1.1) 3 boolean functions which produce y given the inputs mentioned :

1. x_4
2. $(x_4 \wedge x_3) \vee (\sim x_1 \wedge \sim x_2)$
3. $(x_4 \wedge x_3) \vee (x_1 \wedge \sim x_2)$

1.2)

1. 0 errors
2. 1 error
3. 1 error

1.3) Yes, the function is linearly separable.

$X_4 > 0 \Rightarrow y=1$

2.1.a) 2^n

2.1.b) To learn one function from the given concept class, the halving algorithm will make 1 mistake.

The halving algorithm predicts on the basis of the majority.

For any given input other than the required one, the output for the majority will be 0. In all cases other than the required case, the prediction will be correct, and thus no change or updation takes place.

In the required case, the majority will be discarded, as the prediction will be incorrect. This will lead to the discard of all functions, except for the one required function, at which point we can state the function has been learnt.

$1 = N_n < N_0 - (N_0 - 1)$

Since here, majority will consist of all functions but the one that disagrees.

Thus, the final bound on number of iterations will only be 1.

2.1.c) Yes, halving is a mistake bound algorithm for this class. However, the number of records to be traversed to make the mistake is exponential.

2.2) Given, N_i = number of elements at i^{th} iteration where i goes from 0 to n. n = number of iterations of algorithm, M = Number of final perfect experts.

$$
\begin{aligned}
M = N_n &< \tfrac{1}{2} N_{n-1} \\
&< (\tfrac{1}{2})^2 N_{n-2} \\
&< (\tfrac{1}{2})^3 N_{n-3} \\
&\vdots \\
&< (\tfrac{1}{2})^n N_0
\end{aligned}
$$

Thus,

$2^n < N_o/M$ Taking $\log_2$ on both sides and writing N_0 as N to represent number of elements at start.

$n < \text{Log}(N/M)$ thus total number of iterations is less than $\log(N/M)$ or is of the $O(\log N/M)$