H-DES: a Hybrid Differential Equation Solver

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The bottleneck in AI & High Performance Computing

Computing challenges in aerospace...

1 week to **6 months** per intensive simulation for new aircraft designs

... and in many other industries …

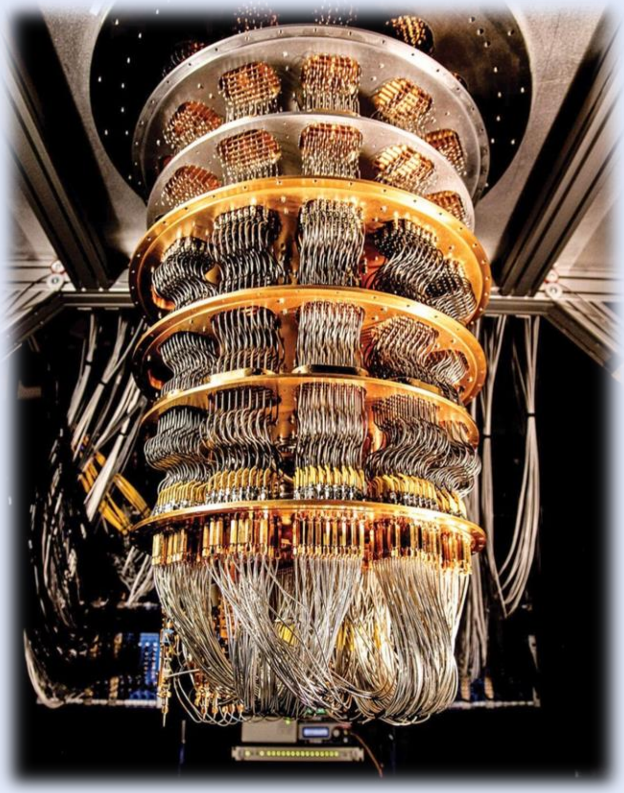
Risk wasting resources on inefficient computations

… with limited current approaches

Classical methods & AI dependence on HPC will slow innovation and impact the environment

Quantum Computing is promising but…

Insufficient quantum computing knowledge among engineers addressing complex problems



Which quantum computer ?

Which algorithms?

Which software, platform, tool, SDK?

How to mitigate errors?

For which use cases?

At which cost?

QUICK: Bringing Quantum Computing to All

Our solution

A bridge between industries
&
quantum computing

Our Customers

NEXT engineering · Agence Innovation Défense · ONERA The French Aerospace Lab · MBDA · Jülich Forschungszentrum · EPITA

Our Hardware Targets & Partners

IBM · IONQ · Quantinuum · rigetti · Quandela* · quobly · |QuEra> · AQT* · PASQAL* · aws · eleQtron

* Available soon

Multi hardware

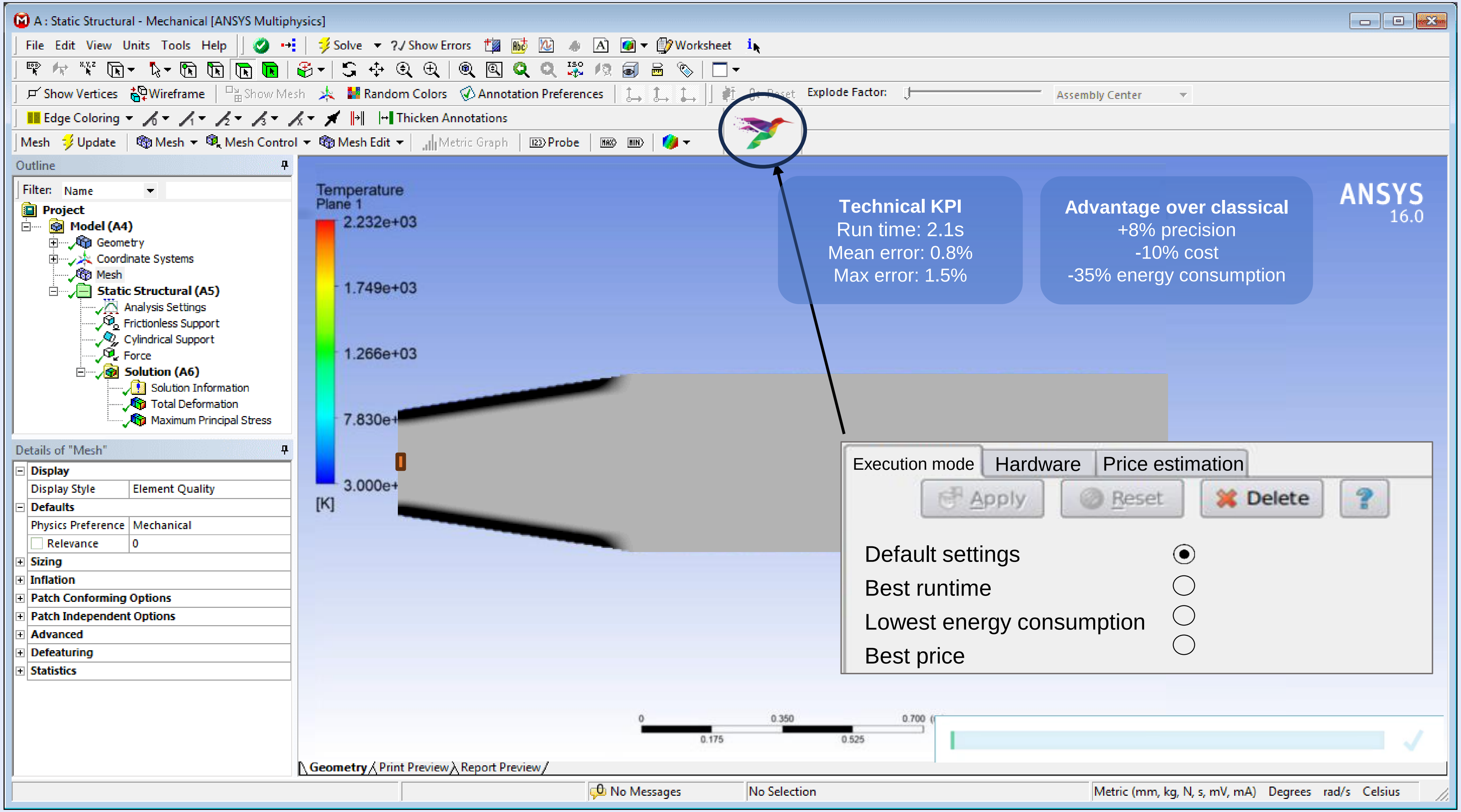
Whatever hardware wins,
Our algorithms will be compatible

QUICK
By ColibrITD

No quantum skills needed to use its advantage
Integrated in favorite simulation software

User friendly

Integrated directly in simulation softwares



What is happening behind

Existing assets

1. API calls from **favorite software** or online **GUI**

Engineer

2. Best algorithm called

H-DES

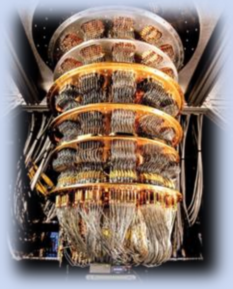
Patent pending

3. Generate Quantum Circuit

MPQP

v0.2

4. QPU choice



QPU

5. Send back results

QUICK by ColibriTD

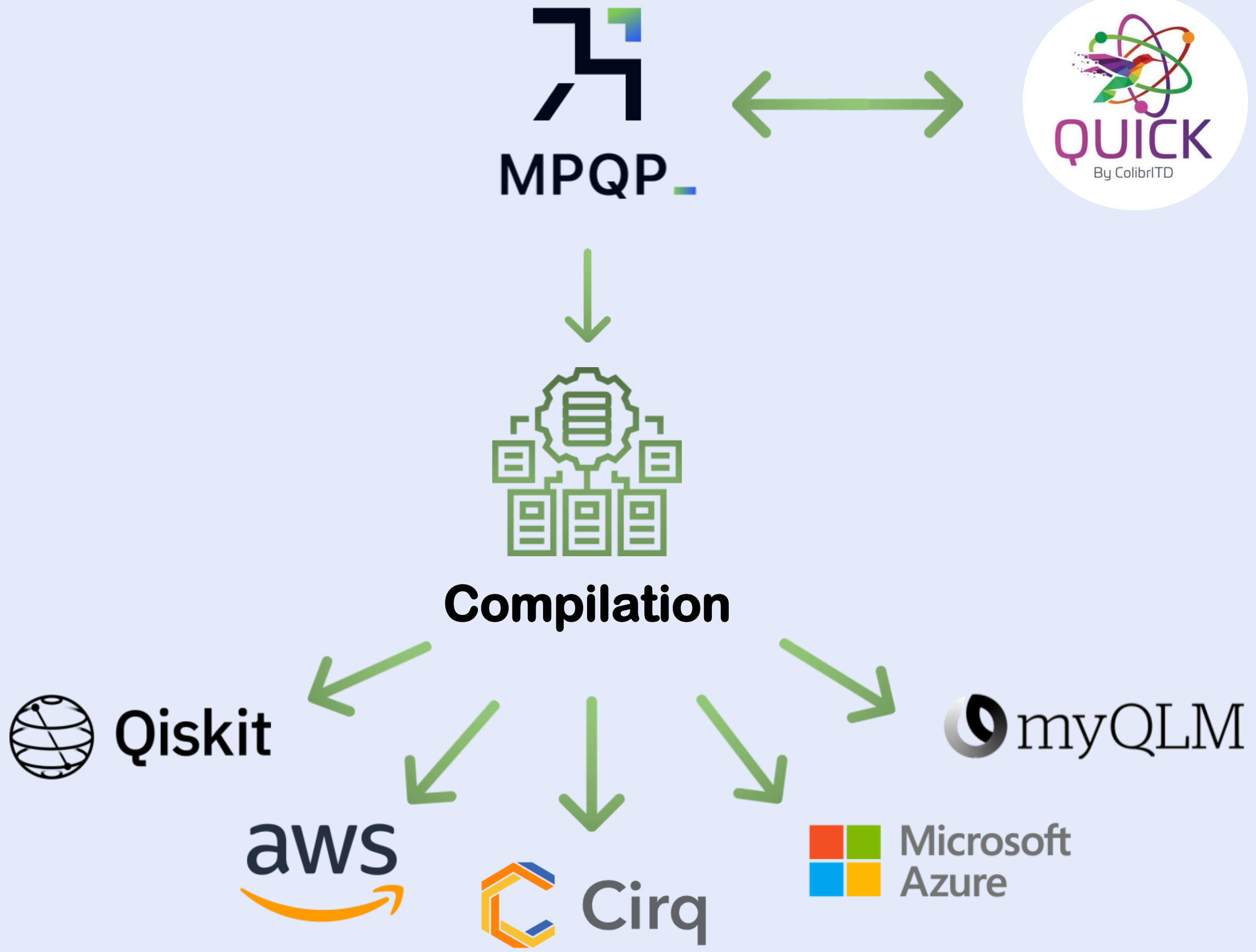
v0.3

6. Package **results** & **KPIs**

MPQP: Multi Platform Quantum Programming

A UNIFIED AND INTUITIVE PROGRAMMING LIBRARY

1. One language, running on all platforms: easy benchmark
2. Changes on provider-side do not affect your code
3. High-level programming framework for education, research and industrial development
4. Combined with QUICK modules



...and more to come!

How to start using it

Checkout our GitHub repo !

```
$ pip install mpqp
```

On GitHub, you can find links to

- Our documentation
- Our Discord server

New release coming soon!

Our Team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& Partners

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icb Laboratoire Interdisciplinaire Carnot de Bourgogne
INSTITUT DE RECHERCHE femto-st SCIENCES & TECHNOLOGIES

Academics

aws

Hardware

IBM

OVHcloud

EVIDEN

Our obsession: solving real-world problems

We focus on solving the underlying mathematical problems behind industrial use cases

Quantum advantage

Energy consumption

Time / Resources

Precision

Focus on Aerospace and Defense

Simulation of combustion

Simulation of fluid dynamics

Simulation of material deformation

Complex phenomena described by systems of nonlinear partial differential equations

Spectral methods

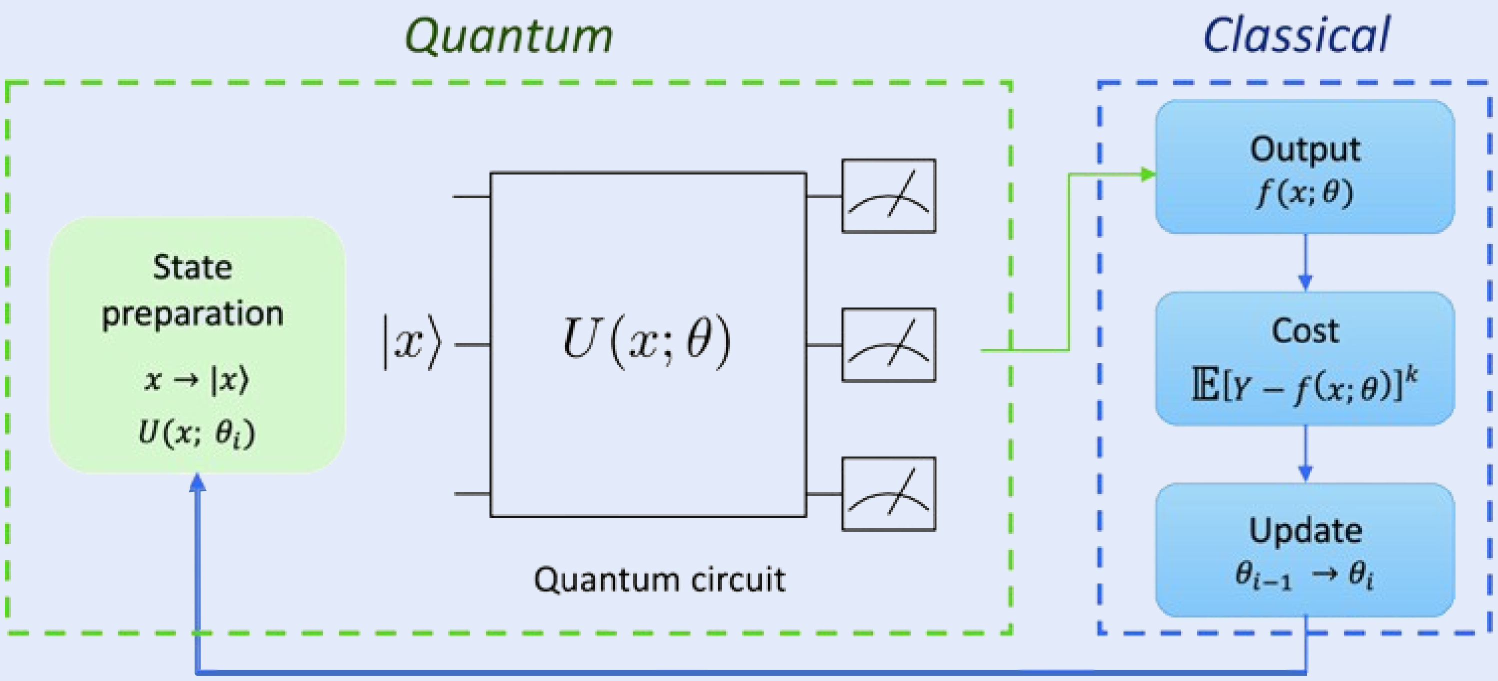
Idea: Approximate the solution of a PDE by a finite linear combination of a chosen set of orthogonal functions $\{P_k\}_k$

$$f(x) = \sum_{k=0}^{C-1} c_k \, P_k(x)$$

An example of widely used basis: Chebyshev polynomials

$$Cheb(k, x) = \begin{cases} cos(k \; arccos \; x) & if \; |x| \leq 1 \\ cosh(k \; arccosh \; x) & if \; x \geq 1 \\ (-1)^k \, cosh\big(k \; arccosh(-x)\big) & if \; x \leq -1 \end{cases}$$

Variational Quantum Algorithms



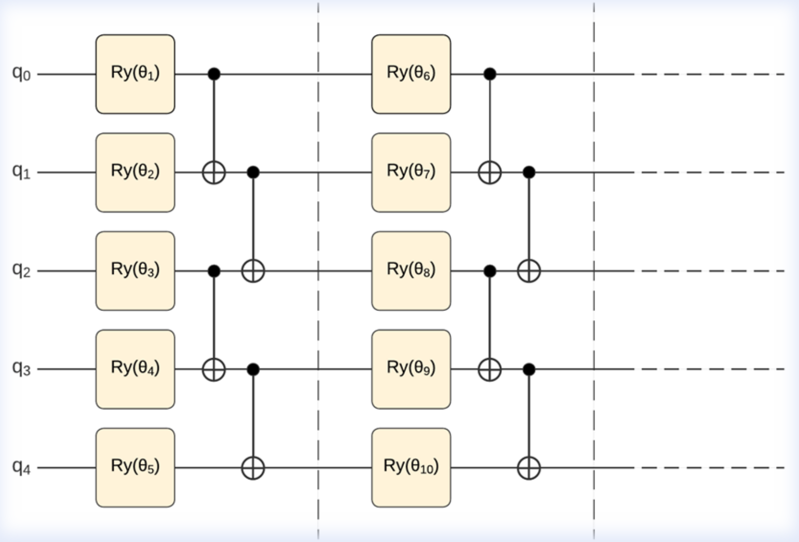
Three main ingredients

- Cost function to encode the problem
- Structure of the parametrized circuit, called Ansatz
- Classical optimizer

- Low depth circuit, better for NISQ
- Uses the best of CPUs/QPUs
- Possibly noise-resilient
- Equivalent of Artificial Neural Networks in ML

The three main components of a VQA

Encoding of the function



$$|\psi_\theta\rangle = U_\theta |0\rangle^{\otimes n} = \sum_{i=0}^{2^n-1} a_i |i\rangle, \quad \text{with} \quad \sum_{i=0}^{2^n-1} |a_i|^2 = 1$$

Example of spectral encoding for 2-qubit state :

$$|a_0|^2 \cdot Cheb(0,x) + |a_1|^2 \cdot Cheb(1,x) - |a_2|^2 \cdot Cheb(0,x) - |a_3|^2 \cdot Cheb(1,x)$$

$$f(x) = \lambda \sum_{i=0}^{2^{n-1}-1} (p_i - p_{i+2^{n-1}}) \cdot Cheb(i,x)$$

Evaluating the function and its derivatives

$$f(x) = \lambda \sum_{i=0}^{2^{n-1}-1} (p_i - p_{i+2^{n-1}}) \cdot Cheb(i,x) = \lambda \langle \psi_\theta | O_c(x) | \psi_\theta \rangle$$

Example of observable for 2-qubit state :

$$O_c(x) = \begin{pmatrix} Cheb(0,x) & 0 & 0 & 0 \\ 0 & Cheb(1,x) & 0 & 0 \\ 0 & 0 & -Cheb(0,x) & 0 \\ 0 & 0 & 0 & -Cheb(1,x) \end{pmatrix}$$

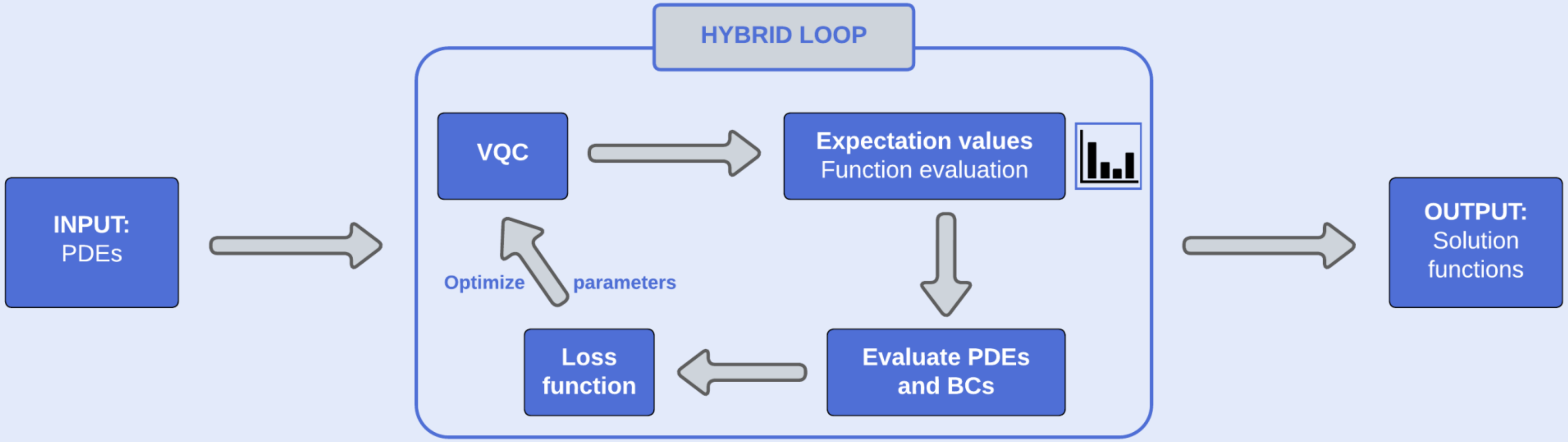
$$O_c(x) = Z \otimes \left(\sum_{i=0}^{2^{n-1}-1} Cheb(i,x) |i\rangle\langle i| \right)$$

For the derivatives, we just change the diagonal coefficients

From solving PDEs to numerical optimization

We use the MSE of the DEs' residuals over a set of sample points $S = \{x_s\}_s$ to define the cost function

$$Cost(\theta) = \underbrace{\frac{1}{n_s} \sum_{e \in E} \sum_{x_s \in S} e(x_s, \theta)^2}_{\text{Error on the DEs}} + \underbrace{\frac{1}{n_{BC}} \sum_{f \in F} \sum_{f_{BC}, x_{BC} \in BC(f)} \left(f(x_{BC}, \theta) - f_{BC}(x_{BC})\right)^2}_{\text{Error on the Boundary conditions}}$$



Easily extendable to PDEs

We consider PDEs made of functions of v variables $X = (x_1, x_2, \dots, x_v)$

We split the basis vectors in small groups of qubits to encode each order or Chebychev polynomials associated with each variable

$$|i\rangle = |i_0\rangle|i_1 i_2 i_3 \dots i_n\rangle = |i_0\rangle|L_1\rangle|L_2\rangle \dots |L_v\rangle$$

$$Cheb(i, X) = \prod_{j=1}^{v} Cheb\left(dec\left(L_j\right), x_j\right)$$

$$O_c(X) = Z \otimes \left(\sum_{i=0}^{2^{n-1}-1} Cheb(i, X)|i\rangle\langle i| \right)$$

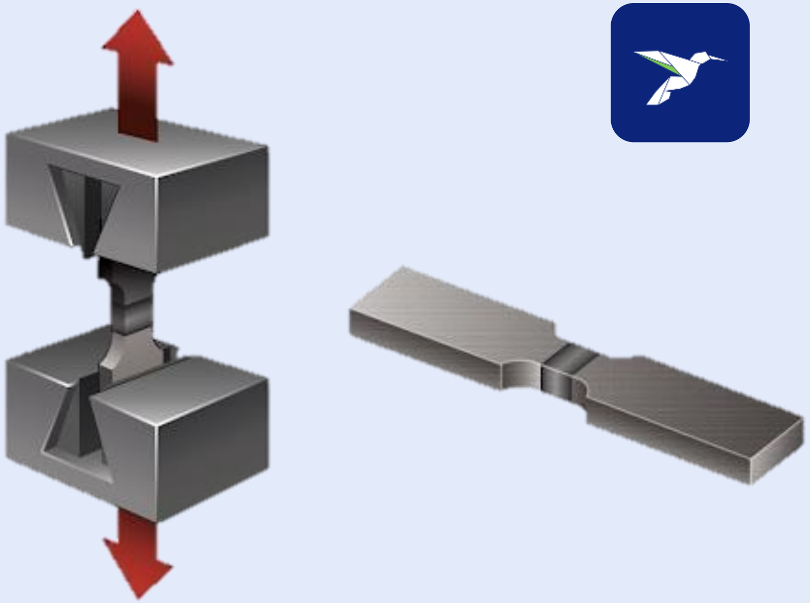
For the derivatives, we can compute the partial derivatives in a clever way reusing the Cheb(L_j,X)s

Example – Tensile test

Used to characterize mechanical properties of a material

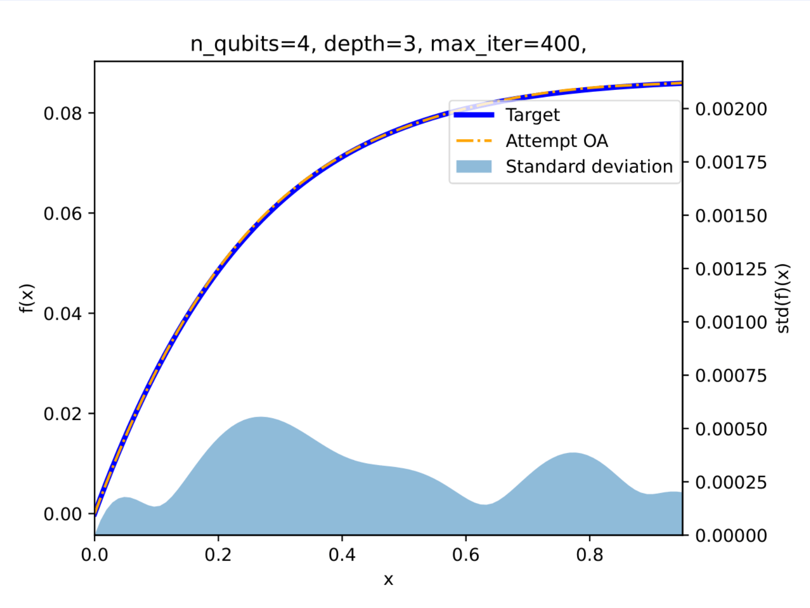
Determine the displacement and the stress-strain relation

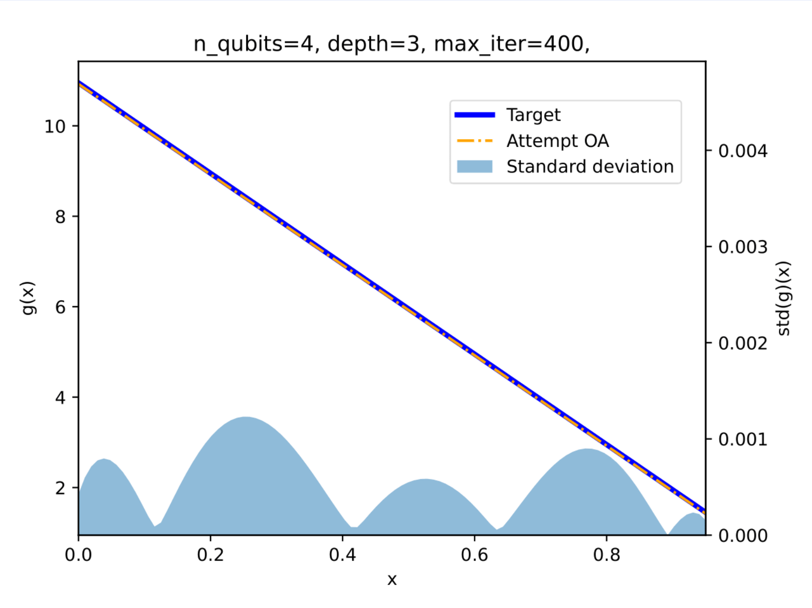
Hypoelastic regime → complex and non-linear elastic responses



$$\begin{cases} \dfrac{du}{dx} = \epsilon_{xx}(\sigma_{xx}) \\ \dfrac{d\sigma_{xx}}{dx} + b_x = 0 \end{cases}$$

$$\epsilon_{xx} = \frac{\sigma_{xx}}{3K} + \frac{2\epsilon_0}{\sqrt{3}}\left(\frac{\sigma_{xx}}{\sqrt{3}\sigma_0}\right)$$





Some differential equations solved with H-DES so far

Linear

1st order

- Continuous function approximation
- Non-homogenous DE with variable coefficients
- *2 coupled* homogenous DEs

2nd order

- Non-homogenous DEs with variable coefficients (e.g. forced harmonic oscillator with time-varying damping)
- 2 coupled homogenous DEs with constant coefficients (e.g. coupled damped harmonic oscillators)

Non-linear

1st order

- *2 coupled* non-homogenous DEs (e.g. material deformation under stress)
- *2 coupled* homogenous DEs (e.g. isothermal pressureless Euler equations with time-dependent damping)
- *3 coupled* non-homogenous DEs (e.g. sub- and super-sonic fluid dynamics in a nozzle)

2nd order

- Homogenous DE (e.g. viscid Burger equation)
- *2 coupled* homogenous stiff DEs (e.g. simplified 1D combustion model)

1D

2D

Advantages of our approach

Takes as input any linear and non-linear, coupled, system,
of ODE/PDE, boundary conditions

Requires low number of qubits to reach a high precision on the solution

No approximation on all the derivatives of the solution functions

Only one circuit to represent the solution function and all its derivatives

Low depth circuits thanks to a VQA approach

Strategies to reduce measurements on the circuit, perfect for NISQ era

Scaling properties

Number of qubits grows linearly with the number of equations and variables, for a given precision

Number of basis function in spectral decomposition grows exponentially with the number of qubits

Number of parameters to optimize grows linearly with the number of qubits

Increasing the number of sample points only affects the classical post-processing thanks to a clever expectation value evaluation

Future directions

Initialization strategy

Extension to stochastic or integro-differential equations

Study the entanglement generated by the Ansatz

Noisy classical optimizers

Other approaches for solving PDEs (Monte Carlo, FTQC, …)

Error mitigations techniques

Leverage symmetries of the PDEs

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Thank you for your attention

Open for collaboration

Academic / Research
Hardware providers
Industrial partners
Investors

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