Project - Influence Line Diagram for Statically Indeterminate Beam

Problem Statement: 1– To find an influence line diagram for the point at distance "K" from the fixed end "A "(as shown in fig. 1).

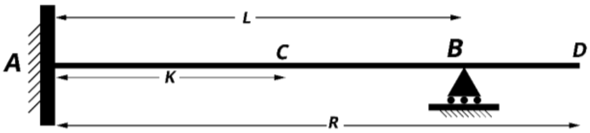

Fig. 1 Cantilever, simply supported Beam

R = Length of Beam,

L = Distance of support from origin from fixed end,

K = Distance of point for which ILD is calculated from fixed end,

EI = flexural rigidity of beam (assumed to be constant)

Solution: The solution will be in three parts for different position of K.

According to the **Müller-Breslau principle**, a response influence line will take a scaled form of the deflected beam shape when resistance to the response at the analysis point is removed and a relative unit deflection is applied for the shear influence line or a unit rotation is applied for the moment influence line is introduced at the analysis point to determine an influence line. To remove shear resistance, a fictitious roller guide is applied. This allows vertical deflection at the analysis point while maintaining resistance to rotation. To remove moment resistance, a fictitious hinge is applied. The hinge allows rotation at the analysis point with zero relative displacement. The resulting deflected shape provides the influence line for a unit point load.

Applying Müller-Breslau principle,

Part 1: K=0,

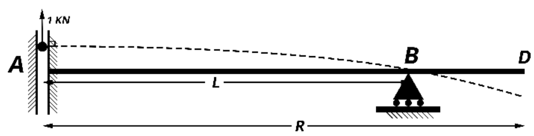

Fig. 2 Influence Line to the shear at C is to the same scale as deflected shape.

When x varies from 0 to K i.e., $\mathbf{0 \leq x < L}$

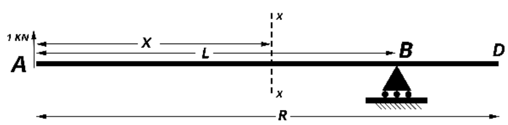

Fig. 3

By Double Integration Method,

M_x= - (L-x)

$EI\ \frac{d^2y}{dx^2}$ = - (L-x)

$EI\frac{dy}{dx}$ = -Lx + $\frac{x^2}{2}$ + C_1 ..... (1)

EI y(x) = -L $\frac{x^2}{2}$ + $\frac{x^3}{6}$ + C_1.x+ C_2 ..... (2)

Putting boundary condition in the equation (1) and (2)

$EI\frac{dy}{dx}$ at x=0 is 0 as there is fixed support. Hence, we get C_1= 0

Again, y(0)=0 as there is fixed support. Hence, we get C_2= $\frac{L^3}{3}$

EI y(x) = -L $\frac{x^2}{2}$ + $\frac{x^3}{6}$ + $\frac{L^3}{3}$ ..... (3)

When x is varies from 0 to K i.e., **L≤ x < R**

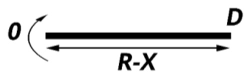

Fig. 5

By Double Integration Method,

M_x=0

$EI\ \frac{d^2y}{dx^2}$ = 0

$EI\frac{dy}{dx}$ = C_5 ..... (4)

EI y(x) = C_5.x + C_6 ..... (5)

Using boundary condition,

When x=L, slope in equation (4) is equal to that of equation (1). Hence, we get C_5= - $\frac{L^2}{2}$

When x=L, y(L)=0 as there no displacement in y in equation (5) at a support, C_6= $\frac{L^3}{2}$

EI y(x) = - $\frac{L^2}{2}$x + $\frac{L^3}{2}$ ..... (6)

To find the function of ILD we have to divide the function in equation (3), and (6) by scale factor which is equal to y(L) in any of equation (3) or (6)

Scale factor = $\frac{L^3}{3EI}$

Hence, the **Influence Line Diagram for shear at C** is

y(x) = (-L $\frac{x^2}{2}$ + $\frac{x^3}{6}$ + $\frac{L^3}{3}$)$\frac{3}{L^3}$ for 0< x <L

y(x) = (- $\frac{L^2}{2}$x + $\frac{L^3}{2}$)$\frac{3}{L^3}$ for L< x <R

Part 2: 0<K<L

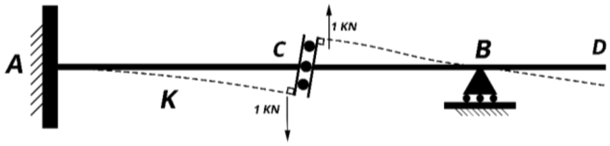

Fig. 2 Influence Line to the shear at C is to the same scale as deflected shape.

When x varies from 0 to K i.e., $\mathbf{0 \leq x < K}$

By Double Integration Method,

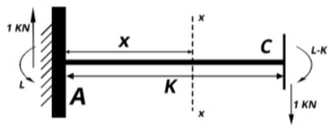

Fig. 3

M_x=x-L

$$EI\ \frac{d^2y}{dx^2} = x - L$$

$$EI\frac{dy}{dx} = \frac{x^2}{2} - Lx + C_1 \qquad ..... (1)$$

$$EI\ y(x) = \frac{x^3}{6} - L\frac{x^2}{2} + C_1x + C_2 \qquad ..... (2)$$

Putting boundary condition in the equation (1) and (2)

$EI\frac{dy}{dx}$ at x=0 is 0 as there is fixed support. Hence, we get C_1=0.

Again, y(0)=0 as there is fixed support. Hence, we get C_2=0.

$$EI\ y(x) = \frac{x^3}{6} - L\frac{x^2}{2} \qquad ..... (3)$$

When x is varies from 0 to K i.e., $\mathbf{K \leq x < L}$

By Double Integration Method,

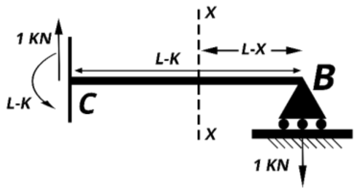

Fig. 4

M_x=x-L

$$EI\ \frac{d^2y}{dx^2} = x - L$$

$$EI\frac{dy}{dx} = \frac{x^2}{2} - Lx + C_3 \qquad ..... (4)$$

$$EI\, y(x) = \frac{x^3}{6} - L\frac{x^2}{2} + C_3.x + C_4 \quad ..... (5)$$

Using boundary condition,

When x=K, slope in equation (1) is equal to that of equation (4). Hence, we get $C_3=C_1=0$.

When x=L, y(L)=0 as there no displacement in y in equation (5) at roller support, $C_4=\frac{L^3}{3}$.

$$EI\, y(x) = \frac{x^3}{6} - L\frac{x^2}{2} + \frac{L^3}{3} \quad ..... (6)$$

When x is varies from 0 to K i.e., **$L \leq x < R$**

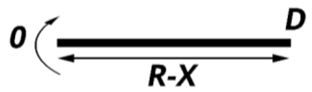

Fig. 5

By Double Integration Method,

$M_x=0$

$$EI\,\frac{d^2y}{dx^2} = 0$$

$$EI\,\frac{dy}{dx} = C_5 \quad ..... (7)$$

$$EI\, y(x) = C_5.x + C_6 \quad ..... (8)$$

Using boundary condition,

When x=L, slope in equation (7) is equal to that of equation (4). Hence, we get $C_5= -\frac{L^2}{2}$

When x=L, y(L)=0 as there no displacement in y in equation (8) at a support, $C_6=\frac{L^3}{2}$.

$$EI\, y(x) = -\frac{L^2}{2}x + \frac{L^3}{2} \quad ..... (9)$$

To find the function of ILD we have to divide the function in equation (3), (6) and (9) by scale factor which is equal to addition of y(k) from equation (3) and (6).

Scale factor $= \frac{L^3}{3EI}$

Hence, the **Influence Line Diagram for shear at C** is

$y(x) = \left(\frac{x^3}{6} - L\frac{x^2}{2}\right)\frac{3}{L^3}$ for $0 < x < K$

$y(x) = \left(\frac{x^3}{6} - L\frac{x^2}{2} + \frac{L^3}{3}\right)\frac{3}{L^3}$ for $K < x < L$

$y(x) = \left(-\frac{L^2}{2}x + \frac{L^3}{2}\right)\frac{3}{L^3}$ for $L < x < R$

Part 3: K=L,

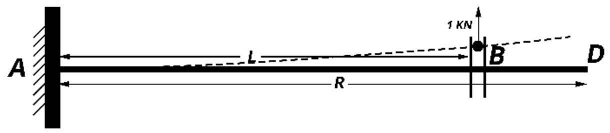

Fig. 2 Influence Line to the shear at C is to the same scale as deflected shape.

When x varies from 0 to K i.e., $\mathbf{0 \leq x < L}$

By Double Integration Method,

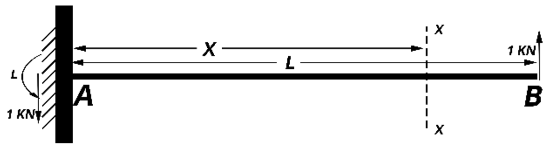

Fig. 3

$M_x = -(x-L)$

$EI \frac{d^2y}{dx^2} = -(x-L)$

$EI \frac{dy}{dx} = -\frac{x^2}{2} + Lx + C_1$ ..... (1)

$EI\, y(x) = -\frac{x^3}{6} + L\frac{x^2}{2} + C_1.x + C_2$ ..... (2)

Putting boundary condition in the equation (1) and (2)

$EI\frac{dy}{dx}$ at x=0 is 0 as there is fixed support. Hence, we get $C_1=0$.

Again, y(0)=0 as there is fixed support. Hence, we get $C_2=0$.

$EI\, y(x) = -\frac{x^3}{6} + L\frac{x^2}{2}$ ..... (3)

When x is varies from 0 to K i.e., $\mathbf{L \leq x < R}$

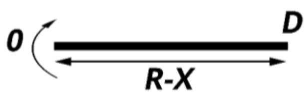

Fig. 5

By Double Integration Method,

$M_x=0$

$EI \frac{d^2y}{dx^2} = 0$

$EI \frac{dy}{dx} = C_5$ ..... (4)

$EI\, y(x) = C_5.x + C_6$ ..... (5)

Using boundary condition,

When x=L, slope in equation (4) is equal to that of equation (1). Hence, we get $C_5 = \frac{L^2}{2}$

When x=L, y(L)=0 as there no displacement in y in equation (5) at a support, $C_6 = -\frac{L^3}{6}$

$EI\,y(x) = \frac{L^2}{2}x - \frac{L^3}{6}$ ..... (6)

To find the function of ILD we have to divide the function in equation (3), and (6) by scale factor which is equal to y(L) in any of equation (3) or (6)

Scale factor $= \frac{L^3}{3EI}$

Hence, the **Influence Line Diagram for shear at C** is

$y(x) = (-\frac{x^3}{6} + L\frac{x^2}{2})\frac{3}{L^3}$ for $0 < x < L$

$y(x) = (\frac{L^2}{2}x - \frac{L^3}{6})\frac{3}{L^3}$ for $L < x < R$

Problem Statement: 2 – To find an influence line diagram for the point at distance "K" from the end "A "(as shown in Fig. 1).

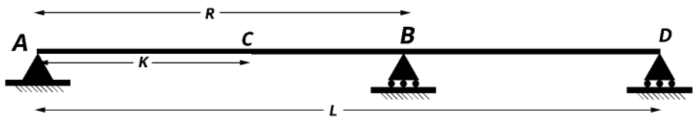

Fig. 1

L = Length of Beam,

R = Distance of middle support from origin from end A,

K = Distance of point for which ILD is calculated from end A,

EI = flexural rigidity of beam (assumed to be constant)

Solution: The solution will be in five parts for different position of K. According to the **Müller-Breslau principle**, a response influence line will take a scaled form
of the deflected beam shape when resistance to the response at the analysis point is removed and a relative unit deflection is applied for the shear influence line or a unit rotation is applied for the moment influence line is introduced at the analysis point to determine an influence line. To remove shear resistance, a fictitious roller guide is applied. This allows vertical deflection at the analysis point while maintaining resistance to rotation. To remove moment resistance, a fictitious hinge is applied. The hinge allows rotation at the analysis point with zero relative displacement. The resulting deflected shape provides the influence line for a unit point load.

Applying Müller-Breslau principle,

PART 1: If 0<K<L,

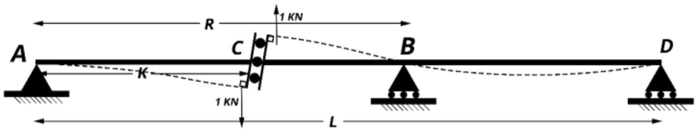

Fig. 2 Removing the support at C and applying the positive shear at C yields the deflected shape. Influence Line to the shear at C is to the same scale as deflected shape.

When x is varies from 0 to K i.e., $\mathbf{R \leq x < L}$

By Double Integration Method,

$M_x = \frac{R}{L-R}(L\text{-}x)$

$EI\ \frac{d^2y}{dx^2} = \frac{R}{L-R}(L\text{-}x)$

$EI\frac{dy}{dx} = \frac{R}{L-R}(Lx\text{-}\frac{x^2}{2}) + C_5$ ..... (1)

$EI\ y(x) = \frac{R}{L-R}(L\frac{x^2}{2}\text{-}\frac{x^3}{6}) + C_5.x + C_6$ ..... (2)

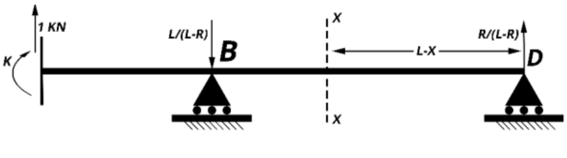

Fig. 3

Using boundary condition,

When x=L and x=R the value of y(L)=0 as there no displacement in y in equation (2) at a support.

We get,

$\frac{R}{L-R}(\frac{L^2}{2} - \frac{L^3}{6}) + C_5.L + C_6 = 0$

$\frac{R}{L-R}(L\frac{R^2}{2} - \frac{R^3}{6}) + C_5.L + C_6 = 0$

Solving the above equations. we get,

$C_5 = \frac{R}{L-R}(L\frac{R^2}{2} - \frac{R^3}{6} - \frac{L^3}{3})$

$C_6 = -\frac{R}{L-R}(\frac{L^3}{3}) - C_5.L$

$EI\ y(x) = = \frac{R}{L-R}(L\frac{x^2}{2} - \frac{x^3}{6}) + C_5.x - \frac{R}{L-R}(\frac{L^3}{3}) - C_5.L$ ..... (3)

When x is varies from 0 to K i.e., $\mathbf{K < x < R}$

By Double Integration Method,

Fig. 4

$M_x = x$

$EI\ \frac{d^2y}{dx^2} = x$

$EI\frac{dy}{dx} = \frac{x^2}{2} + C_3$ ..... (4)

$EI\ y(x) = \frac{x^3}{6} + C_3.x + C_4$ ..... (5)

Using boundary condition,

When x=L, slope in equation (1) is equal to that of equation (4),

we get $C_3 = \frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2}$

When x=L, y(L)=0 as there no displacement in y in equation (7) at roller support,

We get $C_4 = -\frac{R^3}{6} - \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2}\}.R$

$EI\ y(x) = \frac{x^3}{6} + \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2} + C_5 - \frac{R^2}{2}\}.x + -\frac{R^3}{6} - \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2}\}.R$ ..... (6)

When x varies from 0 to K i.e., $\mathbf{0 \leq x < K}$

By Double Integration Method,

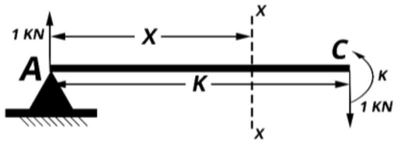

Fig. 5

$M_x = x$

$$EI\ \frac{d^2y}{dx^2} = x$$

$$EI\frac{dy}{dx} = \frac{x^2}{2} + C_1 \qquad \text{..... (7)}$$

$$EI\ y(x) = \frac{x^3}{6} + C_1.x + C_2 \qquad \text{..... (8)}$$

Putting boundary condition in the equation (7) and (8),

EI y(x)=0 at x=0 as there is a support, we get $C_2=0$.

When x=K, slope in equation (4) is equal to that of equation (7), we get $C_1 = C_3$.

$$EI\ y(x) = \frac{x^3}{6} + \{\frac{R}{L-R}(L\frac{R^2}{2} - \frac{R^3}{6}) + C_5 - \frac{R^2}{2}\}.x \qquad \text{..... (9)}$$

To find the function of ILD we have to divide the function in equation (3), (6) and (9) by scale factor which is equal to addition of y(k) from equation (6) and (9).

$$\text{Scale factor} = \frac{\frac{K^3}{6} + \{\frac{R}{L-R}(L\frac{R^2}{2} - \frac{R^3}{6}) + C_5 - \frac{R^2}{2}\}.K + \frac{K^3}{6} + \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2} + C5 - \frac{R^2}{2}\}.K + -\frac{R^3}{6} - \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2}\}.R}{EI}$$

Hence, the **Influence Line Diagram for shear at C from equation** (3), (6) and (9) is

$$y(x) = \frac{\frac{x^3}{6} + \{\frac{R}{L-R}(L\frac{R^2}{2} - \frac{R^3}{6}) + C_5 - \frac{R^2}{2}\}.x}{\text{Scale factor . EI}} \qquad \text{for } 0 < x < K$$

$$y(x) = \frac{\frac{x^3}{6} + \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2} + C5 - \frac{R^2}{2}\}.x - \frac{R^3}{6} - \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2}\}.R}{\text{Scale factor . EI}} \qquad \text{for } K < x < R$$

$$y(x) = \frac{\frac{R}{L-R}(L\frac{x^2}{2} - \frac{x^3}{6}) + C_5.x - \frac{R}{L-R}(\frac{L^3}{3}) - C_5.L}{\text{Scale factor . EI}} \qquad \text{for } R < x < L$$

$$\text{where } C_5 = \frac{R}{L-R}\ (L\frac{R^2}{2} - \frac{R^3}{6} - \frac{L^3}{3})$$

PART 2: K=R,

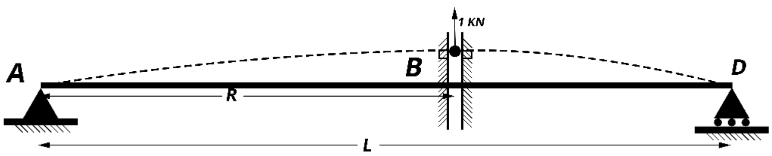

Fig. 6. Placing the roller guide at B and applying the positive shear at B yields the deflected shape. Influence Line to the shear at B is to the same scale as deflected shape

When x is varies from 0 to R i.e., $\mathbf{0 \leq x < R}$

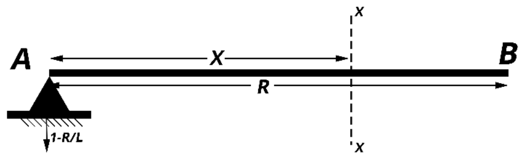

Fig. 7

By Double Integration Method,

$M_x = -x.(1-\frac{R}{L})$

$EI\ \frac{d^2y}{dx^2} = -x.(1-\frac{R}{L})$

$EI\frac{dy}{dx} = -x^2.(1-\frac{R}{L})+C_1$ ..... (1)

$EI\ y(x) =\ -x^3.(1-\frac{R}{L})+C_1.x+C_2$ ..... (2)

Using boundary condition,

When x=0 the value of y(0)=0 as there no displacement in y in equation (2) as it is at a support.

We get, $C_2=0$. ..... (3)

When x is varies from 0 to K i.e., $\mathbf{R< x < L}$

By Double Integration Method,

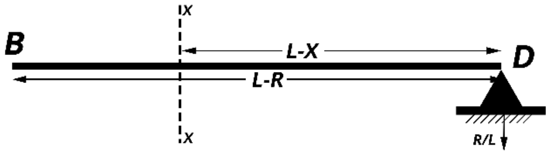

Fig. 8

$M_x= -(L-x).\frac{R}{L}$

$EI\ \frac{d^2y}{dx^2} = -(L-x).\frac{R}{L}$

$EI\frac{dy}{dx} = -(Lx - \frac{x^2}{2}).\frac{R}{L} + C_3.x$ ..... (4)

$EI\ y(x) = -(L\frac{x^2}{2} - \frac{x^3}{6}).\frac{R}{L} + C_3.x + C_4$ ..... (5)

When x=L the value of y(L)=0 as there no displacement in y in equation (3) as it is at a support.

We get,

$C_3.L + C_4 = R.\frac{L^2}{3}$ ..... (6)

Also,

When x=R, slope in equation (1) is equal to that of equation (4) and also y(x) will be equal in equation (2) and (5),

We get $-\frac{R^3}{6}(1 - \frac{R}{L}) + C_1R = -(L\frac{x^2}{2} - \frac{x^3}{6}).\frac{R}{L} + C_3.x + C_4$ ..... (7)

$-(Lx - \frac{x^2}{2}).\frac{R}{L} + C_3.x = -x^2.(1-\frac{R}{L}) + C_1$ ..... (8)

From the above equations (3), (6), (7) and (8)

$C_1 = \frac{RL}{3} + \frac{R^3}{6L} - \frac{R^2}{2}$

$C_2 = 0$

$C_3 = \frac{RL^2}{3} - \frac{R^3}{6}$

$C_4 = -\frac{R^3}{6}$

Hence,

$y(x) = -x^3.(1-\frac{R}{L}) + (\frac{RL}{3} + \frac{R^3}{6L} - \frac{R^2}{2}).x$ for $0 < x < R$ ..... (9)

$y(x) = -\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right)\frac{R}{L} + (\frac{RL^2}{3} - \frac{R^3}{6}).x - \frac{R^3}{6}$ for $R < x < L$ ..... (10)

To find the function of ILD we have to divide the function in equation (9) and (10) by scale factor which is equal to addition of y(k) from equation (9) and (10).

$$\text{Scale factor} = \frac{-R^3.(1-\frac{R}{L}) + \left(\frac{RL}{3} - \frac{R^3}{6L} - \frac{R^2}{2}\right).R}{EI}$$

Hence, the **Influence Line Diagram for shear at C from equation** (9), and (10) is

$$y(x) = \frac{-x3.(1-\frac{R}{L}) + (\frac{RL}{3} + \frac{R^3}{6L} - \frac{R^2}{2}).x}{\text{Scale factor . EI}} \quad \text{for } 0 < x < R$$

$$y(x) = \frac{-\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right)\frac{R}{L} + (\frac{RL^2}{3} - \frac{R^3}{6}).x - \frac{R^3}{6}}{\text{Scale factor . EI}} \quad \text{for } R < x < L$$

PART 3: R< K <L,

By Similarity we can get ILD by replacing x to (L-x), R to (L-R) and K to (L-K) in the part 2.

We get,

Scale factor =

$$\frac{\frac{K^3}{6} + \{\frac{(L-R)}{L-(L-R)}(L\frac{(L-R)^2}{2} - \frac{(L-R)^3}{6}) + C_5 - \frac{(L-R)^2}{2}\}.K + \frac{K^3}{6} + \{\frac{(L-R)}{L-(L-R)}(L(L-R) - \frac{(L-R)^2}{2}) + C_5 - \frac{(L-R)^2}{2} + C5 - \frac{(L-R)^2}{2}\}.K + - \frac{(L-R)^3}{6} - \{\frac{(L-R)}{L-(L-R)}(L(L-R) - \frac{(L-R)^2}{2}) + C_5 - \frac{(L-R)^2}{2}\}.(L-R)}{EI}$$

Hence, the **Influence Line Diagram for shear at C from equation** (8), (14) and (18) is

$$y(x) = \frac{\frac{(L-x)^3}{6} + \{\frac{(L-R)}{L-(L-R)}(L\frac{(L-R)^2}{2} - \frac{(L-R)^3}{6}) + C_5 - \frac{(L-R)^2}{2}\}.(L-x)}{\text{Scale factor . EI}} \qquad \text{for } K< x <L$$

$$y(x) = \frac{\frac{(L-x)^3}{6} + \{\frac{(L-R)}{L-(L-R)}(L(L-R) - \frac{(L-R)^2}{2}) + C_5 - \frac{(L-R)^2}{2} + C5 - \frac{(L-R)^2}{2}\}.(L-x) - \frac{(L-R)^3}{6} - \{\frac{(L-R)}{L-(L-R)}(L(L-R) - \frac{(L-R)^2}{2}) + C_5 - \frac{(L-R)^2}{2}\}.(L-R)}{\text{Scale factor . EI}} \qquad \text{for } R< x < K$$

$$y(x) = \frac{\frac{(L-R)}{L-(L-R)}(L\frac{(L-x)^2}{2} - \frac{(L-x)^3}{6}) + C_5 .(L-x) - \frac{(L-R)}{L-(L-R)}(\frac{L^3}{3}) - C_5.L}{\text{Scale factor . EI}} \qquad \text{for } 0< x <R$$

where $C_5 = \frac{(L-R)}{L-(L-R)} \ (L\frac{(L-R)^2}{2} - \frac{(L-R)^3}{6} - \frac{L^3}{3})$

PART 4: K = L,

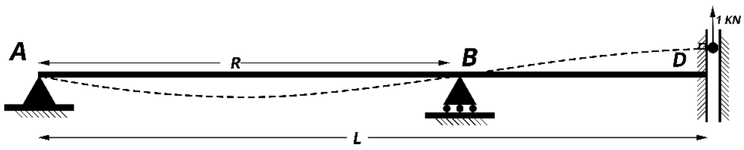

Fig. 9 Placing the roller guide at C and applying the positive shear at C yields the deflected shape. Influence Line to the shear at C is to the same scale as deflected shape.

When x is varies from 0 to R i.e., $\mathbf{0 \leq x < R}$

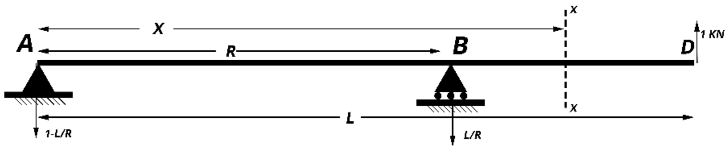

Fig. 4

By Double Integration Method,

$M_x = -x.(1-\frac{R}{L})$

$EI\,\frac{d^2y}{dx^2} = -x.(1-\frac{R}{L})$

$EI\,\frac{dy}{dx} = -x^2.(1-\frac{R}{L})+C_1$ ..... (1)

$EI\,y(x) = -x^3.(1-\frac{R}{L})+C_1.x+C_2$ ..... (2)

Using boundary condition,

When x=0 and x=R the value of y(0)=0 as there no displacement in y in equation (2) as it is at a support.

We get,

$C_1 = (1-\frac{L}{R}).\frac{R^2}{6}$

$C_2 = 0$

Hence,

$EI\,y(x) = -x^3.(1-\frac{R}{L})+(1-\frac{L}{R})\frac{R^2x}{6}$ ..... (3)

When x is varies from 0 to K i.e., $\mathbf{R < x < L}$

By Double Integration Method,

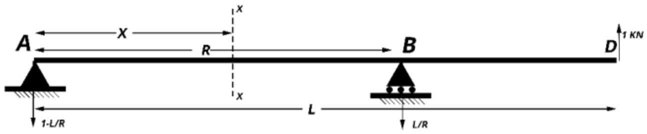

Fig. 10

$M_x = (L-x)$

$EI\ \frac{d^2y}{dx^2} = (L-x)$

$EI\frac{dy}{dx} = Lx - \frac{x^2}{2} + C_3$ ..... (4)

$EI\ y(x) = (L\frac{x^2}{2} - \frac{x^3}{6}) + C_3.x + C_4$ ..... (5)

When x=R the value of y(L)=0 as there no displacement in y in equation (3) as it is at a support.

We get,

$C_3.R + C_4 = -(L\frac{R^2}{2} - \frac{R^3}{6})$ ..... (6)

Also, When x=R, slope in equation (10) is equal to that of equation (16) and also y(x) will be equal,

We get,

$Lx - \frac{x^2}{2} + C_3 = -x^2.(1-\frac{R}{L}) + C_1$ ..... (7)

From the above equations (6) and (7),

$C_3 = \frac{R^2}{6} - 2L\frac{R}{3}$

$C_4 = L\frac{R^2}{6}$

Hence,

$y(x) = -\frac{x^3}{6}.(1-\frac{L}{R}) + ((1-\frac{L}{R}).\frac{R^2}{6}).x$ for $0 < x < R$ ..... (8)

$y(x) = \left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) + (\frac{R^2}{6} - 2L\frac{R}{3}).x + L\frac{R^2}{6}$ for $R < x < L$ ..... (9)

To find the function of ILD we have to divide the function in equation (8) and (9) by scale factor which is equal to addition of y(k) from equation (8) and (9).

$$\text{Scale factor} = \frac{\left(\frac{LR^2}{2} - \frac{R^3}{6}\right) + (\frac{R^2}{6} - 2L\frac{R}{3}).R + L\frac{R^2}{6}}{EI}$$

Hence, the **Influence Line Diagram for shear at C from equation** (8), and (9) is

$$y(x) = \frac{-\frac{x^3}{6}.(1-\frac{L}{R}) + ((1-\frac{L}{R}).\frac{R^2}{6}).x}{\text{Scale factor}.\ EI} \quad \text{for } 0 < x < R$$

$$y(x) = \frac{\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) + (\frac{R^2}{6} - 2L\frac{R}{3}).x + L\frac{R^2}{6}}{\text{Scale factor}.\ EI} \quad \text{for } R < x < L$$

PART 5: If K=0,

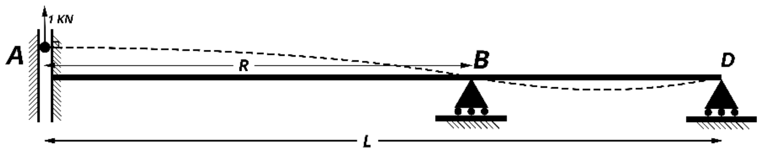

Fig. 11

By Similarity we can get ILD by replacing x to (L-x), R to (L-R) and K to (L-K) in the part 4.

We get,

$$\text{Scale factor} = \frac{\left(\frac{L(L-R)^2}{2} - \frac{(L-R)^3}{6}\right) + \left(\frac{(L-R)^2}{6} - 2L\frac{(L-R)}{3}\right).(L-R) + L\,\frac{(L-R)^2}{6}}{EI}$$

Hence, the **Influence Line Diagram for shear at C from equation** (18), (14) and (8) is

$$y(x) \; = \frac{-\frac{(L-x)^3}{6}.\left(1-\frac{L}{(L-R)}\right)+\left(\left(1-\frac{L}{(L-R)}\right).\frac{(L-R)^2}{6}\right).(L-x)}{\text{Scale facto}(L-R)\;.\;EI} \qquad \text{for } R< x <L$$

$$y(x) = \frac{\left(\frac{L(L-x)^2}{2} - \frac{(L-x)^3}{6}\right) + \left(\frac{(L-R)^2}{6} - 2L\frac{(L-R)}{3}\right).(L-x) + L\,\frac{(L-R)^2}{6}}{\text{Scale facto}(L-R)\;.\;EI} \qquad \text{for } 0< x <R$$