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# **Cascading Failures in Modern Power Systems**

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September 2018

## Abstract

Power systems today are undergoing extreme changes with the integration of renewable generation sources such as solar and wind. With these new technologies come new challenges. Traditional generation sources, such as coal and nuclear, provide inertial support, which helps maintain system stability when an outage occurs. However, solar and wind generation do not, on their own, provide this support, which potentially makes the system more vulnerable to failures. The effect on stability as power engineers introduce more renewables into the generation mix remains an open question.

I will investigate the impact of power system inertia on cascading failures in two ways: the first is by calculating the distribution of cascade size and its probability (explain what you mean there); the second is by analyzing the differences in the results of the same contingency with different inertial values. The latter is important to understand because even if the risk of large cascades does not change, knowing which contingencies lead to large cascades will alter how operators need to plan day-to-day operations and infrastructure updates. You need a sentence here explaining what distribution of cascade size and probability means and saying how you'll approach that. In order to understand the differences in particular contingency scenarios I will look at which lines actually failed, as well as the interaction among the components in the system, in particular the generators. I would like to explore a motif-driven analysis of the final steady-states of contingencies to determine if certain substructures occur more frequently.

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# Chapter 1

## Introduction

Electricity is used every day by the majority of Americans, but is largely consumed without thought. Often people notice electricity, and their consumption of it, when they no longer have access to it; perhaps there is a brief power surge, or even a total blackout, which reminds us that our ability to see in the dark is a fragile luxury, that we often take for granted. Blackouts, or cascading failures, are ubiquitous in power systems, because we use the power system as cost-effectively as possible [10]. The study of cascading failure dynamics in power systems started at the turn of the century, and draws researchers from a multitude of backgrounds such as electrical engineering, network science, and even cyber-security. The ultimate goal for many researchers is to predict and/or prevent blackouts as a way to ensure reliable access to power. However, as I hope becomes clear throughout this paper, I'll show that a better understanding of the underlying mechanisms and dynamics of cascading failures is not only an interesting and challenging problem, but also necessary for any hope in controlling them.

Any research regarding modern power systems must account for the changes occurring due to the reconfiguration of our power sources. We are no longer able to ignore the fact that there is a demonstrable need to replace fuel sources that contribute large amounts of greenhouse

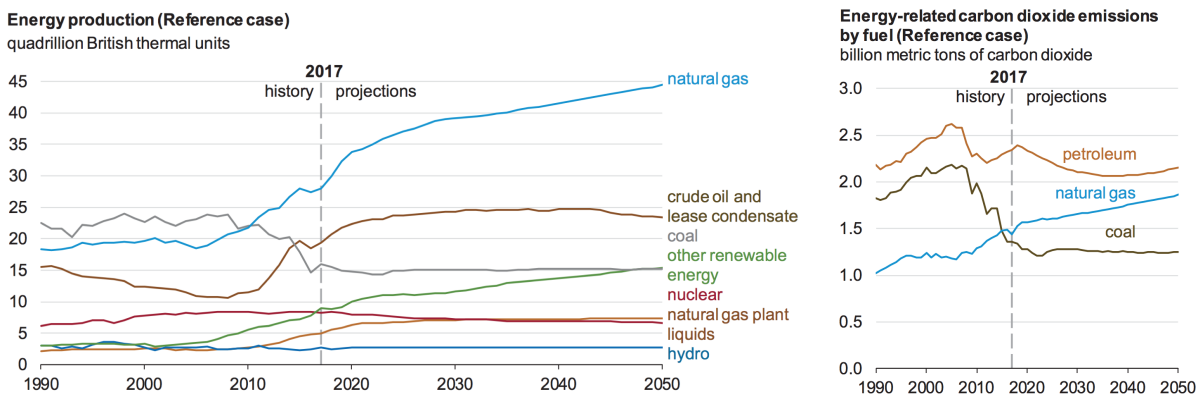


Figure 1.1: **Left:** Projected growth of fuel types, based on current trends in the energy market. **Right:** Amount of greenhouse gas emissions by energy fuel type, based on projected electricity consumption. Figures from the Energy Information Administration 2018 Annual Energy Outlook [3].

gas emissions. According to the Environmental Protection Agency, electricity is one of the largest contributors of greenhouse gases in the US, only slightly below transportation [20]. Up until very recently, the majority of these emissions were due to a significant use of coal as a fuel source for electricity, seen on the right side of Figure 1.1. However, based on the lowering price of natural gas and renewable resources such as wind and solar photovoltaics (PV), it is expected that renewable resources will contribute at least as much as coal by the year 2050<sup>1</sup> (see left panel of Figure 1.1) [3]. If protecting the environment is not enough reason, consider that the cost of For these reasons, it has become more and more important to consider how renewable resources like wind and solar PV alter how our electricity infrastructure functions.

<sup>1</sup>Note: These projections do not consider the Clean Power Plan implemented in 2014.

# Chapter 2

## Power Systems Background

### 2.1 Structure and Organization of Power Systems

It is important to note that some power system studies are dependent on which power system is being studied; other studies pursue a more general understanding of power systems as a whole. In this section we'll discuss characteristics of power systems, with a focus on the United States power system; however, most of these concepts are universal to all power systems throughout the world.

There is one rule that allows a power system to function: power generation and power demand must be instantaneously and continuously balanced. This fact, while simple and concise, is difficult to achieve in practice. Many of the rules and regulations surrounding power systems are to ensure that balance is maintained, or what procedures to follow should an imbalance occur. In general, power systems are divided into transmission systems and distribution systems. The regulations and procedures in part determine the physical structure of a power system, but it is also dependent on engineering choices and how power systems grew historically in a country. In general, there are two main components of a power system: the



distribution system and the transmission system. Distribution systems tend to be tree-like network structures that deliver power to homes and commercial buildings. Transmission systems are highly interconnected networks that exchange power between large regions. The purpose of having a large system is that it makes it easier to balance power across multiple areas. Due to the large areas that transmission systems cover, it is more efficient to use AC power, which allows for larger amounts of power for the same voltage level in DC power. However, for safety reasons, distribution systems usually use DC power. The mathematical distinction between AC and DC power will be discussed in Section 2.2.

## 2.2 Power System Equations

A fundamental part of any power system study is a definition of how power flows from the sources to the sinks in the system. These are referred to as the power flow equations (PFE), and they describe the the output power of every generator (sources) and the power flow on every line given the consumption of power at every bus. For simplicity in our derivation, we assume that the power system we are modeling does not contain any capacitive effects, nor does it contain any transformers. In particular, assuming zero capacitive effects means that current is conserved across the lines, and simplifies the power flow equations. The derivation we present here is adapted from [5]. (ending sentence here)

### 2.2.1 Power Flow Equations

First, we begin with some notation to represent the various components in the system. A power system is represented by a set of buses (or nodes) and lines (edges). Let the set of all buses in the system be represented by  $N$  where  $|N| = n_b$ . In a power system, buses can be generators, loads, or intermediaries. Let the set of generator nodes be represented by  $G$ , where  $|G| = n_g$  and the set of load nodes by  $L$  where  $|L| = n_l$ . Note that  $G$  and  $L$  are

subsets of  $N$ , and a bus in  $G$  can also be in  $L$ . Intermediary buses are buses that do not consume or produce power, but instead help transmit power from the sources to the sinks. Intermediary buses are all buses that are not in  $G$  or  $L$ . Finally, when we refer to lines in the system, we will denote this with a subscript of  $ij$ , which indicates the line that connects bus  $i$  to bus  $j$ .

To begin our derivation, we start by defining the *impedance*,

$$z_{ij}^* = r_{ij} + jx_{ij}, \quad i, j \in N \quad (2.1)$$

where the real part  $r_{ij}$  is the resistance of the line, and the imaginary part  $x_{ij}$  is the reactance of the line. Impedance describes the opposing force of the line to the alternating current (AC) power. It quantifies how difficult it is for power to flow through the line. Often, lines are instead quantified by their *admittance*,

$$y_{ij} = \frac{1}{z_{ij}} = g_{ij} + jb_{ij}, \quad i, j \in N \quad (2.2)$$

where  $g_{ij}$  is the conductance, and  $b_{ij}$  is the susceptance of the line. Using Ohm's law, we can describe the complex current injected into a line by Equation 2.3

$$I_{ij} = y_{ij}(V_i - V_j), \quad i, j \in N \quad (2.3)$$

where  $I$  is the current and  $V$  is the voltage. To write the complex power equation across a line, we introduce  $\theta_i$  as the voltage angle at bus  $i$ , and  $\theta_{ij} = \theta_i - \theta_j$ . Power is equal to the current times the voltage of a component, so the complex power across line  $ij$  is

$$S_{ij} = V_i I_{ij}^* = y_{ij}^* |V_i| e^{j\theta_i} (|V_i| e^{-j\theta_i} - |V_j| e^{-j\theta_j}) \quad (2.4)$$

Often, power flow equations are expressed in polar coordinates, and complex power can then be separated into a real and imaginary part. The real component represents the active power (denoted as  $P$ ), and the imaginary part is the reactive power (denoted as  $Q$ ). Physically speaking, active power is consumed by resistive components (like your phone charger), and reactive power is consumed by reactive components (like your air conditioner). Then, the real and reactive power across a line are defined by

$$P_{ij} = |V_i|^2 g_{ij} - |V_i||V_j|g_{ij} \cos(\theta_{ij}) - |V_i||V_j|b_{ij} \sin(\theta_{ij}) \quad (2.5a)$$

$$Q_{ij} = -|V_i|^2 b_{ij} + |V_i||V_j|b_{ij} \cos(\theta_{ij}) - |V_i||V_j|g_{ij} \sin(\theta_{ij}) \quad (2.5b)$$

Using Kirchoff's law, the real and reactive power at a node is defined as the sum of the real and reactive power of the lines connected to the node.

$$P_i = \sum_{j=1}^N P_{ij} \quad (2.6a)$$

$$Q_i = \sum_{j=1}^N Q_{ij} \quad (2.6b)$$

Equations 2.5a - 2.6b describe the AC power flow equations (ACPFE) in their entirety. However, it is extremely common to linearize these equations, this is known as the DC approximation. For this approximation, the following assumptions are made:

- $|V_i| \approx 1, \quad \forall i \in N$
- $|\theta_{ij}|$  is small such that  $\sin(\theta_{ij}) \approx \theta_{ij}$
- For every line, the resistance is much smaller than the reactance, which means  $g_{ij} \approx 0$  and  $b_{ij} \approx \frac{-1}{x_{ij}}$

Using all of these assumptions, the real power flow across a line becomes  $P_{ij} = -b_{ij}\theta_{ij}$ . Lastly, in the DC approximation we assume the reactive power consumption and production is constant. This, along with the assumptions above, is generally what is meant by “DC approximation” in cascading failure literature in particular.

To solve the system of nonlinear equations a few parameters need to be given *a priori*. Usually, the real power ( $P_i = P_g$ ), and voltage magnitude ( $|V_i| = |V_g|$ ) of each generator is given, which is why they are often referred to as “PV nodes” [9]. Every load node usually has its real power ( $P_i = -P_l$ ) and reactive power ( $Q_i = -Q_l$ ) specified, which is why they are often referred to as “PQ nodes”<sup>1</sup>. Finally a reference node, often called the “slack” node, is defined for the system and its voltage angle is set to zero ( $\theta_i = 0$ ) and is used as a reference angle for every other bus in the system. This node is required because we cannot know all power outputs prior to solving the equations because it depends on the steady-state of the system [47]. Generally, the slack node is chosen to be the largest generator in the system because it will usually have sufficient real power to compensate for any deficits [9].

It is extremely common to only use the power flow equations to study cascading failures for multiple reasons. A large reason is that the addition of other dynamics increases the computational complexity, which can be prohibitive when studying a large number of outage scenarios. However, it is becoming more common to also include generator dynamics when considering cascading failures, because it impacts the stability of the system, and as we will see in Chapter 3, it can change the severity of a cascade. This next section outlines the necessary equations for modeling generator dynamics.

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<sup>1</sup>Note that the negative here means that real and reactive power are consumed at these nodes

### 2.2.2 Generator Dynamics

Generators in power systems are usually described as phase oscillators, and often differences in power system stability studies differ by how loads are represented (*i.e.* whether they are modeled as oscillators or not, and what kind) [47]. Here, we describe the *structure preserving* model, which models load nodes as oscillators, or in other words, the consumption of power at load nodes is dependent upon the frequency of the generators. Regardless of the model being used, all generator dynamics are derived based on Equation 2.7 which defines the fundamental equation of motion for a generator, and is referred to as the *swing equation* in the literature.

$$M_g \dot{\omega}_g + D_g \omega_g = P_{m_g} - P_{e_g}, \quad (2.7)$$

$$\dot{\delta}_g = \omega_g - \omega_R, \quad \forall g \in G$$

$M_g$  and  $D_g$  are the inertia and damping constants of the generator, and are based on the type and mechanical specifications of the generator. The inertia parameter  $M_g$  can also be rewritten such that it has units of seconds, and can be directly related to the kinetic energy stored in the rotor of the generator

$$M_g \omega_r \pi = S_g H_g = K E_g \quad (2.8)$$

where  $S_g$  is the rated power of the generator. The rotor angle,  $\delta_g$ , and frequency,  $\omega_g$ , of the generator are defined with respect to a rotating reference frame with a frequency of  $\omega_R$  (60 Hz in the U.S. and 50 Hz in Europe).  $P_{m_g}$  is the mechanical power input to the machine's rotor, and  $P_{e_g}$  is the electric power demanded by the rest of the system. For the structure preserving model and using the DC approximation assumptions, the equation of motion for generators is represented as a second order Kuramoto oscillator [1].

$$\dot{\omega}_g = -\frac{D_g}{M_g}\omega_g - \frac{1}{M_g}(P_g + \sum_{\forall i \notin G} b_{gi} \sin \delta_{gi}), \quad \forall g \in G \quad (2.9)$$

The load nodes (and all non-generator nodes, assuming they all have a frequency dependence) is then modeled by Equation 2.10

$$\dot{\delta}_i = -\frac{1}{D_i}(P_i + \sum_{\forall j \in N} b_{ij} \sin \delta_{ij}), \quad \forall i \notin G \quad (2.10)$$

where  $D_i$  is the damping term that describes the frequency dependence of the node.

More detailed descriptions of generator dynamics include the governor, which is responsible for controlling the generator frequency, and the excitation system, which controls the internal voltage magnitude of the generator. However, the majority of cascading failure models discussed in Chapter 3 do not consider these systems, and we will explicitly note when they do, so we will not discuss their equations here, and instead point the reader to the publications.

## 2.3 Understanding Power System Performance and Health

In North America, utility companies provide different system analyses to a variety of organizations. This organizational structure is intended to promote safe and reliable flow across North America as more systems became connected to each other after growing independently with various operating conditions. The intent of federal regulation is to create a uniform expectation across all power systems that hopefully makes it easier to have these systems interact with one another. If power systems are not properly connected, failures can propagate throughout the system and cause severe damage, resulting in high financial costs.

Several entities are involved with controlling, regulating, and operating our power systems

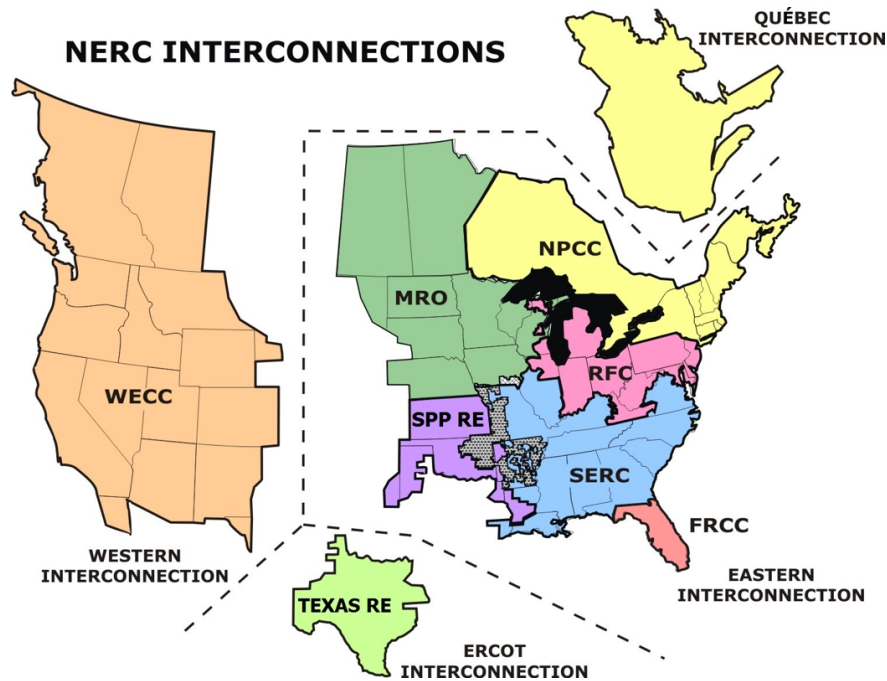


Figure 2.1: The interconnections of North America. Interconnections have independent frequencies because they are connected with DC lines. Image from [43].

so they are safe, secure, and reliable for consumers and producers. In 2005, Congress gave the Federal Energy Regulatory Commission (FERC) the task of overseeing reliability of electric power systems and the power to appoint an organization to perform these tasks [15]. Prior to this, federal involvement in power systems only consisted of price regulation. In 2006, FERC gave the North America Electric Reliability Corporation (NERC) the authority to regulate the reliability and security of the bulk power system in North America [42]. All electric utility companies must follow NERC statutes to ensure proper coordination of power flow. While NERC regulates the entire North American system, it can be broken up into several interconnections where utilities in these interconnections are electrically tied together, as shown in Figure 2.1. These interconnections operate mostly independently of one another. Within each interconnection exist Independent System Operators (ISO) and Regional Transmission Organizations (RTO). Both of these entities have similar functions, but RTOs have greater responsibilities outlined by FERC. ISOs operate their region's grid, oversee the wholesale electricity market, and give reliability planning for the entire region.

RTOs do this and also ensure fair transmission access for utilities [18]. Sometimes, ISOs and RTOs can also be the Balancing Authorities (BA) in their region. BAs are in charge of maintaining the balance between load and generation within their specific areas. Within each balancing area reside transmission operators, which are in charge of operating their local transmission systems. Finally, distribution providers control how power flows from transmission operators to end users.

Reliability, stability, and security are key issues for all of these players. These measures of performance are conducted by each power system entity to determine system risks, upgrades, and instabilities. The following sections describe these measures, and we pay particular attention to stability, which is most impacted by the addition of renewable generation to the system.

### 2.3.1 Reliability

Reliability is generally defined as the probability that an item will perform a required function under a specified state [11]; a power system's reliability is defined by its ability to maintain an equilibrium between power demand and generation under multiple environmental conditions. Reliability is the general goal of FERC [6].

A common goal of reliability studies is to determine the likelihood that a component or system will fail. Depending on modeling goals and assumptions, some cascading failure simulations can provide these type of statistics. For example, one study found that distributed generation sources in a distribution system can increase the reliability especially during islanding, when a system splits into multiple components, but when these distributed sources come from renewable energy the benefits from the distributed generation can be reduced or even negligible [8]. Essential modeling techniques in reliability analysis include fault trees, Markov models, and Monte Carlo simulations [6]. Some cascading failure models use proba-



bility of failure established by reliability models [13], whereas some reliability models can use cascading failure models to determine the likelihood of failure [29]. These types of studies are closely related but cascading failure simulations model how blackouts occur whereas reliability analysis quantifies the likelihood of failure under specific conditions. Reliability is a key concern for power systems, and often, cascading failures occur when various components fail, i.e. become unreliable.

### 2.3.2 Stability

Broadly speaking, a power system's stability is its ability to return to an equilibrium state following a disturbance [35]. The more stable the system is the less likely it will propagate failure. Although instabilities can *precede* cascading they are more likely to occur *during* a cascade. For example, an analysis of blackout data shows that most were a result from untrimmed vegetation, followed by failure of protection equipment [19].

There are generally three categories of stability: voltage stability, frequency stability, and rotor angle stability. First, we introduce voltage stability, which is the system's ability to maintain bus voltages at an acceptable. Voltage stability depends on the system's ability to restore the equilibrium between load supply and load demand, and the driving force tends to be the loads [2]. One particular type of voltage instability that can lead to cascading failures is Fault-Induced Delayed Voltage Recovery (FIDVR). This occurs after a fault has been cleared in the system which leads to low voltages. These low voltages raise the reactive power requirement of motors, particularly air-conditioning units. When the reactive power cannot be met, the AC units stall, causing them to need 5-6 times the pre-contingency reactive power, this decreases the voltage further and can cause cascading failures [40]. In particular, this can cause voltage collapse, which is when the voltage of the nodes start to drop slowly and then quickly until the system cannot recover. These scenarios can cause serious damage to equipment and loss of power for a large population if they are not handled

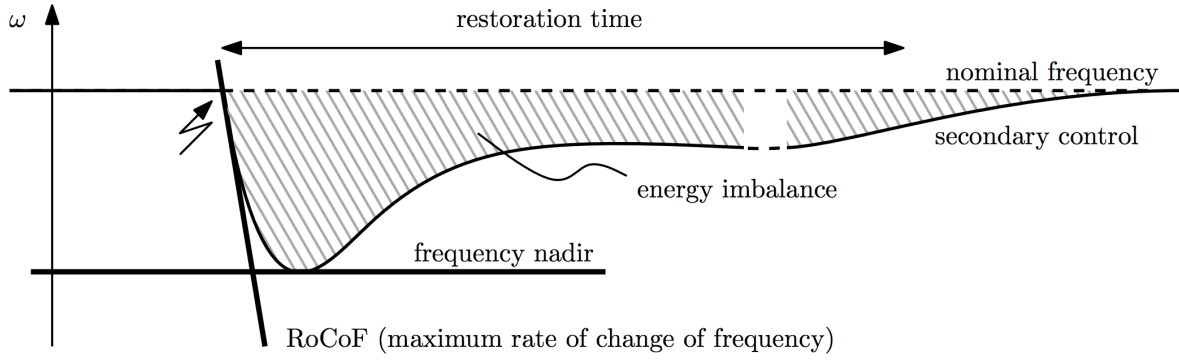


Figure 2.2: An example of a frequency drop. Figure from [23].

properly.

Next, we define frequency stability and rotor angle stability, which are dependent upon generator dynamics. Frequency stability is when the power system maintains a frequency (50 or 60 Hz) within a specified margin (usually  $\pm 0.1$  Hz) after an upset that causes an imbalance between load and generation. An example of a frequency drop is shown in Figure 2.2, with relevant terminology labeled. Large imbalances in load and generation usually occurs due to improper responses of equipment [35]. An example of this occurred in August 2016 when a fire in Southern California caused a fault that exposed improper inverter settings causing unnecessary tripping of equipment for low frequency detection. This in turn caused the frequency of the system to actually drop, which caused the system to lose 1200 megawatts of PV power [41]. The frequency of a system is generally referred to as the center of inertia, and is computed based on the rotor speeds and inertia constants of the generators in the system [37]. This is defined by Equation 2.11

$$\omega_{COI} = \frac{\sum_{g=1}^{n_g} M_g \omega_g}{\sum_{g=1}^{n_g} M_g} \quad (2.11)$$

However, in practice, system frequency is usually measured at some relevant bus in the

system. This is a less accurate measure of system frequency as it is influenced by generators that are closer to the bus, rather than the average frequency of the whole system. Thus, the center of inertia frequency is largely used in simulations. Closely related to frequency stability is rotor angle stability, which refers to a generator's oscillations. When a generator becomes out of sync with other generators due to an instability, usually rebalancing forces from other generators restore it to the proper oscillation speed and phase. This is dependent on the system's ability to restore the equilibrium between the mechanical and electromagnetic torque of the generator. The system becomes unstable when it cannot absorb the kinetic energy caused from the differences in rotor speeds [34].

Frequency stability and rotor angle stability have become a prominent research area the past decade due to the increase in renewable generation sources.

The study of power system instabilities is concerned with identifying the system's *response to* instability. Instabilities are usually involved with cascading failures (such as hidden failures of protection equipment) but may not be the primary cause. Some cascading failure models use various instabilities as a reason for line tripping, thus furthering a cascade, but because of the complexity required, very few employ these dynamics explicitly. Modeling the instabilities themselves is generally thought of as separate from modeling cascading failures because of the level of detail (i.e. ACPFE, DAE) required for instability analysis.

### 2.3.3 Security

A power system's ability to handle instabilities partly determines its robustness, but two systems that are equally stable might not be equally secure as one could have more severe consequences from a failure [24]. NERC requirements determine how secure a power system must be under various types of operating conditions. A canonical regulation requires all transmission systems to be  $n - 1$  secure which means if any one component fails, power can

still flow to all loads [39]. However, there are less security requirements for  $n-k$  contingencies because they are numerous and it is hard to determine which sets of these contingencies have higher risk and should be prevented. Overall, a power system's security measures its ability to maintain reliability and stability due to a specified perturbation.

Reliability, stability, and security analyses have suggested improvements for increasing the resiliency of power systems. But, cascading failures are still inevitable despite our best efforts to avoid them. Understanding how failure propagates through an electrical system and what factors increase the likelihood and size of the blackout is the concern of cascading failure modeling. In the next section, we will review various cascading failure models which highlight various properties of power systems and determine why blackouts are inevitable in modern power systems.

# Chapter 3

## Modeling and Analysis of Cascading Failures

### 3.1 Models

Cascading failures (CF) are a ubiquitous phenomenon across most networks. In social networks, biological networks, or neural networks, cascading failures spread linearly, or through adjacent nodes. However, in power systems or other flow networks, cascading failures spread non-locally, in that when one edge or node is removed, the next removed component is not necessarily connected to the most recently failed component. This is a consequence of the power flow equations discussed in Chapter 2.

Cascading failure models can be generally classified based on which dynamics are modeled and also by how those dynamics are modeled. In general, almost all CF models choose line overloads as the representative failure. Overload failures occur when a component in the system has some capacity for transmitting flow, and the dynamics of the system cause the flow on the component to exceed its capacity. However, other types of failures, more specific

to power systems, are also explored in the CF literature. For example, hidden failures occur when a protective component of the system operates at an incorrect time or fails to operate when it is necessary [13]. For the models described in the rest of this section, most of the failures that are represented are overload failures, but some do consider generator or frequency failures and will be discussed when appropriate.

We will cover three basic categories of CF models relevant to this thesis work: pure topological models, models that only consider power flow, and models that consider transient and power flow dynamics. Topological models take the simplest approach, as they do not model any power system dynamics, while power flow and transient dynamics based models contain more dynamical details.

### 3.1.1 Topological Models

We use the term Topological Models here to describe cascading failure models that do not inherently describe power flow dynamics. Ever since Watts and Strogatz [55] asserted that power systems are “small-world” networks based on their characteristic path length and clustering coefficient, the network science community has given considerable attention to power systems and cascading failures because “small-world” networks tend to have larger propagations of failure due to their topological properties<sup>1</sup>.

Ever since Watts produced a canonical model for cascades on networks, the contagion model, first described in [54], spreading phenomena has become a ubiquitous research area in the network science community [30][45][46]. The contagion model is a linear spreading model, where nodes can only become infected if neighboring nodes are infected. In power systems,

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<sup>1</sup>There has been some disagreement on topological properties of power systems. Power systems have been shown to have exponential degree distributions in some cases, and power law degree distribution in others [4][12]. It is unsurprising that different power systems made with different engineering choices would have different network characteristics, but even studies of the same power system have yielded different network properties [26][55].

neighboring nodes of failed components are usually not the next components to fail, and a large amount of research shows that the spread of failures in a power system is inherently non-local [29][56]. Part of the problem with the contagion model is it assumes information travels on all paths equally. In power systems, two paths of equal length that connect a pair of nodes are not necessarily electrically equivalent; power may not flow equally between them. Additionally, due to non-linear interactions in power systems, there are no modeling assumptions or changes in the contagion model that could simulate cascading failures that occur in power systems [22].

Despite these drawbacks, purely topological models have found similar insights into CF as the power flow and transient dynamic models. In [31], the authors found that a small number of nodes in the Western Interconnection can cause a total collapse of the system. The authors concluded that upgrades to these components could significantly reduce the risk of CF for the system. Motter et. al [38] describe a model in which components have a load defined by the number of shortest paths that go through the component. The capacity for each component is based on the initial topology of the network, plus an additional fraction. When nodes are randomly removed, this causes the load to redistribute (as the shortest paths between two nodes might have changed), and can cause further failures. Out of all the topological models used, this one is most closely related to the power flow based CF described in the next section. However, because topological models do not use any power flow dynamics (and assume all paths are equal), these models are less useful for identifying specific upgrades to the system, and are more well-suited for understanding the fundamental drivers of CF [27]. Therefore, most CF models used today fall into the next two categories described below.

### 3.1.2 Power Flow Models

A significant number of CF models fall under the power flow based model category. This is because they are the simplest to model, while maintaining some sense of reality. The most prominent of these is the OPA model [17]. OPA is run over many “days”, where each day the following occurs: (I) the initial power flow of the system is solved, using the DC approximation, (II) an initial set of lines are removed at the beginning of the day based on a uniform probability, (III) power flow is re-solved with these lines removed, if any line is overloaded after resolving, it is removed with some probability, and the cascade continues in this way until there are no longer overloaded lines. At the end of each “day” all lines that failed due to overload are upgraded, in that their capacity is increased, and the load that needs to be served is increased by some fraction. The important concept that followed from this model is that power systems are consistently operated at a critical level. When the system limits are pushed too far, failures occur, resulting in system upgrades. However, these upgrades allow for more power to be served for the increasing demand, which inevitably will result in more failures, causing the system to again be in a critical state<sup>2</sup>. OPA was the first model to introduce the idea that by running a power system in the most economical fashion (using all components at their maximum capacity, when necessary), also pushes it towards more critical regions.

A similar model to OPA is introduced by Hines et. al, called DCSIMSEP [28]. In this model, rather than probabilistic failure, specific  $n - k$  contingencies are studied. This model uses a standard re-dispatching technique for generators when the system becomes islanded, and additionally models protection equipment in a more detailed fashion. DCSIMSEP identifies a minimal set of lines that result in a CF using a random chemistry algorithm. A set of line failures is considered minimal if all lines in the set must fail for a blackout to occur. Power flow based models are the most prominent models for CF analysis because it more

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<sup>2</sup>Critical here does not necessarily mean imminent failure, rather it means the system is near the edge of a transition, and a small perturbation, such as a line failure, could push it towards an undesirable state.



accurately describes the dynamics of the system, and are less computationally expensive than the transient and power flow models.

### 3.1.3 Transient and Power Flow Based Models

Fewer models of CF exist that include transient dynamics. This is due to the higher complexity of solving not only the power flow equations, but also the differential equations associated with generators (Equation 2.7). However, these models are perhaps the most important when we start to consider the effects of adding large amounts of renewable generation to power systems, as the interactions among components will inevitably change. Power flow based models do not describe these effects and thus are inadequate when we consider this question.

COSMIC [53] takes as input a sequence of discrete events, such as line outages, to apply to the system. The model first initializes and solves the differential and algebraic equations for the system. Then the exogenous events are processed. If the system separates into two islands then these two systems are resolved for and integrated separately. If the system stays connected then the admittance matrix is updated and the ACPFE are recomputed. The simulation is then integrated until the end simulation time is reached or a discrete threshold, such as percent load shed or power lost, is crossed. During this integration, lines that exceed limit rates are tripped, and generators that exceed stability limits are disconnected. **Summary sentence here!!**

**Clean up this section!** The model we plan on using in this research is a phase transition model that was first presented by [16] and then adapted by [59]. This model uses the generator swing equation (2.9) and load equation (2.10), along with the DC power flow equations. However, in order to create a continuous dynamical system, line overloads must be represented by a continuous variable, rather than a discrete one as is used in the COSMIC model. Let  $\eta_{ij}$  represent the line status, where  $\eta_{ij} \approx 1$  is on, and  $\eta_{ij} \approx 0$  is off. Let  $c_{ij}$  represent the

fractional amount of flow on a line, such that  $c_{ij} = 1$  means the line has the maximum amount of flow, more specifically  $c_{ij} = \frac{b_{ij}(1 - \cos \delta_{ij})}{X_{ij}}$  which defines the fraction of the reactance energy stored in the line. In order to model the automatic removal of a line, the derivative of the line status is defined as  $\dot{\eta}_{ij} = f(\eta_{ij}) - c_{ij}$  where  $f(\eta_{ij}) = \frac{1}{a}(\frac{1}{\eta_{ij}} - \frac{1}{1-\eta_{ij}}) + a\eta_{ij} - b$  is a function that has three equilibriums: a stable equilibrium at  $\eta_{ij} \approx 1$ , a stable equilibrium at  $\eta_{ij} \approx 0$ , and an unstable equilibrium at which point  $\eta_{ij}$  will evolve to 0. In practice,  $f(\eta_{ij})$  could be any function with the desired equilibriums, in fact in [16] they use a totally different function, but we will remain consistent with the model presented in [59] for this work. Thus, the final equation describing the evolution of the line status is

$$\dot{\eta}_{ij} = 10(f(\eta_{ij}) - \frac{b_{ij}(1 - \cos \delta_{ij})}{X_{ij}}) \quad (3.1)$$

Equation 2.9 remains unchanged, but we modify the load voltage angle equation by incorporating the line status variable

$$\dot{\delta}_i = -\frac{1}{D_i}(P_i + \sum_{j=1}^{n_b} b_{ij}\eta_{ij} \sin \delta_{ij}) \quad (3.2)$$

What makes the continuous phase-space transition model appealing over other models is that these equations can be derived from a Hamiltonian-like system with the energy function

$$\begin{aligned} \Psi(\mathbf{x}) = & \sum_{g=1}^{n_g} (\frac{1}{2} M_g \omega_g^2 + \sum_{j \notin G}^{n_b} b_{ij}(1 - \cos \delta_{ij})) \\ & + \sum_{i \notin G}^{n_b} \sum_{j \notin G}^{n_b} b_{ij}(1 - \cos \delta_{ij})\eta_{ij} - \sum_{\forall i,j \in E}^{|E|} X_{ij} F(\eta_{ij}) \\ & + \sum_{i=2}^{n_g+n_b} P_i \delta_i \end{aligned} \quad (3.3)$$

where the first line represents the kinetic energy provided by the generators and the reactive

energy of the lines connecting the generators to the system, the second term is the reactive energy on all other lines subtracting out the total possible reactive energy, and the last line represents the real power at every node in the system.

## 3.2 Types of Analysis

Cascading failure analysis is generally trying to answer three broad questions: (I) how can we build a more resilient system, or what about how it is built effects the cascading failure dynamics? (II) Due to the large number of potential failures that could occur in the system, which ones should we care about? (III) How do system components, and their interactions, effect the size of a cascade? We will briefly cover the results surrounding these questions and identify key areas where more research needs to be done.

### 3.2.1 Topology and CF Dynamics

However, the structural properties alone do not govern all the dynamics of the system. The way power flows in the system is extremely dependent on how each of the components are made.

With this in mind, [25] defines a power grid’s adjacency matrix in terms of its electrical properties, which is encompassed in the impedance matrix or the inverse admittance matrix (Section 2.2). With this, the network has a scale-free structure, that is there are a few “electrically central” nodes that most of the power flows through. Furthering this research, [27] uses various network measures to successively weaken test systems. The measures included betweenness centrality, maximum flow loss, minimum flow loss, and degree centrality. Each component is removed based on its centrality measure; this allows a comparison of these “attack strategies” to determine which measure indicates vulnerability. However, vulnerability

can be defined in multiple ways. In [27] the authors explore blackout size, connectivity loss, and increase in path length as three potential vulnerability measures. Depending on which vulnerability metric is used, the best attack strategy differs. This implies that topological methods alone cannot provide enough detail on cascading failures.

Topological information has been shown to be crucial when we consider islanding, restoration, or system upgrades [48][58]. For example, in [49] a DC model with a “small-world” network generator is used to understand the importance of grid topology to islanding. Using a similar cascade model as OPA, the authors found that increasing “short-cut” links in the grid increases how much load can be served, but also makes the grid more susceptible to islanding. This helps explain why transmission systems are built with redundant lines. Furthermore, stability studies using Kuramoto oscillator models have shown that nodes in “dead-ends” of the network topology are more likely to become unstable during perturbations [36]. These models use reasonably accurate formulations of cascade propagation that are not contained in the topological models previously discussed.

### 3.2.2 $N - k$ Contingency Analysis

Current standards require utilities to ensure  $N - 1$  security of the power system, so a failure of any single component should not result in other failures [39]. While this suppresses the likelihood of cascades from any single outage, it does not change the probability of large blackouts [50]. Many cascades start with some  $n - k$  contingency where  $k$  components fail simultaneously. Some of these combinations do not lead to cascading failures. It is common for system operators to maintain a list of extremely risky  $n - k$  contingencies. But determining these sets is non-trivial.

Exploring all possible  $n - k$  contingencies and determining which of them are most dangerous is computationally expensive because there are  $n!/(n - k)!$  possible combinations to

explore. Significant research has been done to determine risky sets of outages. One solution identifies the minimal set of  $n - k$  contingencies that result in system failure using a “random chemistry” algorithm [21]. The minimal set is defined as a set of outages that cause a large blackout, but without any one of those outages, the blackout would not have occurred. The cascading failure simulation this algorithm uses first initializes the power flow using the DCPFE, applies the  $n - k$  contingency to the system, checks for a user-defined system failure, updates the protection system, and then advances times. Similar approaches to the  $n - k$  problem also only continue simulating cascades that pose some pre-defined risk factor [14][19][32]. Alternatively, [52] quantifies  $n - k$  contingencies based on how much power flow is redistributed after applying those contingencies to a base case DC power flow solution. However, there is no agreement in the community over which metric best identifies risky contingencies.

### 3.2.3 Critical Components

Regardless of whether all  $n - k$  contingencies are simulated, most researchers use the result of these simulations to determine the risk each component poses to large cascading failures. In general, the more often a component is involved in a large failure, the more likely upgrading that component decreases the overall system risk to large failures [51], but the effect of load increase over time means the system is always in a critical state [44]. Most studies have found that the failure of highly connected lines in the system generally leads to larger cascades [33][61][60].

something something

Finally, a large body of research in cascading failure analysis surrounds identifying which  $k$ -line failures are most crucial to model in detail due to their severity or their probability. There are several approaches to this problem, all of which reduce the combinatorial search

space to some extent. (then talk about all the ways people have approached this problem because it is numerous). Even WECC maintains a list a of important paths to consider when looking at any transmission model.

Part of the reason the order and timing of line failures matters is because of the differences in how power flow will redistribute. This becomes more important as models start to consider more detailed information such as generator dynamics. In [yang and motter paper], it was shown that a few  $n - 2$  contingencies cause significant differences in outcomes for the Iceland test case system.

# Chapter 4

## Inertia and Cascading Failures

Hopefully after reading Chapter 3, it is clear that numerous models exist to quantify a power system's risk to cascading failure, but very little research has been done to understand the role of a power system's inertia during cascading failures. Cascading failures and power system inertia have largely been treated as separate issues in the literature, but it is intuitive to believe that decreasing the system's ability to respond to instabilities will change its security from larger cascades. A thorough understanding of the interplay between the transient dynamics and cascade response is specifically lacking from the literature. For the proposed work, I will address what I believe to be two separate issues in regards to inertia: the first is how *total* system inertia affects cascade dynamics, the second is how the *distribution* of inertia affects cascades.

First, we need to understand how total system inertia impacts stability and risk of failure. As a reminder, total system inertia is a weighted average of every generator's inertia constant and its rated apparent power output

$$H_{tot} = \frac{\sum_{i=1}^{n_g} H_i S_{B_i}}{\sum_{i=1}^{n_g} S_{B_i}}$$

and is independent of the system's topology. This parameter is related to a power system's total kinetic energy, and decreasing inertia lowers a system's ability to respond quickly to failures, and maintain a balance between power generation and consumption. The motivation for studying how total system inertia influences stability comes from recent regulations on a maximum amount of inertia-less generation at any given time, for example in Ireland [CITE]. It is unclear what value of total system inertia is necessary to maintain stability, thus we will explore the effect of decreasing total system inertia on the system's risk to cascading failure. This will be investigated by incrementally reducing total system inertia and running cascading failure simulations ( $n - 1$  and  $n - 2$  contingencies to start) and observing how the distribution of failures changes as total system inertia changes. While this is not how a power system's total inertia will actually change, it separates out the effect of decreasing inertia from where inertia is actually located on the grid.

Second, we would like to understand how the distribution of inertia changes power system stability. Consider two scenarios shown in Figure ???. The first illustrates a network with evenly distributed inertia, while the second illustrates a case where all inertia generators are in one area of the system, while all inertia-less generators are in another area of the power system. We wish to explore the difference in stability between these two cases. where all generators with inertia are at one end of a system, and all generators without inertia are at another end of a system. This allows us to specifically address the interplay between topology and dynamics. In particular, is there a relation between the severity of a cascade and the distribution of inertia on the system? To address this question we will need to first quantify what the distribution of inertia is on a power system. Let  $D_P$  be the diameter of the power system in question, and  $\sigma(i, g)$  be the shortest path between a bus  $i$  and a generator  $g$ , then we define the total kinetic energy absorbed at bus  $i$  as

$$KE_i = \frac{1}{\sum_{\forall g \in G} KE_g} \sum_{\forall g \in G} KE_g \frac{\sigma(i, g)}{D_P} \quad (4.1)$$



We can quantify the severity of a cascade in a multitude of ways, the more prominent choices in the literature are cascade size (in terms of number of failed lines, or amount of load shed). We will use the number of failed lines in this work, but we also choose to explore a few other measures which include the amount

## 4.1 Model for Cascading Failure

To study the effect of inertia on power system stability during cascading failures, it is important to use a model that not only accurately reflects the cascading failure dynamics, but also the transient dynamics of the generators. For this reason, we choose to run the cascading failure simulations using the continuous phase-space (CPS) model developed in [59] and [Cite DEMARCO]. It is important to note that this model assumes constant voltage magnitudes at every bus, and does not solve for reactive power. We previously discussed that voltage stability is very important for maintaining reliability, and thus this is a limitation of the CPS model. For this reason, we will compare results obtained with the continuous model to the results from PSLF, a commercial software created by General Electric, which is an industry standard for testing power system stability. We expect that the results of some cascades will be different between the two models, but we hypothesize that the continuous model should never be worse than the PSLF model, as the PSLF model accounts for more potential instabilities than the continuous model. The purpose of using the continuous model is to understand the usefulness of the lyapunov energy function to the understanding of cascade severity and outcome. However, should the results differ significantly (i.e. in terms of the number of outages, or size of frequency deviations), then we will use the final results from PSLF, because it is the industry standard. Additionally, the comparison of these two models will serve as an exploration into the accuracy of the DC power flow approximation compared to the AC power flow equations.

## 4.2 Power Networks to Use

This section is going to explain which power system test cases I am going to use and why.

In order to study inertial impacts, I will use two specific test cases. The first is a model of the Central Illinois power system, and the second is a model of the ERCOT test case. Both of these models were created using population information of specific zip codes for creating load data, and statistical information about generators from the Energy Information Administration [7]. The generator dynamic data, such as damping and inertial constants, are drawn from a distribution of known inertia constants for generators of different types, such as coal or nuclear [57].

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