Relative Changes in removal of constraints in 3D Sudokus

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Video: https://youtu.be/dTnoKvseOFI

Github Link:

https://github.com/samarthbhargav/constraint-removal-3d-sudoku

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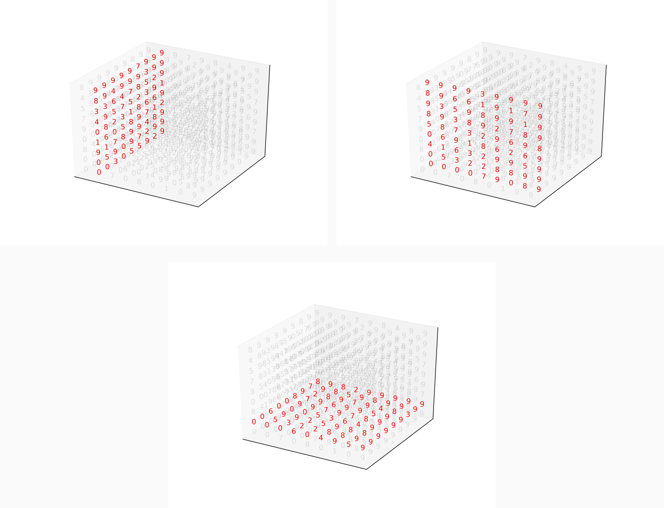
Example



The 3 views of a 3D Sudoku

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- The `z` direction constraint: In the z direction, all n numbers should be present, and should appear exactly once
- The `box` constraint: In the x-y plane, each cell of size $\sqrt{n} \times \sqrt{n}$ should contain all n numbers exactly once. These cells are arranged just like the box constraint of a 2D Sudoku: n boxes tiled in a $\sqrt{n} \times \sqrt{n}$ grid

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Hypothesis

The *relative change* in **complexity** between subsequent degrees of constraints will be the same.

Dataset

- Sourced from `http://www.menneske.no/sudoku3d/eng/`
- 5354 3D Sudokus

Dataset

- Sourced from `http://www.menneske.no/sudoku3d/eng/`
- 5354 3D Sudokus
- Block constraint on the x-y plane

PICOSAT

- Easy to use - it has `python` bindings!

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SAT Solver

PICOSAT

- Easy to use - it has `python` bindings!
- Fast
- Deterministic

Metric

The `level` metric is defined as the total number of levels divided by the number of decisions. The lower the level, the more complex the Sudoku.

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Encoding

At Most One: $\wedge_{i=1}^{n-1} \wedge_{j=1}^{n} (\neg x_i \vee \neg x_j)$

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At Least One: $\vee_{i=1}^{n} x_i$

Exactly One: $\left(\wedge_{i=1}^{n-1} \wedge_{j=1}^{n} (\neg x_i \vee \neg x_j)\right) \wedge (\vee_{i=1}^{n} x_i)$

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Setup - 1

- `all`: All 4 constraints were enforced
- `3-constraint`: Only 3 constraints out of 4 were enforced
- `2-constraint`: Only 2 constraints out of 4 were enforced
- `1-constraint`: Only 1 constraint out of 4 was enforced

Setup - 2

- Ran all combinations of the constraints
- 15 combinations $\times$ 5354 $=$ 80310

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- 15 combinations $\times$ 5354 $=$ 80310
- Some puzzles time out!
- Capped at 30 seconds of computation time

Setup - 3

`level` metrics were then averaged per constraint combination

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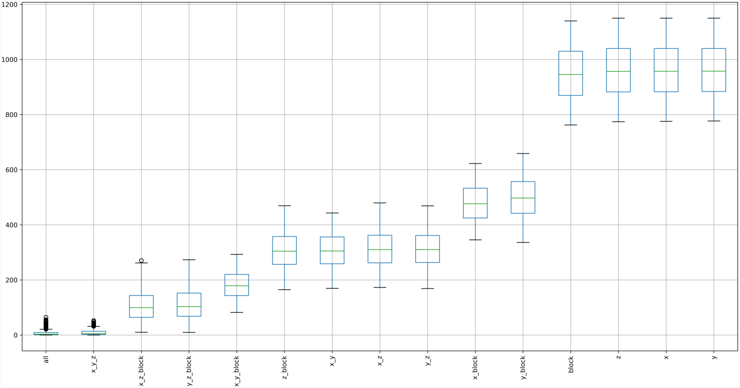
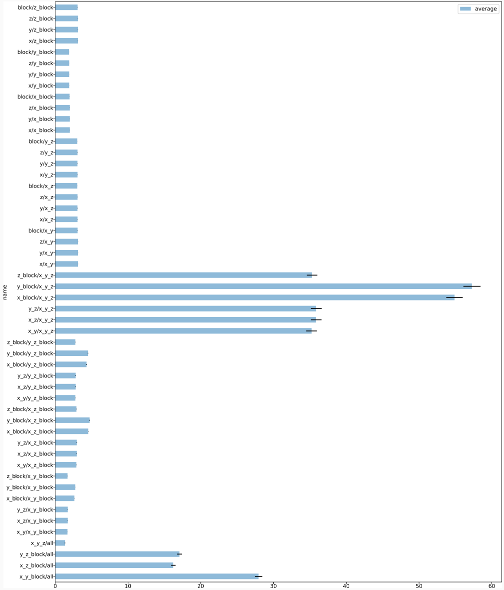
Results

Constraint	Timeouts	Average `level`
`all`	367	6.507894
`x-y-block`	0	181.843220
`x-z-block`	0	105.605603
`y-z-block`	0	111.127232
`x-y-z`	2980	8.734983
`x-y`	0	307.721815
`x-z`	0	313.153474
`y-z`	0	313.309301
`x-block`	0	479.311524
`y-block`	0	500.234516
`z-block`	0	308.272768
`x`	0	960.703399
`y`	0	960.182013
`z`	0	960.736272
`block`	0	948.700504

Box plot for `level`



The average `level` for all constraints



Relative change in the average `level`

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Conclusions - 1

Since there is a large variation in these levels, we can conclude that our original hypothesis is false

Conclusions - 2

Considering the average of the `level` metrics for the 2 have overlapping uncertainty bounds, these can be considered close

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Considering the timeouts, the results indicate that the removal of the `block` constraint appears to make the problem **more** difficult for the picosat solver

Conclusions - 3

$\implies$ The search space is far more nuanced than we assumed
The most difficult constraint combination was not the combination of all of the constraints, but was found to be the combination of all 3 row constraints

Future Work - 1

- Optimized encoding schemes

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- Optimized encoding schemes
- Larger dataset

Future Work - 1

- Optimized encoding schemes
- Larger dataset
- Remove or increase runtime cap

Future Work - 2

- Higher-dimensional Sudokus

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Thank you!