ISM

Assignment - 2

Section - 1
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Answers:-

1) Ans.

Given total population is 200,

Let n_1 and n_2 be the sample sizes from population

So, n_1 = 30 % of 200 = 200×0.30
= 60

n_2 = 70 % of 200 = 200×0.70
= 140

Let P_1 and P_2 be the sample proportions from n_1 and n_2

Given, 40 from n_1 got success

80 from n_2 got success

$P_1 = \frac{x_1}{n_1} = \frac{40}{60} = 0.667 \Rightarrow q_1 = 1 - P_1 = 0.333$

$P_2 = \frac{x_2}{n_2} = \frac{80}{140} = 0.571 \Rightarrow q_2 = 1 - P_2 = 0.429$

This is a problem of test of ~~sig~~ difference between two samples proportions - for large samples taken.

Null Hypothesis:- (H_0) $\Rightarrow P_1 = P_2$

H_0: The first question is good enough in discriminating students being examined here.

Alternate Hypothesis:- (H_1) : $P_1 \neq P_2$ [It's a two-tailed test]

The first ~~hypoth~~ question is not good enough in discriminating students being examined here.

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Level of significance: $\alpha = 0.05$

Test Statistic: Perform normal z test, by using the method of pooling, we pool the two sample proportions P_1 and P_2 into a single proportion P.

$$P = \frac{n_1 P_1 + n_2 P_2}{n_1 + n_2} = \frac{60 \times 0.667 + 140 \times 0.571}{60 + 140}$$

$$P = 0.6$$

$$q = 1 - P = 1 - 0.6 = 0.4$$

Standard Error of difference (S.E) $= \sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$

$$= \sqrt{0.6 \times 0.4 \left(\frac{1}{60} + \frac{1}{140}\right)}$$

$$SE = 0.0755991$$

$$Z = \frac{P_1 - P_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = \frac{\left(\frac{40}{60}\right) - \left(\frac{80}{140}\right)}{0.0755991}$$

$$\cancel{\frac{0.667 - 0.571}{0.0755}} \quad Z' = 1.2597 \simeq 1.26$$

Decision :- Z_{tab} at $\alpha = 0.05$ is 1.96

So, $Z_{cal} < Z_{tab}$

$$\boxed{1.26 < 1.96}$$

Hence, we ~~cannot~~ cannot reject the null hypothesis (H_0) at 5% level of significance. So, we can conclude that the first question is good enough in discriminating ability of students being examined.

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2)
Ans. Given $n_1 = 12$, $n_2 = 10$

Sample mean $\bar{x}_1 = 85$, $\bar{x}_2 = 81$

Sample standard deviation $S_1 = 4$ and $S_2 = 5$
↳ Repectively for material and material 2

Null Hypothesis (H_0): $\mu_1 - \mu_2 = 2$

Alternate Hypothesis (H_1): $\mu_1 - \mu_2 > 2$

Level of significance (α) = 0.05

Degrees of freedom (Dof) = $12 + 10 - 2 = 20$

Critical Region: As here both n_1 and n_2 are smaller than 30, so, unpaired T test has to be used.

$t_{\alpha,20} = t_{1-0.05,20} = 1.725$

So, reject region (RR): $T > 1.725$

where $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - d_0}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$S_p = \sqrt{\frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1+n_2-2}}$$

$$S_p = \sqrt{\frac{(11)\times 4^2 + 9\times 5^2}{12+10-2}} = \sqrt{20.05}$$

$= 4.477722$

$S_p \approx 4.48$

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$$t = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \quad (\because \text{here } \Delta_0 = d_0 = 2)$$

$$= \frac{(85-81)-2}{4.477722 \times \sqrt{\frac{1}{12}+\frac{1}{10}}}$$

$t_{calc} = 1.043162944$

Decision:-

$t_{cal} < t_{\alpha,20}$

$1.0431 < 1.725$

Since, t falls under the accept region, so we cannot reject ~~the~~ the null hypothesis H_0 i.e. at 0.05 level of significance. So, we can't conclude that the abrasive wear of material 1 exceeds that of material 2 by more than 2 units.

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3) Ans

Given total number of students = 500

No. of students failed = 220

No. of students secured third class = 170

No. of students placed ~~in~~ second class = 90

No. of students got first class = 20

Null Hypothesis:- H_0: The observed results commensurate with the general examination results.

Expected frequencies are in the ratio of 4:3:2:1

Total frequency = 500

To get the expected frequencies for the given ratio, we divide the total frequency 500 in the ratio of 4:3:2:1

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$500 \times \frac{4}{100} = 200$

$500 \times \frac{3}{100} = 150$

$500 \times \frac{2}{100} = 100$

$500 \times \frac{1}{100} = 50$

Let's tabulate the data

n	Class	observed Frequency (O_i)	Expected Frequency (E_i)	$(O_i - E_i)$	$(O_i - E_i)^2$	$\frac{(O_i-E_i)^2}{E_i}$
1	Failed	220	200	20	400	2
2	Third	170	150	20	400	2.666666667
3	Second	90	100	-10	100	1
4.	First	20	50	-30	900	18
	Total	500	500			23.666666667

We are applying chi-square test here,

$\chi^2_{calculated} = 23.666666667$

≈ 23.667

Here, n = 4 observations

So, Degrees of freedom (d.f) = n-1
= 4-1 = 3

~~Tabulate value~~ Table value of χ^2 at 5% (0.05) level of significance with dof = 3 is $\chi^2 = 7.815$

Decision:- We can observe that the

χ^2 calculated > χ^2 tabulated

23.667 > 7.815

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Therefore, H0 : Null hypothesis is rejected
We can conclude that the observed results are not commensturate with the general examination.

4)

Ans. From the given data, we derive the null and alternate hypothesis,

Null Hypothesis (H0) : $\mu_1 = \mu_2 = \mu_3 = \mu_4$
There is no significant different between the technicians, i.e the difference among 4 sample means can be attributed to chance.

Alternate Hypothesis (H_1) : There is a significant difference among the four means.

Level of Signifiance : 1% ⇒ $\alpha = 0.01$

We solve this using ANNOVA (one way), since the analysis of variance.

X ⇒ means technicians

X_1	X_2	X_3	X_4	X_1^2	X_2^2	X_3^2	X_4^2
5	17	9	9	25	289	81	81
12	12	11	13	144	144	121	169
9	15	6	7	81	225	36	49
8	14	14	10	64	196	196	100
11	17	10	11	121	289	100	121
$\Sigma X_1 =$ 45	ΣX_2 75	ΣX_3 50	ΣX_4 50	ΣX_1^2 435	ΣX_2^2 1143	ΣX_3^2 1534	ΣX_4^2 1520

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Test statistic

here $N = 20$ (No: of Observations)

T = Total value of observations.

$= 45 + 75 + 50 + 50$

$T = \underline{220}$

Correction Factor $\Rightarrow CF = \frac{T^2}{N} = \frac{(220)^2}{20}$

$CF = \underline{2420}$

Total sum of Squares: $TSS = \Sigma x_1^2 + \Sigma x_2^2 + \Sigma x_3^2 + \Sigma x_4^2 - CF$

$= 435 + 1143 + 534 + 520 - 2420$

$= 2632 - 2420$

$TSS = \underline{212}$

Sum of Squares ~~between~~ ~~treat~~ (SST):

$SST = \frac{(\Sigma x_1)^2}{N_1} + \frac{(\Sigma x_2)^2}{N_1} + \frac{(\Sigma x_3)^2}{N_1} + \frac{(\Sigma x_4)^2}{N_1} - C.F$

$= \frac{45^2}{5} + \frac{75^2}{5} + \frac{50^2}{5} + \frac{50^2}{5} - 2420$

$= \frac{12650}{5} - 2420$

$SST = \underline{110}$

Sum of Squares due to Error:- $SSE = TSS - SST$

$= 212 - 110$

$SSE = \underline{102}$

here, $K = 4$ ($\because$ 4 technicians)

ANOVA Table

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Mean sum of Squares due to ~~mistakes~~ Technicians : $MST = \frac{SST}{K-1}$

Mean sum of squares due to Error = $MSE = \frac{SSE}{n-K}$

Source of variation	Sum of Squares	Degree of freedom	Mean sum of Squares	F-Ratio
Technicians ~~Mistakes~~ Between Samples	$SST = 110$	$K-1$ $4-1$ 3	$MST = \frac{SST}{K-1}$ $= 36.6666667$	$F_0 = \frac{MST}{MSE}$ $= \frac{36.6667}{6.375}$ $= 5.751339$
Error Within samples	$SSE = 102$	$n-K$ $20-4$ 16	$MSE = \frac{SSE}{n-K}$ $= 6.375$	

$\therefore F_0 = 5.751339$

Critical value:-

here, degrees of $(3,16)$

Tab $F_c(3,16) = 5.29$ at $\alpha = 0.01$

Here, $F_0 > F_c$

$5.751339 > 5.29$

Decision: $\therefore$ We reject the null hypothesis

$\therefore$ There is a significant difference among four means.