Introduction to Statistical Methods
(S1-23_AIMLCZC418) – Assignment 2
AIML Section- 1

Each question carries 2.5 Marks (2.5 x 4 = 10 Marks)

Questions:

1. Based on their total scores, 200 candidates of civil service examination are divided into two groups, the upper 30% and the remaining 70%. The first question of the examination is considered as a sample. Among the first group, 40 had the correct answer, whereas among the second group, 80 had the correct answer. Based on these results, can one conclude that the first question is not good at discriminating the ability of the students being examined here? (Take $\alpha = 0.05$)

Solution:

Ans:

Given that $n_1 = 60, n_2 = 140, p_1 = \frac{40}{60} = 0.667, p_2 = \frac{80}{140} = 0.571$

$x_1 = 40, x_2 = 80$

$P = \frac{x_1+x_2}{n_1+n_2} = \frac{120}{200} = 0.6$

Q = 1-Q = 0.4

Null hypothesis H_0 : $P_1 = P_2$

Alternate hypothesis H_1: $P_1 \neq P_2$ (Two tailed test)

Level of significance $\propto = 0.05$

Here the tailed test is two tailed test,

Tabulated value of Z at 5% LOS is 1.96 .

Test Statistic:

$$Z_{cal} = \frac{p_1 - p_2}{\sqrt{(PQ)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.667-0.571}{\sqrt{(0.6)(0.4)\left(\frac{1}{60}+\frac{1}{140}\right)}} = 1.27$$

Here $|Z_{cal}| = |1.27| < Z_{tab} = 1.96$ at α = 5 % LOS.

Decision : we accept the Null hypothesis at α = 5 % LOS.

i.e, the first question is good enough in discriminating the ability of the students of both groups.

2. An experiment was performed to compare the abrasive wear of two different laminated materials. Twelve pieces of material 1 were tested by exposing each piece to a machine measuring wear. Ten pieces of material 2 were similarly tested. In each case, the depth of wear was observed. The samples of material 1 gave an average (coded) wear of 85 units with a sample standard deviation of 4, while the samples of material 2 gave an average of 81 with a sample standard deviation of 5. Can we conclude at 0.05 level of significance that the abrasive wear of material 1 exceeds that of material 2 by more than 2 units? Assume the populations to be approximately normal with equal variances.

Solution: Let μ_1 and μ_2 represent the population means of the abrasive wear for material 1 and material 2, respectively.

1. H_0: $\mu_1 - \mu_2 = 2$.
2. H_1: $\mu_1 - \mu_2 > 2$.
3. $\alpha = 0.05$.
4. Critical region: $t > 1.725$, where $t = \frac{(\bar{x}_1 - \bar{x}_2) - d_0}{s_p\sqrt{1/n_1 + 1/n_2}}$ with $v = 20$ degrees of freedom.
5. Computations:

$$\bar{x}_1 = 85, \quad s_1 = 4, \quad n_1 = 12,$$
$$\bar{x}_2 = 81, \quad s_2 = 5, \quad n_2 = 10.$$

Hence

$$s_p = \sqrt{\frac{(11)(16) + (9)(25)}{12 + 10 - 2}} = 4.478,$$
$$t = \frac{(85 - 81) - 2}{4.478\sqrt{1/12 + 1/10}} = 1.04,$$
$$P = P(T > 1.04) \approx 0.16. \qquad \text{(See Table A.4.)}$$

6. Decision: Do not reject H_0. We are unable to conclude that the abrasive wear of material 1 exceeds that of material 2 by more than 2 units.

3. A sample analysis of examination results of 500 students was made. It was found that 220 students had failed, 170 had secured a third class, 90 were placed in second class and 20 got a first class. Do these figures commensurate with the general examination result which is in ratio of 4:3:2:1 for the various categories respectively? Test at 5% Level of Significance.

Solution:

H_0: The observed results commensurate with the general examination results.

H_1: The observed results not commensurate with the general examination results.

Expected frequencies are in the ratio of 4:3:2:1

Total frequency = 500

If we divide total frequency 500 in the ratio 4:3:2:1, We get the Expected frequencies

As 200$\left(500\ X\ \frac{4}{10}\right)$, 150$\left(500\ X\ \frac{3}{10}\right)$,100$\left(500\ X\ \frac{2}{10}\right)$,50$\left(500\ X\ \frac{1}{10}\right)$

S. NO	(Oi)	(Ei)	(Oi-Ei)	$(O-E)^2$	$(O-E)^2$/Ei
1	220	200	20	400	2
2	170	150	20	400	2.667
3	90	100	10	100	1
4	20	50	30	900	18

The Chi-square value is

$$\chi^2 = \sum_1^4 \frac{(O-E)^2}{E} = 2 + 2.667 + 1 + 18$$

$$= 23.667$$

Dof is 4-1 = 3 and Level of significance ∝ = 0.05

χ^2 tab value is 7.81

χ^2 calculated = 23.667 > χ^2 tab = 7.81 at ∝ = 0.05 for 3 dof

Decision : we reject the Null hypothesis H_0 at α = 5 % LOS.

i.e, we accept H_1.

i.e, The observed results not commensurate with the general examination results

4. The following are the numbers of mistakes made in 5 successive days for 4 technicians working for a photographic laboratory:

Technician I	Technician II	Technician III	Technician IV
5	17	9	9
12	12	11	13
9	15	6	7
8	14	14	10
11	17	10	11

Test at the level of significance $\alpha = 0.01$ whether the differences among the 4-sample means can be attributed to chance.

Solution:

Ho: There is no difference among sample means.

H1: There is difference among sample means.

Data Summary				
Groups	N	Mean	Std. Dev.	Std. Error
Group 1	5	9	2.7386	1.2247
Group 2	5	15	2.1213	0.9487
Group 3	5	10	2.9155	1.3038
Group 4	5	10	2.2361	1

ANOVA Summary					
Source	Degrees of Freedom DF	Sum of Squares SS	Mean Square MS	F-Stat	P-Value
Between Groups	3	110	36.6667	5.7516	0.0072
Within Groups	16	102.0005	6.375		
Total:	19	212.0005			

.

F(3,16)= 5.29. i.e Calculated F > F table value.

Therefore, Ho is rejected

i.e., There is difference among sample means.