Introduction to Statistical Methods

(S1-23_ AIMLCZC 418) – Assignment 1

AIML Section - 2

Each question carries 02 Marks (2 x 5 = 10 Marks)
Duration: 13th December 2023 – 29th December 2023
1) Submissions are individual
2) Solve these on paper, scan, and upload
3) Plagiarism results in zero marks
4) Write your name, BITS ID and Section on each page

1. In a garden, there are 39 plants. The heights (in cm) of 5 randomly selected plants are 38, 51, 46, 79, and 57. Calculate the standard deviation of their heights.

Solution:
Given that, Number of observations = 5
Hence, the mean of 5 observations is:
Mean = (38 + 51 + 46 + 79 + 57)/5 = **54.2**
Now, the standard deviation is calculated as follows:
Standard Deviation, SD = $\sqrt{[(\Sigma(x_i - \bar{x})^2) / (N-1)]}$
Now, substitute the values in the formula, we get
S.D= $\sqrt{(51-54.2)^2+(38-54.2)^2+(79-54.2)^2+(46-54.2)^2+(57-54.2)^2 / 4}$
On solving the above expression, we get
S.D = **15.5**
Hence, the standard deviation (S.D) of their heights is 15.5.

Now by the question we have,

$$P(A_1) = \frac{2000}{2000+2500+4000} = \frac{2000}{8500} = \frac{4}{17}$$

$$P(A_2) = \frac{2500}{2000+2500+4000} = \frac{2500}{8500} = \frac{5}{17}$$

$$P(A_3) = \frac{4000}{2000+2500+4000} = \frac{4000}{8500} = \frac{8}{17}$$

Again, $P(X/A_1) = 3/100$, $P(X/A_2) = 4/100$ and $P(X/A_3) = 2.5/100$

Now, event X occurs if one of the mutually exclusive and exhaustive events A_1, A_2 and A_3 occurs. Therefore, using Bayes' theorem formula we get,

$$P(\frac{A_2}{X}) = \frac{P(A_2).P(\frac{X}{A_2})}{P(A_1).P(\frac{X}{A_1}) + P(A_2).P(\frac{X}{A_2}) + P(A_3).P(\frac{X}{A_3})} = \frac{\frac{5}{17}.\frac{4}{100}}{\frac{4}{17}.\frac{3}{100} + \frac{5}{17}.\frac{4}{100} + \frac{8}{17}.\frac{25}{100}}$$

$$= \frac{20}{12+20+20} = \frac{20}{52} = \frac{5}{13}$$

2. Three professors, Prof. Anand, Prof. Bhanu Prakash and Prof. Chandra Shekhar appear in an interview for the post of Managing Director in a reputed company. Their chances of getting selected are 2/9 , 4/9 and 1/3 respectively. The probabilities that they introduce a new policy in the company are 3/10 , 1/2 and 4/5 respectively.

i. What is the probability that new policy is introduced in the company?
ii. What is the probability the new policy introduced is done by Prof. Anand?

Given that, $P(A) = \frac{2}{9}$, $P(B) = \frac{4}{9}$, $P(C) = \frac{1}{3}$

Let x be the event that a new policy is introduced in the Company.

Hence,

$$P(x/A) = \frac{3}{10}, \quad P(x/B) = \frac{1}{2}, \quad P(x/c) = \frac{4}{5}$$

(i) By using theorem on total probability, Probability that new policy is introduced in the Company is given by

$$P(x) = P(A)\cdot P(x/A) + P(B)\cdot P(x/B) + P(C)\cdot P(x/C)$$

$$= \frac{2}{9} \times \frac{3}{10} + \frac{4}{9} \times \frac{1}{2} + \frac{1}{3} \times \frac{4}{5}$$

$$= \frac{5}{9}$$

$$\therefore P(x) = \frac{5}{9}$$

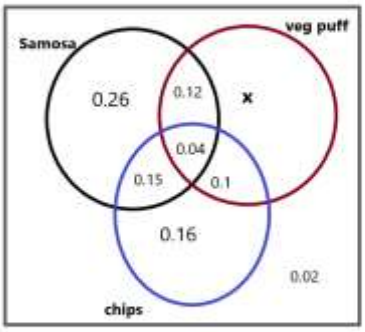
(ii) By Baye's theorem, the probability that the new policy introduced is done by Prof. Anand is given by

$$P(A/x) = \frac{P(A)\cdot P(x/A)}{P(x)} = \frac{2/9 \times 3/10}{5/9}$$

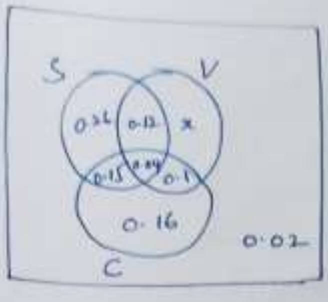
$$= \frac{3}{25}$$

$$\therefore P(A/x) = \frac{3}{25}$$

3. In a certain school there are 500 students. The students of the school buy samosa, chips and vegpuffs from their canteen. The following data provides the probability of the student's preference of food sold in the canteen.



i.Find x and What is the probability that students buy at least one of the items in the canteen?
ii.What is the probability that a student buys chips if the student had already bought samosa?
ii.What is the probability that a student buys veg puffs if he had already bought chips?
v.What is the probability that a student buys veg puffs if he had already bought samosa?



(i) Since the total probability = 1

$$0.26 + 0.12 + 0.04 + 0.15 + x + 0.1 + 0.16 + 0.02 = 1$$

$$\Rightarrow x + 0.85 = 1$$

$$\Rightarrow x = 0.15$$

$$\therefore P(S) = 0.26 + 0.12 + 0.15 + 0.04$$

$$= 0.57$$

$$P(V) = 0.12 + 0.15 + 0.04 + 0.1$$

$$= 0.41$$

$$P(C) = 0.16 + 0.15 + 0.1 + 0.04$$

$$= 0.45$$

$$P(S \cap V) = 0.16$$

$$P(S \cap C) = 0.19$$

$$P(V \cap C) = 0.14$$

$$P(S \cap V \cap C) = 0.04$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$\therefore P(S \cup V \cup C) = 0.57 + 0.41 + 0.45 - 0.16 - 0.19 - 0.14 + 0.04$$
$$= 0.98$$

ii) $P(C|S) = \dfrac{P(C \cap S)}{P(S)}$

$$= \frac{0.19}{0.57} = \frac{19}{57} = \frac{1}{3} = 0.33$$

iii) $P(V|C) = \dfrac{P(V \cap C)}{P(C)}$

$$= \frac{0.14}{0.45} = \frac{14}{45} = 0.31$$

iv) $P(V|S) = \dfrac{P(V \cap S)}{P(S)} = \dfrac{0.16}{0.57} = \dfrac{16}{57} = 0.28$

4. In a multiplex movie theatre, the seats are divided in to three sections namely Classic, Prime and Comfort.45% of the seats are in Classic section, 30% of the seats are in Prime section and 25% of the seats are in Comfort section. In every section few seats are ‘Preferred seats’ which can be used to book for management. 2% of the seats in Classic Section,12% of the seats in Prime section and 1% of the seats in Comfort section are preferred seats. Then

i. What is the probability that you sit in Prime section if you have a preferred seat?

ii. What is the probability that you sit in Comfort section if you don’t have a preferred seat?

Q2 Solution

$P(CL) = 45\% = 0.45$

$P(Pr) = 30\% = 0.3$

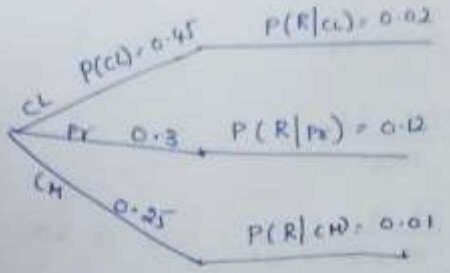
$P(CM) = 25\% = 0.25$

Let 'R' be the no. of preferred seats in all sections

then $P(R|CL) = 0.02$

$P(R|Pr) = 0.12$

$P(R|CM) = 0.01$



R $P(Px|R) = ?$

$P(R) = 0.45 \times 0.02 + 0.3 \times 0.12 + 0.25 \times 0.01$

$= 0.009 + 0.036 + 0.0025$

$= 0.0475$

$$P(Px|R) = \frac{P(Px \cap R)}{P(R)}$$

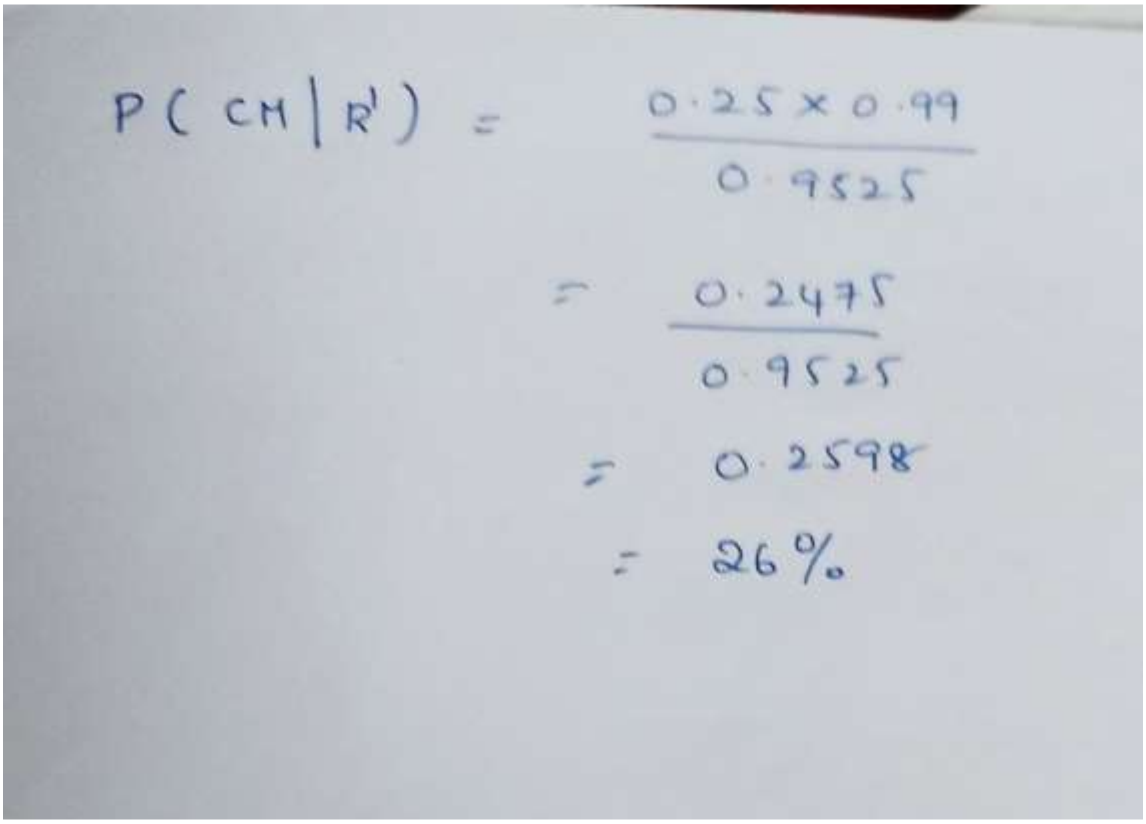
$$= \frac{0.3 \times 0.12}{0.0475} = \frac{0.036}{0.0475} = 0.7579$$

$= 76\%$

(ii) $P(R') = 0.45 \times 0.98 + 0.3 \times 0.88 + 0.25 \times 0.99$

$= 0.441 + 0.264 + 0.2475$

$= \cancel{0.9485}$ 0.9525



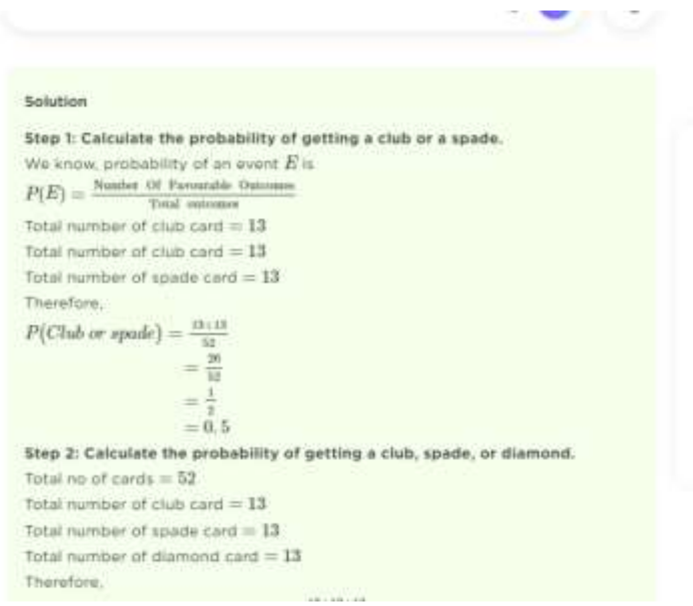
$$P(CH \mid R') = \frac{0.25 \times 0.99}{0.9525}$$

$$= \frac{0.2475}{0.9525}$$

$$= 0.2598$$

$$= 26\%$$

5. A standard deck of cards contains 52 cards. One card is selected from the deck.
Compute the probability of randomly selecting a club or spade, compute the probability of randomly selecting a club or spade or diamond and compute the probability randomly of randomly selecting a six or heart.



Solution

Step 1: Calculate the probability of getting a club or a spade.

We know, probability of an event E is

$P(E) = \frac{\text{Number Of Favourable Outcomes}}{\text{Total outcomes}}$

Total number of club card $= 13$

Total number of club card $= 13$

Total number of spade card $= 13$

Therefore,

$$\begin{aligned} P(\text{Club or spade}) &= \frac{13+13}{52} \\ &= \frac{26}{52} \\ &= \frac{1}{2} \\ &= 0.5 \end{aligned}$$

Step 2: Calculate the probability of getting a club, spade, or diamond.

Total no of cards $= 52$

Total number of club card $= 13$

Total number of spade card $= 13$

Total number of diamond card $= 13$

Therefore,

Therefore,

$$P(\text{Diamond or spade or club}) = \frac{13+13+13}{52} = \frac{39}{52} = 0.75$$

Step 3: Calculate the probability of getting a six or a heart.

Total number of card $= 52$

Total number of six card $= 4$

Probability of getting a six is $P(A) = \frac{4}{52}$

Total number of heart card $= 13$

Probability of getting a heart is $P(B) = \frac{13}{52}$

Probability of getting six of heart is $P(A \cap B) = \frac{1}{52}$

Probability of getting six or heart is $P(A \cup B)$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$

$\Rightarrow P(A \cup B) = \frac{4}{13}$

Hence, the probability of randomly selecting a club or spade is 0.5, the probability of randomly selecting a club or spade or diamond is $.0.75$, the probability randomly of randomly selecting a six or heart is $\frac{4}{13}$

ALL THE BEST

----ALL THE BEST----