Studying the Response of RLC Circuits to Sinusoidal Inputs Using Simulink

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IMPORTANT

Provide your hash value here. Θ is the value you calculated from your group number – see Instruction sheet:

1.78, group value = 78

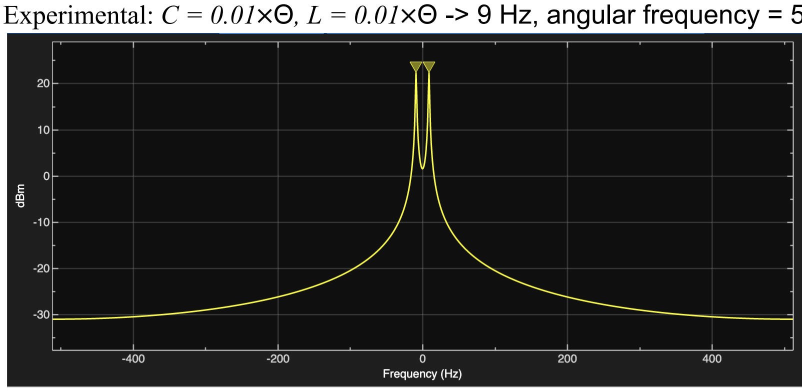
Include all other requested screenshots and answers to the back of this answer sheet.

1. Natural frequency of an RLC Circuit

1.1 Exercise 1.1

Calculate the natural frequency for the following systems. (1 pt)

1. Experimental: $C = 0.01 \times \Theta$, $L = 0.01 \times \Theta$ -> 9 Hz, angular frequency = 56.55 rad/s



Peaks

Zero-Order Hold	P1	P2	P3
Frequency (Hz)	9.0000e+0	-9.0000e+0	
Power (dBm)	2.2380e+1	2.2380e+1	

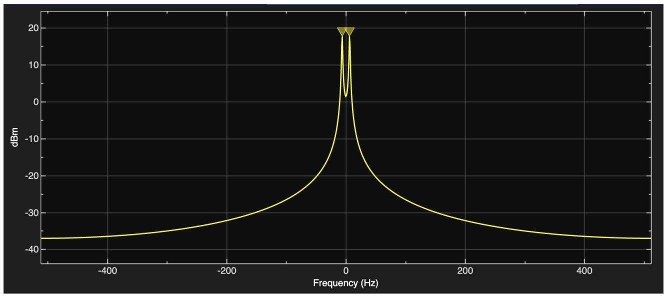
$C = 0.0178$, $L = 0.0178$

$w_n = \frac{1}{2\pi\sqrt{LC}}$

$= 8.94$

$\approx 9 Hz$

2. Experimental: $C = 0.02 \times \Theta$, $L = 0.01 \times \Theta$ -> 6Hz, angular frequency = 37.7 rad/s



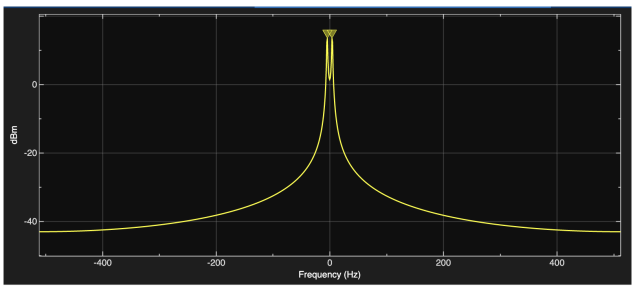
Peaks

Zero-Order Hold	P1	P2	P3
Frequency (Hz)	6.0000e+0	-6.0000e+0	
Power (dBm)	1.7734e+1	1.7734e+1	

$C = 0.0356$, $L = 0.0178$

$$W_n = \frac{1}{2\pi\sqrt{LC}} = 6.322\,Hz$$

3. Experimental: $C = 0.04 \times \Theta$, $L = 0.01 \times \Theta$ -> 4Hz, angular frequency = 25 rad/s



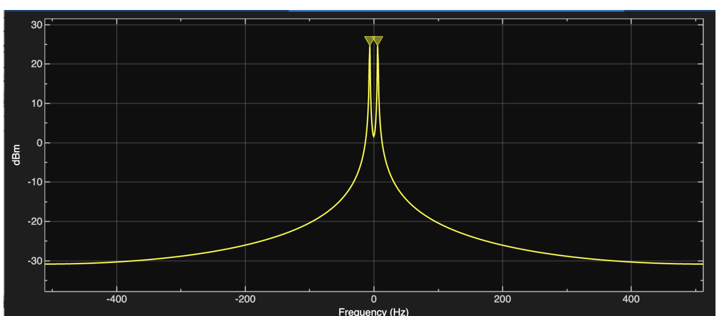
Peaks

Zero-Order Hold	P1	P2	P3
Frequency (Hz)	4.0000e+0	-4.0000e+0	
Power (dBm)	1.3476e+1	1.3476e+1	

$C = 0.0712$, $L = 0.0178$

$$W_n = \frac{1}{2\pi\sqrt{LC}} = 4.47\,Hz$$

4. Experimental: $C = 0.01 \times \Theta$, $L = 0.02 \times \Theta$ -> 6Hz, angular frequency = 37.7 rad/s



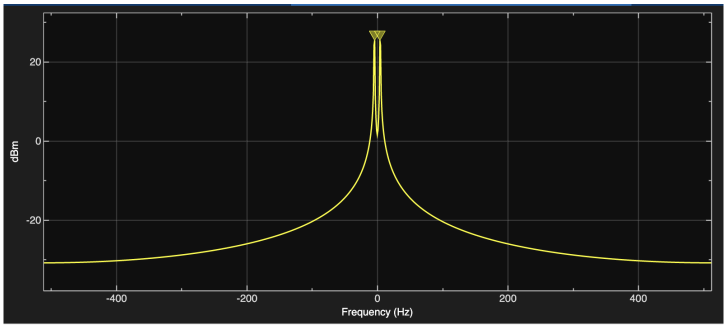
Peaks

Zero-Order Hold	P1	P2	P3
Frequency (Hz)	6.0000e+0	-6.0000e+0	
Power (dBm)	2.4611e+1	2.4611e+1	

$C = 0.0178$, $L = 0.0356$

$$W_n = \frac{1}{2\pi\sqrt{LC}} = 6.322\,Hz$$

5. Experimental: $C = 0.01 \times \Theta$, $L = 0.04 \times \Theta$ -> 4Hz, angular frequency = 25 rad/s



Peaks

Zero-Order Hold	P1	P2	P3
Frequency (Hz)	4.0000e+0	-4.0000e+0	
Power (dBm)	2.5390e+1	2.5390e+1	

$C = 0.0178,\ L = 0.0712$

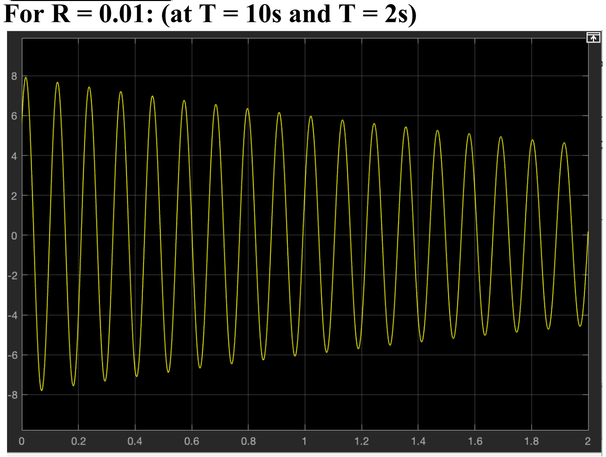
$$W_n = \frac{1}{2\pi\sqrt{LC}} = 4.47\text{Hz}$$

Conclusion (1.1): There is about a +- difference of 0.3Hz between calculated values and Simulink, showing accurate representations of the RLC circuit

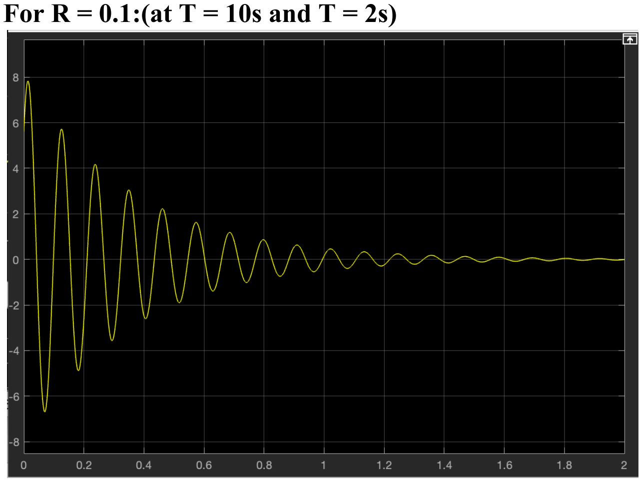
1.2 Exercise 1.2

a. *Vary the resistance values to show that damping factor increases as you increase the resistance (set C=0.01×Θ and L=0.01×Θ). Save your screenshot and attach to this document.* (1 pt)

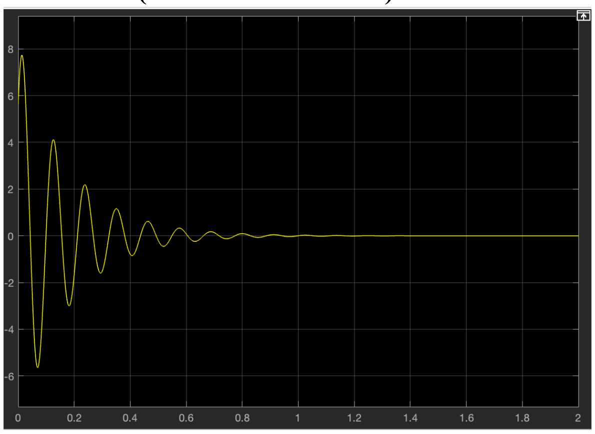
For R = 0.01: (at T = 10s and T = 2s)



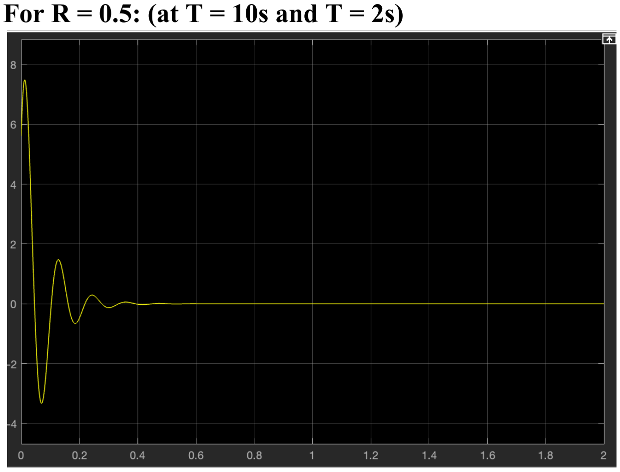
For R = 0.1:(at T = 10s and T = 2s)



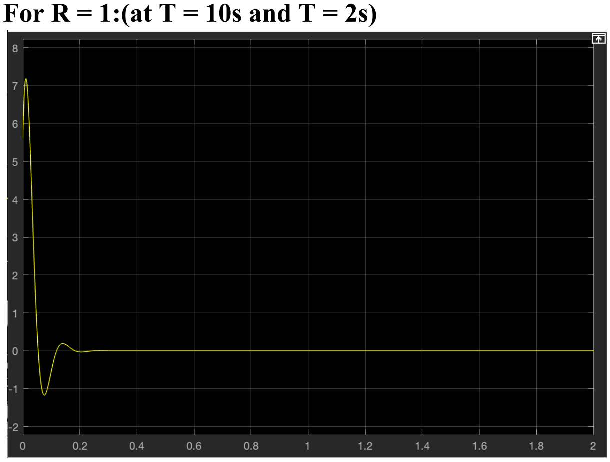
For R = 0.2:(at T = 10s and T = 2s)



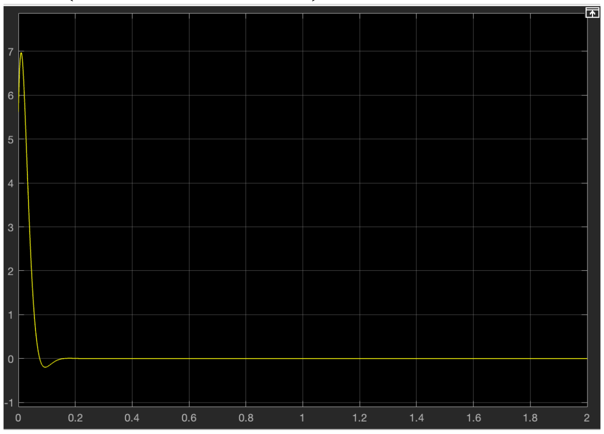
For R = 0.5: (at T = 10s and T = 2s)



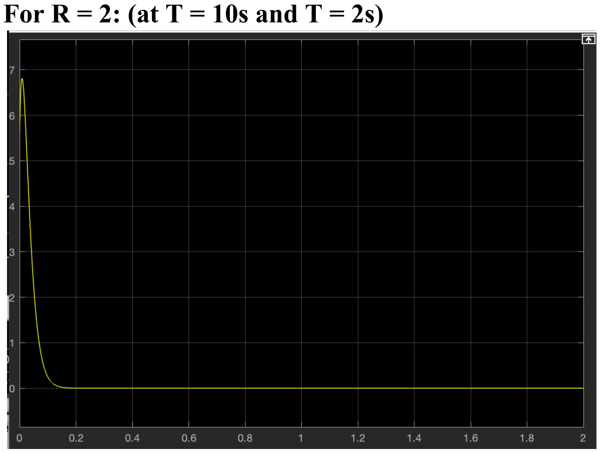
For R = 1:(at T = 10s and T = 2s)



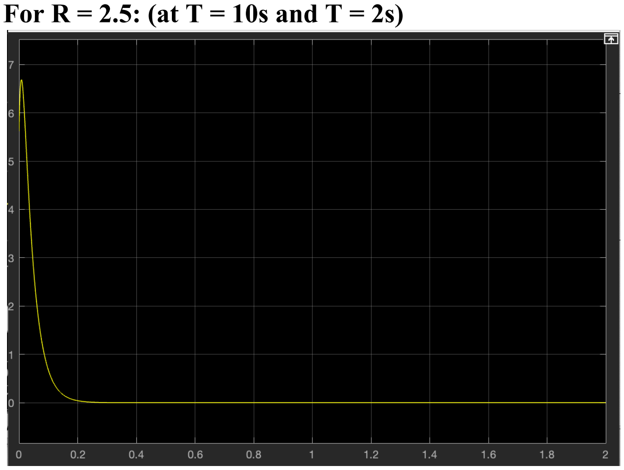
For R = 1.5: (at T = 10s and T = 2s)



For R = 2: (at T = 10s and T = 2s)



For R = 2.5: (at T = 10s and T = 2s)



b. *At what resistance does the system transition from underdamped to overdamped? (Keep L=0.01×Θ and C=0.01×Θ)* (0.5 pt)
At R = 2ohms, we are critically damped. This is when the system transitions from under to overdamped.

c. *How would underdamped to overdamped transition change if you increase L to 0.02×Θ and C to 0.02×Θ?* (0.5 pt)

Conclusion (1.2):

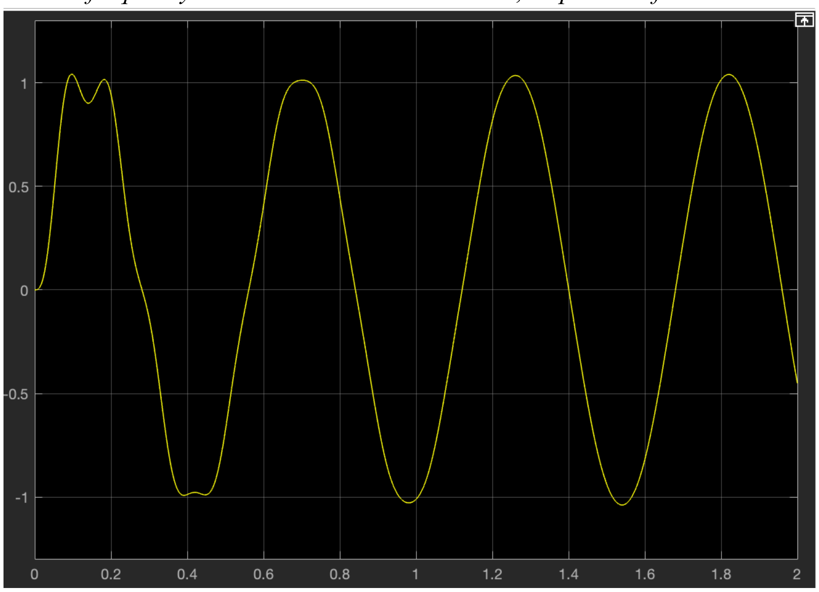
Transition remains the exact same since L = C, and R = 2*sqrt(L/C) if we care about the value of R that makes the system critically damped. Since L and C are the same, and they both increase by the same factor of 2, the result in the square root remains 1, and R remains at 2.

2. RLC circuit response to an external voltage source

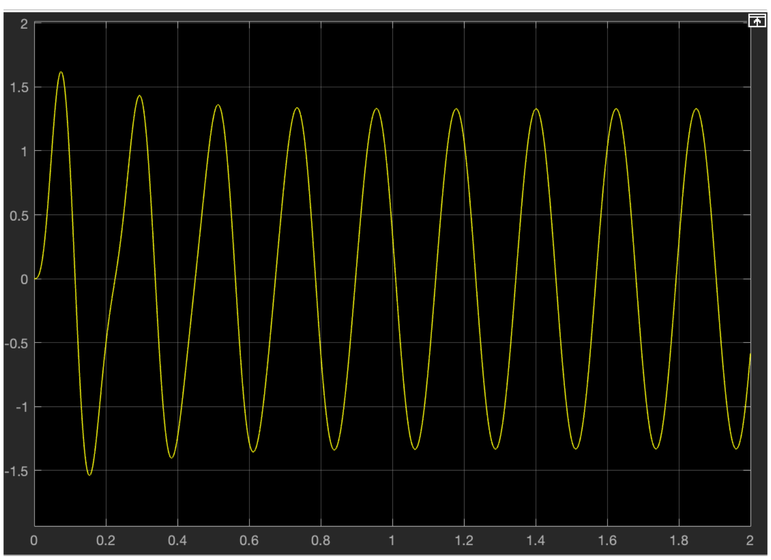
2.1 Exercise 2.1

a. *Set the amplitude of your voltage source to 1 and measure the amplitude of the response of the circuit for the following input frequencies.*(1 pt)

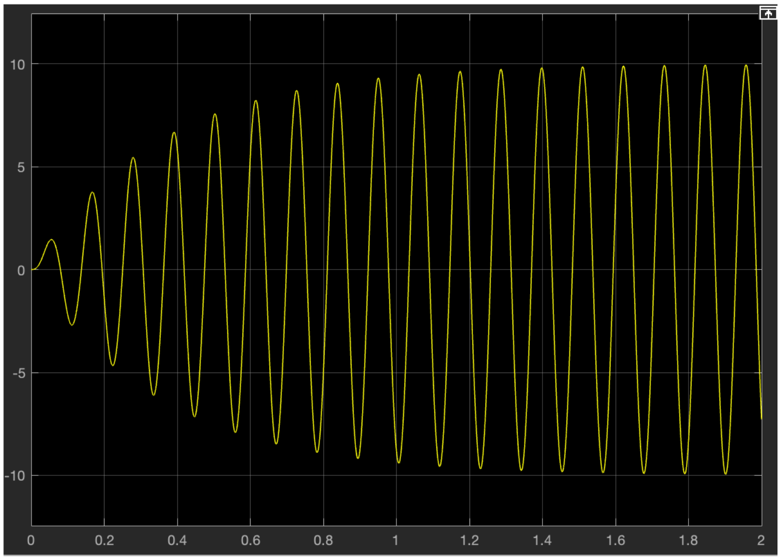
1. *Natural frequency / 5 = 1.78Hz or 11.23 rad/sec, amplitude of 1.041V*



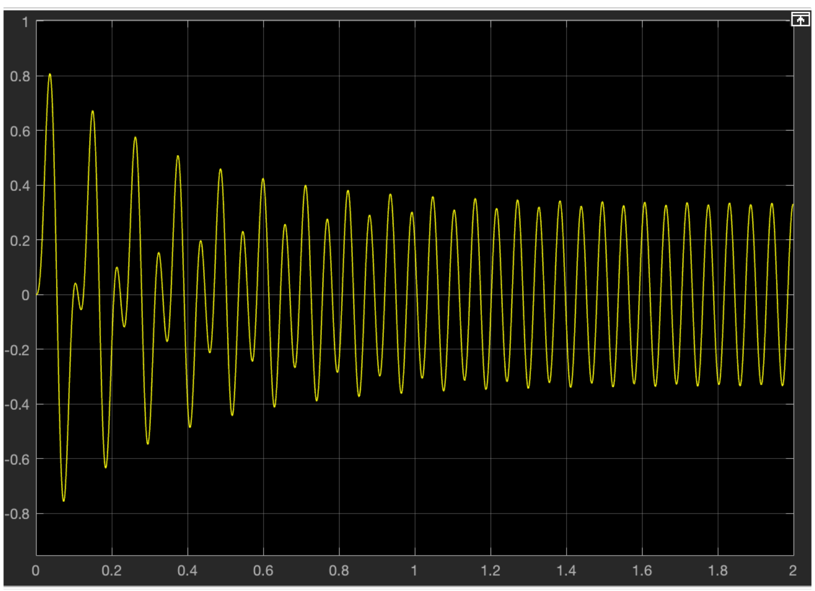
2. *Natural frequency / 2 = 4.47 Hz or 28.08 rad/sec, , amplitude of 1.3V*



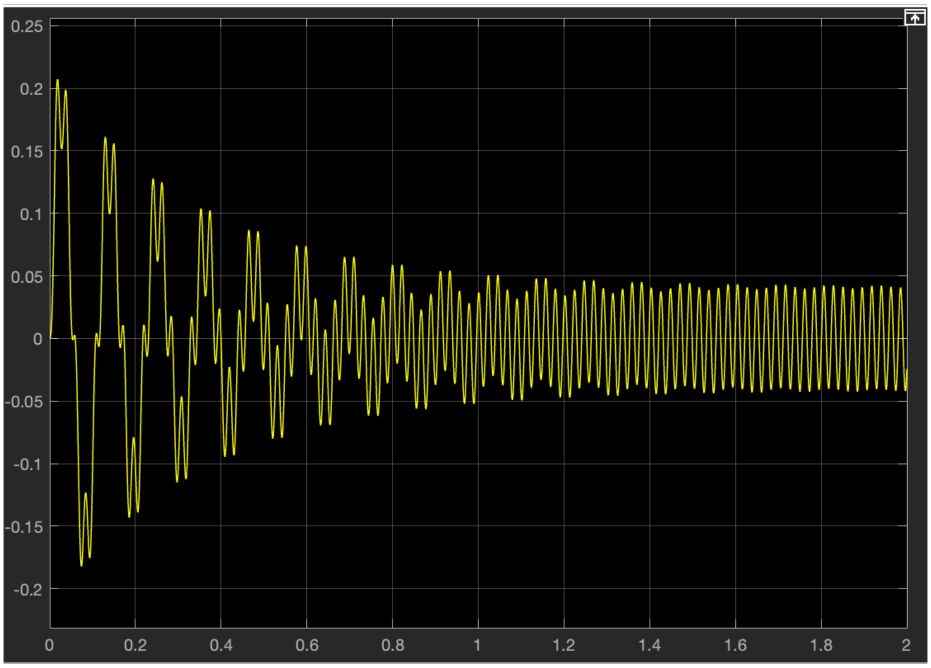
3. *Natural frequency = 8.94 Hz or 56.17 rad/sec, amplitude of 10V*



4. *Natural frequency * 2 = 17.88 Hz or 112.34 rad/sec, amplitude of 0.3V*



5. *Natural frequency * 5 = 44.7 Hz or 280.86 rad/sec, amplitude of 0.04V*



Conclusion (2.1):

At frequencies significantly below the circuit's natural frequency (for instance if we divide by 5 or 2), the RLC circuit operates in a manner that allows the input signal to be transferred to the output with little to no modification in strength.

When the frequency of the input signal begins to approach the circuit's resonant frequency, the circuit enhances the signal, resulting in a boosted output amplitude.

Beyond this resonant frequency, the output amplitude undergoes a steep reduction.

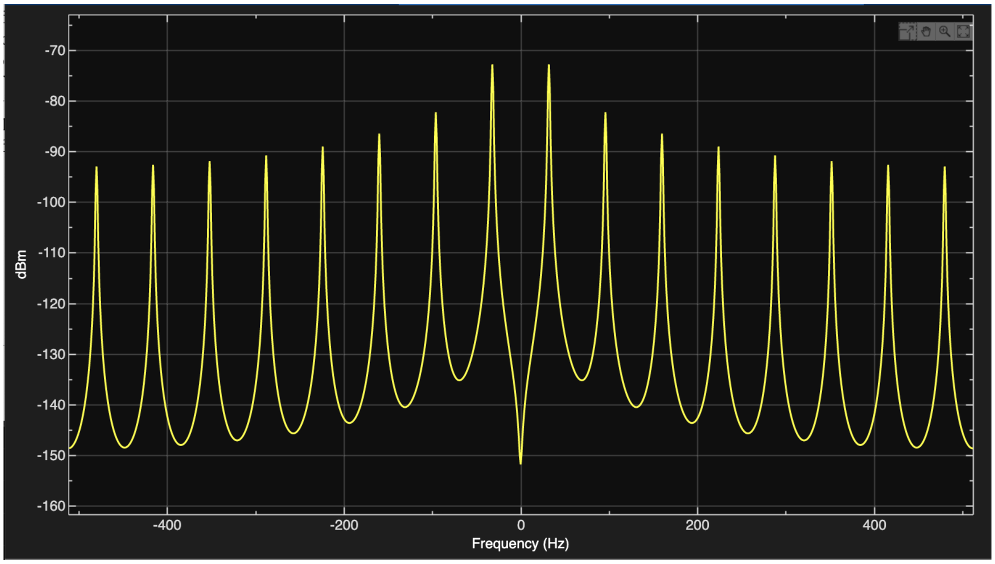
This pattern of behavior is indicative of a second-order low pass filter. Such a filter permits signals with frequencies below its resonant frequency to pass with minimal interference, resulting in an output that closely matches the input in terms of amplitude. Near the resonant frequency, the filter tends to amplify the signal due to the inherent resonance effect associated with second-order low pass filters. Above the resonant frequency, the filter becomes highly effective at reducing the amplitude of the signal, thereby curtailing higher frequency components.

2.2 Exercise 2.2

Include a screenshot demonstrating the square wave Simulink model and provide an explanation of the peaks in the spectrum analyzer. (TA to check box) (1 pt)

Conclusion (2.2):

As can be seen from the spectrum analyzer, the peaks occur once every 64Hz, given by a sampling rate of 1/32/2 periods = 1/64, since F = 1/T. The value is held for one interval because of the zero order hold module, which means we will peak at those frequencies. A square wave is essentially an aggregate of numerous sine waves with different frequencies. The visible peaks in the spectrum are the harmonics, which are unique frequencies that are odd multiples (1st, 3rd, 5th, and so on) of the fundamental frequency of the square wave. The spaces between these peaks highlight the discrete nature of these harmonics – they are not a continuous spread but individual frequencies that collectively shape the square wave.



3. Applying Fourier Series in circuit analysis

3.1 Exercise 3.1

a. *Use a square wave with 1/32 sec period. Read the frequency of the first 4 peaks on the frequency spectrum, and record the results below.* (0.5 pt)
32Hz, 96Hz, 160Hz, and 224Hz

b. *Calculate the first 4 terms of the Fourier series for the square wave using the equation provided, and write down the frequency and amplitude of each term from the Fourier approximation below.* (0.5 pt) part 3.1 sample rate = $\frac{1}{64}$, Period = $\frac{1}{32}$

$$\text{Square}(t) = \frac{4}{\pi} \sum_{n=1,3,5,7\ldots}^{\infty} \frac{1}{n} \sin \frac{2\pi n t}{T}$$

· Sine wave 1 (n=1)

$F = \frac{1}{T} = 32\,Hz$

$A = \frac{4}{\pi} \approx 1.27$

Sine wave 2 (n=3)

$F = 3 \cdot \frac{1}{T} = 96\,Hz$

$A = \frac{4}{3\pi} = 0.4244$

· Sine wave 3 (n=5)

$F = 5 \times \frac{1}{T} = 160\,Hz$

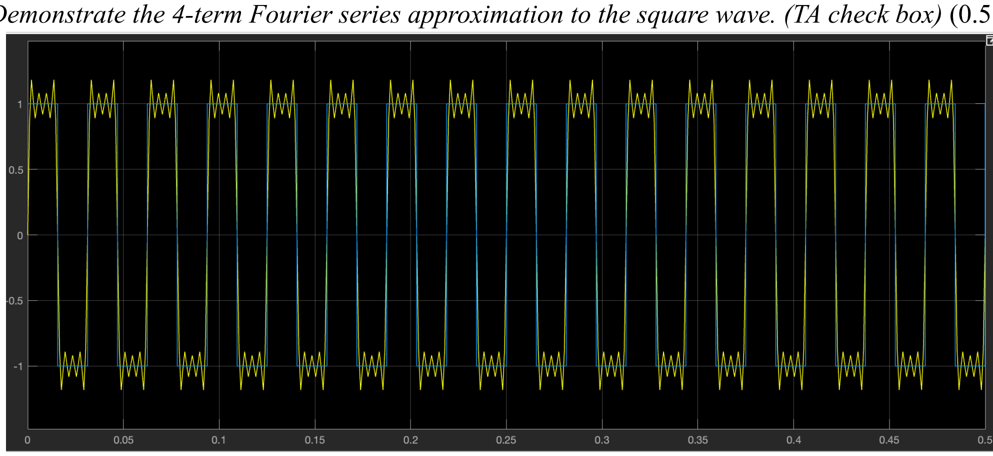
$A = \frac{4}{5\pi} = 0.2546$

· Sine wave 4 (n=7)

$F = 7 \times \frac{1}{T} = 224\,Hz$

$A = \frac{4}{7\pi} = 0.1822$

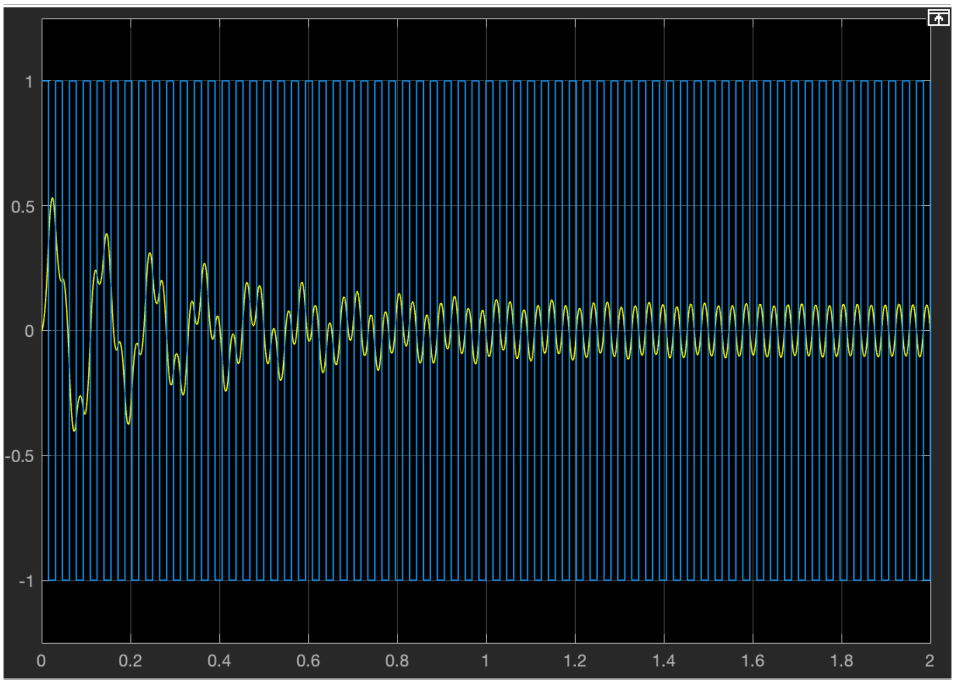
c. *Demonstrate the 4-term Fourier series approximation to the square wave. (TA check box)* (0.5 pt)



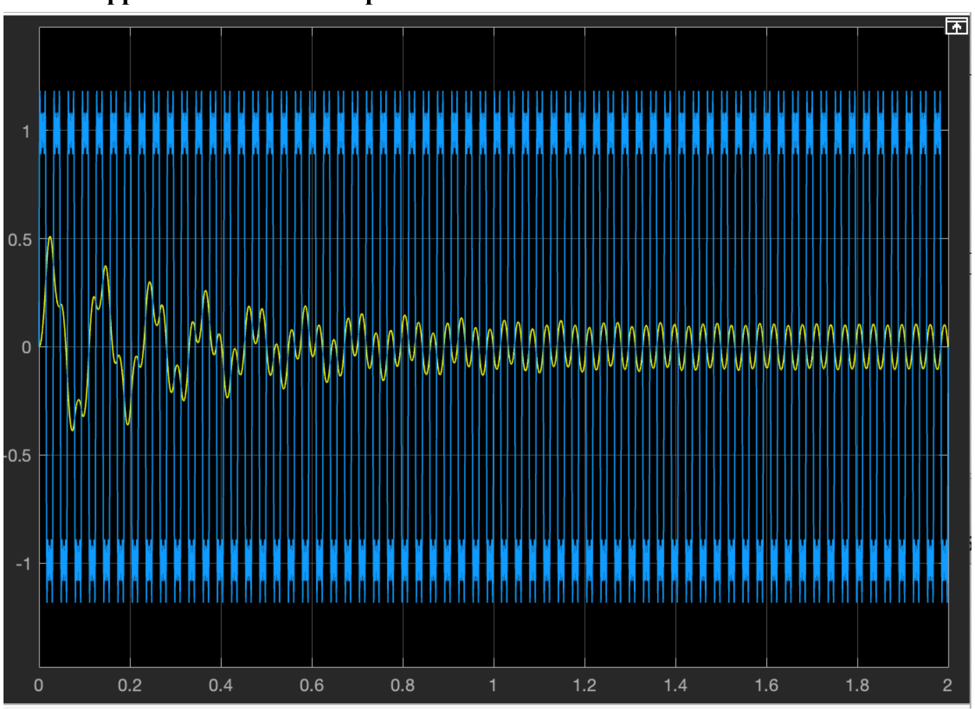
3.2 Exercise 3.2

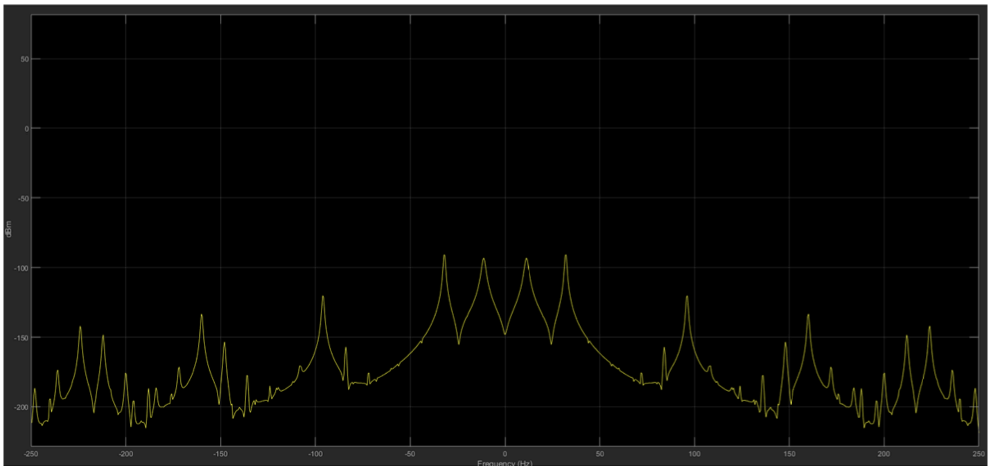
Compare the response of RLC circuit to the 4-term Fourier series approximation to that of the square wave and include a screenshot. (TA check box) (1 pt)

Square wave RLC response:



Fourier approximation RLC response:



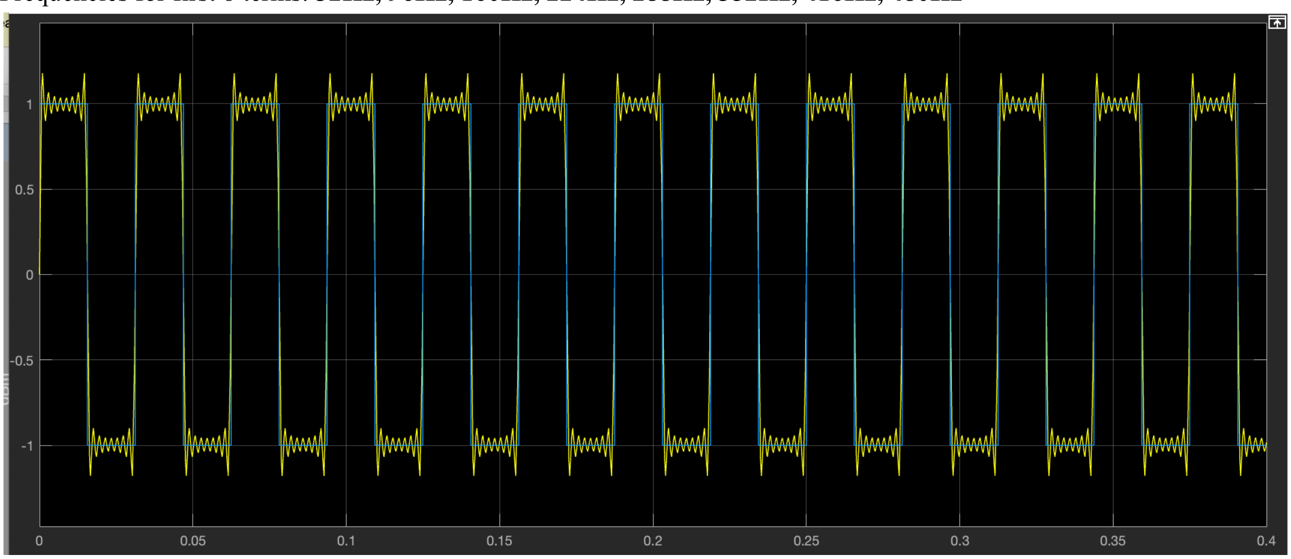


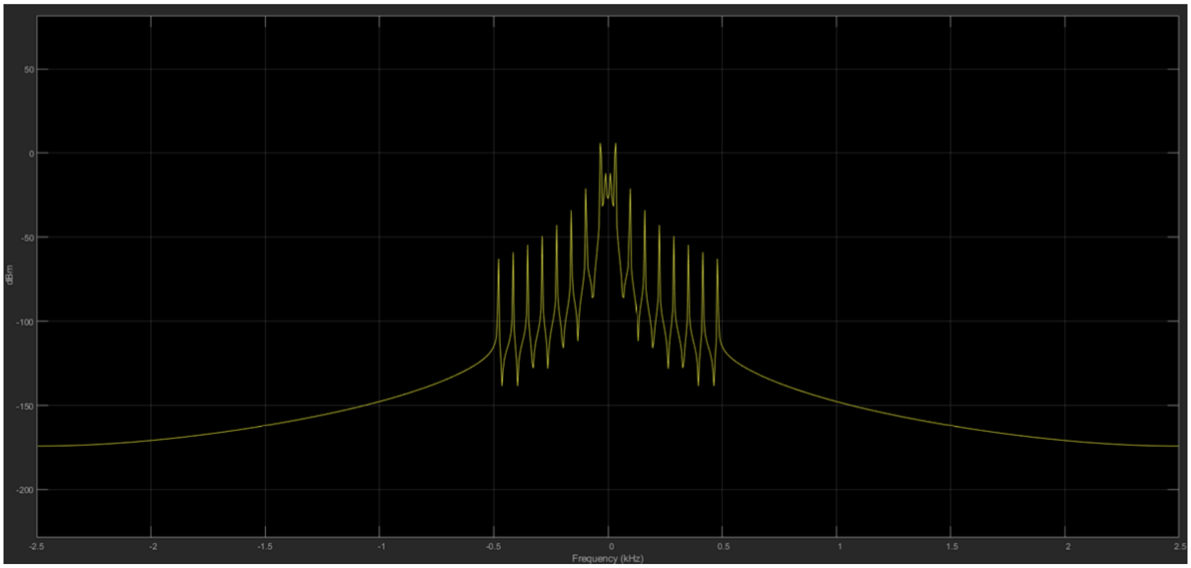
Conclusion (3.2):

The output signal, when driven by a square wave input, is a sinusoid with an amplitude of 0.1758 V and a frequency of 32 Hz. Similarly, using a 4-term approximation as the input produces a sinusoidal output with a slightly different amplitude of 0.1757 V, but the frequency remains constant at 32 Hz.

3.3 Exercise 3.3

a. *Compare the 8-term Fourier series approximation to the square wave and include a screenshot. (TA check box)* (0.5 pt)
Frequencies for first 8 terms: **32Hz, 96Hz, 160Hz, 224Hz, 288Hz, 352Hz, 416Hz, 480Hz**





Sample rate $= \frac{1}{64}$, Period $= \frac{1}{32}$

$$\text{Square}(t) = \frac{4}{\pi} \sum_{n=1,3,5,7\ldots}^{\infty} \frac{1}{n} \sin \frac{2\pi n t}{T}$$

· Sine wave 1 (n=1)

$F = \frac{1}{T} = 32\,Hz$

$A = \frac{4}{\pi} = 1.27$

· Sine wave 2 (n=3)

$F = 3 \cdot \frac{1}{T} = 96\,Hz$

$A = \frac{4}{3\pi} = 0.4244$

· Sine wave 3 (n=5)

$F = 5 \times \frac{1}{T} = 160\,Hz$

$A = \frac{4}{5\pi} = 0.2546$

· Sine wave 4 (n=7)

$F = 7 \times \frac{1}{T} = 224\,Hz$

$A = \frac{4}{7\pi} = 0.1822$

· Sine wave 5 (n=9)

288Hz

$4/9\pi$

· Sine wave 6 (n=11)

352Hz

$4/11\pi$

· Sine wave 7 (n=13)

416Hz

$4/13\pi$

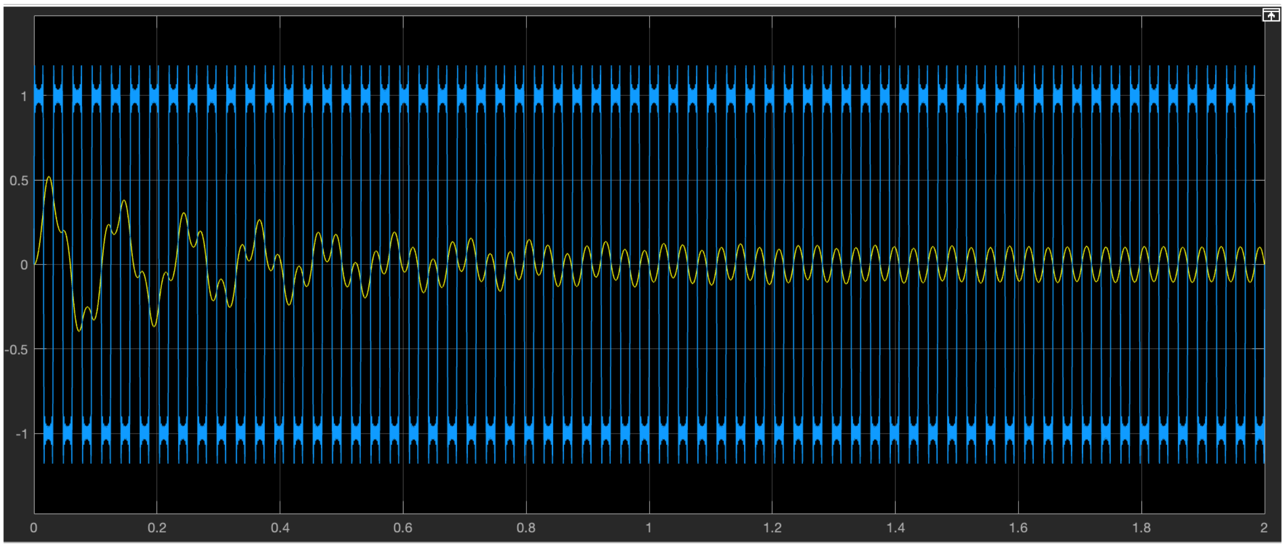
· Sine wave 8 (n=15)

480Hz

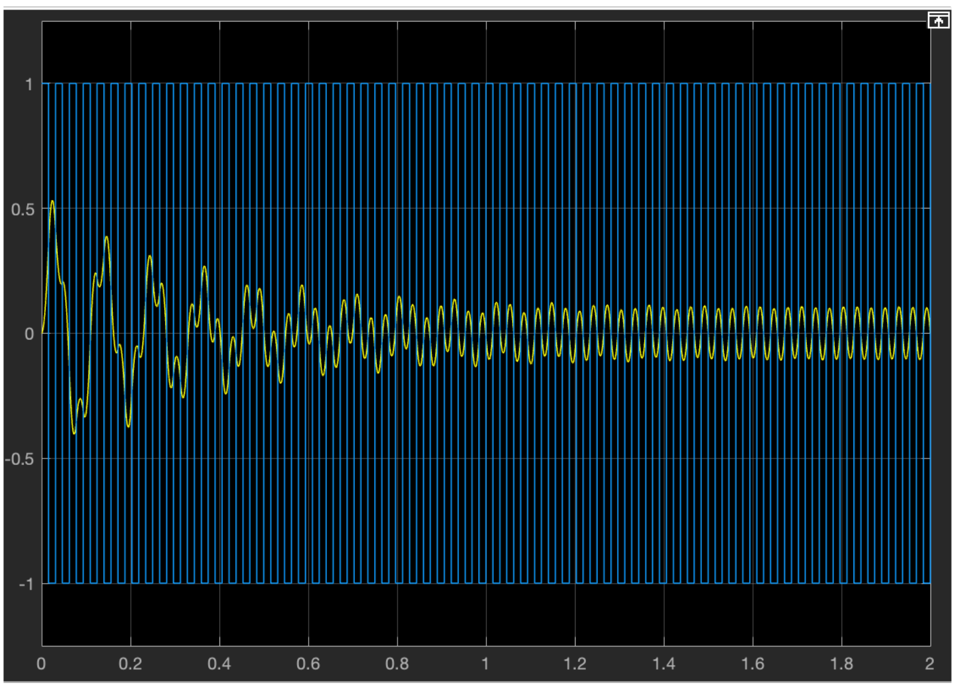
$4/15\pi$

b. *Compare the response of RLC circuit to the 8-term Fourier series approximation to that of the square wave and include a screenshot. (TA check box)* (1pt)

8-term Fourier series RLC response:



Square wave RLC response



c. *Does the 8-term Fourier series approximate the square wave better, or does the output response with the 8-term Fourier series approximate the output response for the square wave better? Include an answer and an explanation of your answer. (TA check box)* (1pt)

Using an 8-term approximation enhances the resemblance to a true square wave, exhibiting less ripple than a 4-term approximation. Despite these differences in waveform smoothness, the overall time response of the actual square wave and both approximations (4-term and 8-term) remains the same. However, in the frequency domain, the 4-term and 8-term approximations show fewer peaks and reduced noise. This reduction is due to the limited number of harmonics in these approximations compared to the countless harmonics present in an ideal square wave.

While the 8 term Fourier series is a better approximation of the square wave, the Fourier series output response of the

RLC circuit input may not be as close to the output response of the RLC circuit with the square wave due to the filtering effects of the circuit. The system's response will modify the input signal by acting as a filter and has different impedances at different frequencies of the input signal's harmonics. The characteristics of the RLC circuit (damping, resonant frequency, etc) causes some harmonics to be amplified while others are attenuated, so the output may not retain the exact same shape of the 8 term Fourier series square wave approximation.