

Introduction to Statistical Methods

(S1-23 AIMLCZC418) – Assignment 1

AIML Section- 1

Each question carries 02 Marks (2 x 5 = 10 Marks)

Duration: 13th December 2023 – 29th December 2023

1) Submissions are individual

2) Solve these on paper, scan, and upload

3) Plagiarism results in zero marks

4) Write your name, BITS ID and Section on each page

1. Suppose the average marks scored by six students are 9 with variance 11.6666 and if the marks of 4 students are 4, 8, 10, 12 then the marks of remaining two students are _____

Solution: Let x and y are the ages of remaining two persons. Therefore,

$$4+8+10+12+x+y=9 \times 6, \quad x+y=54-34=20.$$

$$\text{variance} = \frac{(4^2+8^2+10^2+12^2+x^2+y^2)}{6} - \text{mean}^2. \text{ This gives } x^2+y^2=232.$$

On solving above equations, we get $x=6, y=14$

2. Validate the following and Justify

a. The probability that a person visits Reliance Mart is 0.2 and that he visits Croma is 0.25. The probability of visiting Reliance or Croma is 0.60.

b. $P\left(\frac{\bar{A}}{B}\right) = 1 - P\left(\frac{A}{B}\right)$

(a) $P(A) = 0.15$ and $P(B) = 0.20$; $P(A \cup B) = 0.50$

we know

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\begin{aligned}\Rightarrow P(A \cap B) &= P(A) + P(B) - P(A \cup B) \\ &= 0.15 + 0.20 - 0.5 \\ &= -0.15 < 0\end{aligned}$$

but Probability of any event is lies between $0 \leq 1$

\therefore This is not a valid statement.

(b) $B = (B \cap A) \cup (B \cap \bar{A})$

$$P(B) = P(B \cap A) + P(B \cap \bar{A})$$

$$P(B) = P(B)P(A/B) + P(B)P(\bar{A}/B)$$

$$\Rightarrow 1 = P(A/B) + P(\bar{A}/B)$$

$$\Rightarrow P(\bar{A}/B) = 1 - P(A/B)$$

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3. A manufacturer has three machine operators A, B and C. The first operator A produce 1% defective items, whereas the other two operators B and C produce 5% and 7% defective items respectively. A is on the job for 50% of the time, B is on the job for 30% of the time. A defective item is produced, what is the probability that it was produced by A, B, C? Based on this write your observations.

$P(A) = \frac{1}{2}$
 $P(B) = \frac{3}{10}$
 $P(C) = \frac{2}{5}$

$P(D/A) = 0.01$
 $P(A) \cdot P(D/A) = (0.5)(0.01)$
 $= 0.005$

$P(D/B) = 0.05$
 $P(B) \cdot P(D/B) = (0.3)(0.05)$
 $= 0.015$

$P(D/C) = 0.07$
 $P(C) \cdot P(D/C) = (0.2)(0.07)$
 $= 0.014$

(i) $P(D) = 0.005 + 0.015 + 0.014$
 $= 0.034$

$P(A/D) = \frac{P(A)P(D/A)}{P(D)}$
 $= \frac{0.005}{0.034} = 0.147$

(ii) $P(B/D) = \frac{P(B)P(D/B)}{P(D)}$
 $= \frac{0.015}{0.034} = 0.4411$

(iii) $P(C/D) = \frac{P(C)P(D/C)}{P(D)}$
 $= \frac{0.014}{0.034} = 0.4117$

4.)If A and B are two events with probability $P(A)=0.38, P(B)=0.63, P(A \cup B)=0.78$, Then find

$$P(A/B), P(B/\bar{A}), P(A \cap \bar{B}), P(\bar{A} \cup \bar{B})$$

Sol:- We know

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$= 0.38 + 0.63 - 0.78$$

$$= 0.23$$

$$\text{Now } P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.23}{0.63} = \frac{23}{63}$$

$$P(\bar{A} \cap B) = P(B) - P(A \cap B)$$

$$= 0.63 - 0.23 = 0.40$$

$$P(B/\bar{A}) = \frac{P(B \cap \bar{A})}{P(\bar{A})} = \frac{0.40}{0.62} = \frac{40}{62} = \frac{20}{31}$$

$$P(A \cap \bar{B}) = P(A) - P(A \cap B) = 0.38 - 0.23 = 0.15$$

$$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) \quad (\text{De Morgan's law})$$

$$= 1 - P(A \cap B)$$

$$= 1 - 0.23 = 0.77$$

5. 1300 families with 2 children were selected randomly, and the following data were recorded:

Number of boys in a family	2	1	0
Number of families	325	761	214

Compute the probability of a family, chosen at random, having

(i) 2 boys (ii) 1 boy (iii) No boy

Also, check whether the sum of these probabilities is 1.

Solution: Total numbers of families = 1300

(i) Numbers of families having 2 boys = 325

Probability = Numbers of families having 2 boys/Total numbers of families

$$P = 325/1300$$

$$P = 25/100$$

(ii) Numbers of families having 1 boy = 761

Probability = Numbers of families having 1 boy/Total numbers of families

$$P = 761/1300$$

$$P = .5853$$

(iii) Numbers of families having no boys = 214

Probability = Numbers of families having 0 boy/Total numbers of families

$$= 214/1300 = 107/650 = 0.1646$$

Sum of the probability = $(25/100) + (761/1300) + (107/650)$

$$= (325+761+214)/1300 = 1300/1300 = 1$$

Yes, the sum of these probabilities is 1.

ALL THE BEST

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