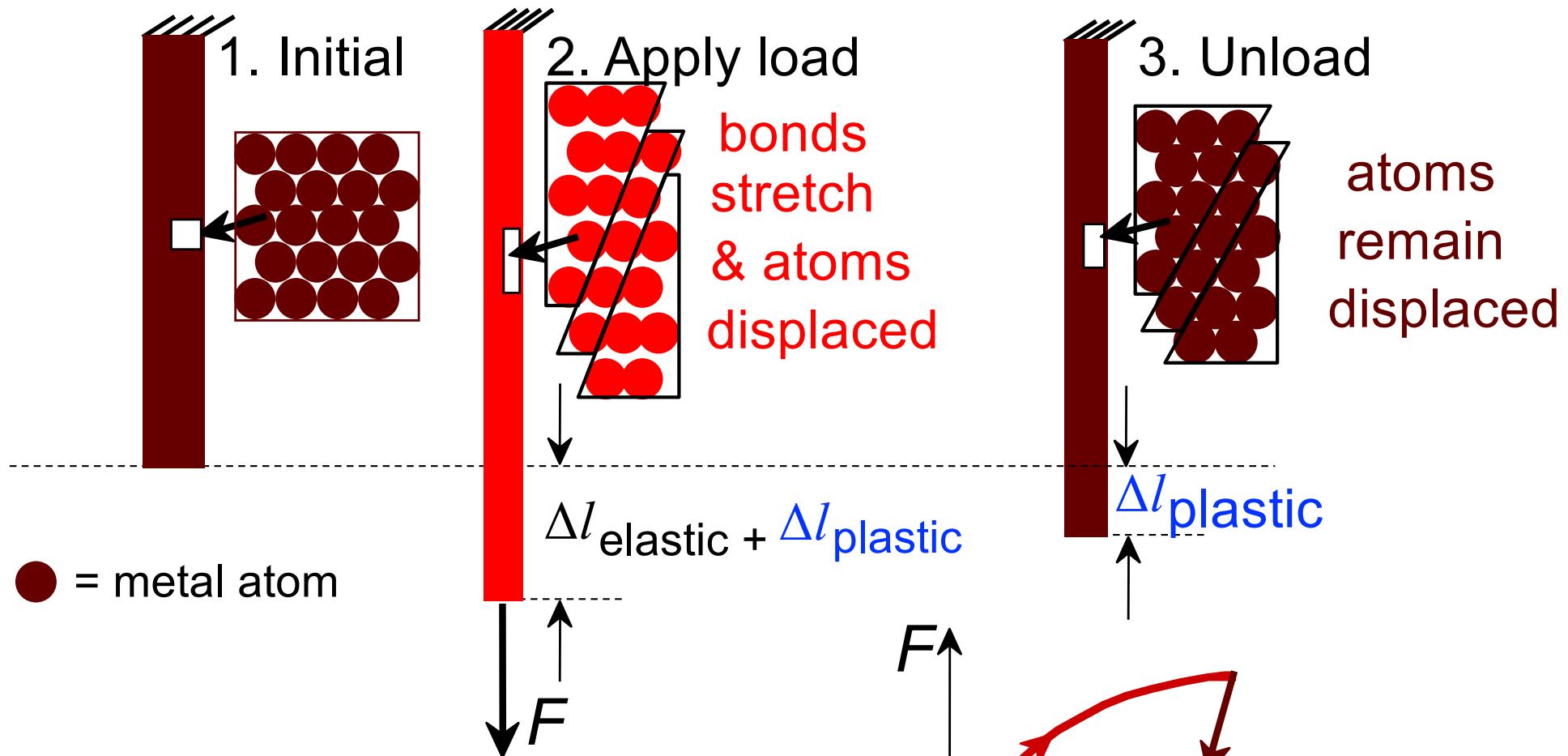
### The applied force



<https://www.youtube.com/watch?v=tuOlM3P7ygA>

# **Mechanical Behavior - Metals**

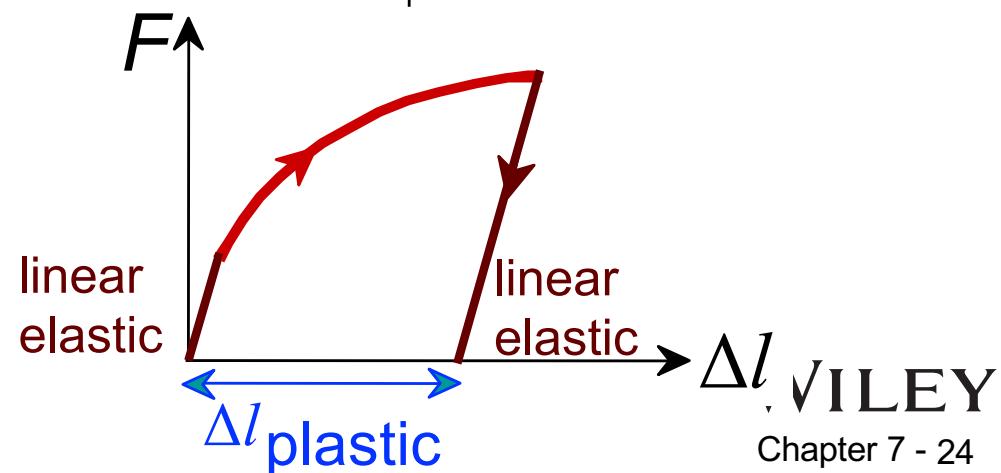
# Plastic Deformation (塑性變形) (Metals)



Plastic deformation is  
permanent and non-recoverable.

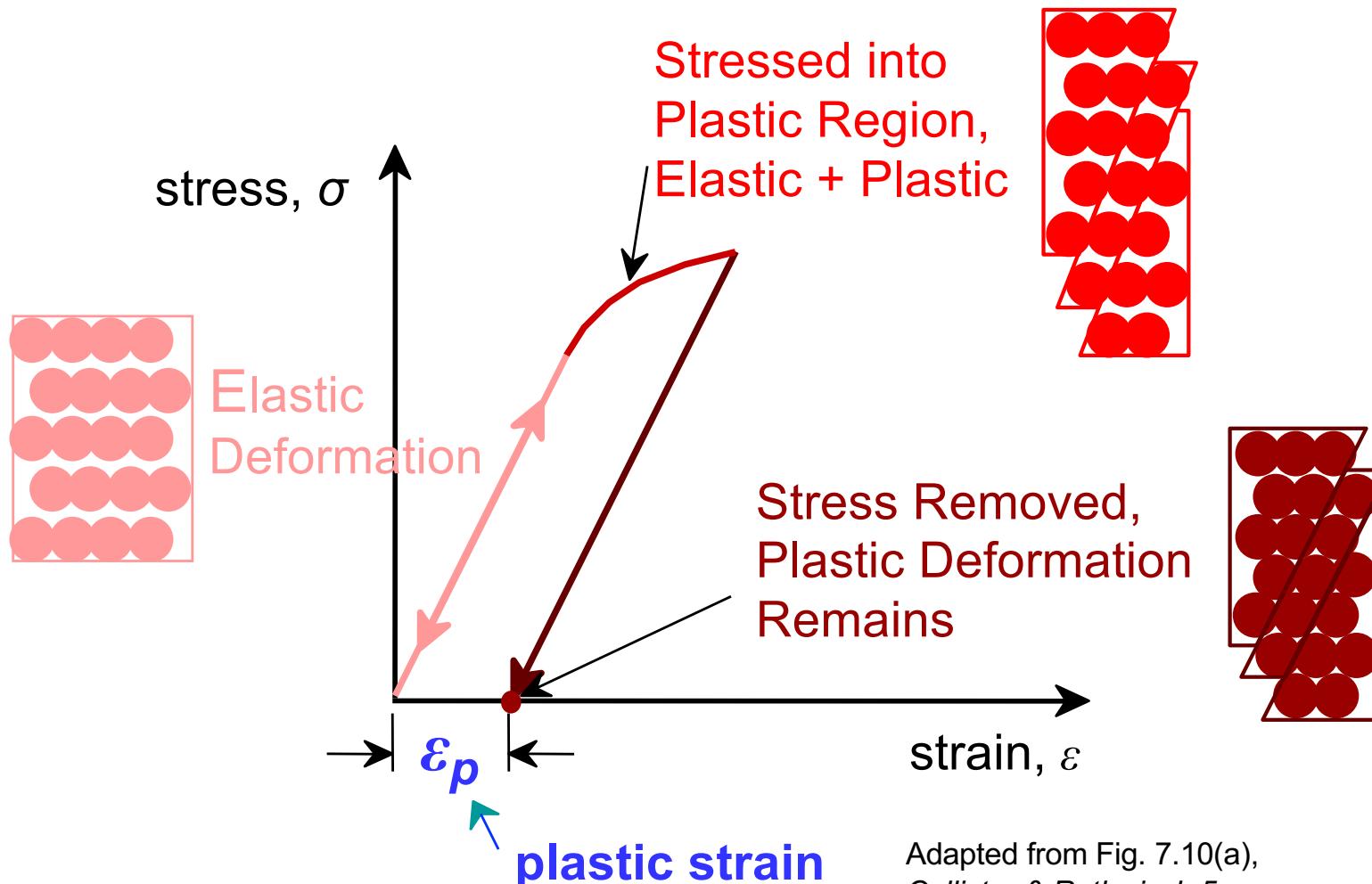
永久

不可逆



# Plastic Deformation (塑性變形)

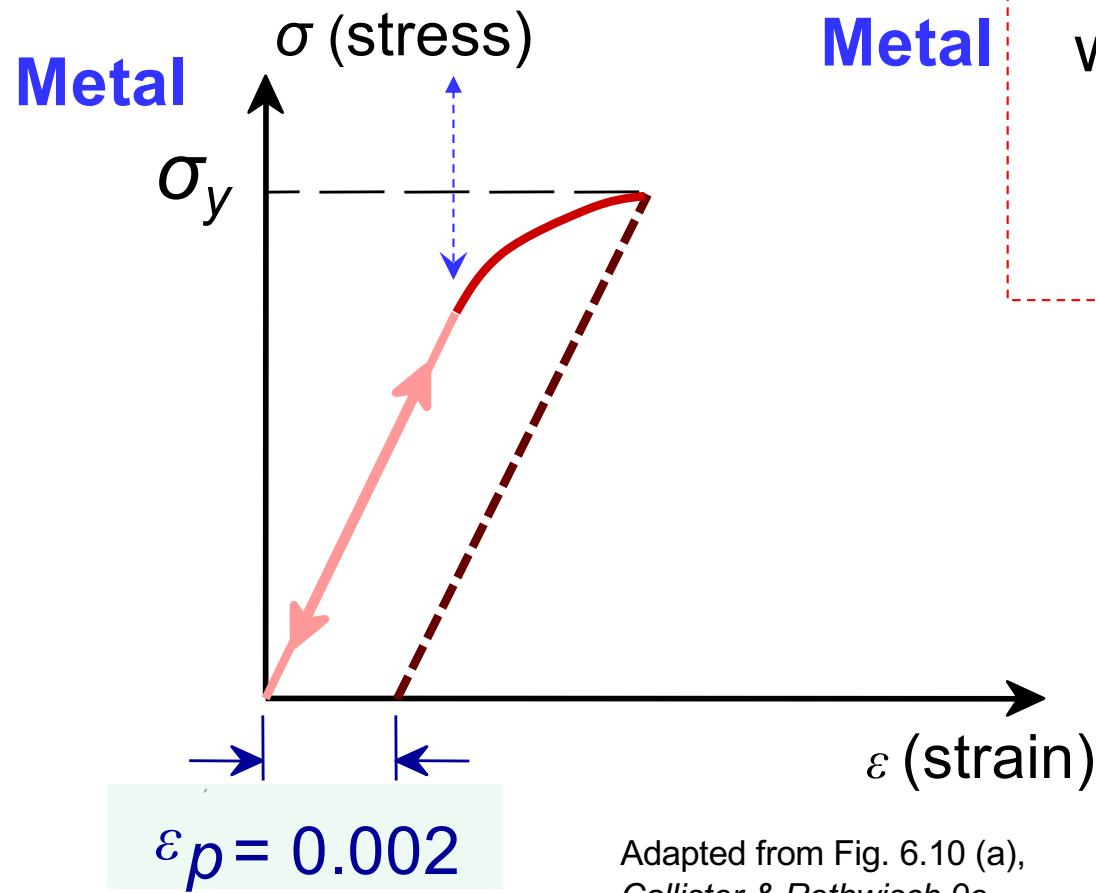
- Plastic Deformation is permanent and nonrecoverable
- Stress-strain plot for simple tension test:



Adapted from Fig. 7.10(a),  
Callister & Rethwisch 5e.

# Yield Strength (降伏強度)

- Transition from elastic to plastic deformation is gradual  
*逐漸的*
- Yield strength = stress at which *noticeable* plastic deformation has occurred  
*明顯的塑性變化*



**Metal**

when  $\varepsilon_p = 0.002$  (0.2%)

$\sigma_y$  = yield strength

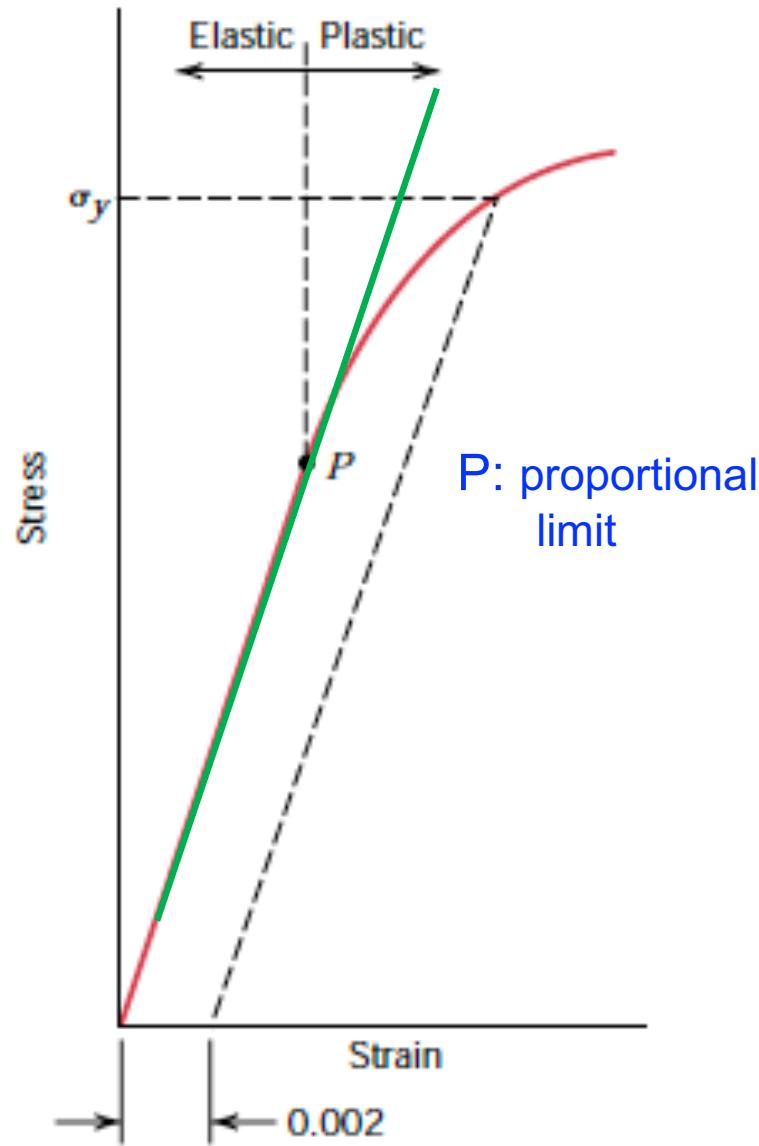
Note: for 5 cm sample

$$\varepsilon = 0.002 = \Delta z / z$$

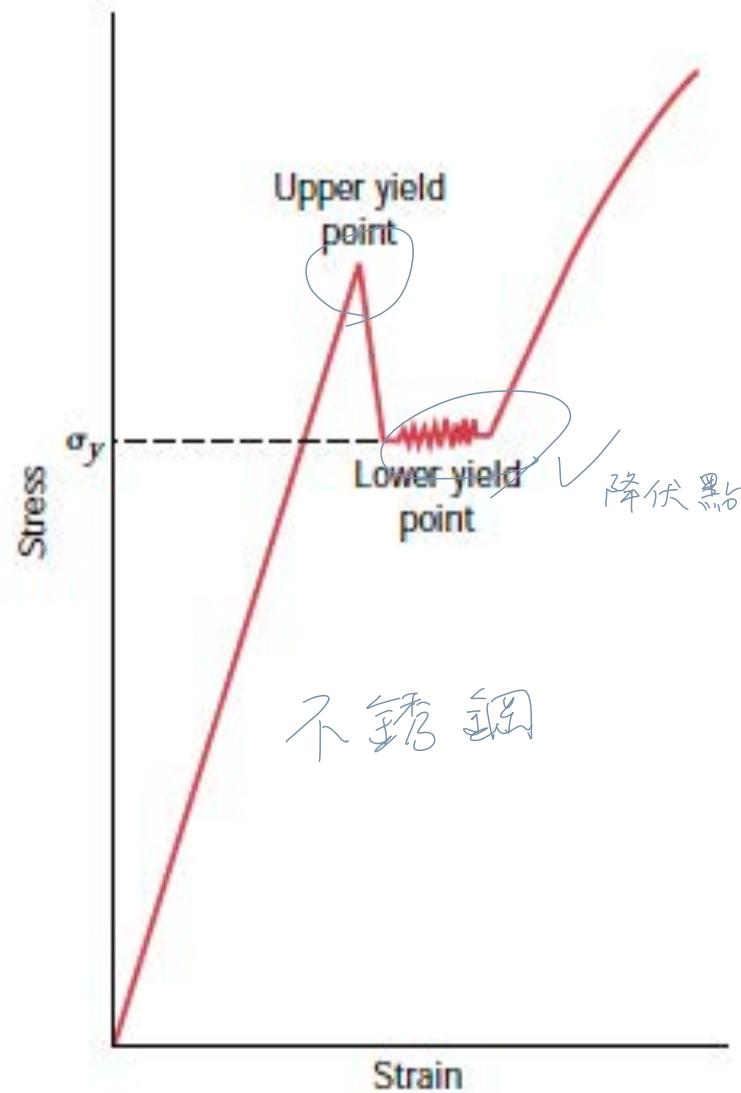
$$\Delta z = 0.01 \text{ cm}$$

Adapted from Fig. 6.10 (a),  
Callister & Rethwisch 9e.

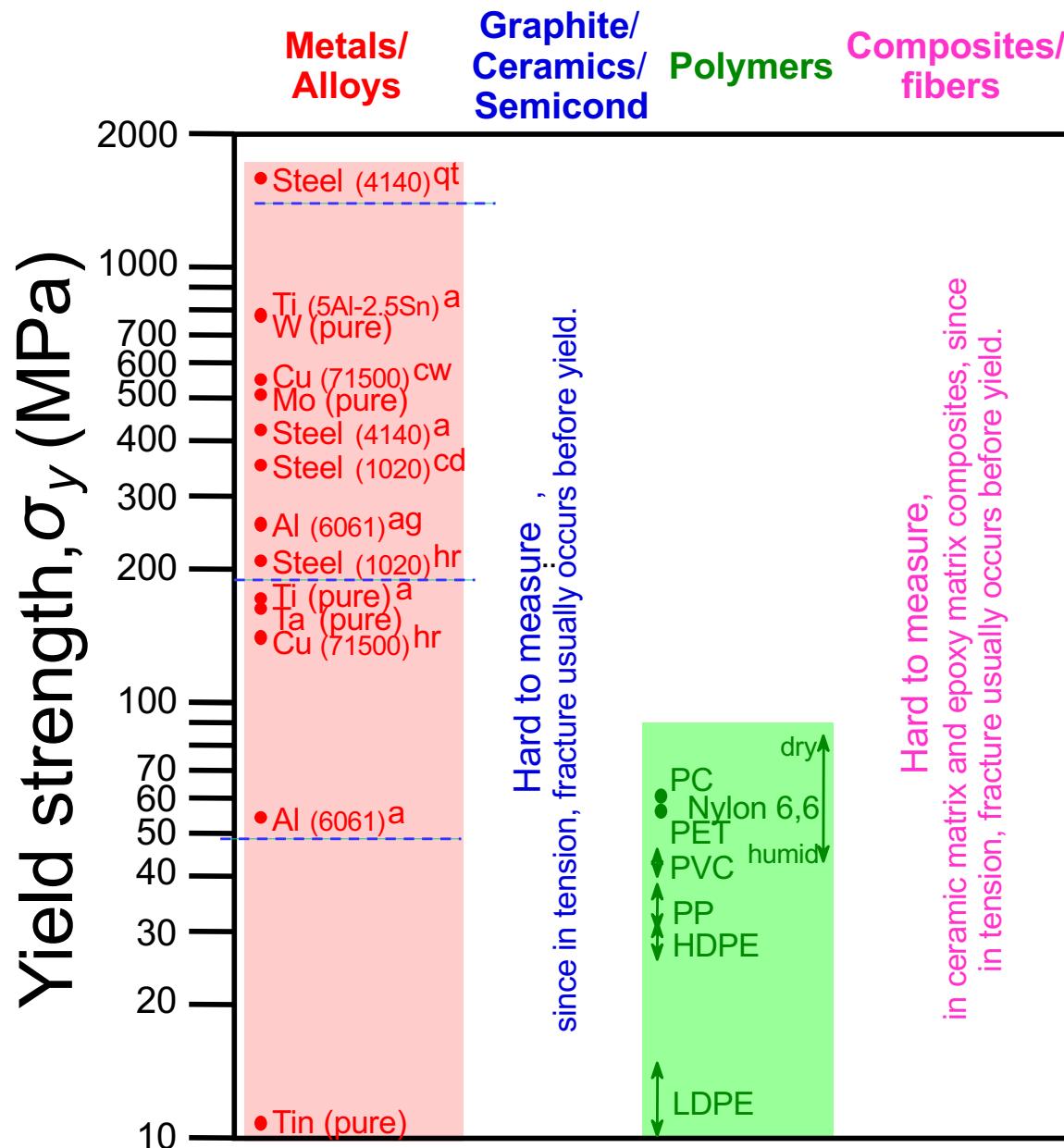
## Typical stress-strain behavior for a metal



The stress–strain behavior found for **some steels** demonstrating the yield point phenomenon.



# Yield Strength – Comparison of Material Types



Room temperature values

Based on data in Table B.4,  
*Callister & Rethwisch 5e.*

a = annealed

hr = hot rolled

ag = aged

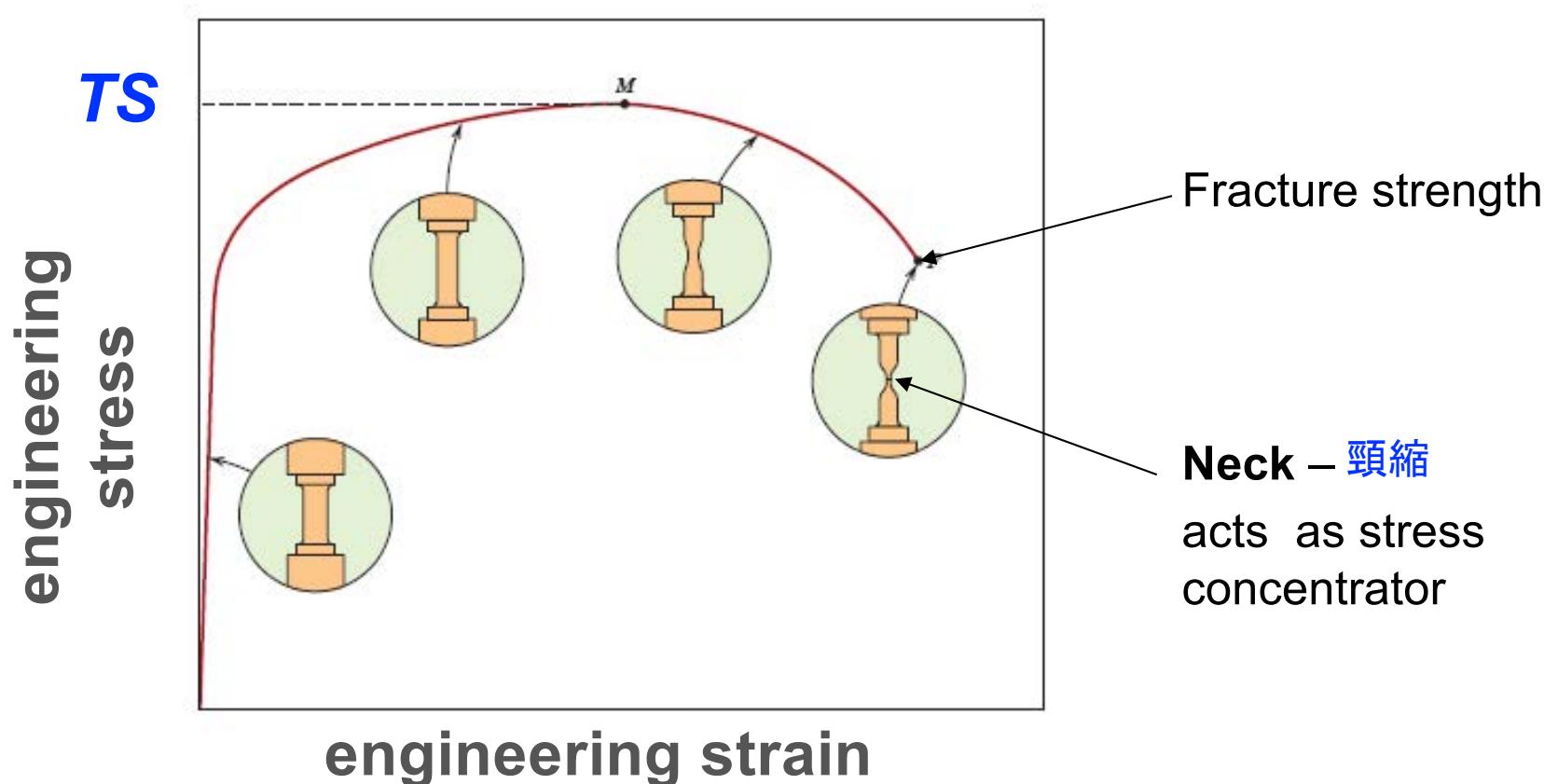
cd = cold drawn

cw = cold worked

qt = quenched & tempered

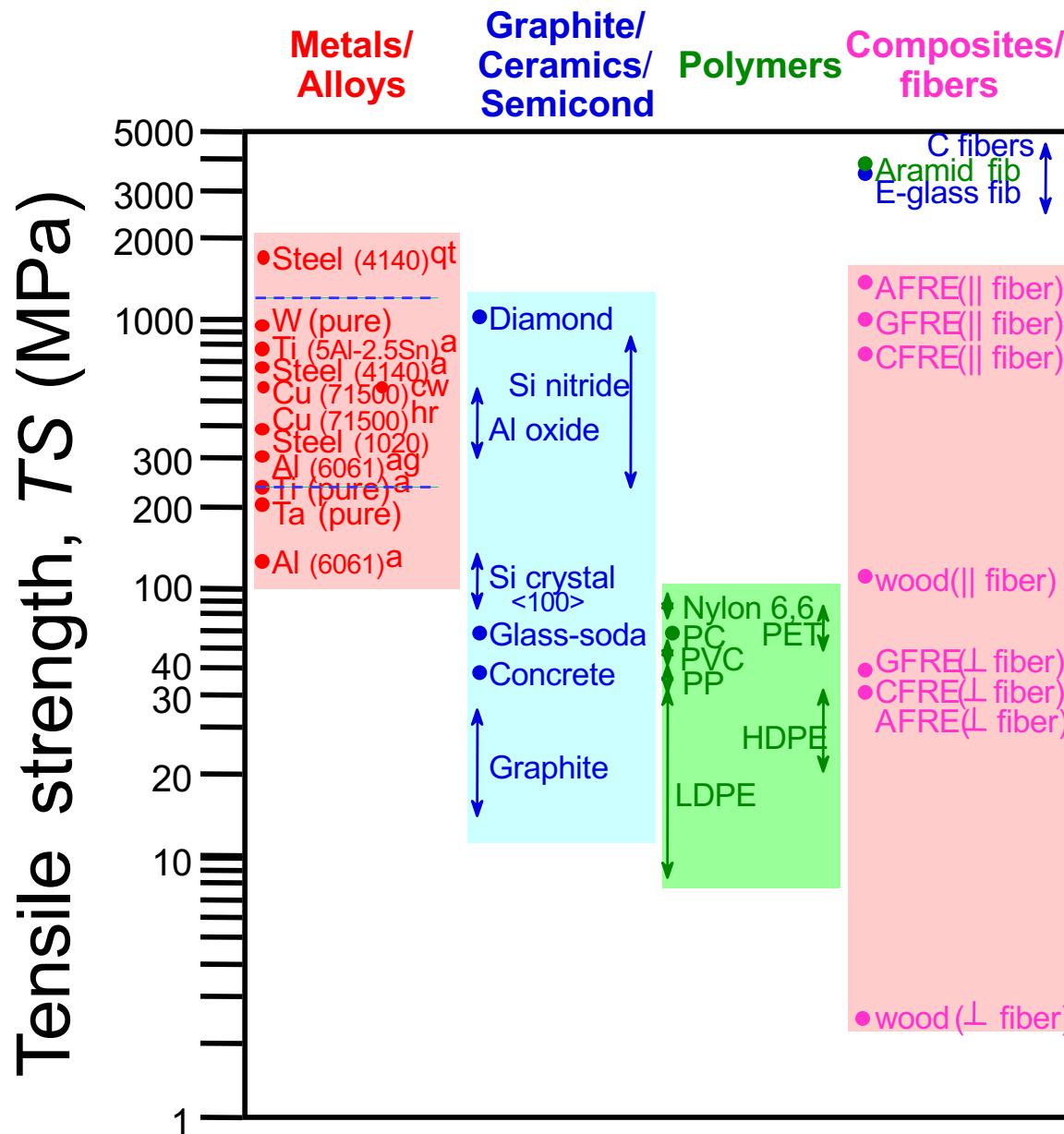
# Tensile Strength (抗拉強度)

- Tensile strength ( $TS$ ) = **maximum stress** on engineering stress-strain curve.



- Metals: Maximum on stress-strain curve appears at the onset of noticeable **necking**

# Tensile Strength: Comparison of Material Types



Room temperature values

Based on data in Table B4,  
Callister & Rethwisch 5e.

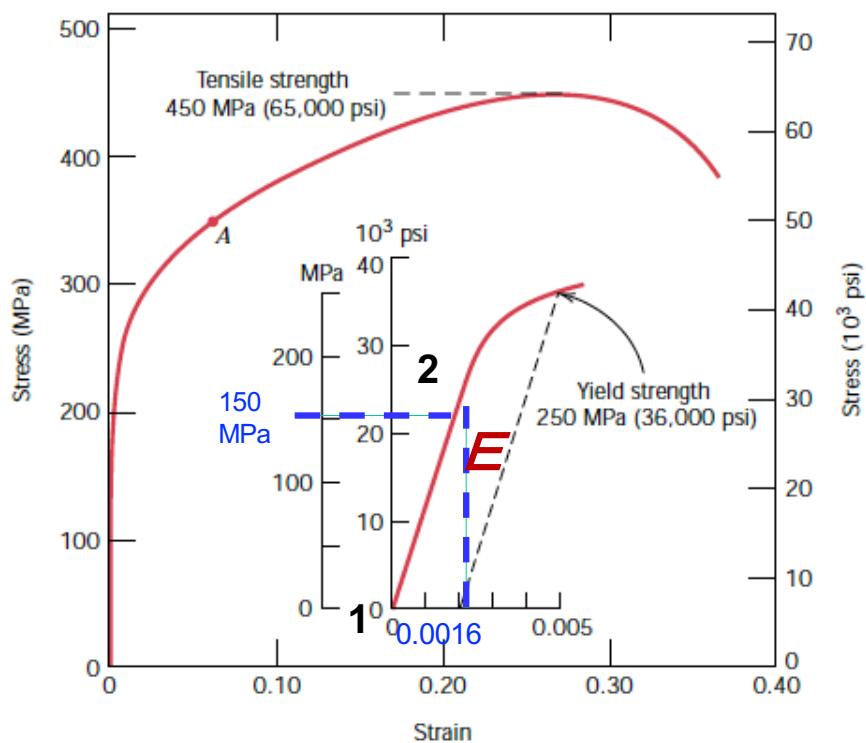
a = annealed  
hr = hot rolled  
ag = aged  
cd = cold drawn  
cw = cold worked  
qt = quenched & tempered

AFRE, GFRE, & CFRE = aramid, glass, & carbon fiber-reinforced epoxy composites, with 60 vol% fibers.

From the tensile stress-strain behavior for the brass specimen shown in Figure 7.12, determine the following:

1/4  
1. 伸長率

- (a) The modulus of elasticity.
- (b) The yield strength at a strain offset of 0.002.



$$E = \text{slope} = \frac{\Delta\sigma}{\Delta\epsilon} = \frac{\sigma_2 - \sigma_1}{\epsilon_2 - \epsilon_1} \quad (7.10)$$

$$E = \frac{(150 - 0) \text{ MPa}}{0.0016 - 0} = 93.8 \text{ GPa} (13.6 \times 10^6 \text{ psi})$$

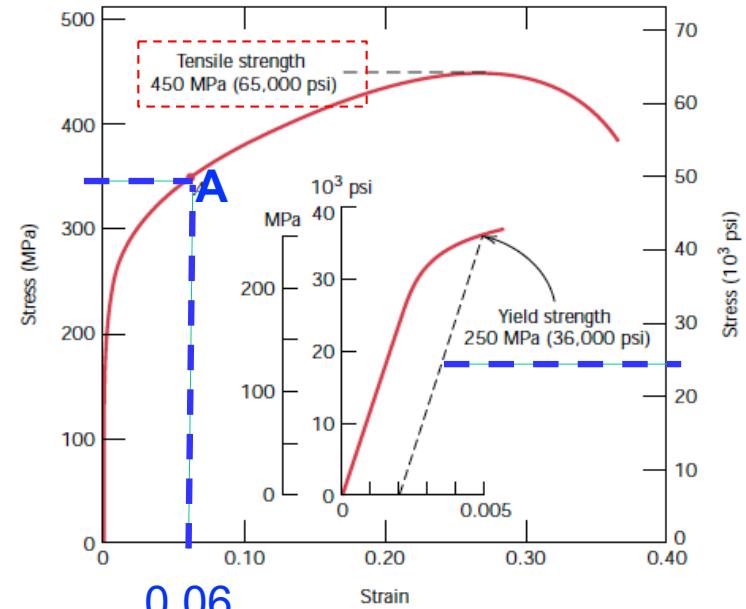
(b) The 0.002 strain offset line is constructed as shown with the stress-strain curve is at approximately which is the yield strength of the brass.

(c) The maximum load that can be sustained by the using Equation 7.1, in which  $\sigma$  is taken to be the tensile strength, 450 MPa (65,000 psi). Solving for  $F$ , the maxi

$$\sigma = \frac{F}{A_0}$$

$$F = \sigma A_0 = \sigma \left( \frac{d_0}{2} \right)^2 \pi$$

$$= (450 \times 10^6 \text{ N/m}^2) \left( \frac{12.8 \times 10^{-3} \text{ m}}{2} \right)^2 \pi = 57,900 \text{ N (13,000 lb}_f\text{)}$$



(d) To compute the change in length,  $\Delta l$ , in Equation 7.2, it is first necessary to determine the strain that is produced by a stress of 345 MPa. This is accomplished by locating the stress point on the stress-strain curve, point A, and reading the corresponding strain from the strain axis, which is approximately 0.06. Inasmuch as  $l_0 = 250$  mm, we have

$$\Delta l = \epsilon l_0 = (0.06)(250 \text{ mm}) = 15 \text{ mm (0.6 in.)}$$

$$\epsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0}$$

# Ductility (延展性)

- Ductility = amount of plastic deformation at failure:
- Specification of ductility
  - Percent elongation (%EL): (伸長率)

If: fracture length

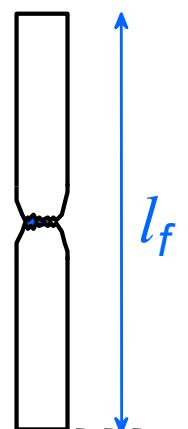
-- Percent reduction in area (%RA): (斷面收縮率)

Af: cross-sectional area

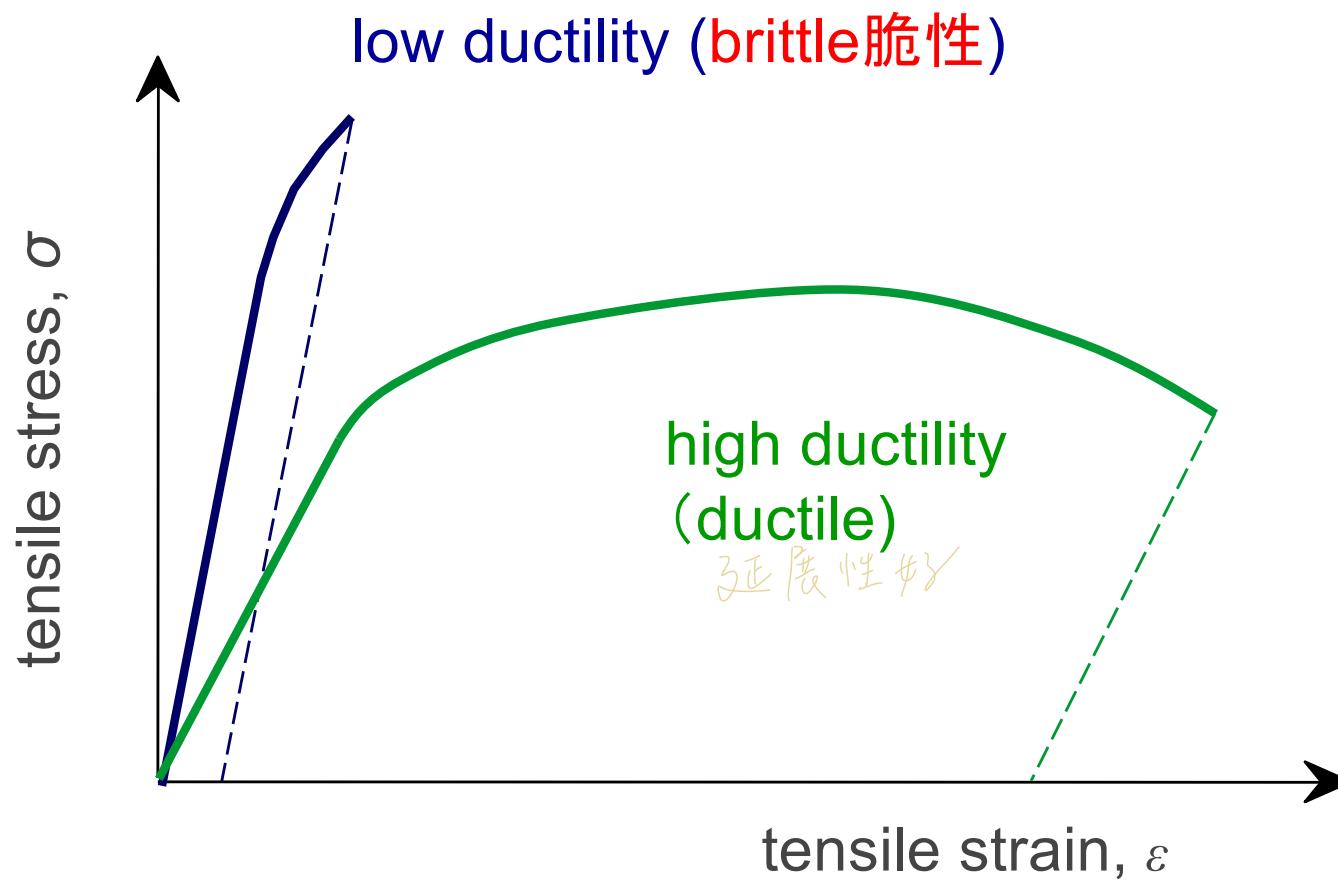
at the point of fracture

$$\%EL = \frac{l_f - l_0}{l_0} \times 100$$

$$\%RA = \frac{A_0 - A_f}{A_0} \times 100$$



- **Brittle** materials are approximately considered to be those having a fracture strain < 5%.



| Material   | Yield Strength |           | Tensile Strength |           | Ductility, %EL<br>[in 50 mm<br>(2 in.)] <sup>a</sup> |
|--|----------------|-----------|------------------|-----------|--|
|  | MPa            | ksi       | MPa              | ksi       |  |
| <i>Metal Alloys<sup>b</sup></i>                            |                |           |                  |           |  |
| Molybdenum   | 565            | 82        | 655              | 95        | 35   |
| Titanium   | 450            | 65        | 520              | 75        | 25   |
| Steel (1020)   | 180            | 26        | 380              | 55        | 25   |
| Nickel   | 138            | 20        | 480              | 70        | 40   |
| Iron   | 130            | 19        | 262              | 38        | 45   |
| Brass (70 Cu–30 Zn)  | 75             | 11        | 300              | 44        | 68   |
| Copper   | 69             | 10        | 200              | 29        | 45   |
| Aluminum   | 35             | 5         | 90               | 13        | 40   |
| <i>Ceramic Materials<sup>c</sup></i>                       |                |           |                  |           |  |
| Zirconia ( $\text{ZrO}_2$ ) <sup>d</sup>                   | —              | —         | 800–1500         | 115–215   | —  |
| Silicon nitride ( $\text{Si}_3\text{N}_4$ )                | —              | —         | 250–1000         | 35–145    | —  |
| Aluminum oxide ( $\text{Al}_2\text{O}_3$ )                 | —              | —         | 275–700          | 40–100    | —  |
| Silicon carbide ( $\text{SiC}$ )                           | —              | —         | 100–820          | 15–120    | —  |
| Glass–ceramic (Pyroceram)                                  | —              | —         | 247              | 36        | —  |
| Mullite ( $3\text{Al}_2\text{O}_3\text{--}2\text{SiO}_2$ ) | —              | —         | 185              | 27        | —  |
| Spinel ( $\text{MgAl}_2\text{O}_4$ )                       | —              | —         | 110–245          | 16–36     | —  |
| Fused silica ( $\text{SiO}_2$ )                            | —              | —         | 110              | 16        | —  |
| Magnesium oxide ( $\text{MgO}$ ) <sup>e</sup>              | —              | —         | 105              | 15        | —  |
| Soda-lime glass  | —              | —         | 69               | 10        | —  |
| <i>Polymers</i>  |                |           |                  |           |  |
| Nylon 6,6  | 44.8–82.8      | 6.5–12    | 75.9–94.5        | 11.0–13.7 | 15–300   |
| Polycarbonate (PC)   | 62.1           | 9.0       | 62.8–72.4        | 9.1–10.5  | 110–150  |
| Poly(ethylene terephthalate) (PET)                         | 59.3           | 8.6       | 48.3–72.4        | 7.0–10.5  | 30–300   |
| Poly(methyl methacrylate) (PMMA)                           | 53.8–73.1      | 7.8–10.6  | 48.3–72.4        | 7.0–10.5  | 2.0–5.5  |
| Poly(vinyl chloride) (PVC)                                 | 40.7–44.8      | 5.9–6.5   | 40.7–51.7        | 5.9–7.5   | 40–80  |
| Phenol-formaldehyde  | —              | —         | 34.5–62.1        | 5.0–9.0   | 1.5–2.0  |
| Polystyrene (PS)   | 25.0–69.0      | 3.63–10.0 | 35.9–51.7        | 5.2–7.5   | 1.2–2.5  |
| Polypropylene (PP)   | 31.0–37.2      | 4.5–5.4   | 31.0–41.4        | 4.5–6.0   | 100–600  |
| Polyethylene–high density (HDPE)                           | 26.2–33.1      | 3.8–4.8   | 22.1–31.0        | 3.2–4.5   | 10–1200  |
| Polytetrafluoroethylene (PTFE)                             | 13.8–15.2      | 2.0–2.2   | 20.7–34.5        | 3.0–5.0   | 200–400  |
| Polyethylene–low density (LDPE)                            | 9.0–14.5       | 1.3–2.1   | 8.3–31.4         | 1.2–4.55  | 100–650  |

<sup>a</sup>For polymers, percent elongation at break.

<sup>b</sup>Property values are for metal alloys in an annealed state.

<sup>c</sup>The tensile strength of ceramic materials is taken as flexural strength (Section 7.10).

<sup>d</sup>Partially stabilized with 3 mol%  $\text{Y}_2\text{O}_3$ .

<sup>e</sup>Sintered and containing approximately 5% porosity.

Table\_7-2

## Stress–strain behavior for iron at three temperatures

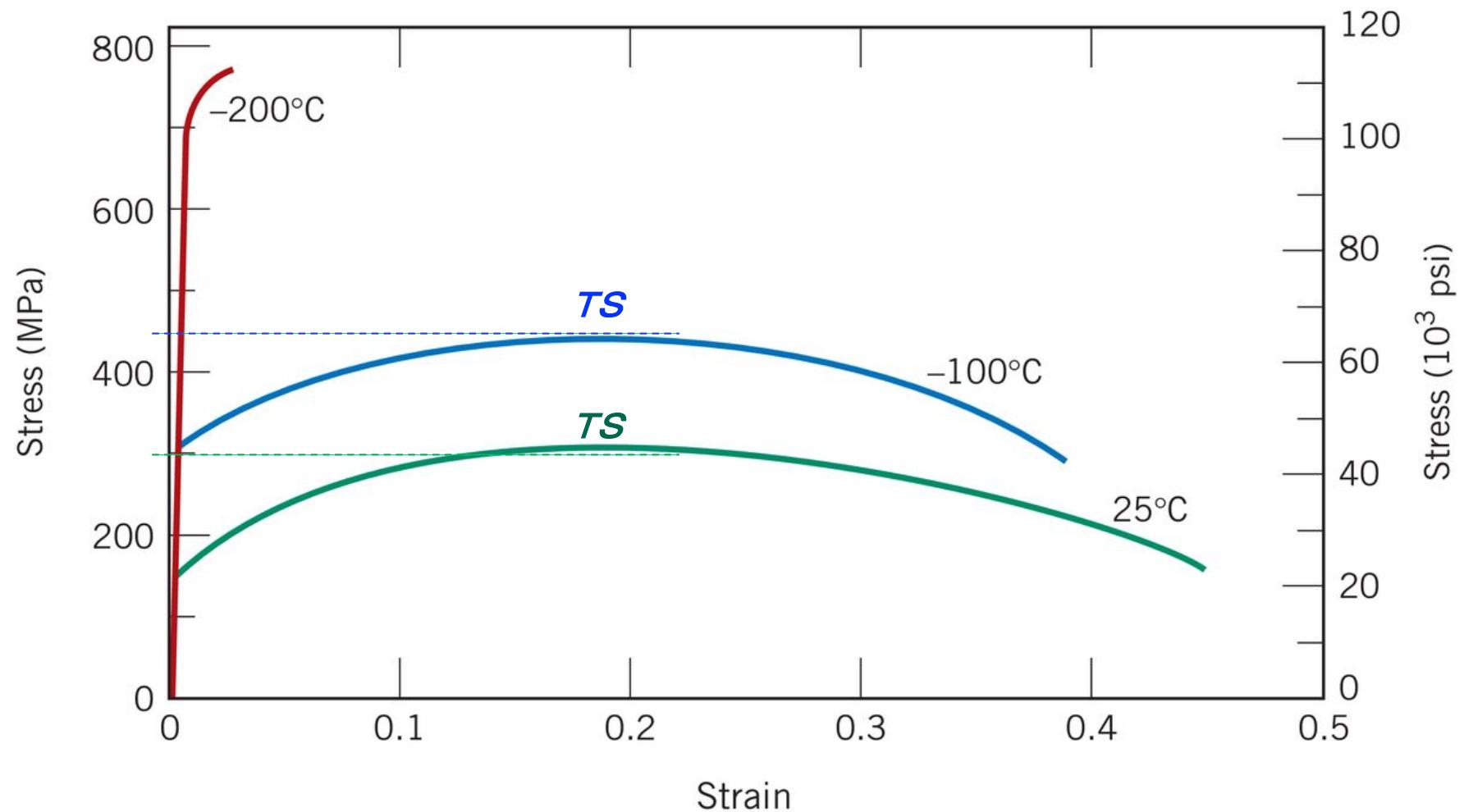


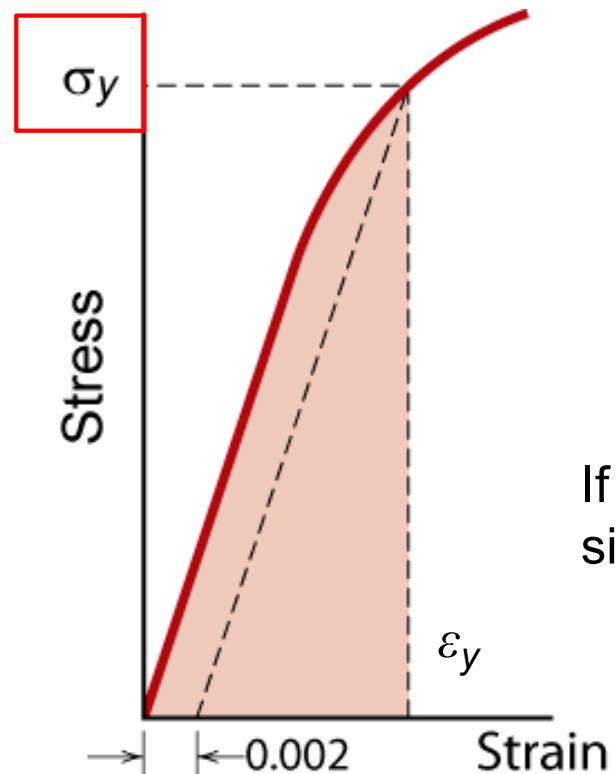
FIGURE 7.14

Engineering stress–strain behavior for iron at three temperatures.



# Resilience (彈性能)

- Resilience—ability of a material to absorb energy **during elastic deformation**
- Energy recovered when load released
- Resilience specified by modulus of resilience,  $U_r$  (彈性能係數)



If assume a linear stress-strain curve this  
simplifies to

$$U_r = \text{Area under stress-strain curve}$$

to yielding =  $\int_0^{\varepsilon_y} \sigma \, d\varepsilon$

↓ 未受應力的狀態

$$U_r \approx \frac{1}{2} \sigma_y \varepsilon_y$$

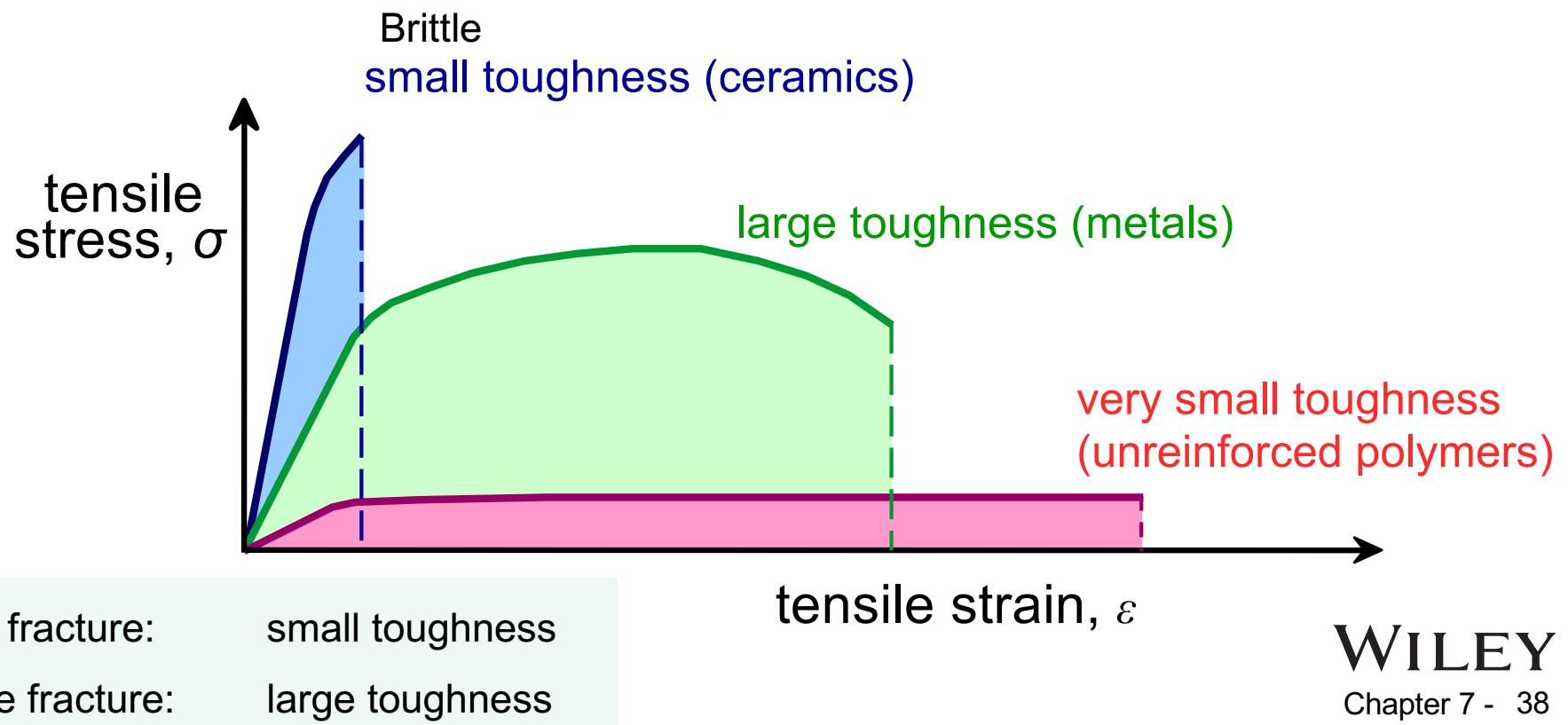
↓ 降伏點的應變

線性彈性行為下的彈性能係數

Fig. 7.15, Callister & Rethwisch 5e.

# Toughness (韌性)

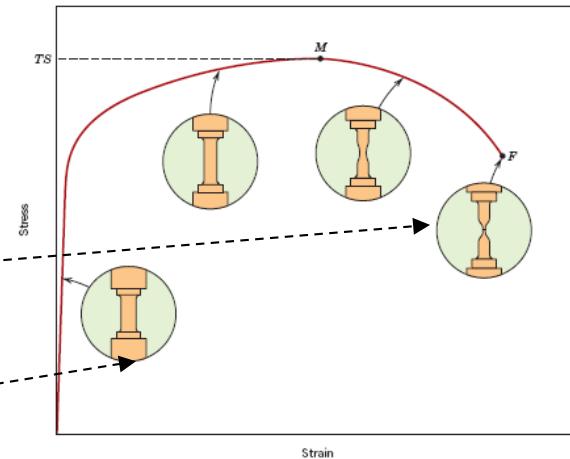
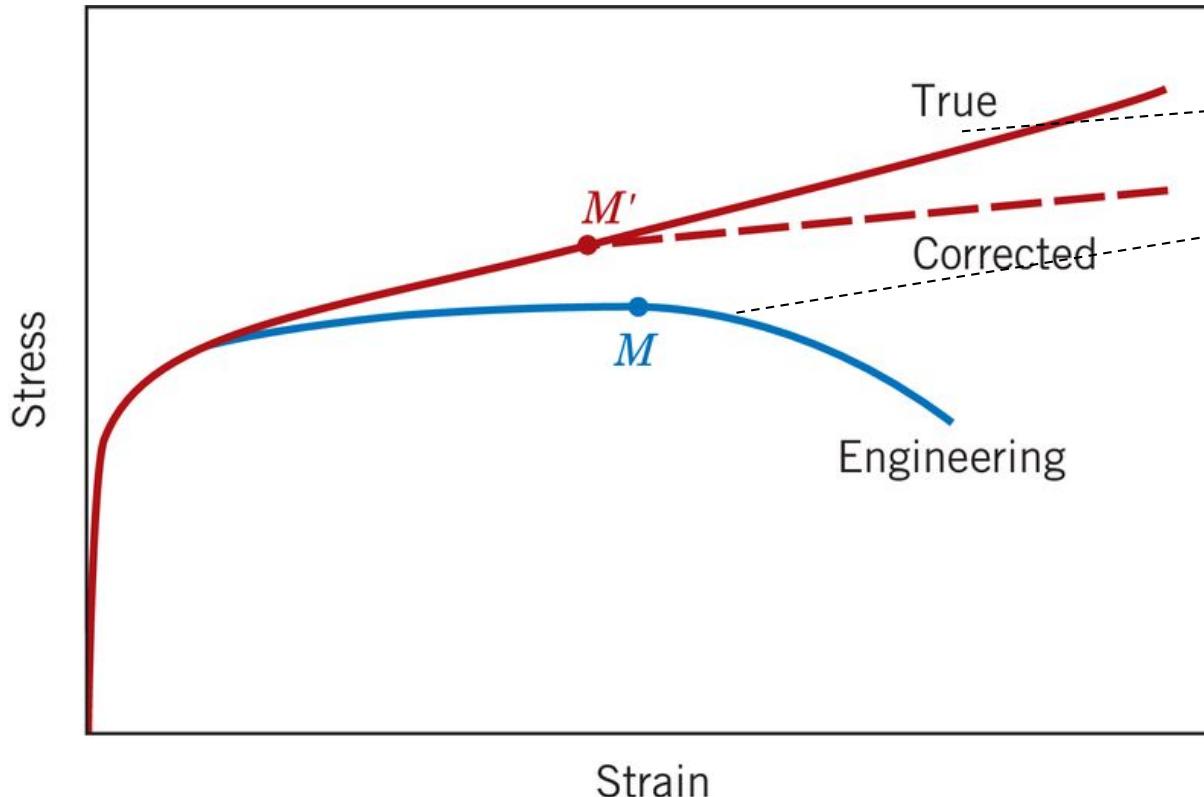
- Toughness of a material is expressed in several contexts
- Toughness = amount of energy absorbed before fracture
- Approximate by area under the stress-strain curve — units of energy per unit volume



# True Stress & Strain

- **True stress**  $\sigma_T = F/A_i$
- **True strain**  $\varepsilon_T = \ln(\ell_i/\ell_o)$

where  $A_i$  = instantaneous cross-sectional area



Conversion Equations:  
valid only to the **onset of necking** 紧缩

$$\sigma_T = \sigma(1 + \varepsilon)$$

$$\varepsilon_T = \ln(1 + \varepsilon)$$

## Ductility and True-stress-at-Fracture Computations



A cylindrical specimen of steel having an original diameter of 12.8 mm (0.505 in.) is tensile tested to fracture and found to have an engineering fracture strength  $f_f$  of 460 MPa (67,000 psi). If its cross-sectional diameter at fracture is 10.7 mm (0.422 in.), determine:

- (a) The ductility in terms of percent reduction in area.

(a) Ductility is computed using Equation 7.12, as

$$\%RA = \frac{\left(\frac{12.8 \text{ mm}}{2}\right)^2 \pi - \left(\frac{10.7 \text{ mm}}{2}\right)^2 \pi}{\left(\frac{12.8 \text{ mm}}{2}\right)^2 \pi} \times 100$$

$\left(\frac{12.8}{2}\right)^2$

$A_f$

$A_o$

$$= \frac{128.7 \text{ mm}^2 - 89.9 \text{ mm}^2}{128.7 \text{ mm}^2} \times 100 = 30\%$$

$$\%RA = \left( \frac{A_0 - A_f}{A_0} \right) \times 100$$

(b) The true stress at fracture.

$$F = \sigma_f A_0 = (460 \times 10^6 \text{ N/m}^2)(128.7 \text{ mm}^2) \left( \frac{1 \text{ m}^2}{10^6 \text{ mm}^2} \right) = 59,200 \text{ N}$$

$$\sigma_T = \frac{F}{A_I}$$

Thus, the true stress is calculated as

$$\sigma_T = \frac{F}{A_f} = \frac{59,200 \text{ N}}{(89.9 \text{ mm}^2) \left( \frac{1 \text{ m}^2}{10^6 \text{ mm}^2} \right)}$$
$$= 6.6 \times 10^8 \text{ N/m}^2 = \underline{\underline{660 \text{ MPa}}} \quad (95,700 \text{ psi})$$

# Elastic Strain Recovery

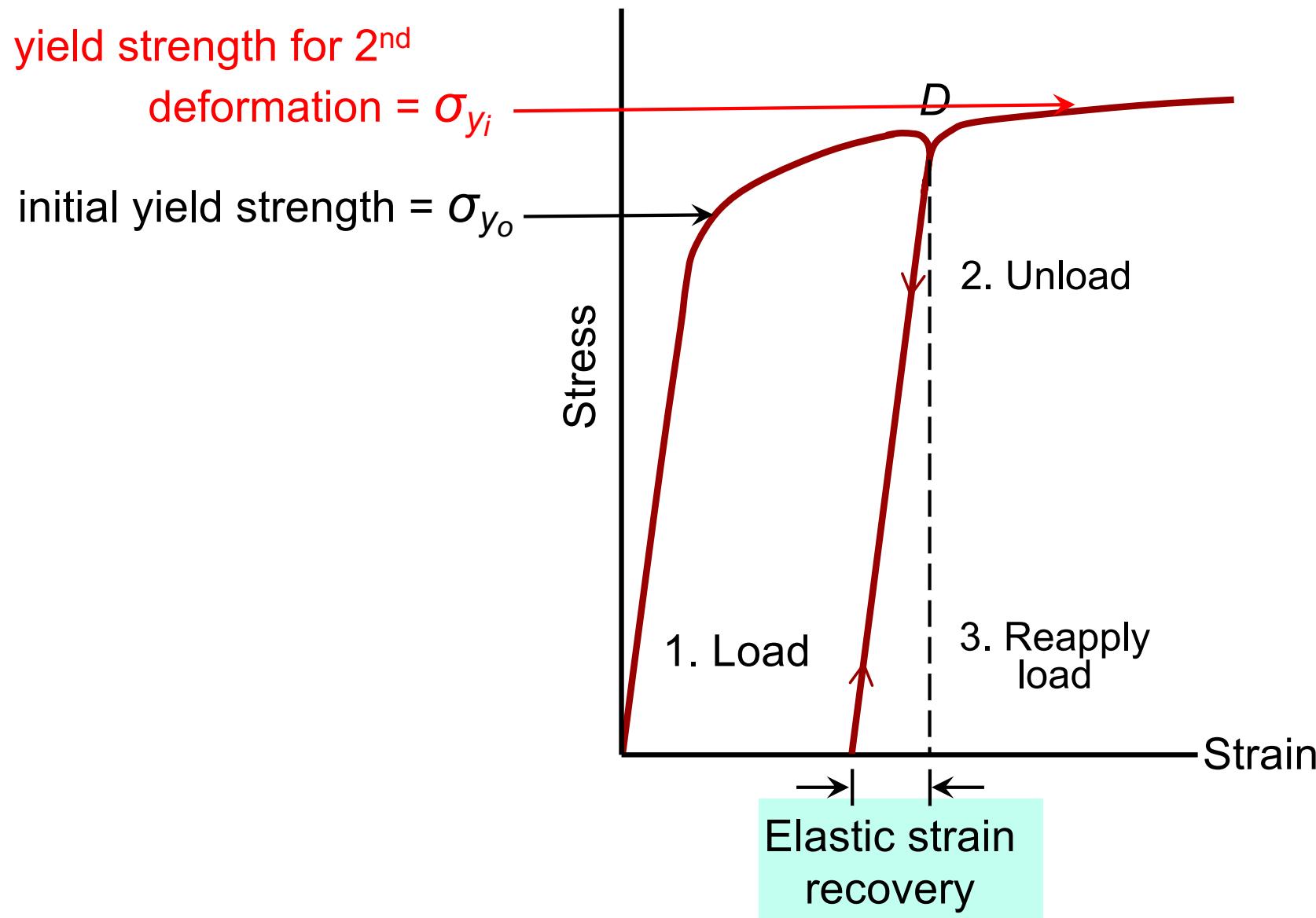


Fig. 7.17, Callister & Rethwisch 5e.

# MECHANICAL BEHAVIOR— Ceramics

Ceramic materials are more brittle than metals.

Why is this so?

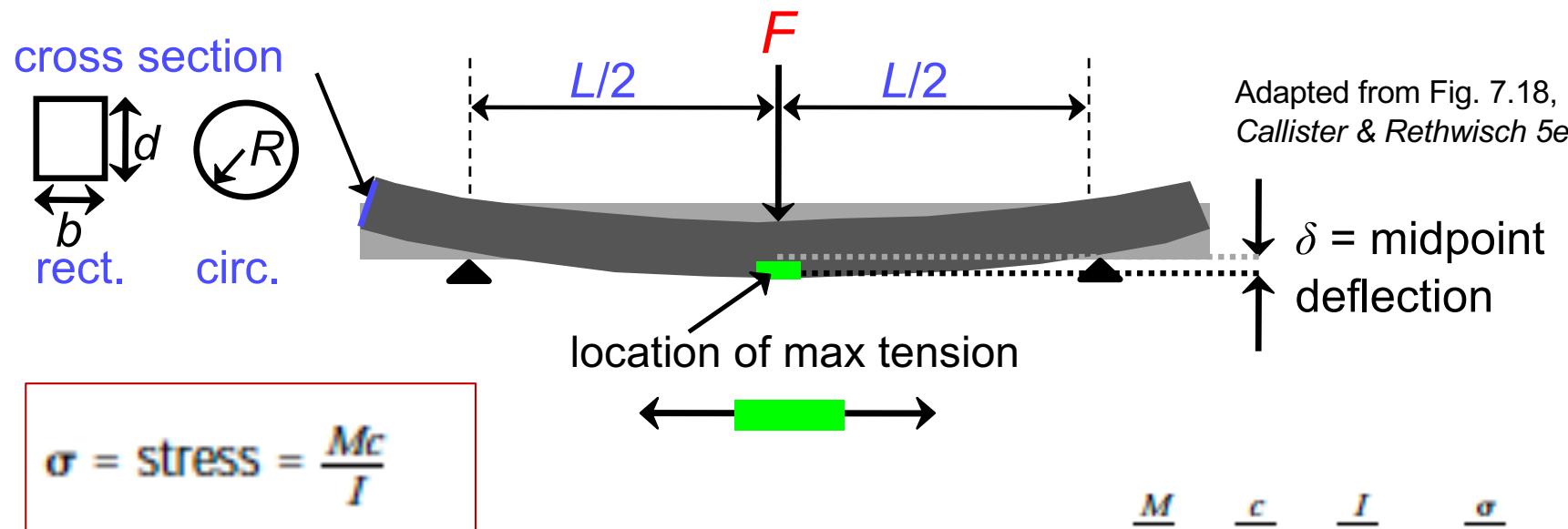
- Consider mechanism of deformation
  - In crystalline, by dislocation motion
  - In highly ionic solids, dislocation motion is difficult
    - few slip systems
    - resistance to motion of ions of like charge (e.g., anions) past one another

# Flexural Tests – Measurement of Elastic Modulus

抗彎強度

CERAMICS

- Room **T** behavior is usually elastic, with brittle failure.
- **3-Point Bend Testing** often used. 三點負荷彎曲測試
  - tensile tests are difficult for brittle materials.



where  $M$  = maximum bending moment 最大彎矩

$c$  = distance from center of specimen  
to outer fibers 試片中心到外緣位置

$I$  = moment of inertia of cross section 截面的慣性矩

$F$  = applied load 作用力

|             | $\frac{M}{c}$  | $\frac{c}{I}$ | $\frac{I}{L}$       | $\frac{\sigma}{\sigma}$ |
|-------------|----------------|---------------|---------------------|-------------------------|
| Rectangular | $\frac{FL}{4}$ | $\frac{d}{2}$ | $\frac{bd^3}{12}$   | $\frac{3FL}{2bd^2}$     |
| Circular    | $\frac{FL}{4}$ | $R$           | $\frac{\pi R^4}{4}$ | $\frac{FL}{\pi R^3}$    |

# Flexural Tests – Measurement of Flexural Strength (抗彎強度)

- **Flexural strength ( $F_f$ ):**

CERAMICS

- The stress at fracture using this flexure test.
- an important mechanical parameter for brittle ceramics.

$$\sigma_{fs} = \frac{3F_f L}{2bd^2} \quad (\text{rect. cross section})$$

$$\sigma_{fs} = \frac{F_f L}{\pi R^3} \quad (\text{circ. cross section})$$

$F_f$ : loaded at fracture

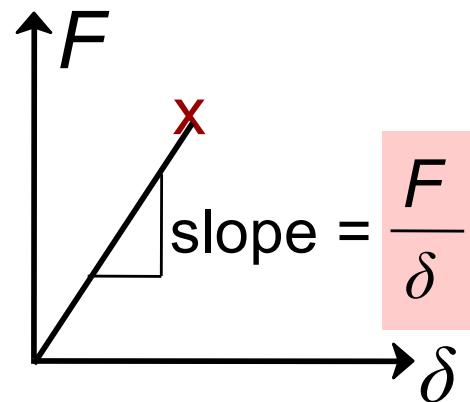
L: distance between point

- Typical values:

| Material          | $\sigma_{fs}$ (MPa) |
|-------------------|---------------------|
| Si nitride        | 250-1000            |
| Si carbide        | 100-820             |
| Al oxide          | 275-700             |
| glass (soda-lime) | 69                  |

Data from Table 7.2, Callister & Rethwisch

- Determine elastic modulus (E) according to:



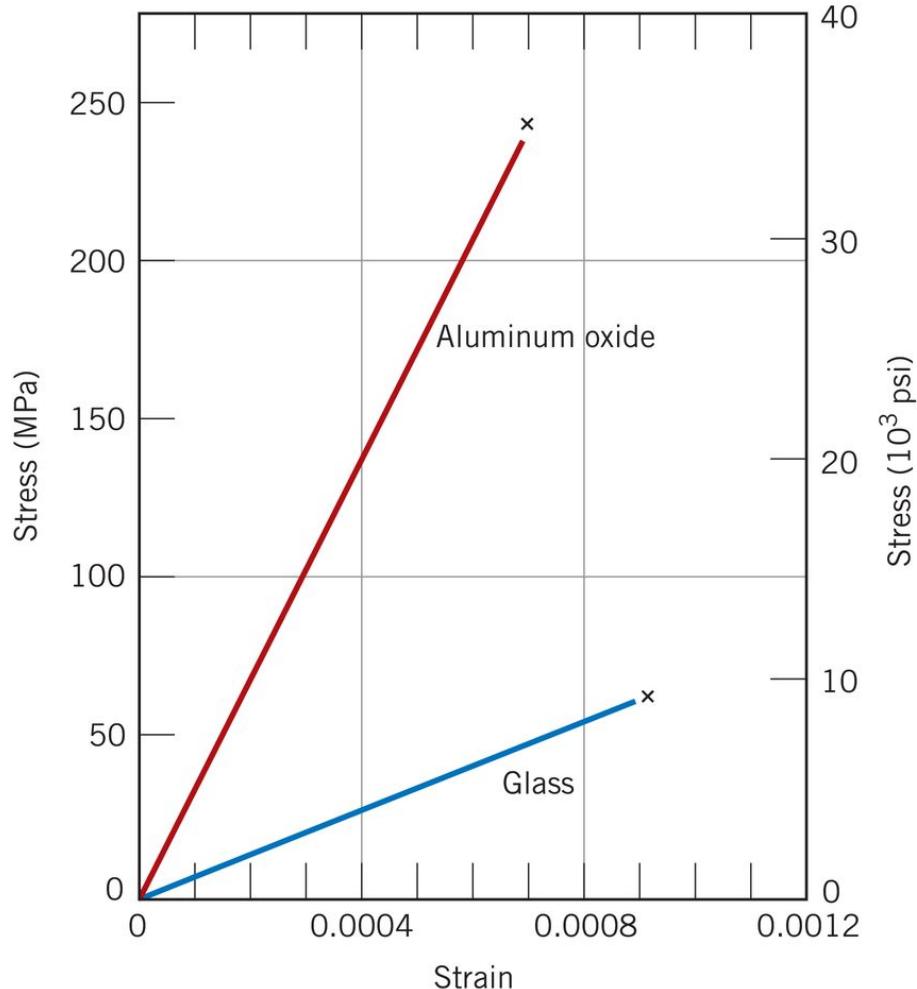
$$E = \frac{F}{\delta} \frac{L^3}{4bd^3} \quad (\text{rectangle cross section})$$

$$E = \frac{F}{\delta} \frac{L^3}{12\pi R^4} \quad (\text{circle cross section})$$

linear-elastic behavior

- **Flexural strength:** the **stress at fracture** using this flexure test, modulus of rupture, fracture strength, or the bend strength,
- an important mechanical parameter for brittle ceramics.

# Flexural Tests – Measurement of Elastic Modulus



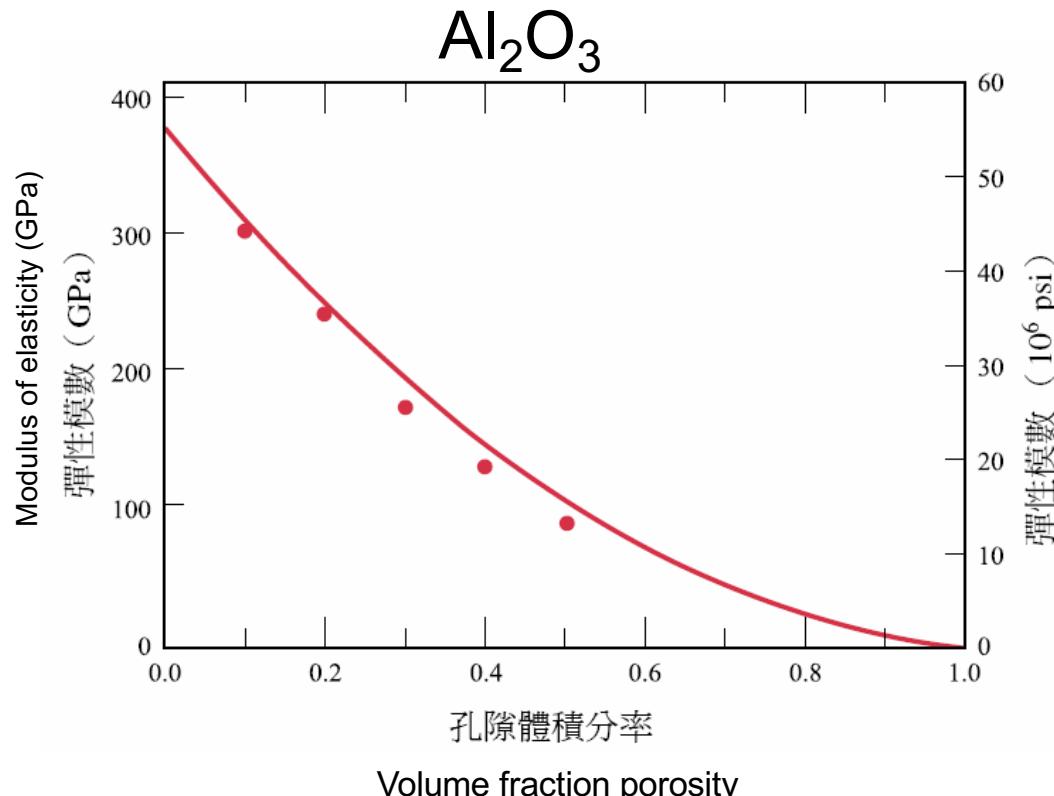
Typical stress–strain behavior to fracture for aluminum oxide and glass.

- Typical values:

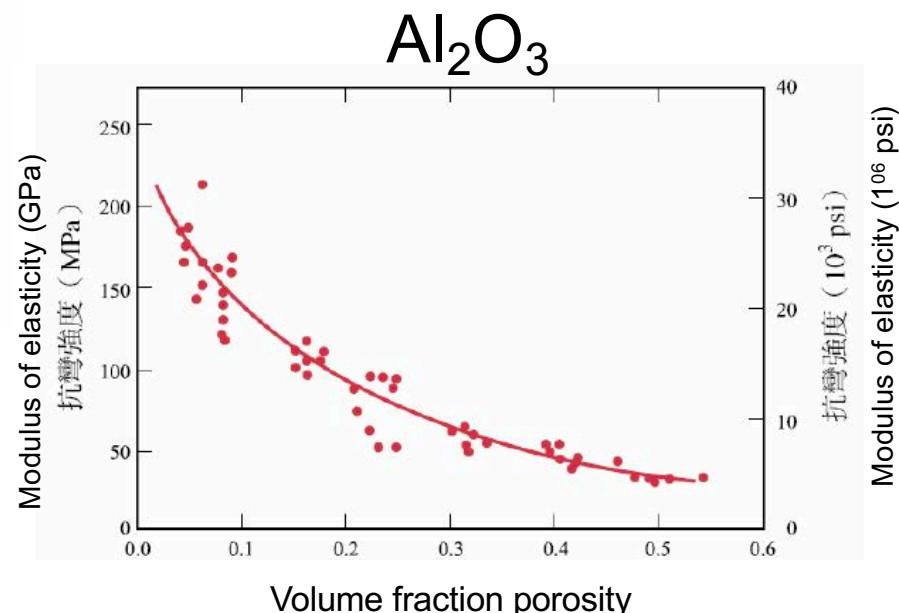
| Material          | $\sigma_{fs}$ (MPa) | $E$ (GPa) |
|-------------------|---------------------|-----------|
| Si nitride        | 250-1000            | 304       |
| Si carbide        | 100-820             | 345       |
| Al oxide          | 275-700             | 393       |
| glass (soda-lime) | 69                  | 69        |

Data from Table 7.2, Callister & Rethwisch 5e.

# Influence of Porosity on The Mechanical Properties of Ceramics)

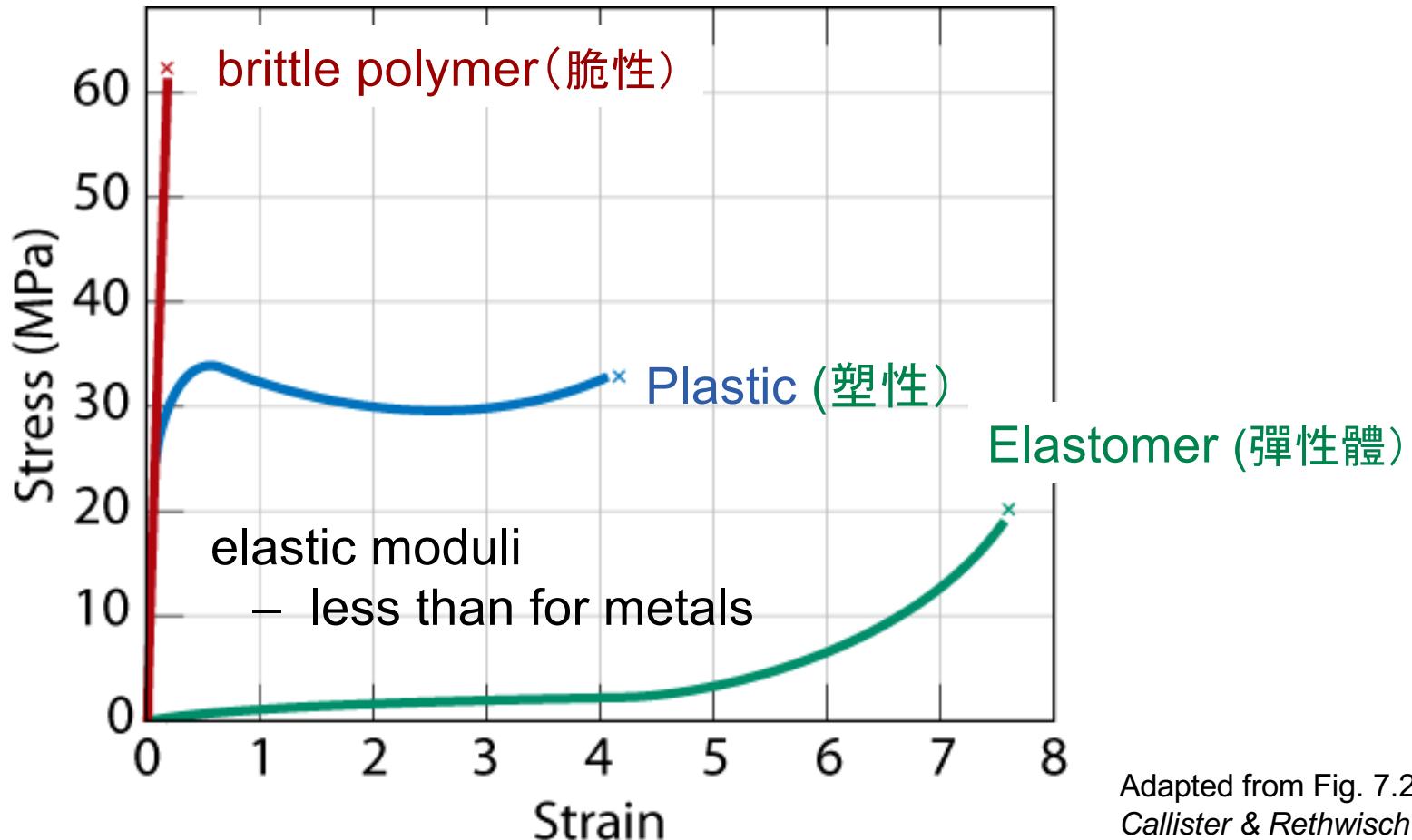


$$E = E_0 (1 - 1.9P + 0.9P^2) \quad (7.21)$$



$$\sigma_{fs} = \sigma_0 \exp(-nP)$$

# Mechanical Properties of Polymers – Stress-Strain Behavior

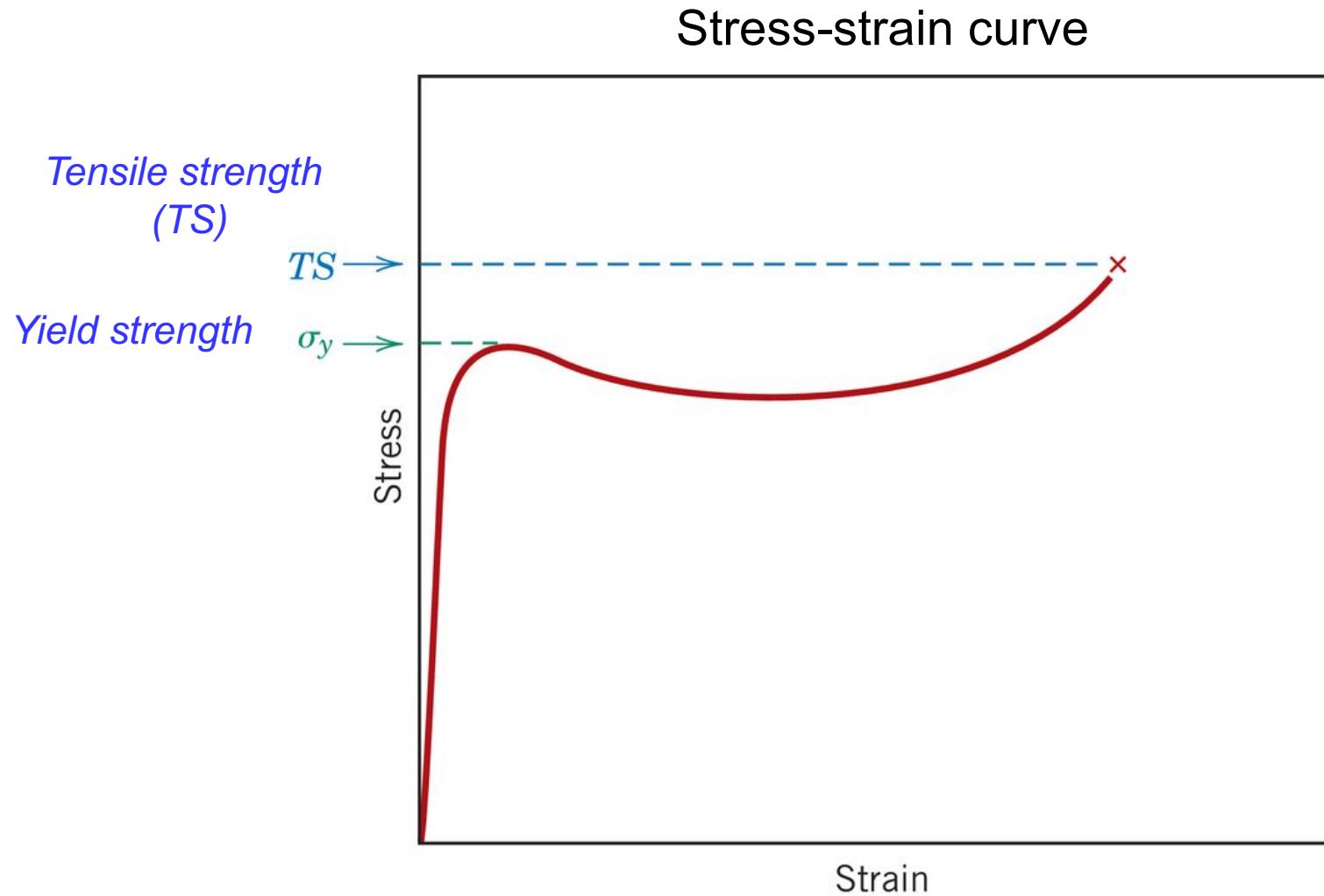


Adapted from Fig. 7.22,  
Callister & Rethwisch 5e.

- Fracture strengths of polymers ~ 10% of those for metals
- Deformation strains for polymers > 1000%
  - for most metals, deformation strains < 10%

# Mechanical Properties of Polymers

## Plastic polymer



| Material   | Yield Strength |           | Tensile Strength |           | Ductility, %EL<br>[in 50 mm<br>(2 in.)] <sup>a</sup> |
|--|----------------|-----------|------------------|-----------|--|
|  | MPa            | ksi       | MPa              | ksi       |  |
| <i>Metal Alloys<sup>b</sup></i>                            |                |           |                  |           |  |
| Molybdenum   | 565            | 82        | 655              | 95        | 35   |
| Titanium   | 450            | 65        | 520              | 75        | 25   |
| Steel (1020)   | 180            | 26        | 380              | 55        | 25   |
| Nickel   | 138            | 20        | 480              | 70        | 40   |
| Iron   | 130            | 19        | 262              | 38        | 45   |
| Brass (70 Cu–30 Zn)  | 75             | 11        | 300              | 44        | 68   |
| Copper   | 69             | 10        | 200              | 29        | 45   |
| Aluminum   | 35             | 5         | 90               | 13        | 40   |
| <i>Ceramic Materials<sup>c</sup></i>                       |                |           |                  |           |  |
| Zirconia ( $\text{ZrO}_2$ ) <sup>d</sup>                   | —              | —         | 800–1500         | 115–215   | —  |
| Silicon nitride ( $\text{Si}_3\text{N}_4$ )                | —              | —         | 250–1000         | 35–145    | —  |
| Aluminum oxide ( $\text{Al}_2\text{O}_3$ )                 | —              | —         | 275–700          | 40–100    | —  |
| Silicon carbide (SiC)                                      | —              | —         | 100–820          | 15–120    | —  |
| Glass–ceramic (Pyroceram)                                  | —              | —         | 247              | 36        | —  |
| Mullite ( $3\text{Al}_2\text{O}_3\text{--}2\text{SiO}_2$ ) | —              | —         | 185              | 27        | —  |
| Spinel ( $\text{MgAl}_2\text{O}_4$ )                       | —              | —         | 110–245          | 16–36     | —  |
| Fused silica ( $\text{SiO}_2$ )                            | —              | —         | 110              | 16        | —  |
| Magnesium oxide ( $\text{MgO}$ ) <sup>e</sup>              | —              | —         | 105              | 15        | —  |
| Soda–lime glass  | —              | —         | 69               | 10        | —  |
| <i>Polymers</i>  |                |           |                  |           |  |
| Nylon 6,6  | 44.8–82.8      | 6.5–12    | 75.9–94.5        | 11.0–13.7 | 15–300   |
| Polycarbonate (PC)   | 62.1           | 9.0       | 62.8–72.4        | 9.1–10.5  | 110–150  |
| Poly(ethylene terephthalate) (PET)                         | 59.3           | 8.6       | 48.3–72.4        | 7.0–10.5  | 30–300   |
| Poly(methyl methacrylate) (PMMA)                           | 53.8–73.1      | 7.8–10.6  | 48.3–72.4        | 7.0–10.5  | 2.0–5.5  |
| Poly(vinyl chloride) (PVC)                                 | 40.7–44.8      | 5.9–6.5   | 40.7–51.7        | 5.9–7.5   | 40–80  |
| Phenol-formaldehyde  | —              | —         | 34.5–62.1        | 5.0–9.0   | 1.5–2.0  |
| Polystyrene (PS)   | 25.0–69.0      | 3.63–10.0 | 35.9–51.7        | 5.2–7.5   | 1.2–2.5  |
| Polypropylene (PP)   | 31.0–37.2      | 4.5–5.4   | 31.0–41.4        | 4.5–6.0   | 100–600  |
| Polyethylene–high density (HDPE)                           | 26.2–33.1      | 3.8–4.8   | 22.1–31.0        | 3.2–4.5   | 10–1200  |
| Polytetrafluoroethylene (PTFE)                             | 13.8–15.2      | 2.0–2.2   | 20.7–34.5        | 3.0–5.0   | 200–400  |
| Polyethylene–low density (LDPE)                            | 9.0–14.5       | 1.3–2.1   | 8.3–31.4         | 1.2–4.55  | 100–650  |

<sup>a</sup>For polymers, percent elongation at break.

<sup>b</sup>Property values are for metal alloys in an annealed state.

<sup>c</sup>The tensile strength of ceramic materials is taken as flexural strength (Section 7.10).

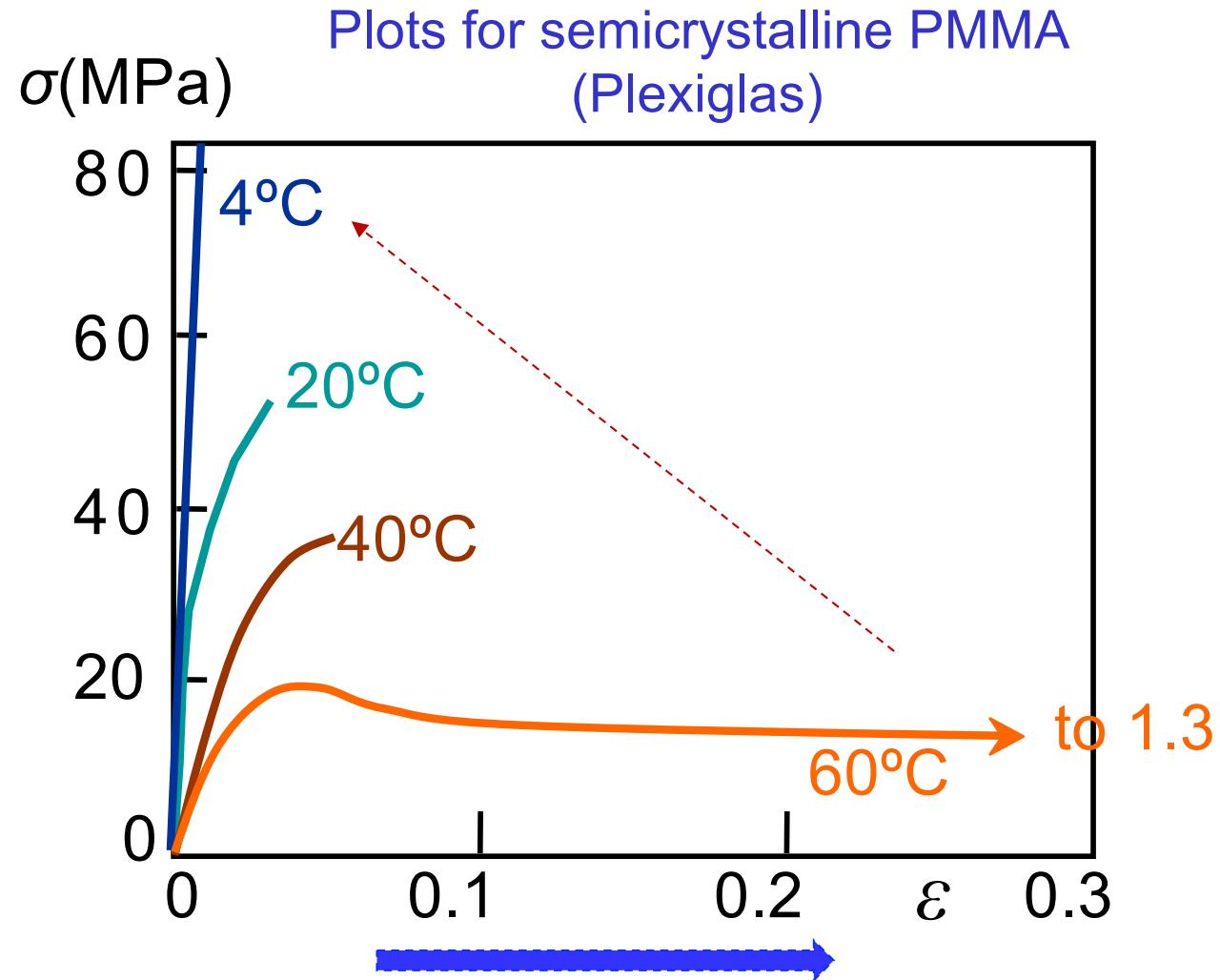
<sup>d</sup>Partially stabilized with 3 mol%  $\text{Y}_2\text{O}_3$ .

<sup>e</sup>Sintered and containing approximately 5% porosity.

Table\_7-2

# Influence of Temperature & Strain Rate on Thermoplastics

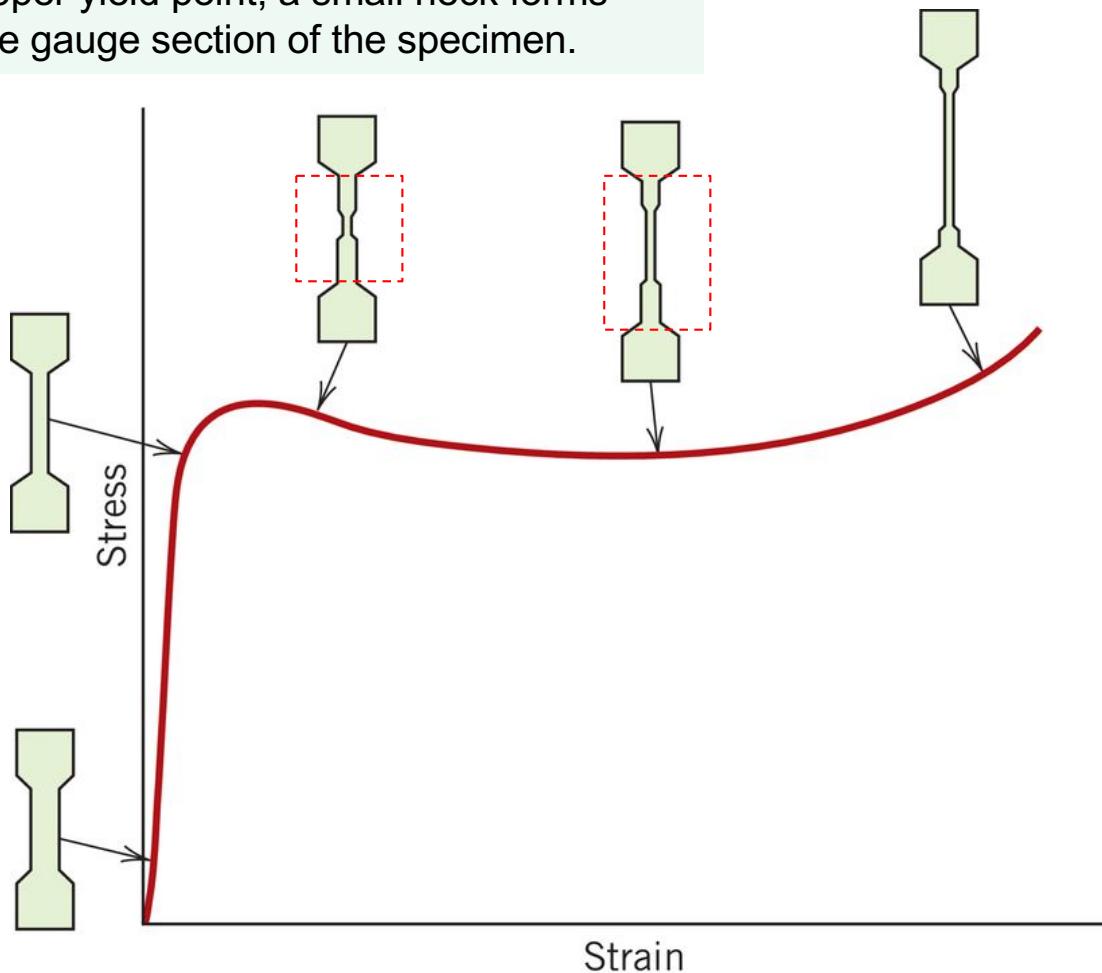
- Decreasing  $T$ ...
  - increases  $E$
  - increases  $TS$
  - decreases  $\%EL$
- Increasing strain rate...
  - same effects as decreasing  $T$ .



# Macroscopic Deformation

## Semi-crystalline polymer

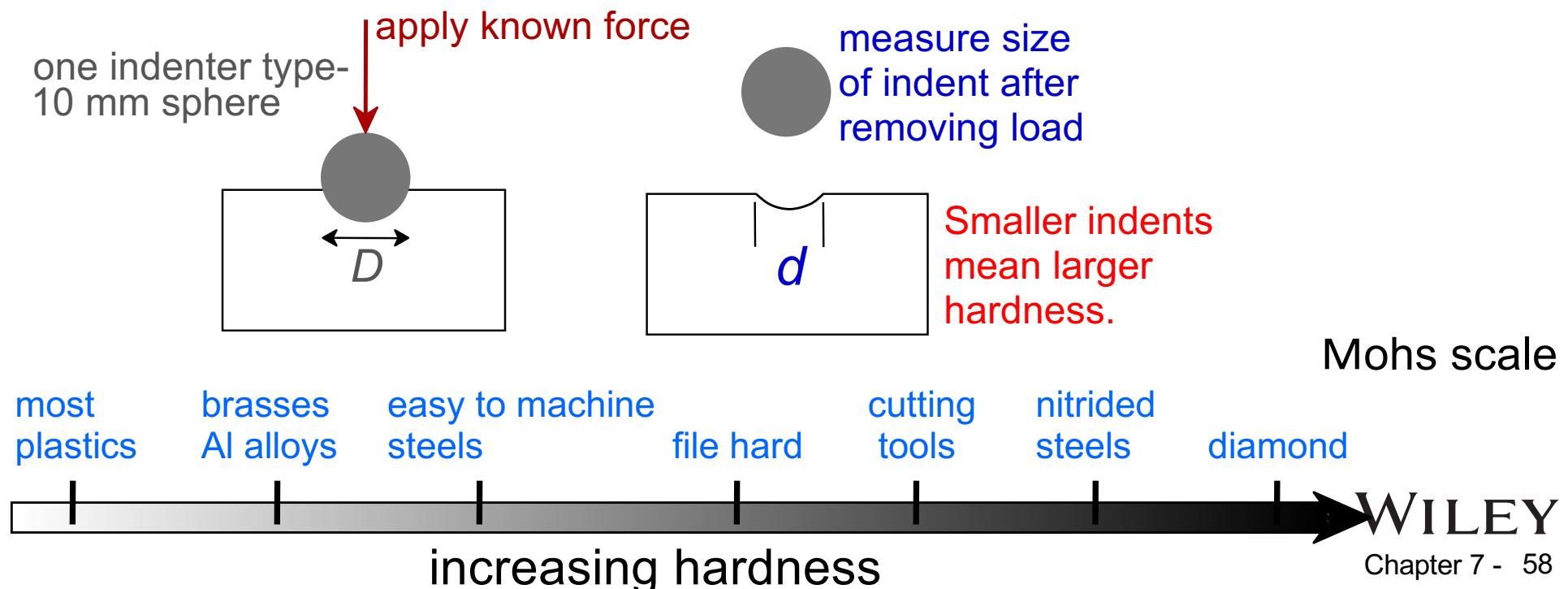
At the upper yield point, a small neck forms within the gauge section of the specimen.



# Hardness (硬度)

↓不考硬度

- Measure of **resistance to surface plastic deformation** — dent or scratch.
- Large hardness means:
  - high resistance to deformation from compressive loads.
  - better wear properties.



# Measurement of Hardness

## Rockwell Hardness (洛氏硬度)

- Several scales—combination of load magnitude, indenter size

|                                   | Indenters   | Loads  |
|-----------------------------------|---|--|
| Rockwell and superficial Rockwell | {<br>Diamond cone:<br>$\frac{1}{16}$ , $\frac{1}{8}$ , $\frac{1}{4}$ , $\frac{1}{2}$ in.<br>diameter<br>steel spheres | {<br>60 kg<br>100 kg<br>150 kg } Rockwell<br><br>{<br>15 kg<br>30 kg<br>45 kg } Superficial Rockwell |

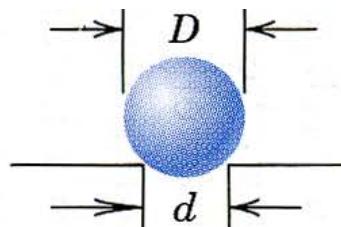
- Examples:
  - Rockwell A Scale – 60 kg load/diamond indenter
  - Superficial Rockwell 15T Scale – 15 kg load/ 1/16 in. indenter
- Rockwell hardness designation: (hardness reading) HR
- Examples: 57 HRA; 63 HR15T
- Hardness range for each scale: 0–130 HR;  
useful range: 20–100 HR

# Measurement of Hardness (cont.)

## Brinell Hardness 勃氏硬度

- Single scale
- Brinell hardness designation: (hardness reading) HB

10-mm sphere  
of steel or  
tungsten carbide



$$HB = \frac{2P}{\pi D[D - \sqrt{D^2 - d^2}]}$$

- $P$  = load (kg)
- $500 \text{ kg} \leq P \leq 3000 \text{ kg}$  (50 kg increments)

- **Relationships—Brinell hardness & tensile strength**

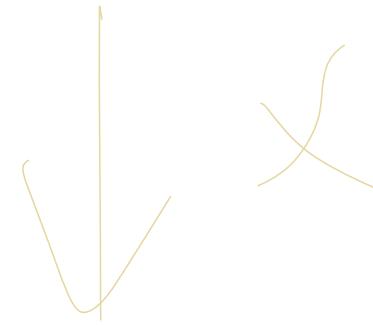
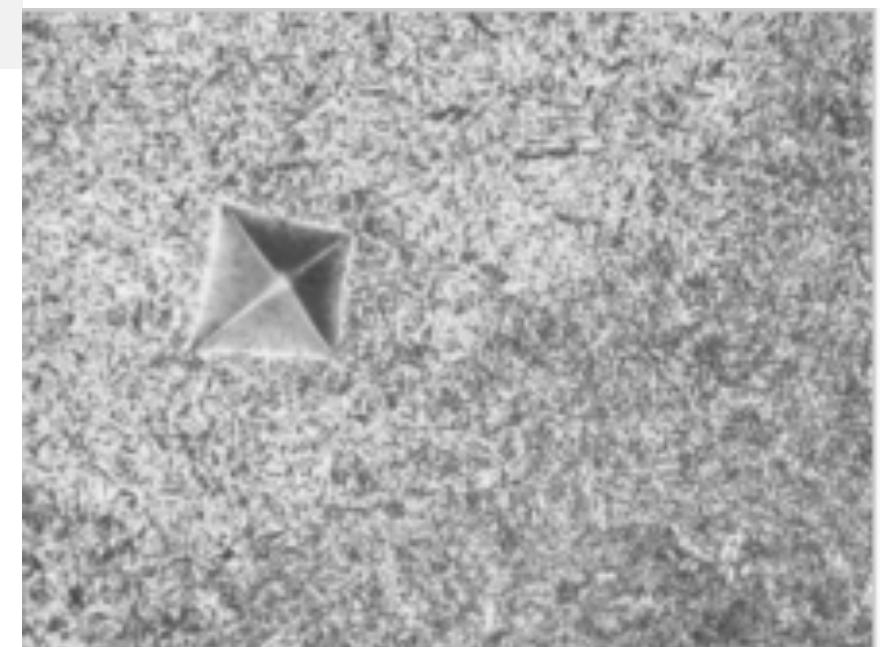
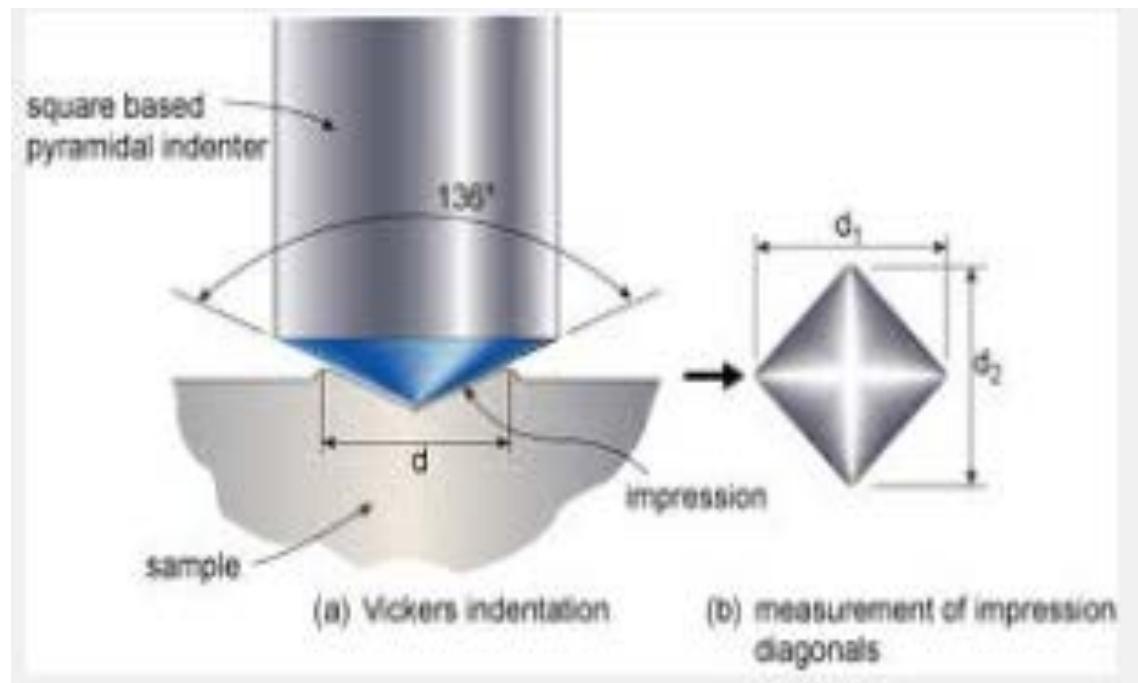
- $TS \text{ (psia)} = 500 \times HB$
- $TS \text{ (MPa)} = 3.45 \times HB$

# Knoop and Vickers Microhardness Tests

## 諾普和維氏微小硬度試驗

- Knoop and Vickers (also called diamond pyramid).
- For each test a **very small diamond indenter** having pyramidal geometry is forced into the surface of the specimen.
- Applied loads are much smaller (ranging 1 - 1000 g).
- The resulting impression is observed under a microscope and measured;
- This measurement is then converted into a hardness number.
- Careful specimen surface preparation (grinding and polishing) may be necessary to ensure a well-defined indentation that may be accurately measured.
- The Knoop and Vickers hardness numbers are designated by **HK** and **HV**, respectively,
- Both are well suited for measuring the hardness of small, selected specimen regions; furthermore, Knoop is used for testing brittle materials such as ceramics.

# Hardness (硬度)



| Test                              | Indenter  | Shape of Indentation |          |   | Formula for Hardness Number <sup>a</sup>       |
|-----------------------------------|---|----------------------|----------|---|--|
|                                   |   | Side View            | Top View | Load  |  |
| Brinell                           | 10-mm sphere of steel or tungsten carbide   |                      |          | $P$   | $HB = \frac{2P}{\pi D [D - \sqrt{D^2 - d^2}]}$ |
| Vickers microhardness             | Diamond pyramid   |                      |          | $P$   | $HV = 1.854 P/d_1^2$                           |
| Knoop microhardness               | Diamond pyramid   |                      |          | $P$   | $HK = 14.2 P/l^2$                              |
| Rockwell and superficial Rockwell | {Diamond cone; $\frac{1}{16}$ -, $\frac{1}{8}$ -, $\frac{1}{4}$ -, $\frac{1}{2}$ - in. diameter tungsten carbide spheres} |                      |          | $\left. \begin{array}{l} 60 \text{ kg} \\ 100 \text{ kg} \\ 150 \text{ kg} \end{array} \right\} \text{Rockwell}$ $\left. \begin{array}{l} 15 \text{ kg} \\ 30 \text{ kg} \\ 45 \text{ kg} \end{array} \right\} \text{Superficial Rockwell}$ |  |

<sup>a</sup>For the hardness formulas given,  $P$  (the applied load) is in kg and  $D$ ,  $d$ ,  $d_1$ , and  $l$  are all in mm.

**Source:** Adapted from H. W. Hayden, W. G. Moffatt, and J. Wulff, *The Structure and Properties of Materials*, Vol. III, *Mechanical Behavior*. Copyright © 1965 by John Wiley & Sons, New York. Reprinted by permission of John Wiley & Sons, Inc.

## Rockwell Hardness Scales

| <i>Scale Symbol</i> | <i>Indenter</i>          | <i>Major Load (kg)</i> |
|---------------------|--------------------------|------------------------|
| A                   | Diamond                  | 60                     |
| B                   | $\frac{1}{16}$ -in. ball | 100                    |
| C                   | Diamond                  | 150                    |
| D                   | Diamond                  | 100                    |
| E                   | $\frac{1}{8}$ -in. ball  | 100                    |
| F                   | $\frac{1}{16}$ -in. ball | 60                     |
| G                   | $\frac{1}{16}$ -in. ball | 150                    |
| H                   | $\frac{1}{8}$ -in. ball  | 60                     |
| K                   | $\frac{1}{8}$ -in. ball  | 150                    |

## Superficial Rockwell Hardness Scales

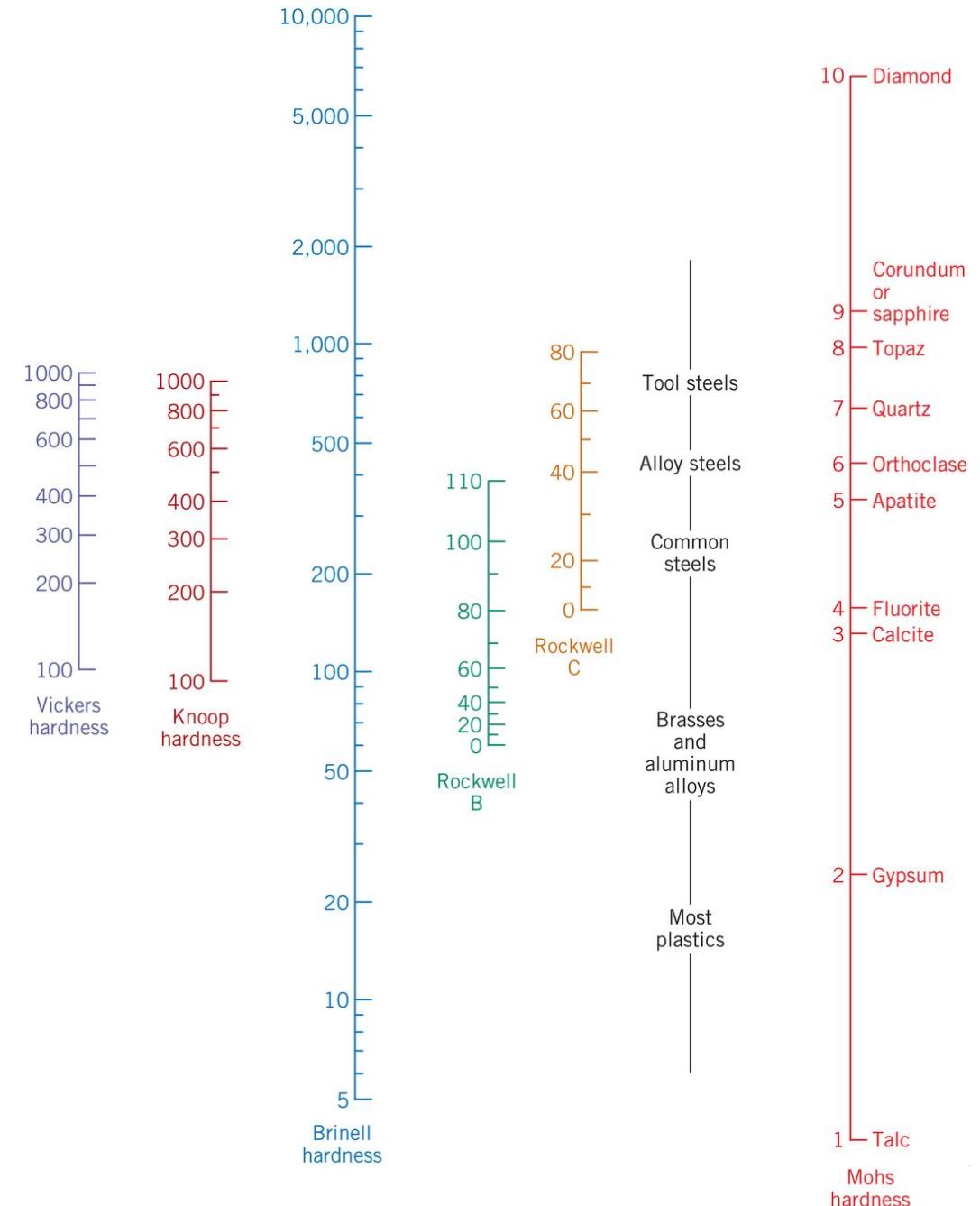
**Table 7.5b Superficial Rockwell Hardness Scales**

| <i>Scale Symbol</i> | <i>Indenter</i>         | <i>Major Load (kg)</i> |
|---------------------|-------------------------|------------------------|
| 15N                 | Diamond                 | 15                     |
| 30N                 | Diamond                 | 30                     |
| 45N                 | Diamond                 | 45                     |
| 15T                 | $\frac{1}{16}$ in. ball | 15                     |
| 30T                 | $\frac{1}{16}$ in. ball | 30                     |
| 45T                 | $\frac{1}{16}$ in. ball | 45                     |
| 15W                 | $\frac{1}{8}$ in. ball  | 15                     |
| 30W                 | $\frac{1}{8}$ in. ball  | 30                     |
| 45W                 | $\frac{1}{8}$ in. ball  | 45                     |

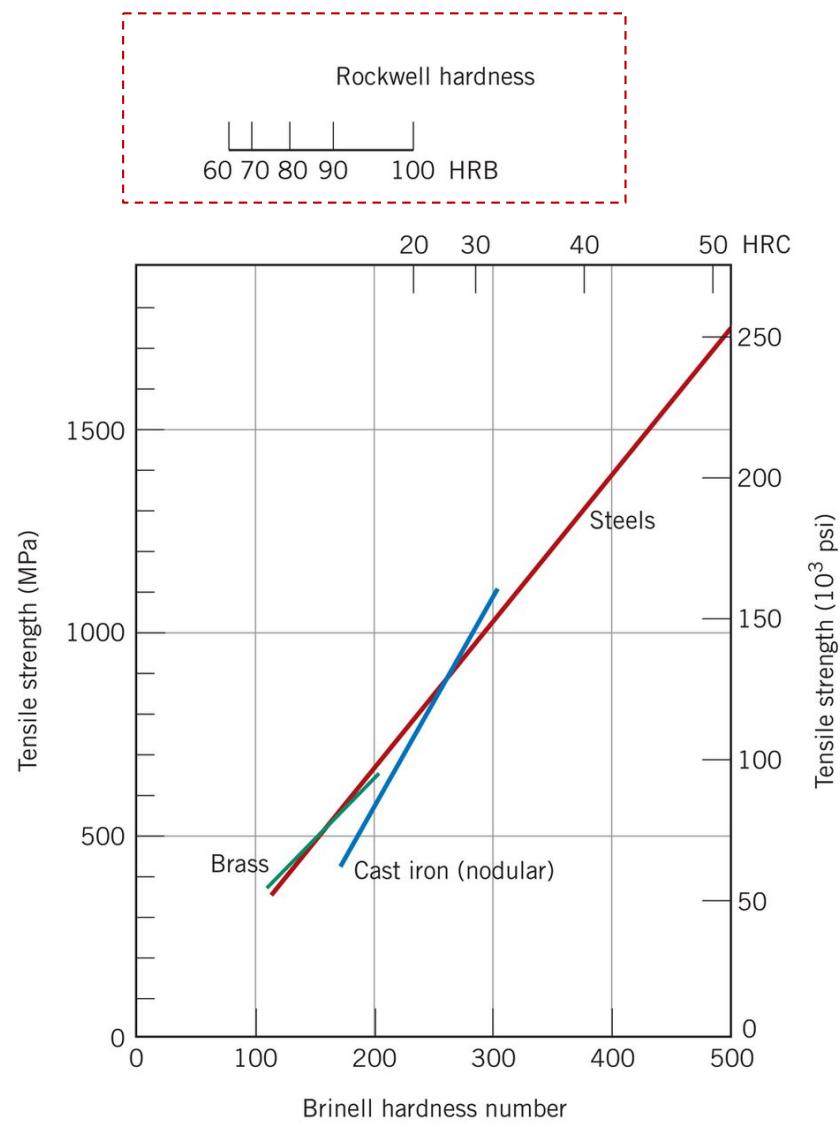
# Hardness Conversion

$$TS \text{ (MPa)} = 3.45 \times HB$$

$$TS \text{ (psi)} = 500 \times HB$$



Adapted with permission from ASM International, *ASM Handbook: Mechanical Testing and Evaluation*, Volume 8, 2000, pg. 936.



Data taken from *Metals Handbook: Properties and Selection: Irons and Steels*, Vol. 1, 9th edition, B. Bardes (Editor), American Society for Metals, 1978, pp. 36 and 461; and *Metals Handbook: Properties and Selection: Nonferrous Alloys and Pure Metals*, Vol. 2, 9th edition, H. Baker (Managing Editor), American Society for Metals, 1979, p. 327.

Relationships between hardness and tensile strength for steel, brass, and cast iron.

# Design/Safety Factors

- Because of design uncertainties allowances must be made to protect against unanticipated failure.
- For structural applications, **to protect against possibility of failure**—use working stress,  $\sigma_w$ , and a **factor of safety**,  $N$

$$\sigma_w = \frac{\sigma_y}{N}$$

yield strength

Depending on application,  
 $N$  is between 1, 2 and 4

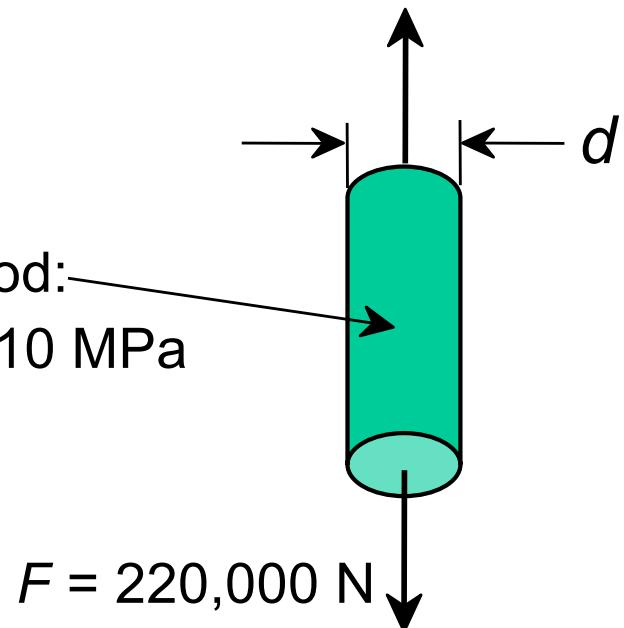
# Design/Safety Factors

## Example Problem:

A cylindrical rod, to be constructed from a **steel** that has a yield strength of 310 MPa, is to withstand a load of 220,000 N without yielding. Assuming a value of 4 for  $N$ , specify a suitable bar diameter.

$$\frac{220,000 \text{ N}}{\pi \left( \frac{d}{2} \right)^2} = \frac{\sigma_y}{N}$$

Steel rod:  
 $\sigma_y = 310 \text{ MPa}$



Solving for the rod diameter  $d$  yields

$$d = 0.060 \text{ m} = 60 \text{ mm}$$

A tensile testing apparatus is to be constructed that must withstand a maximum load of 220,000 N (50,000 lb<sub>f</sub>). The design calls for two cylindrical support posts, each of which is to support half of the maximum load. Furthermore, plain-carbon (1045) steel ground and polished shafting rounds are to be used; the minimum yield and tensile strengths of this alloy are 310 MPa (45,000 psi) and 565 MPa (82,000 psi), respectively. Specify a suitable diameter for these support posts.

$$\sigma_w = \frac{\sigma_y}{N}$$

$$= \frac{310 \text{ MPa}}{5} = 62 \text{ MPa (9000 psi)}$$

$$\sigma = \frac{F}{A_o}$$

$$\begin{aligned} d &= 2 \sqrt{\frac{F}{\pi \sigma_w}} \\ &= 2 \sqrt{\frac{110,000 \text{ N}}{\pi (62 \times 10^6 \text{ N/m}^2)}} \\ &= 4.75 \times 10^{-2} \text{ m} = 47.5 \text{ mm (1.87 in.)} \end{aligned}$$

$$A_o = \left(\frac{d}{2}\right)^2 \pi = \frac{F}{\sigma_w}$$

Therefore, the diameter of each of the two rods should be 47.5 mm or 1.87 in.

# Stress-strain curves of Metal

A material is loaded above its strength, it deforms plastically or fractures

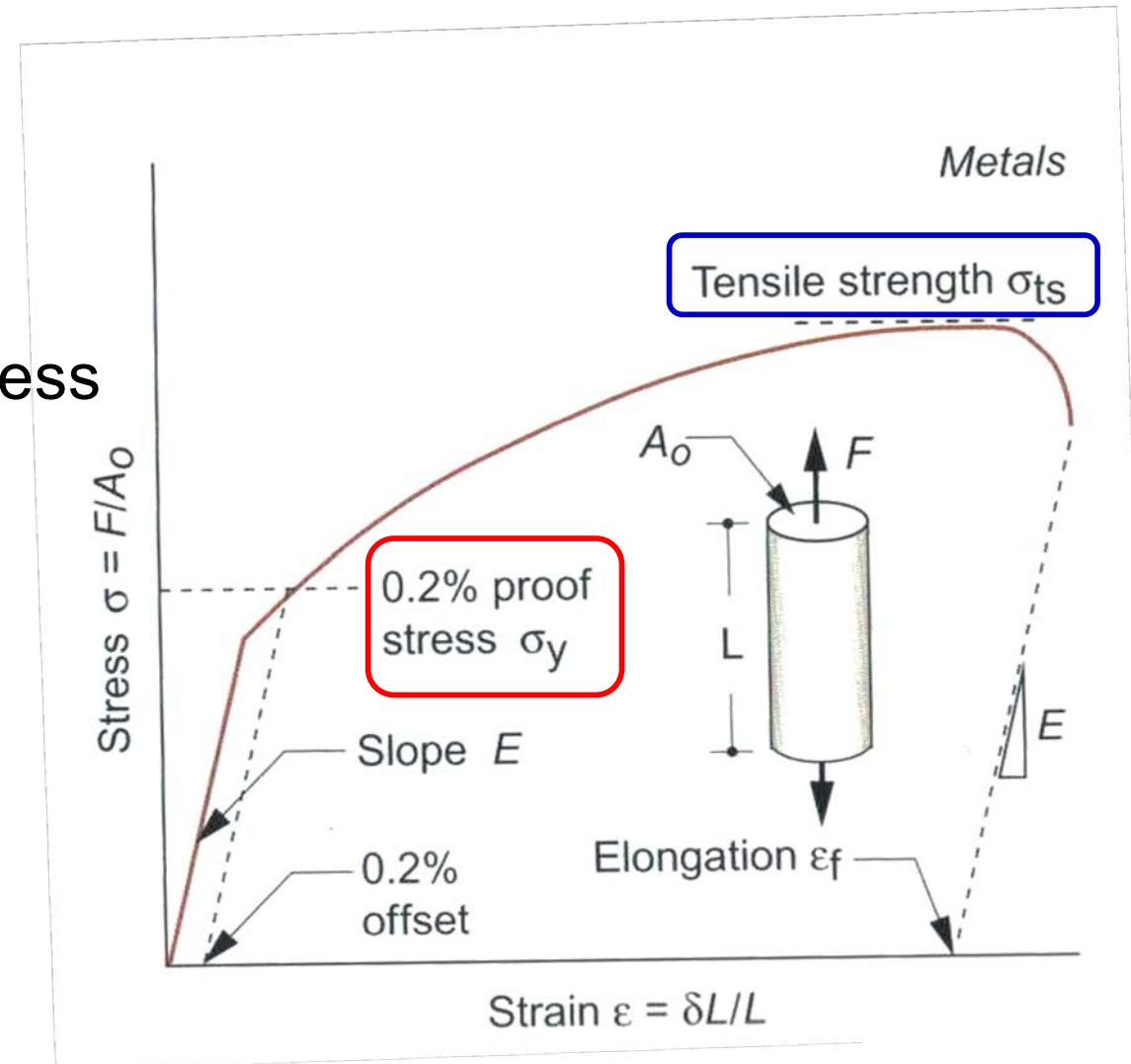
**Yield strength**

- 0.2% proof stress

$$\sigma_y$$

**Tensile strength**

$$\sigma_{ts}$$



# Stress-strain curves of Polymer

Yield strength - 1% strain

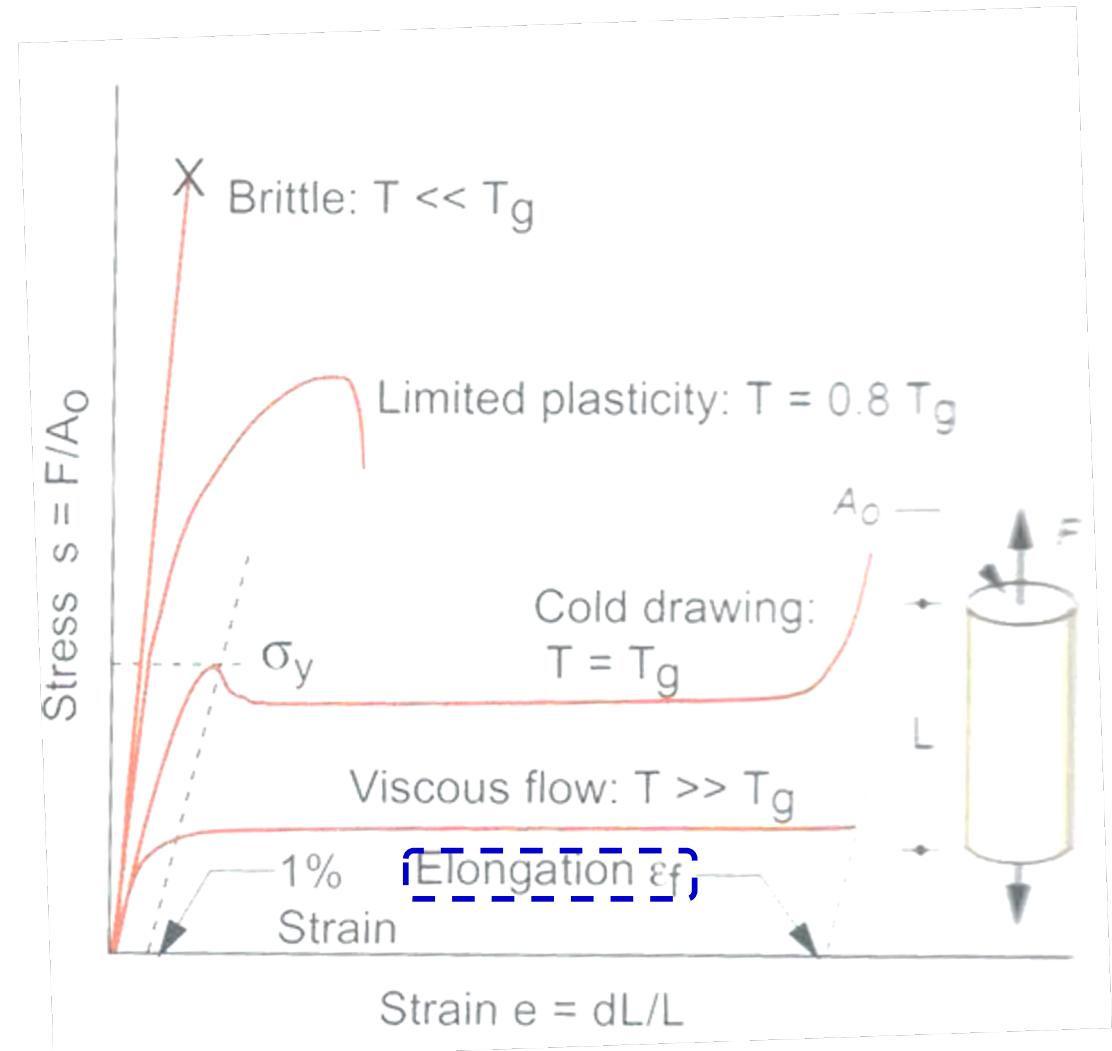
$$\sigma_y$$

Ductility

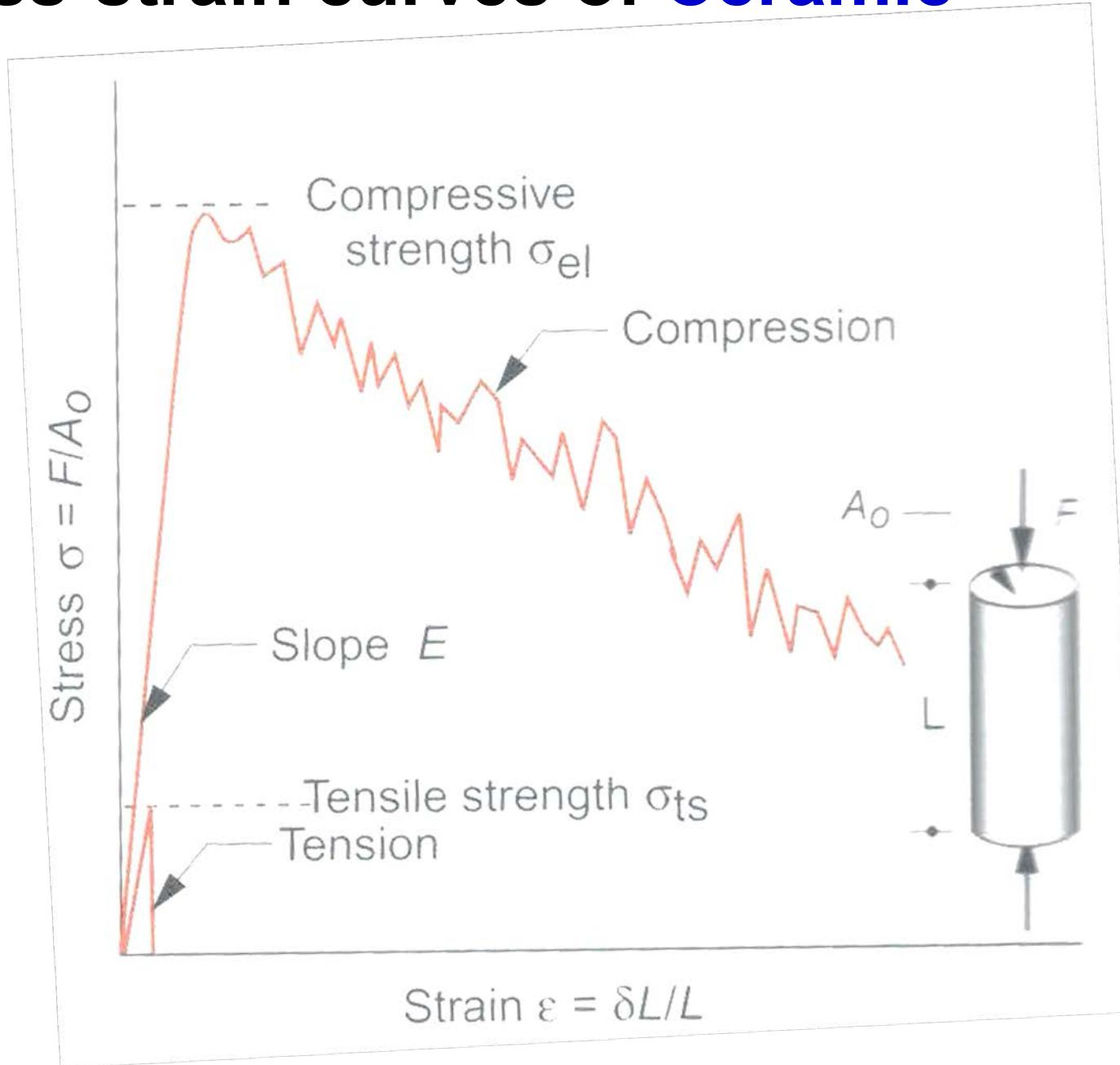
A measure of how much plastic strain a material can tolerate

$$\text{Elongation } \epsilon_f$$

the tensile strain at break



# Stress-strain curves of Ceramic



# Summary

- Applied mechanical force — normalized to **stress**
- Degree of deformation — normalized to **strain**
- **Elastic deformation:**
  - **non-permanent**; occurs at low levels of stress
  - stress-strain behavior is **linear**
- **Plastic deformation**
  - **permanent**; occurs at higher levels of stress
  - stress-strain behavior is nonlinear
- **Stiffness** - a material's **resistance to elastic deformation**
  - elastic (or Young's) modulus

# Summary (cont.)

- **Strength** - a material's resistance to plastic deformation  
yield and tensile strengths
- **Ductility** - amount of plastic deformation at failure percent  
elongation, reduction in area
- **Hardness** - **resistance to localized surface deformation**  
& compressive stresses
  - Rockwell, Brinell hardnesses