Introduction to Statistical Methods

S1-23_AIMLCZC418 – Assignment 2

AIML Section - 3

Each question carries 2.5 Marks (4 x 2.5 =10Marks)

1. A company has the head office at Chennai and a branch at Coimbatore. The personnel director wanted to know if the workers at the two places would like the introduction of a new plan of work and a survey was conducted for this purpose. Out of a sample of 500 workers at Chennai, 62% favoured the new plan. At Coimbatore out of a sample of 400 workers, 41% were against the new plan. Is there any significant difference between the two groups in their attitude towards the new plan at 5% level?

SOLUTION:

Solution : Given : $n_1 = 500, \quad p_1 = 0.62$

$n_2 = 400, \quad p_2 = 1 - 0.41 = 0.59$

$$P = \frac{n_1p_1 + n_2p_2}{n_1 + n_2} = \frac{(500)(0.62) + (400)(0.59)}{500 + 400} = 0.607$$

$$Q = 1 - P = 1 - 0.607 = 0.393$$

1. The parameter of interest is P_1, P_2
2. $H_o : P_1 = P_2$ [there is no significant difference between the two groups in their attitude towards the new plan]
3. $H_1 : P_1 \neq P_2$ [Two-tailed]
4. $\alpha = 0.05$
5. The test statistic is $Z = \dfrac{p_1 - p_2}{\sqrt{PQ\left(\dfrac{1}{n_1} + \dfrac{1}{n_2}\right)}}$
6. Reject H_o if $|Z| > 1.96$ at 5% level.
7. **Computations :**

$$Z = \frac{0.62 - 0.59}{\sqrt{(0.607)(0.393)\left(\frac{1}{500} + \frac{1}{400}\right)}} = \frac{0.03}{\sqrt{0.00107}} = 0.917$$

Conclusion: Since $|Z|=0.917< 1.96$, so we accept H_0 at 5% Level of Significance.

Hence, there is no significant difference between the two groups in their attitude towards the new plan.

2. A soap manufacturing company was distributing a particular brand of soap through a large number of retail shops. Before a heavy advertisement campaign, the mean sales per week per shop was 140 dozen. After the campaign a sample of 26 shops was taken and the mean sales was found to be 147 dozen with a standard deviation of 16 dozen. Is the advertisement effective?

SOLUTION:

Solution:

1) Given: $n = 26$, $\bar{x} = 147$ dozens, $s = 16$ dozens,

$\mu = 140$ dozens.

2) $H_0 : \mu = 140$ (The advertisement campaign is not effective)

3) $H_1 : \mu > 140$ (The advertisement is effective)

4) $\alpha = 0.05$, d.f $= n-1 = 26-1 = 25$

$\therefore \nu = 25$

5) The test statistic, $t = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}}$

$$\therefore t_{cal} = \frac{147-140}{16/\sqrt{25}} = \frac{35}{16} = 2.1875$$

6) t-value for $\alpha = 5\%$, $\nu = 25$ is 1.708 (from table)

7) Conclusion: since $|t_{cal}| > t_\alpha$, we reject H_0.

$\therefore$ we conclude that the advertising campaign is effective.

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3. In 120 throws of a single die, the following distribution of faces was observed.

Face	1	2	3	4	5	6
Frequency	30	25	18	10	22	15

Can you say that the die is biased?

SOLUTION:

Solution : **Given** : $n = 6$

1. The variable of interest is to test if the die is biased.
2. H_o : The die is unbiased.
3. H_1 : The die is biased.
4. $\alpha = 0.05$, d.f $= n - 1 = 6 - 1 = 5$
5. **The test statistic is** $\chi^2 = \Sigma \frac{(O-E)^2}{E}$
6. **Reject** H_o if $\chi^2 > 11.07$ [from table]
7. Computions :

On the assumption H_o, the expected frequency for each face $=$ $120 \times \frac{1}{6} = 20$

Face	O	E	O – E	$(O-E)^2$
1	30	20	10	100
2	25	20	5	25
3	18	20	–2	4
4	10	20	–10	100
5	22	20	2	4
6	15	20	–5	25
	120	120		258

$$\chi_c^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$= 258/ 20 = 12.9$

Conclusion: Since calculated chi-square (12.9)> 11.07 (table value), **we reject H_0 at 5% Level of Significance**. Hence, the die can be regarded as biased.

4. In a comparison of the cleaning action of four detergents, 20 pieces of white cloth were first soiled with India ink. The clothes were then washed under controlled conditions with 5 pieces washed by each of the detergents. Unfortunately, three pieces of cloth were 'lost' in the course of the experiment. Whiteness readings, made on the 17 remaining pieces of cloth are shown below:

Detergent			
A	**B**	**C**	**D**
77	74	73	76
81	66	78	85
61	58	57	77
76		69	64
69		63	

Assuming all whiteness readings to be normally distributed with common variance, test the hypothesis of no difference between the four brands as regards mean whiteness readingafter washing.

SOLUTION:

Solution

H_0: no difference in mean readings $\quad \mu_i = \mu$ all i

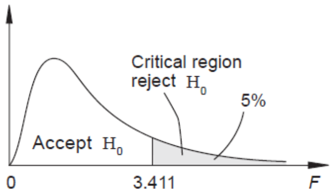
H_1: a difference in mean readings $\quad \mu_i \neq \mu$ some i

Significance level, $\alpha = 0.05$ (say)

Degrees of freedom, $v_1 = k - 1 = 3$

and $v_2 = n - k = 17 - 4 = 13$

Critical region is $\quad F > 3.411$



	A	B	C	D	Total
n_i	5	3	5	4	$17 = n$
T_i	364	198	340	302	$1204 = T$

$$\sum\sum x_{ij}^2 = 86362$$

$$SS_T = 86362 - \frac{1204^2}{17} = 1090.47$$

$$SS_B = \left(\frac{364^2}{5} + \frac{198^2}{3} + \frac{340^2}{5} + \frac{302^2}{4}\right) - \frac{1204^2}{17} = 216.67$$

$$SS_W = 1090.47 - 216.67 = 873.80$$

The anova table is now as follows.

Source of variation	Sum of squares	Degrees of freedom	Mean square	*F* ratio
Between detergents	216.67	3	72.22	1.07
Within detergents	873.80	13	67.22	
Total	1090.47	16		

The F ratio of 1.07 does not lie in the critical region.

Thus there is no evidence, at the 5% significance level, to suggest a difference between the four brands as regards mean whiteness after washing.