Introduction to Statistical Methods

S1-23_AIMLCZC418 – Assignment 2

AIML Section - 4

Each question carries 2.5 Marks (4 x 2.5 =10Marks)

1. The article "Effect of Internal Gas Pressure on the Compression Strength of Beverage Cans and Plastic Bottles" (*J. of Testing and Evaluation*, 1993: 129–131) includes the accompanying data on compression strength (lb) for a sample of 15-oz aluminium cans filled with strawberry drink and another sample filled with cola.

Does the data suggest that the extra carbonation of cola results in a higher average compression strength?

	sample size	sample mean	sample SD
Strawberry drink	15	540	21
Cola	15	554	15

SOLUTION:

Sol: Given that $n_1 = 15, n_2 = 15, \bar{x}_1 = 540, \bar{x}_2 = 554, s_1 = 21$ *and* $s_2 = 15$.

Null hypothesis $H_0 : \mu_1 = \mu_2$

Alternate hypothesis H_1: $\mu_1 < \mu_2$ (one tailed test)

Level of significance $\propto = 0.05$

t-table value for 28 d.f is $t_{tab} = t_\alpha = 1.701$

Test Statistic:

$$t_{cal} = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{540 - 554}{\sqrt{\frac{21^2}{15} + \frac{15^2}{15}}} = -2.1$$

Here $|t_{cal}| = |-2.1| > t_{tab} = 1.701$ at α = 5 % LOS.

Decision : we reject the Null hypothesis at α = 5 % LOS.

i.e, we accept H_1 .

2. An automobile tyre manufacturer claims that the average life of a particular grade of tyre is more than 20,000 km. A random sample of 16 tyres is having mean 22,000 km with a standard deviation of 5000 km. Validate the claim of the manufacturer at 5% LoS.

SOLUTION:

H0 (Null Hypothesis) : $\mu <= 20000$
H1 (Alternate Hypothesis): $\mu > 20000$ (less than mean one tailed test)
LoS (α) = 5% (Take 5% if not given in question)

n = 16 (Sample size)
$\bar{x}$ = 22000 (Sample mean)
s = 5000 (sample Standard deviation)

n < 30 hence will go with t-test
step 1:
calculate t value from the t-test formula:
$t = (\bar{x} - \mu)/ (s/\sqrt{n})$
$t = (22000 - 20000) / 5000/\sqrt{16}$
t = 1.60

step 2:
get t critical value from t-table for α = 5% and degree of freedom = 16-1 = 15.
t critical value = 1.753

step 3:
check if t calculate < t critical then accept the null hypothesis else reject the null hypothesis. Here, t calculated 1.60 < t critical 1.753, hence will accept the null hypothesis.

Conclusion: from the data given, it is significantly proven that average life of the tyres is more than 20000. ie., **accept null hypothesis**.

3. The severity of a disease and blood group were studied in a research projest. The findings are given in the following table, known as the m xn contingency table. Can this severity of the condition and blood group are associated. Severity of a disease classified by blood group in 1500 patients.

Condition	Blood Groups				Total
	O	A	B	AB	
Severe	51	40	10	9	110
Moderate	105	103	25	17	250
Mild	384	527	125	104	1140
Total	540	670	160	130	1500

SOLUTION:

Solution:

H_0: The two attributes severity of the condition and blood groups are not associated.

H_1: The two attributes severity of the condition and blood groups are associated.

Calculation of Expected frequencies

Condition	Blood Groups				Total
	O	A	B	AB	
Severe	39.6	49.1	11.7	9.5	110
Moderate	90.0	111.7	26.7	21.7	250
Mild	410.4	509.2	121.6	98.8	1140
Total	540	670	160	130	1500

Test statistic:

$$\chi^2 = \sum_{i=j}^{m} \sum_{j=1}^{n} \frac{\left(o_{ij} - E_{ij}\right)}{E_{ij}} \sim \chi^2 (m-1)(n-1) \text{ df}$$

Here m= 3.,n =4

Calculations:

O_i	E_i	$O_i E_i$	$(O_i E_i)^2$	$(O_i E_i)^2/E_i$
51	39.6	11.4	129.96	3.2818
40	49.1	-9.1	82.81	1.6866
10	11.7	-1.7	2.89	0.2470
9	9.5	-0.5	0.25	0.0263
105	90.0	15	225.00	2.5000
103	111.7	-8.7	75.69	0.6776
25	26.7	-1.7	2.89	0.1082
17	21.7	-4.7	22.09	1.0180
384	410.4	-26.4	696.96	1.6982
527	509.2	17.8	316.84	0.6222
125	121.6	3.4	11.56	0.0951
104	98.8	5.2	27.04	0.2737
				12.2347

$\therefore \chi^2 = 12.2347$

Table value:

χ^2 (3-1)(4-1)= χ^2 (6) at 5 % l.os)= 12.59

Inference

$\chi^2_{cal} < \chi^2_{tab}$

We accept the null hypothesis.

(i.e) the two attributes severity of the condition and blood group are independent.

2x2 – contingency table

$$\chi^2 = \frac{N(ad-bc)^2}{(a+b)(c+d)(a+c)(b+d)} \sim \chi^2 \text{ (2-1) (2-1) df } = \chi^2 \text{ (1) df}$$

4. A random sample is selected from each of three makes of ropes and their breaking strength(pounds) are measured with the following results:Test whether the breaking strength of ropes differs significantly. Note: Take LOS as 5 %

I	II	III
70	100	60
72	110	65
75	108	57
80	112	84
83	113	87
	120	73
	107	

SOLUTION

① Sol

H_0: ~~the~~ the breaking strength of the roaps does not differ significantly.

$H_0: \mu_1 = \mu_2 = \mu_3$.

for simplifying calculations let us take 80 as common (origin) and the new values given below

I	II	III
−10	20	−20
−8	30	−15
−5	28	−23
0	32	4
3	33	7
.	40	−7
	27	

$T_1 = -20$ $T_2 = 210$ $T_3 = -54$.

$G = \Sigma\Sigma y_{ij}$ = Grand Total $= -20 + 210 - 54 = 136$.

correction factor = 1027.56.

Sum of squares $= SST = \Sigma\Sigma y_{ij}^2 - \frac{G^2}{N} = 7992 - 1027.56$
$= 6964.44$.

tween Sum of squares = 5838.44.

Error Sum of squares = 1126.

Source of variation	df	SS	Mean square	F ratio
Btween	2	5838.44	2919.22	$F = \frac{2919.22}{75.07} = 38.89$
Error	15	1126	75.07	
total	17	6994.44		