

Hidden Naive Bayes

Harry Zhang

Faculty of Computer Science
University of New Brunswick, Canada
E3B 5A3, hzhang@unb.ca

Liangxiao Jiang

Faculty of of Computer Science
China University of Geosciences
Wuhan, China 430074

Jiang Su

Faculty of Computer Science
University of New Brunswick, Canada
E3B 5A3, k4km1@unb.ca

Abstract

The conditional independence assumption of naive Bayes essentially ignores attribute dependencies and is often violated. On the other hand, although a Bayesian network can represent arbitrary attribute dependencies, learning an optimal Bayesian network from data is intractable. The main reason is that learning the optimal structure of a Bayesian network is extremely time consuming. Thus, a Bayesian model without structure learning is desirable. In this paper, we propose a novel model, called *hidden naive Bayes* (HNB). In an HNB, a hidden parent is created for each attribute which combines the influences from all other attributes. We present an approach to creating hidden parents using the average of weighted one-dependence estimators. HNB inherits the structural simplicity of naive Bayes and can be easily learned without structure learning. We propose an algorithm for learning HNB based on conditional mutual information. We experimentally test HNB in terms of classification accuracy, using the 36 UCI data sets recommended by Weka (Witten & Frank 2000), and compare it to naive Bayes (Langley, Iba, & Thomas 1992), C4.5 (Quinlan 1993), SBC (Langley & Sage 1994), NBTree (Kohavi 1996), CL-TAN (Friedman, Geiger, & Goldszmidt 1997), and AODE (Webb, Boughton, & Wang 2005). The experimental results show that HNB outperforms naive Bayes, C4.5, SBC, NBTree, and CL-TAN, and is competitive with AODE.

Introduction

A Bayesian network consists of a structural model and a set of conditional probabilities. The structural model is a directed graph in which nodes represent attributes and arcs represent attribute dependencies. Attribute dependencies are quantified by conditional probabilities for each node given its parents. Bayesian networks are often used for classification problems, in which a learner attempts to construct a classifier from a given set of training examples with class labels. Assume that A_1, A_2, \dots, A_n are n attributes (corresponding to attribute nodes in a Bayesian network). An example E is represented by a vector (a_1, a_2, \dots, a_n) , where a_i is the value of A_i . Let C represent the class variable (corresponding to the class node in a Bayesian network). We

use c to represent the value that C takes and $c(E)$ to denote the class of E . The Bayesian classifier represented by a Bayesian network is defined in Equation 1.

$$c(E) = \arg \max_{c \in C} P(c)P(a_1, a_2, \dots, a_n|c). \quad (1)$$

Assume that all attributes are independent given the class; that is,

$$P(E|c) = P(a_1, a_2, \dots, a_n|c) = \prod_{i=1}^n P(a_i|c). \quad (2)$$

The resulting classifier is called a naive Bayesian classifier, or simply naive Bayes:

$$c(E) = \arg \max_{c \in C} P(c) \prod_{i=1}^n P(a_i|c). \quad (3)$$

Figure 1 shows graphically the structure of naive Bayes. In naive Bayes, each attribute node has the class node as its parent, but does not have any parent from attribute nodes. Because the values of $P(a_i|c)$ can be easily estimated from training examples, naive Bayes is easy to construct. It is also, however, surprisingly effective (Kononenko 1990; Langley, Iba, & Thomas 1992; Domingos & Pazzani 1997).

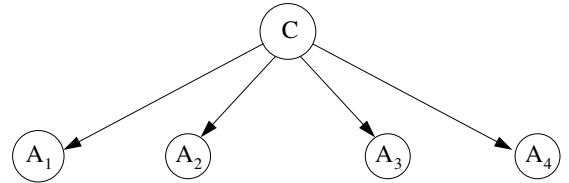


Figure 1: An example of naive Bayes

Naive Bayes is the simplest form of Bayesian networks. It is obvious that the conditional independence assumption in naive Bayes is rarely true. Extending its structure is a direct way to overcome the limitation of naive Bayes, since attribute dependencies can be explicitly represented by arcs. Tree Augmented naive Bayes (TAN) is an extended tree-like naive Bayes (Friedman, Geiger, & Goldszmidt 1997), in which the class node directly points to all attribute nodes

and an attribute node can have only one parent from another attribute node. Figure 2 shows an example of TAN. TAN is a specific case of general Augmented naive Bayesian networks, or simply Augmented naive Bayes (ANB), in which the class node also directly points to all attribute nodes, but there is no limitation on the arcs among attribute nodes (except that they do not form any directed cycle).

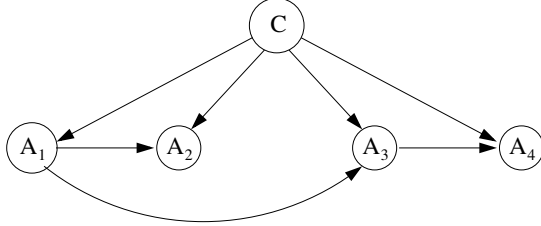


Figure 2: An example of TAN

Learning an optimal ANB is equivalent to learning an optimal Bayesian network, which has been proved to be NP-hard (Chickering 1996). In fact, the most time consuming step in learning a Bayesian network is learning the structure (structure learning). In practice, imposing restrictions on the structures of Bayesian networks, such as TAN, leads to acceptable computational complexity and a considerable improvement over naive Bayes. One main issue in learning TAN is that only one attribute parent is allowed for each attribute, ignoring the influences from other attributes. In addition, in a TAN learning algorithm, structure learning is also unavoidable.

Certainly, a model that avoids structure learning, and is still able to represent attribute dependencies to some extent, is desirable. In this paper, we present a new model *hidden naive Bayes* (HNB). HNB creates a hidden parent for each attribute, which represents the influences from all other attributes. Our experimental results show that HNB demonstrates remarkable accuracy compared to other state-of-the-art algorithms.

The rest of the paper is organized as follows. We first introduce the related work. Then we present our new model *hidden naive Bayes*, followed by the description of our experimental setup and results in detail. We make a conclusion and outline the main directions for future research.

Related Work

Numerous techniques have been proposed to improve or extend naive Bayes, mainly in two approaches: selecting attribute subsets in which attributes are conditionally independent, and extending the structure of naive Bayes to represent attribute dependencies.

The idea of selecting a subset of attributes or forming new attributes is to convert the data to a new form that satisfies the conditional independence assumption. Of the proposed techniques, *selective naive Bayes* (SBC) by Langley and Sage (1994) demonstrates a remarkable improvement over naive Bayes. SBC uses forward selection to find a good subset of attributes, and then uses this subset to construct a naive Bayes.

As discussed in the previous section, ANB is an example of the second approach. Learning restricted ANB, such as TAN, is a reasonable trade-off between model optimality and computational complexity. In fact, *Friedman et al.* (1997) show that the result of searching for an optimal Bayesian network may not be better than the result of just searching for an optimal TAN. They propose a TAN learning algorithm based on conditional mutual information between two attributes given the class variable, called CL-TAN in this paper. CL-TAN is an extension of the *ChowLiu* algorithm (Chow & Liu 1968). The conditional mutual information is defined as

$$I_P(X; Y|Z) = \sum_{x,y,z} P(x,y,z) \log \frac{P(x,y|z)}{P(x|z)P(y|z)}, \quad (4)$$

where x , y , and z are the values of variables X , Y , and Z respectively. In CL-TAN, $I_P(A_i; A_j|C)$ between each pair of attributes is computed, and a complete undirected weighted graph is built, in which nodes are attributes A_1, \dots, A_n , and the weight of an edge connecting A_i to A_j is set to $I_P(A_i; A_j|C)$. Then, a maximum weighted spanning tree is constructed. Finally, the undirected tree is converted to directed, and a node labeled by C that points to all attribute nodes is added. Keogh and Pazzani (1999) propose a cross-validation-based TAN learning algorithm *SuperParent*, and show that the *SuperParent* algorithm outperforms the CL-TAN learning algorithm in classification accuracy. However, its training time complexity is also significantly higher than that of CL-TAN.

Kohavi (1996) presents a model NBTree to combine a decision tree with naive Bayes. In an NBTree, a local naive Bayes is deployed on each leaf of a traditional decision tree, and an example is classified using the local naive Bayes on the leaf into which it falls. The experiments show that NBTree outperforms naive Bayes significantly in accuracy.

The most recent work on improving naive Bayes is AODE (averaged one-dependence estimators) (Webb, Boughton, & Wang 2005). In AODE, an ensemble of one-dependence classifiers are learned and the prediction is produced by aggregating the predictions of all qualified classifiers. The notion of x -dependence is introduced by Sahami (Sahami 1996). An x -dependence estimator means that the probability of an attribute is conditioned by the class variable and at most x other attributes, which corresponds to an ANB with at most x attribute parents. In AODE, a one-dependence classifier is built for each attribute, in which the attribute is set to be the parent of all other attributes. Their experimental results show that AODE performs surprisingly well compared to other classification algorithms. For example, AODE outperforms the *SuperParent* algorithm significantly.

Some other, more sophisticated, naive Bayes-based learning algorithms with high time complexity have also been proposed. Zhang (2004) proposes a model, *hierarchical naive Bayes*, in which hidden variables are introduced to alleviate the conditional independence assumption. A *hierarchical naive Bayes* is a tree-like Bayesian network in which internal nodes are hidden variables, and leaf nodes are attributes. Learning *hierarchical naive Bayes* has a high com-

putational complexity. Zheng and Webb (2000) propose an approach of lazy learning, the lazy Bayesian rule (LBR). The time complexity for learning LBR is also quite high.

Hidden Naive Bayes

As discussed in previous sections, naive Bayes ignores attribute dependencies. On the other hand, although a Bayesian network can represent arbitrary attribute dependencies, it is intractable to learn it from data (Chickering 1996). Thus, learning restricted structures, such as TAN, is more practical. However, only one parent is allowed for each attribute in TAN, even though several attributes might have the similar influence on it. Our motivation is to develop a new model that can avoid the intractable computational complexity for learning an optimal Bayesian network and still take the influences from all attributes into account. Our idea is to create a hidden parent for each attribute, which combines the influences from all other attributes. This model is called *hidden naive Bayes* (HNB).

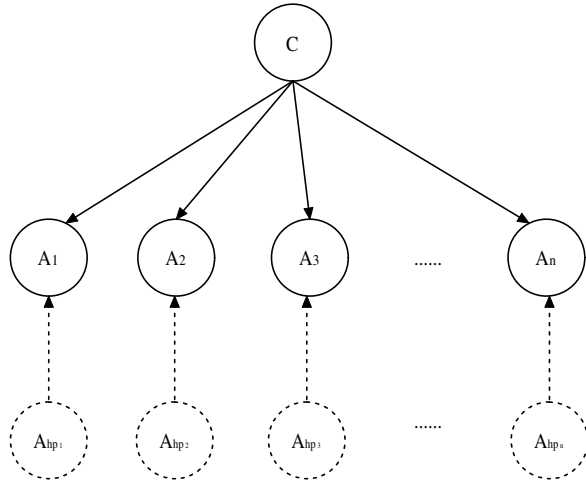


Figure 3: The structure of HNB

Figure 3 gives the structure of an HNB. In Figure 3, C is the class node, and is also the parent of all attribute nodes. Each attribute A_i has a hidden parent A_{hpi} , $i = 1, 2, \dots, n$, represented by a dashed circle. The arc from the hidden parent A_{hpi} to A_i is also represented by a dashed directed line, to distinguish it from regular arcs.

The joint distribution represented by an HNB is defined as follows.

$$P(A_1, \dots, A_n, C) = P(C) \prod_{i=1}^n P(A_i | A_{hpi}, C), \quad (5)$$

where

$$P(A_i | A_{hpi}, C) = \sum_{j=1, j \neq i}^n W_{ij} * P(A_i | A_j, C), \quad (6)$$

and $\sum_{j=1, j \neq i}^n W_{ij} = 1$. The hidden parent A_{hpi} for A_i is essentially a mixture of the weighted influences from all other attributes.

The classifier corresponding to an HNB on an example $E = (a_1, \dots, a_n)$ is defined as follows.

$$c(E) = \arg \max_{c \in C} P(c) \prod_{i=1}^n P(a_i | a_{hpi}, c). \quad (7)$$

In an HNB, attribute dependencies are represented by hidden parents of attributes. The way of defining hidden parents determines the capability of representing attribute dependencies. In Equation 6, one-dependence estimators $P(A_i | A_j, C)$ are used to define hidden parents. Recall that at most one attribute parent is allowed for each attribute in TAN, and the influences from other attributes have to be ignored. In an HNB, on the other hand, the influences from all other attributes can be represented and a weight is used to represent the importance of an attribute. Thus, intuitively, HNB is a more accurate and expressive model than TAN with respect to representing attribute dependencies.

If we have an order of attributes: A_1, \dots, A_n , $P(A_i | A_{hpi}, C)$ can be thought of as an approximation of $P(A_i | A_1, \dots, A_{i-1})$. In Equation 6, the approximation is based on one-dependence estimators. However, in principle, arbitrary x -dependence estimators can be used to define hidden parents. If $x = n - 1$, any Bayesian network is representable by an HNB. Thus, theoretically, an HNB is equivalent to a Bayesian network in terms of expressive power. In practice, however, we would prefer a simple way to define hidden parents in order to make the learning process simple and efficient.

From Equation 5 and 6, we can see that the approach to determining the weights W_{ij} , $i, j = 1, \dots, n$ and $i \neq j$, is crucial for learning an HNB. There are two general approaches to doing it: performing a cross-validation based search, or directly computing the estimated values from data. We adopt the latter, and use the conditional mutual information between two attributes A_i and A_j as the weight of $P(A_i | A_j, C)$. More precisely, in our implementation, W_{ij} is defined in Equation 8.

$$W_{ij} = \frac{I_P(A_i; A_j | C)}{\sum_{j=1, j \neq i}^n I_P(A_i; A_j | C)}, \quad (8)$$

where $I_P(A_i; A_j | C)$ is the conditional mutual information defined in Equation 4.

Learning an HNB is quite simple and mainly about estimating the parameters in the HNB from the training data. The learning algorithm for HNB is depicted as follows.

Algorithm HNB(D)

Input: a set D of training examples

Output: an hidden naive Bayes for D

for each value c of C

 Compute $P(c)$ from D .

for each pair of attributes A_i and A_j

for each assignment a_i, a_j , and c to A_i, A_j , and C

 Compute $P(a_i, a_j | c)$ from D

for each pair of attributes A_i and A_j

 Compute $I_P(A_i, A_j | C)$

for each attribute A_i

 Compute $W_i = \sum_{j=1, j \neq i}^n I_P(A_i; A_j | C)$

for each attribute A_j and $j \neq i$
 Compute $W_{ij} = \frac{I_P(A_i, A_j|C)}{W_i}$

From the algorithm above, we know that the training process of HNB is similar to CL-TAN, except no structure learning. A three-dimensional table of probability estimates for each attribute-value, conditioned by each other attribute-value and each class is generated. To create the hidden parent of an attribute, HNB needs to compute the conditional mutual information $I_P(A_i; A_j|C)$ for each pair of attributes. The time complexity for computing weights using Equation 8 is $O(n^2)$. Thus, the training time complexity of HNB is $O(tn^2 + kn^2v^2)$, where t is the number of training examples, n is the number of attributes, k is the number of classes, and v is the average number of values for an attribute. At classification time, given an example, Equation 7 is used, and it takes $O(kn^2)$.

Compared to CL-TAN, which has a training time complexity of $O(tn^2 + kn^2v^2 + n^2 \log n)$ and classification time complexity of $O(kn)$, HNB does not have structure learning with the time complexity of $O(n^2 \log n)$ in CL-TAN. Thus, the training time complexity of HNB is lower than that of CL-TAN.

Compared to AODE, which has a training time complexity of $O(tn^2)$ and classification time complexity of $O(kn^2)$, HNB needs more training time and same classification time. HNB, however, has an explicit semantics. Roughly speaking, a hidden parent for an attribute can be seen as aggregating the influences from all other attributes that with higher influences are assigned higher weights. Such an explicit semantics makes HNB understandable. In real-world applications, the comprehensibility of a model is important for decision making. Actually, the weights in HNB can be assigned by human experts, which allows an effective interaction between human experts and the learning program. In addition, we should notice that AODE is an ensemble learning method, in which a collection of models is built and their predictions are combined, whereas only a single model is learned in HNB.

Experiments and Results

We ran our experiments on all the 36 data sets recommended by Weka (Witten & Frank 2000), which are described in Table 1. All these data sets are from the UCI repository (Blake & Merz 2000). We downloaded these data sets in the format of *arff* from the main web of Weka.

In the preprocessing stages of data sets, we used the filter of *ReplaceMissingValues* in Weka to replace the missing values of attributes. Numeric attributes were discretized by the filter of *Discretize* in Weka using unsupervised ten-bin discretization. Thus, all attributes were treated as nominal. Moreover, it is well-known that, if the number of values of an attribute is almost equal to the number of examples in a data set, this attribute does not contribute any information to classification. So we used the filter of *Remove* in Weka to delete this type of attribute.

In our experiments, we used the Laplace estimation to avoid the zero-frequency problem. More precisely, we esti-

Table 1: Description of the data sets used in the experiments.

data set	size	attributes	classes	missing
anneal	898	39	6	Y
anneal.ORIG	898	39	6	Y
audiology	226	70	24	Y
autos	205	26	7	Y
balance-scale	625	5	3	N
breast-cancer	286	10	2	Y
breast-w	699	10	2	Y
colic	368	23	2	Y
colic.ORIG	368	28	2	Y
credit-a	690	16	2	Y
credit-g	1000	21	2	N
diabetes	768	9	2	N
Glass	214	10	7	N
heart-c	303	14	5	Y
heart-h	294	14	5	Y
heart-statlog	270	14	2	N
hepatitis	155	20	2	Y
hypothyroid	3772	30	4	Y
ionosphere	351	35	2	N
iris	150	5	3	N
kr-vs-kp	3196	37	2	N
labor	57	17	2	Y
letter	20000	17	26	N
lymphography	148	19	4	N
mushroom	8124	23	2	Y
primary-tumor	339	18	21	Y
segment	2310	20	7	N
sick	3772	30	2	Y
sonar	208	61	2	N
soybean	683	36	19	Y
splice	3190	62	3	N
vehicle	846	19	4	N
vote	435	17	2	Y
vowel	990	14	11	N
waveform-5000	5000	41	3	N
zoo	101	18	7	N

mated the probabilities $P(c)$, $P(a_i|c)$, and $P(a_i|a_j, c)$ using Laplace estimation as follows.

$$\hat{P}(c) = \frac{n_c + 1}{t + k},$$

$$\hat{P}(a_i|c) = \frac{n_{ic} + 1}{n_c + v_i},$$

$$\hat{P}(a_i|a_j, c) = \frac{n_{ijc} + 1}{n_{jc} + v_i},$$

where t is the total number of training examples, k is the number of classes, v_i is the number of values of attribute A_i , n_c is the number of examples in class c , n_{ic} is the number of examples in class c and with $A_i = a_i$, n_{jc} is the number of examples in class c and with $A_j = a_j$, and n_{ijc} is the number of examples in class c and with $A_i = a_i$ and $A_j = a_j$.

We conducted experiments to compare HNB to naive Bayes (Langley, Iba, & Thomas 1992), C4.5 (Quinlan 1993), SBC (Langley & Sage 1994), NBTree (Kohavi 1996), CL-TAN (Friedman, Geiger, & Goldszmidt 1997), and AODE

(Webb, Boughton, & Wang 2005) in classification accuracy. We implemented HNB and SBC within the Weka framework (Witten & Frank 2000), and used the implementation of C4.5(J48), NBTree, CL-TAN, and AODE in Weka. In all experiments, the accuracy of an algorithm on a data set was obtained via 10 runs of ten-fold cross validation. Runs with the various algorithms were carried out on the same training sets and evaluated on the same test sets. Finally, we conducted a two-tailed *t*-test with a 95% confidence level to compare our algorithm with other algorithms.

Table 2 shows the accuracies of the algorithms on each data set, and the average accuracy and standard deviation on all data sets are summarized at the bottom of the table. Table 3 shows the results of the two-tailed *t*-test, in which each entry *w/t/l* means that the algorithm in the corresponding row wins in *w* data sets, ties in *t* data sets, and loses in *l* data sets, compared to the algorithm in the corresponding column.

The detailed results displayed in Table 2 and Table 3 show that the performance of HNB is competitive with the state-of-the-art classification algorithms compared in the paper. Now, we summarize the highlights briefly as follows:

1. HNB achieves a significant improvement over naive Bayes (16 wins and 2 losses).
2. HNB outperforms SBC (10 wins and 3 losses), C4.5 (10 wins and 4 losses), CL-TAN (10 wins and 3 losses), and NBTree (8 wins and 4 losses).
3. HNB is competitive with AODE (3 wins and 2 losses). Considering that HNB is a single understandable classifier in contrast to an ensemble of classifiers in AODE, HNB is overall more effective.

Conclusions

In this paper, we proposed a novel model *hidden Naive Bayes* (HNB) by adding a hidden parent for each attribute on naive Bayes. Our experimental results show that HNB has a better overall performance compared to the state-of-the-art algorithms. Considering the simplicity and comprehensibility of HNB, HNB is a promising model that could be used in many real world applications.

The HNB that we implemented is based on one-dependence estimators. It could be generalized to arbitrary dependence estimators. Thus, HNB can be seen as a general model in which structure learning plays a less important role than in Bayesian networks. In defining and learning an HNB, how to learn the weights is crucial. Currently, we use conditional mutual information to estimate the weights directly from data. We believe that the use of more sophisticated methods, such as EM, could improve the performance of the current HNB and make its advantage stronger. This is one direction for our future research.

References

- Blake, C., and Merz, C. J. 2000. UCI repository of machine learning databases. In *Dept of ICS, University of California, Irvine*. <http://www.ics.uci.edu/mllearn/MLRepository.html>.
- Chickering, D. M. 1996. Learning Bayesian networks is NP-Complete. In Fisher, D., and Lenz, H., eds., *Learning from Data: Artificial Intelligence and Statistics V*. Springer-Verlag. 121–130.
- Chow, C. K., and Liu, C. N. 1968. Approximating discrete probability distributions with dependence trees. *IEEE Trans. on Information Theory* 14:462–467.
- Domingos, P., and Pazzani, M. 1997. Beyond independence: Conditions for the optimality of the simple Bayesian classifier. *Machine Learning* 29:103–130.
- Friedman, N.; Geiger, D.; and Goldszmidt, M. 1997. Bayesian network classifiers. *Machine Learning* 29:131–163.
- Keogh, E. J., and Pazzani, M. J. 1999. Learning augmented Naive Bayes classifiers. In *Proceedings of the Seventh International Workshop on AI and Statistics*.
- Kohavi, R. 1996. Scaling up the accuracy of naive-bayes classifiers: A decision-tree hybrid. In *Proceedings of the Second International Conference on Knowledge Discovery and Data Mining*. AAAI Press. 202–207.
- Kononenko, I. 1990. Comparison of inductive and naive Bayesian learning approaches to automatic knowledge acquisition. In Wielinga, B., ed., *Current Trends in Knowledge Acquisition*. IOS Press.
- Langley, P., and Sage, S. 1994. Induction of selective Bayesian classifiers. In *Proceedings of Uncertainty in Artificial Intelligence 1994*. Morgan Kaufmann.
- Langley, P.; Iba, W.; and Thomas, K. 1992. An analysis of Bayesian classifiers. In *Proceedings of the Tenth National Conference of Artificial Intelligence*. AAAI Press. 223–228.
- Quinlan, J. 1993. *C4.5: Programs for Machine Learning*. Morgan Kaufmann: San Mateo, CA.
- Sahami, M. 1996. Learning limited dependence bayesian classifiers. In *Proceedings of the Second International Conference on Knowledge Discovery and Data Mining*. AAAI Press. 335–338.
- Webb, G. I.; Boughton, J.; and Wang, Z. 2005. Not so naive bayes: Aggregating one-dependence estimators. *Journal of Machine Learning* 58(1):5–24.
- Witten, I. H., and Frank, E. 2000. *Data Mining – Practical Machine Learning Tools and Techniques with Java Implementation*. Morgan Kaufmann.
- Zhang, N. L. 2004. Hierarchical latent class models for cluster analysis. *Journal of Machine Learning Research* 5:697–723.
- Zheng, Z., and Webb, G. I. 2000. Lazy learning of bayesian rules. *Journal of Machine Learning* 41(1):53–84.

Table 2: Experimental results on classification accuracy.

Datasets	C4.5	NB	SBC	CL-TAN	NBTree	AODE	HNB
anneal	98.65±0.97	94.32±2.23	96.94±2.03	97.65±1.48	98.4±1.53	96.74±1.72	97.74±1.28
anneal.ORIG	90.36±2.51	88.16±3.06	89.68±2.92	91.66±2.34	91.27±3.03	88.79±3.17	89.87±2.2
audiology	77.22±7.69	71.4±6.37	74.15±7	62.97±6.25	76.66±7.47	71.66±6.42	69.04±5.83
autos	81.54±8.32	63.97±11.35	68.69±11.27	74±9.65	74.75±9.44	73.38±10.24	75.49±9.89
balance-scale	64.14±4.16	91.44±1.3	91.44±1.3	85.57±3.33	91.44±1.3	89.78±1.88	89.14±2.05
breast-cancer	75.26±5.04	72.94±7.71	72.53±7.52	67.95±6.8	71.66±7.92	72.53±7.15	73.09±6.11
breast-w	94.01±3.28	97.3±1.75	96.58±2.19	94.46±2.54	97.23±1.76	97.11±1.99	95.67±2.33
colic	84.31±6.02	78.86±6.05	83.37±5.56	79.71±6.31	82.5±6.51	80.9±6.19	81.44±6.12
colic.ORIG	80.79±5.66	74.21±7.09	74.83±6.17	70.59±7.03	74.83±7.82	75.3±6.6	75.66±5.19
credit-a	85.06±4.12	84.74±3.83	85.36±3.99	83.06±4.75	84.86±3.92	85.91±3.78	85.8±4.1
credit-g	72.61±3.49	75.93±3.87	74.76±3.85	74.95±4.09	75.54±3.92	76.42±3.86	76.29±3.45
diabetes	73.89±4.7	75.68±4.85	76±5.24	74.83±4.43	75.28±4.84	76.37±4.35	76±4.6
glass	58.14±8.48	57.69±10.07	56.19±9.73	59.72±9.69	58±9.42	61.13±9.79	59.02±8.67
heart-c	79.14±6.44	83.44±6.27	81.12±7.15	78.46±8.03	81.1±7.24	82.48±6.96	82.31±6.82
heart-h	80.1±7.11	83.64±5.85	80.19±7.03	80.89±6.7	82.46±6.26	84.06±5.85	83.21±5.88
heart-statlog	79.78±7.71	83.78±5.41	80.85±7.61	78.74±6.98	82.26±6.5	83.67±5.37	82.7±5.89
hepatitis	81.12±8.42	84.06±9.91	82.51±8.48	82.72±8.23	82.9±9.79	84.82±9.75	83.92±9.43
hypothyroid	93.24±0.44	92.79±0.73	93.46±0.5	92.99±0.69	93.05±0.65	93.53±0.62	93.48±0.47
ionosphere	87.47±5.17	90.86±4.33	91.25±4.14	92.74±3.86	89.18±4.82	92.08±4.24	92±4.32
iris	96±4.64	94.33±6.79	96.67±4.59	91.73±8.16	95.27±6.16	94.47±6.22	93.93±5.92
kr-vs-kp	99.44±0.37	87.79±1.91	94.34±1.3	93.53±1.47	97.81±2.05	91.01±1.67	92.36±1.3
labor	84.97±14.24	96.7±7.27	82.63±12.69	88.33±11.89	95.6±8.39	95.3±8.49	92.73±11.16
letter	81.31±0.78	70.09±0.93	70.71±0.9	81.09±0.84	83.49±0.81	85.54±0.68	84.68±0.73
lymph	78.21±9.74	85.97±8.88	80.24±9.58	83.69±9.23	82.21±8.95	86.25±9.43	83.9±9.31
mushroom	100±0	95.52±0.78	99.7±0.22	99.51±0.26	100±0	99.95±0.07	99.94±0.1
primary-tumor	41.01±6.59	47.2±6.02	44.49±6.76	44.51±6.38	45.84±6.61	47.67±6.3	47.66±6.21
segment	93.42±1.67	89.03±1.66	90.65±1.77	93.88±1.55	92.64±1.61	92.94±1.4	93.76±1.47
sick	98.16±0.68	96.78±0.91	97.51±0.72	97.55±0.72	97.86±0.69	97.51±0.73	97.77±0.68
sonar	71.09±8.4	76.35±9.94	69.78±9.74	74.28±9.68	71.4±8.8	79.04±9.42	81.75±8.4
soybean	92.63±2.72	92.2±3.23	92.03±3.14	93.69±2.8	92.3±2.7	93.28±2.84	93.88±2.47
splice	94.17±1.28	95.42±1.14	94.95±1.29	95.31±1.15	95.42±1.14	96.12±1	95.86±1.08
vehicle	70.74±3.62	61.03±3.48	60.98±3.62	71.94±3.49	68.91±4.58	71.62±3.6	72.15±3.41
vote	96.27±2.79	90.21±3.95	95.59±2.76	93.01±3.95	94.78±3.32	94.52±3.19	94.43±3.18
vowel	75.57±4.58	66.09±4.78	68.59±4.5	93.06±2.86	88.01±3.71	89.52±3.12	91.34±2.92
waveform-5000	72.64±1.81	79.97±1.46	81.17±1.45	80.17±1.79	81.62±1.76	84.24±1.59	83.79±1.54
zoo	92.61±7.33	94.37±6.79	94.04±7.34	95.24±6.22	94.55±6.54	94.66±6.38	97.73±4.64
Mean	82.64±4.75	82.34±4.78	82.33±4.89	83.17±4.88	84.47±4.78	85.01±4.61	84.99±4.42

Table 3: Summary of experimental results: classification accuracy comparisons.

	C4.5	NB	SBC	CL-TAN	NBTree	AODE
NB	8/15/13					
SBC	5/21/10	11/23/2				
CL-TAN	5/22/9	11/20/5	5/24/7			
NBTree	7/27/2	12/24/0	8/28/0	8/26/2		
AODE	13/17/6	13/22/1	11/23/2	12/21/3	5/26/5	
HNB	10/22/4	16/18/2	10/23/3	10/23/3	8/24/4	3/31/2