ISM Assignment - 1

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Solutions:-

1. A] Given, total population (N) = 6

average (or) mean (μ) = 9

variance (σ^2) = 11.6666

The marks of 4 students are 4, 8, 10, 12

To find the marks of remaining two students, from the given data, $\Rightarrow$ average = $\frac{\text{Total sum of population}}{\text{Number of population}} = \frac{\Sigma x_i}{N}$

let the other two students be a_1 and a_2, so

$$9 = \frac{4+8+10+12+a_1+a_2}{6}$$

$$54 = 34 + a_1 + a_2 \Rightarrow a_1 + a_2 = 20 \longrightarrow (1)$$

$\Rightarrow$ Variance $\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{N}$

$$11.6666 = \frac{(4-9)^2+(8-9)^2+(10-9)^2+(12-9)^2+(a_1-9)^2+(a_2-9)^2}{6}$$

$$69.9996 = 25+1+1+9+(a_1-9)^2+(a_2-9)^2$$

$(\because (a-b)^2 = a^2+b^2-2ab)$

$$= 36 + a_1^2 + 81 - 2.9.a_1 + a_2^2 + 81 - 2.9.a_2$$

$$= 198 + a_1^2 + a_2^2 - 18a_1 - 18a_2$$

$$69.9996 = 198 + a_1^2 + a_2^2 - 18(a_1+a_2) \longrightarrow (2)$$

Substituting eq. (1) in (2)

$$= 198 + a_1^2 + a_2^2 - 18 \times 20$$

$$69.9996 = -162 + a_1^2 + a_2^2$$

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$a_1^2 + a_2^2 = 231.9996 \rightarrow (3)$

since it is here, we rearrange this as

$a_1^2 = 231.9996 - a_2^2$

$a_1 = \sqrt{231.9996 - a_2^2}$ (∵ from eq (1), $a_1 + a_2 = 20$, $a_1 = 20 - a_2$)

$20 - a_2 = \sqrt{231.9996 - a_2^2}$

Squaring on both sides,

$(20 - a_2)^2 = 231.9996 - a_2^2$

$(20)^2 + a_2^2 - 2 \cdot 20 \cdot a_2 = 231.9996 - a_2^2$

$400 + a_2^2 - 40a_2 = 231.9996 - a_2^2$

$2a_2^2 - 40a_2 + 400 = 231.9996$

$2a_2^2 - 40a_2 + 168.0004 = 0 \rightarrow (4)$

eq (4) is in form of quadratic eqn, solving this
[∵ $ax^2 + bx + c = 0$]

Roots of eq (4) are, $a_2 = 14$

$a_2 = 6.0004 \simeq 6$

So substituting $a_2 = 14$ (or) 6 in eq (1) $a_1 + a_2 = 20$

$a_1 + 14 = 20$ if $a_1 + 6 = 20$

$a_1 = 6$ $a_1 = 14$

So, $a_1 = 6$ and $a_2 = 14$

The marks of remaining two students are 6 and 14 //

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2.

a) Given, the probability that a person visits Reliance Mart

$P(R) = 0.2$

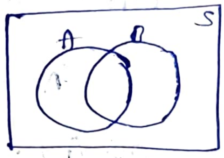
the probability that he visits Croma is $P(C) = 0.25$

the probability of visiting Reliance (or) Croma is $P(R \cup C) = 0.60$

from the given data,

using additive Rules, theorem

If A, B are two events, then



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

here, $P(R \cup C) = P(R) + P(C) - P(R \cap C)$

$\therefore P(R \cap C) = P(R) + P(C) - P(R \cup C)$

$= 0.2 + 0.25 - 0.60$

$P(R \cap C) = -0.15$!

Here the probability is negative, but it shouldn't be negative

since, probability of any event lies between 0 to 1

$\Rightarrow 0 \leq P(E) \leq 1$

Also, the probability of union of two events cannot exceed the sum of their individual probabilities.

$P(A) + P(B) > P(A \cup B)$

If A, B are mutually exclusive then, $P(A) + P(B) = P(A \cup B)$

But here $P(R \cup C) > P(R) + P(C)$

$0.60 > 0.20 + 0.25$

$0.60 > 0.45$

So, as per the additive Rules of probability theorems,

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given $P(R) = 0.4$, $P(C) = 0.25$, $P(R \cup C) = 0.60$ is invalid.

$\because$ $P(R \cup C) \not\leq P(R) + P(C)$

Since it violates above Rule and $[P(E) \geq 0, \leq 1]$

b) Given $P(\bar{A}/B) = 1 - P(A/B) \longrightarrow (1)$

Given, A and B are two events

$\bar{A}$ represents the complement of event A; A doesn't occur

$P(\bar{A}/B)$ = conditional prob. that $\bar{A}$ occurs given B occured

$P(A/B)$ = conditional prob. that A occurs given B occured

To justify eq(1),

$$P(\bar{A}|B) = 1 - P(A/B)$$

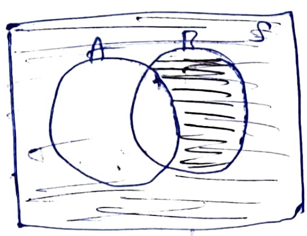
$$= 1 - \frac{P(A \cap B)}{P(B)}$$

($\because$ By conditional Prob, $P(A|B) = \frac{P(A \cap B)}{P(B)}$) $\rightarrow (2)$

$$= \frac{\cancel{P(B)}}{\cancel{P(B)}} - \frac{P(A \cap B)}{P(B)}$$

$$= 1 - \frac{P(A \cap B)}{P(B)}$$

$$\boxed{P(\bar{A}/B) = 1 - P(A/B)}$$



also, from eq(2),

$$P(\bar{A}/B) = \frac{P(\bar{A} \cap B)}{P(B)}$$

from venn diagram, $\Rightarrow P(\bar{A} \cap B) = P(B) - P(A \cap B)$

So, $P(\bar{A}|B) = \frac{P(B) - P(A \cap B)}{P(B)} = 1 - \frac{P(A \cap B)}{P(B)}$

here, $\boxed{P(\bar{A}/B) = 1 - P(A/B)}$ ($\because$ from eq. 2)

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another way is, from eq ①, $P(\bar{A}/B) + P(A/B) = 1$

[from venn diagram, $P(\bar{A} \cap B) + P(A \cap B) = P(B)$]

$$\frac{P(\bar{A} \cap B)}{P(B)} + \frac{P(A \cap B)}{P(B)}$$

$$\frac{P(\bar{A} \cap B) + P(A \cap B)}{P(B)} = \frac{\cancel{P(B)}}{\cancel{P(B)}} = 1$$

Hence justified.

3.

A) Given data,

Probability of A on the job $P(A) = 0.50$

Probability of B on the job $P(B) = 0.30$

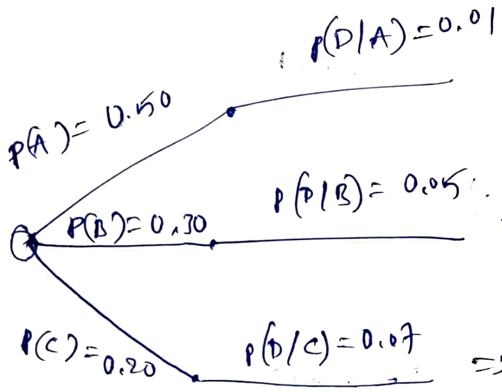
Probability of C on the job $P(C) = 0.20$

($\because P(A) + P(B) + P(C) = 1$)

Probability of A produce defective $P(D/A) = 0.01$

Probability of B produce defective $P(D/B) = 0.05$

Probability of C produce defective $P(D/C) = 0.07$



$\Rightarrow P(A) \cdot P(D/A) = 0.50 \times 0.01 = 0.005$

$\Rightarrow P(B) \cdot P(D/B) = 0.30 \times 0.05 = 0.015$

$\Rightarrow P(C) \cdot P(D/C) = 0.20 \times 0.07 = 0.014$

To find the probability of defective from the three operators is given by Total Probability theorem:

$$P(A) = \sum_{i=1}^{k} P(B_i)\, P(A/B_i)$$

here,

$$P(D) = P(A) \cdot P(D/A) + P(B) \cdot P(D/B) + P(C) \cdot P(D/C)$$

$$= 0.005 + 0.015 + 0.014.$$

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Hence, probability of defective $P(D) = 0.034$

To find the probability of producing a defective item from producer/operator A, B, C is found by Baye's theorem.

$$P(B_r/A) = \frac{P(B_r)\, P(A|B_r)}{\sum_{i=1}^{k} P(B_i) \cdot P(A|B_i)}$$

Here, Probability of defective item produced by A is

$$P(A|D) = \frac{P(A)\, P(D|A)}{P(D)} \quad (P(D) = \text{total Probability})$$

$$P(A|D) = \frac{0.50 \times 0.01}{0.034} = 0.147$$

lly,

$$P(B|D) = \frac{P(B) \cdot P(D|B)}{P(D)}$$

$$= \frac{0.30 \times 0.05}{0.034} = 0.441$$

$$P(C|D) = \frac{P(C) \cdot P(D|C)}{P(D)}$$

$$= \frac{0.20 \times 0.07}{0.034} = 0.412$$

Based on these, we can observe that, the probability of getting defective item from operator B is higher compared to other two operators, the prob. is; so more defectives may come from B.

$$\boxed{P(B|D) = 0.441}$$

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and the probability of getting defectives from A is very low,

i.e. $\boxed{P(A|D) = 0.147}$

So, less defectives produce from operator A.

4.

A) Given, A, B are two events,

$P(A) = 0.38, \quad P(B) = 0.63, \quad P(A \cup B) = 0.78$

We know, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = 0.38 + 0.63 - P(A \cap B)$

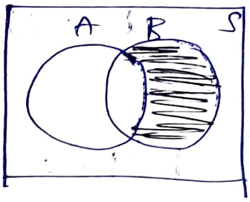
$P(A \cap B) = 0.38 + 0.63 - 0.78$

$P(A \cap B) = \underline{0.23}$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.23}{0.63}$

$\boxed{P(A|B) = 0.36507}$

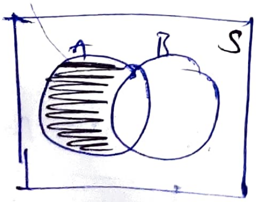
$P(B|\bar{A}) = \dfrac{P(B \cap \bar{A})}{P(\bar{A})} = \dfrac{P(B) - P(A \cap B)}{1 - P(A)}$



$= \dfrac{0.63 - 0.23}{1 - 0.38}$ (∵ from venn diagram)

$\boxed{P(B|\bar{A}) = 0.64516}$

$P(A \cap \bar{B}) = P(A) - P(A \cap B)$



$= 0.38 - 0.23$

$\boxed{P(A \cap \bar{B}) = 0.15}$

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$P(\bar{A} \cup \bar{B}) = P(A \cap B)'$ ($\because$ By using probability Rules) $P(A \cap B') = P(A' \cup B')$

$= 1 - P(A \cap B)$ $\because P(A') = 1 - P(A)$

$= 1 - 0.23$

$\boxed{P(\bar{A} \cup \bar{B}) = 0.77}$

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5. A) From given data,

Total population $N = 1300$

No. of families with 2 boys $= 325$

No. of families with 1 boy $= 761$

No. of families with 0 boys $= 214$

The probability of a family, chosen at random having;

(i) $P(2\text{ Boys}) = \dfrac{\text{No. of families with 2 boys}}{\text{Total no. of families}}$

$P(2\text{ Boys}) = \dfrac{325}{1300} \Rightarrow \boxed{P(2\text{ Boys}) = 0.25} \rightarrow (1)$

(ii) $P(1\text{ Boy}) = \dfrac{\text{No. of families with 1 boy}}{\text{Total no. of families}}$

$= \dfrac{761}{1300} \Rightarrow \boxed{P(1\text{ Boy}) = 0.585} \rightarrow (2)$

(iii) $P(0\text{ Boys}) = \dfrac{\text{No. of families with 0 boys}}{\text{Total no. of families}}$

$= \dfrac{214}{1300} \Rightarrow \boxed{P(0\text{ Boys}) = 0.165} \rightarrow (3)$

Adding these probabilities, (1) + (2) + (3)

$\boxed{0.25 + 0.585 + 0.165 = 1}$ $\therefore$ Hence equal to 1