A Project

ON

Influence line Diagram for Indeterminate Beam

Using Muller Breslau Principle

Prepared in partial fulfillment of the

Study Project

Course No. - **CE F266**

Submitted By

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Prepared for

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Table of Content

Introduction

The project shows the computation of the influence line diagram using the Muller Breslau principle for indeterminate beams and to build a code to compute values and graphs. According to the **Müller-Breslau principle**, an influence line will be a scaled form of the deflected beam shape, when resistance to the response is removed at the analysis point, and a relative unit deflection is applied for the shear influence line, or a unit rotation is applied for the moment influence line is introduced at the analysis point to determine an influence line. In order to remove shear resistance, a fictitious roller guide is applied, which allows vertical deflection at the analysis point while maintaining resistance to rotation. A fictitious hinge is applied to remove moment resistance. The hinge allows rotation at the analysis point with zero relative displacements. The resulting deflected shape provides the influence line for a unit point load.

There are various approaches to solve the beam's deflection after applying the Muller Breslau principle.

- Castigliano's method
- Moment-area theorem
- Conjugate beam method
- Double Integration
- Force Method

In the project double integration method is used for calculating bending, and also, there is another approach used in problem 3 and 4 by taking deflection of the beam as a cubic equation then assigning the boundary condition to the equations to find the coefficients of equations. The second approach makes all the calculation much easier in coding and calculation perspective.

Java and JavaFX handled for calculation and graph representation. User can give inputs in terms of length of the beam, distance of support, and distance analysis point. The output will be a graphical line chart representation with thousand intervals and can be modified to find any position. All calculation verified using STAAD.Pro.

Problem Statement: 1

To find an influence line diagram of shear force for the point at distance "K" from the fixed end "A "(as shown in fig. 1).

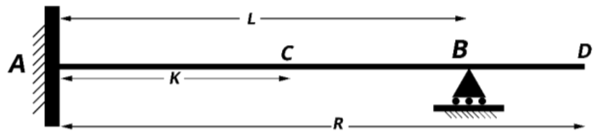


Fig. 1 Cantilever, simply supported Beam

R = Length of Beam,

L = Distance of support from origin from fixed end,

K = Distance of point for which ILD is calculated from fixed end,

EI = flexural rigidity of beam (assumed to be constant)

Solution: The solution will be in three parts for different position of K.

Applying Müller-Breslau principle,

Part 1: K=0,

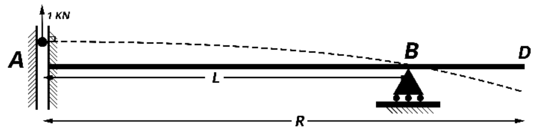


Fig. 2 Influence Line to the shear at C is to the same scale as deflected shape.

When x varies from 0 to K i.e., $\mathbf{0 \leq x < L}$

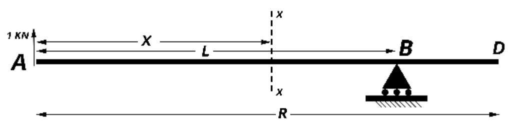


Fig. 3

By Double Integration Method,

M_x= - (L-x)

$$EI \frac{d^2y}{dx^2} = -(L-x)$$

$$EI \frac{dy}{dx} = -Lx + \frac{x^2}{2} + C_1 \qquad \text{..... (1)}$$

$$EI\, y(x) = -L\frac{x^2}{2} + \frac{x^3}{6} + C_1 x + C_2 \qquad \text{..... (2)}$$

Putting boundary condition in the equation (1) and (2)

$EI\frac{dy}{dx}$ at x=0 is 0 as there is fixed support. Hence, we get $C_1= 0$

Again, y(0)=0 as there is fixed support. Hence, we get $C_2= \frac{L^3}{3}$

$EI\, y(x) = -L\frac{x^2}{2} + \frac{x^3}{6} + \frac{L^3}{3}$ ..... (3)

When x is varies from 0 to K i.e., $\mathbf{L \le x < R}$

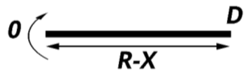


Fig. 5

By Double Integration Method,

$M_x=0$

$EI\, \frac{d^2y}{dx^2} = 0$

$EI\frac{dy}{dx} = C_5$ ..... (4)

$EI\, y(x) = C_5 x + C_6$ ..... (5)

Using boundary condition,

When x=L, slope in equation (4) is equal to that of equation (1). Hence, we get $C_5= -\frac{L^2}{2}$

When x=L, y(L)=0 as there no displacement in y in equation (5) at a support, $C_6= \frac{L^3}{2}$

$EI\, y(x) = -\frac{L^2}{2}x + \frac{L^3}{2}$ ..... (6)

To find the function of ILD we have to divide the function in equation (3), and (6) by scale factor which is equal to y(L) in any of equation (3) or (6)

Scale factor = $\frac{L^3}{3EI}$

Hence, the **Influence Line Diagram for shear at C** is

$y(x) = (-L\frac{x^2}{2} + \frac{x^3}{6} + \frac{L^3}{3})\frac{3}{L^3}$ for $0 < x < L$

$y(x) = (-\frac{L^2}{2}x + \frac{L^3}{2})\frac{3}{L^3}$ for $L < x < R$

Part 2: 0<K<L

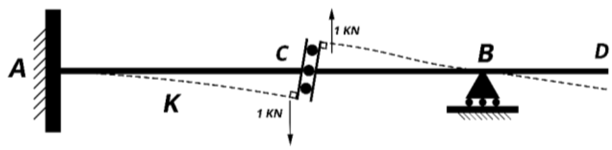


Fig. 2 Influence Line to the shear at C is to the same scale as deflected shape.

When x varies from 0 to K i.e., $\mathbf{0 \leq x < K}$

By Double Integration Method,

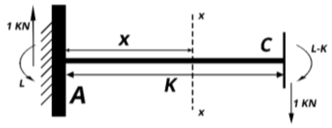


Fig. 3

M_x=x-L

EI $\frac{d^2y}{dx^2} = x - L$

EI $\frac{dy}{dx} = \frac{x^2}{2} - Lx + C_1$ ..... (1)

EI $y(x) = \frac{x^3}{6} - L\frac{x^2}{2} + C_1.x + C_2$ ..... (2)

Putting boundary condition in the equation (1) and (2)

EI$\frac{dy}{dx}$ at x=0 is 0 as there is fixed support. Hence, we get C_1=0.

Again, y(0)=0 as there is fixed support. Hence, we get C_2=0.

EI $y(x) = \frac{x^3}{6} - L\frac{x^2}{2}$ ..... (3)

When x is varies from 0 to K i.e., $\mathbf{K \leq x < L}$

By Double Integration Method,

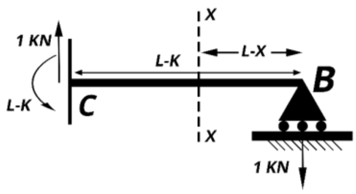


Fig. 4

M_x=x-L

EI $\frac{d^2y}{dx^2} = x - L$

EI $\frac{dy}{dx} = \frac{x^2}{2} - Lx + C_3$ ..... (4)

EI $y(x) = \frac{x^3}{6} - L\frac{x^2}{2} + C_3.x + C_4$ ..... (5)

Using boundary condition,

When x=K, slope in equation (1) is equal to that of equation (4). Hence, we get $C_3 = C_1 = 0$.

When x=L, y(L)=0 as there no displacement in y in equation (5) at roller support, $C_4 = \frac{L^3}{3}$.

$\text{EI } y(x) = \frac{x^3}{6} - L\frac{x^2}{2} + \frac{L^3}{3}$ ..... (6)

When x is varies from 0 to K i.e., $L \leq x < R$

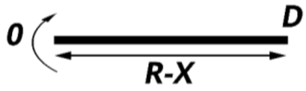


Fig. 5

By Double Integration Method,

$M_x = 0$

$\text{EI } \frac{d^2y}{dx^2} = 0$

$\text{EI} \frac{dy}{dx} = C_5$ ..... (7)

$\text{EI } y(x) = C_5 x + C_6$ ..... (8)

Using boundary condition,

When x=L, slope in equation (7) is equal to that of equation (4). Hence, we get $C_5 = -\frac{L^2}{2}$

When x=L, y(L)=0 as there no displacement in y in equation (8) at a support, $C_6 = \frac{L^3}{2}$.

$\text{EI } y(x) = -\frac{L^2}{2}x + \frac{L^3}{2}$ ..... (9)

To find the function of ILD we have to divide the function in equation (3), (6) and (9) by scale factor which is equal to addition of y(k) from equation (3) and (6).

Scale factor $= \frac{L^3}{3EI}$

Hence, the **Influence Line Diagram for shear at C** is

$y(x) = (\frac{x^3}{6} - L\frac{x^2}{2})\frac{3}{L^3}$ for $0 < x < K$

$y(x) = (\frac{x^3}{6} - L\frac{x^2}{2} + \frac{L^3}{3})\frac{3}{L^3}$ for $K < x < L$

$y(x) = (-\frac{L^2}{2}x + \frac{L^3}{2})\frac{3}{L^3}$ for $L < x < R$

Part 3: K=L,

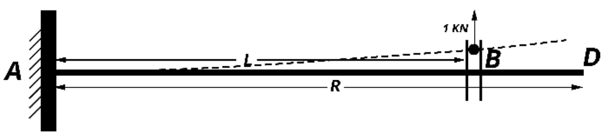


Fig. 2 Influence Line to the shear at C is to the same scale as deflected shape.

When x varies from 0 to K i.e., $0 \leq x < L$

By Double Integration Method,

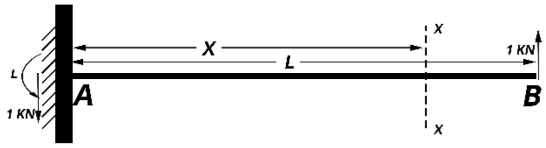


Fig. 3

M_x= -(x-L)

EI $\frac{d^2y}{dx^2}$ = -(x-L)

EI $\frac{dy}{dx} = -\frac{x^2}{2} + Lx + C_1$ ..... (1)

EI $y(x) = -\frac{x^3}{6} + L\frac{x^2}{2} + C_1.x + C_2$ ..... (2)

Putting boundary condition in the equation (1) and (2)

EI$\frac{dy}{dx}$ at x=0 is 0 as there is fixed support. Hence, we get C_1=0.

Again, y(0)=0 as there is fixed support. Hence, we get C_2=0.

EI $y(x) = -\frac{x^3}{6} + L\frac{x^2}{2}$ ..... (3)

When x is varies from 0 to K i.e., $\mathbf{L \leq x < R}$

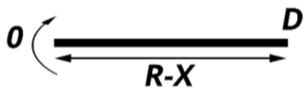


Fig. 5

By Double Integration Method,

M_x=0

EI $\frac{d^2y}{dx^2} = 0$

EI $\frac{dy}{dx} = C_5$ ..... (4)

EI $y(x) = C_5.x + C_6$ ..... (5)

Using boundary condition,

When x=L, slope in equation (4) is equal to that of equation (1). Hence, we get $C_5 = \frac{L^2}{2}$

When x=L, y(L)=0 as there no displacement in y in equation (5) at a support, $C_6 = -\frac{L^3}{6}$

EI $y(x) = \frac{L^2}{2}x - \frac{L^3}{6}$ ..... (6)

To find the function of ILD we have to divide the function in equation (3), and (6) by scale factor which is equal to y(L) in any of equation (3) or (6)

Scale factor $= \frac{L^3}{3EI}$

Hence, the **Influence Line Diagram for shear at C** is

$y(x) \ = (-\frac{x^3}{6} + L\frac{x^2}{2})\frac{3}{L^3}$ for $0 < x < L$

$y(x) = (\frac{L^2}{2}x - \frac{L^3}{6})\frac{3}{L^3}$ for $L < x < R$

Problem Statement: 2

To find an influence line diagram of shear force for the point at distance “K” from the end “A “(as shown in Fig. 1).

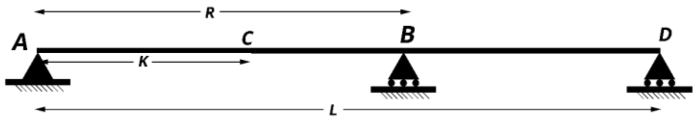


Fig. 1

L = Length of Beam,

R = Distance of middle support from origin from end A,

K = Distance of point for which ILD is calculated from end A,

EI = flexural rigidity of beam (assumed to be constant)

Solution: The solution will be in five parts for different position of K.
Applying Müller-Breslau principle,

PART 1: If 0<K<L,

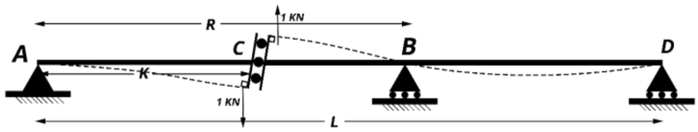


Fig. 2 Removing the support at C and applying the positive shear at C yields the deflected shape. Influence Line to the shear at C is to the same scale as deflected shape.

When x is varies from 0 to K i.e., $\mathbf{R \leq x < L}$

By Double Integration Method,

$M_x = \frac{R}{L-R}(L-x)$

$EI\,\frac{d^2y}{dx^2} = \frac{R}{L-R}(L-x)$

$EI\,\frac{dy}{dx} = \frac{R}{L-R}\left(Lx-\frac{x^2}{2}\right) + C_5$ ..... (1)

$EI\,y(x) = \frac{R}{L-R}\left(L\frac{x^2}{2}-\frac{x^3}{6}\right) + C_5.x + C_6$ ..... (2)

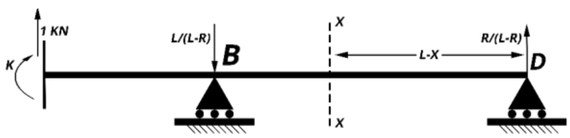


Fig. 3

Using boundary condition,

When x=L and x=R the value of y(L)=0 as there no displacement in y in equation (2) at a support.

We get,

$\frac{R}{L-R}(\frac{L^2}{2} - \frac{L^3}{6}) + C_5.L + C_6 = 0$

$\frac{R}{L-R}(L\frac{R^2}{2} - \frac{R^3}{6}) + C_5.L + C_6 = 0$

Solving the above equations. we get,

$C_5 = \frac{R}{L-R}\ (L\frac{R^2}{2} - \frac{R^3}{6} - \frac{L^3}{3})$

$C_6 = - \frac{R}{L-R}(\frac{L^3}{3}) - C_5.L$

$EI\ y(x) = = \frac{R}{L-R}(L\frac{x^2}{2} - \frac{x^3}{6}) + C_5.x\ - \frac{R}{L-R}(\frac{L^3}{3}) - C_5.L$ ..... (3)

When x is varies from 0 to K i.e., $\mathbf{K < x < R}$

By Double Integration Method,

Fig. 4

M_x=x

$EI\ \frac{d^2y}{dx^2} = x$

$EI\frac{dy}{dx} = \frac{x^2}{2} + C_3$ ..... (4)

$EI\ y(x) = \frac{x^3}{6} + C_3.x + C_4$ ..... (5)

Using boundary condition,

When x=L, slope in equation (1) is equal to that of equation (4),

we get $C_3 = \frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2}$

When x=L, y(L)=0 as there no displacement in y in equation (7) at roller support,

We get $C_4 = - \frac{R^3}{6} - \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2}\}.R$

$EI\ y(x) = \frac{x^3}{6} + \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2} + C_5 - \frac{R^2}{2}\}.x + - \frac{R^3}{6} - \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2}\}.R$ ..... (6)

When x varies from 0 to K i.e., $\mathbf{0 \leq x < K}$

By Double Integration Method,

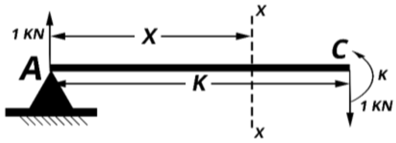


Fig. 5

$M_x = x$

$$EI\,\frac{d^2y}{dx^2} = x$$

$$EI\,\frac{dy}{dx} = \frac{x^2}{2} + C_1 \qquad \text{..... (7)}$$

$$EI\,y(x) = \frac{x^3}{6} + C_1.x + C_2 \qquad \text{..... (8)}$$

Putting boundary condition in the equation (7) and (8),

EI y(x)=0 at x=0 as there is a support, we get $C_2=0$.

When x=K, slope in equation (4) is equal to that of equation (7), we get $C_1 = C_3$.

$$EI\,y(x) = \frac{x^3}{6} + \{\frac{R}{L-R}(L\frac{R^2}{2} - \frac{R^3}{6}) + C_5 - \frac{R^2}{2}\}.x \qquad \text{..... (9)}$$

To find the function of ILD we have to divide the function in equation (3), (6) and (9) by scale factor which is equal to addition of y(k) from equation (6) and (9).

$$\text{Scale factor} = \frac{\frac{K^3}{6} + \{\frac{R}{L-R}(L\frac{R^2}{2} - \frac{R^3}{6}) + C_5 - \frac{R^2}{2}\}.K + \frac{K^3}{6} + \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2} + C5 - \frac{R^2}{2}\}.K + - \frac{R^3}{6} - \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2}\}.R}{EI}$$

Hence, the **Influence Line Diagram for shear at C from equation** (3), (6) and (9) is

$$y(x) = \frac{\frac{x^3}{6} + \{\frac{R}{L-R}(L\frac{R^2}{2} - \frac{R^3}{6}) + C_5 - \frac{R^2}{2}\}.x}{\text{Scale factor . EI}} \qquad \text{for } 0 < x < K$$

$$y(x) = \frac{\frac{x^3}{6} + \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2} + C5 - \frac{R^2}{2}\}.x - \frac{R^3}{6} - \{\frac{R}{L-R}(LR - \frac{R^2}{2}) + C_5 - \frac{R^2}{2}\}.R}{\text{Scale factor . EI}} \qquad \text{for } K < x < R$$

$$y(x) = \frac{\frac{R}{L-R}(L\frac{x^2}{2} - \frac{x^3}{6}) + C_5 .x - \frac{R}{L-R}(\frac{L^3}{3}) - C_5.L}{\text{Scale factor . EI}} \qquad \text{for } R < x < L$$

where $C_5 = \frac{R}{L-R}\left(L\frac{R^2}{2} - \frac{R^3}{6} - \frac{L^3}{3}\right)$

PART 2: K=R,

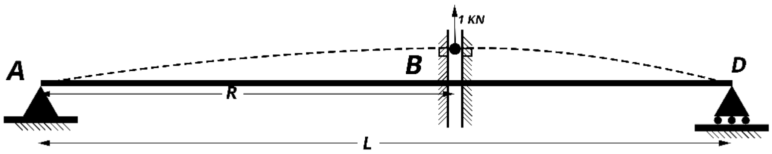


Fig. 6. Placing the roller guide at B and applying the positive shear at B yields the deflected shape. Influence Line to the shear at B is to the same scale as deflected shape

When x is varies from 0 to R i.e., $\mathbf{0 \leq x < R}$

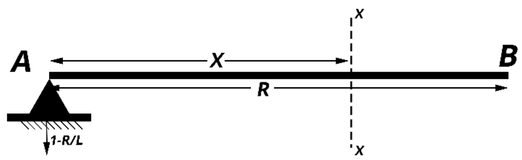


Fig. 7

By Double Integration Method,

M_x = -x.(1- $\frac{R}{L}$)

EI $\frac{d^2y}{dx^2}$ = -x.(1- $\frac{R}{L}$)

EI $\frac{dy}{dx}$ = -x^2.(1- $\frac{R}{L}$)+C_1 ..... (1)

EI y(x) = -x^3.(1- $\frac{R}{L}$)+C_1.x+C_2 ..... (2)

Using boundary condition,

When x=0 the value of y(0)=0 as there no displacement in y in equation (2) as it is at a support.

We get, C_2=0. ..... (3)

When x is varies from 0 to K i.e., $\mathbf{R < x < L}$

By Double Integration Method,

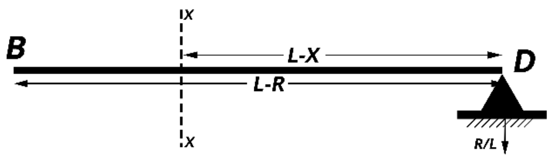


Fig. 8

M_x= -(L-x).$\frac{R}{L}$

EI $\frac{d^2y}{dx^2}$ = -(L-x).$\frac{R}{L}$

EI $\frac{dy}{dx}$ = -(Lx $-\frac{x^2}{2}$). $\frac{R}{L}$ + C_3.x ..... (4)

EI y(x) = -(L$\frac{x^2}{2}-\frac{x^3}{6}$). $\frac{R}{L}$ + C_3.x +C_4 ..... (5)

When x=L the value of y(L)=0 as there no displacement in y in equation (3) as it is at a support.

We get,

C_3.L +C_4 = R.$\frac{L^2}{3}$ ..... (6)

Also,

When $x=R$, slope in equation (1) is equal to that of equation (4) and also $y(x)$ will be equal in equation (2) and (5),

We get $-\frac{R^3}{6}(1-\frac{R}{L}) + C_1R = -(L\frac{x^2}{2} - \frac{x^3}{6}).\frac{R}{L} + C_3.x + C_4$ ..... (7)

$-(Lx - \frac{x^2}{2}).\frac{R}{L} + C_3.x = -x^2.(1-\frac{R}{L}) + C_1$ ..... (8)

From the above equations (3), (6), (7) and (8)

$C_1 = \frac{RL}{3} + \frac{R^3}{6L} - \frac{R^2}{2}$

$C_2 = 0$

$C_3 = \frac{RL^2}{3} - \frac{R^3}{6}$

$C_4 = -\frac{R^3}{6}$

Hence,

$y(x) = -x^3.(1-\frac{R}{L}) + (\frac{RL}{3} + \frac{R^3}{6L} - \frac{R^2}{2}).x$ for $0 < x < R$ ..... (9)

$y(x) = -\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right)\frac{R}{L} + (\frac{RL^2}{3} - \frac{R^3}{6}).x - \frac{R^3}{6}$ for $R < x < L$ ..... (10)

To find the function of ILD we have to divide the function in equation (9) and (10) by scale factor which is equal to addition of $y(k)$ from equation (9) and (10).

$$\text{Scale factor} = \frac{-R^3.(1-\frac{R}{L}) + \left(\frac{RL}{3} - \frac{R^3}{6L} - \frac{R^2}{2}\right).R}{EI}$$

Hence, the **Influence Line Diagram for shear at C from equation** (9), and (10) is

$$y(x) = \frac{-x3.(1-\frac{R}{L}) + (\frac{RL}{3} + \frac{R^3}{6L} - \frac{R^2}{2}).x}{\text{Scale factor . EI}} \quad \text{for } 0 < x < R$$

$$y(x) = \frac{-\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right)\frac{R}{L} + (\frac{RL^2}{3} - \frac{R^3}{6}).x - \frac{R^3}{6}}{\text{Scale factor . EI}} \quad \text{for } R < x < L$$

PART 3: R< K <L,

By Similarity we can get ILD by replacing x to (L-x), R to (L-R) and K to (L-K) in the part 2.

We get,

Scale factor =

$$\frac{\frac{K^3}{6} + \{\frac{(L-R)}{L-(L-R)}(L\frac{(L-R)^2}{2} - \frac{(L-R)^3}{6}) + C_5 - \frac{(L-R)^2}{2}\}.K + \frac{K^3}{6} + \{\frac{(L-R)}{L-(L-R)}(L(L-R) - \frac{(L-R)^2}{2}) + C_5 - \frac{(L-R)^2}{2} + C5 - \frac{(L-R)^2}{2}\}.K + -\frac{(L-R)^3}{6} - \{\frac{(L-R)}{L-(L-R)}(L(L-R) - \frac{(L-R)^2}{2}) + C_5 - \frac{(L-R)^2}{2}\}.(L-R)}{EI}$$

Hence, the **Influence Line Diagram for shear at C from equation** (8), (14) and (18) is

$$y(x) = \frac{\frac{(L-x)^3}{6} + \{\frac{(L-R)}{L-(L-R)}(L\frac{(L-R)^2}{2} - \frac{(L-R)^3}{6}) + C_5 - \frac{(L-R)^2}{2}\}.(L-x)}{\text{Scale factor . EI}} \qquad \text{for } K< x <L$$

$$y(x) = \frac{\frac{(L-x)^3}{6} + \{\frac{(L-R)}{L-(L-R)}(L(L-R) - \frac{(L-R)^2}{2}) + C_5 - \frac{(L-R)^2}{2} + C5 - \frac{(L-R)^2}{2}\}.(L-x) - \frac{(L-R)^3}{6} - \{\frac{(L-R)}{L-(L-R)}(L(L-R) - \frac{(L-R)^2}{2}) + C_5 - \frac{(L-R)^2}{2}\}.(L-R)}{\text{Scale factor . EI}} \qquad \text{for } R< x < K$$

$$y(x) = \frac{\frac{(L-R)}{L-(L-R)}(L\frac{(L-x)^2}{2} - \frac{(L-x)^3}{6}) + C_5.(L-x) - \frac{(L-R)}{L-(L-R)}(\frac{L^3}{3}) - C_5.L}{\text{Scale factor . EI}} \qquad \text{for } 0< x <R$$

where $C_5 = \frac{(L-R)}{L-(L-R)}\ (L\frac{(L-R)^2}{2} - \frac{(L-R)^3}{6} - \frac{L^3}{3})$

PART 4: $K = L$,

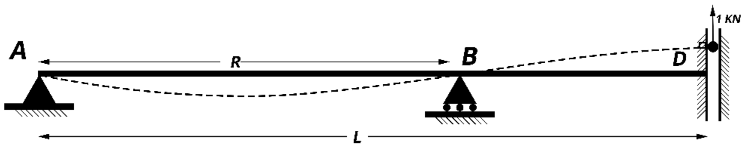


Fig. 9 Placing the roller guide at C and applying the positive shear at C yields the deflected shape. Influence Line to the shear at C is to the same scale as deflected shape.

When x is varies from 0 to R i.e., $\mathbf{0 \leq x < R}$

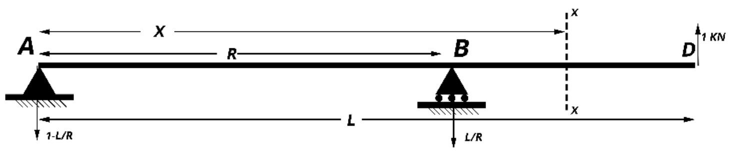


Fig. 4

By Double Integration Method,

$M_x = -x.(1-\frac{R}{L})$

$EI\ \frac{d^2y}{dx^2} = -x.(1-\frac{R}{L})$

$EI\frac{dy}{dx} = -x^2.(1-\frac{R}{L})+C_1$ ..... (1)

$EI\ y(x) = -x^3.(1-\frac{R}{L})+C_1.x+C_2$ ..... (2)

Using boundary condition,

When x=0 and x=R the value of y(0)=0 as there no displacement in y in equation (2) as it is at a support.

We get,

$C_1 = (1 - \frac{L}{R}).\frac{R^2}{6}$

$C_2 = 0$

Hence,

$$EI\ y(x) = -x^3.(1-\frac{R}{L}) + (1-\frac{L}{R})\frac{R^2x}{6} \qquad ..... (3)$$

When x is varies from 0 to K i.e., **$R < x < L$**

By Double Integration Method,

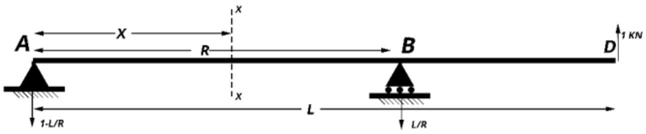


Fig. 10

$M_x = (L-x)$

$$EI\ \frac{d^2y}{dx^2} = (L\text{-}x)$$

$$EI\frac{dy}{dx} = Lx - \frac{x^2}{2} + C_3 \quad ..... (4)$$

$$EI\ y(x) = (L\frac{x^2}{2} - \frac{x^3}{6}) + C_3.x + C_4 \quad ..... (5)$$

When x=R the value of y(L)=0 as there no displacement in y in equation (3) as it is at a support.

We get,

$$C_3.R + C_4 = -(L\frac{R^2}{2} - \frac{R^3}{6}) \quad ..... (6)$$

Also, When x=R, slope in equation (10) is equal to that of equation (16) and also y(x) will be equal,

We get,

$$Lx - \frac{x^2}{2} + C_3 = -x^2.(1-\frac{R}{L}) + C_1 \ ..... (7)$$

From the above equations (6) and (7),

$$C_3 = \frac{R^2}{6} - 2L\frac{R}{3}$$

$$C_4 = L\frac{R^2}{6}$$

Hence,

$$y(x) = -\frac{x^3}{6}.(1-\frac{L}{R}) + ((1-\frac{L}{R}).\frac{R^2}{6}).x \qquad \text{for } 0 < x < R \qquad ..... (8)$$

$$y(x) = \left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) + (\frac{R^2}{6} - 2L\frac{R}{3}).x + L\frac{R^2}{6} \qquad \text{for } R < x < L \qquad ..... (9)$$

To find the function of ILD we have to divide the function in equation (8) and (9) by scale factor which is equal to addition of y(k) from equation (8) and (9).

$$\text{Scale factor} = \frac{\left(\frac{LR^2}{2} - \frac{R^3}{6}\right) + (\frac{R^2}{6} - 2L\frac{R}{3}).R + L\,\frac{R^2}{6}}{EI}$$

Hence, the **Influence Line Diagram for shear at C from equation** (8), and (9) is

$$y(x) = \frac{-\frac{x^3}{6}.(1-\frac{L}{R})+(\,(1-\frac{L}{R}).\frac{R^2}{6}).x}{\text{Scale factor . EI}} \qquad \text{for } 0 < x < R$$

$$y(x) = \frac{\left(\frac{Lx^2}{2} - \frac{x^3}{6}\right) + (\frac{R^2}{6} - 2L\frac{R}{3}).x + L\,\frac{R^2}{6}}{\text{Scale factor . EI}} \qquad \text{for } R < x < L$$

PART 5: If K=0,

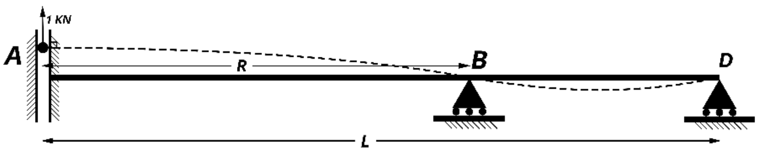


Fig. 11

By Similarity we can get ILD by replacing x to (L-x), R to (L-R) and K to (L-K) in the part 4.

We get,

$$\text{Scale factor} = \frac{\left(\frac{L(L-R)^2}{2} - \frac{(L-R)^3}{6}\right) + (\frac{(L-R)^2}{6} - 2L\frac{(L-R)}{3}).(L-R) + L\,\frac{(L-R)^2}{6}}{EI}$$

Hence, the **Influence Line Diagram for shear at C from equation** (8), and (9) is

$$y(x) = \frac{-\frac{(L-x)^3}{6}.(1-\frac{L}{(L-R)})+(\,(1-\frac{L}{(L-R)}).\frac{(L-R)^2}{6}).(L-x)}{\text{Scale facto(L}-\text{R) . EI}} \qquad \text{for } R < x < L$$

$$y(x) = \frac{\left(\frac{L(L-x)^2}{2} - \frac{(L-x)^3}{6}\right) + (\frac{(L-R)^2}{6} - 2L\frac{(L-R)}{3}).(L-x) + L\,\frac{(L-R)^2}{6}}{\text{Scale facto(L}-\text{R) . EI}} \qquad \text{for } 0 < x < R$$

Problem Statement: 3

To find an influence line diagram of Bending Moment for the point at distance "K" from the fixed end "A "(as shown in fig. 1).

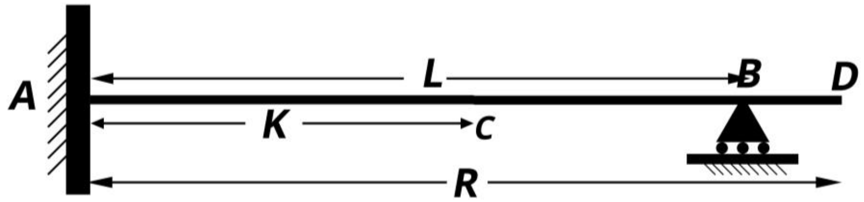


Fig. 1 Cantilever, simply supported Beam

R = Length of Beam,

L = Distance of support from origin from fixed end,

K = Distance of point for which ILD is calculated from fixed end,

EI = flexural rigidity of beam (assumed to be constant)

Solution: The solution will be in three parts for different position of K.

Applying Müller-Breslau principle,

Part 1: K=0

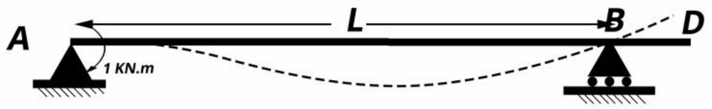


Fig. 2 Influence Line to the bending moment at 0 is to the same scale as deflected shape.

When x varies from 0 to K i.e., $\mathbf{0 \leq x < L}$

As we know the M_x is a linear function of x(observation from the above problems and can be proved for any given beam)

By Double Integration it can be proved that deflection of beam is either a cubic function or a linear function

Let the equation for the deflected beam be,

$y(x)=A_1x^3+B_1x^2+C_1x+D_1$ ..... (1)

The above equation will satisfy the following boundary condition

$y(0)= D_1 = 0$ ..... (2)

$y(L) = A_1L^3+B_1L^2+C_1L+D_1 = 0$ ..... (3)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = 1$ at x=0 ..... (4)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = 0$ at x=L ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ L^3 & L^2 & L & 1 \\ 0 & 2 & 0 & 0 \\ 6L & 2 & 0 & 0 \end{pmatrix}\begin{pmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

The value for the constants is calculated using inverse and multiplication matrix methods in java program.

When x varies from 0 to K i.e., **$L \leq x < R$**

Let the equation for the deflected beam be,

$y(x)=A_3x^3+B_3x^2+C_3x+D_3$ ..... (1)

The above equation will satisfy the following boundary condition

$y(L)= A_3L^3+B_3L^2+C_3L+D_3 = 0$ ..... (2)

derivatives will be equal at x=K

$3A_3L^2+2B_3L+C_3 = 3A_1L^2+2B_1L+C_1$ ..... (3)

$\frac{d^2y}{dx^2} = 6A_3x+2B_3 = 0$ at x=L ..... (4)

$\frac{d^2y}{dx^2} = 6A_3x+2B_3 = 0$ at x=R ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} L^3 & L^2 & L & 1 \\ 3L^2 & 2L & 1 & 0 \\ 6L & 2 & 0 & 0 \\ 6R & 2 & 0 & 0 \end{pmatrix}\begin{pmatrix} A_3 \\ B_3 \\ C_3 \\ D_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3A_1K^2 + 2B_1K + C_1 \\ 0 \\ 0 \end{pmatrix}$$

The values for the constant is calculated using java.

After computation of all the constant for the cubic equations, Scale factor is calculated

Scale factor = $(3A_1K^2 + 2B_1K + C_1)$

Hence, the **Influence Line Diagram for shear at C from equation**

$y(x)=(A_1x^3+B_1x^2+C_1x+D_1)/(3A_1K^2 + 2B_1K + C_1)$

$y(x)=(A_3x^3+B_3x^2+C_3x+D_3)/(3A_1K^2 + 2B_1K + C_1)$

Part 2: 0<K<L

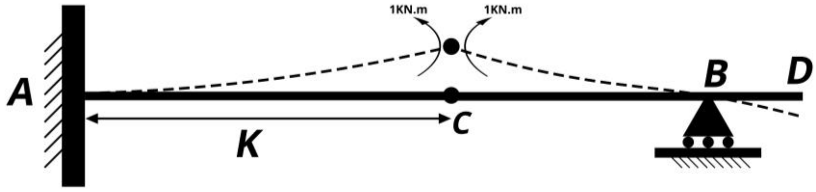


Fig. 2 Influence Line to the bending moment at C is to the same scale as deflected shape.

When x varies from 0 to K i.e., $\mathbf{0 \leq x < K}$

As we know the M_x is a linear function of x(observation from the above problems and can be proved for any given beam)

By Double Integration it can be proved that deflection of beam is either a cubic function or a linear function

Let the equation for the deflected beam be,

$y(x)=A_1x^3+B_1x^2+C_1x+D_1$ ..... (1)

The above equation will satisfy the following boundary condition

$y(0)= D_1 = 0$ ..... (2)

$\frac{dy}{dx} = 0$ at x=0 ..... (3)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = \frac{L}{L-K}$ at x=0 ..... (4)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = 1$ at x=K ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \\ 6K & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 6A_1x + 2B_1 \\ 6A_1x + 2B_1 \end{pmatrix}$$

The value for the constants is calculated using inverse and multiplication matrix methods in java program.

When x varies from 0 to K i.e., $\mathbf{K \leq x < L}$

Let the equation for the deflected beam be,

$y(x)=A_2x^3+B_2x^2+C_2x+D_2$ ..... (1)

The above equation will satisfy the following boundary condition

$y(L)= A_2L^3+B_2L^2+C_2L+D_2 = 0$ ..... (2)

displacement will be equal at x=K

$A_2K^3+B_2K^2+C_2K+D_2 = A_1K^3+B_1K^2+C_1K+D_1$ ..... (3)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = 0$ at x=L ..... (4)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = 1$ at x=K ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} L^3 & L^2 & L & 1 \\ K^3 & K^2 & K & 1 \\ 6L & 2 & 0 & 0 \\ 6K & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} A_2 \\ B_2 \\ C_2 \\ D_2 \end{pmatrix} = \begin{pmatrix} 0 \\ A_1K^3 + B_1K^2 + C_1K + D_1 \\ 0 \\ 1 \end{pmatrix}$$

The values for the constant is calculated using java.

When x varies from 0 to K i.e., **$L \leq x < R$**

Let the equation for the deflected beam be,

$y(x)=A_3x^3+B_3x^2+C_3x+D_3$ ..... (1)

The above equation will satisfy the following boundary condition

$y(L)= A_3L^3+B_3L^2+C_3L+D_3 = 0$ ..... (2)

derivatives will be equal at x=K

$3A_3K^2+2B_3K+C_3 = 3A_2K^2+2B_2K+C_2$ ..... (3)

$\frac{d^2y}{dx^2} = 6A_3x+2B_3 = 0$ at x=L ..... (4)

$\frac{d^2y}{dx^2} = 6A_3x+2B_3 = 0$ at x=R ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} L^3 & L^2 & L & 1 \\ 3K^2 & 2K & 1 & 0 \\ 6L & 2 & 0 & 0 \\ 6R & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} A_3 \\ B_3 \\ C_3 \\ D_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3A_2K^2 + 2B_2K + C_2 \\ 0 \\ 0 \end{pmatrix}$$

The values for the constant is calculated using java.

After computation of all the constant for the cubic equations, Scale factor is calculated

Scale factor = $(3A_1K^2 + 2B_1K + C_1) - (3A_2K^2 + 2B_2K + C_2)$

Hence, the **Influence Line Diagram for shear at C from equation**

$y(x)=(A_1x^3+B_1x^2+C_1x+D_1)/((3A_1K^2 + 2B_1K + C_1) - (3A_2K^2 + 2B_2K + C_2))$

$y(x)=(A_2x^3+B_2x^2+C_2x+D_2)/((3A_1K^2 + 2B_1K + C_1) - (3A_2K^2 + 2B_2K + C_2))$

$y(x)=(A_3x^3+B_3x^2+C_3x+D_3)/((3A_1K^2 + 2B_1K + C_1) - (3A_2K^2 + 2B_2K + C_2))$

Part 3: K=L

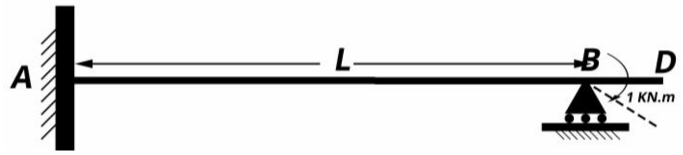


Fig. 2 Influence Line to the bending moment at L is to the same scale as deflected shape.

When x varies from 0 to K i.e., $\mathbf{0 \leq x < L}$

As we know the M_x is a linear function of x(observation from the above problems and can be proved for any given beam)

By Double Integration it can be proved that deflection of beam is either a cubic function or a linear function

Let the equation for the deflected beam be,

$y(x)=A_1x^3+B_1x^2+C_1x+D_1$ ..... (1)

The above equation will satisfy the following boundary condition

$y(0)= D_1 = 0$ ..... (2)

$y(L) = A_1L^3+B_1L^2+C_1L+D_1 = 0$ ..... (3)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = 0$ at x=0 ..... (4)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = 0$ at x=L ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ L^3 & L^2 & L & 1 \\ 0 & 2 & 0 & 0 \\ 6L & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

The value for the constants is calculated using inverse and multiplication matrix methods in java program.

When x varies from 0 to K i.e., $\mathbf{L \leq x < R}$

Let the equation for the deflected beam be,

$y(x)=A_3x^3+B_3x^2+C_3x+D_3$ ..... (1)

The above equation will satisfy the following boundary condition

$y(L)= A_3L^3+B_3L^2+C_3L+D_3 = 0$ ..... (2)

$\frac{dy}{dx} = 3A_3L^2+2B_3L+C_3 = -1$ at x=0 ..... (3)

$\frac{d^2y}{dx^2} = 6A_3x+2B_3 = 0$ at x=L ..... (4)

$\frac{d^2y}{dx^2} = 6A_3x+2B_3 = 0$ at x=R ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} L^3 & L^2 & L & 1 \\ 3L^2 & 2L & 1 & 0 \\ 6L & 2 & 0 & 0 \\ 6R & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} A_3 \\ B_3 \\ C_3 \\ D_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \\ 0 \\ 0 \end{pmatrix}$$

The values for the constant is calculated using java.

After computation of all the constant for the cubic equations, Scale factor is calculated

Scale factor = 1

Hence, the **Influence Line Diagram for shear at C from equation**

y(x)=0

$y(x)=(A_3x^3+B_3x^2+C_3x+D_3)$

Problem Statement: 4

To find an influence line diagram of Bending Moment for the point at distance "K" from the fixed end "A "(as shown in fig. 1).

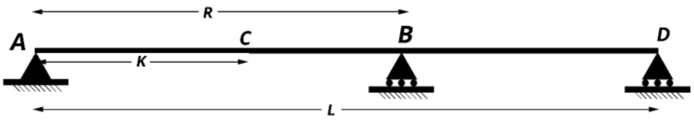


Fig. 1 Cantilever, simply supported Beam

R = Length of Beam,

L = Distance of support from origin from fixed end,

K = Distance of point for which ILD is calculated from fixed end,

EI = flexural rigidity of beam (assumed to be constant)

Solution: The solution will be in three parts for different position of K.

Applying Müller-Breslau principle,

Part 1: K=0

Moment at support will be zero when a unit load act upon beam.

Part 2: 0<K<R

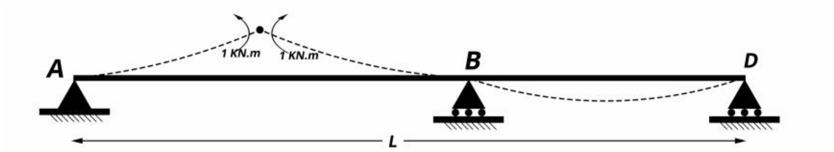


Fig. 2 Influence Line to the bending moment at C is to the same scale as deflected shape.

When x varies from 0 to K i.e., $\mathbf{R \leq x < L}$

As we know the M_x is a linear function of x(observation from the above problems and can be proved for any given beam)

By Double Integration it can be proved that deflection of beam is either a cubic function or a linear function

Let the equation for the deflected beam be,

$y(x)=A_1x^3+B_1x^2+C_1x+D_1$ ..... (1)

The above equation will satisfy the following boundary condition

$y(R)= A_1R^3+B_1R^2+C_1R+D_1 = 0$ ..... (2)

$y(L)= A_1L^3+B_1L^2+C_1L+D_1 = 0$ ..... (2)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = \frac{R}{K}$ at x=R ..... (4)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = 0$ at x=L ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} L^3 & L^2 & L & 1 \\ R^3 & R^2 & R & 1 \\ 6L & 2 & 0 & 0 \\ 6R & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} A_2 \\ B_2 \\ C_2 \\ D_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ R/K \end{pmatrix}$$

The value for the constants is calculated using inverse and multiplication matrix methods in java program.

When x varies from 0 to K i.e., **$K \leq x < R$**

Let the equation for the deflected beam be,

$y(x)=A_2x^3+B_2x^2+C_2x+D_2$ ..... (1)

The above equation will satisfy the following boundary condition

$y(L)= A_2R^3+B_2R^2+C_2R+D_2 = 0$ ..... (2)

Slope will be equal at x=K

$3A_2R^2+2B_2R+C_2 = 3A_3R^2+2B_3R+C_3$ ..... (3)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = 0$ at x=R ..... (4)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = 1$ at x=K ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} R^3 & R^2 & R & 1 \\ 3R^2 & 2R & 1 & 0 \\ 6R & 2 & 0 & 0 \\ 6K & 2 & 0 & 0 \end{pmatrix} \begin{pmatrix} A_2 \\ B_2 \\ C_2 \\ D_2 \end{pmatrix} = \begin{pmatrix} 0 \\ A_3R^2 + B_3R + C_3 \\ R/K \\ 1 \end{pmatrix}$$

The values for the constant is calculated using java.

When x varies from 0 to K i.e., **$0 \leq x < K$**

Let the equation for the deflected beam be,

$y(x)=A_3x^3+B_3x^2+C_3x+D_3$ ..... (1)

The above equation will satisfy the following boundary condition

$y(L)= A_3L^3+B_3L^2+C_3L+D_3 = 0$ ..... (2)

displacements will be equal at x=K

$A_1K^3 + B_1K^2 + C_1K + D_1 = A_2K^3 + B_2K^2 + C_2K + D_2$ ..... (3)

$\frac{d^2y}{dx^2}$ = 6A$_3$x+2B$_3$ = 0 at x=0 ..... (4)

$\frac{d^2y}{dx^2}$ = 6A$_3$x+2B$_3$ = 1 at x=K ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 \\ 6K & 2 & 0 & 0 \\ R^3 & R^2 & R & 1 \end{pmatrix}\begin{pmatrix} A_3 \\ B_3 \\ C_3 \\ D_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ A_2K^3 + B_2K^2 + C_2K + D_2 \end{pmatrix}$$

The values for the constant is calculated using java.

After computation of all the constant for the cubic equations, Scale factor is calculated

Scale factor = $(3A_1K^2 + 2B_1K + C_1) - (3A_2K^2 + 2B_2K + C_2)$

Hence, the **Influence Line Diagram for shear at C from equation**

y(x)=($A_1x^3+B_1x^2+C_1x+D_1$)/($(3A_1K^2 + 2B_1K + C_1) - (3A_2K^2 + 2B_2K + C_2)$)

y(x)=($A_2x^3+B_2x^2+C_2x+D_2$)/($(3A_1K^2 + 2B_1K + C_1) - (3A_2K^2 + 2B_2K + C_2)$)

y(x)=($A_3x^3+B_3x^2+C_3x+D_3$)/($(3A_1K^2 + 2B_1K + C_1) - (3A_2K^2 + 2B_2K + C_2)$)

Part 3: K=R

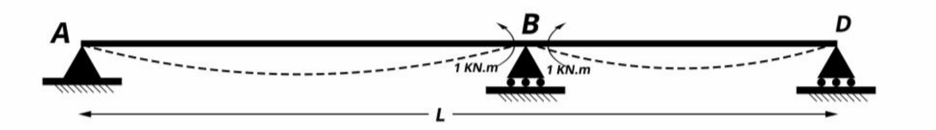


Fig. 2 Influence Line to the bending moment at L is to the same scale as deflected shape.

When x varies from 0 to K i.e., **$0 \leq x < L$**

As we know the M_x is a linear function of x(observation from the above problems and can be proved for any given beam)

By Double Integration it can be proved that deflection of beam is either a cubic function or a linear function

Let the equation for the deflected beam be,

y(x)=$A_1x^3+B_1x^2+C_1x+D_1$ ..... (1)

The above equation will satisfy the following boundary condition

y(0)= D_1 = 0 ..... (2)

y(R) = $A_1R^3+B_1R^2+C_1R+D_1$ = 0 ..... (3)

$\frac{d^2y}{dx^2}$ = 6A$_1$x+2B$_1$ = 0 at x=0 ..... (4)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = 0$ at x=R ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ R^3 & R^2 & R & 1 \\ 0 & 2 & 0 & 0 \\ 6R & 2 & 0 & 0 \end{pmatrix}\begin{pmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1/2 \\ 1 \end{pmatrix}$$

The value for the constants is calculated using inverse and multiplication matrix methods in java program.

When x varies from 0 to K i.e., **$L \leq x < R$**

Let the equation for the deflected beam be,

$y(x)=A_3x^3+B_3x^2+C_3x+D_3$ ..... (1)

The above equation will satisfy the following boundary condition

$y(R)= A_3R^3+B_3R^2+C_3R+D_3 = 0$ ..... (2)

$y(L)= A_3L^3+B_3L^2+C_3L+D_3 = 0$ ..... (3)

$\frac{d^2y}{dx^2} = 6A_3x+2B_3 = -1/2$ at x=L ..... (4)

$\frac{d^2y}{dx^2} = 6A_3x+2B_3 = 1$ at x=R ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} R^3 & R^2 & R & 1 \\ L^3 & L^2 & L & 1 \\ 6L & 2 & 0 & 0 \\ 6R & 2 & 0 & 0 \end{pmatrix}\begin{pmatrix} A_3 \\ B_3 \\ C_3 \\ D_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -1/2 \\ 1 \end{pmatrix}$$

The values for the constant is calculated using java.

After computation of all the constant for the cubic equations, Scale factor is calculated

Scale factor = $3A_1K^2+2B_1K+C_1 - 3A_2K^2+2B_2K+C_2$

Hence, the **Influence Line Diagram for shear at C from equation**

$y(x)=(A_1x^3+B_1x^2+C_1x+D_1)/((3A_1K^2 + 2B_1K + C_1) - (3A_2K^2 + 2B_2K + C_2))$

$y(x)=(A_2x^3+B_2x^2+C_2x+D_2)/((3A_1K^2 + 2B_1K + C_1) - (3A_2K^2 + 2B_2K + C_2))$

Part 4: R<K<L

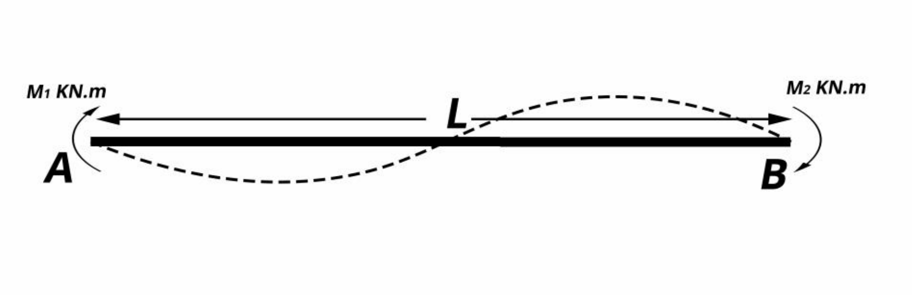
Replacing x by L-x in part 2 we will get solution for the part 4.

Part 5: K=L

Moment at support will be zero when a unit load act upon beam.

Problem Statement: 4

To find arbitrary beam's deflection with known moment (using **slope deflection method)**, after applying Muller Breslau principle.



We can prove that Internal Moment will be a linear function of x

By Double Integration we can say deflection of beam is either a cubic function or a linear function

Let the equation for the deflected beam be,

$y(x)=A_1x^3+B_1x^2+C_1x+D_1$ ..... (1)

The above equation will satisfy the following boundary condition

$y(0)= D_1 = 0$ ..... (2)

$y(L) = A_1L^3+B_1L^2+C_1L+D_1 = 0$ ..... (3)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = M_1$ at x=0 ..... (4)

$\frac{d^2y}{dx^2} = 6A_1x+2B_1 = M_2$ at x=L ..... (5)

The equation (2), (3), (4), & (5) we can solve for y(x) using matrix method

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ L^3 & L^2 & L & 1 \\ 0 & 2 & 0 & 0 \\ 6L & 2 & 0 & 0 \end{pmatrix}\begin{pmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ M_1 \\ M_2 \end{pmatrix}$$

From the above matrix we can find coefficient of cubic equation hence we can find beam's deflection after applying Muller Breslau Principle.

It can be concluded that we can find ILD for any beam after finding out end moments using methods such as Slope Deflection.