PH108 : Basics of Electricity & Magnetism
ANSWER ALL QUESTIONS : 30 marks

Mid Semester exam
February 24, 2018

1. Answer the following questions with *very* brief reasoning. (None of them should require more than 5-6 lines)

   (a) Compute the unit vector normal to the surface described by $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$, where a, b, c are constants and x, y, z are the cartesian co-ordinates.

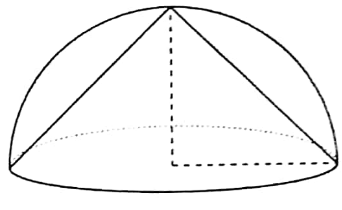
   (b) Let r denote the distance from the origin. Verify if $u(r) = \frac{1}{r}$ is a solution of the Laplace's equation in 2 dimension ?

   (c) Can a static configuration of charges produce an electric field given by the equation ?

$$\vec{E} \propto r^k \hat{r} + r\cos\theta\hat{\theta} + r\sin\phi\hat{\phi}$$

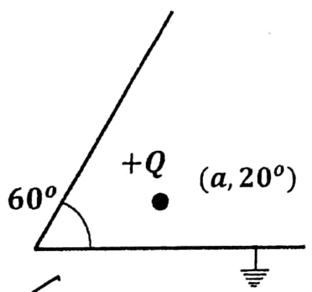
   (d) Consider a closed but arbitrary surface. Integrate the vector area element $d\vec{S}$ over the entire closed surface. What should be the value $\int_{surf} d\vec{S}$?

   (e) Now suppose you have two arbitrary open surfaces with an exactly similar edge (like the cone and the hemisphere sharing a base in the figure). Show that $\int d\vec{S}$ over the curved surfaces of the two should be same in magnitude.



5× 2 = 10 marks

2. Two infinite conducting planes meet at an angle of 60^o, as shown in the figure. Both of them are perpendicular to the xy plane and are kept at ground potential. A point charge $+Q$ is placed at the point such that its polar co-ordinates are $(a, 20^o)$. Find the location and magnitude of the set of image charges that may be used to solve for the potential $V(x, y)$ in the region between the two planes. Give the locations in (r, θ) form and state the "sign" of each charge clearly.

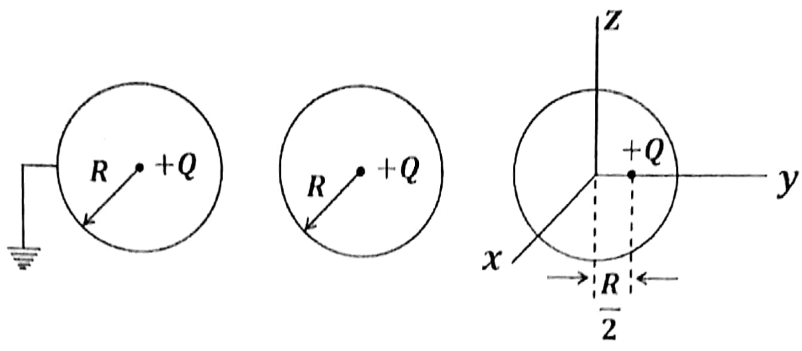


5 marks -5.

3. Consider a sphere of radius R filled with negative charge of uniform density, the total charge being equivalent to that of two electrons. Embed in this jelly two protons (treat them as point charges) so that the total charge of the system is zero. Assume that the charge density of the electron "jelly" is not altered because of the presence of the protons. Where must the protons be located so that the force on each on them is zero?

5 marks

4. A point charge Q is kept inside a hollow conducting spherical shell.



(i) (ii) (iii)

(a) What is the potential $V(r)$ for $r < R$ and $r > R$ if the charge is at the center and the shell is grounded, as in fig (i) ?

(b) How much is the charge on the spherical shell under this condition?

(c) Consider the situation shown in fig(ii). The shell is <u>not</u> grounded. What is the potential $V(r)$ for $r < R$ and $r > R$?

(d) Consider the situation shown in fig (iii) where the shell is <u>not</u> connected to ground and the point charge is at a distance $R/2$ from the center of the sphere. What is the potential $V(x, y, z)$ for $r < R$ and $r > R$?

Note: You may use standard results for image configurations without deriving them, wherever required.

(1+1+1+2 = 5 marks)

5. A conducting sphere of radius R is kept at a fixed potential $V = 0$ by "grounding" it. A point charge Q is brought slowly from infinity to a distance x from the center of the sphere ($x > R$). (You may use known results for standard image configurations without deriving them.)

(a) Calculate the work done in the process and express it in terms of Q, R, x and the fundamental constants only. Your answer should not contain any unevaluated integrals, derivatives etc.

(b) Compare this with the apparent potential energy of the charge in its final position and its image charge. What is the ratio ?

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(4+1 = 5 marks)

Useful formulae:

$$\nabla\phi = \frac{1}{h_1}\frac{\partial\phi}{\partial u_1}\hat{u}_1 + \frac{1}{h_2}\frac{\partial\phi}{\partial u_2}\hat{u}_2 + \frac{1}{h_3}\frac{\partial\phi}{\partial u_3}\hat{u}_3$$

$$\nabla.\vec{F} = \frac{1}{h_1h_2h_3}\left[\frac{\partial(h_2h_3F_1)}{\partial u_1} + \frac{\partial(h_3h_1F_2)}{\partial u_2} + \frac{\partial(h_1h_2F_3)}{\partial u_3}\right]$$

$$\nabla\times\vec{F} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\hat{u}_1 & h_2\hat{u}_2 & h_3\hat{u}_3 \\ \frac{\partial}{\partial u_1} & \frac{\partial}{\partial u_2} & \frac{\partial}{\partial u_3} \\ h_1F_1 & h_2F_2 & h_3F_3 \end{vmatrix}$$

Spherical polar(r, θ, ϕ) $h_1 = 1$ $h_2 = r$ $h_3 = r\sin\theta$

Cylindrical polar(r, ϕ, z) $h_1 = 1$ $h_2 = r$ $h_3 = 1$