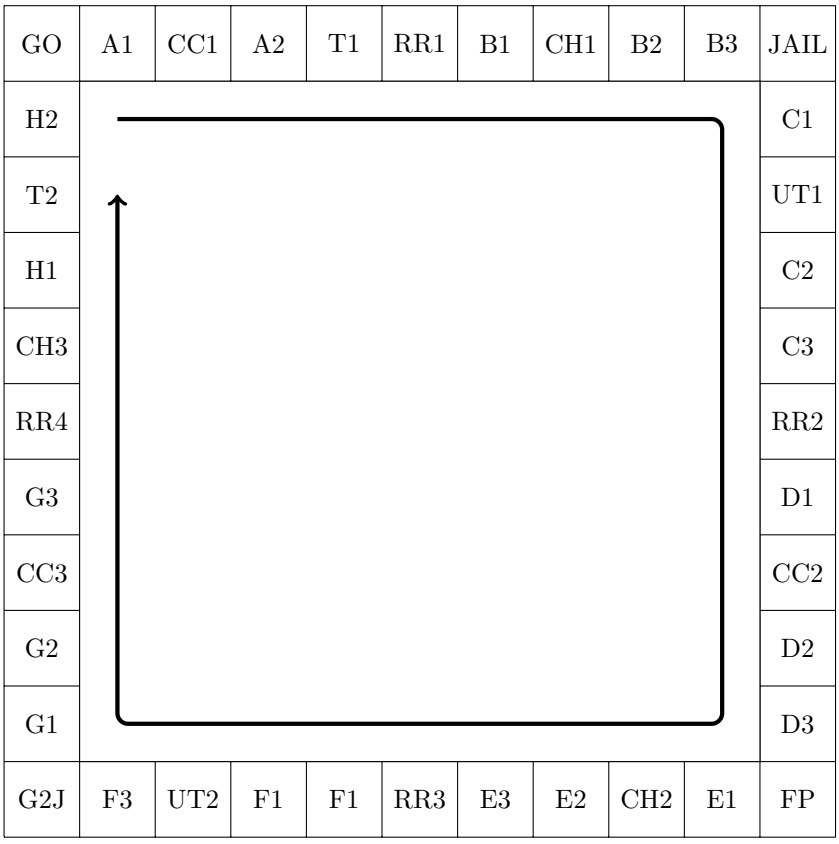
Homework Assignment 4

STA 141A

Due Tuesday, November 21 by 5:00 pm

Description

The game Monopoly is played on a board with 40 squares, set up as shown below.



Players begin the game on the GO square. On each turn, the player rolls two 6-sided dice. The sum of the dice determines the number of squares they advance (in a clockwise direction) on that turn.

Without any further rules, we would expect players to visit each square with equal probability. However, landing on G2J (go to jail), CC (community chest), and CH (chance) changes this distribution. When a player lands on G2J, they must immediately move to the JAIL square.

At the beginning of the game, the CC and CH cards are shuffled. When a player lands on CC or CH they take a card from the top of the respective pile and, after following the instructions, it is returned to the bottom of the pile. There are 16 cards in each pile, but for this assignment we are only concerned with cards that order a movement. Any card not concerned with movement will be ignored and the player will remain on the CC/CH square. The relevant cards are:

- Community Chest (2/16)
    1. Advance to GO
    2. Go to JAIL
- Chance (10/16)
    1. Advance to GO
    2. Go to JAIL
    3. Go to C1
    4. Go to E3
    5. Go to H2
    6. Go to R1
    7. Go to next RR (railroad)
    8. Go to next RR
    9. Go to next UT (utility)
    10. Go back 3 squares

In addition to G2J and CC/CH, if a player rolls doubles on three consecutive turns, they proceed directly to jail. A "double" is a roll where the two dice are equal, such as rolling 2 fives or 2 sixes.

The goal of this assignment is to estimate the long-term probabilities of landing on each square. That is, to compute for each square the probabilty that the player will end a turn that square if they play the game for infinitely many turns.

This assignment is based on a Project Euler exercise.

Questions

Use R to find answers to all of the following questions (that is, don't do any by hand or by point-and-click). Save your code in an R script.

1. Write a function `simulate_monopoly()` that simulates n turns by a player in a game of Monopoly using two d-sided dice. The inputs to your function should be n and d. The output of your function should be a length $n + 1$ vector of positions, encoded as numbers from 0 to 39.
2. Write a function `estimate_monopoly()` that uses your simulation to estimate the long-term probabilities of ending a turn on each Monopoly square. What are the 3 most likely squares to end a turn on if you play Monopoly with 6-sided dice? What if you play with 4-sided dice? Display graphically the long-term probabilities for 3, 4, 5, and 6-sided dice.
3. Use $k = 1,000$ simulations with $n = 10,000$ turns each to estimate the standard error for the long-term probability of ending a turn in jail.
4. Use the non-parametric bootstrap with $b = 1,000$ samples from a simulation of $n = 10,000$ turns to estimate the standard error for the long-term probability of ending a turn in jail.
    (a) How does the boostrap estimate compare to the simulation estimate? Which do you think is more accurate? Explain.
    (b) Which is faster to compute: the bootstrap estimate or the simulation estimate? Explain why there is a difference.
5. Display graphically the standard errors for the long-term probabilities for 3, 4, 5, and 6-sided dice (use the same settings you used in question 2). Discuss why some probabilities have much larger standard errors than others.

6. What happens to the standard errors for the long-term probability estimates as n increases? Why does this happen?
7. Please list the names of your final project group members here. Some details about the final project are at the end of this document.

Assemble your answers into a report. Please do not include any raw R output. Instead, present your results as neatly formatted[1] tables or graphics, and write something about each one. You must **cite your sources**. Your report should be **no more than 3 pages** including graphics, but excluding code and citations.

What To Submit

Submit a digital copy on Canvas. The digital copy must contain your report (as a PDF) and your code (as one or more R scripts).

Additionally, submit a printed copy to the box in the statistics department office[2]. The printed copy must contain your report and your code (in an appendix). Please print double-sided to save trees. It is your responsibility to make sure the graphics are legible in the printed copy!

Relevant Functions

All of the functions from previous assignments, as well as random number generation functions and control flow statements.

Final Project

- The final project will begin on November 27th and will be due on December 5th.
- The final project will be comprehensive and require many topics you have learned this quarter.
- You will work in groups of 3 and can choose your own group members.
- The instructor and TAs will be available for questions/help only for the week of Nov 26th. Starting December 2nd the instructor and TAs will no longer answer questions about the final project (including on Piazza). The Piazza will remain open until the end of the quarter so you can still help each other out.

[1] See the graphics checklist on Canvas.
[2] 4th floor of Mathematical Sciences Building