COL215 Digital Logic and System Design

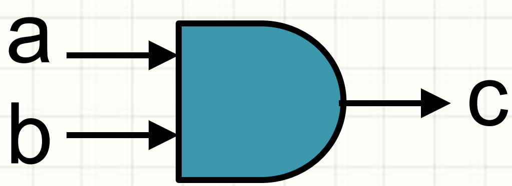
Combinational Circuits continued

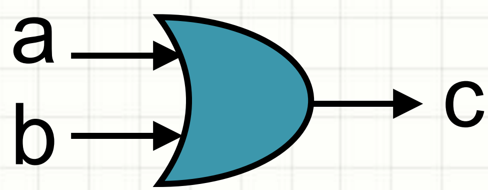
28 July 2017

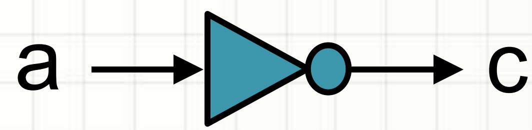
Lecture Outline

- Logic gates, truth tables, boolean algebra
- Gates
  - All types of gates
  - Minimum set of gates
- SoP, PoS and canonical forms
  - From truth tables
  - From arbitrary circuits or expressions
  - Notations

Building Blocks: Gates







c = a AND b

$c = a \cdot b$

a	b	c
0	0	0
0	1	0
1	0	0
1	1	1

c = a OR b

$c = a + b$

a	b	c
0	0	0
0	1	1
1	0	1
1	1	1

c = NOT a

$c = \overline{a}$ or $c = a'$

a	c
0	1
1	0

Boolean Algebra

Set B, operations . & +, identity 0 & 1, inverse ’

$a + a = a$	$a . a = a$
$a + b = b + a$	$a . b = b . a$
$a + (b+c) = (a+b) + c$	$a . (b.c) = (a.b) . c$
$a . (b+c) = (a.b) + (a.c)$	$a + (b.c) = (a+b) . (a+c)$
$a . 1 = a$	$a + 0 = a$
$a . a' = 0$	$a + a' = 1$

Derived results

$(a')' = a$

$a . 0 = 0$ $\quad$ $a + 1 = 1$

$(a + b)' = a' . b'$ $\quad$ $(a . b)' = a' + b'$

$x + (x . y) = x$ $\quad$ $x . (x + y) = x$

$x + (x' . y) = x + y$ $\quad$ $x . (x' + y) = x . y$

$(x.y) + (y.z) + (x'.z) = (x.y) + (x'.z)$

$(x+y) . (y+z) . (x'+z) = (x+y). (x'+z)$

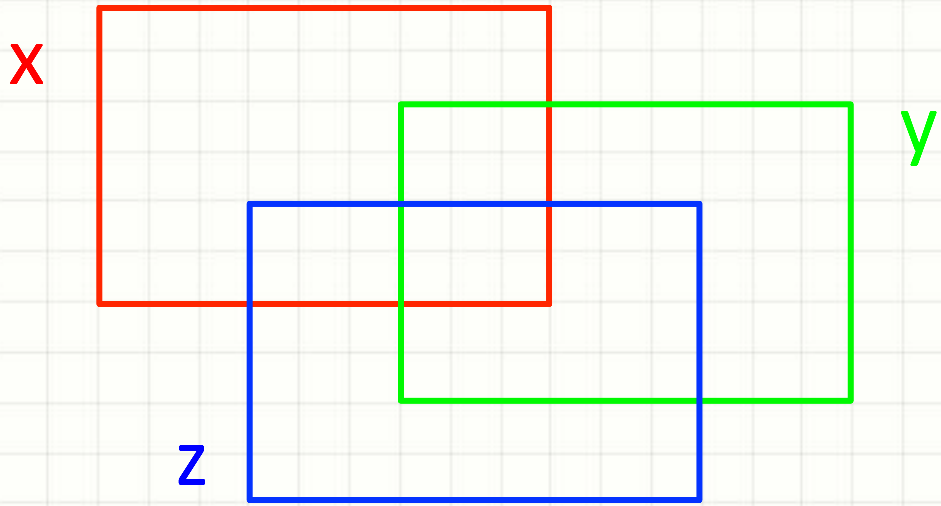
Comparing two circuits/expressions

Is $(x.y) + (y.z) + (x'.z) = (x.y) + (x'.z)$?

1. Transform LHS into RHS (or vice versa) using boolean algebra
2. Compare truth tables of LHS and RHS
3. Use Venn diagrams

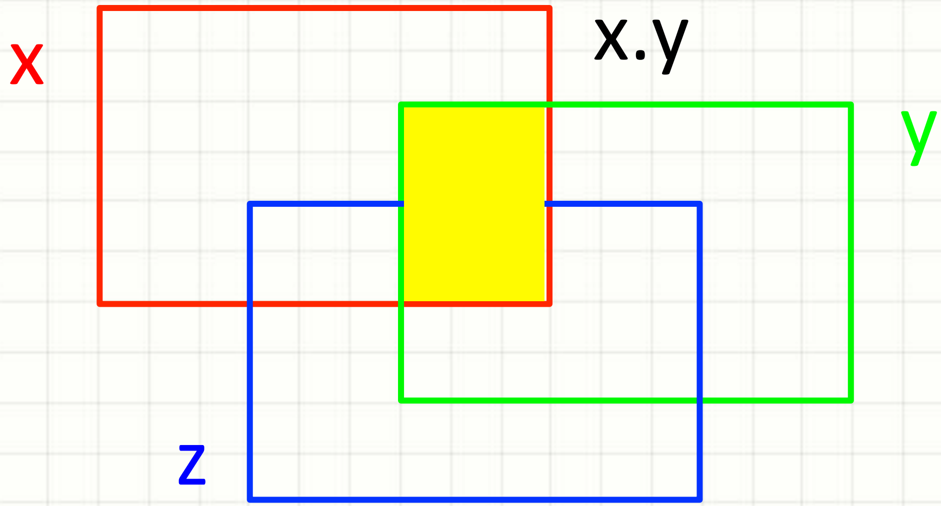
Venn diagram example

Compare (x.y) + (y.z) + (x’.z) with (x.y) + (x’.z)



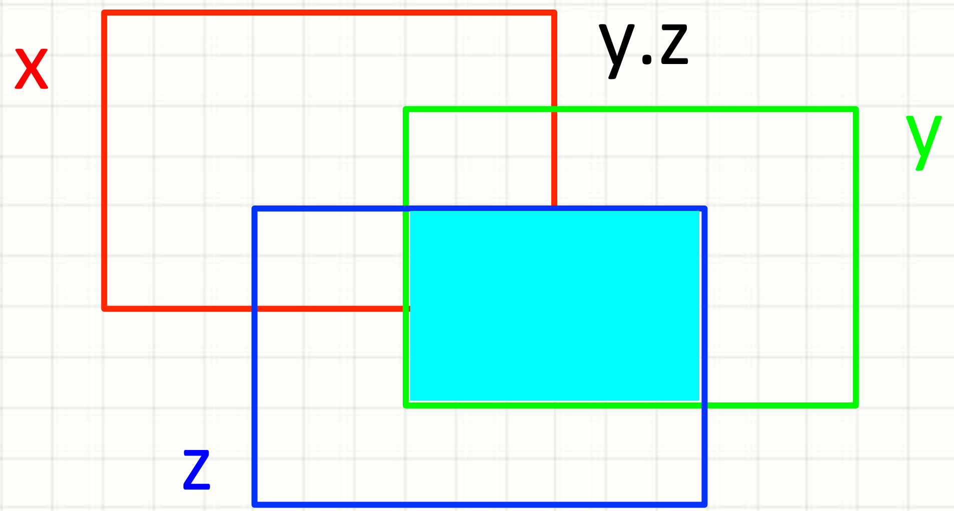
Venn diagram example

Compare (x.y) + (y.z) + (x'.z) with (x.y) + (x'.z)



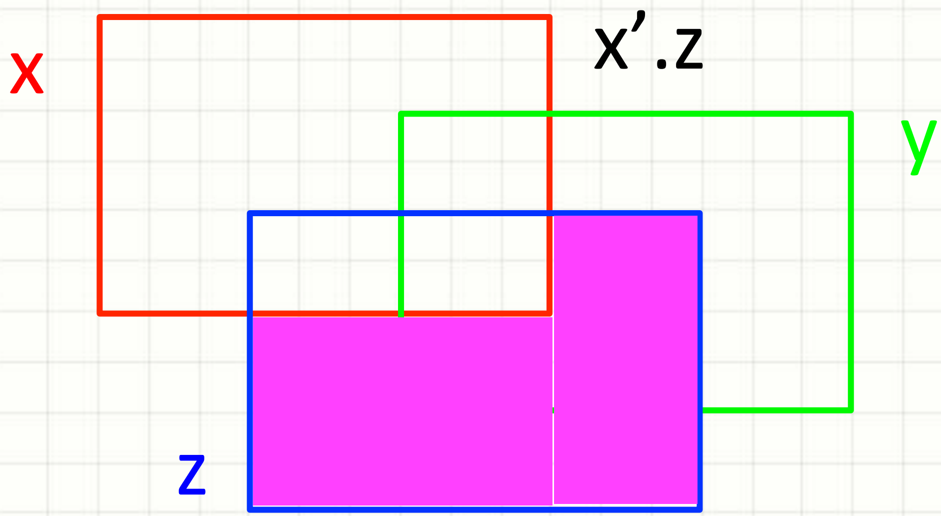
Venn diagram example

Compare $(x.y) + (y.z) + (x'.z)$ with $(x.y) + (x'.z)$



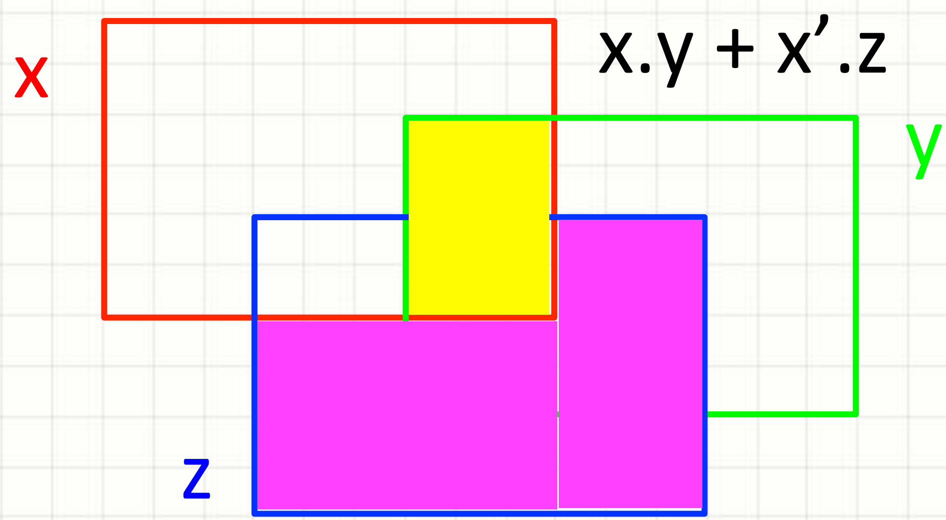
Venn diagram example

Compare $(x.y) + (y.z) + (x'.z)$ with $(x.y) + (x'.z)$



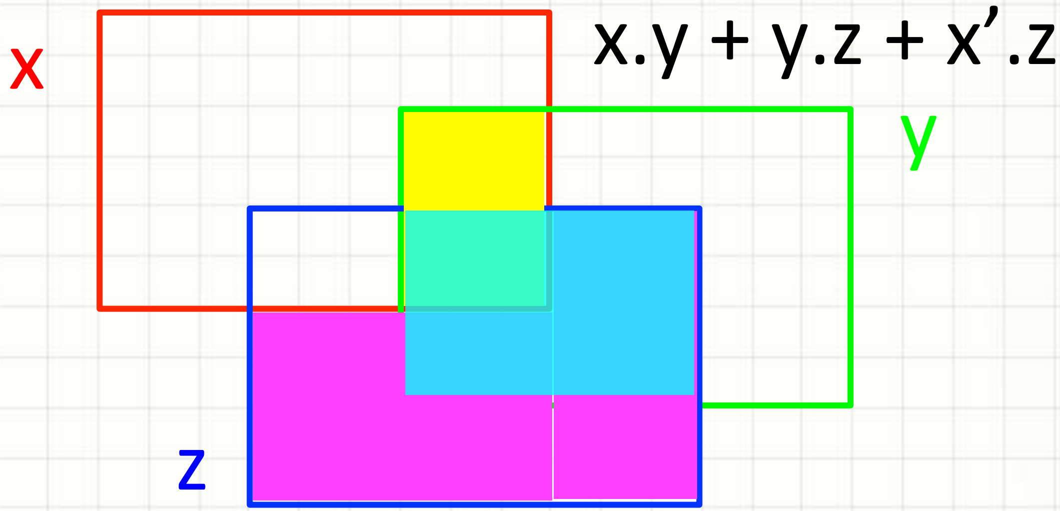
Venn diagram example

Compare (x.y) + (y.z) + (x’.z) with (x.y) + (x’.z)



Venn diagram example

Compare $(x.y) + (y.z) + (x'.z)$ with $(x.y) + (x'.z)$

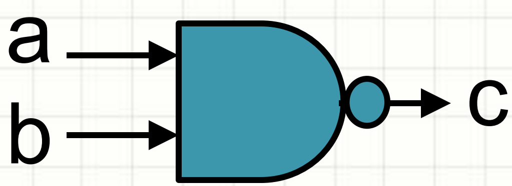


Boolean algebra with $|B| > 2$

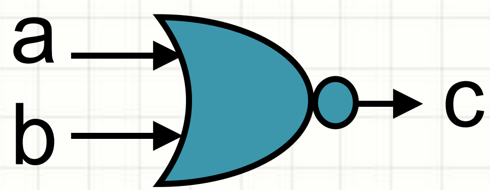
Example:

- Let S be a finite set
- B = powerset (S)
- 0 = { } 1 = S
- AND = set intersection OR = set union
- NOT = set complement

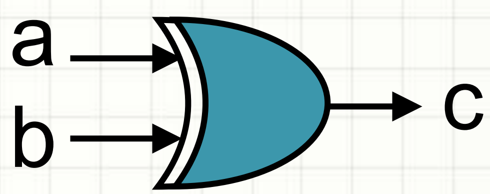
Are there other types of gates?



c = a NAND b



c = a NOR b



c = a XOR b

a	b	c
0	0	1
0	1	1
1	0	1
1	1	0

a	b	c
0	0	1
0	1	0
1	0	0
1	1	0

a	b	c
0	0	0
0	1	1
1	0	1
1	1	0

How many types of gates exist?

Any 2 input gate is defined by a truth table of the form:

a	b	c
0	0	t_0
0	1	t_1
1	0	t_2
1	1	t_3

Number of possible values of t_3, t_2, t_1, t_0 is

$2^4 = 16$

0	a nor b	a’ and b	a’
a and b’	b’	a xor b	a nand b
a and b	a xnor b	b	a’ or b
a	a or b’	a or b	1

How many types of gates exist?

Any 2 input gate is defined by a truth table of the form:

a	b	c
0	0	t_0
0	1	t_1
1	0	t_2
1	1	t_3

Number of possible values of

t_3, t_2, t_1, t_0 is

$2^4 = 16$

0	a nor b	(a <= b)’	a’
(a => b)’	b’	a xor b	a nand b
a and b	a ≡ b	b	a => b
a	a <= b	a or b	1

Generalize to n inputs

n input truth table :

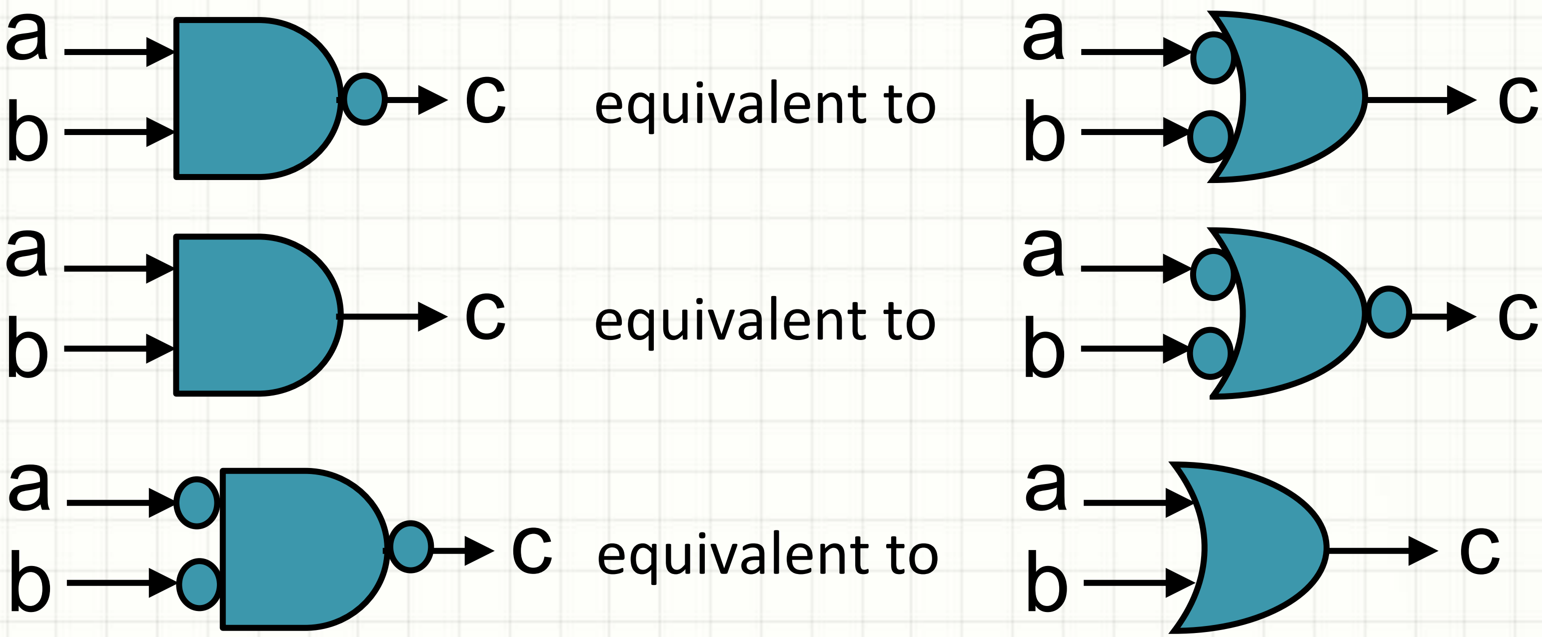
Number of rows = 2^n

Number of possible values of output column is $\mathbf{2}^{2^n}$

a_0 a_1 . . . a_n	c

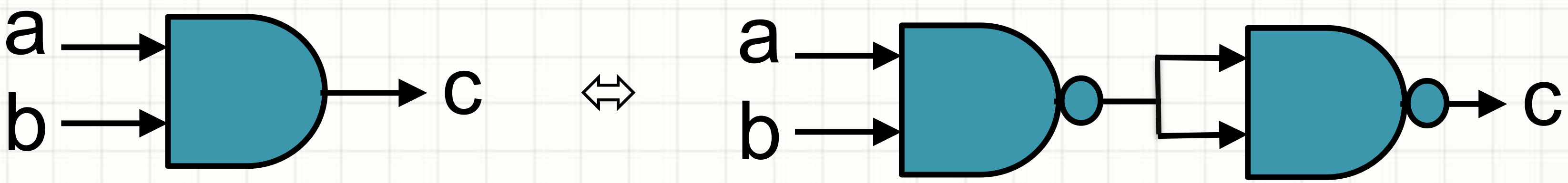
Do we need all types of gates?

Any function can be expressed as SoP or PoS => {AND, OR, NOT} is a functionally complete set.

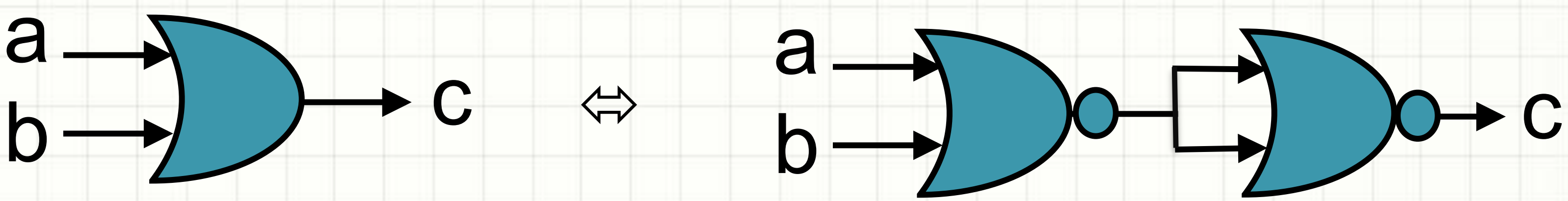


With NOT, only one of {AND, OR} is sufficient. => {AND, NOT}, {OR, NOT} are functionally complete

Further . . .

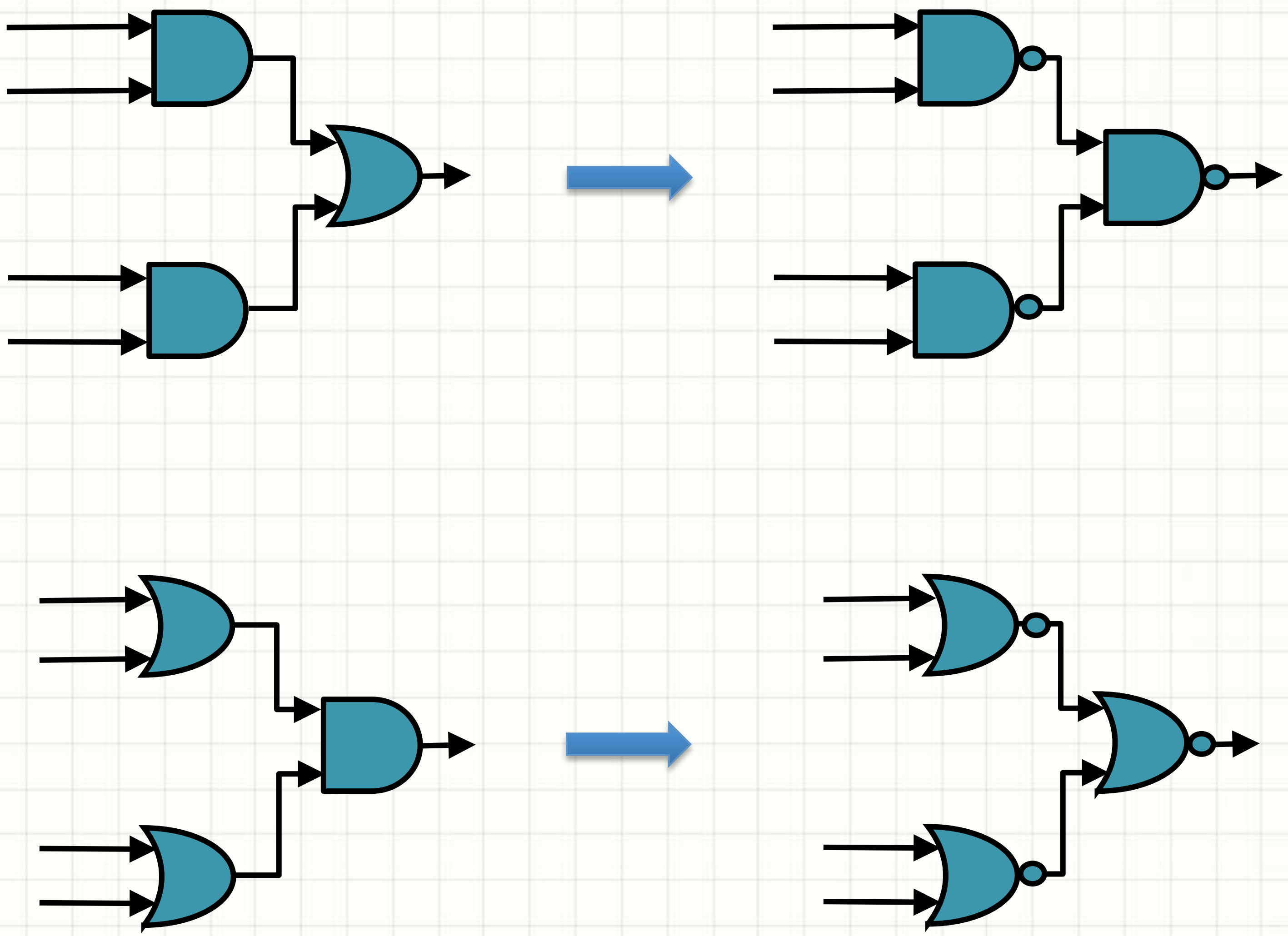


=> {NAND} is functionally complete set.

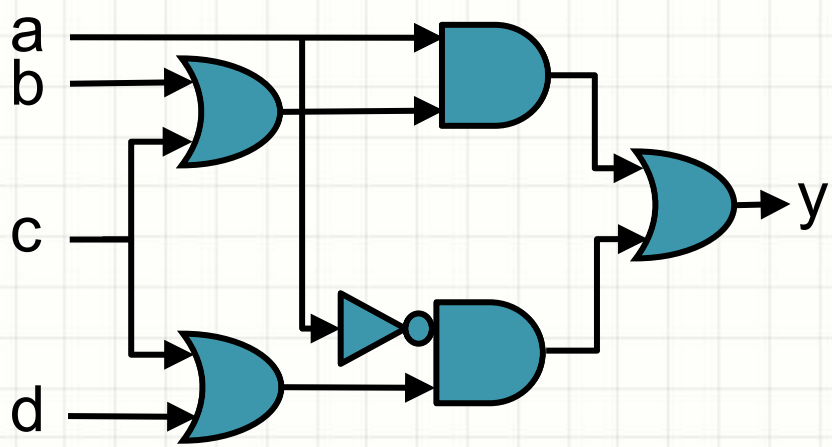


=> {NOR} is functionally complete set.

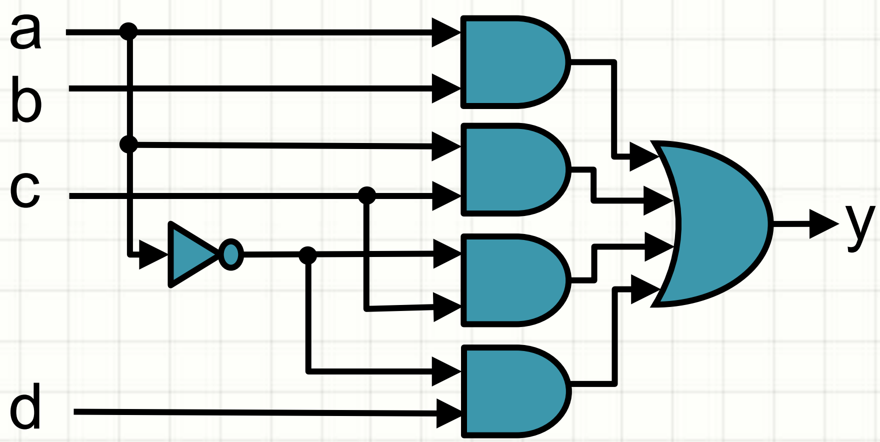
SOP to NAND-NAND, POS to NOR-NOR



Multi-level and SoP forms



y = (a . (b + c)) + (a’ . (c + d))

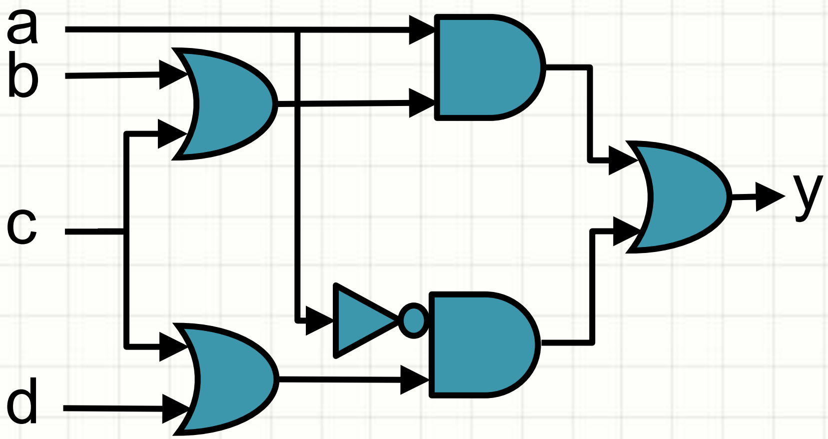


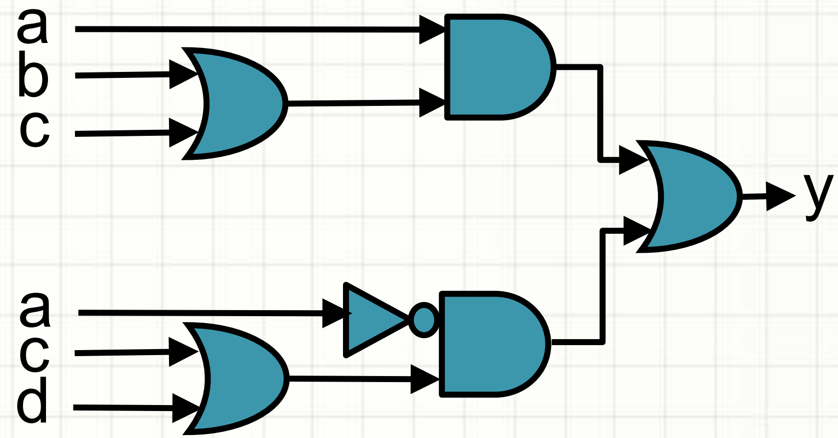
y = (a . b) + (a . c) + (a’ . c) + (a’ . d)

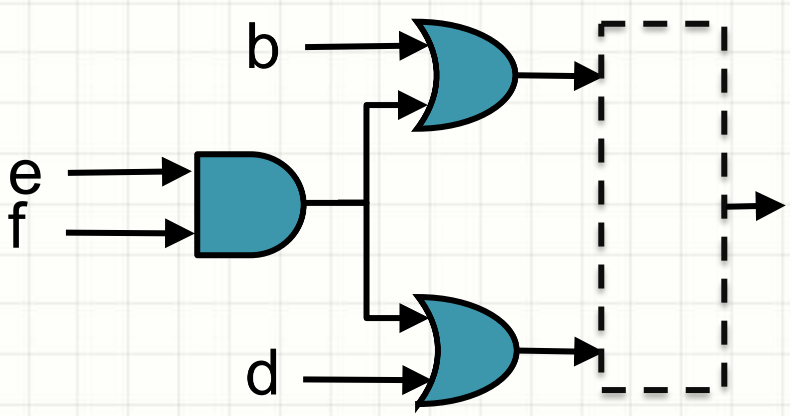
a	b	c	d	y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

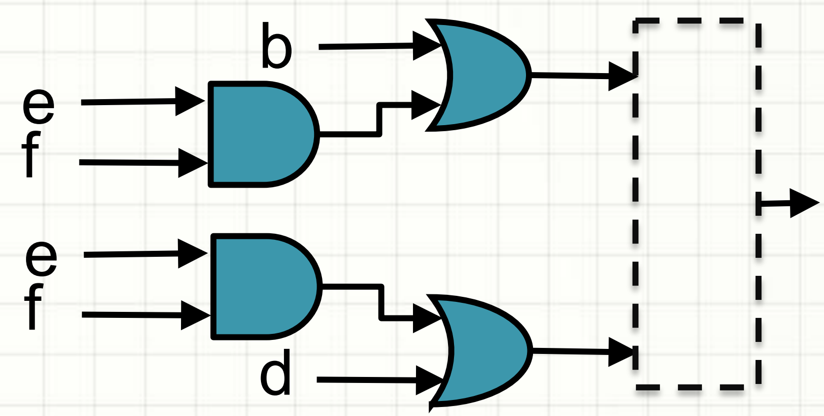
Arbitrary Circuit to SoP/PoS

Convert to tree



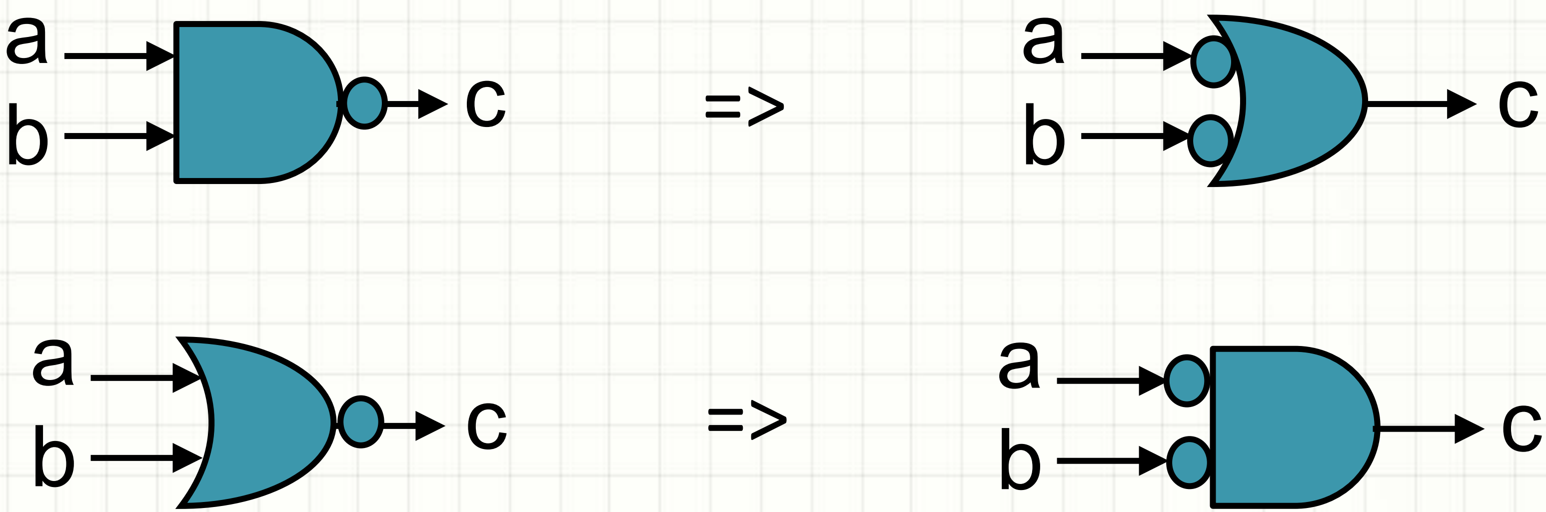






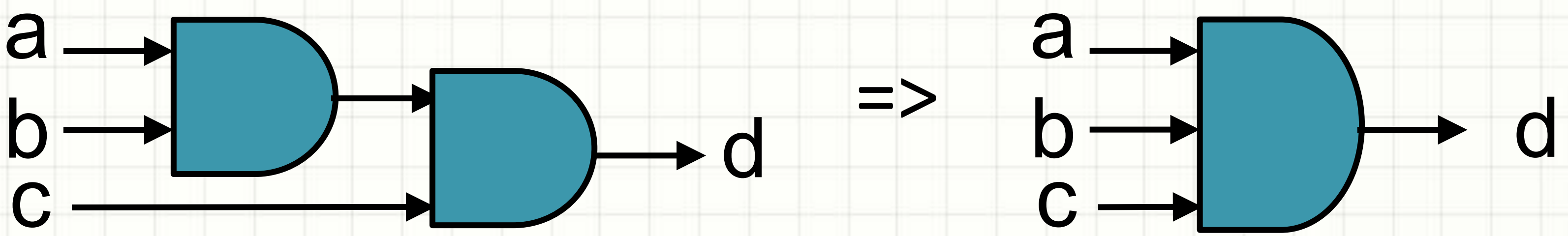
Arbitrary Circuit to SoP/PoS

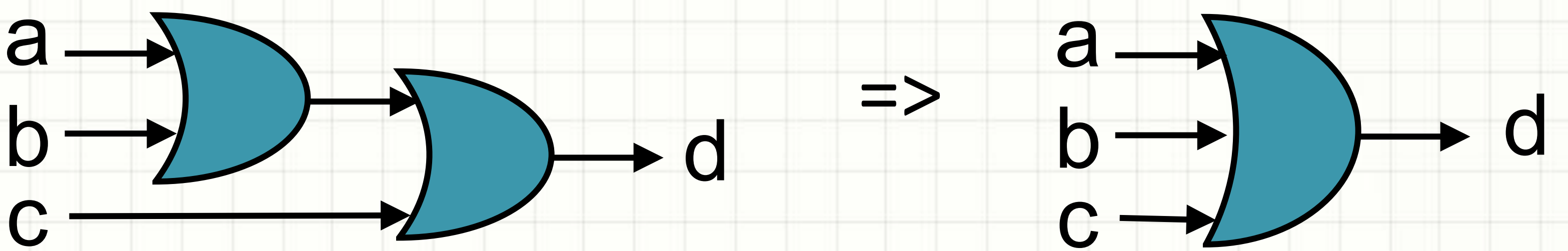
Move inverters to leaves



Arbitrary Circuit to SoP/PoS

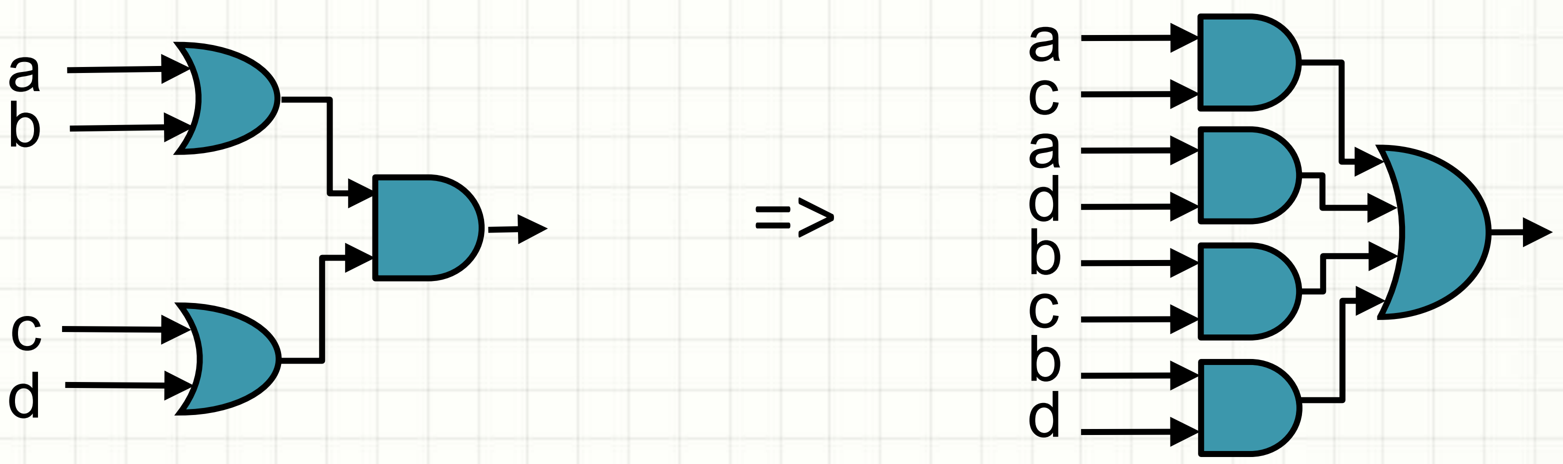
Merge gates



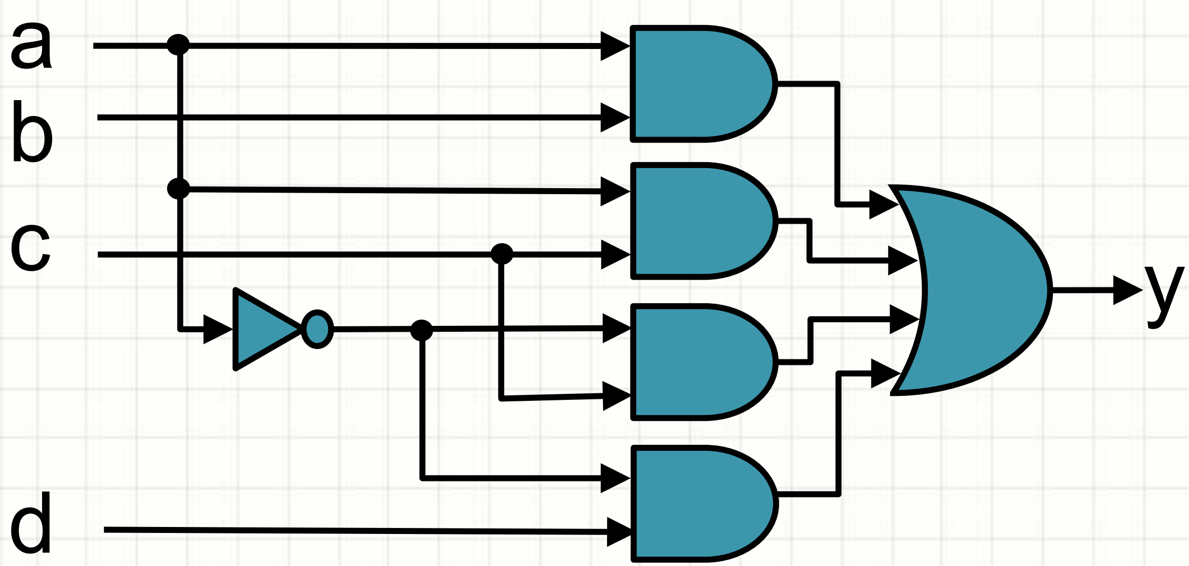


Arbitrary Circuit to SoP/PoS

PoS to SoP and vice versa (locally)



Canonical SoP form

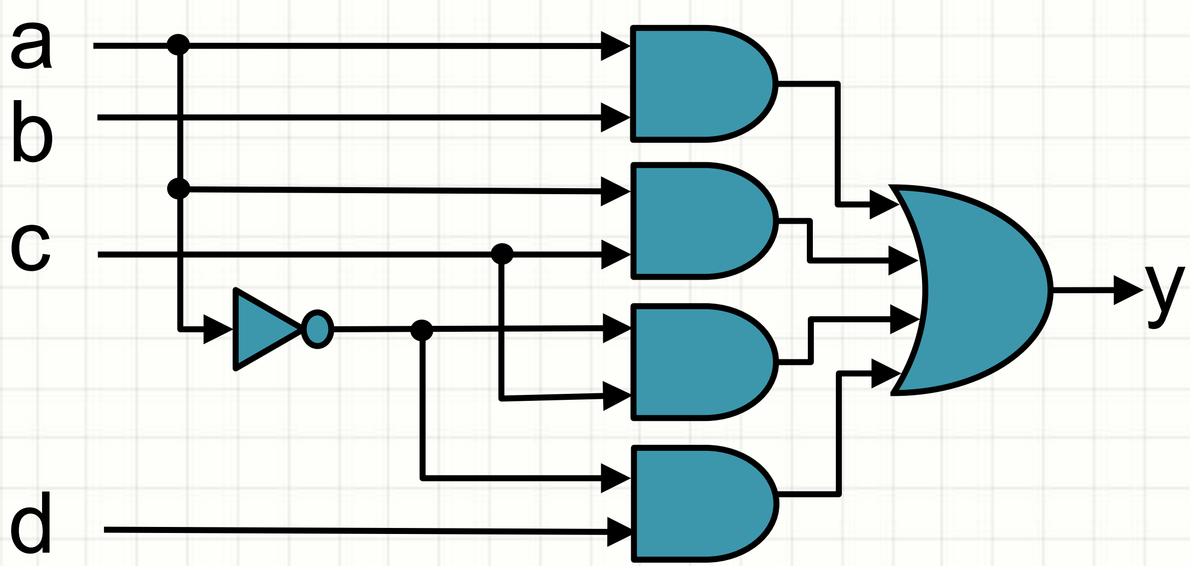


a	b	c	d	y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Canonical Sum of Product (SOP) form

$y = (a'.b'.c'.d) + (a'.b'.c.d') + (a'.b'.c.d)$

Canonical SoP form

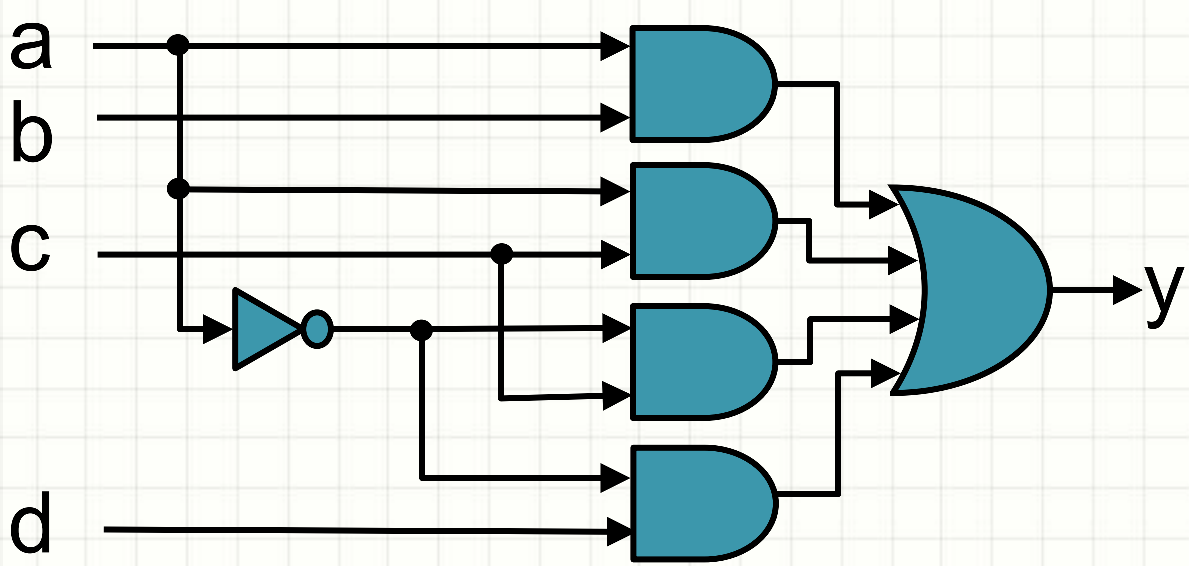


a	b	c	d	y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Canonical Sum of Product (SOP) form

y = (a'.b'.c'.d) + (a'.b'.c .d') + (a'.b'.c .d)
+ (a'.b .c'.d) + (a'.b .c .d') + (a'.b .c .d)

Canonical SoP form

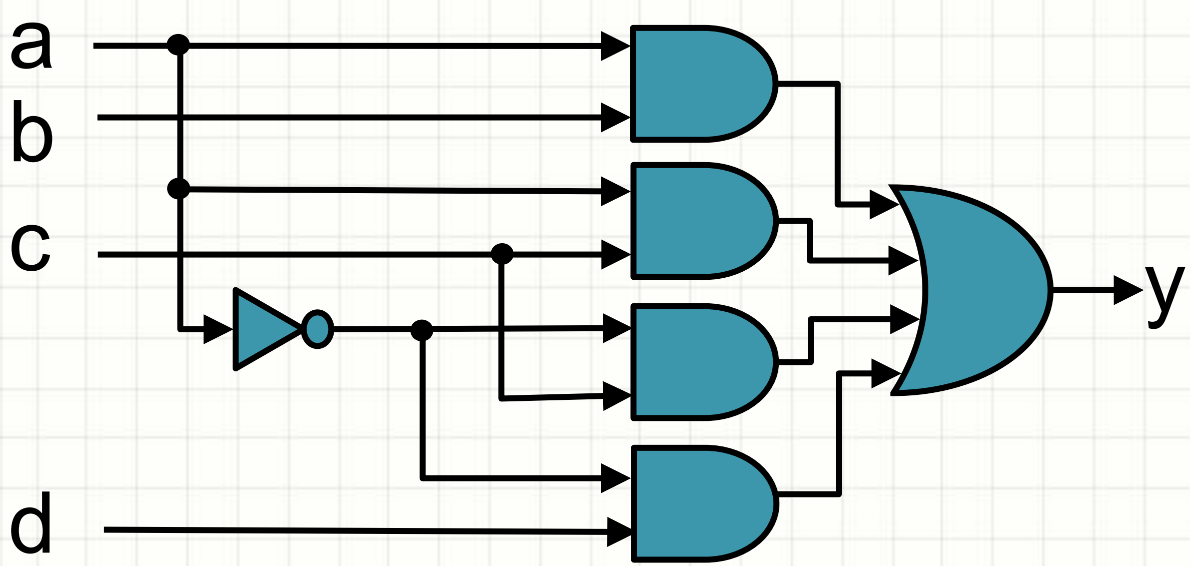


Canonical Sum of Product (SOP) form

$$
\begin{aligned}
y = &(a'.b'.c'.d) + (a'.b'.c.d') + (a'.b'.c.d) \\
&+ (a'.b.c'.d) + (a'.b.c.d') + (a'.b.c.d) \\
&+ (a.b'.c.d') + (a.b'.c.d) + (a.b.c'.d')
\end{aligned}
$$

a	b	c	d	y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Canonical SoP form



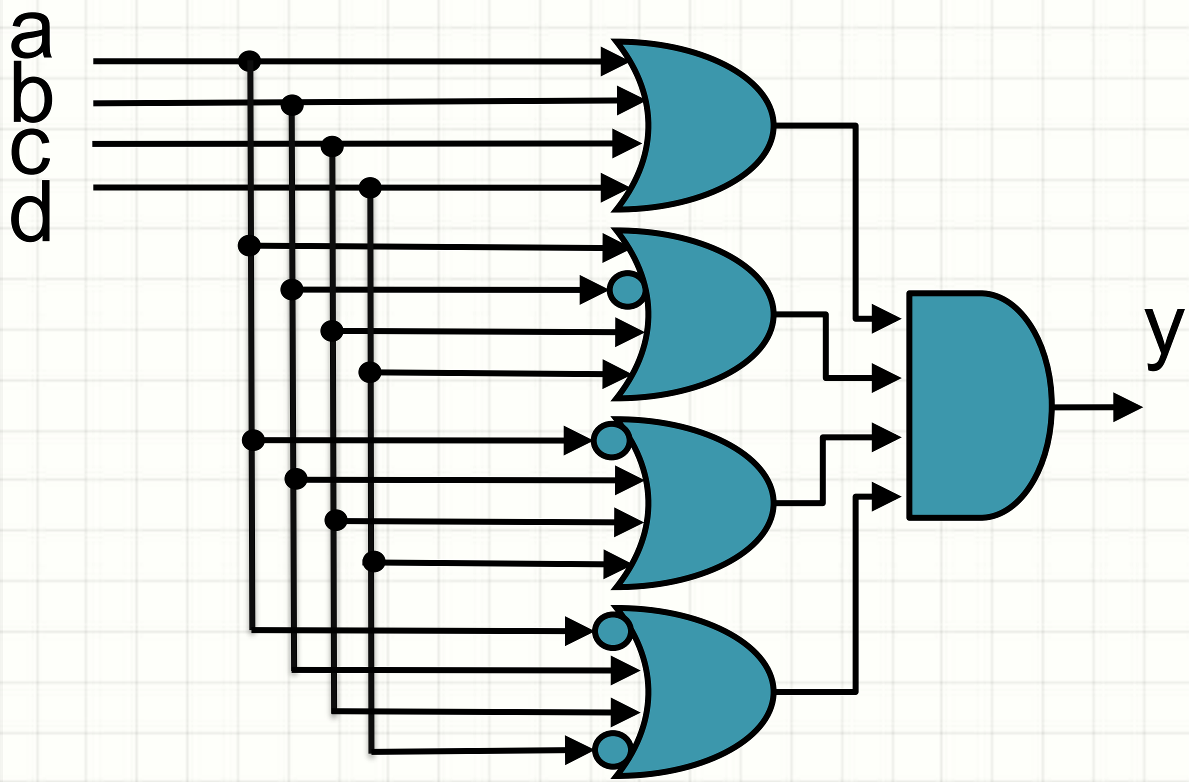
a	b	c	d	y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Canonical Sum of Product (SOP) form

$$y = (a'.b'.c'.d) + (a'.b'.c.d') + (a'.b'.c.d)$$
$$+ (a'.b.c'.d) + (a'.b.c.d') + (a'.b.c.d)$$
$$+ (a.b'.c.d') + (a.b'.c.d) + (a.b.c'.d')$$
$$+ (a.b.c'.d) + (a.b.c.d') + (a.b.c.d)$$

These are “min terms”

Canonical POS form



$y' = a'b'c'd' + a'b\,c'd' + a\,b'c'd' + a\,b'c'd$

$y = (a + b + c + d)\,.\,(a + b' + c + d)$

$\quad .\,(a' + b + c + d)\,.\,(a' + b + c + d')$

These are "max terms".

a	b	c	d	y
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Numbering min/max terms

	a	b	c	d	y
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

Notation for min/max terms

Min terms

$m_0 = a'.b'.c'.d'$

$m_1 = a'.b'.c'.d$

$m_2 = a'.b'.c\ .d'$

$m_3 = a'.b'.c\ .d$

and so on

Max terms

$M_0 = a + b + c + d$

$M_1 = a + b + c + d'$

$M_2 = a + b + c' + d$

$M_3 = a + b + c' + d'$

and so on

	a	b	c	d	y
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

Notation for canonical SOP form

Canonical SOP form

$y = (a'.b'.c'.d\ ) + (a'.b'.c\ .d') + (a'.b'.c\ .d\ )$
$\quad + (a'.b\ .c'.d\ ) + (a'.b\ .c\ .d') + (a'.b\ .c\ .d\ )$
$\quad + (a\ .b'.c\ .d') + (a\ .b'.c\ .d\ ) + (a\ .b\ .c'.d')$
$\quad + (a\ .b\ .c'.d\ ) + (a\ .b\ .c\ .d') + (a\ .b\ .c\ .d\ )$

$y = f\,(a, b, c, d) = m_1 + m_2 + m_3 +$
$\quad m_5 + m_6 + m_7 +$
$\quad m_{10} + m_{11} + m_{12} +$
$\quad m_{13} + m_{14} + m_{15}$
$= \Sigma\,(m_1, m_2, m_3, m_5, m_6, m_7,$
$\quad m_{10}, m_{11}, m_{12}, m_{13}, m_{14}, m_{15})$

$= \Sigma\, m\,(1, 2, 3, 5, 6, 7, 10, 11, 12, 13, 14, 15)$

	a	b	c	d	y
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

Notation for canonical POS form

Canonical POS form

$y = (a + b + c + d) \cdot (a + b' + c + d)$
$\cdot (a' + b + c + d) \cdot (a' + b + c + d')$

$y = f(a, b, c, d) = M_0 \cdot M_4 \cdot M_8 \cdot M_9$

$= \Pi (M_0, M_4, M_8, M_9)$

$= \Pi M (0, 4, 8, 9)$

	a	b	c	d	y
0	0	0	0	0	0
1	0	0	0	1	1
2	0	0	1	0	1
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	1
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

QUESTIONS?