Measuring Solar Surface and Internal Flows using Helioseismology

A Thesis

Submitted to the
Tata Institute of Fundamental Research, Mumbai
for the degree of Doctor of Philosophy
in Physics

by

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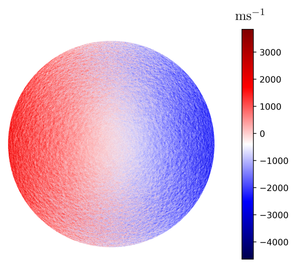
Declaration

This thesis is a presentation of my original research work. Wherever contributions of others are involved, every effort is made to indicate this clearly, with due reference to the literature, and acknowledgment of collaborative research and discussions.

The work was done under the guidance of Professor Shravan Hanasoge, at the Tata Institute of Fundamental Research, Mumbai.



[candidate's name and signature]

In my capacity as the supervisor of the candidate's thesis, I certify that the above statements are true to the best of my knowledge.

[guide's name and signature]

Date:

Collaborators

This thesis is based on work done in collaboration with several people.

1. The work presented in Chapter 2 was done in collaboration with Srijan Bharati Das and Jeroen Tromp from Princeton University, USA and Martin Woodard from Nortwest Research Associates, Colorado, USA, and is based on the publication that appeared in print as **ApJS 253**, 47 (March 2021) — (Kashyap et al., 2021)

2. The work presented in Chapter **??** was done in collaboration with Srijan Bharati Das, Deniz Oktay and Jeroen Tromp from Princeton University, USA and is based on the manuscript that has been submitted to the Astrophysical Journal.

Acknowledgments

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Dedicated to Appa, Amma and Anju.

Measuring Solar Surface and Internal Flows using Helioseismology

Abstract

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Contents

Publications

This thesis presents results that have been published in the following journal articles and arXiv preprints. The primary contributors of the papers are indicated using a * symbol next to the author's name.

Publications contributing to this thesis

1. Kashyap et al. (2021): Samarth G Kashyap*, Srijan Bharati Das*, Martin F Woodard, Shravan Hanasoge, Jeroen Tromp, "*Inferring Solar Differential Rotation through Normal-mode Coupling Using Bayesian Statistics*", The Astrophysical Journal Supplement Series, **253**: 47, March 2021. (Reproduced by permission of the AAS.)

2. Kashyap and Hanasoge (2021): Samarth G Kashyap*, Shravan Hanasoge, "*Characterizing Solar Surface Convection Using Doppler Measurements*", The Astrophysical Journal, **916**: 87, August 2021. (Reproduced by permission of the AAS.)

3. Das et al. (2022): Srijan Bharati Das*, Samarth G Kashyap*, Deniz Oktay, Shravan Hanasoge, Jeroen Tromp, "*Inference of near-surface solar rotation using mode-coupling*", submitted to The Astrophysical Journal, July 2022

4. Kashyap et al. (2022): Samarth G Kashyap*, Shravan Hanasoge, "*Meridional circulation from normal-mode coupling I: Modelling the Center-to-Limb systematic*", submitted to The Astrophysical Journal, August 2022.

Other publications

The following report was published during the period of the PhD and is peripherally related to the subject matter, but is not included in the thesis.

1.

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Chapter 1

Introduction

The Sun is a main-sequence star that can be spatially resolved and investigated in great detail, thanks to its proximity to us. A greater understanding of stellar structure and interior can be sought, by using the information of solar structure and interior as a benchmark. Nuclear reactions at the solar interior produce energy, which are transmitted out through the optically thin outer surface of the Sun through electromagnetic radiation. This radiation enables the observation of the solar surface and it reveals the rich dynamics of the Sun's evolution – from time-scales of seconds to decades. The subsurface of the Sun being optically thick, obfuscates an easy understanding of the solar interior. However, solar oscillations, which propagate through the solar interior and show up on the solar surface, are sensitive to the conditions of the solar interior. A detailed analysis of these oscillations has enabled a suprisingly precise imaging of the solar interior.

Stellar models are typically sherically symmetric i.e. the properties are purely a function of the stellar radius. The ingredients that go into modelling a star, which affect the structure and evolution are its mass m, initial helium abundance Y_0 and initial heavy metal abundace Z_0. Convection in stars is modelled by employing the mixing-length parameter α. Such a model is evolved over time to obtain to reach the observed temperature and luminosity. For the Sun, independent estimates of mass, radius, luminosity and age exist, which can be used to determine the helium abundance Y_0 and the mixing-length parameter α, iteratively. Such a solar model is termed as a standard solar model (SSM) and a popular choice of an SSM is Model S (Christensen-Dalsgaard et al., 1996). Such a model is spherically symmetric, non-rotation, non-magnetic, adiabatic, isotropic and static (SNRNMAIS). However, we have observed the Sun to be magnetically active, rotating and highly dynamic. It is also to be noted that the SSMs reproduce the observed solar oscillation power spectrum quite well – thus enabling a perturbative analysis of deviations from Model S. This means that propagation of seismic waves in the solar interior can be used to measure flows in the interior, given that the flows are not strong enough to alter the physics of wave propagation itself. In this thesis, we explore and establish the method of estimating flows in the solar interior through the measurement of distortion of helioseismic modes.

Solar activity is the term used to describe a range of magnetic phenomena, both short and long-lived, such as flares, sunspots, coronal mass ejections, etc (Usoskin, 2017). Solar activity is known to periodically alternate between periods of high and low acitivity – well characterized by various indices of acitivity, such as the *global sunspot number* (GSN; Hoyt and Schatten, 1998), the *flare index* (Özgüç et al., 2003), the F10.7 *index* (Tapping and Charrois, 1994), *sunspot area* (Baranyi et al., 2001), etc. The period of this activity cycle (*solar cycle*) is approximately 11 years (Hathaway, 2015). The strength and variation of observed solar activity is thought to be driven by flow fields in the convective envelope – particularly differential rotation, meridional circulation and convection (Charbonneau, 2020; Fan, 2009). Thus, understanding the physics that governs the evolution and sustenance of the activity cycle of the Sun is one of the long-standing challenges in astrophysics. Thus, it necessitates imaging its internal layers and accurately characterizing the flows in the interior.

Modelling the solar dynamo, which is the mechanism that maintains the Sun's magnetic field, is a real challenge. Cowling's anti-dynamo theorem (Cowling, 1933) states that a purely axisymmetric flow field cannot by itself sustain an axisymmetric magnetic field against Ohmic dissipation. The mechanism of breaking of axisymmetry was provided by Parker (1955) – who proposed that convective upflows at different latitudes would experience different cyclonic twists because of Coriolis force, thus breaking axisymmetry and providing a mechanism to bypass Cowling's theorem. Subsequent attempts to model the dynamo led to mean-field electrodynamics. However, dynamo models are "yet to recover from the three-way punch" (Charbonneau, 2020), namely, buoyancy effects on magnetic fields, questions regarding magnetic diffusivity, and the observed solar differential rotation, which is markedly different from models that produce solar-like dynamos. Hence, precise estimates of solar internal flows provide crucial inputs to constrain dynamo models.

Chapter 2

Measuring rotation from mode-coupling

Published as: Samarth G Kashyap, Srijan Bharati Das, Martin F Woodard, Shravan Hanasoge, Jeroen Tromp, "*Inferring Solar Differential Rotation through Normal-mode Coupling Using Bayesian Statistics*", The Astrophysical Journal Supplement Series, **253,** 47, (March 2021) — Kashyap et al. (2021)

2.1 Introduction

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Chapter 3

Measuring surface convection

Published as: Samarth G Kashyap, Shravan Hanasoge, "*Characterizing Solar Surface Convection Using Doppler Measurements*", The Astrophysical Journal, **916,** 87, (August 2021) — Kashyap and Hanasoge (2021)

3.1 Introduction

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Chapter 4

Conclusions

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Appendix A

Appendices for Chapter 2

A.1 Spherical harmonics symmetry relations

Consider a time-varying, real-valued scalar field on a sphere $\phi(\theta, \phi, t)$. The spherical harmonic components are given by

$$\phi^{l,|m|}(t) = \int_\Omega \mathrm{d}\Omega Y^{*l,|m|}(\theta, \phi)\phi(\theta, \phi, t) = (-1)^{|m|} \int_\Omega \mathrm{d}\Omega Y^{l,-|m|}\phi(\theta, \phi, t) = (-1)^{|m|}\phi^{*l,-|m|}(t) \tag{A.1}$$

where $d\Omega$ is the area element, the integration being performed over the entire surface of the sphere. After performing a temporal Fourier transform, we have

$$\phi^{l,|m|}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}t e^{-i\omega t}\phi^{l,|m|}(t) = (-1)^{|m|}\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}t e^{-i\omega t}\phi^{*l,-|m|}(t) \tag{A.2}$$

$$\phi^{*l,-|m|}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}t e^{i\omega t}\phi^{*l,-|m|}(t) = (-1)^{|m|}\phi^{l,|m|}(-\omega) \implies \phi^{l,-|m|}(\omega) = (-1)^{|m|}\phi^{*l,|m|}(-\omega) \tag{A.3}$$

Appendix B

Appendices for Chapter 3

B.1 Vector Spherical Harmonics

The vector spherical harmonic components are given by

$$\vec{Y}_{lm}(\theta,\phi) \equiv \hat{\mathbf{r}} Y_{lm}(\theta,\phi) \tag{B.1}$$

$$\vec{\Psi}_{lm}(\theta,\phi) \equiv \vec{\nabla}_h Y_{lm}(\theta,\phi) \tag{B.2}$$

$$\vec{\Phi}_{lm}(\theta,\phi) \equiv \hat{\mathbf{r}} \times \vec{\nabla}_h Y_{lm}(\theta,\phi), \tag{B.3}$$

where $Y_{lm}(\theta,\phi)$ are spherical harmonics and $\hat{\mathbf{r}}$ is the radial unit vector and $\vec{\nabla}_h$ is the horizontal gradient operator given by

$$\vec{\nabla}_h = \hat{\theta}\frac{\partial}{\partial\theta} + \hat{\phi}\frac{1}{\sin\theta}\frac{\partial}{\partial\phi}. \tag{B.4}$$

The vector spherical harmonics are orthogonal. The orthonormality may be expressed in compact form if we define $\Lambda^0_{lm} \equiv \vec{Y}_{lm}$, $\Lambda^1_{lm} = \vec{\Psi}_{lm}$ and $\Lambda^2_{lm} = \vec{\Phi}_{lm}$.

$$\int d\Omega \vec{\Lambda}^i{}_{lm} \cdot \vec{\Lambda}^j{}^*_{l'm'} = N_{il}\delta^{ij}\delta_{ll'}\delta_{mm'}, \tag{B.5}$$

where N_{il} is a normalization constant, $N_{0l} = 1, N_{1l} = N_{2l} = l(l+1)$ and $d\Omega = sin\theta d\theta d\phi$ is the surface element, integration being performed over the entire surface of the Sun.

Appendix C

Open source software contribution from the thesis

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