Date

Tutorial - 4

1. $T(n) = 3T\left(\frac{n}{2}\right) + n^2$.

$T(n) = aT\left(\frac{n}{2}\right) + f(n)$

$a \geq 1, \quad b > 1$

On comparing.

$a = 3, \quad b = 2, \quad f(n) = n^2$

Now.

$c = \log_b a = \log_2 3 = 1.584$

$n^c = n^{1.584} < n^2$

$\therefore f(n) > n^c$

$\therefore T(n) = \theta(n^2)$.

2. $T(n) = 4T(n/2) + n^2$.

$a \geq 1, \quad b > 1$

$a = 4, \quad b = 2, \quad f(n) = n^2$

$c = \log_2 4 = 2$

$\therefore n^c = n^2 = f(n) = n^2$

$\therefore T(n) = \theta(n^2 \log_2 n)$.

3. $T(n) = T(n/2) + 2^n$

$a = 1, \quad b = 2$

$f(n) = 2^n$

$c = \log_b a = \log_2 4 = 0$

Date

$n^c = n^0 = 1$

$f(n) > n^c$

$T(n) = \theta(2^n)$

4. $T(n) = 2^n T(n/2) + n^n$.

$a = 2^n$

$b = 2$, $f(n) = n^n$

$c = \log_b a = \log_2 2^n = n$

$n^c = n^n$

$\therefore f(n) = n^c$

$T(n) = \theta(n^2 \log_2 n)$

5. $T(n) = 16T\left(\frac{n}{4}\right) + n$.

$a = 16$, $b = 4$

$f(n) = n$

$c = \log_4 16 = \log_4 4^2 = 2$

$n^2 = n^2$

$f(n) < n^c$

$T(n) = \theta(n^2)$.

6. $T(n) = 2T(n/2 + n\log n)$.

$a = 2$, $b = 2$

$f(n) = n \log n$.

$c = \log_2 2 = 1$

$n^c = n^1 = n$

$n \log n > n$

$f(n) > n^c$

Mohan

Date

$T(n) = \Theta(n \log n)$.

7. $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$.

$a = 2, \; b = 2, \; f(n) = n/\log n$

$c = \log_2 2 = 1$

$n^c = n^1 = n$.

since $\frac{n}{\log n} < n$

$\therefore f(n) < n^c$

$\therefore T(n) = O(n)$

8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$.

$a = 2, \; b = 4, \; f(n) = n^{0.51}$

$c = \log_b a = \log_4 2 = 0.5$

$n^c = n^{0.5}$

since $n^{0.5} < n^{0.51}$

$f(n) > n^c$

$T(n) = \Theta(n^{0.51})$.

9. $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$.

$a = 0.5, \; b = 2$.

According to master method, $a \geq 1$, but here a is 0.5. So we cannot apply masters theorm.

Mohan

Date

10. $T(n) = 16T\left(\frac{n}{4}\right) + n!$

$a = 16, \quad b = 4, \quad f(n) = n!$

$c = \log_b a = \log_4 16 = 2$

Now $n^c = n^2$

As $n! > n^2$

$\therefore T(n) = \Omega(n!)$.

11. $4T\left(\frac{n}{2}\right) + \log n.$

$a = 4, \quad b = 2, \quad f(n) = \log n.$

$c = \log_b a = \log_2 4 = 2$

$\therefore n^c = n^2$

$f(n) = \log n.$

Since $\log n < n^2$

$\therefore f(n) < n^c$

$\therefore T(n) = \Theta(n^c)$

$= \Theta(n^2).$

12. $T(n) = \text{sqrt}(n)\,T(n/2) + \log n.$

$a = \sqrt{n}, \quad b = 2$

$c = \log_b a = \log_2 \sqrt{n} = \frac{1}{2}\log_2 n.$

$\therefore \frac{1}{2}\log_2 n < \log(n).$

$f(n) > n^c$

$T(n) = \Theta(f(n)) = \Theta(\log(n))$

Mohan

Date

13. $T(n) = 3T\left(\frac{n}{2}\right) + n.$

$a = 3, \quad b = 2, \quad f(n) = n.$

$c = \log_b a = \log_2 3 = 1.5849.$

$n^c = n^{1.5849}.$

$n < n^{1.5849}.$

$f(n) = n^c$

$T(n) = \theta(n^{1.5849}).$

14. $T(n) = 3T(n/3) + \text{sqrt}(n).$

$a = 3, b = 3.$

$c = \log_b a = \log_3 3 = 1.$

$n^c = n^1 = n$

As $\text{sqrt}(n) < n$

$\therefore f(n) < n^c$

$T(n) = \theta(n).$

15. $T(n) = 4T(n/2) + (n).$

$a = 4, \quad b = 2$

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$c^n < n^2$

$f(n) < n^c$

$T(n) = \theta(n^2).$

Mohan

Date

16. $T(n) = 3T(n/4) + n\log n$.

$a = 3, b = 4, f(n) = n\log n$.

$c = \log_b a = \log_4 3 = 0.792$.

$n^c = n^{0.792}$.

$\therefore n^{0.792} < n\log n$.

$\therefore T(n) = \theta(n\log n)$.

17. $T(n) = 3T(n/3) + n/2$.

$a = 3, \quad b = 3$.

$c = \log_b a = \log_3 3 = 1$

$f(n) = n/2$.

$\therefore n^c = n^1 = n$

As $n/2 < n$

$f(n) = n^c$.

$\therefore T(n) = \theta(n)$.

18. $T(n) = 6T(n/3) + n^2\log n$.

$a = 6, \quad b = 3$

$c = \log_b a = \log_3 6 = 0.6309$.

$n^c = n^{1.6309}$.

As, $n^{1.6309} < n^2\log n$.

$\therefore T(n) = \theta(n^2\log n)$.

19. $T(n) = 4T(n/2) + n\log n$.

$a = 4, \quad b = 2, \quad f(n) = \frac{n}{\log n}$.

Mohan

Date

$c = \log_b a = \log_2 4 = 2$

$n^c = n^2$

$\therefore \frac{n}{\log n} < n^2$

$\therefore T(n) = \theta(n^2)$.

20. $T(n) = 64\,T(n/8) - n^2 \log n$.

$a = 64$, $b = 8$.

$c = \log_b a = \log_8 64 = \log_8 (8)^2$

$c = 2$

$n^c = n^2$

$n^2 \log n > n^2$

$\therefore T(n) = \theta(n^2 \log n)$.

21. $T(n) = 7T(n/3) + n^2$.

$a = 7$, $b = 3$, $f(n) = n^2$

$c = \log_b a = \log_3 7 = 1.7712$.

$n^c = n^{1.7712}$

$\Rightarrow n^{1.7712} < n^2$

$\therefore T(n) = \theta(n^2)$.

22. $T(n) = T(n/2) + n(2 - \cos n)$.

$a = 1$, $b = 2$

$c = \log_b a = \log_2 1 = 0$.

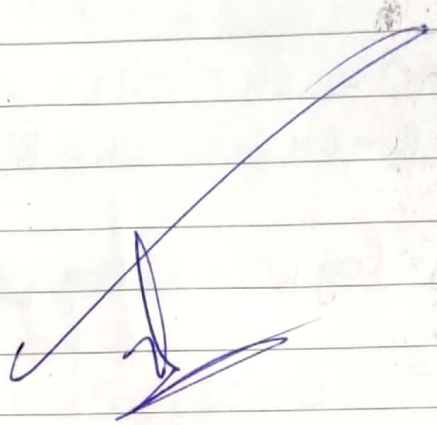
Mohan

Date

$\therefore n^c = n^0 = 1$

$\therefore n(2 - \cos n) \geq n^c$

$\therefore T(n) = \Theta(n(2 - \cos n))$.



Mohan