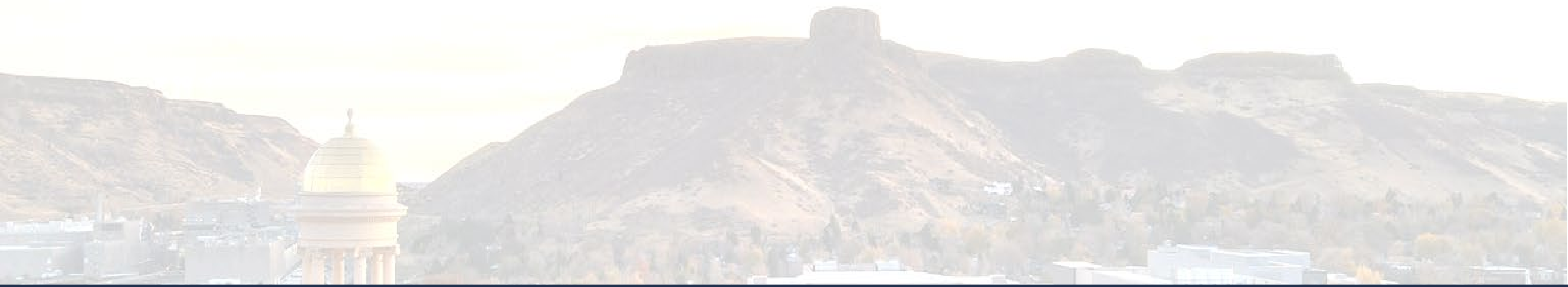
Shear Wave Velocity Modeling **using Transdimensional Bayesian Inversion** of Surface Wave Dispersion Data

Samara Omar, Jeffrey Shragge, Matthew Siegfried

MINES



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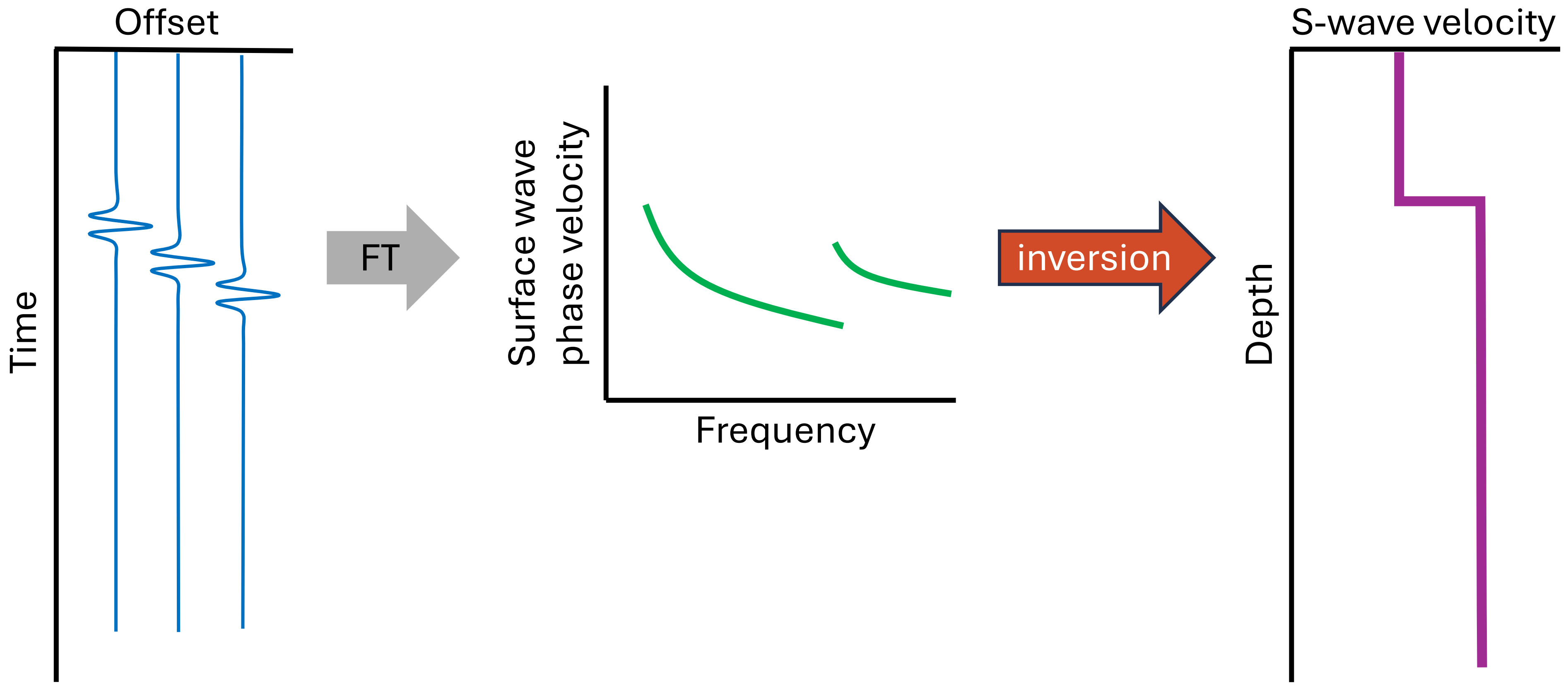
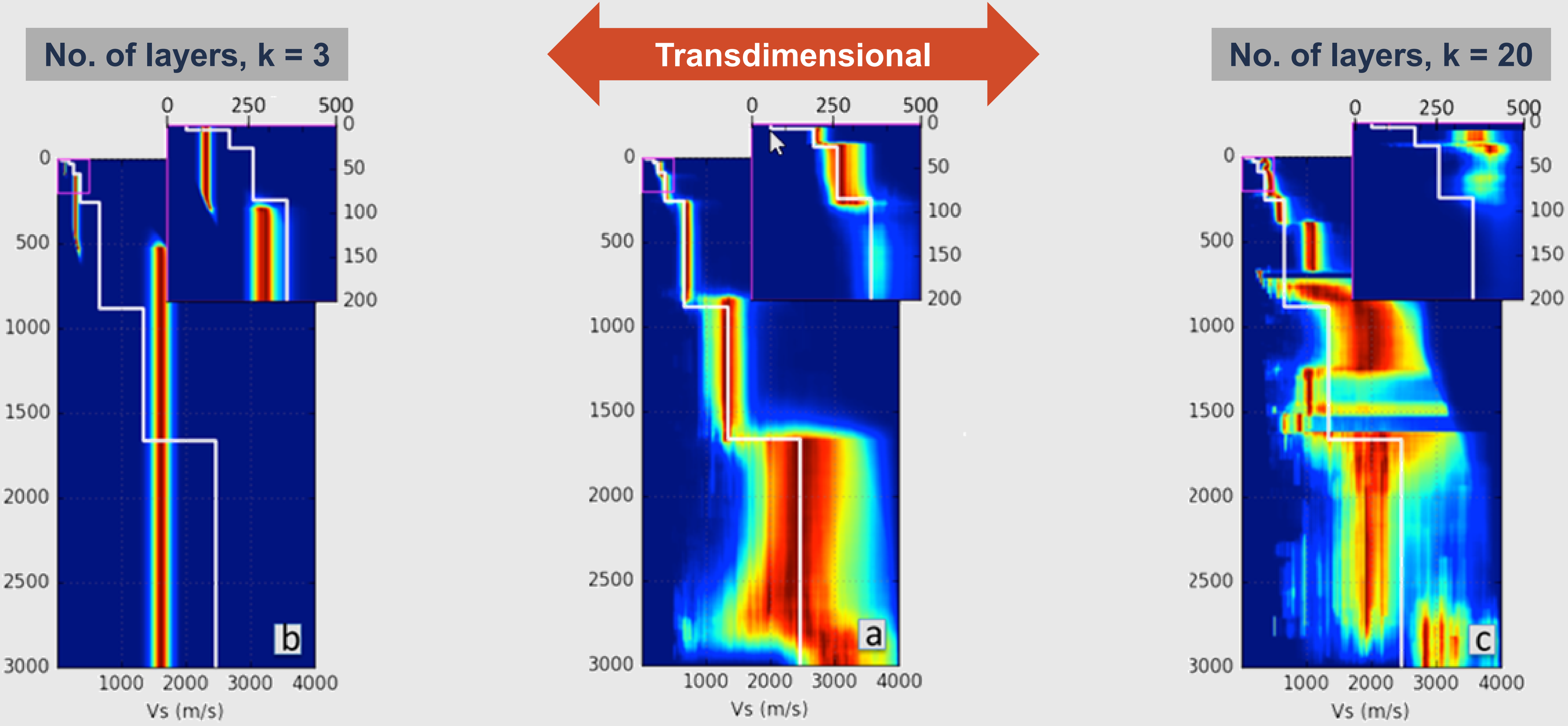
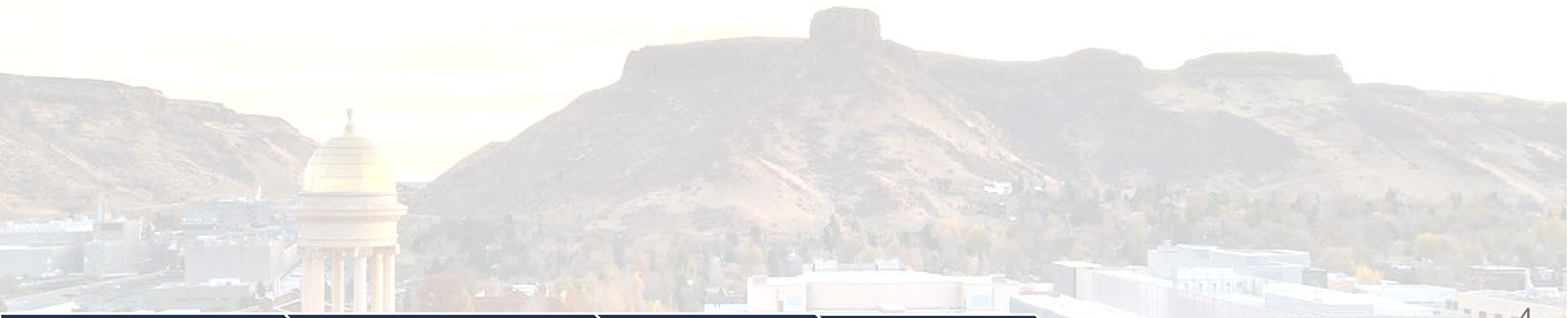




Inversion of surface wave ellipticity data
for a 10-layer model representative of the Jakarta Basin.

Cipta et al., 2018

Transdimensional Bayesian Inversion



Challenges of the Inverse Problem

- Strong non-linearity
- Strongly ill-posed

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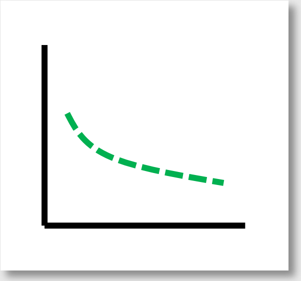
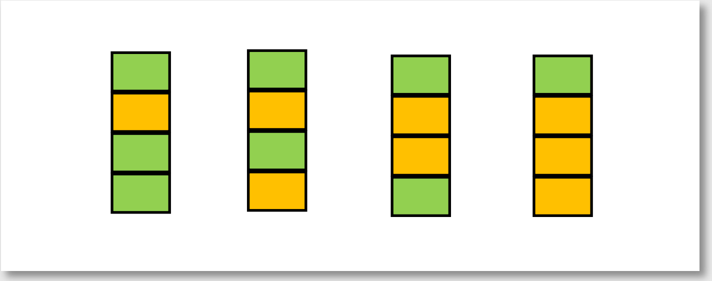
data (V_r-f)



model (V_s)



Challenges of the Inverse Problem

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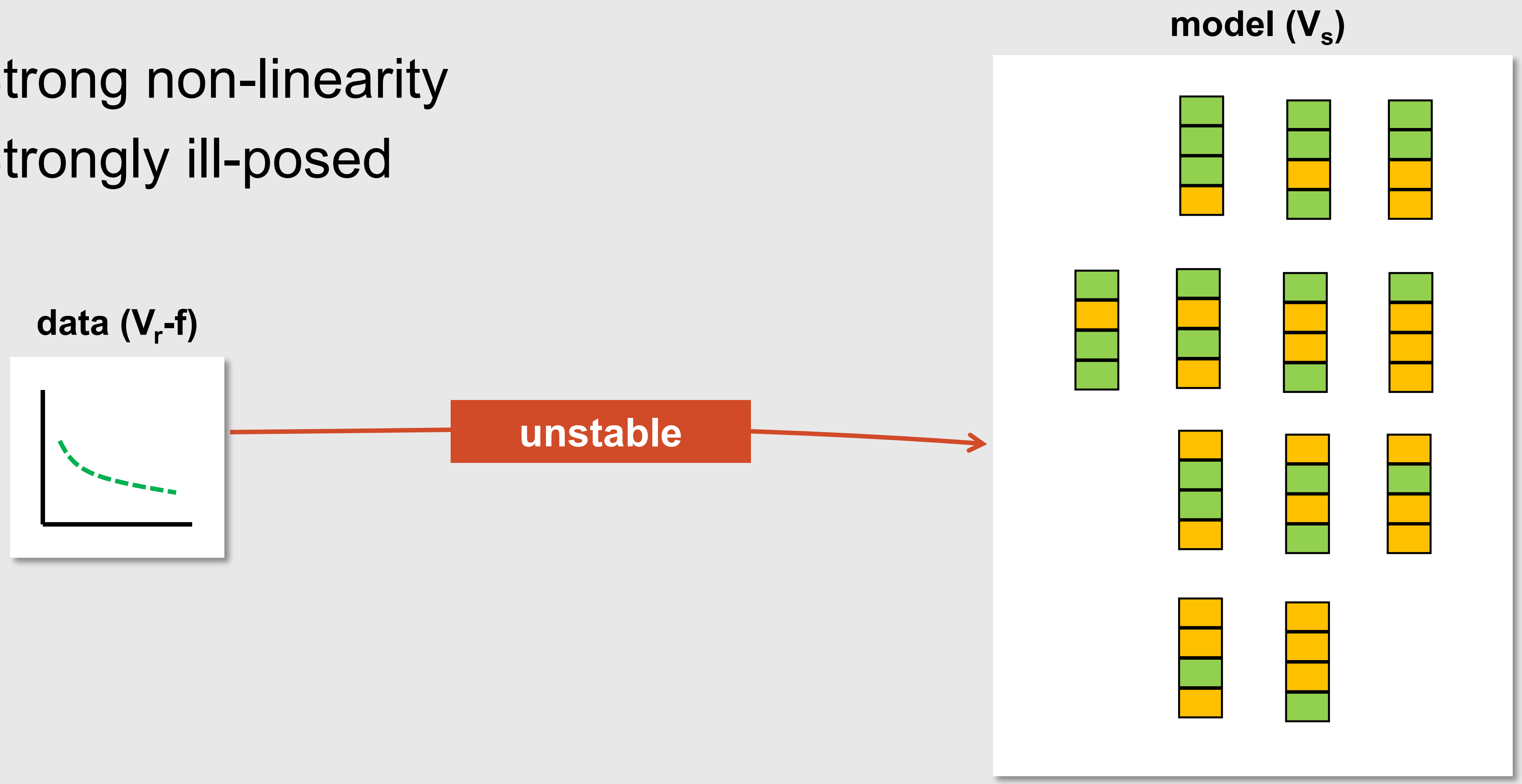
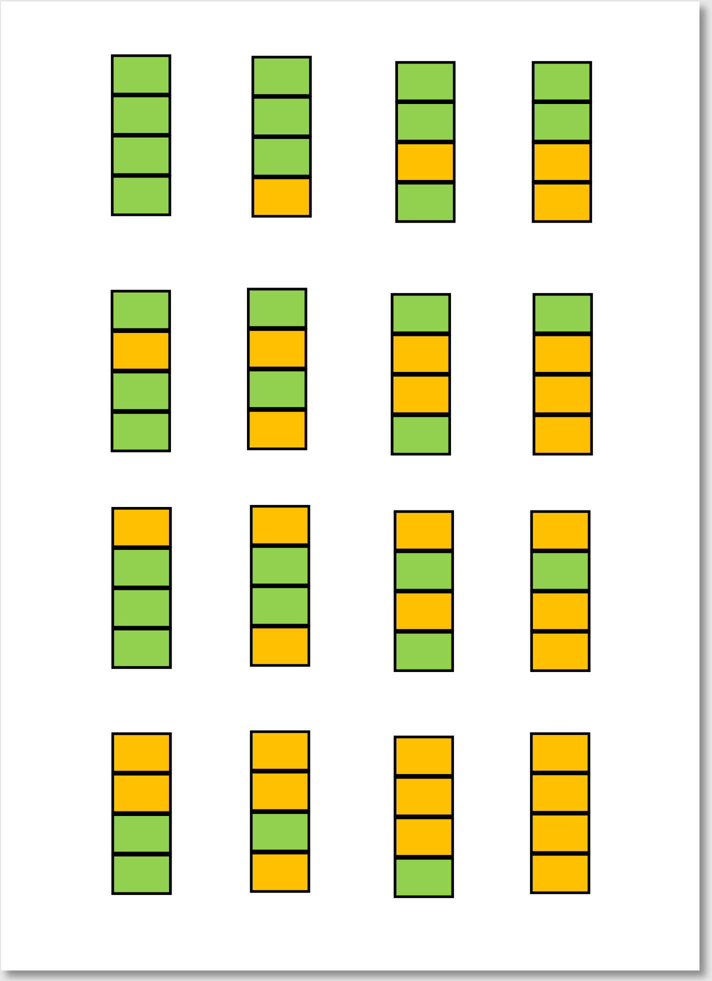


Overcoming Ill-posedness

No. of layers, k = 4
Options for V_s = 2



Overcoming Ill-posedness

- Regularization e.g. norm damping
- Simplify Model Parameterization
- Spatial Smoothing
- etc

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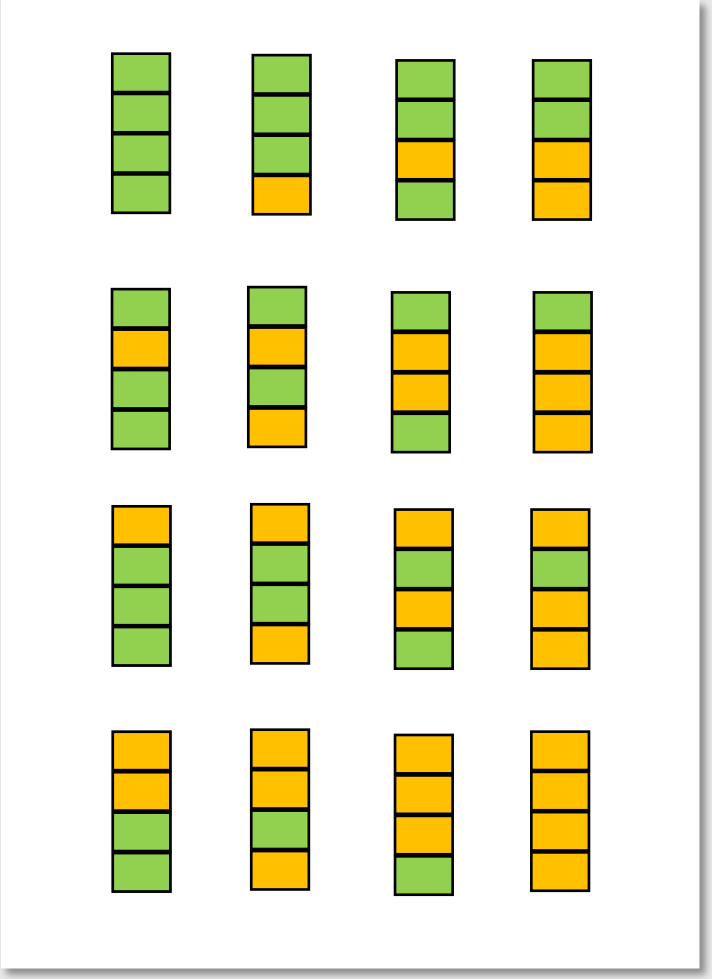

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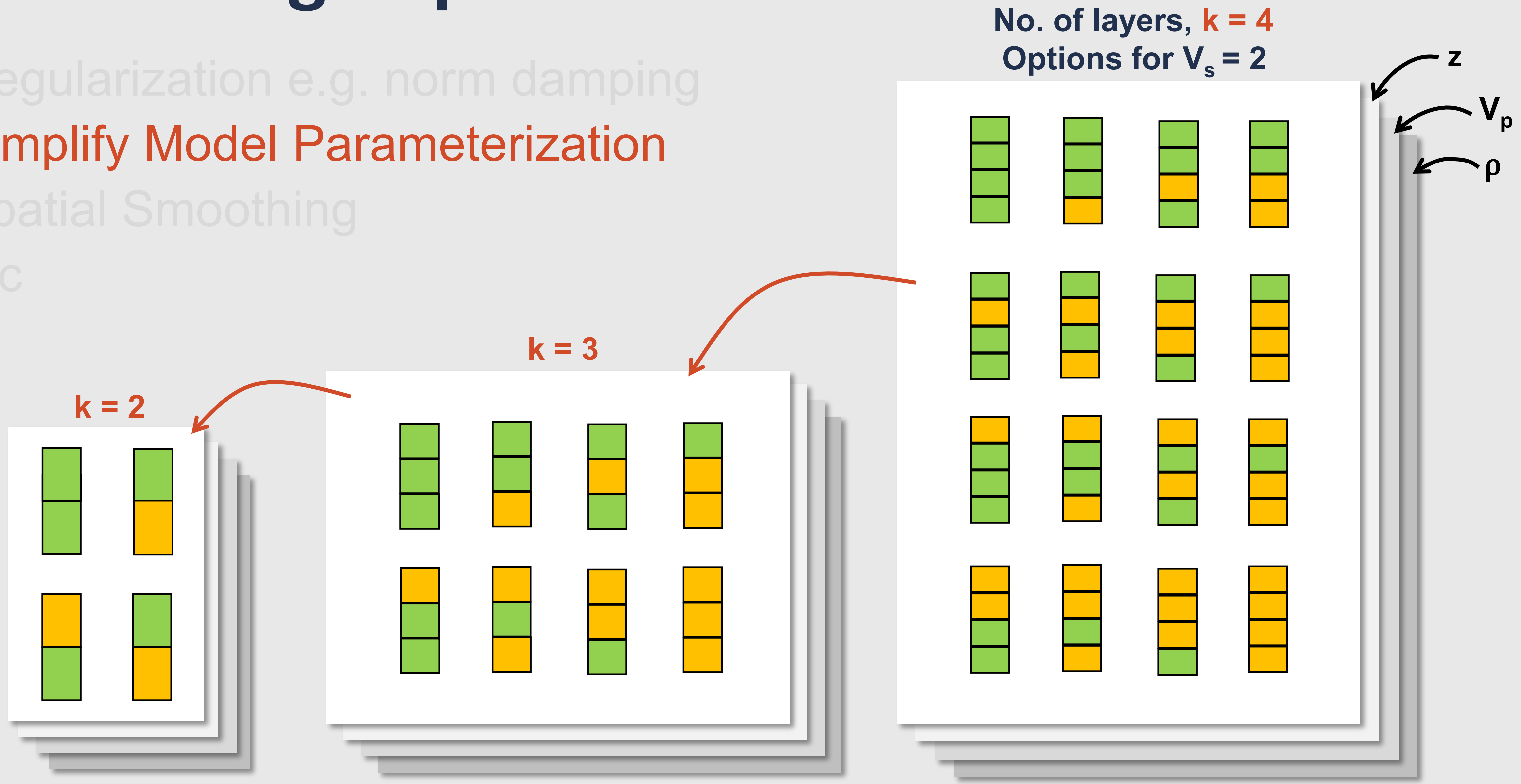
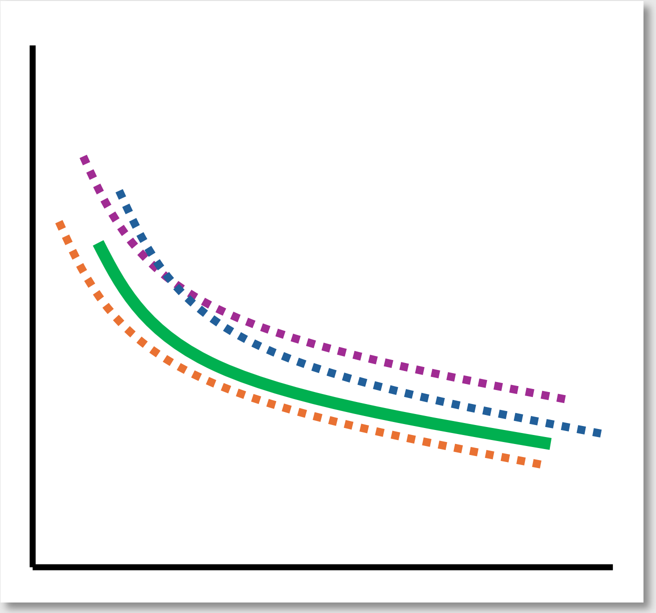
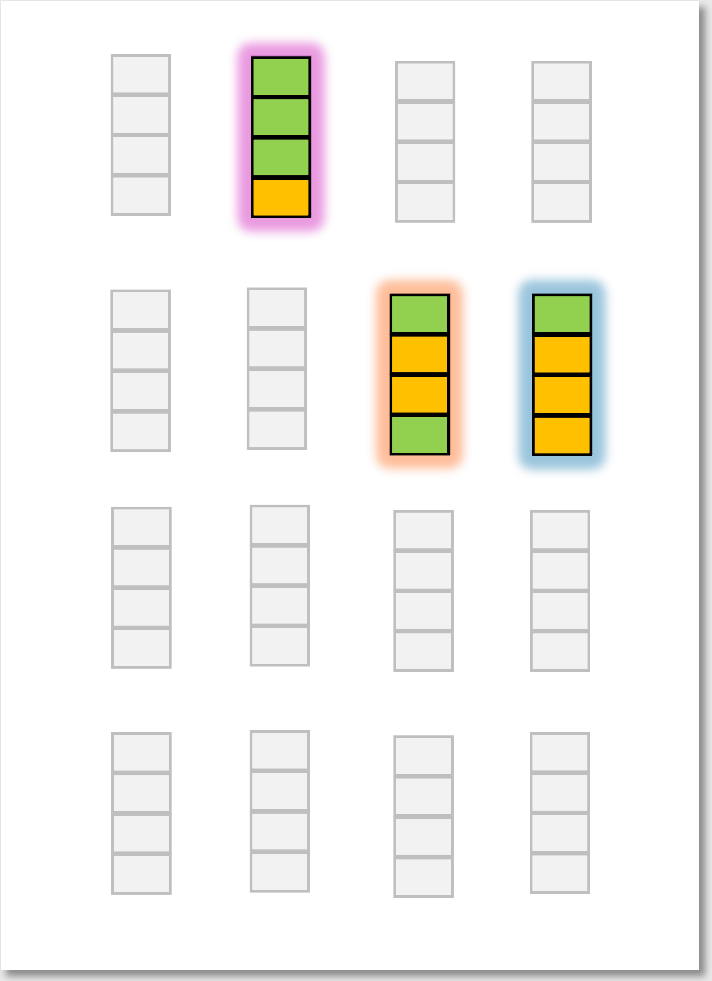


Choosing the best model...

Observed vs Calculated data (V_r-f)



V_s



What we need from an inversion...

1. Quantification of uncertainty in model solutions

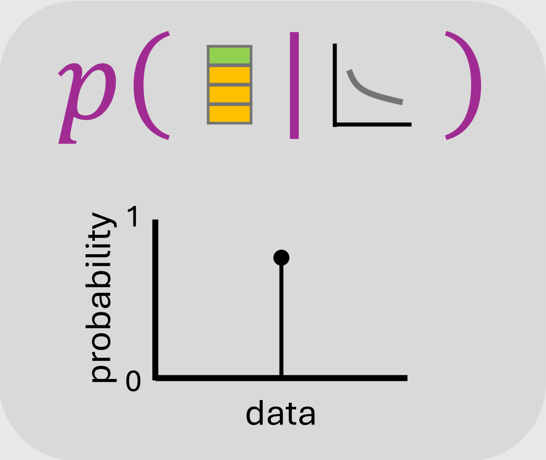
2. Explore multi-dimensional model solution space

Bayesian Inversion – Surface Wave Context

$$\underbrace{p(m|d)}_{\text{posterior}} = \frac{\overbrace{p(d|m)}^{\text{likelihood}} \times \overbrace{p(m)}^{\text{prior}}}{\underbrace{p(d)}_{\text{evidence}}}$$

Bayesian Inversion – Surface Wave Context

$$\overset{\text{posterior}}{p(m|d)} = \frac{\overset{\text{likelihood}}{p(d|m)} \times \overset{\text{prior}}{p(m)}}{\underset{\text{evidence}}{p(d)}}$$



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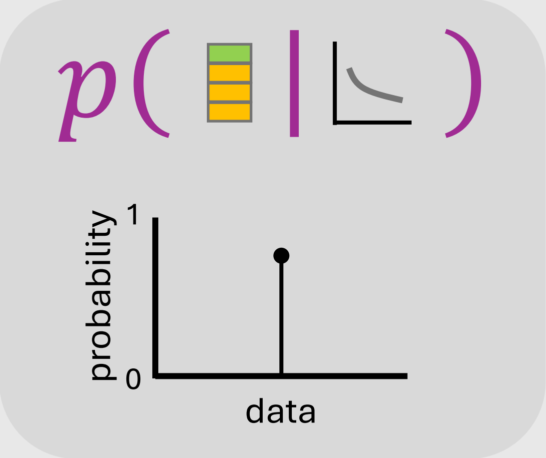

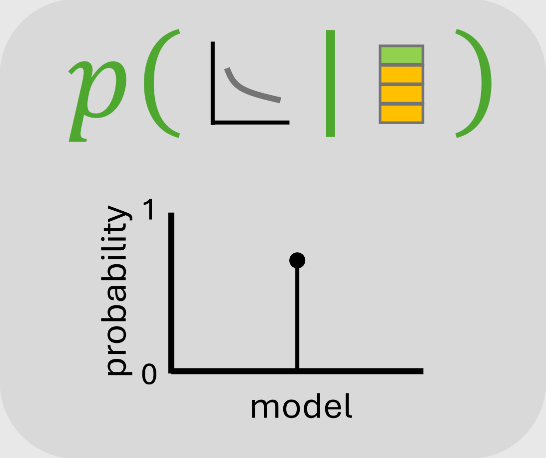

via forward modeling

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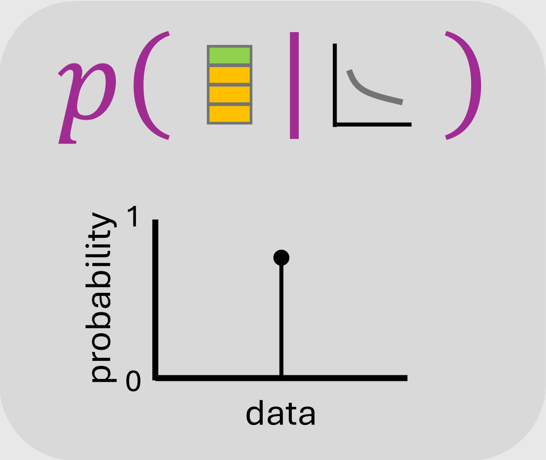
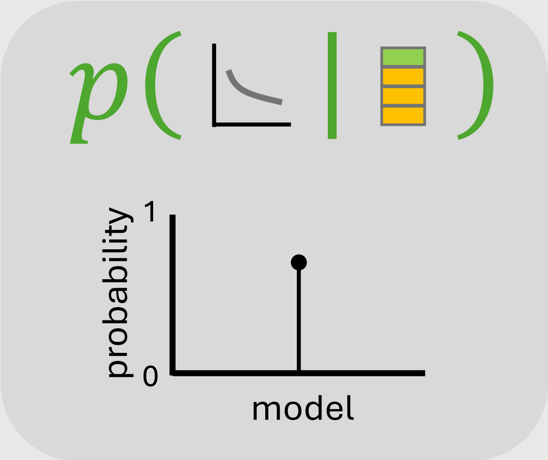
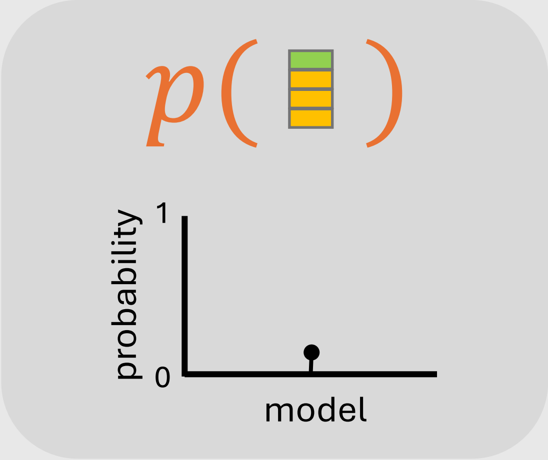
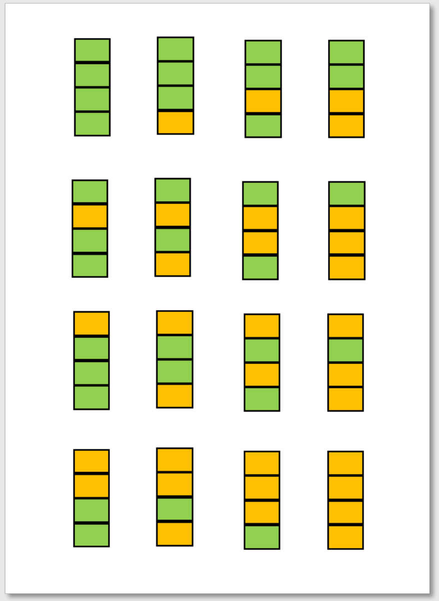




via forward modeling



background



Bayesian Inversion – Surface Wave Context

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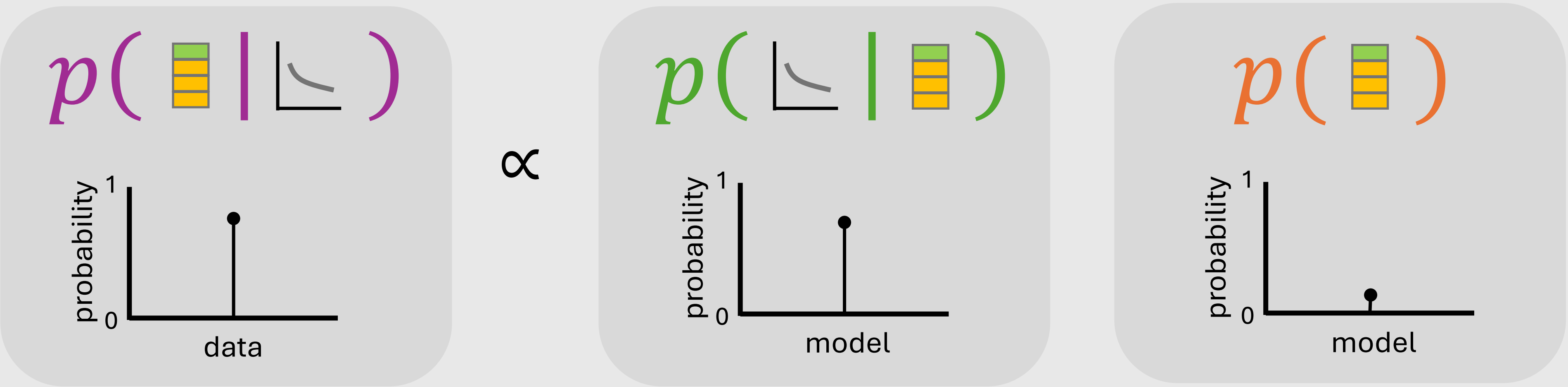
$$\textit{posterior} \propto \textit{likelihood} \times \textit{prior}$$



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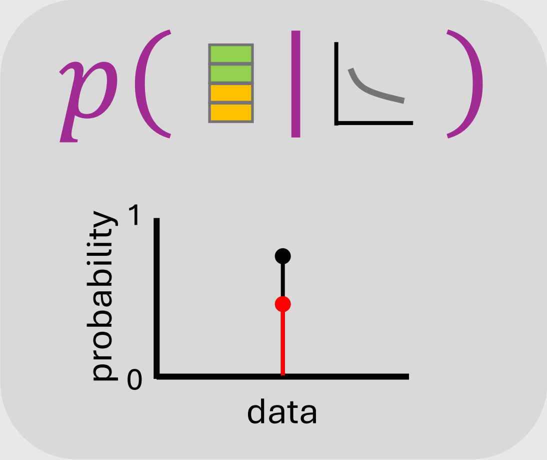
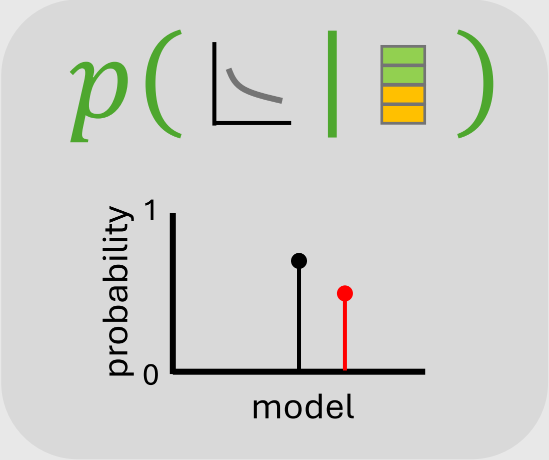
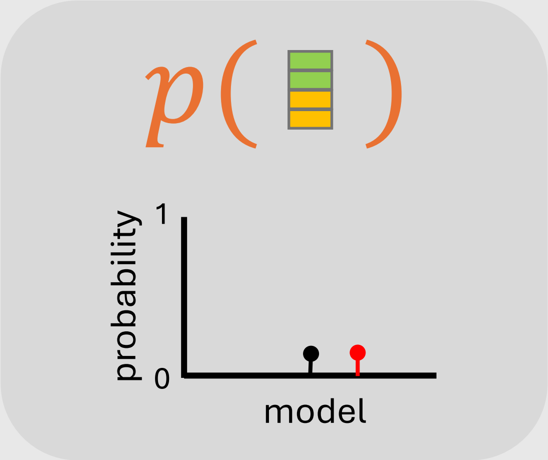
Another model ...



$\propto$



via forward modeling



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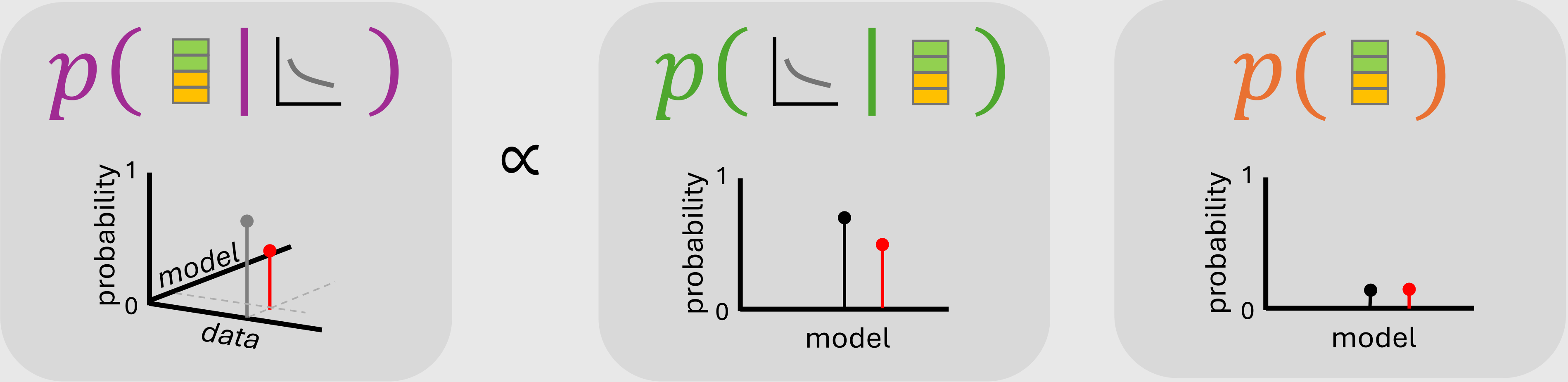


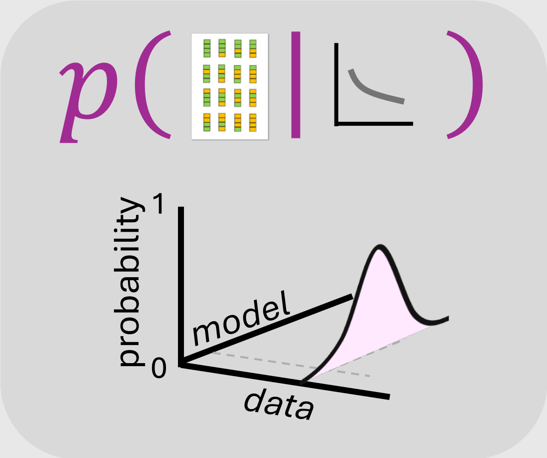
via forward modeling

background

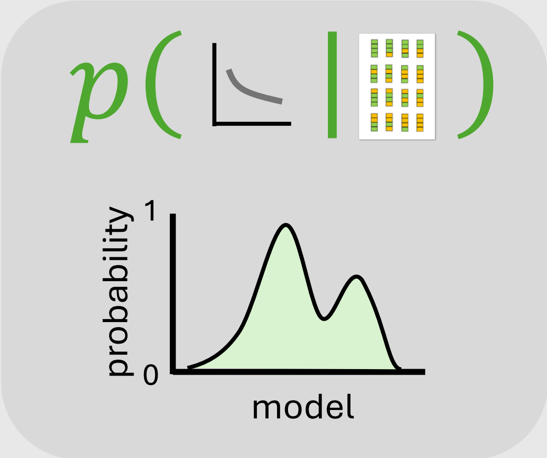
Bayesian Inversion – Surface Wave Context

$$posterior \propto likelihood \times prior$$

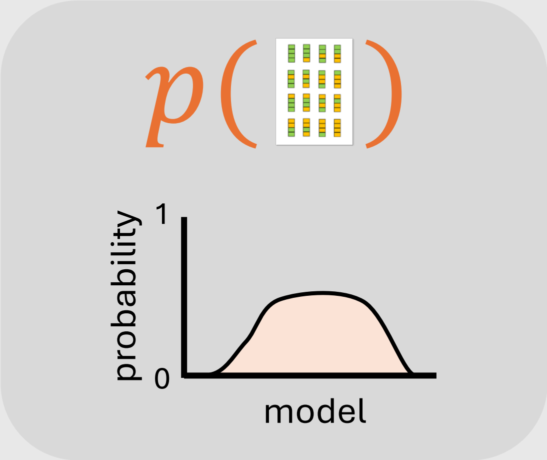
Several models ...



$\propto$



via forward modeling

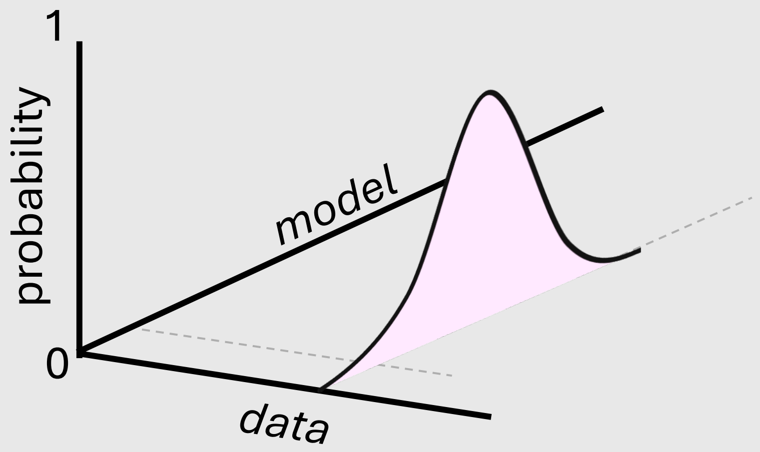


background

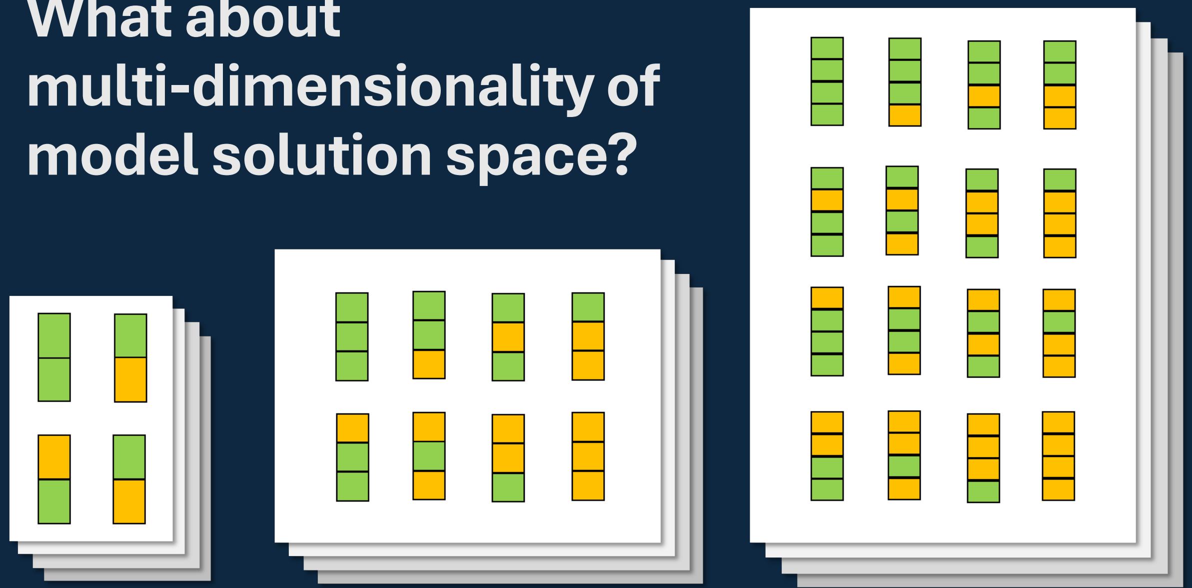
Bayesian Inversion

Outcome:

- Ensemble of model solutions
- Quantification of uncertainty

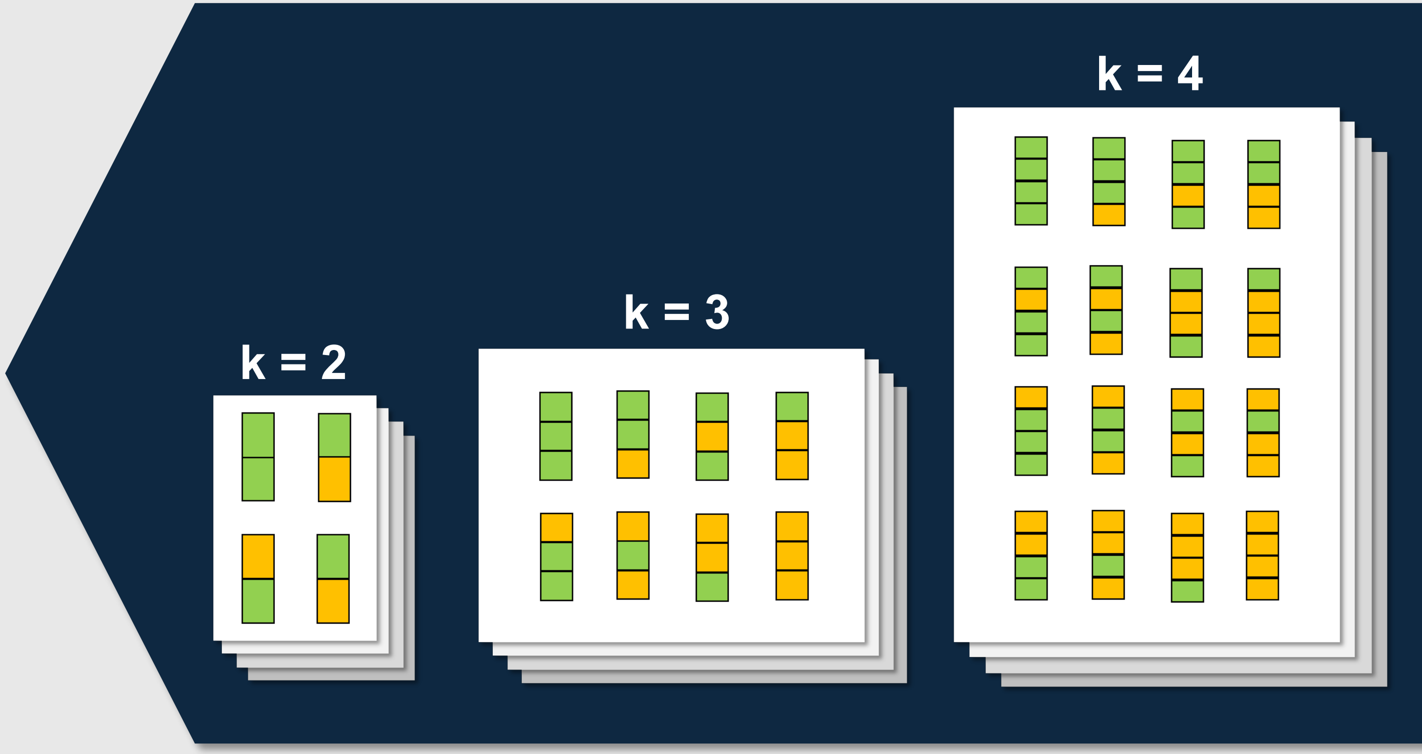


What about multi-dimensionality of model solution space?



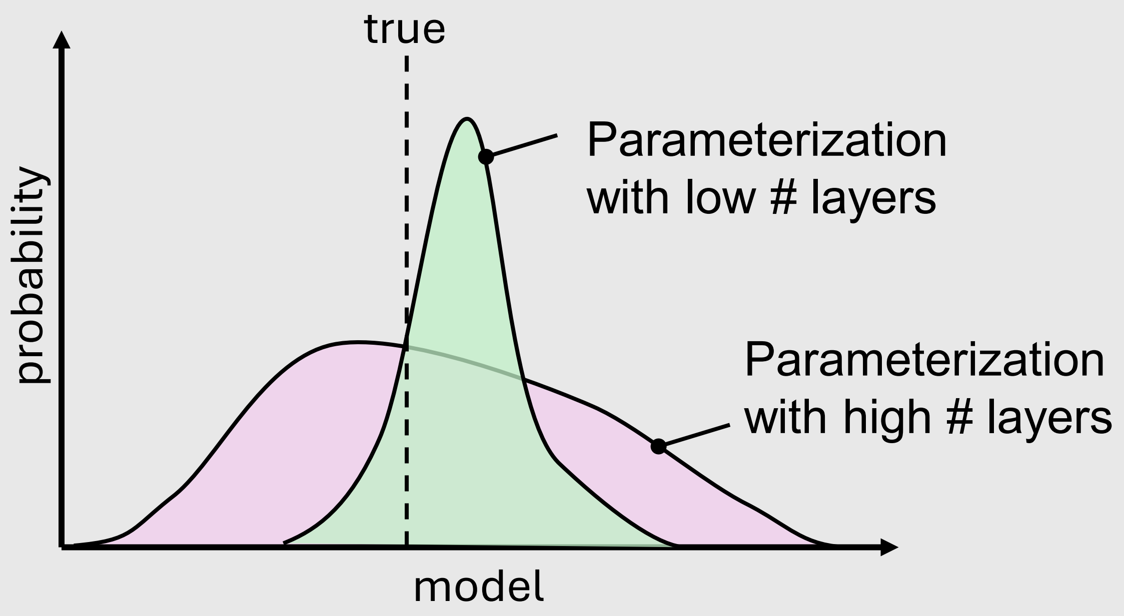
Transdimensional Bayesian Inversion

- No. of model parameters is unknown
- Allows data to constrain the maximum allowable complexity



Transdimensional Bayesian Inversion

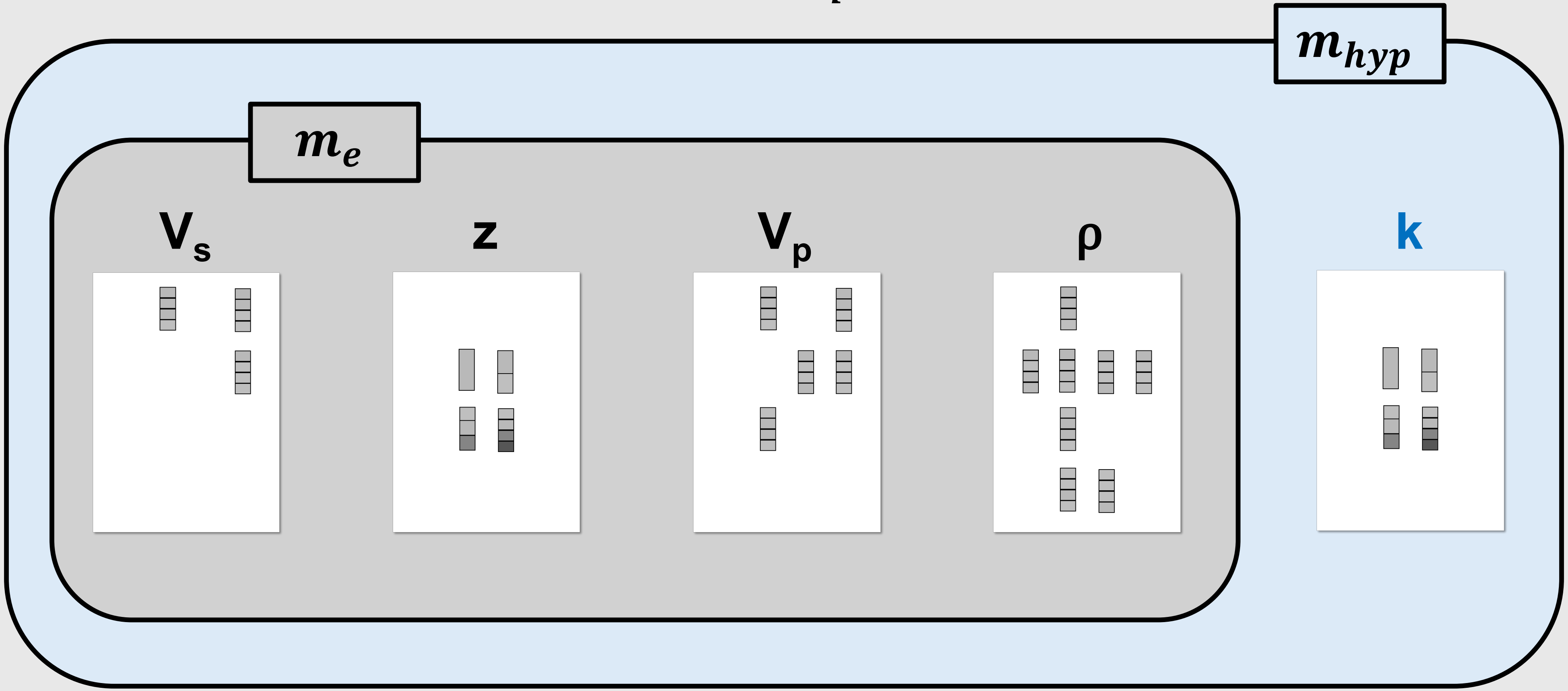
As Bayesian sampling is naturally parsimonious – no need to worry about overfitting with complex models



After Dettmer, 2010 and MacKay, 2003

Model Parameterization for TDBayes

$$m\,[\,V_s, z, V_p, \rho, k\,]$$



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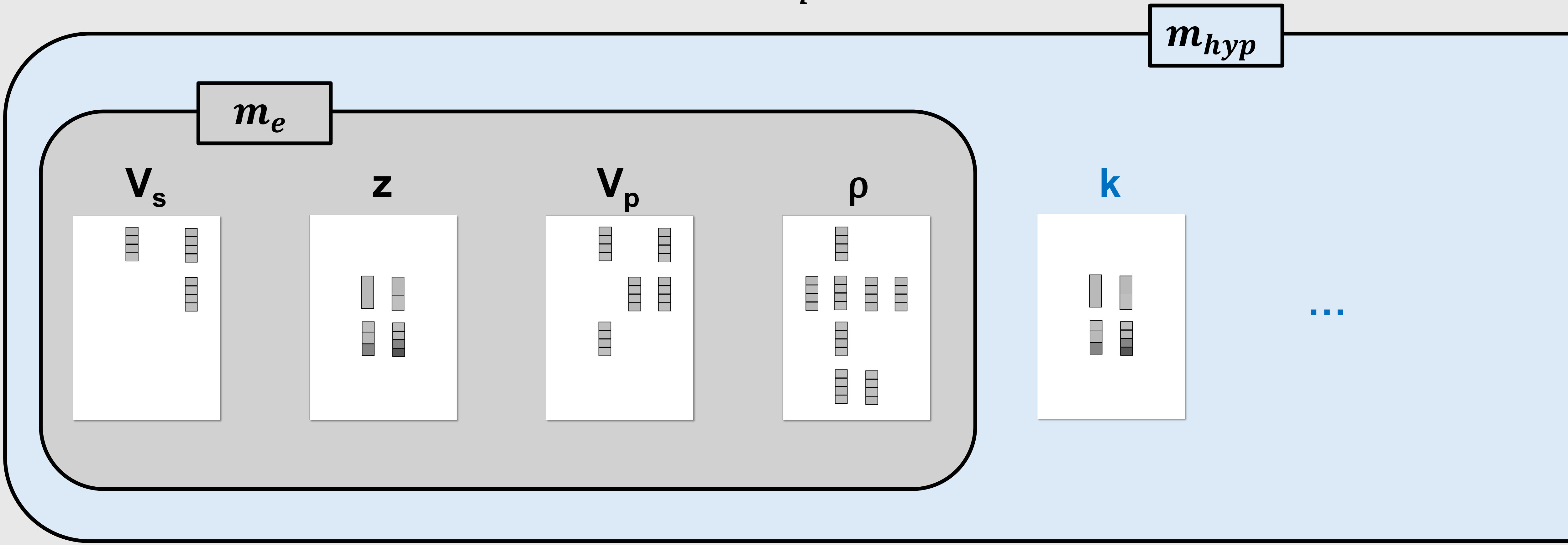


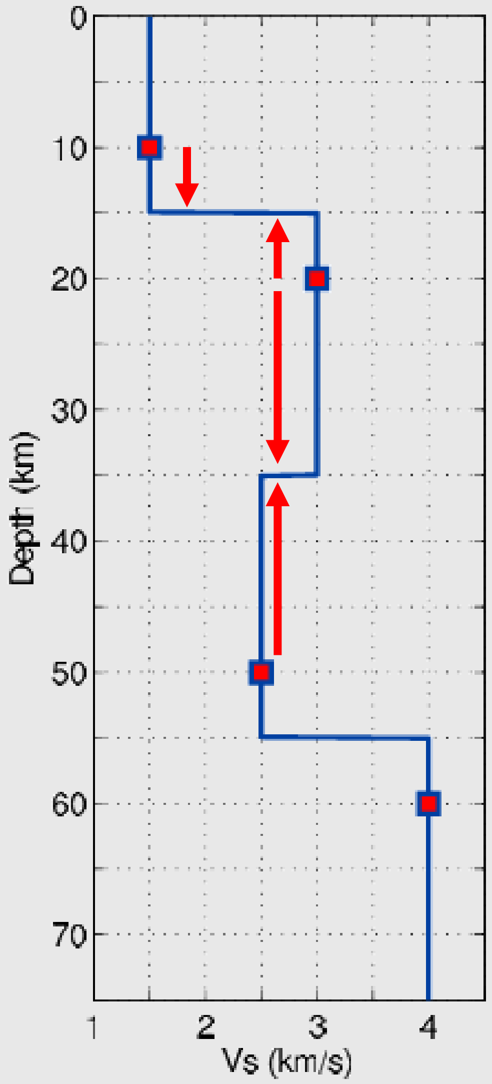
Hyperparameterization

$m\ [\ m_e, k, c, n]$

k = number of layers

c = Voronoi nuclei locations (**c**entroid)

n = elements of **n**oise covariance



Bodin et al, 2012

Transdimensional Bayes Theorem

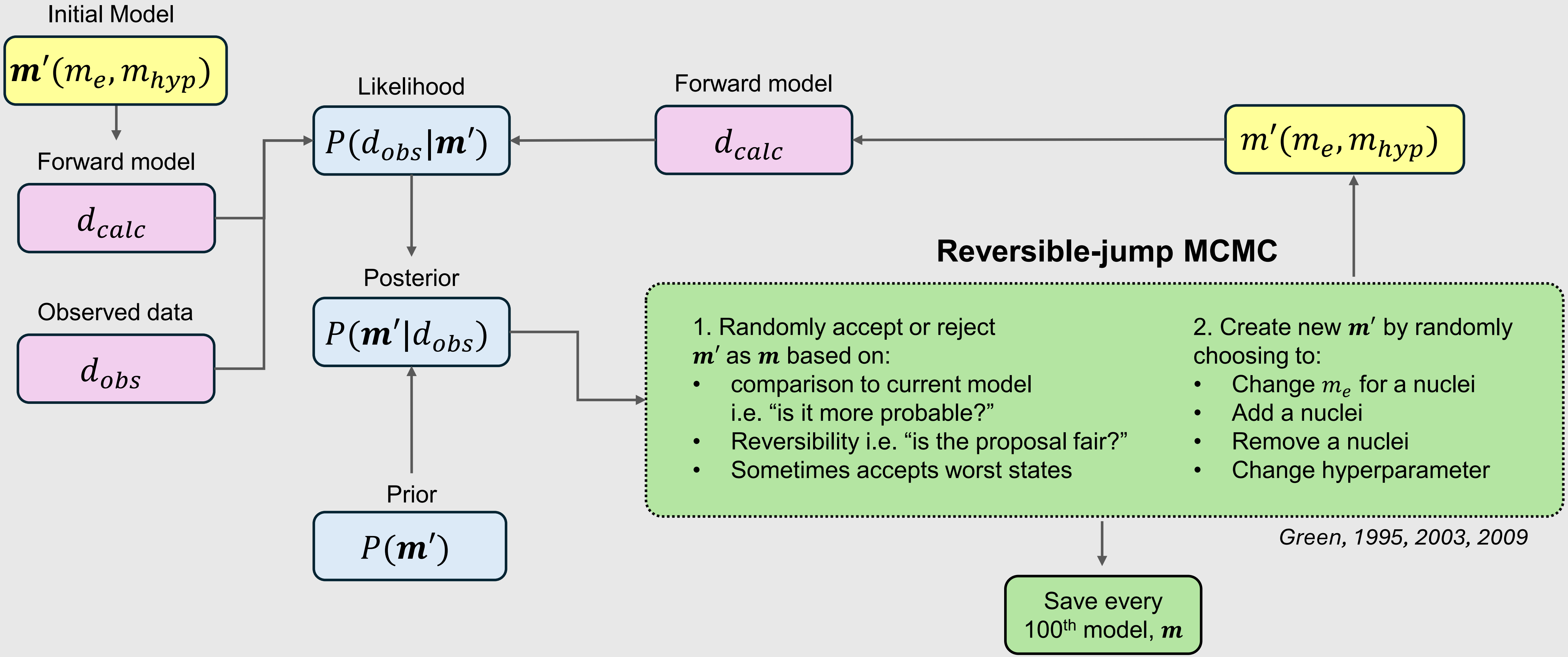
posterior — *likelihood* — *prior*

$$p(m_e, m_{hyp} | d) \propto p(d | m_e, m_{hyp}) \times p(m_e | m_{hyp})$$

Mahalanobis distance to account for correlated data errors (noise)

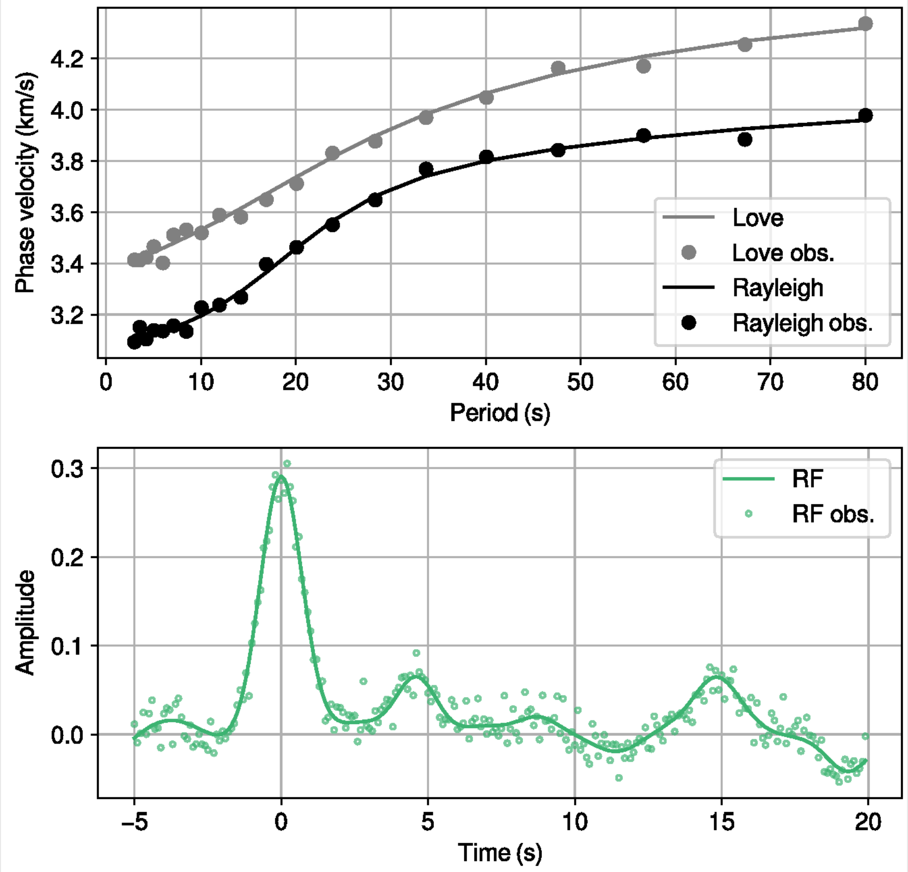
Independent priors, therefore separable

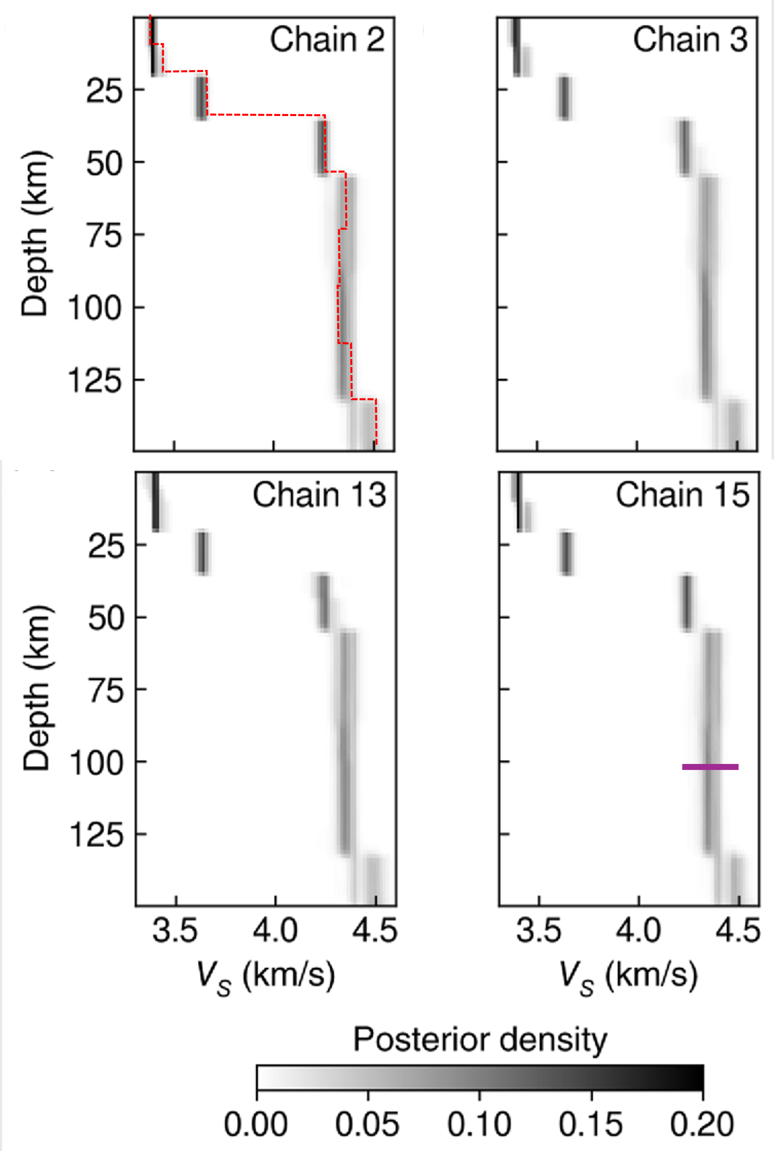
Transdimensional Bayes Inversion

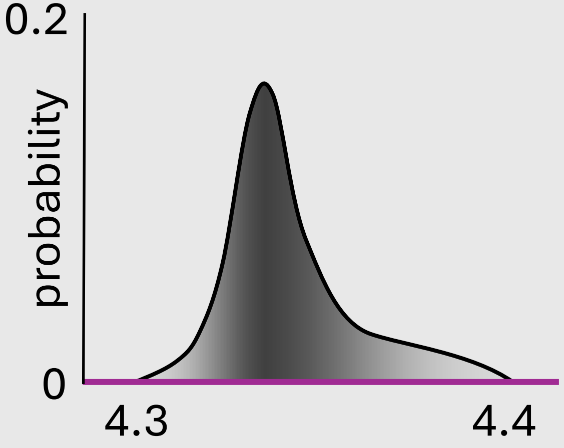


TD Bayes Example

Magrini, He and Sambridge, 2025







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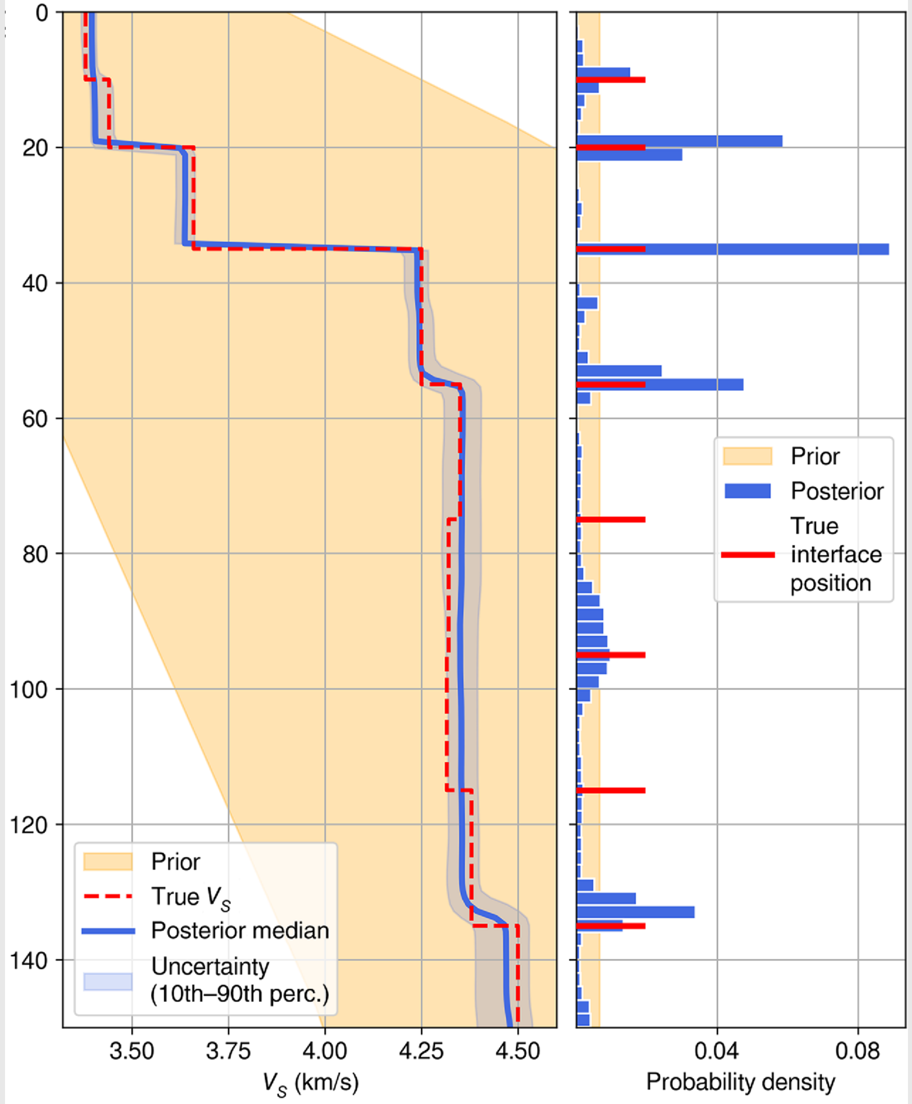

TD Bayes

- Frees solutions from fixed layer assumptions—especially powerful for complex or poorly understood geologies

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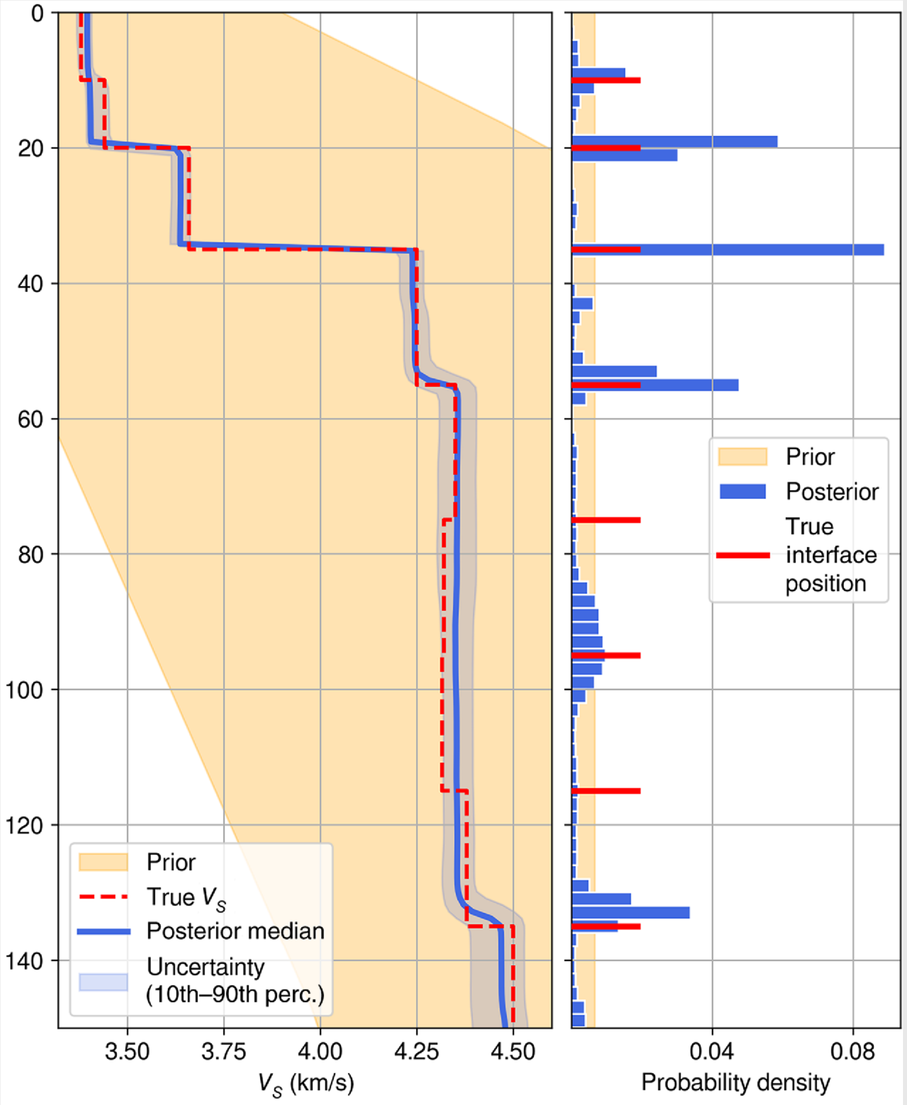
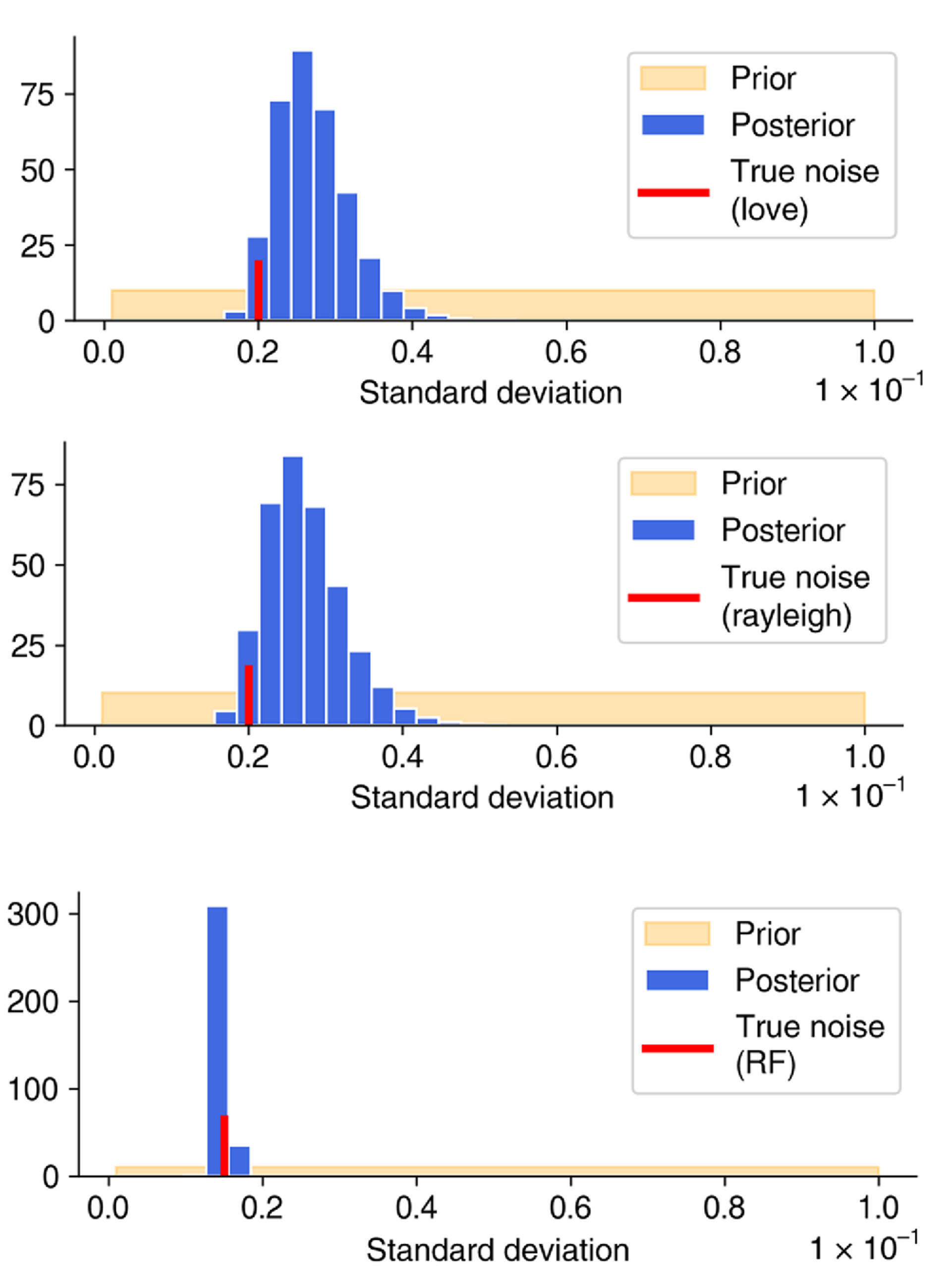


TD Bayes

Magrini, He and Sambridge, 2025

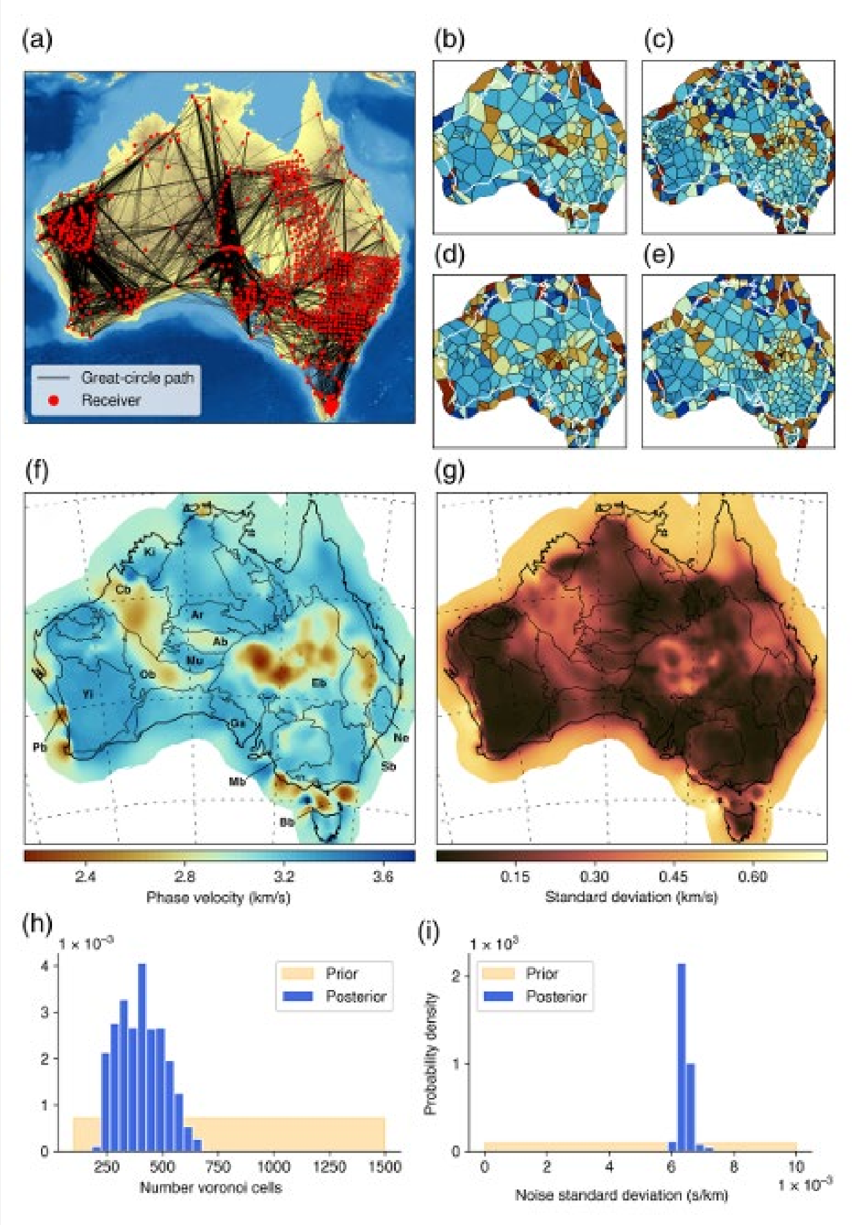
- Frees solutions from fixed layer assumptions—especially powerful for complex or poorly understood geologies

- Well-suited for joint inversion: accommodates disparate datasets with flexible, dataset-specific treatment

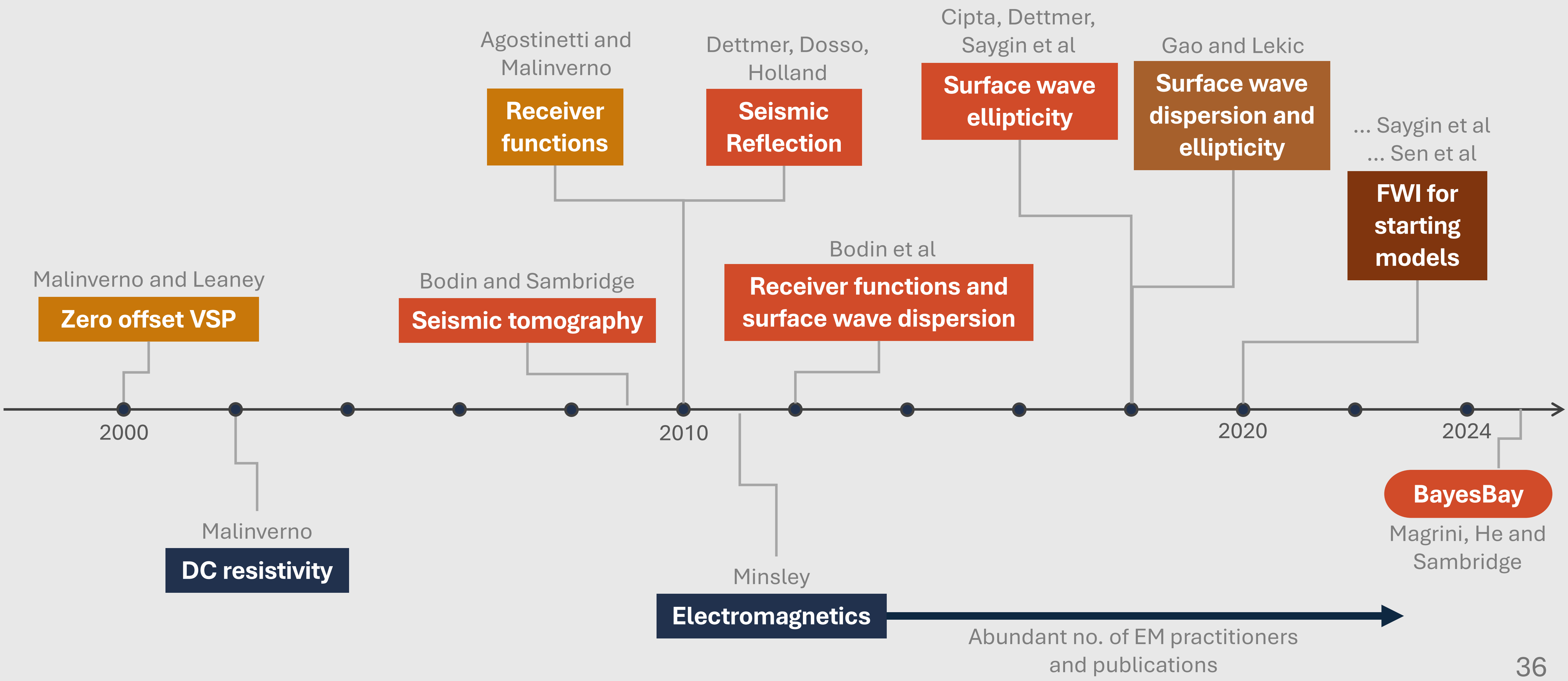


TD Bayes

- Frees solutions from fixed layer assumptions—especially powerful for complex or poorly understood geologies
- Well-suited for joint inversion: accommodates disparate datasets with flexible, dataset-specific treatment
- Scales to 2-D and 3-D problems, enabling mapping of spatially varying subsurface properties



Geophysical Applications of TD Bayes



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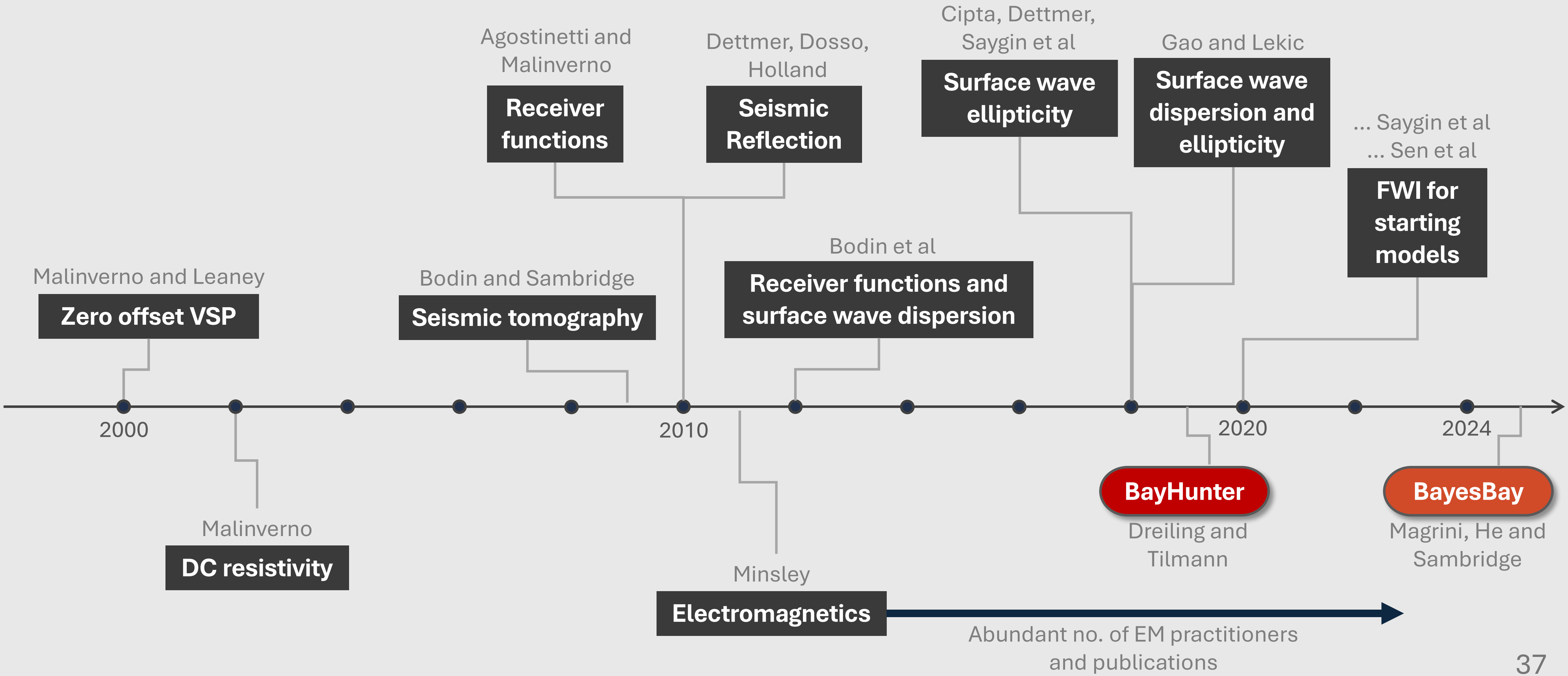

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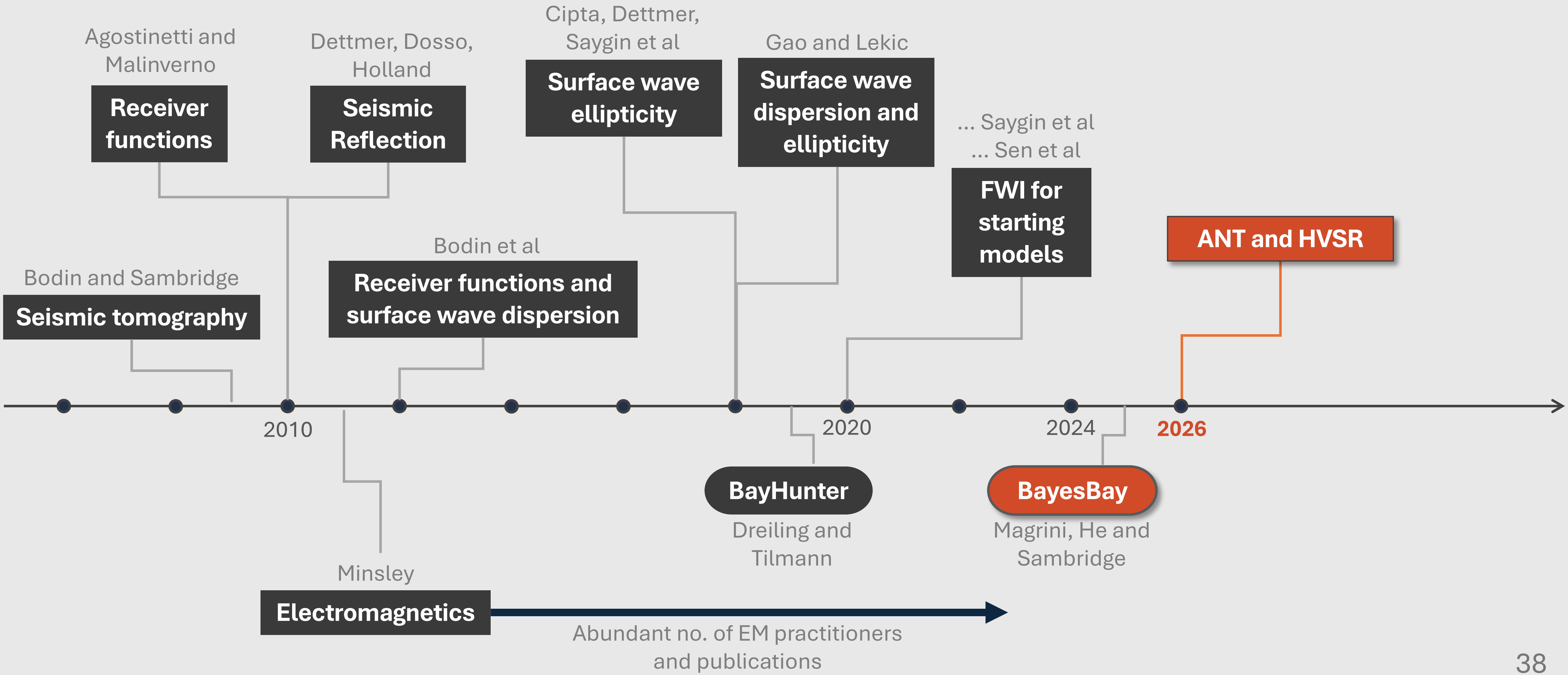
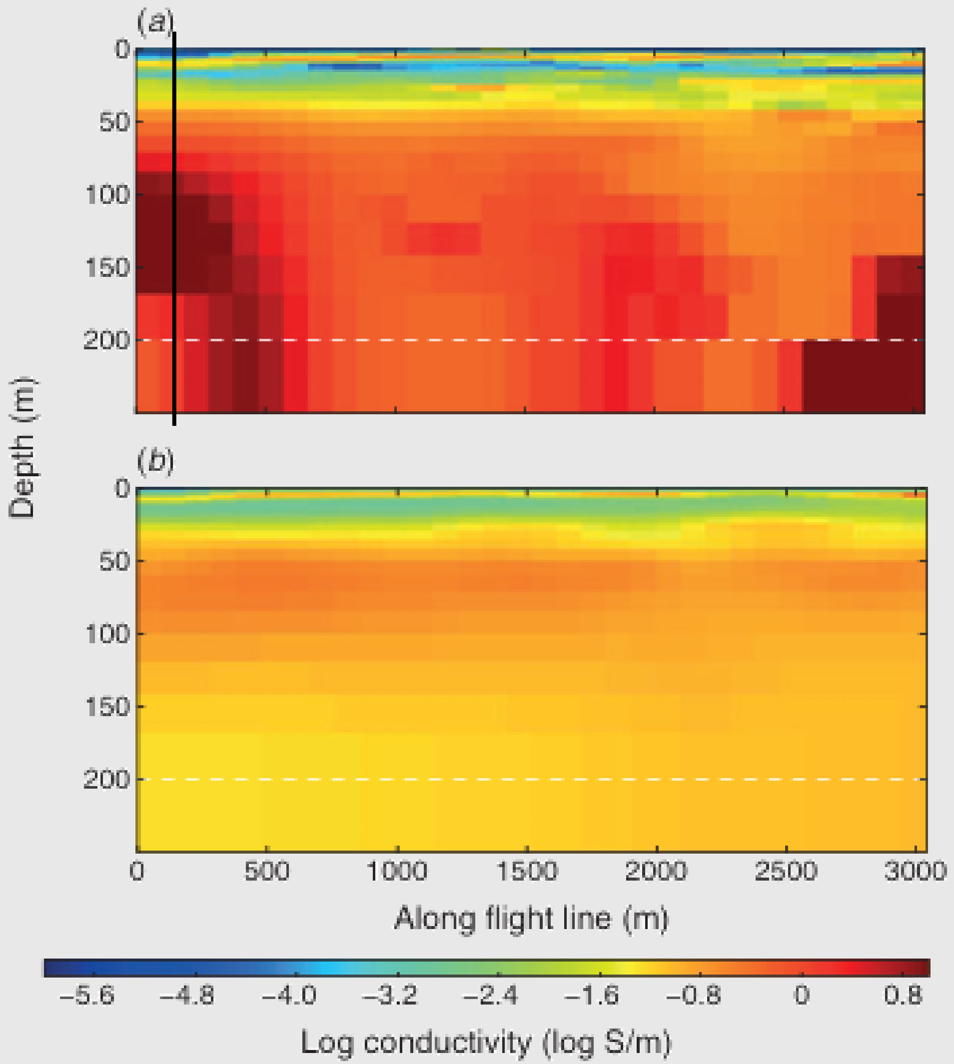


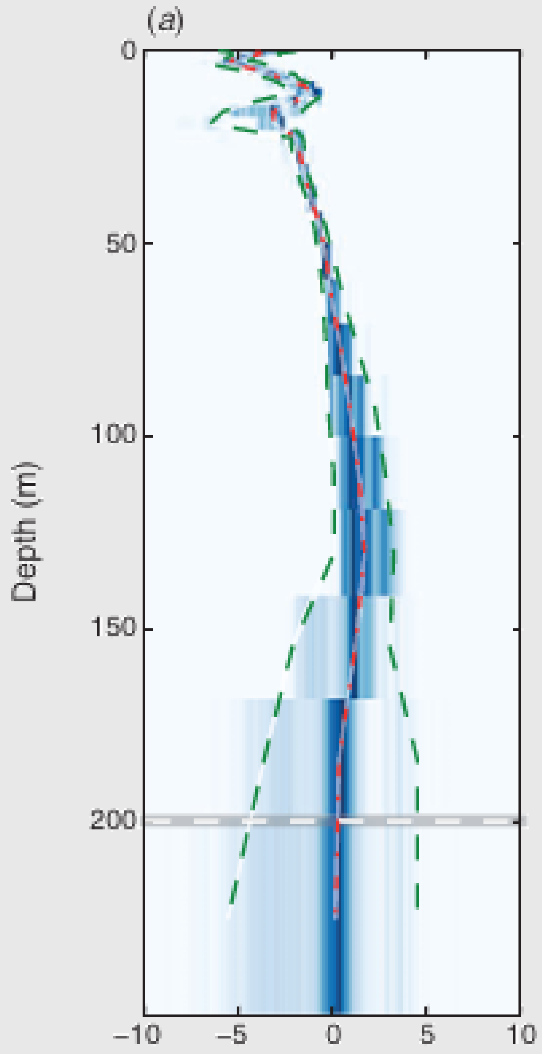
Thank You

somar@mines.edu

Trans-dimensional Bayesian

Deterministic





Example using airborne electromagnetic data for building 2D conductivity profiles

Hawkins et al, 2018

EARTH MODEL PARAMETERS
OBS DATA and FUNCTIONS FOR CALC DATA
HYPERPARAMETERS
rjMCMC PARAMETERS

VORONOI POSITIONS (km)		
vmin	0	Should cover depths of investigation
vmax	0.25	
perturb_std	0.001	Large enough to climb, small enough to not jump

VELOCITY (km/s)		
vs_position	[0 , .25]	Define depth control points for defining a depth-dependent Vs prior
vs_min	[1.4, 1.4]	
vs_max	[2.4, 2.4]	
vs_perturb_std	0.1	

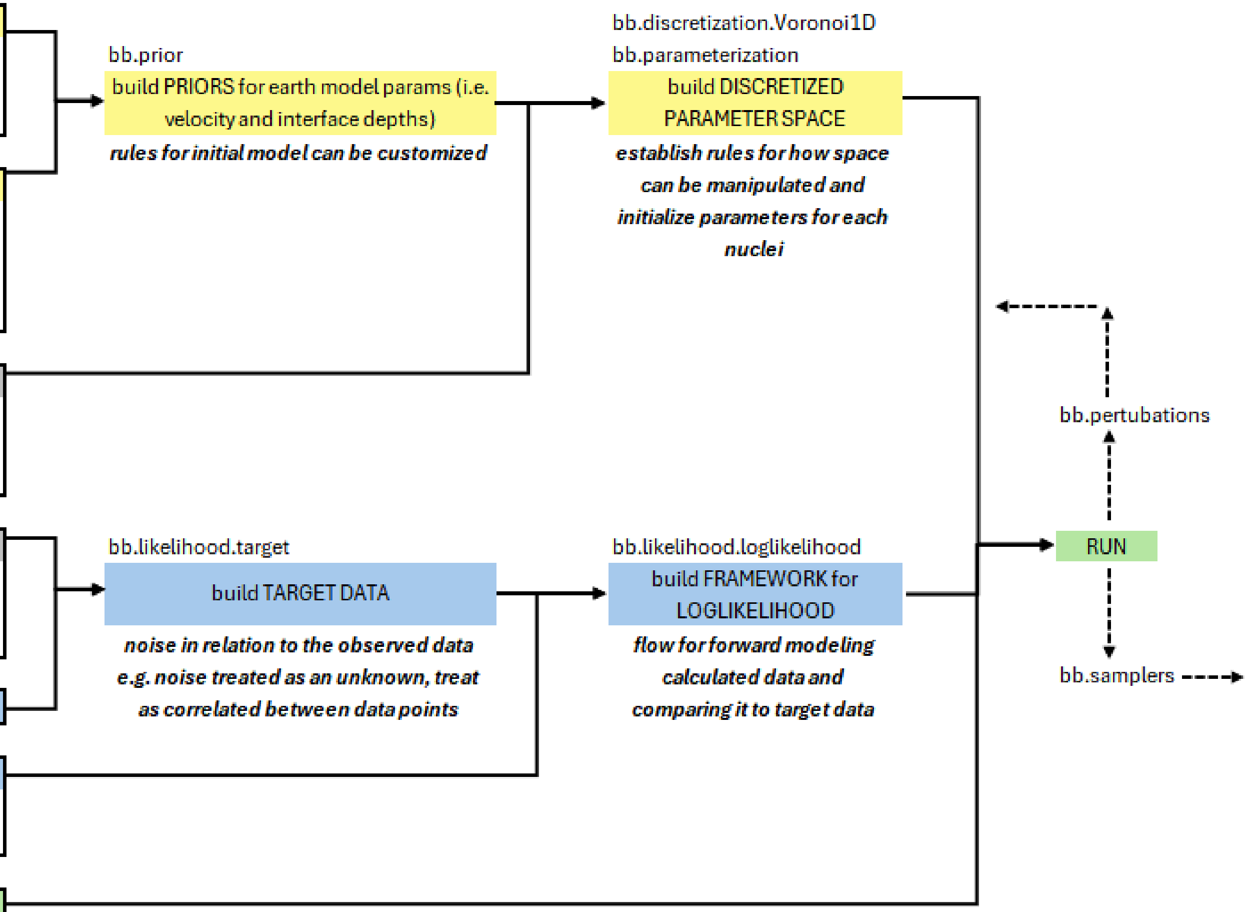
TD PARAMETERS		
n_dimensions	None	If set, then not transdimensional, and n_dim_min and _max are ignored
n_dimensions_min	2	
n_dimensions_max	8	Too high can lead to too many variations in model space and posteriors can be difficult to interpret

NOISE (units of data)*		
std_min	0.005	Want a range that is within an acceptable misfit
std_max	0.05	
std_perturb_std	0.005	

OBSERVED DATA*		

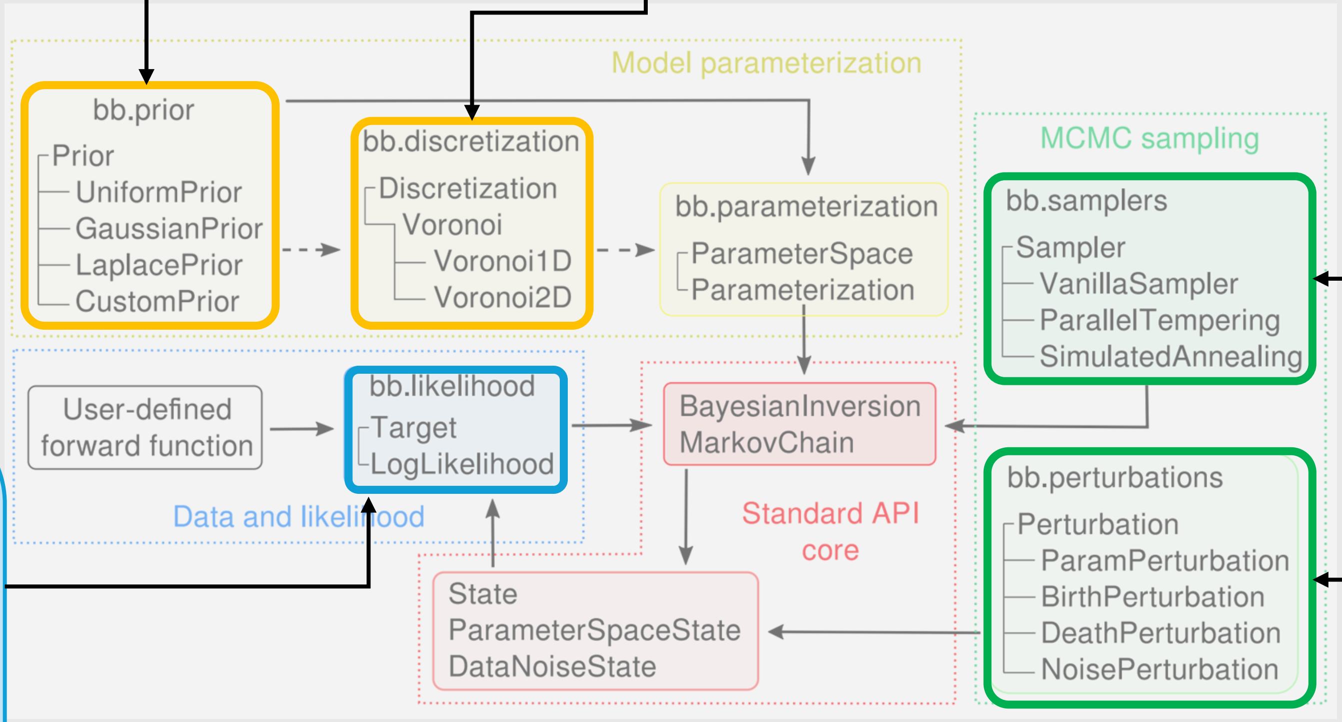
FORWARD MODELER*		
Fucntions for Vp and Rho		
Vr to Vs	disba	

MCMC PARAMETERS		
samplers*		Vanilla or accelerated with parallel tempering, simulated annealing e.g. communicate between chains
n_chains	20	A single chain can dominate median or posterior if this is too small
n_iterations	100000	
burnin_iterations	50	
save_every	100	



bb.prior
User designs/defines a prior for each “free” model parameter

bb.discretization
User defines the number of basis functions (that is, linking the discretization to the dimensionality of “free” parameters)



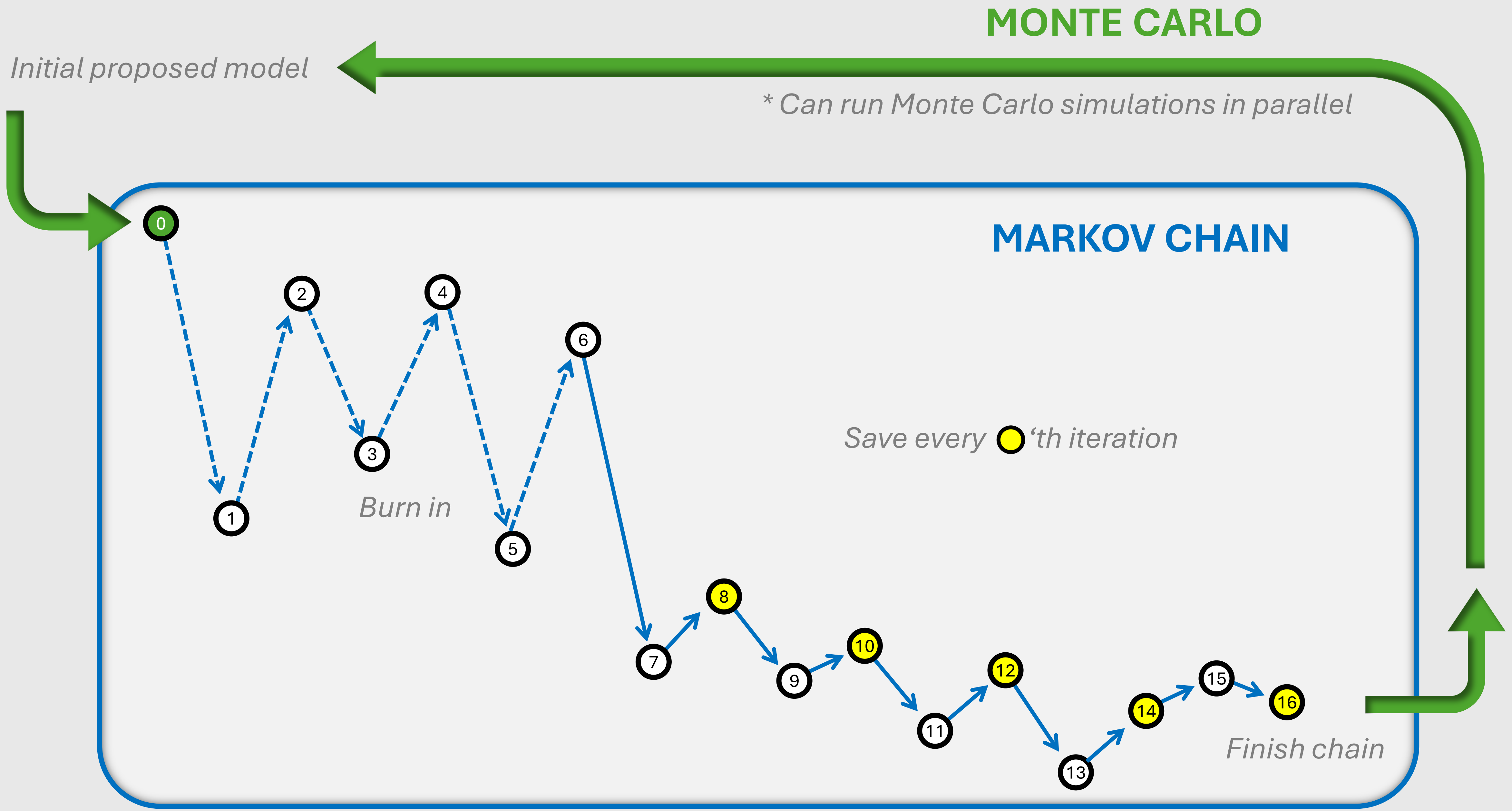
bb.samplers
User can define how to sample each chain
For example, can sample incrementally or can choose an advanced sampler which communicates between chains etc

bb.perturbations
Model is randomly updated using rj-MCMC

Target = each obs data type

bb.likelihood
Target - User defines forward function for each data type

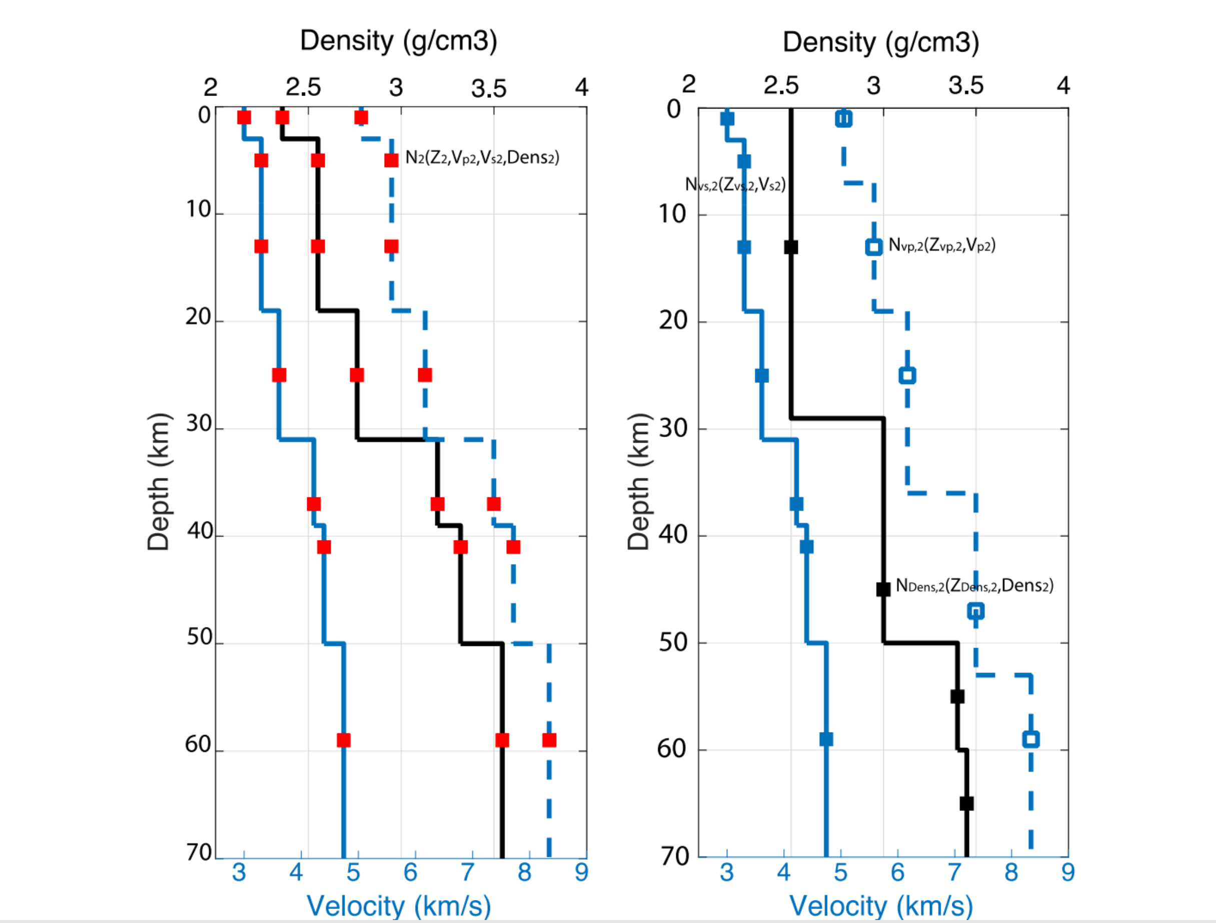
Loglikelhood - Using observed data (target), and forward calculated data, calculates Loglikelihood function for each MonteCarlo iteration



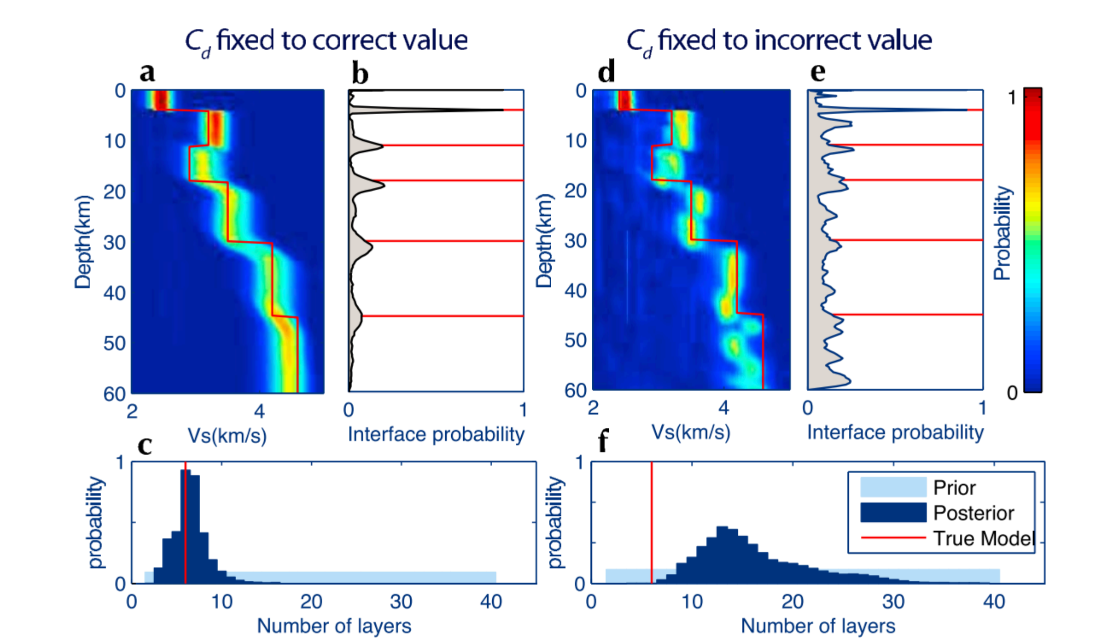
2025 example shown: 24 Markov chains in parallel for 600,000 iterations, saving every 150th model after an initial burn-in of 150,000 iterations.

Model parameter Independence

ATTACHED vs DETACHED NUCLEI



NOISE HYPERPARAMETERIZATION



Acceptance Probability

$$\alpha(m'|m) = \frac{P(d|m')}{P(d|m)} \times \frac{P(m')}{P(m)} \times \frac{q(m|m')}{q(m'|m)} \times |J|$$

Likelihood ratio *Prior ratio* *Proposal ratio* *Jacobian*

Posterior

'reversibility'

scaling between dimensions

Define likelihood: $p(d|m)$

1. Forward model $\boldsymbol{d}_{calc}$
2. Define a misfit
   - based on Mahalanobis distance which accounts for correlated data errors

$$\Phi(m) = (\boldsymbol{d}_{calc} - \boldsymbol{d}_{obs})^T \boldsymbol{C}_{\boldsymbol{e}}^{-\boldsymbol{1}} (\boldsymbol{d}_{calc} - \boldsymbol{d}_{obs})$$

Need a PDF

Assume Gaussian:

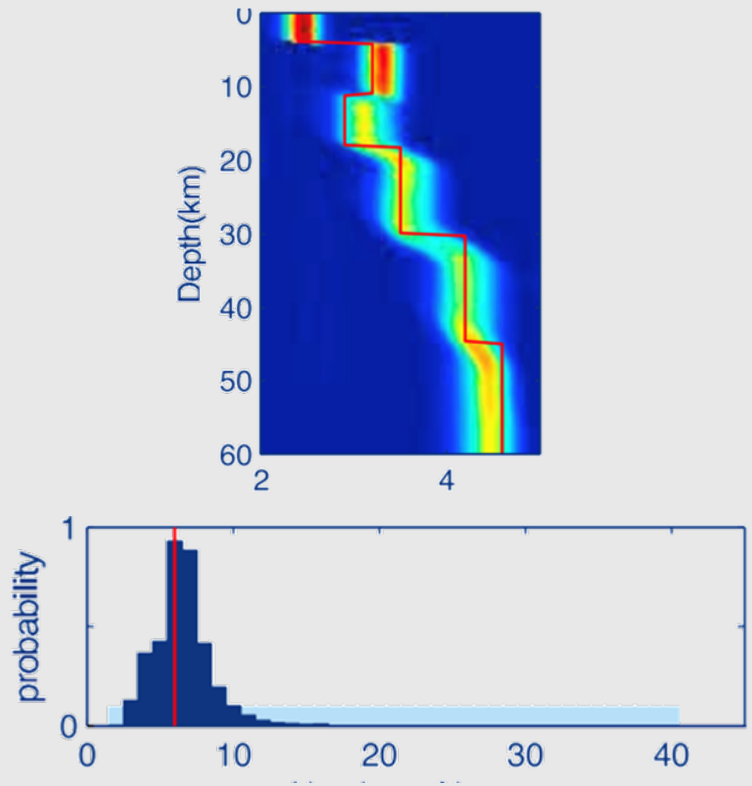
$$P(\boldsymbol{d}_{obs}|\boldsymbol{m}) = \frac{1}{\sqrt{(2\pi)^n |\boldsymbol{C}_{\boldsymbol{e}}|}} e^{-\frac{\Phi(\boldsymbol{m})}{2}}$$

, n = no. of data points

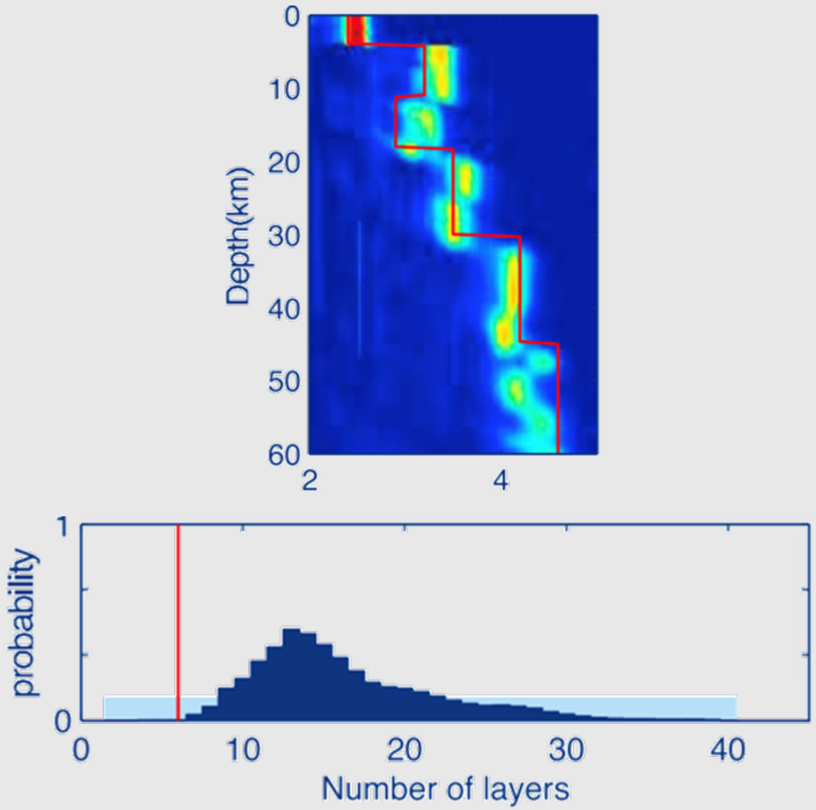
The Covariance Matrix: $\boldsymbol{C_e}$

Bodin et al, 2012

$\boldsymbol{C_e}$ correct



$\boldsymbol{C_e}$ wrong



can fix $\boldsymbol{C_e}$ based on expectation that correlation between data points decay exponentially with increasing distance

OR

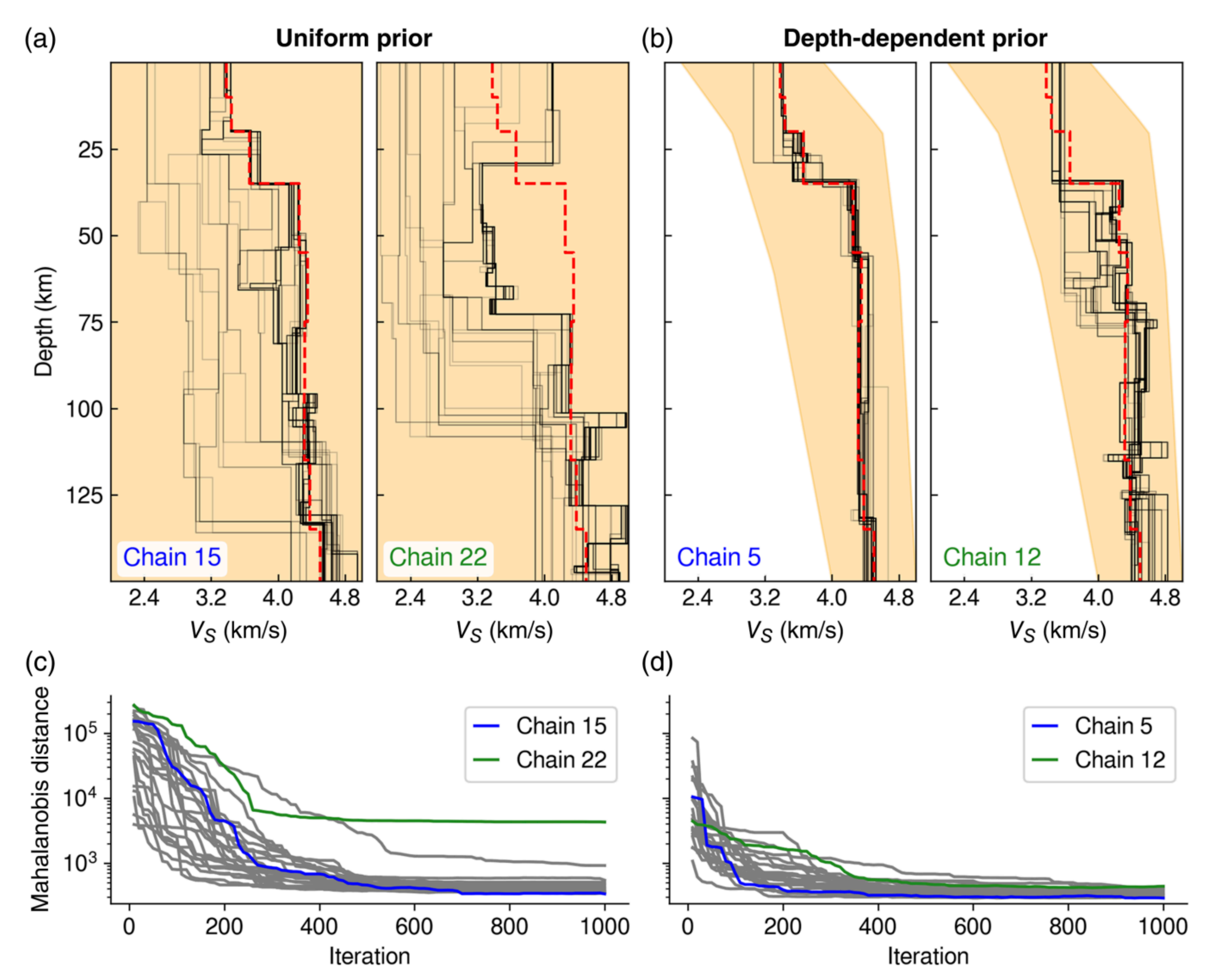
can solve for $\boldsymbol{C_e}$ as an unknown hyperparameter *(Hierarchical bayes)*

Define Prior: $P(\boldsymbol{m})$

- Probability of model parameters being ‘true’
- Assume independent parameters therefore separable:

$$P(\boldsymbol{m}) = P(\boldsymbol{m_e}|k)P(c|k)P(k)P(n)$$

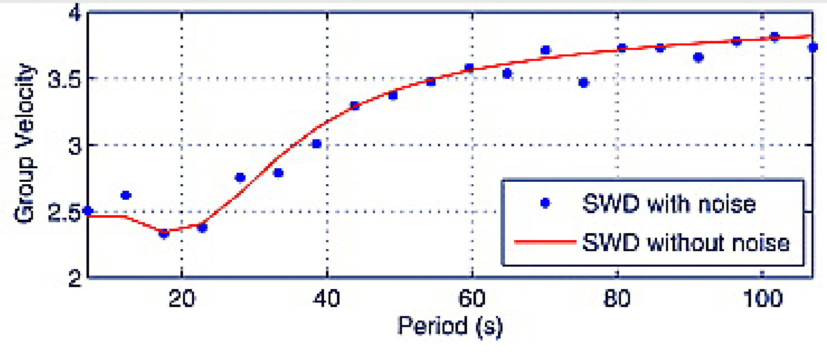
- Cautious to not bias outcome → use a uniform distribution

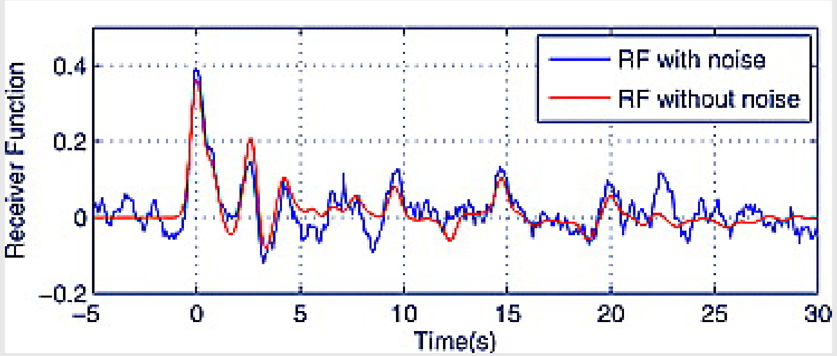


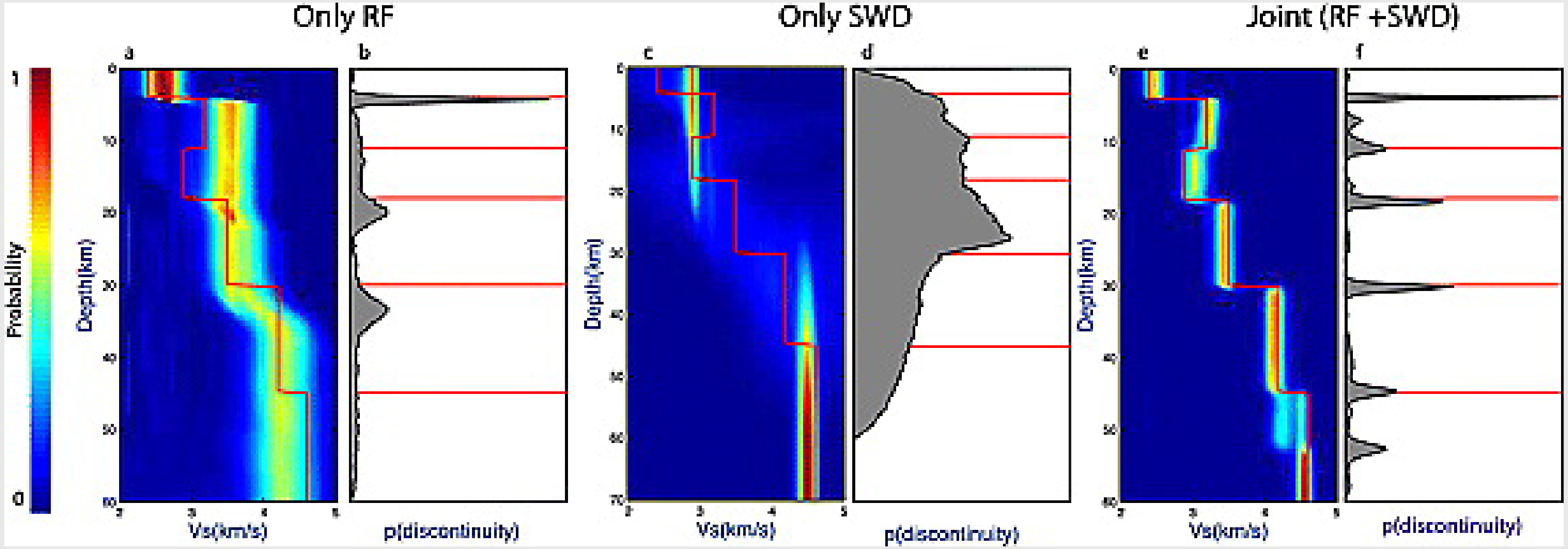
A “well-informed” prior can help reduce the number of iterations in the burn-in period…potentially speeding up MCMC convergence and helping computational cost

Synthetic Data Example

Bodin, Thomas, et al. "Transdimensional inversion of receiver functions and surface wave dispersion." *Journal of geophysical research: solid earth* 117.B2 (2012).

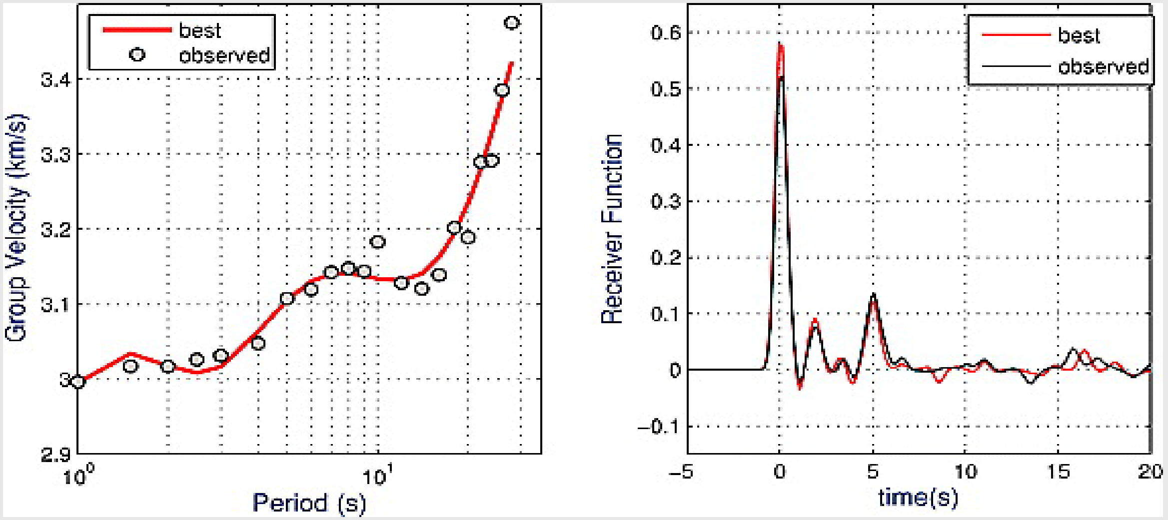


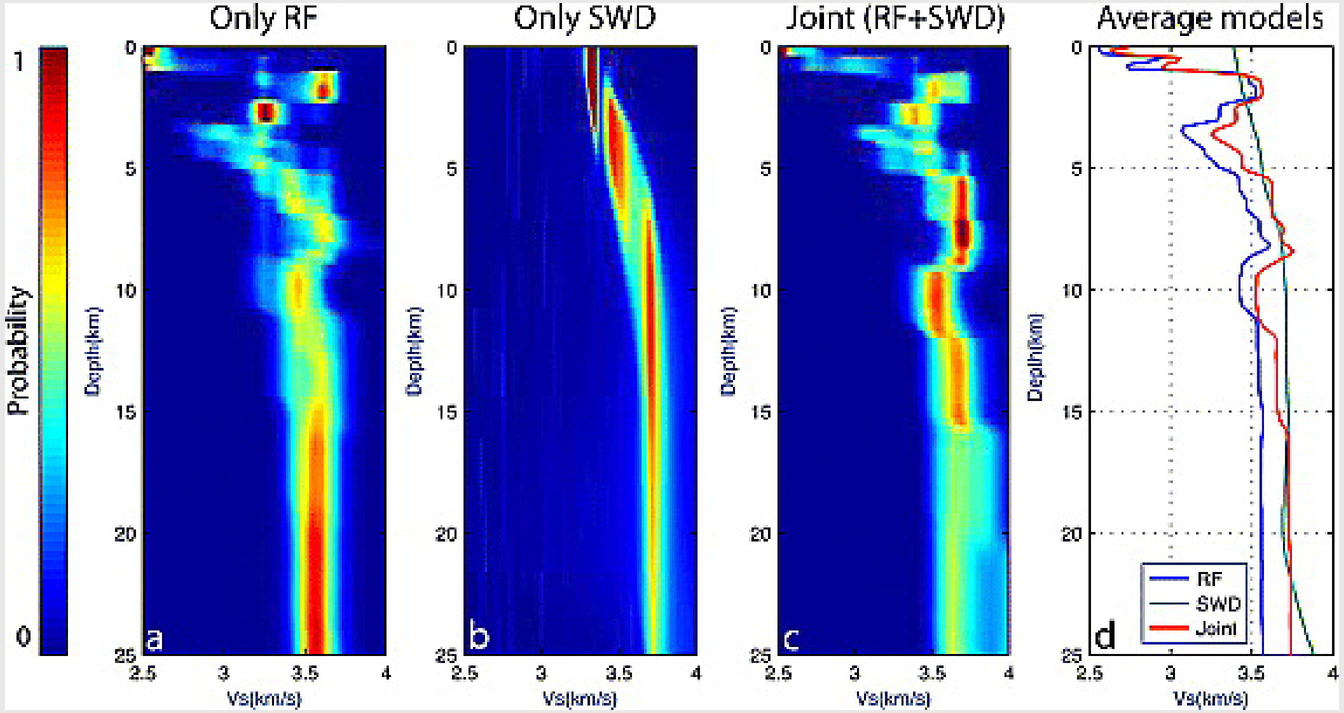




Field Data Example

Bodin, Thomas, et al. "Transdimensional inversion of receiver functions and surface wave dispersion." *Journal of geophysical research: solid earth* 117.B2 (2012).





Transdimensional Bayes Theorem

$$p(m_e, k|d) \;\propto\; p(d|m_e, k) \;\; p(m_e|k)$$

$$p(m_e, k|d) = p(m_e|d, k)\, p(k|d)$$

'variational bayes'

'fixed bayes'

Information provided by the data on the dimension alone

Killingbeck, 2019