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# Math for Machine Learning

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## **Linear algebra - Week 1**



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# System of Linear Equations

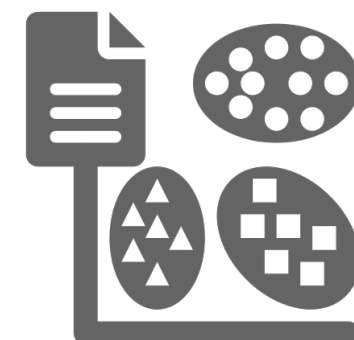
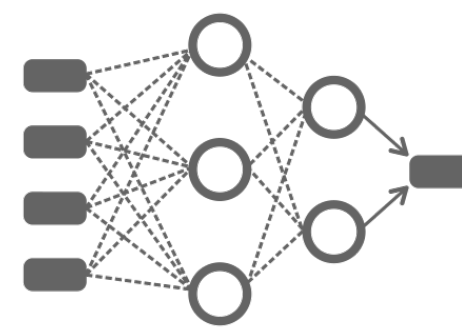
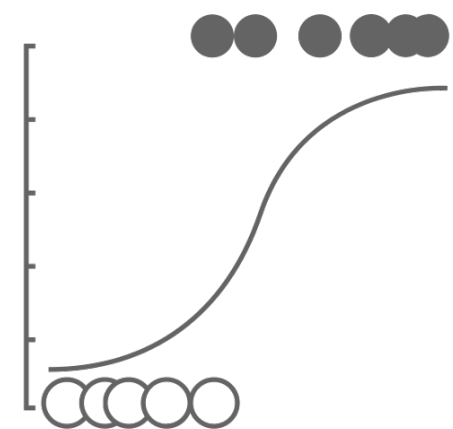
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## **Linear Algebra Applied I**

# Machine Learning



# Machine Learning



Don't worry about  
the math!

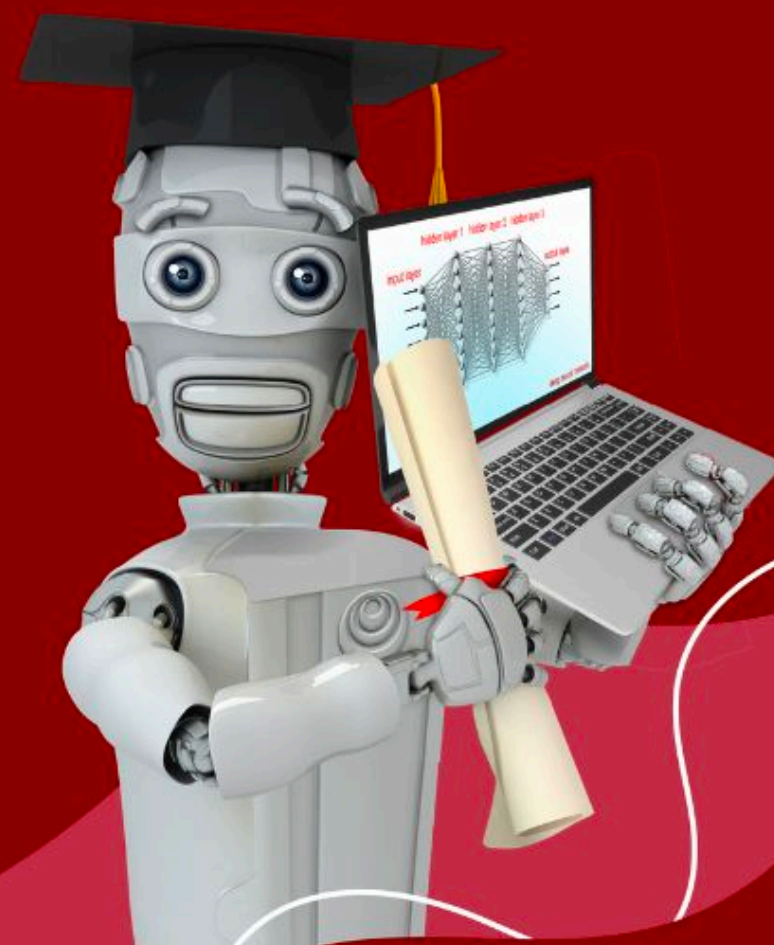


Don't worry about  
the machine learning!

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**Machine Learning  
Specialization**

**Enroll now**





# Linear Algebra and Machine Learning

## Linear Regression

*Supervised Machine Learning*



# Linear Algebra and Machine Learning

Input



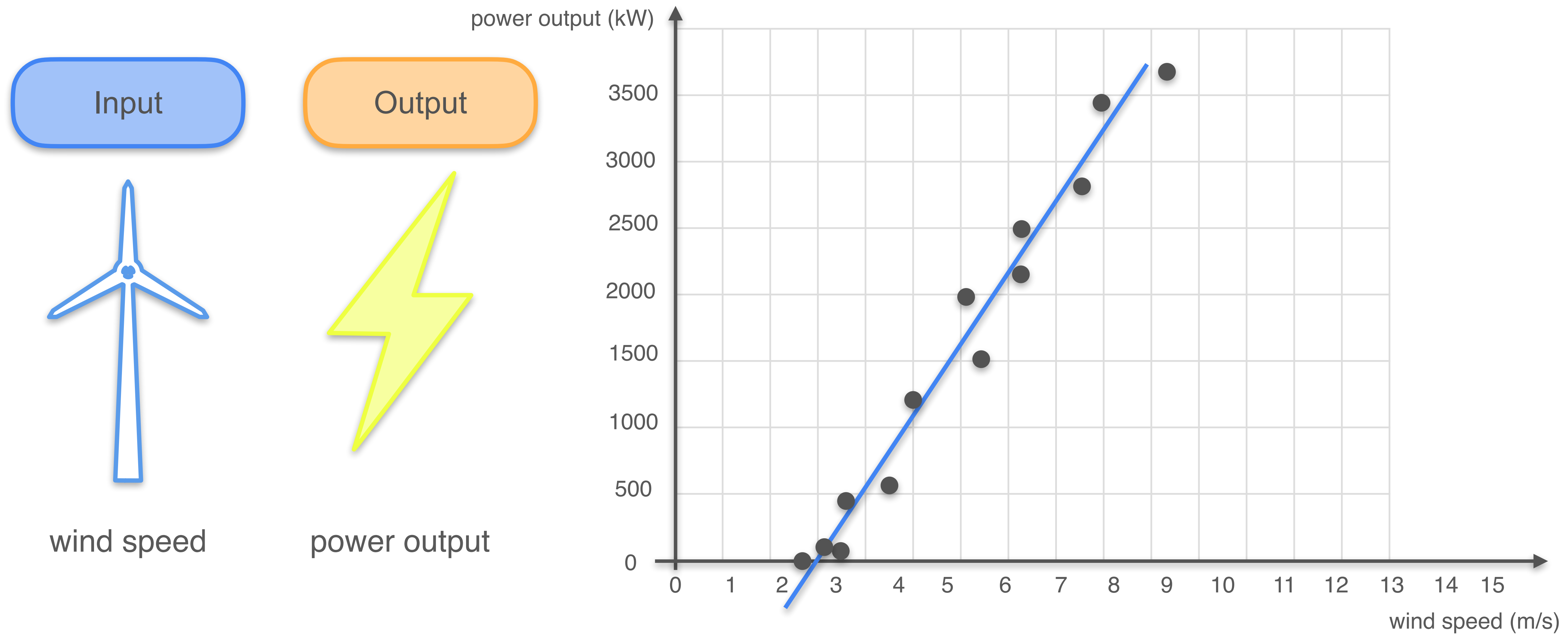
wind speed

Output



power output

# Linear Algebra and Machine Learning





# Linear Algebra and Machine Learning



$$m \times \text{wind speed} + b = \text{power output}$$

5m/s                      1500kW



# Linear Algebra and Machine Learning

Input



wind speed

Input



temperature

Output



power output

# Linear Algebra and Machine Learning



# Linear Algebra and Machine Learning

Input



wind speed

Input



temperature

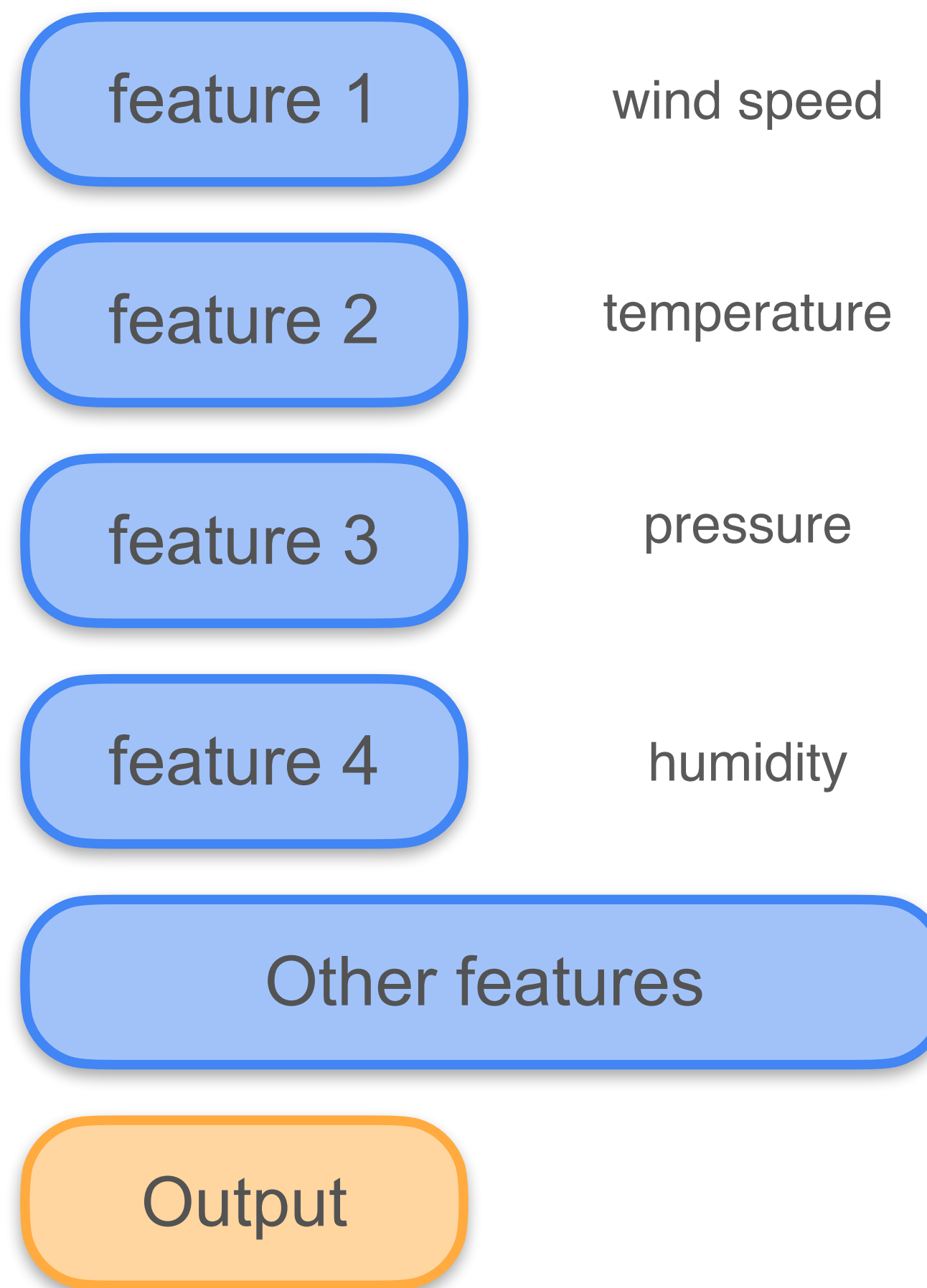
Output



power output



# Linear Algebra and Machine Learning



# Linear Algebra and Machine Learning





# Linear Algebra and Machine Learning

$$w_1 x_1 + w_2 x_2 + \dots + w_n x_n + b = y$$


**TARGET**

# Linear Algebra and Machine Learning

$$w_1 x_1^{(1)} + w_2 x_2^{(1)} + \dots + w_n x_n^{(1)} + b = y^{(1)}$$

$$w_1 x_1^{(2)} + w_2 x_2^{(2)} + \dots + w_n x_n^{(2)} + b = y^{(2)}$$

$$w_1 x_1^{(3)} + w_2 x_2^{(3)} + \dots + w_n x_n^{(3)} + b = y^{(3)}$$

System of Linear Equations

$$\vdots$$
$$w_1 x_1^{(m)} + w_2 x_2^{(m)} + \dots + w_n x_n^{(m)} + b = y^{(m)}$$



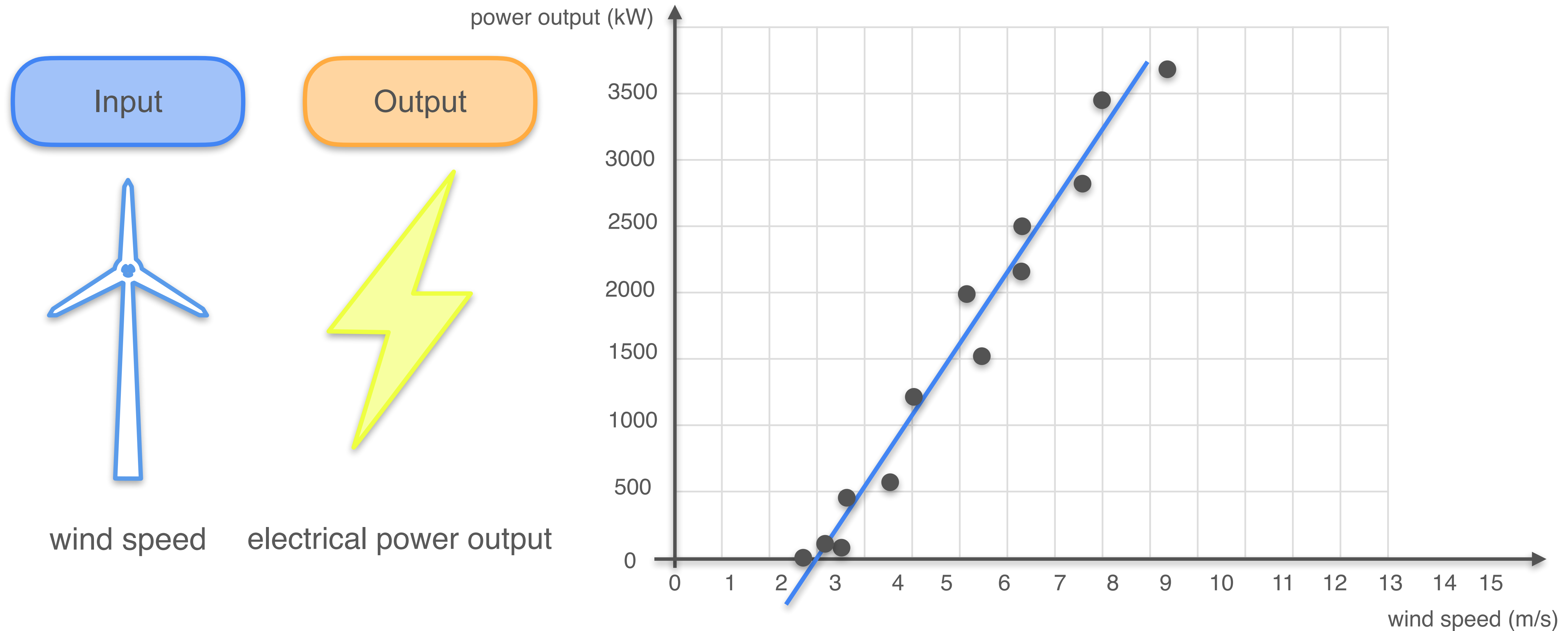
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# System of Linear Equations

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## **Linear Algebra Applied II**

# Linear Algebra and Machine Learning



# Linear Algebra and Machine Learning

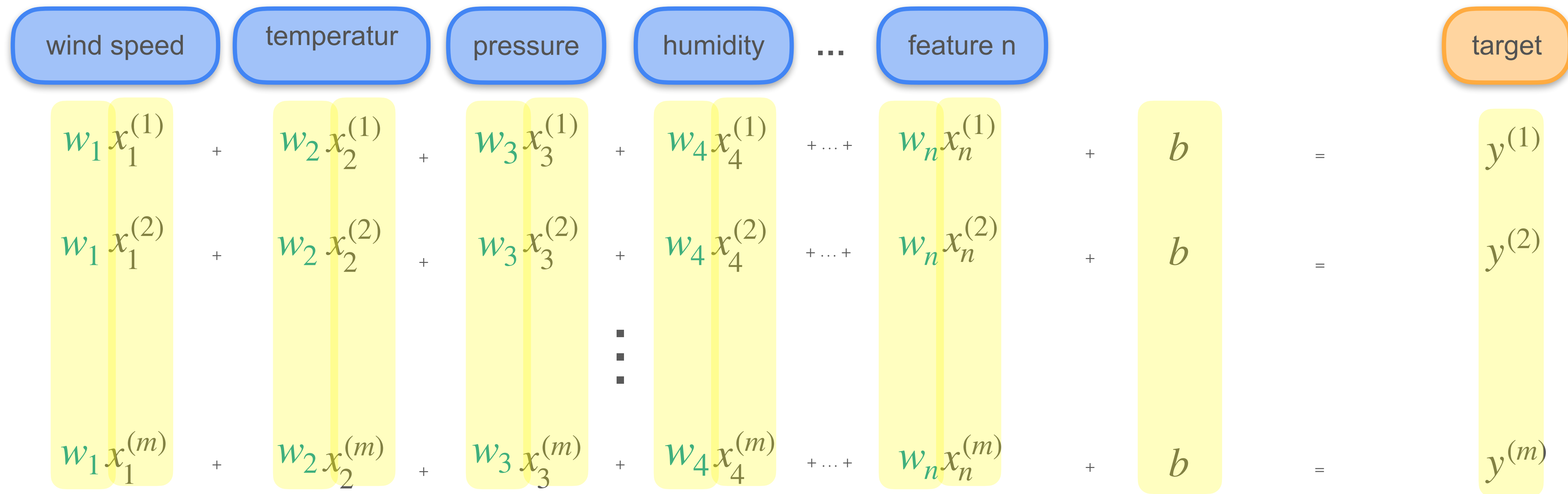


# Linear Algebra and Machine Learning

wind speed	temperatur	pressure	humidity	...	feature n		target							
$w_1 x_1^{(1)}$	+	$w_2 x_2^{(1)}$	+	$w_3 x_3^{(1)}$	+	$w_4 x_4^{(1)}$	+	...	+	$w_n x_n^{(1)}$	+	$b$	=	$y^{(1)}$
$w_1 x_1^{(2)}$	+	$w_2 x_2^{(2)}$	+	$w_3 x_3^{(2)}$	+	$w_4 x_4^{(2)}$	+	...	+	$w_n x_n^{(2)}$	+	$b$	=	$y^{(2)}$
				■										
				■										
				■										
$w_1 x_1^{(m)}$	+	$w_2 x_2^{(m)}$	+	$w_3 x_3^{(m)}$	+	$w_4 x_4^{(m)}$	+	...	+	$w_n x_n^{(m)}$	+	$b$	=	$y^{(m)}$



# Linear Algebra and Machine Learning



# Linear Algebra and Machine Learning

$$\begin{array}{c} \mathbf{w} \\ \left[ \begin{array}{cccc} w_1 & w_2 & w_3 & w_4 & \dots & w_n \end{array} \right] \\ \text{vector} \end{array} \cdot \begin{array}{c} X \\ \left[ \begin{array}{cccc} x_1^{(1)} & x_2^{(1)} & x_3^{(1)} & x_4^{(1)} & \dots & x_n^{(1)} \\ x_1^{(2)} & x_2^{(2)} & x_3^{(2)} & x_4^{(2)} & \dots & x_n^{(2)} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ x_1^{(m)} & x_2^{(m)} & x_3^{(m)} & x_4^{(m)} & \dots & x_n^{(m)} \end{array} \right] \\ \text{matrix} \end{array} + \begin{array}{c} b \\ \left[ \begin{array}{cccc} y^{(1)} & y^{(2)} & \dots & y^{(m)} \end{array} \right] \\ \text{vector} \end{array} = Y$$

# Linear Algebra and Machine Learning



# Plan for the Week

Common vector and matrix operations



# Plan for the Week

Systems of Linear Equations

Representing systems as vectors and matrices

Computing the determinant of matrices

# Check your Knowledge

Linear  
Algebra

Your algebra score added to your calculus score minus your probability score was 6

---

Calculus

Your algebra score minus your calculus score plus double your probability score was 4.

---

Probability &  
Statistics

Four times your algebra score minus double your calculus score added to your probability score was 10

**Represent these statements as a system of linear equations.**



# Check your Knowledge

a

Linear  
Algebra

Your algebra score added to your calculus score minus your probability score was 6

$$a + c - p = 6$$

c

Calculus

Your algebra score minus your calculus score plus double your probability score was 4.

$$a - c + 2p = 4$$

p

Probability &  
Statistics

Four times your algebra score minus double your calculus score added to your probability score was 10

$$4a - 2c + p = 10$$

**Represent these statements as a system of linear equations.**

# Check your Knowledge

What are the weights,  $w$ ?  $a, c, p$

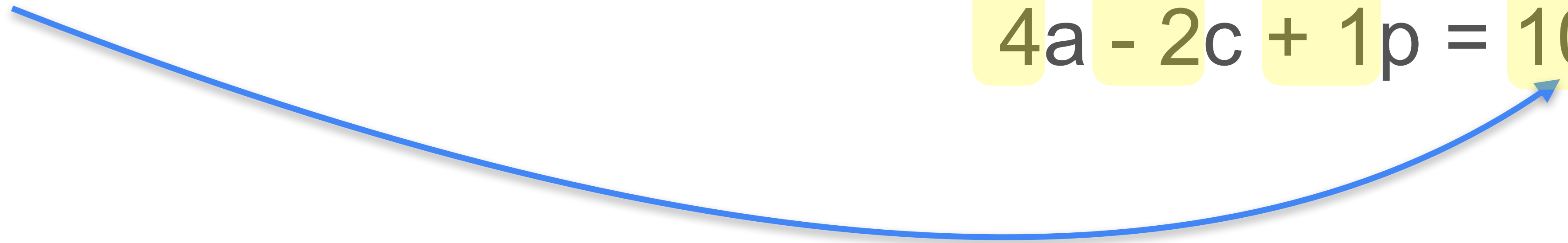
$$1a + 1c - 1p = 6$$

What are the features,  $x$ ?

$$1a - 1c + 2p = 4$$

The targets,  $y$ ? 6, 4, 10

$$4a - 2c + 1p = 10$$



# Check your Knowledge

Is this system singular or non-singular?

$$a + c - p = 6$$

Can you solve this system of equations?

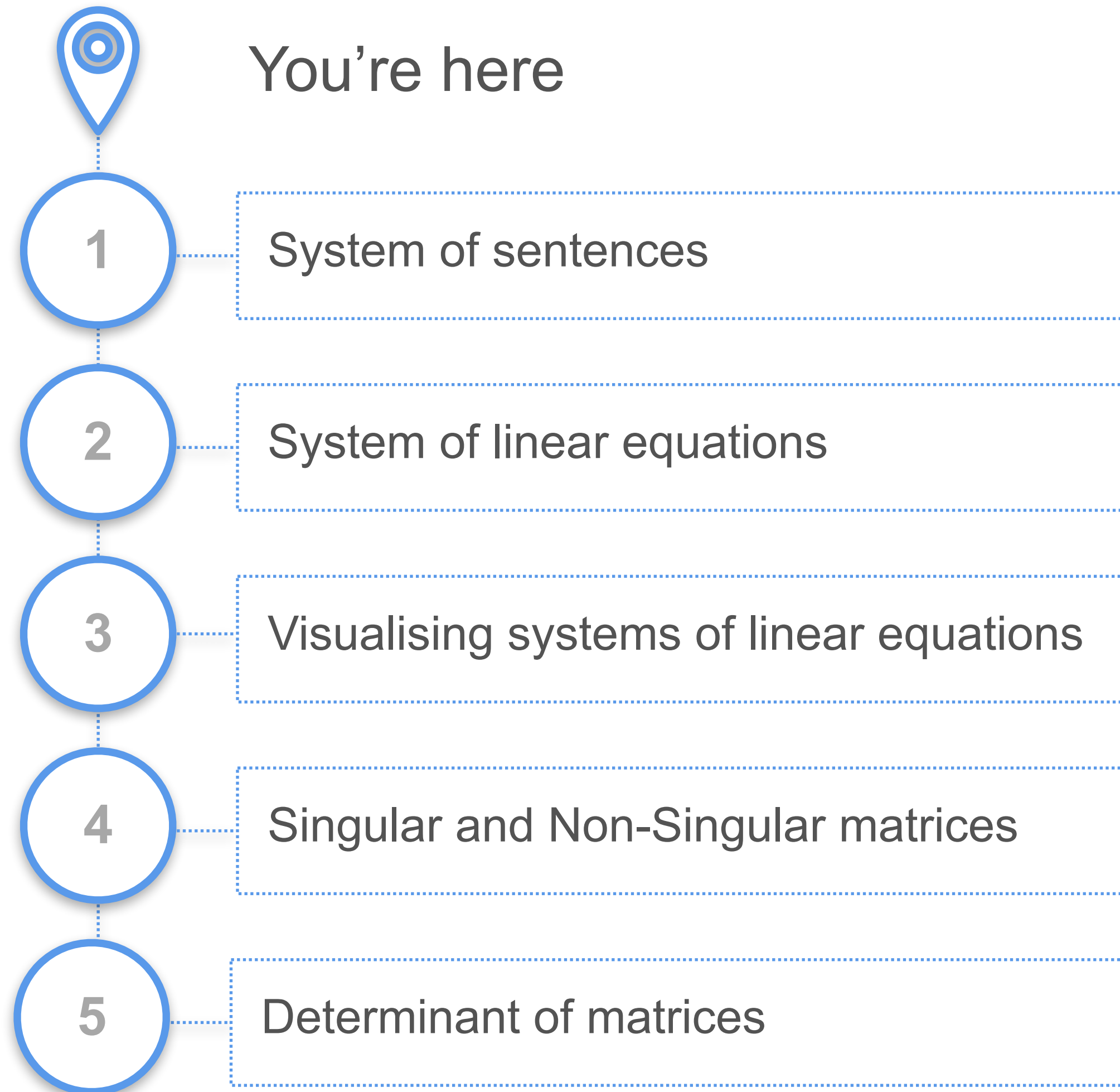
$$a - c + 2p = 4$$

Can you represent this system as a matrix and a vector?

$$4a - 2c + p = 10$$

Can you calculate the determinant of that matrix?

# What to expect





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# System of Linear Equations

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**System of sentences**

# Systems of sentences

## System 1

 The dog is **black**  
 The cat is **orange**

Complete

Non-singular


## System 2

 The dog is **black**  
 The dog is **black**

Redundant

Singular

## System 3

 The dog is **black**  
 The dog is **white**

Contradictory

Singular



# Systems of sentences

## System 1



Complete

Non-singular

## System 2



Redundant

Singular

## System 3



Redundant

Singular

## System 4



Contradictory

Singular

# Quiz: Systems of sentences

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.

**Problem 1:**

What color is the bird?

**Problem 2:**

Is this system singular or non-singular?

# Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



**Solution 1:**

The bird is red. 

**Solution 2:**

It is non-singular.   



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# System of Linear Equations

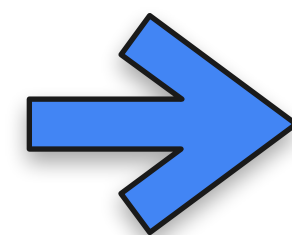
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**System of equations**

# Sentences → Equations

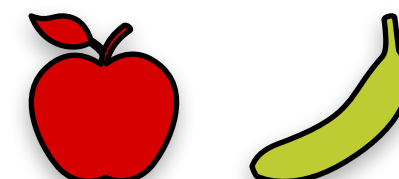
## Sentences

Between the dog and the cat, one is black.



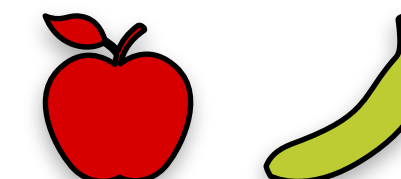
## Sentences with numbers

The price of an apple and a banana is \$10.



## Equations

$$a + b = 10$$



# Quiz: Systems of equations 1

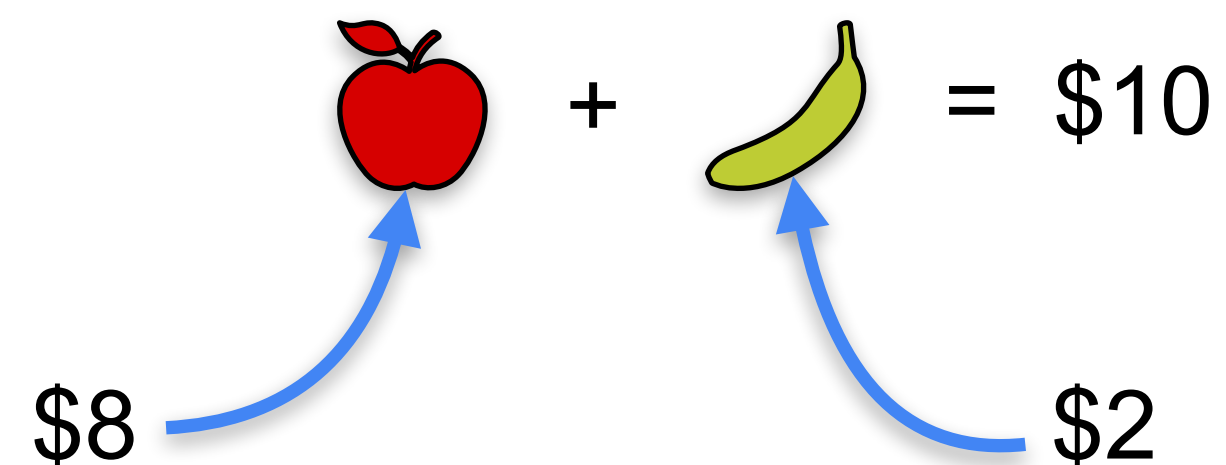
You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought an apple and two bananas and they cost \$12.

**Question:** How much does each fruit cost?

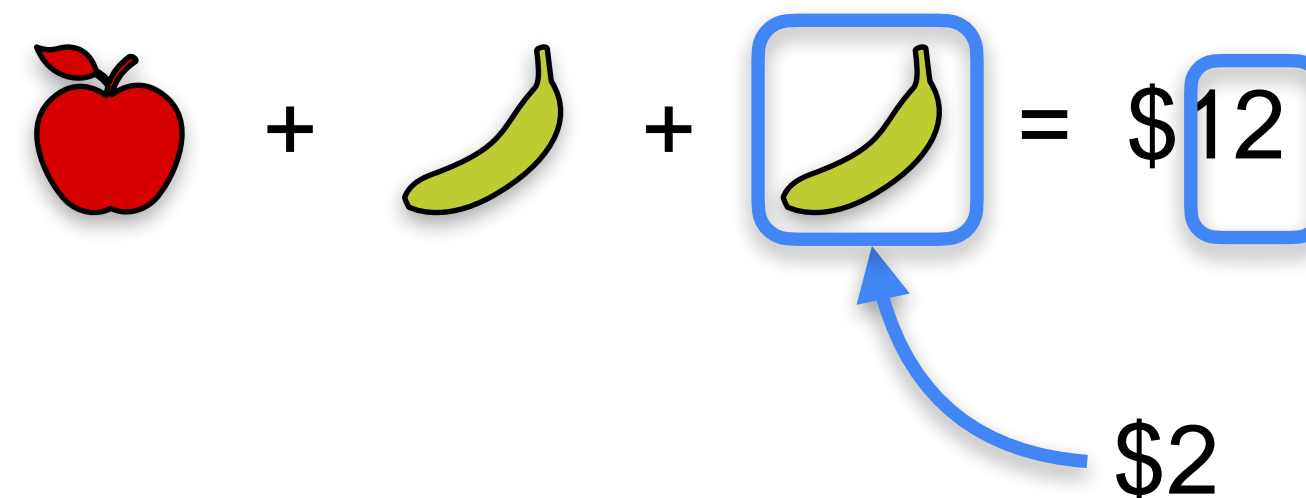
# Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.


$$\text{Apple} + \text{Banana} = \$10$$

\$8      \$2

- **Day 2:** You bought an apple and two bananas and they cost \$12.


$$\text{Apple} + \text{Banana} + \boxed{\text{Banana}} = \$12$$

\$2

- **Solution:** An apple costs \$8, a banana costs \$2.



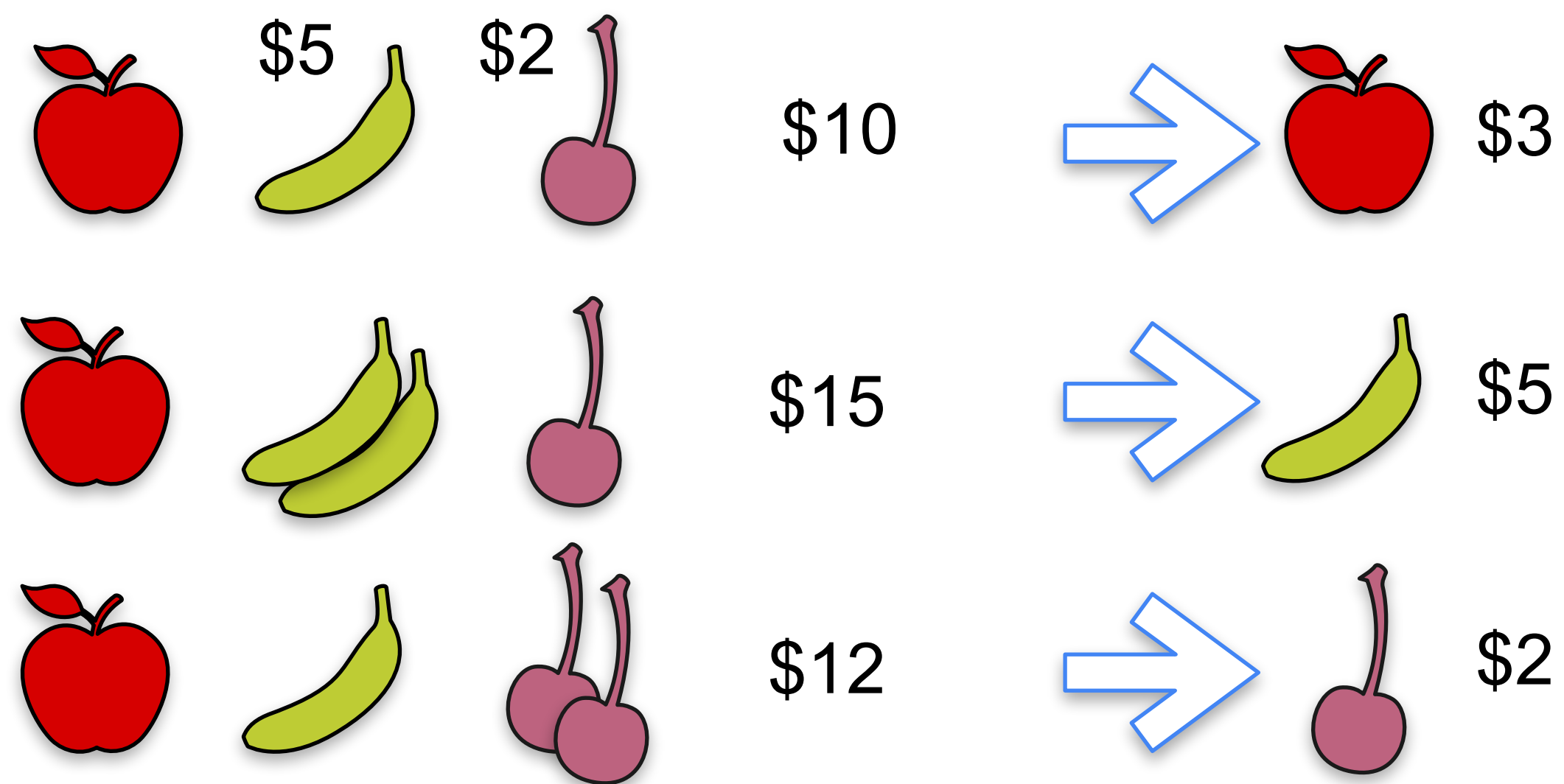
# Quiz: Systems of equations 2

**Problem 1:** You're trying to figure out the price of apples, bananas, and cherries at the store. You go three days in a row, and bring this information.

- **Day 1:** You bought an apple, a banana, and a cherry, and paid \$10.
- **Day 2:** You bought an apple, two bananas, and a cherry, and paid \$15.
- **Day 3:** You bought an apple, a banana, and two cherries, and paid \$12.

How much does each fruit cost?

# Solution: Systems of equations 2



## System of equations 1

$$\begin{aligned}a + b + c &= 10 \\a + 2b + c &= 15 \\a + b + 2c &= 12\end{aligned}$$

## Solution

$$\begin{aligned}a &= 3 \\b &= 5 \\c &= 2\end{aligned}$$

# Quiz: Systems of equations 3

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$20.

**Question:** How much does each fruit cost?

# Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$\text{🍏🍏} + \text{🍌🍌} = \$20$$

Same thing!!!



8      2

5      5

8.3    1.7

0      10

Infinitely many solutions!

# Quiz: Systems of equations 4

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$24.

**Question:** How much does each fruit cost?

# Solution: Systems of equations 4

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍏} + \text{🍌} = \$10 \quad \Rightarrow \quad \text{🍏🍏} + \text{🍌🍌} = \$20$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

$$\text{🍏🍏} + \text{🍌🍌} = \$24$$

Contradiction!

No solutions!

# Systems of equations

## System 1

$$a + b = 10$$

$$a + 2b = 12$$

**Unique solution:**

$$a = 8$$

$$b = 2$$

**Complete**

**Non-singular**

## System 2

$$a + b = 10$$

$$2a + 2b = 20$$

**Infinite solutions**

$$\begin{array}{l} a = 8, 7, 6, \dots \\ b = 2, 3, 4 \end{array}$$

**Redundant**

**Singular**

## System 3

$$a + b = 10$$

$$2a + 2b = 24$$

**No solution**

**Contradictory**

**Singular**

# Quiz: More systems of equations

## System 1

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$



# Solutions: More systems of equations

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

## Infinitely many sols.

$$\begin{aligned}c &= 5 \\a + b &= 5 \\(0, 5, 5), (1, 4, 5), (2, 3, 5), \dots\end{aligned}$$

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

## No solutions

$$\begin{aligned}\text{From 1st and 2nd:} \\c &= 5 \\\text{From 2nd and 3rd:} \\c &= 3\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

## Infinitely many solutions

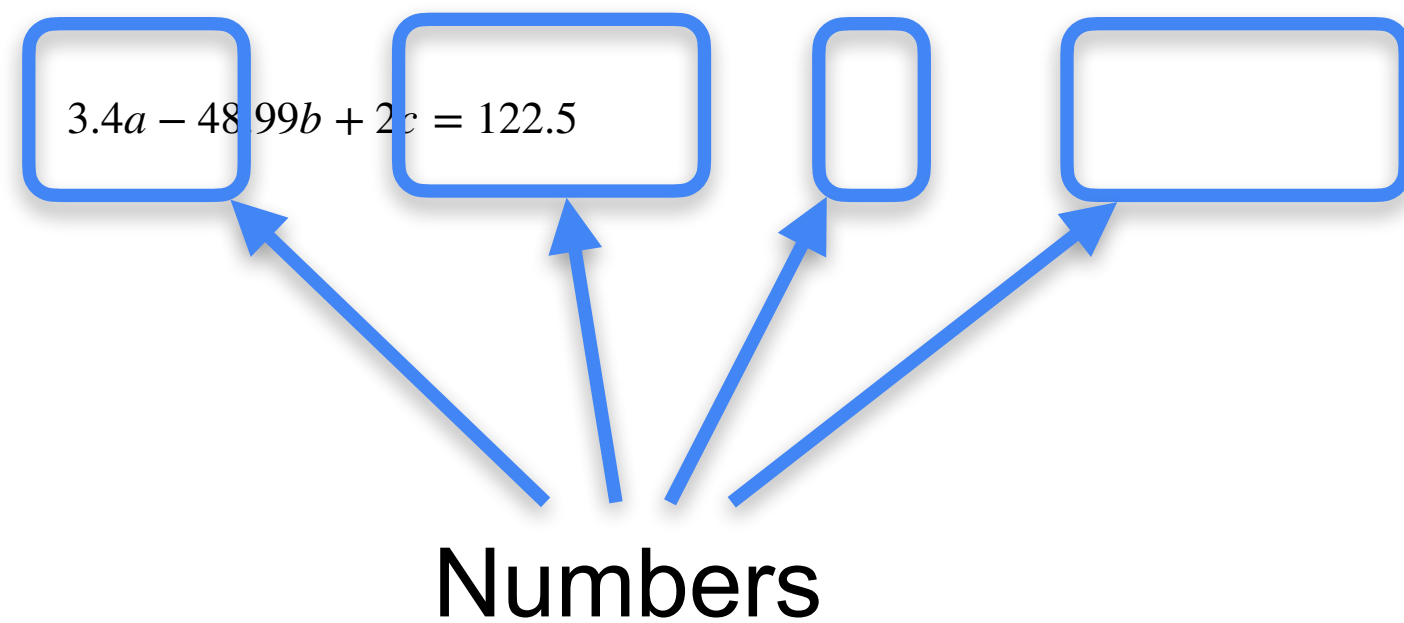
$$\begin{aligned}\text{Any 3 numbers that add} \\ \text{to 10 work.} \\(0, 0, 10), (2, 7, 1), \dots\end{aligned}$$

# What is a linear equation?

## Linear

$$a + b = 10$$

$$2a + 3b = 15$$



## Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

$$2^a - 3^b = 0$$

$$ab^2 + \frac{b}{a} - \frac{3}{b} - \log(c) = 4^a$$



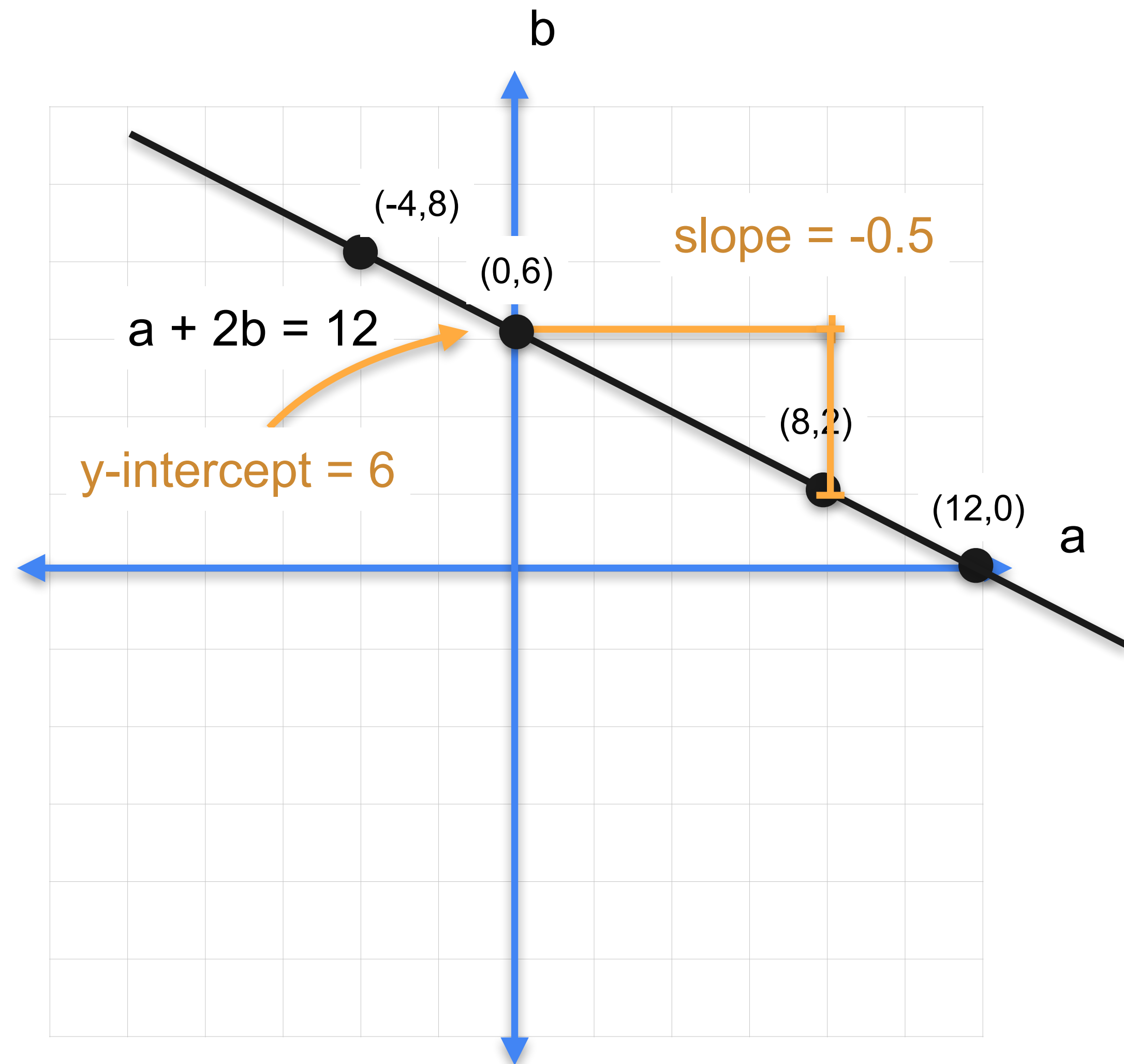
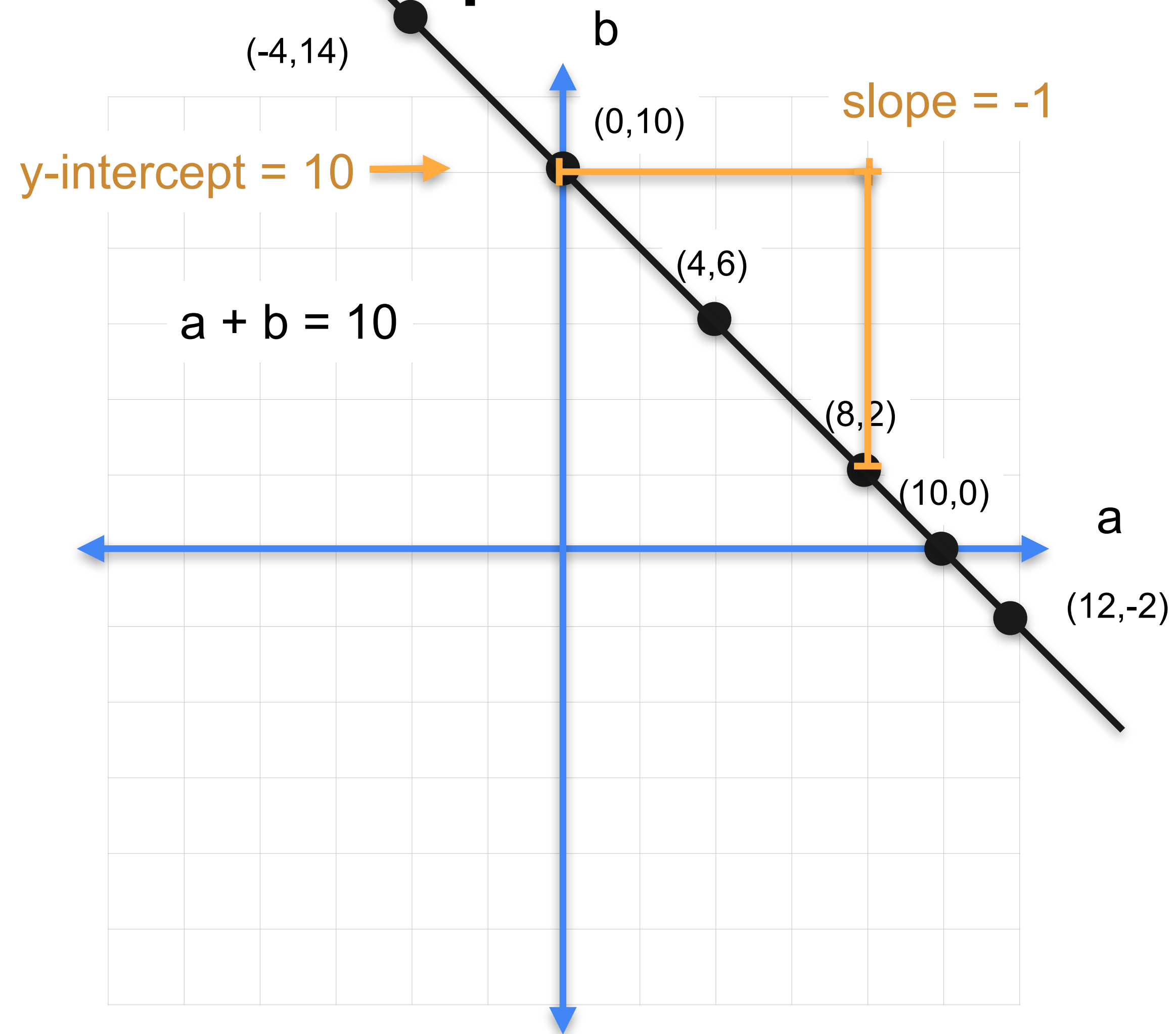
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# System of Linear Equations

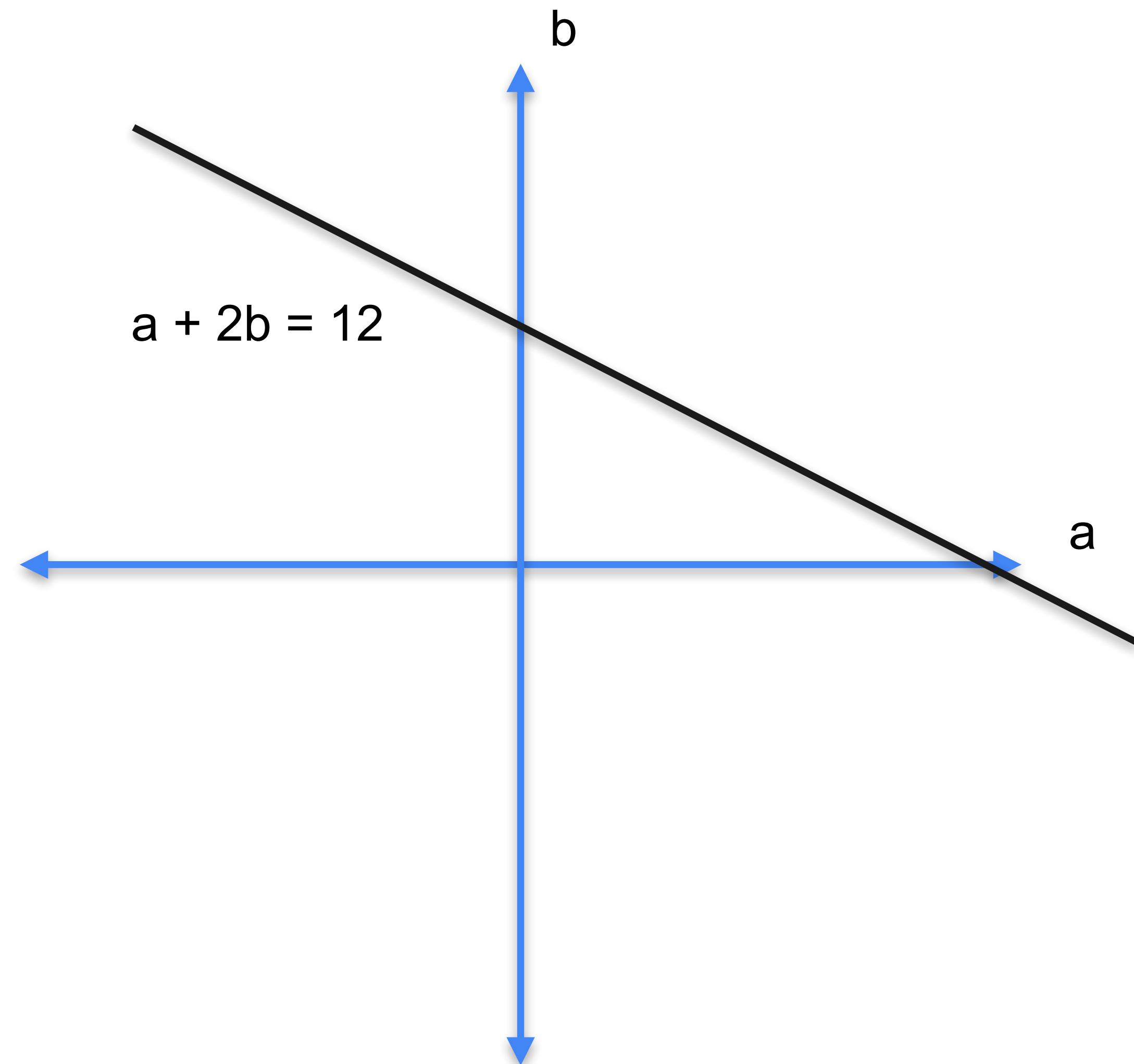
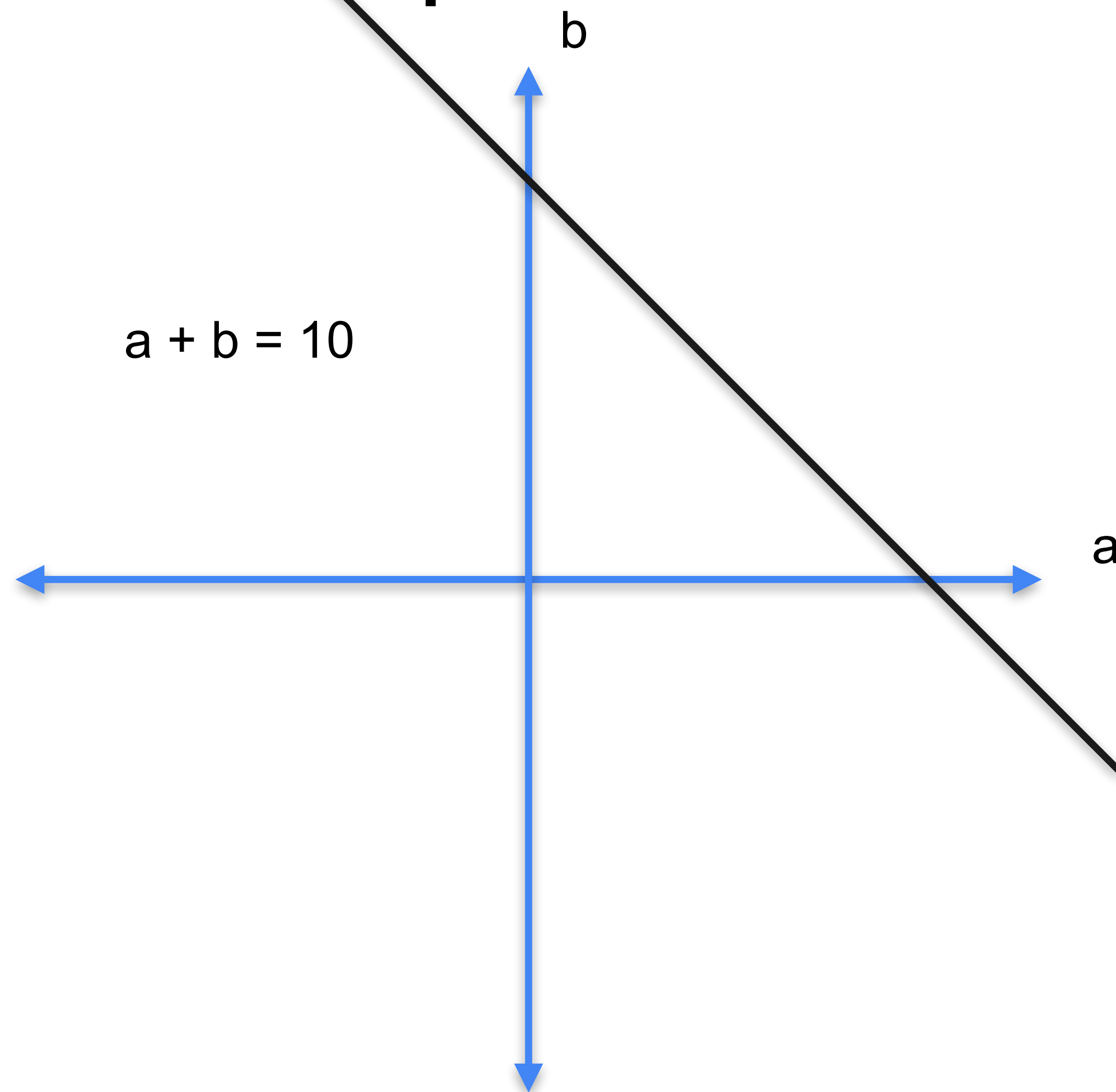
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**System of equations as lines  
and planes**

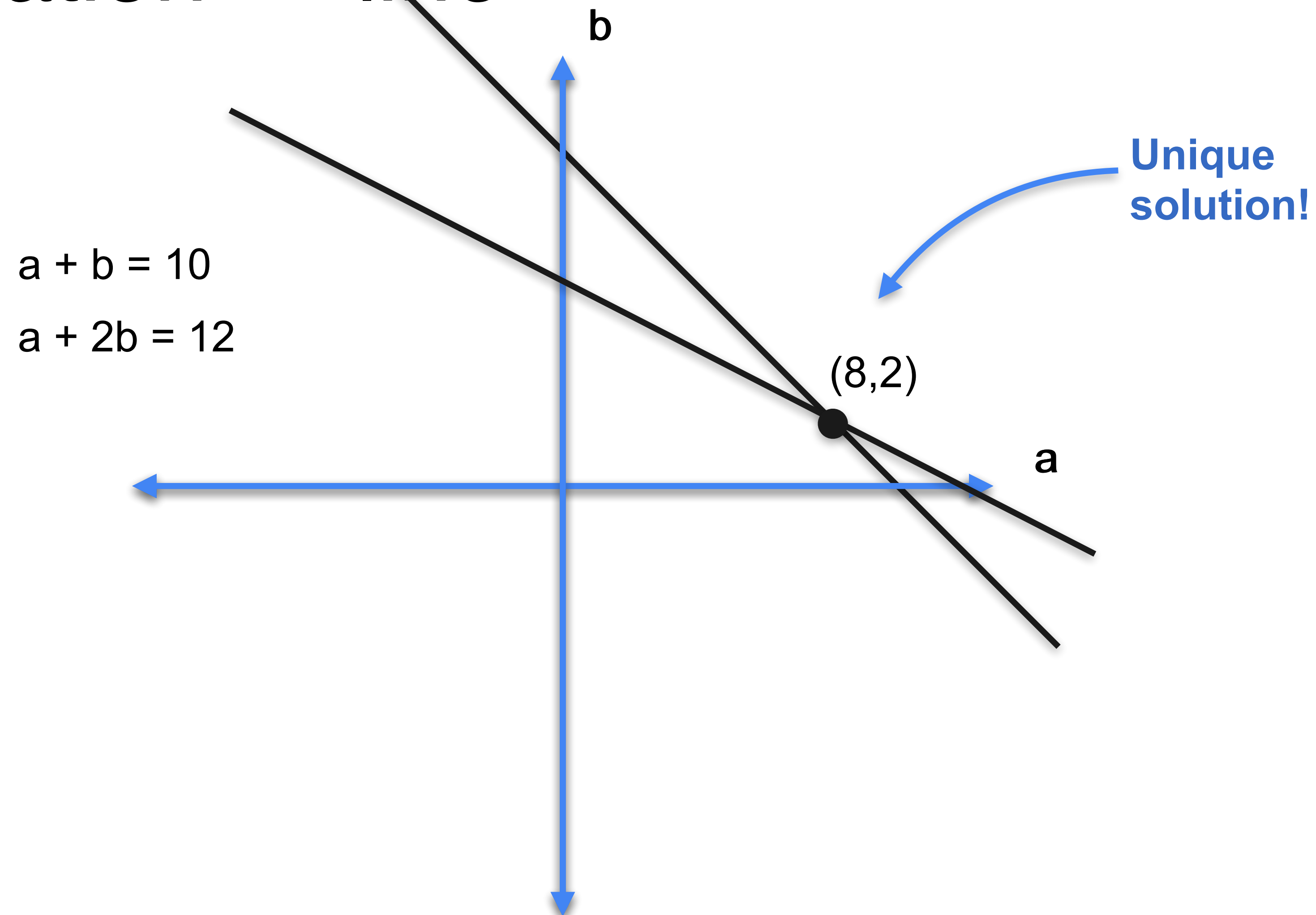
# Linear equation $\rightarrow$ line



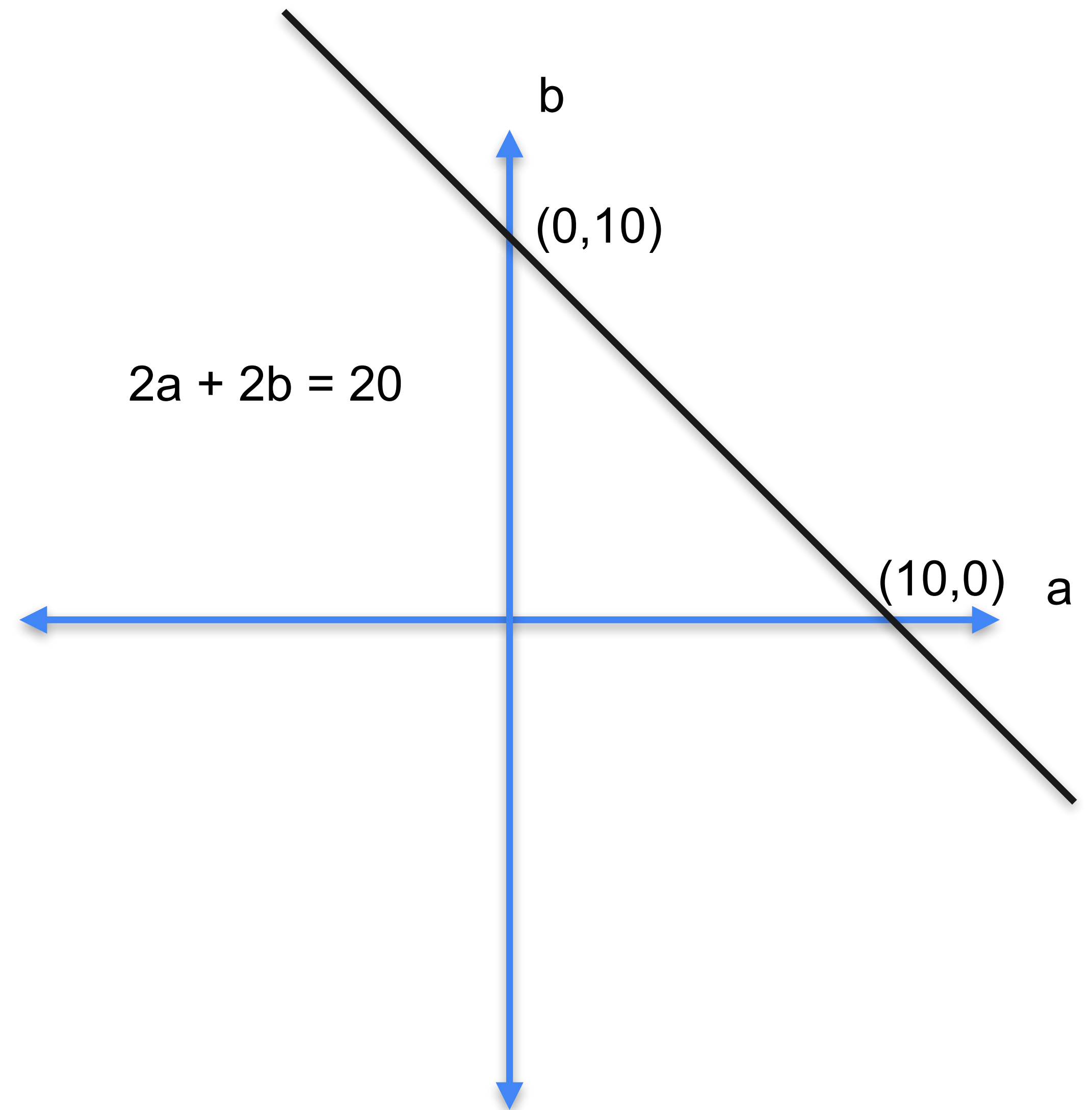
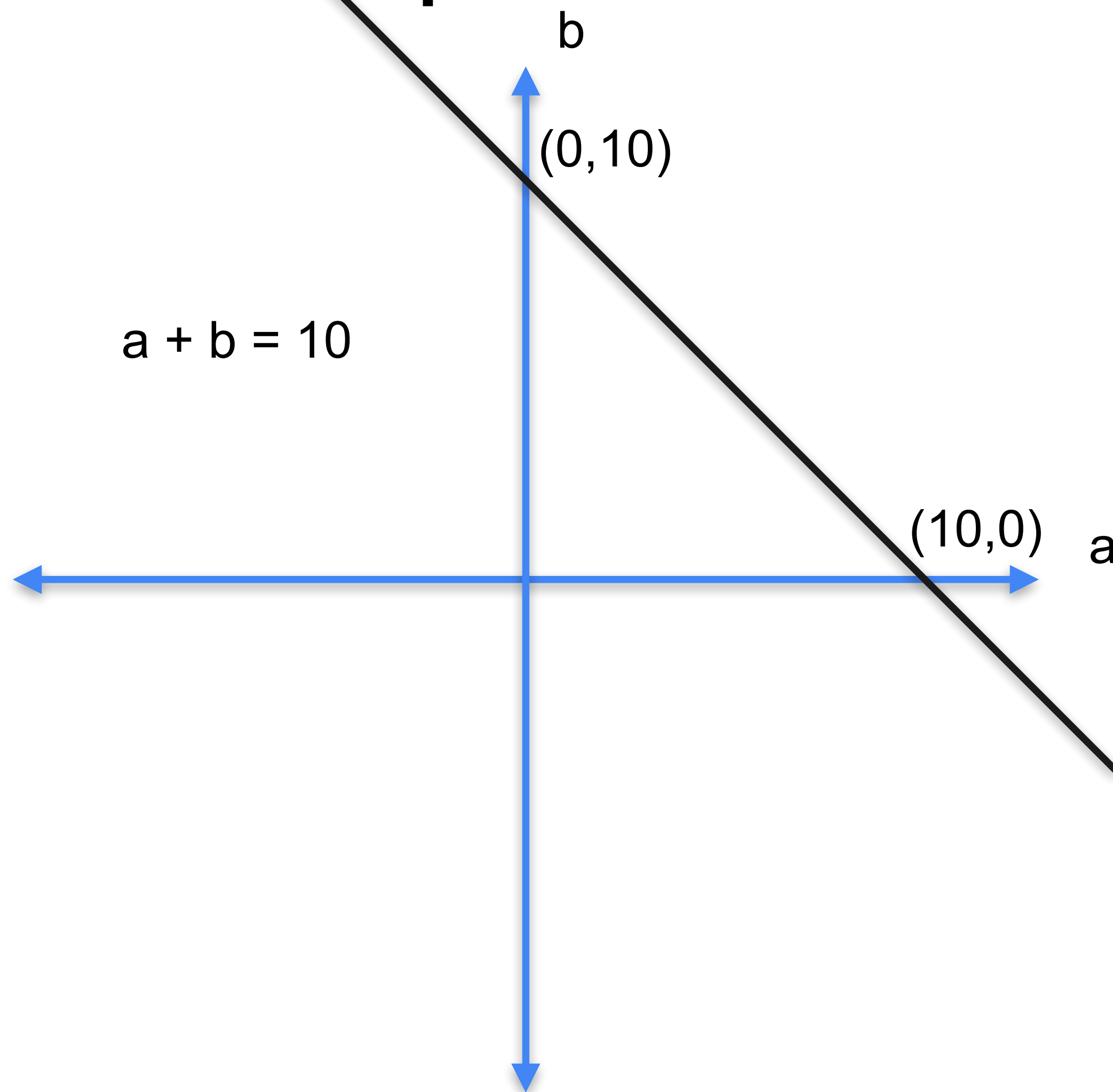
# Linear equation $\rightarrow$ line



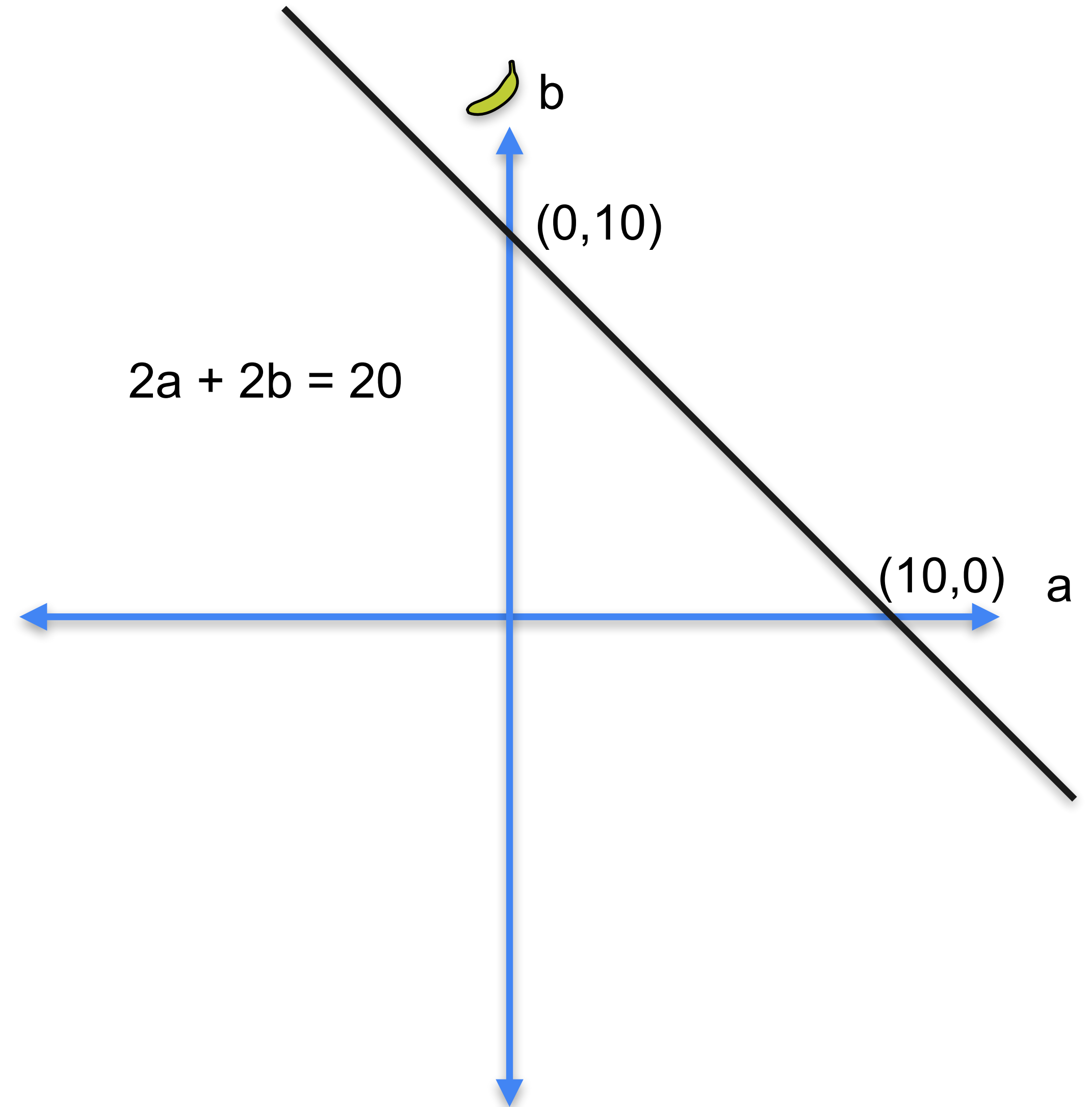
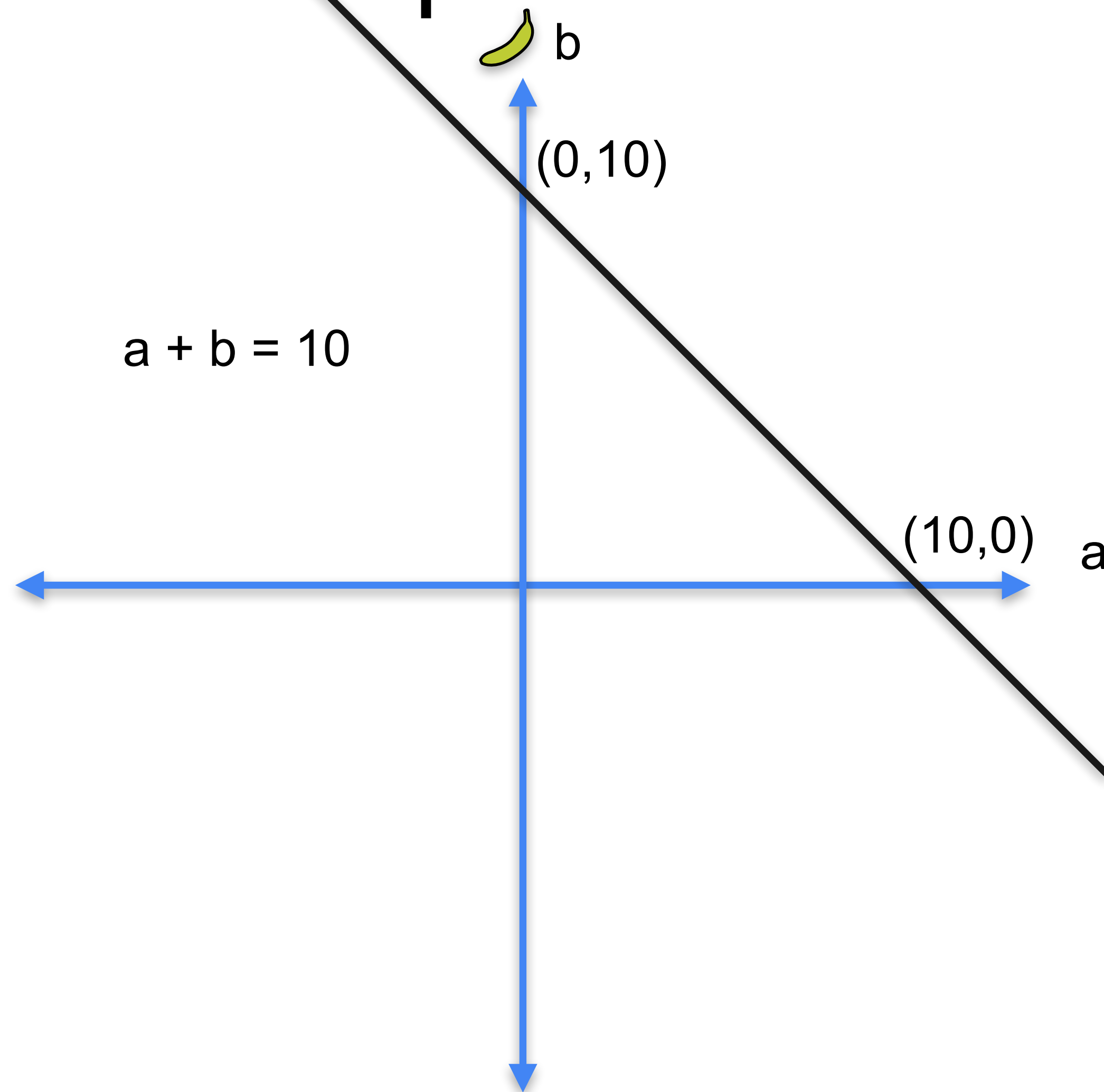
# Linear equation $\rightarrow$ line



# Linear equation $\rightarrow$ line



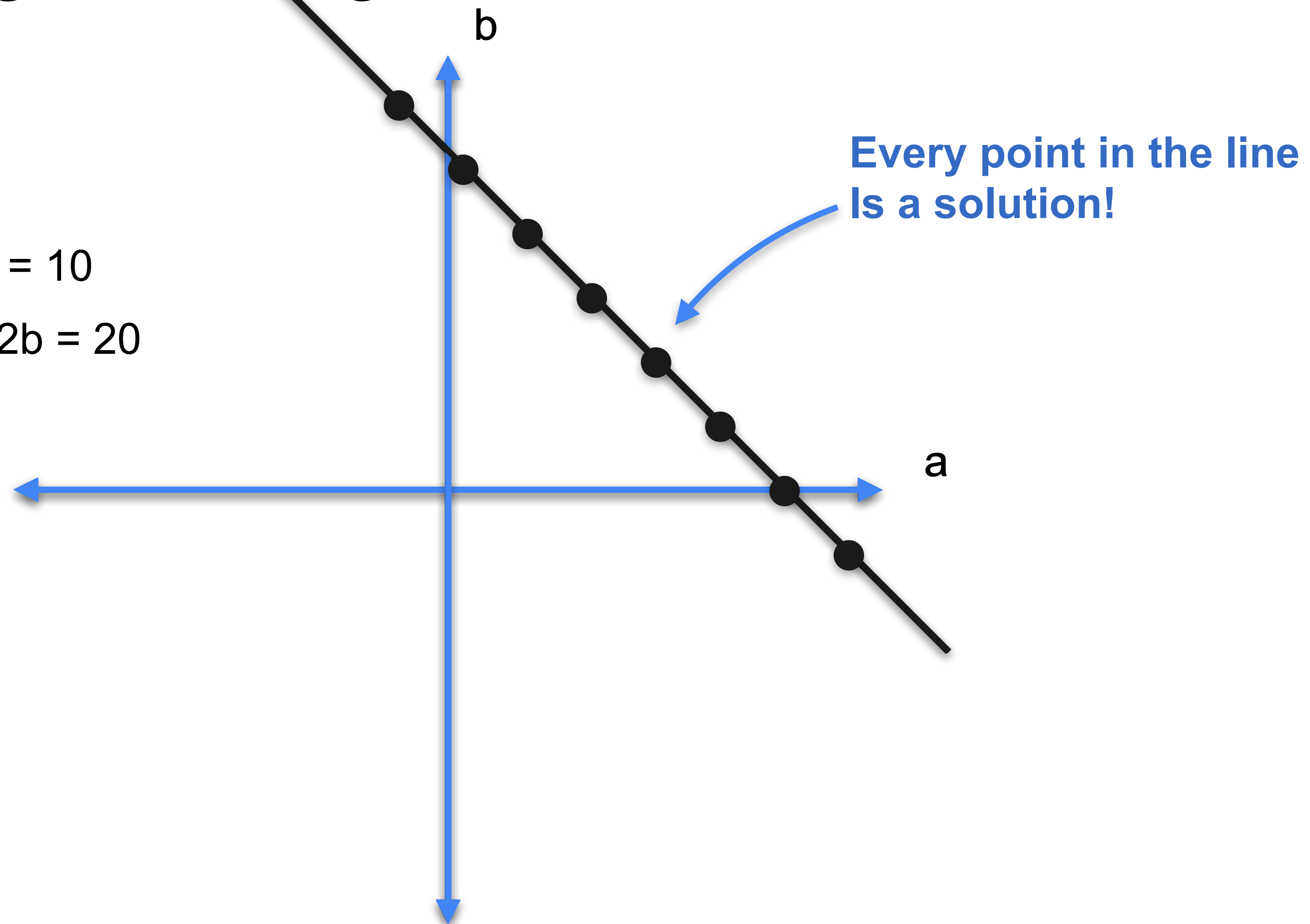
# Linear equation $\rightarrow$ line



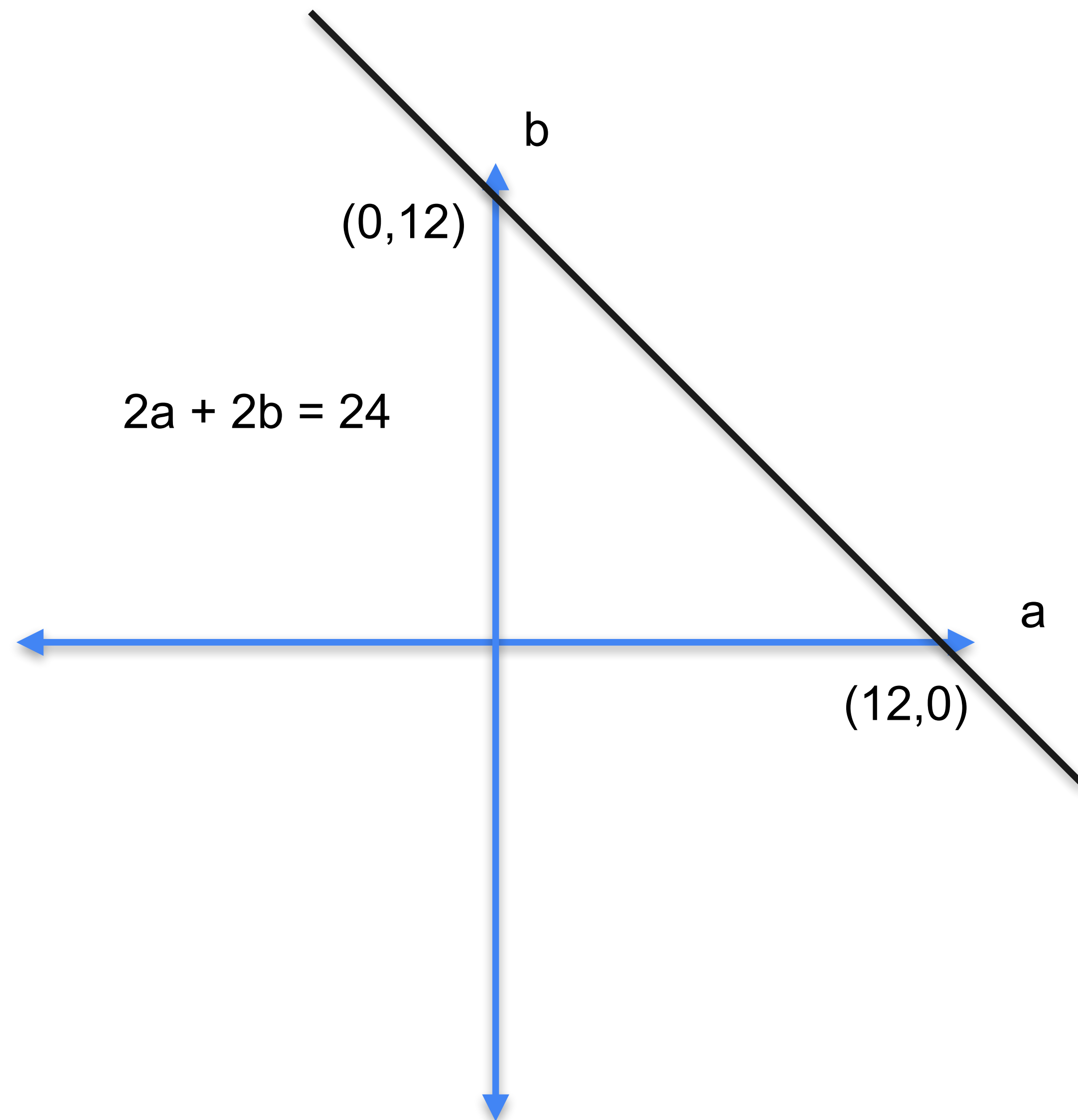
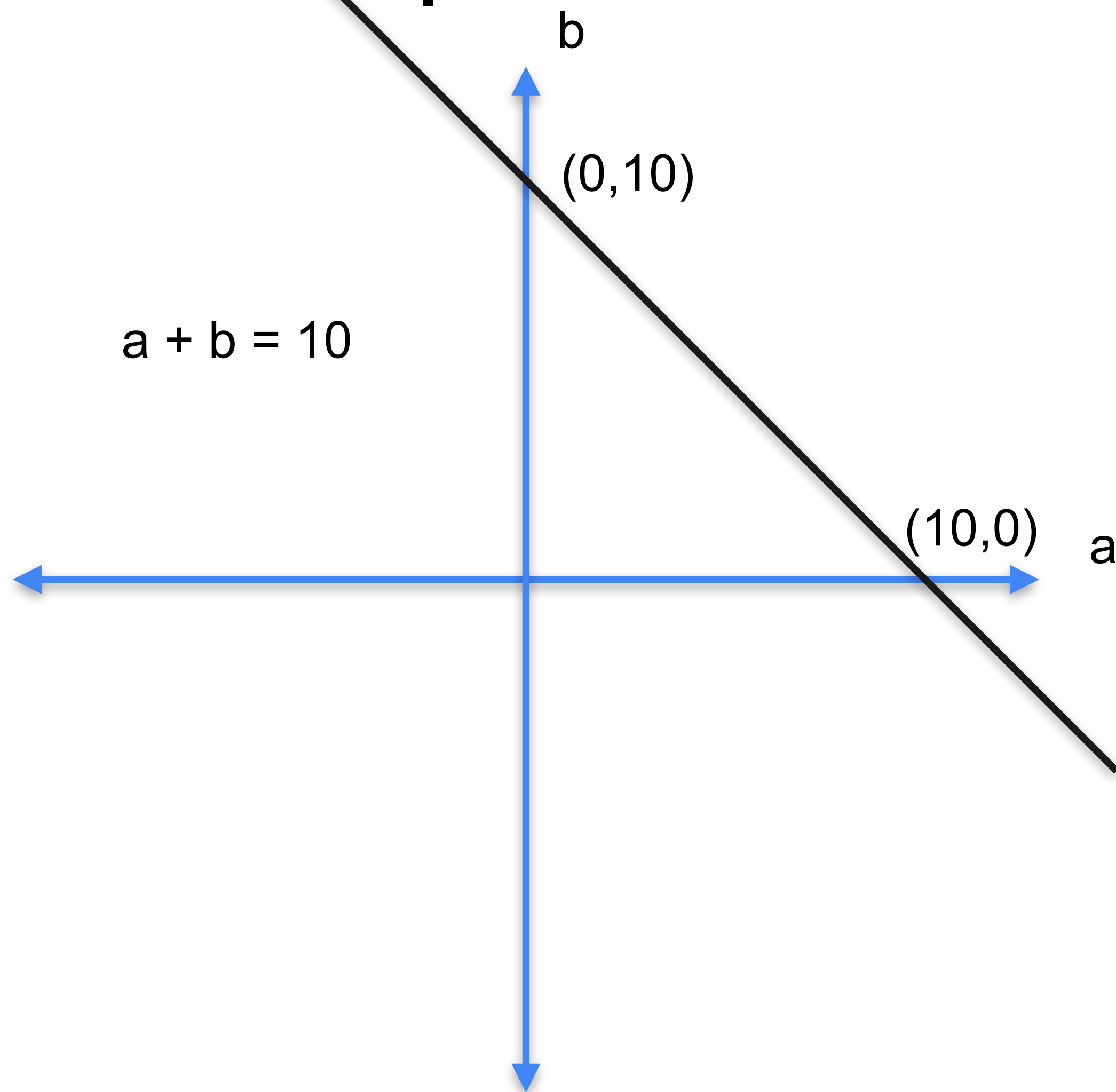


# Linear equation → line

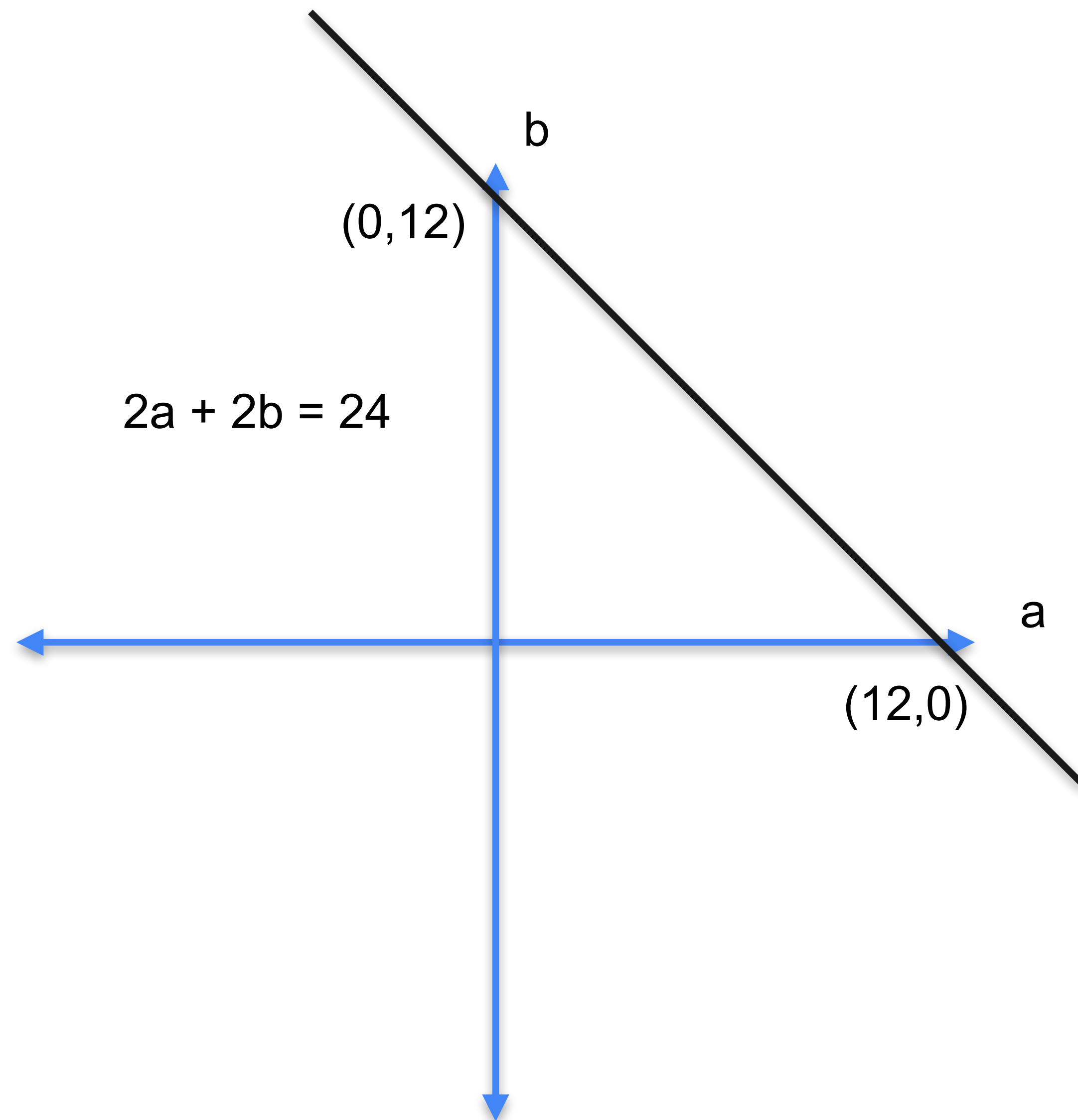
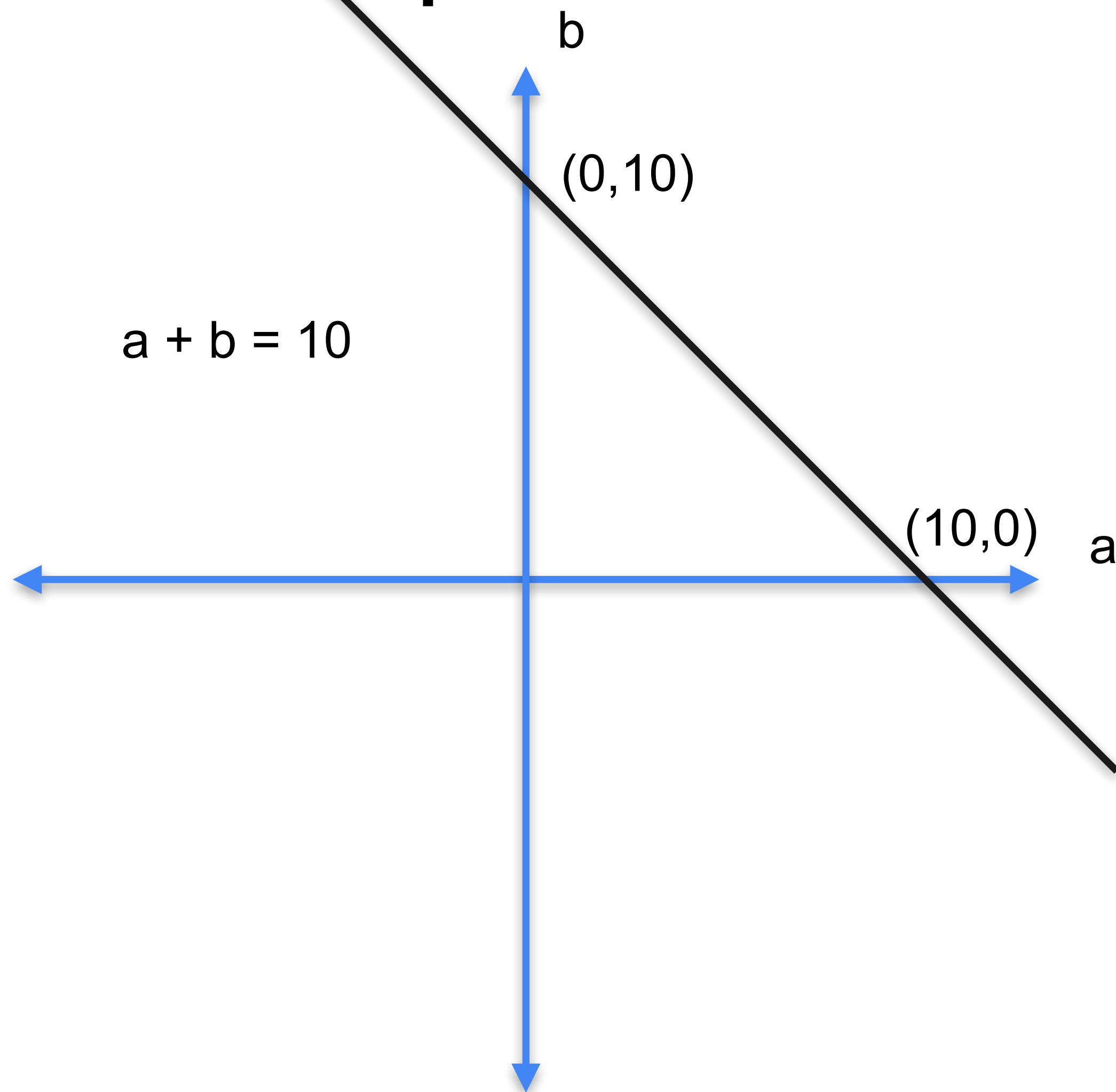
$$a + b = 10$$
$$2a + 2b = 20$$



# Linear equation $\rightarrow$ line



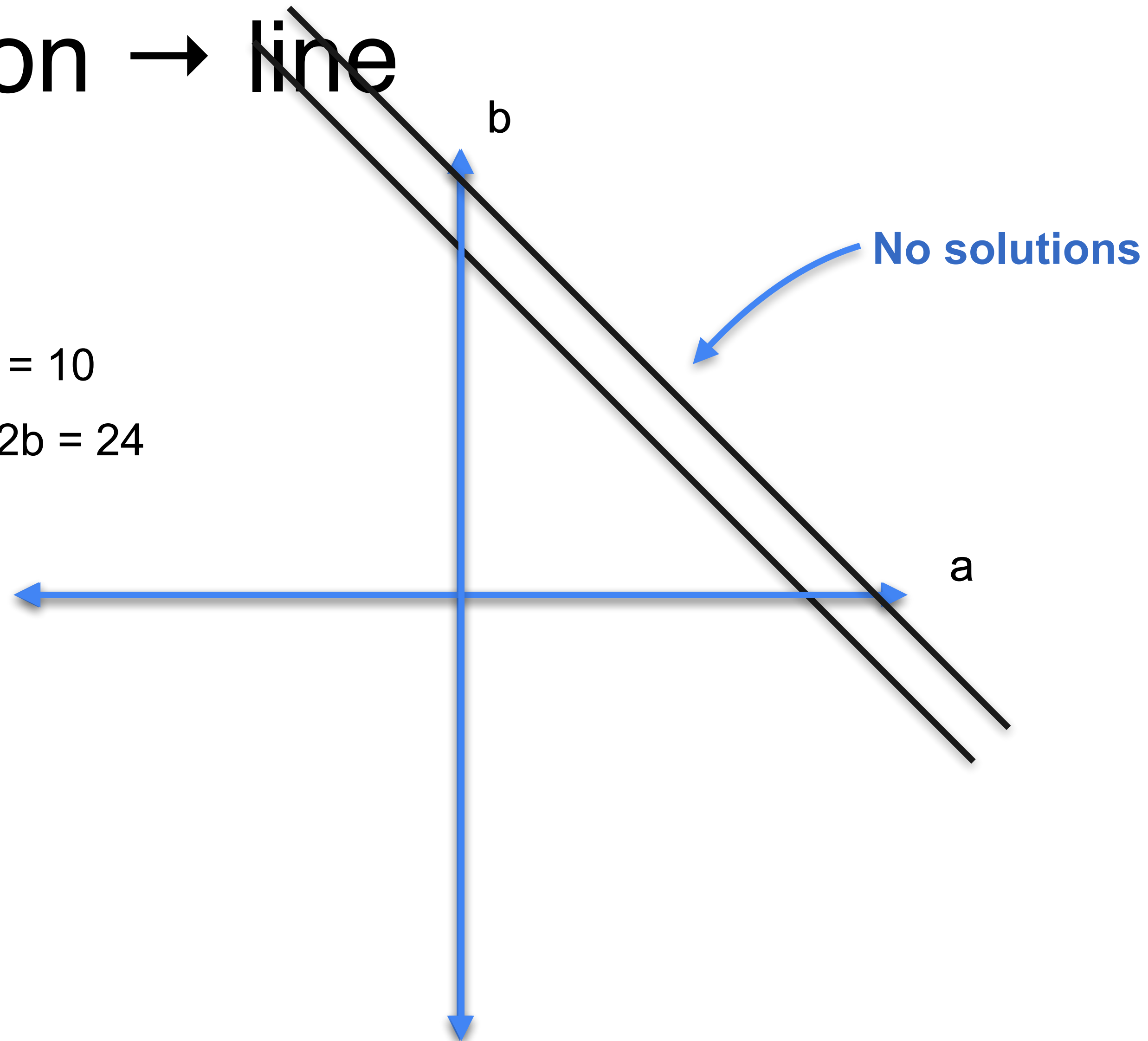
# Linear equation $\rightarrow$ line



# Linear equation → line

$$a + b = 10$$

$$2a + 2b = 24$$

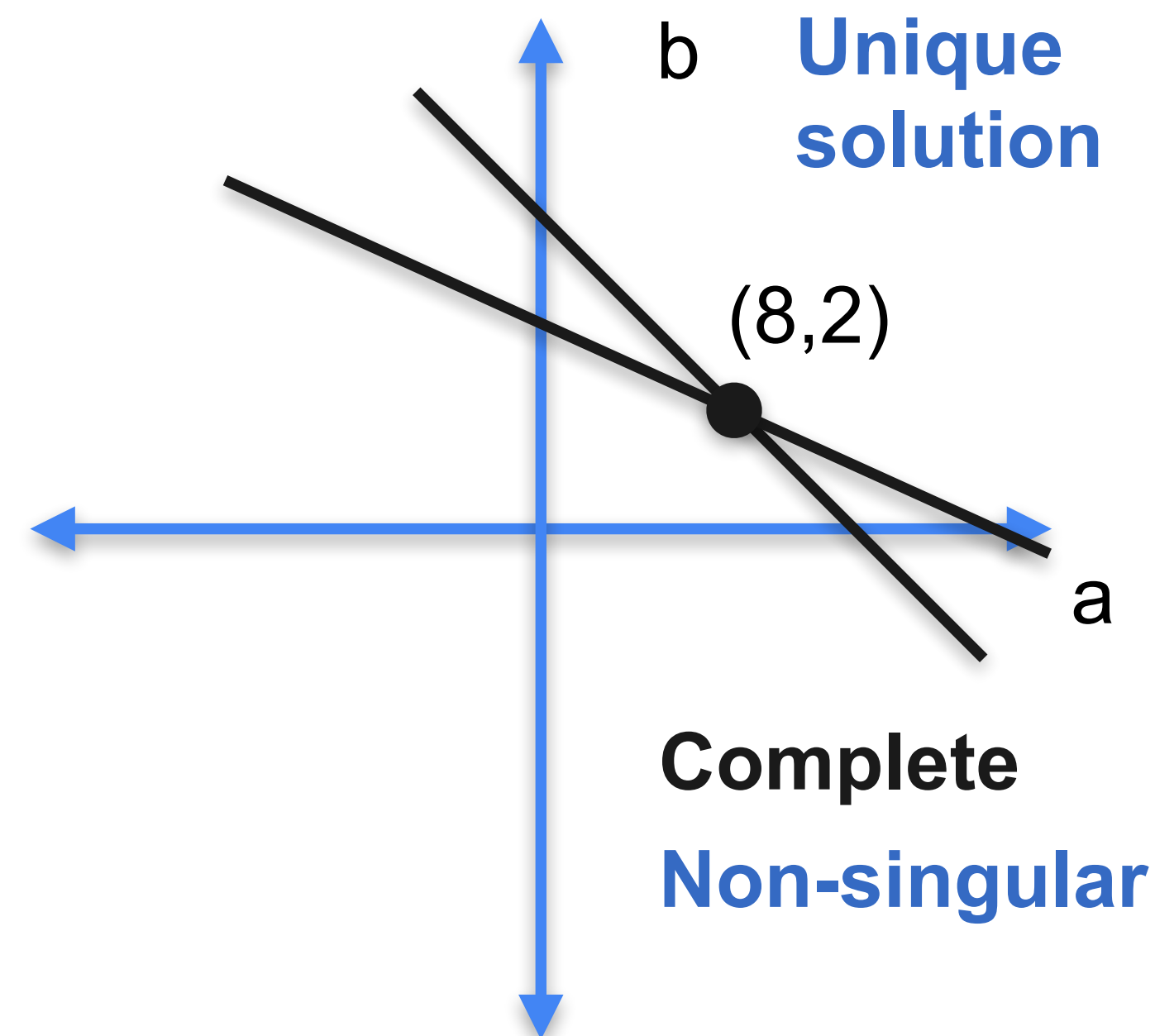


# Systems of equations as lines

System 1

$$a + b = 10$$

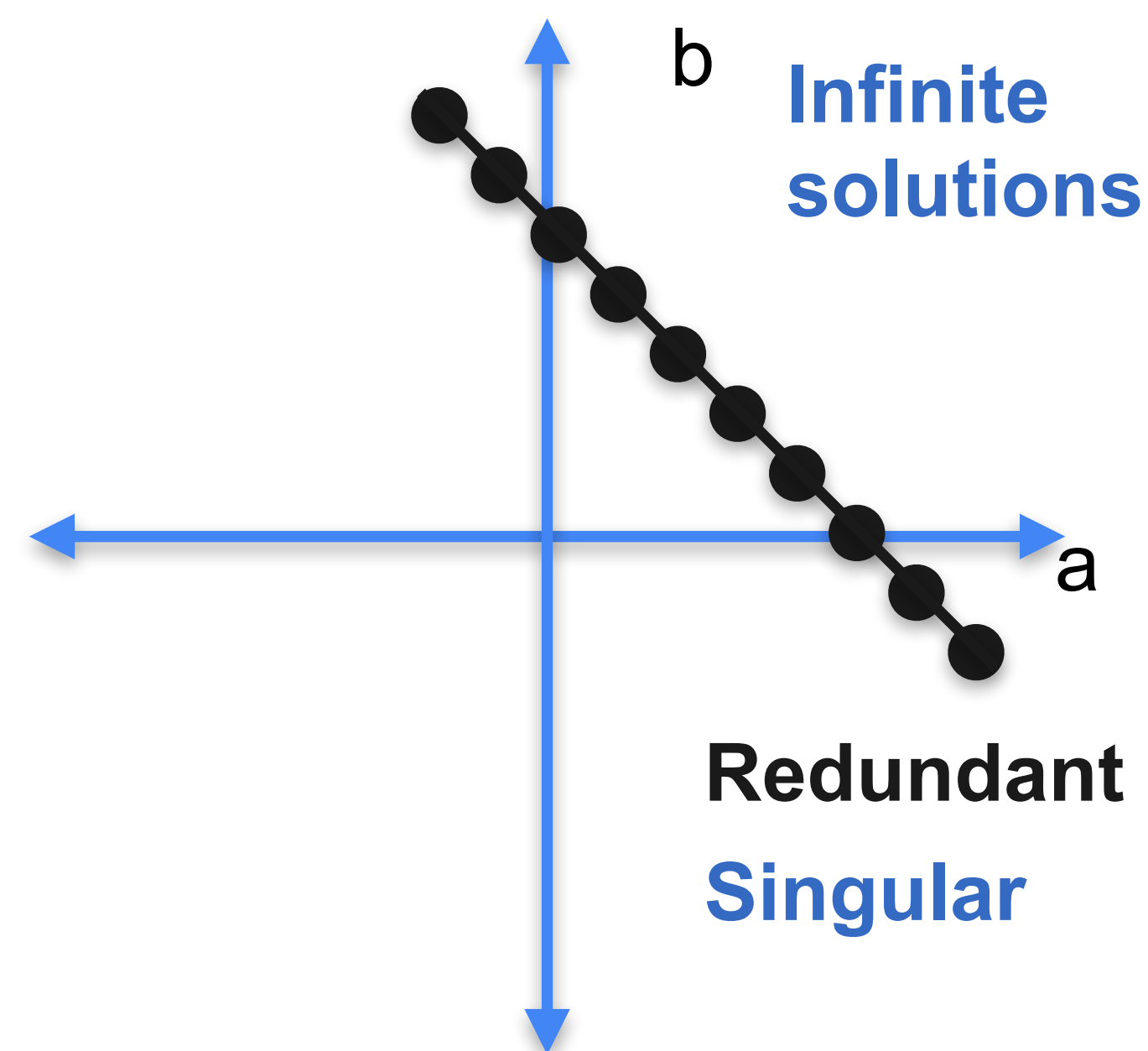
$$a + 2b = 12$$



System 2

$$a + b = 10$$

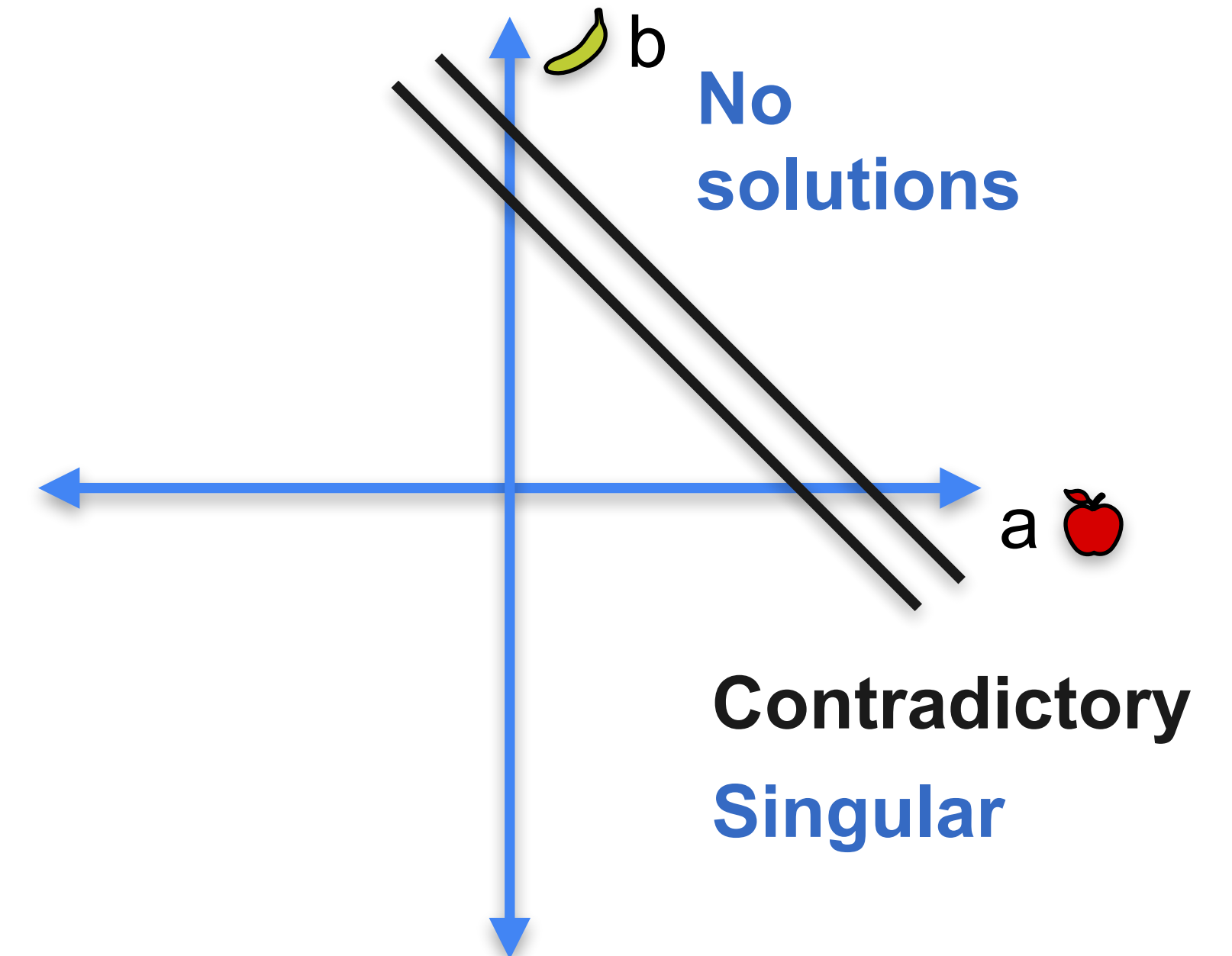
$$2a + 2b = 20$$



System 3

$$a + b = 10$$

$$2a + 2b = 24$$



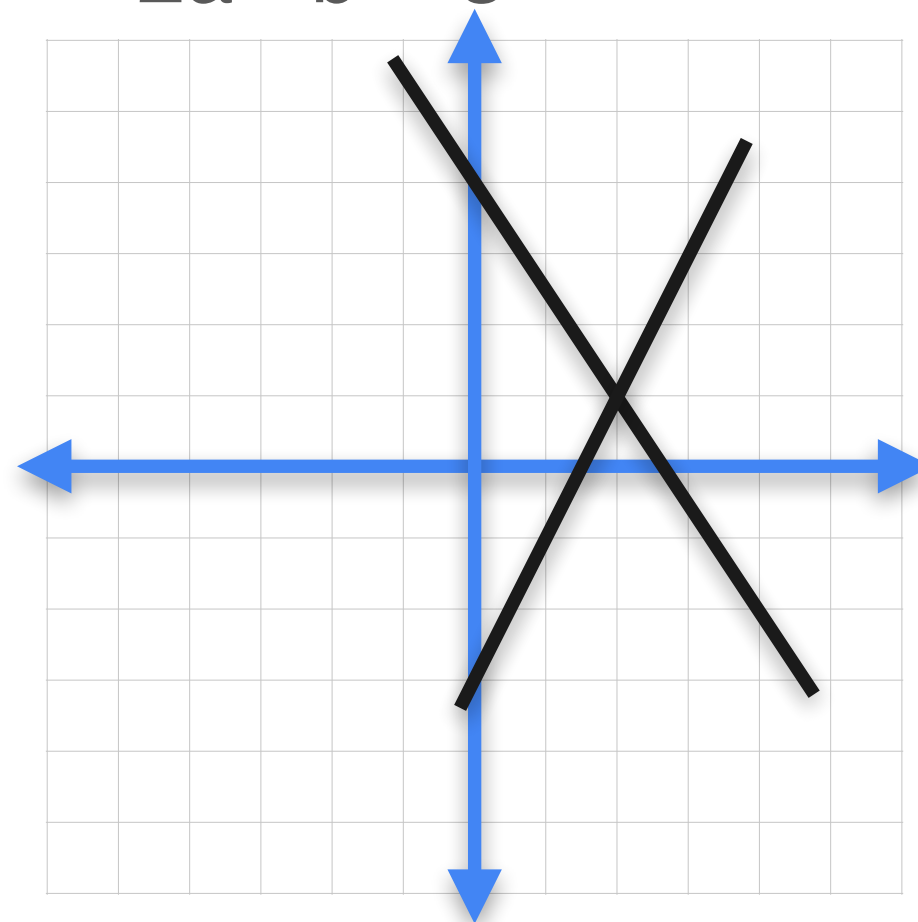
# Quiz

## Problem 1

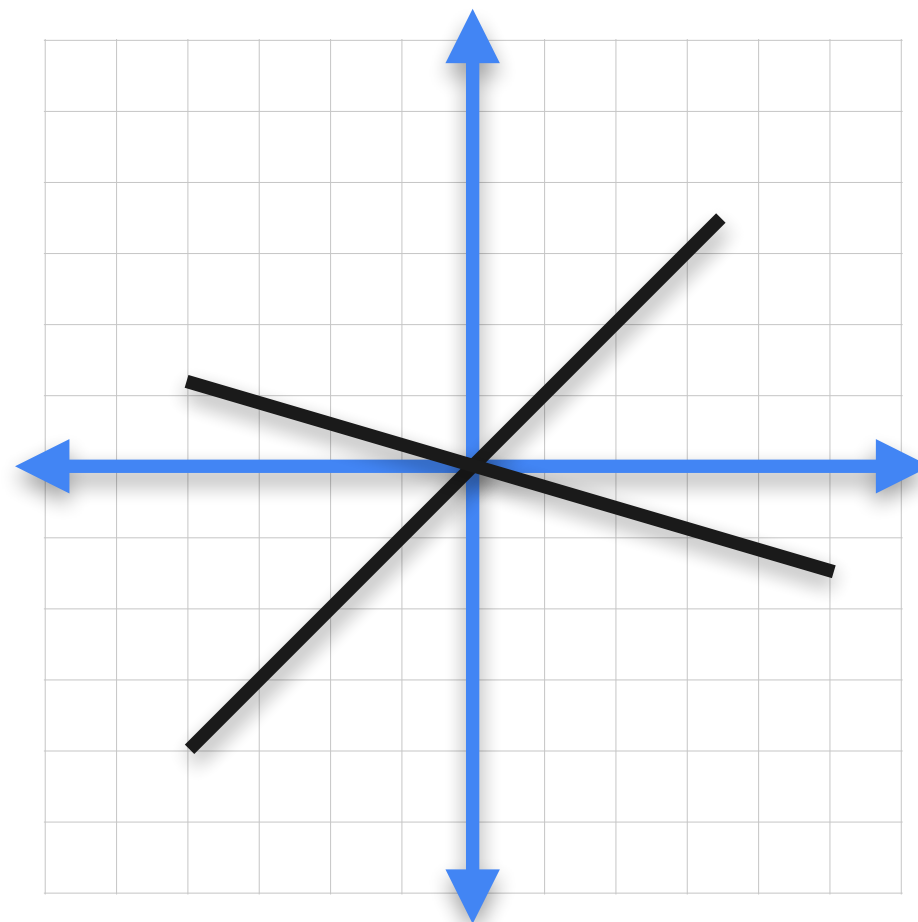
Which of the following plots corresponds to the system of equations:

- $3a + 2b = 8$
- $2a - b = 3$

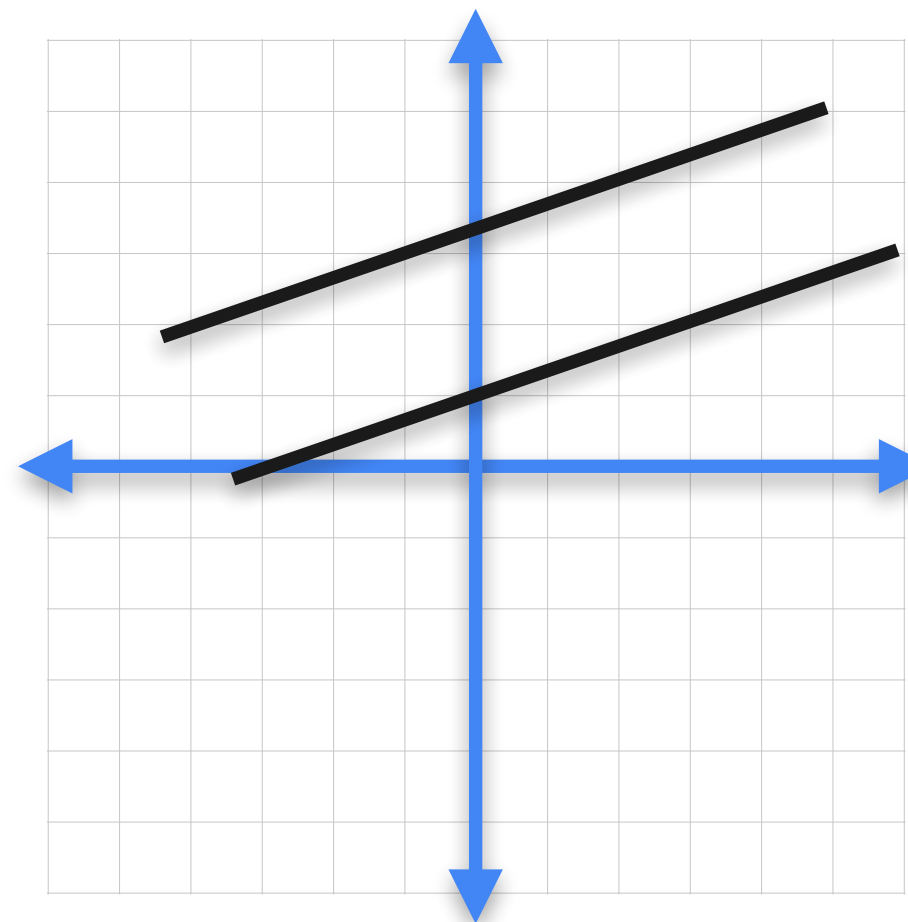
a)



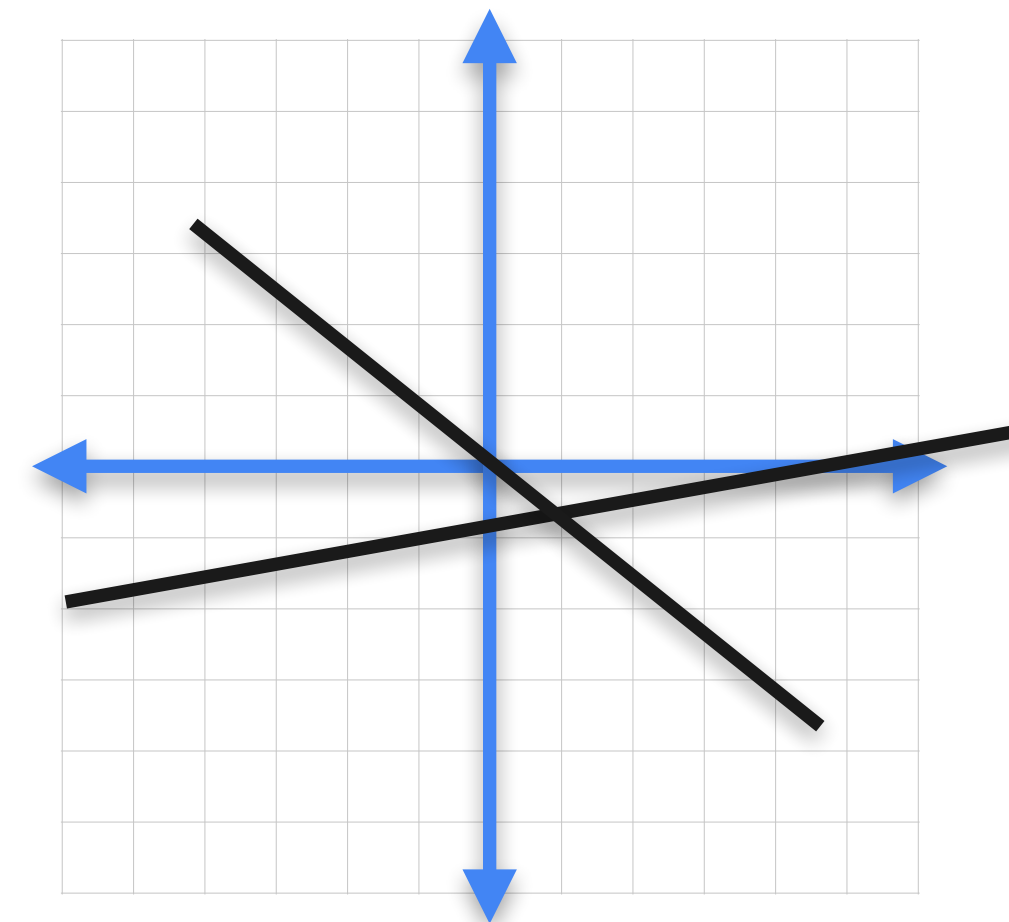
b)



c)



d)

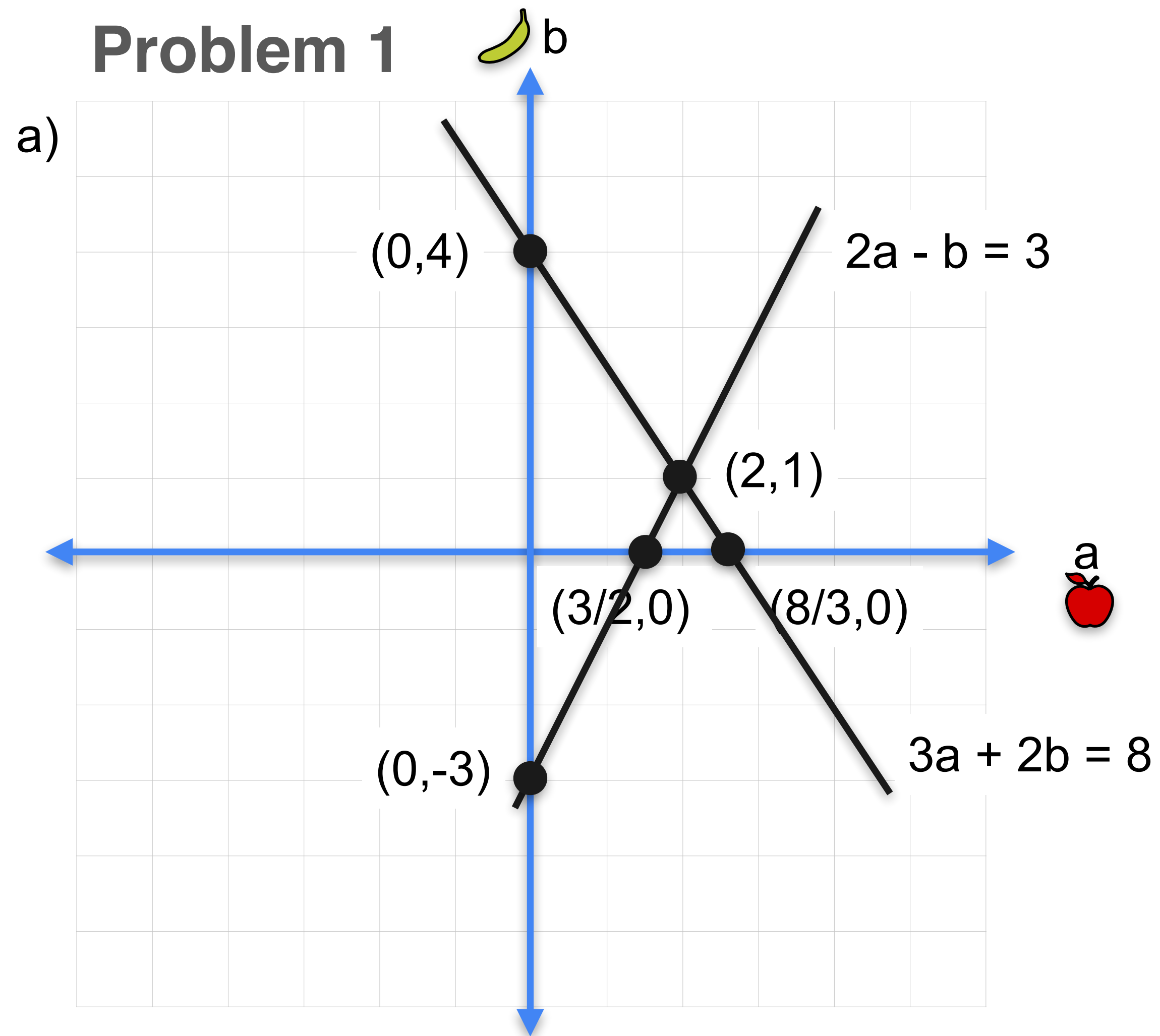


## Problem 2

Is this system singular or non-singular?

# Solution

## Problem 1



## Problem 2

Since the lines cross at a unique point, the system is non-singular.

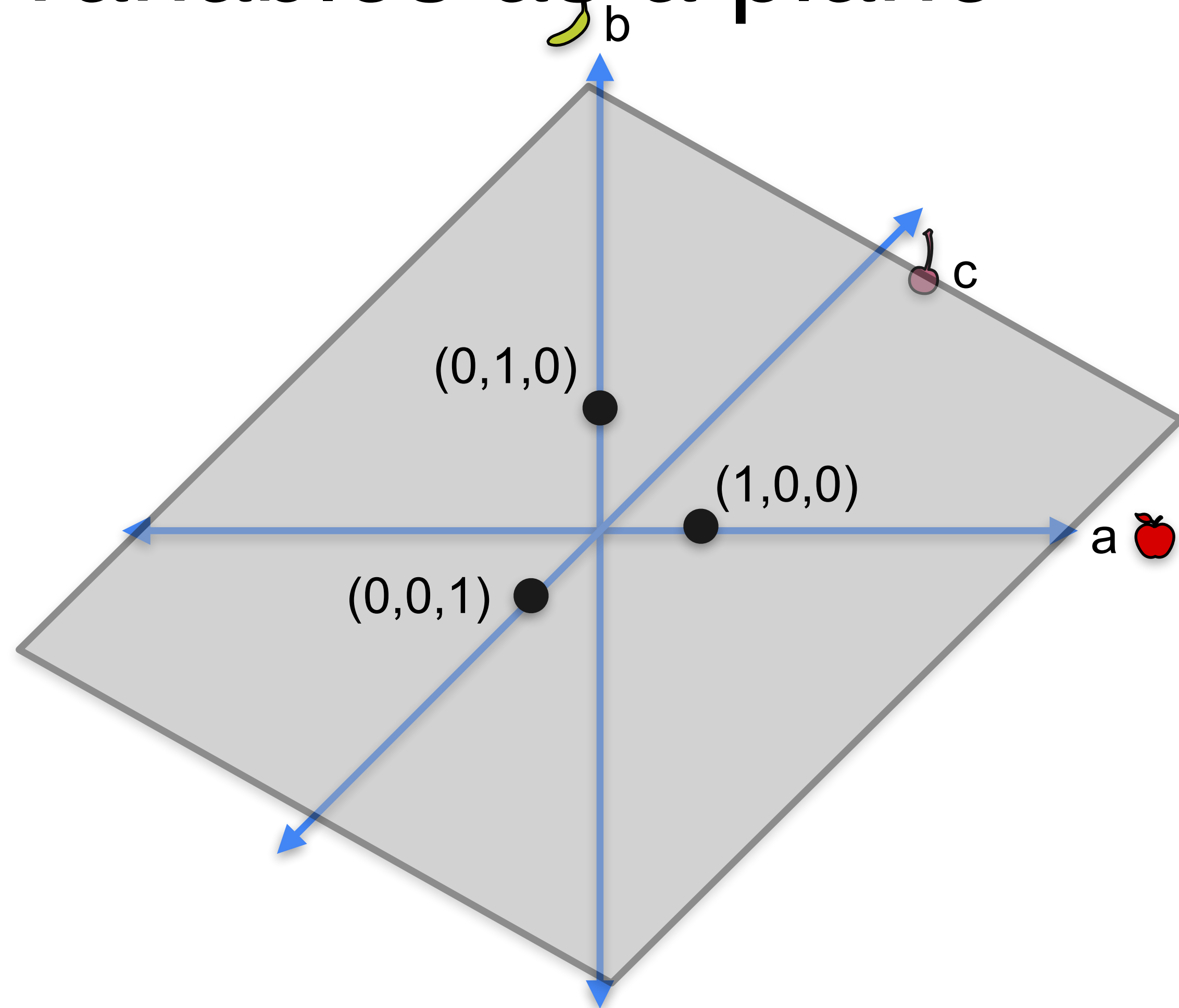
# Linear equation in 3 variables as a plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$

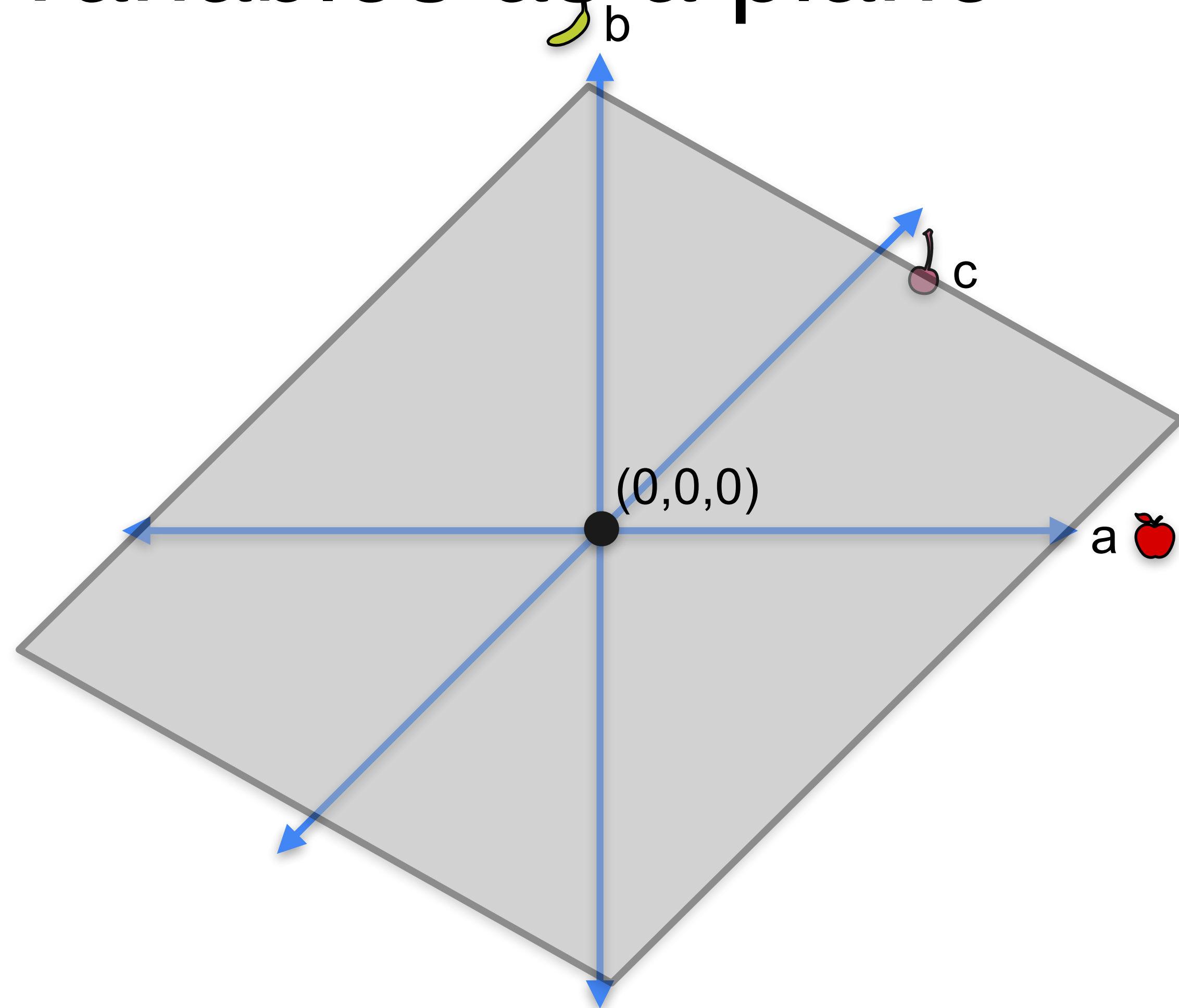




# Linear equation in 3 variables as a plane

$$3a - 5b + 2c = 0$$

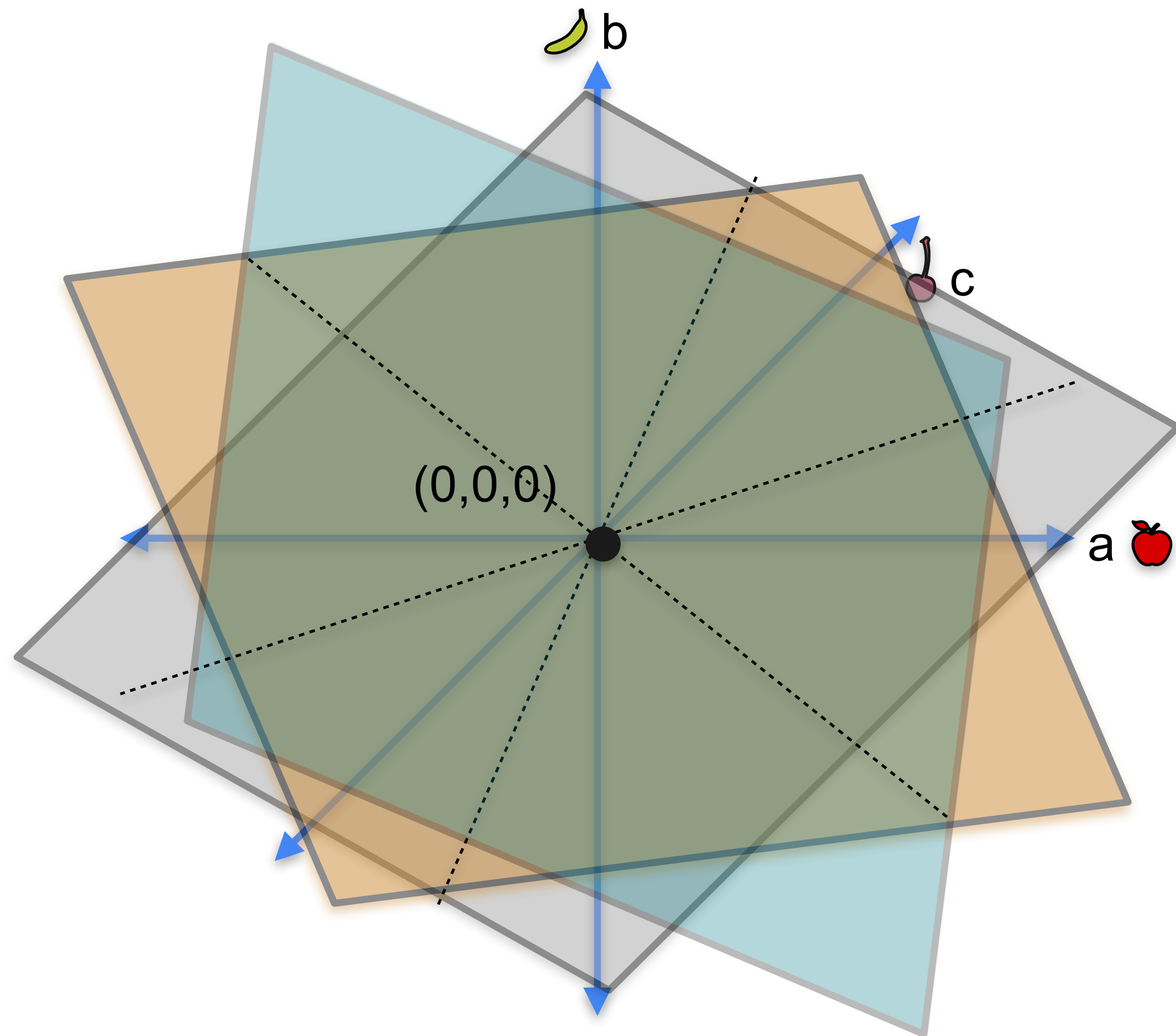
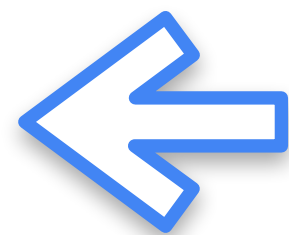
$$3(0) + 5(0) + 2(0) = 0$$



# System 1

## System 1

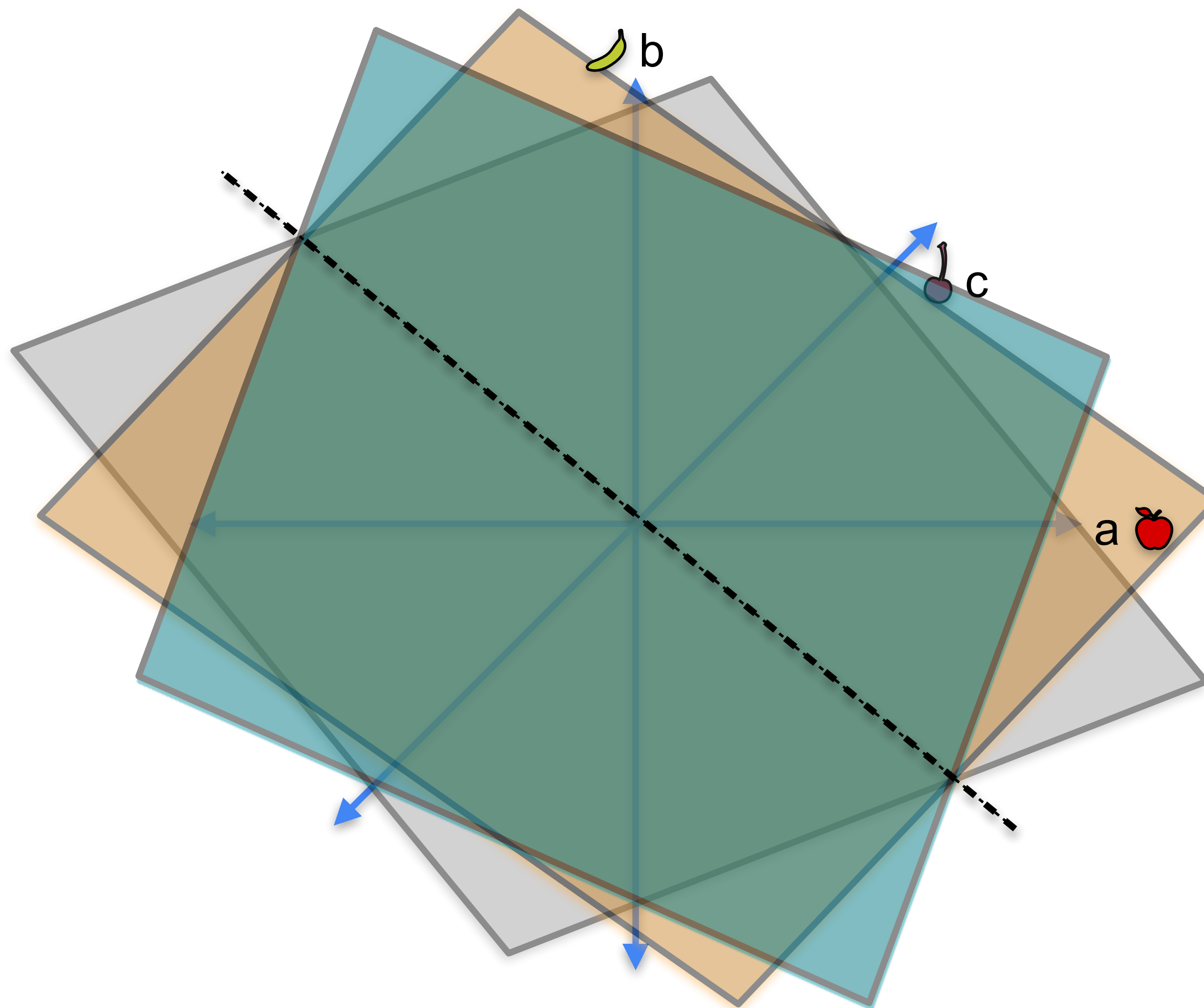
- $a + b + c = 0$
- $a + 2b + c = 0$
- $a + b + 2c = 0$



# System 2

## System 2

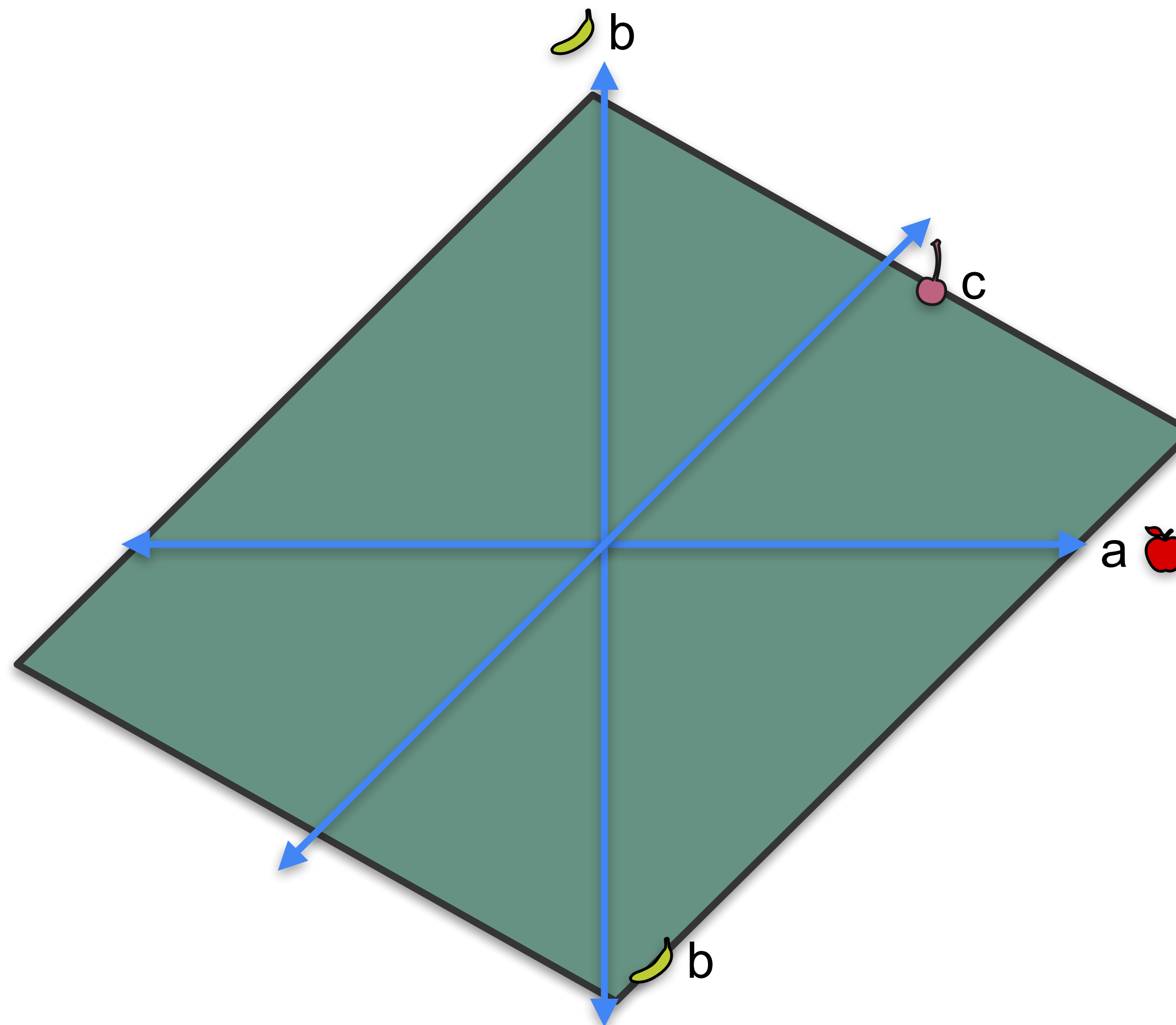
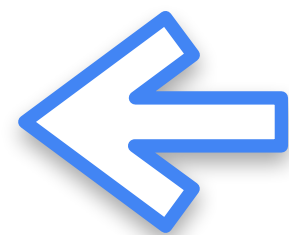
- $a + b + c = 0$
- $a + b + 2c = 0$
- $a + b + 3c = 0$



# System 3

## System 3

- $a + b + c = 0$
- $2a + 2b + 2c = 0$
- $3a + 3b + 3c = 0$





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# System of Linear Equations

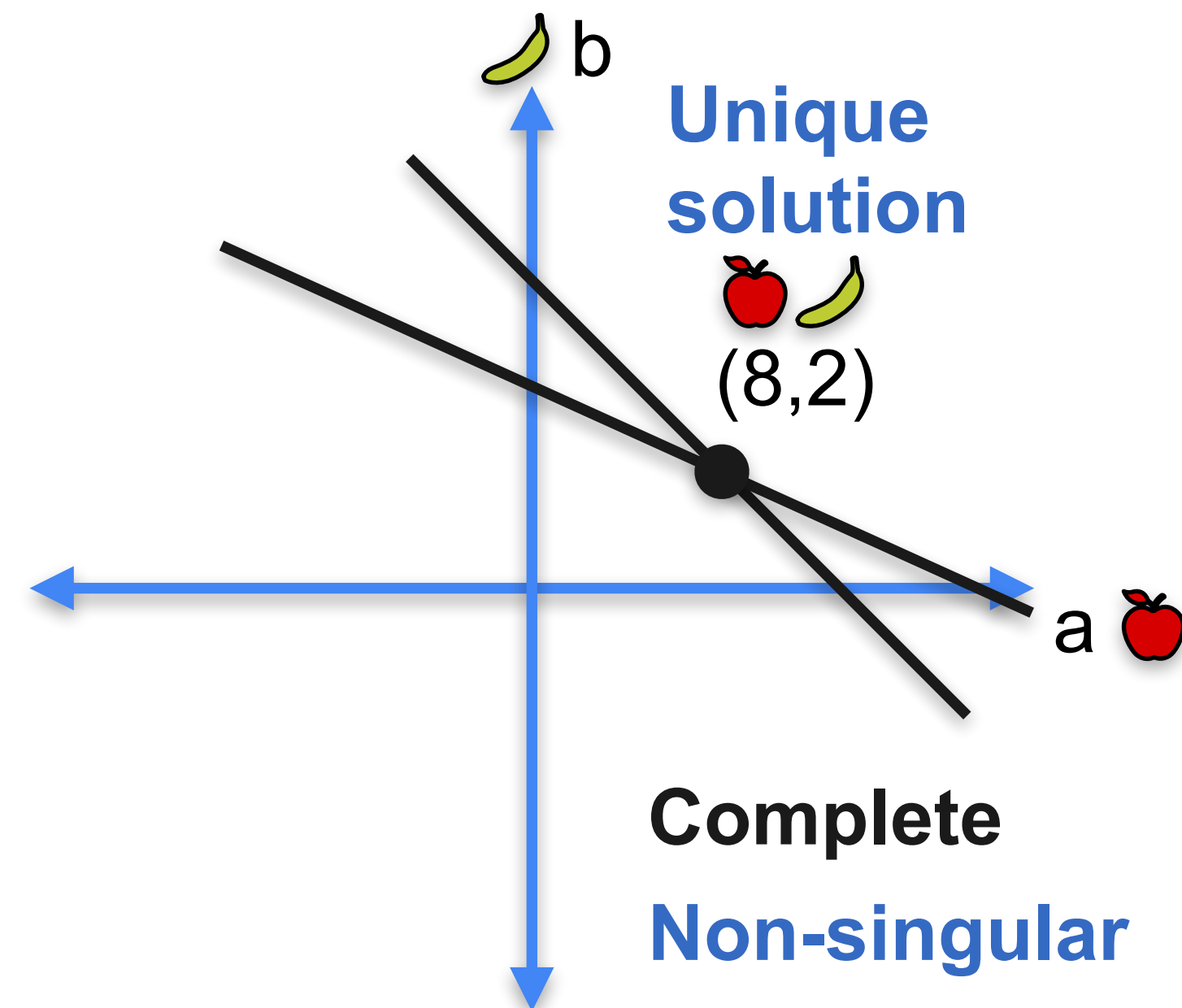
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**A geometric notion of  
singularity**

# Systems of equations as lines

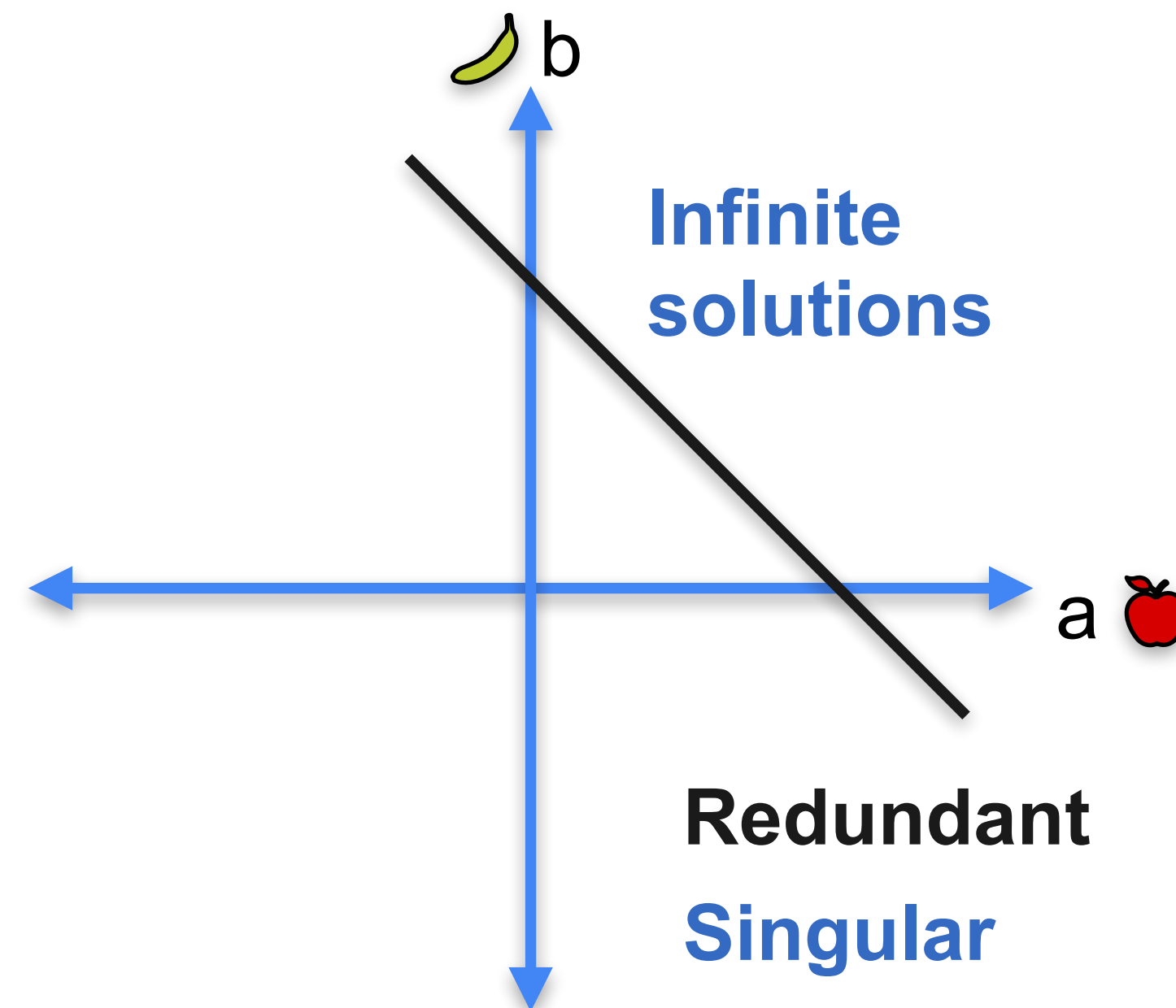
System 1

$$\begin{array}{l} a + b = 10 \\ a + 2b = 12 \end{array}$$



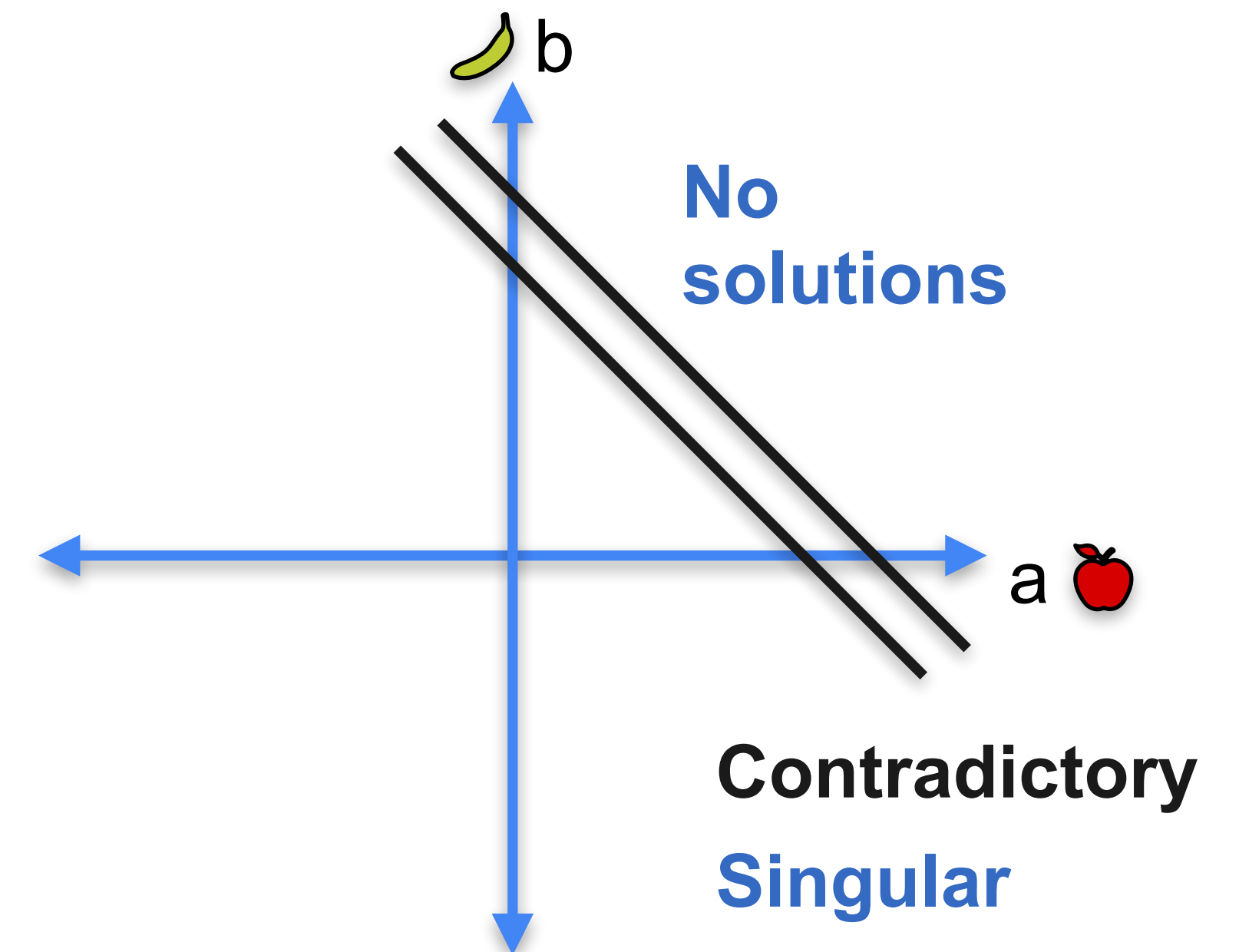
System 2

$$\begin{array}{l} a + b = 10 \\ 2a + 2b = 20 \end{array}$$



System 3

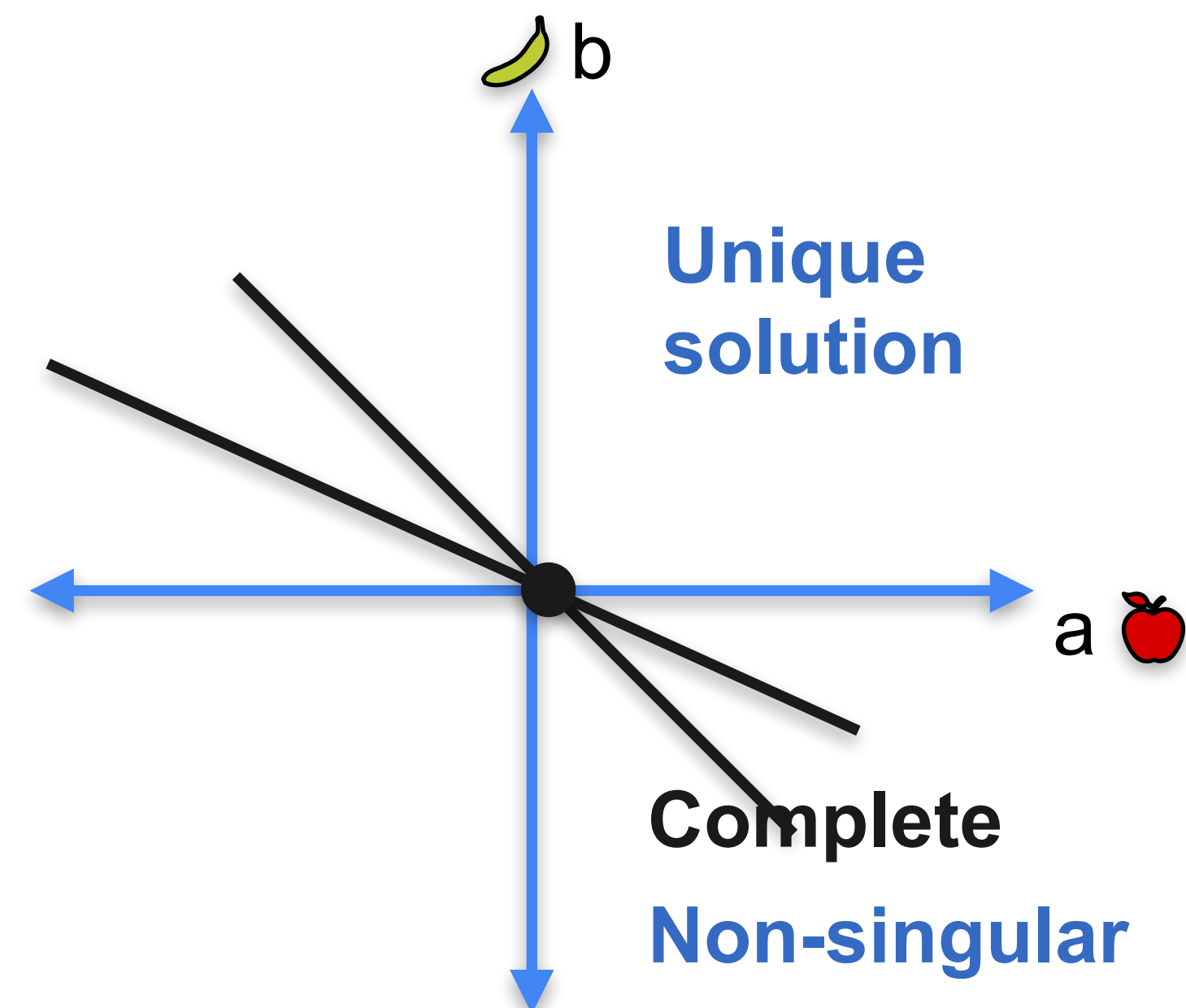
$$\begin{array}{l} a + b = 10 \\ 2a + 2b = 24 \end{array}$$



# Systems of equations as lines

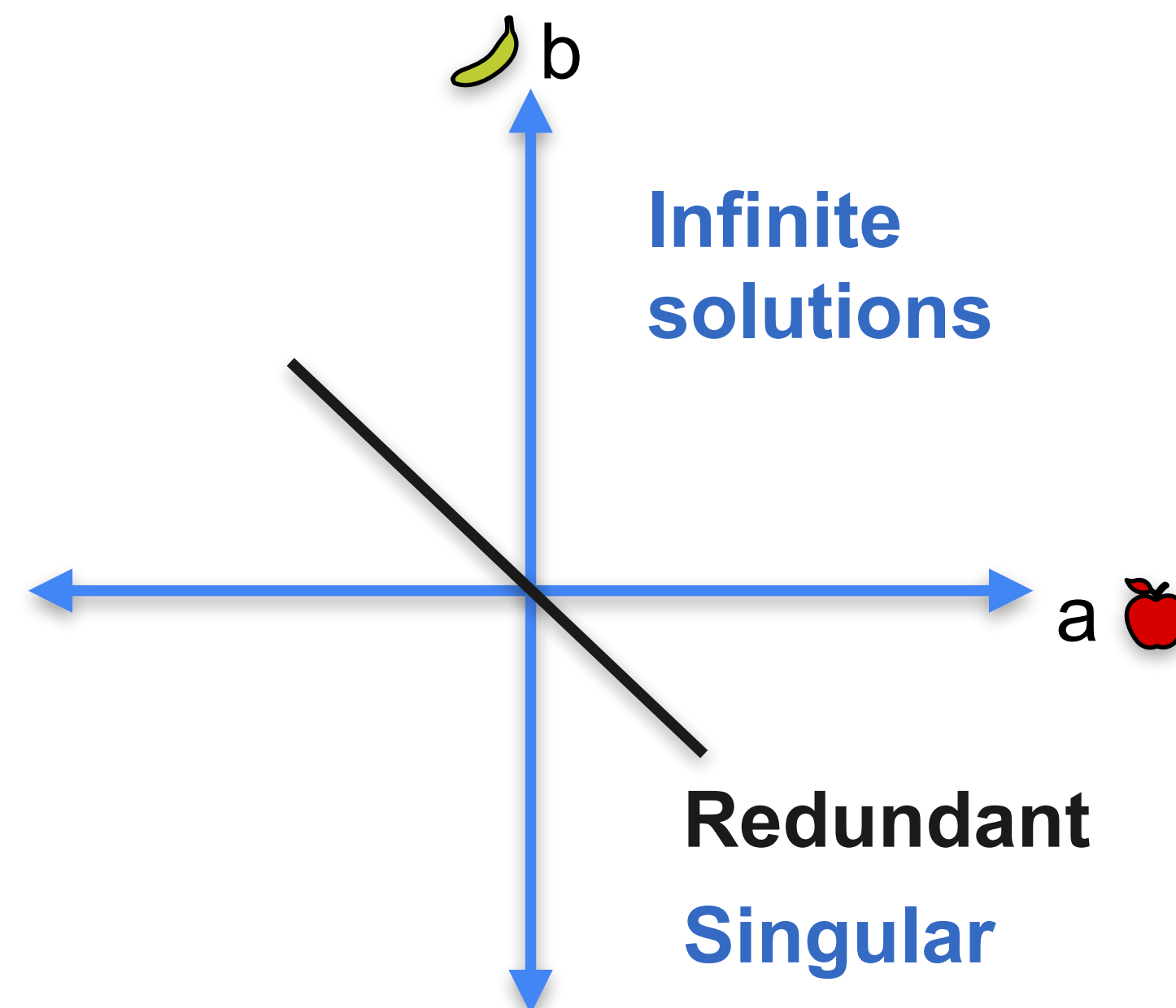
System 1

$$\begin{array}{l} a + b = 0 \\ a + 2b = 0 \end{array}$$



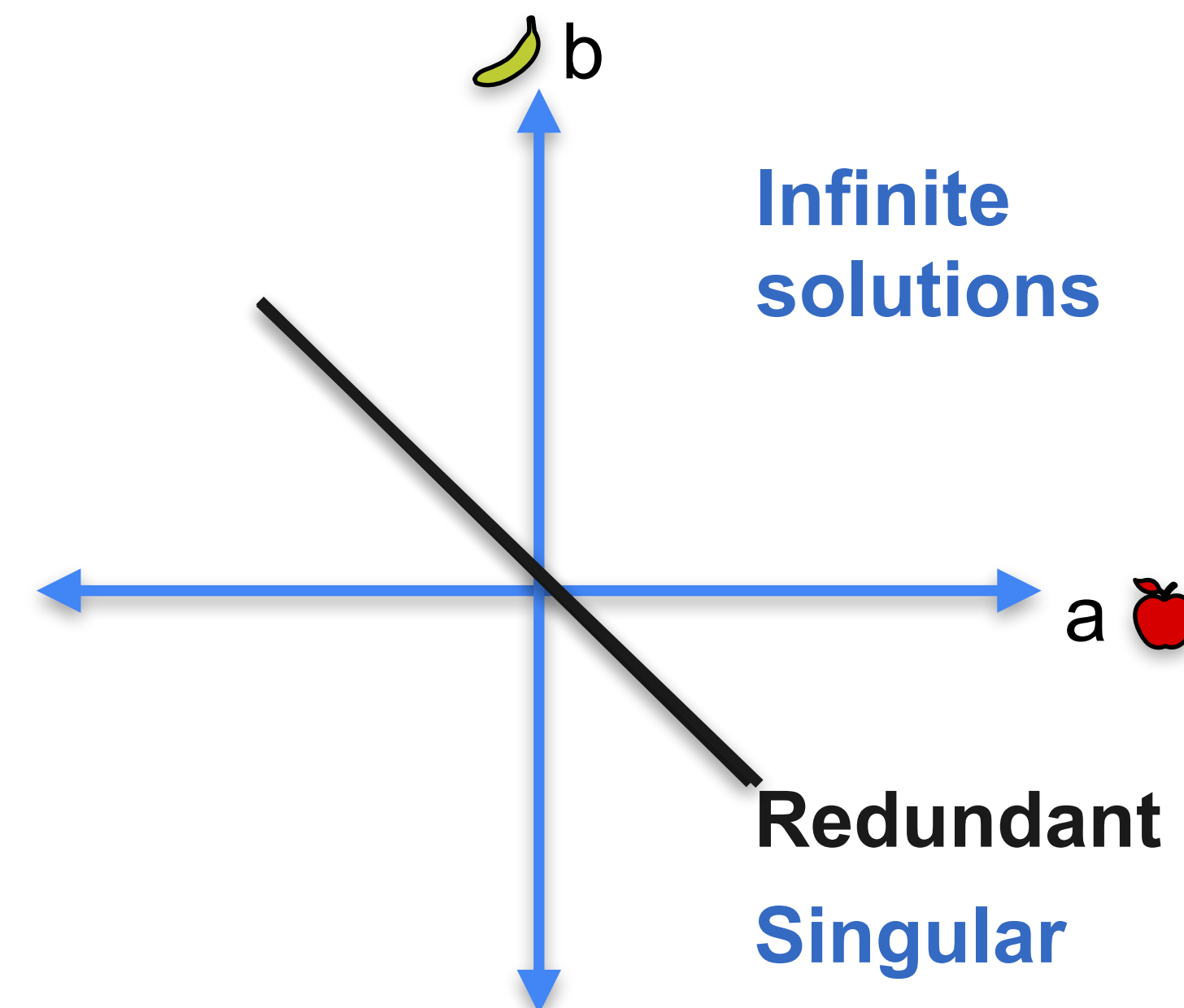
System 2

$$\begin{array}{l} a + b = 0 \\ 2a + 2b = 0 \end{array}$$



System 3

$$\begin{array}{l} a + b = 0 \\ 2a + 2b = 0 \end{array}$$





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# System of Linear Equations

---

**Singular vs non-singular  
matrices**



# Systems of equations as matrices

## System 1

$$\begin{array}{c} a + b = 0 \\ \text{🍏} + \text{🍌} \end{array}$$

$$\begin{array}{c} a + 2b = 0 \\ \text{🍏} + \text{🍌🍌} \end{array}$$

**Non-singular  
system**

🍏	🍌
1	1
1	2

**Non-singular  
matrix**

**(Unique solution)**

## System 2

$$\begin{array}{c} a + b = 0 \\ \text{🍏} + \text{🍌} \end{array}$$

$$\begin{array}{c} 2a + 2b = 0 \\ \text{🍏🍏} + \text{🍌🍌} \end{array}$$

**Singular  
system**

🍏	🍌
1	1
2	2

**Singular  
matrix**

**(Infinitely many solutions)**

# Constants don't matter for singularity

## System 1

$$\begin{aligned}a + b + c &= 10 \\a + 2b + c &= 15 \\a + b + 2c &= 12\end{aligned}$$

Unique solution

Complete

Non-singular

## System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

Infinite solutions

Redundant

Singular

## System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

No solutions

Contradictory

Singular

## System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 15 \\3a + 3b + 3c &= 20\end{aligned}$$

Infinite solutions

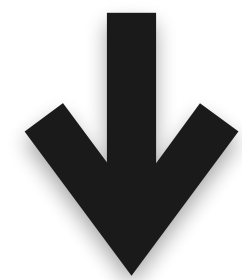
Redundant

Singular

# Constants don't matter for singularity

## System 1

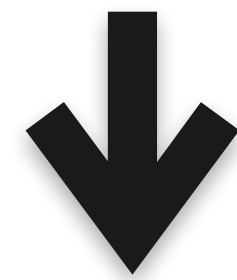
$$\begin{aligned}a + b + c &= 10 \\ a + 2b + c &= 15 \\ a + b + 2c &= 12\end{aligned}$$



$$\begin{aligned}a + b + c &= 0 \\ a + 2b + c &= 0 \\ a + b + 2c &= 0\end{aligned}$$

## System 2

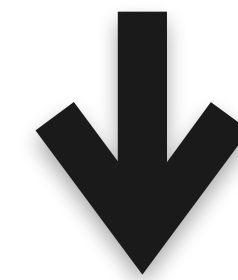
$$\begin{aligned}a + b + c &= 10 \\ a + b + 2c &= 15 \\ a + b + 3c &= 20\end{aligned}$$



$$\begin{aligned}a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0\end{aligned}$$

## System 3

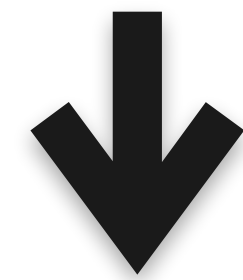
$$\begin{aligned}a + b + c &= 10 \\ a + b + 2c &= 15 \\ a + b + 3c &= 18\end{aligned}$$



$$\begin{aligned}a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0\end{aligned}$$

## System 4

$$\begin{aligned}a + b + c &= 10 \\ 2a + 2b + 2c &= 20 \\ 3a + 3b + 3c &= 30\end{aligned}$$



$$\begin{aligned}a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0\end{aligned}$$

# Constants don't matter for singularity

## System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

## System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

## System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

# Constants don't matter for singularity

**System 1**

$$\begin{aligned}a + b + c &= 0 \\a + 2b + c &= 0 \\a + b + 2c &= 0\end{aligned}$$

1	1	1
1	2	1
1	1	2

**Non-singular**

**System 2**

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

1	1	1
1	1	2
1	1	3

**Singular**

**System 3**

$$\begin{aligned}a + b + c &= 0 \\a + b + 2c &= 0 \\a + b + 3c &= 0\end{aligned}$$

**System 4**

$$\begin{aligned}a + b + c &= 0 \\2a + 2b + 2c &= 0 \\3a + 3b + 3c &= 0\end{aligned}$$

1	1	1
2	2	2
3	3	3

**Singular**



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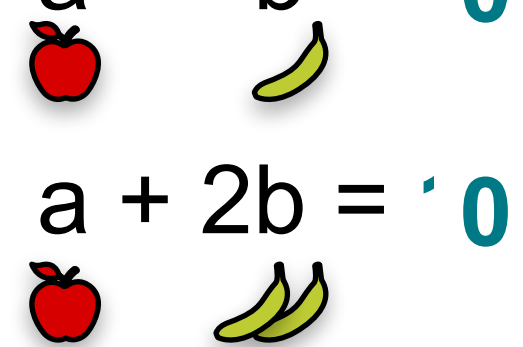
# System of Linear Equations

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**Linear dependence and  
independence**

# Linear dependence between rows

## Non-singular

$$\begin{array}{l} a + b = 0 \\ a + 2b = 0 \end{array}$$


No equation is a multiple of the other one

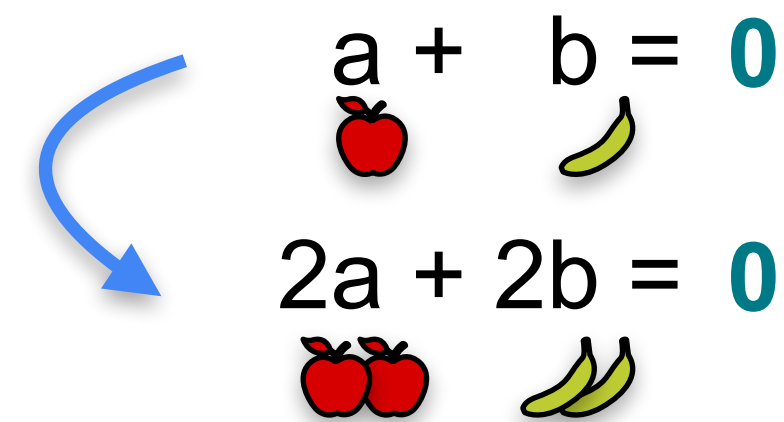


1	1
1	2


No row is a multiple of the other one

Rows are  
*linearly independent*

## Singular system


$$\begin{array}{l} a + b = 0 \\ 2a + 2b = 0 \end{array}$$

Second equation is a multiple of the first one



1	1
2	2

Second row is a multiple of the first row

Rows are  
*linearly dependent*

# Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} \xrightarrow{\quad} a + 0b + 0c = 1 \\ \xrightarrow{\quad} 0a + b + 0c = 2 \\ \xrightarrow{\quad} \hline a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**



# Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array}$$
$$\begin{array}{l} a + b + c = 0 \\ + \quad 2a + 2b + 2c = 0 \\ \hline 3a + 3b + 3c = 0 \end{array}$$

1	1	1
2	2	2
3	3	3

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

# Linear dependence and independence

The diagram illustrates the relationship between three linear equations and their corresponding matrix rows. On the left, three equations are listed:

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

Below these equations is a 3x3 matrix representing the coefficients:

1	1	1
1	1	2
1	1	3

On the right, the equations are manipulated to show that the second equation is the average of the first and third:

$$\begin{aligned} &a + b + c = 0 \\ &+ \quad a + b + 3c = 0 \\ &\hline &2a + 2b + 4c = 0 \\ &\quad \quad \quad \div 2 \\ &a + b + 2c = 0 \end{aligned}$$

Blue arrows indicate the flow of information: one arrow points from the first equation to the first equation in the manipulation, another from the third equation to the third equation in the manipulation, and a third from the second equation to the final result of the manipulation. This shows that the second equation is derived from the first and third, indicating linear dependence.

Average of Row 1 and Row 3 is Row 2  
Row 2 **depends** on rows 1 and 3

Rows are **linearly dependent**

# Linear dependence and independence

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$



No relations between equations

1	1	1
1	2	1
1	1	2

No relations between rows

Rows are **linearly independent**

# Quiz: Linear dependence and independence

**Problem:** Determine if the following matrices have linearly dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

# Solution: Linear dependence and independence

**Problem:** Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

**Dependent (singular)**

1	1	1
1	1	2
0	0	-1

$$\text{Row1} - \text{Row2} = \text{Row3}$$

**Dependent (singular)**

1	1	1
0	2	2
0	0	3

No relations

**Independent  
(Non-singular)**

1	2	5
0	3	-2
2	4	10

$$2\text{Row1} = \text{Row3}$$

**Dependent (singular)**



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# System of Linear Equations

---

## **The determinant**

# Linear dependence between rows



Non-singular matrix

	
1	1
1	2

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times ? = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

Rows linearly independent


Singular matrix

	
1	1
2	2

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

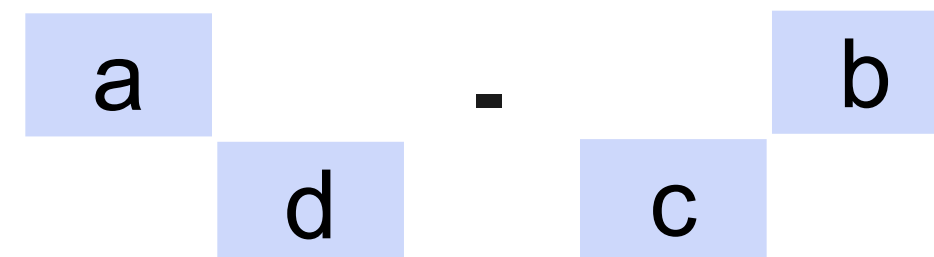
Rows linearly dependent

# Determinant



a	b
c	d

**Determinant**  $= ad - bc$



$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

**Matrix is singular if**

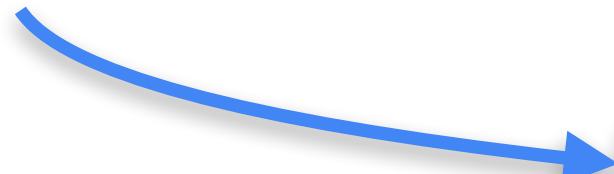
a	b
---	---

 $\cdot k =$ 

c	d
---	---

**Determinant**

$$ad = bc$$


$$ad - bc = 0$$



# Determinant

Non-singular matrix

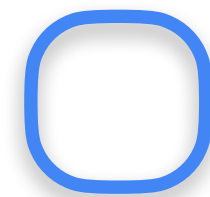


1	1
1	2

Determinant

$$\begin{array}{cc} 1 & 1 \\ 2 & 1 \end{array} -$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$



Singular matrix

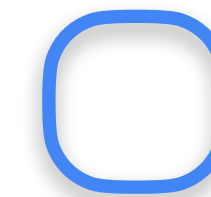


1	1
2	2

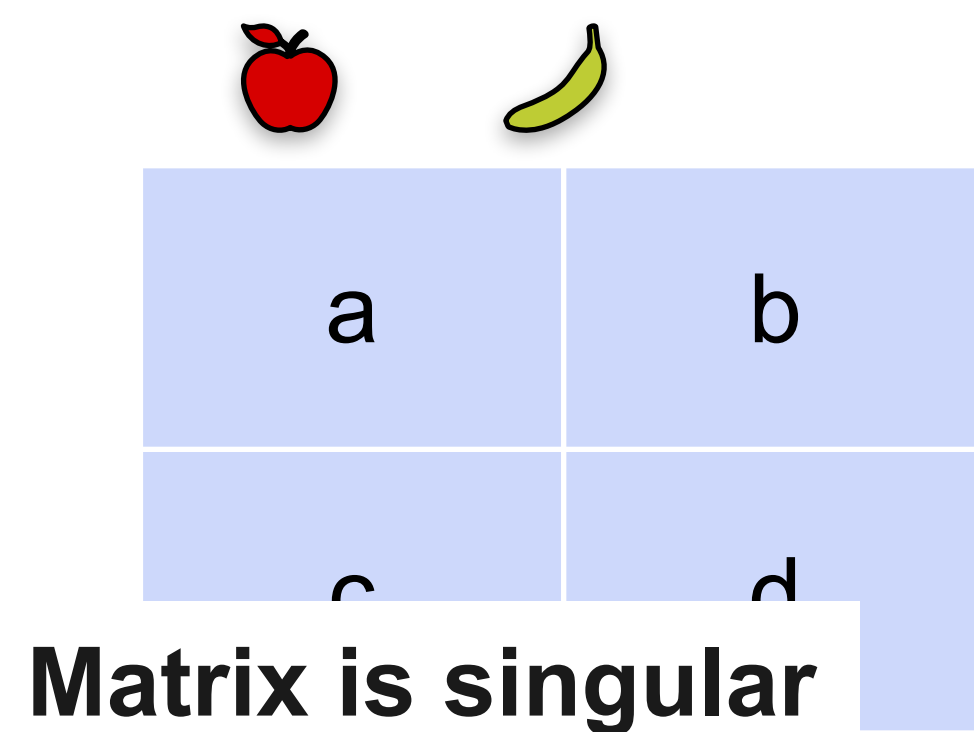
Determinant

$$\begin{array}{cc} 1 & 1 \\ 2 & 2 \end{array} -$$

$$1 \cdot 2 - 2 \cdot 1 = 0$$



# Determinant and singularity



$$ad - bc$$

**Determinant is zero**

# Quiz: Determinant

**Problem 1:** Find the determinant of the following matrices

**Matrix 1**

5	1
-1	3

2	-1
-6	3

Are these matrices singular or non-singular?

# Solutions: Determinant

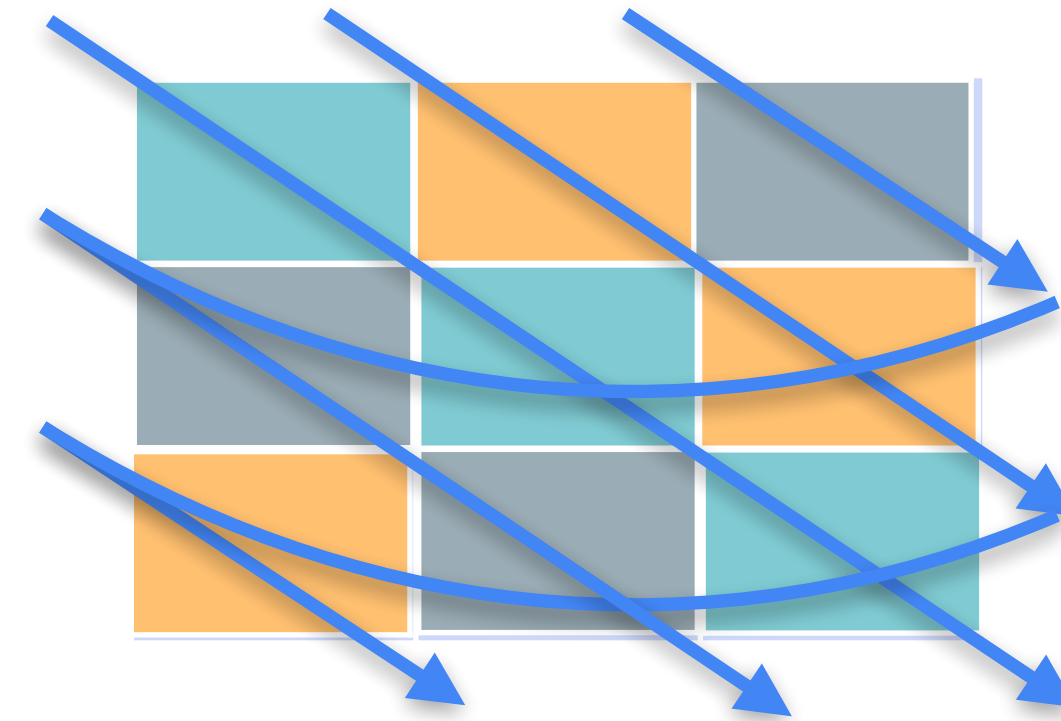
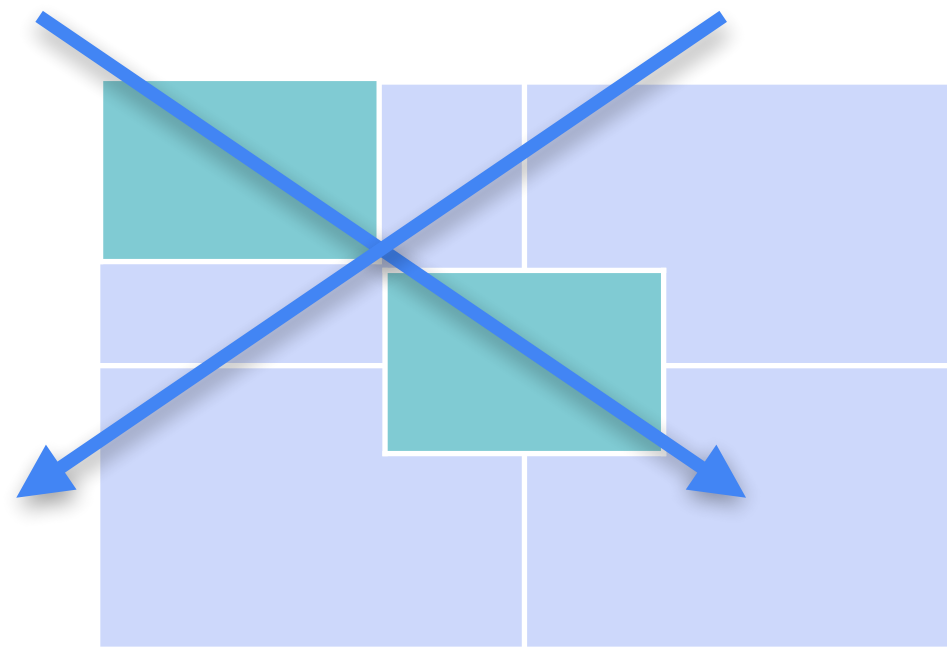
**Matrix 1:**  $\det = 5 \cdot 3 - 1 \cdot (-1) = 15 + 1 = 16$

5	1	<b>Non-singular</b>
-1	3	

**Matrix 2:**  $\det = 2 \cdot 3 - (-1) \cdot (-6) = 6 - 6 = 0$

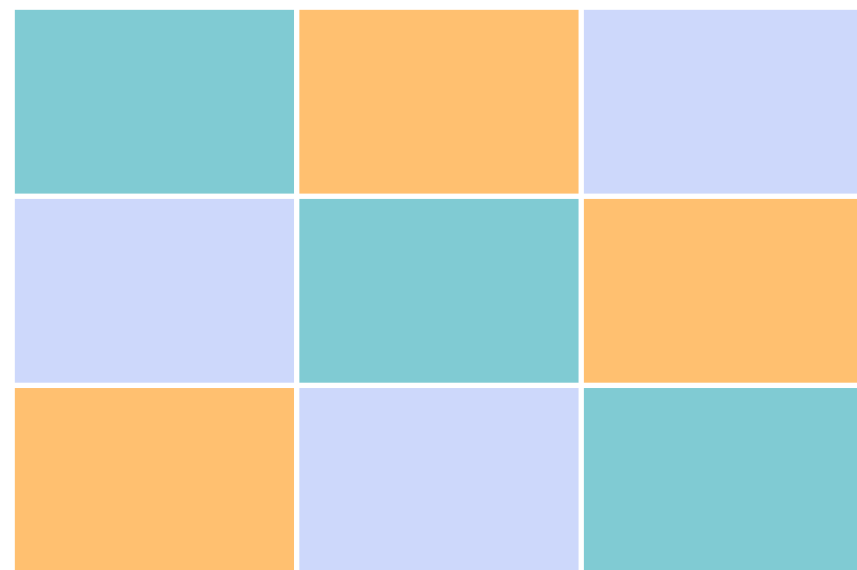
2	-1	<b>Singular</b>
-6	3	

# Diagonals in a 3x3 matrix

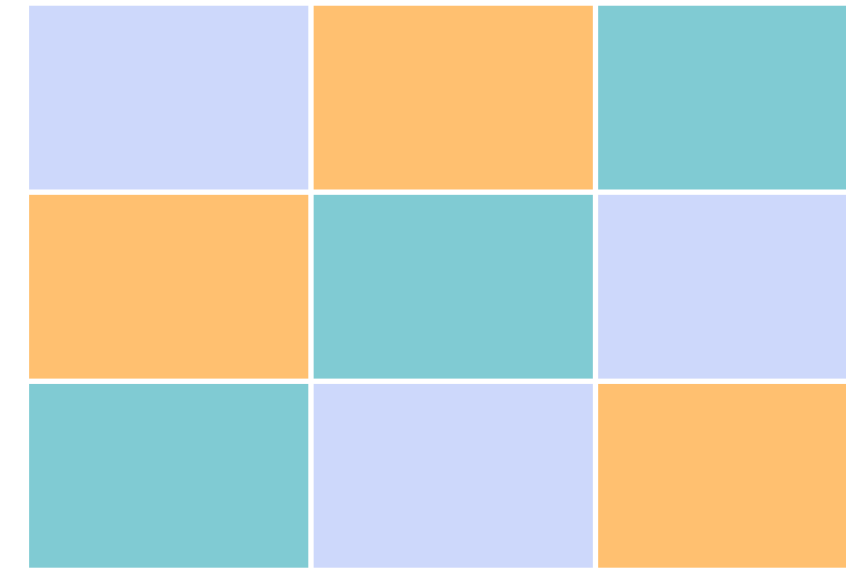


# Determinant

Add



Subtract



# The determinant

1	1	1
1	2	1
1	1	2

# The determinant

1	1	1
1	2	1
1	1	2

1		
	2	
		2

$$+ 1 \cdot 2 \cdot 2$$

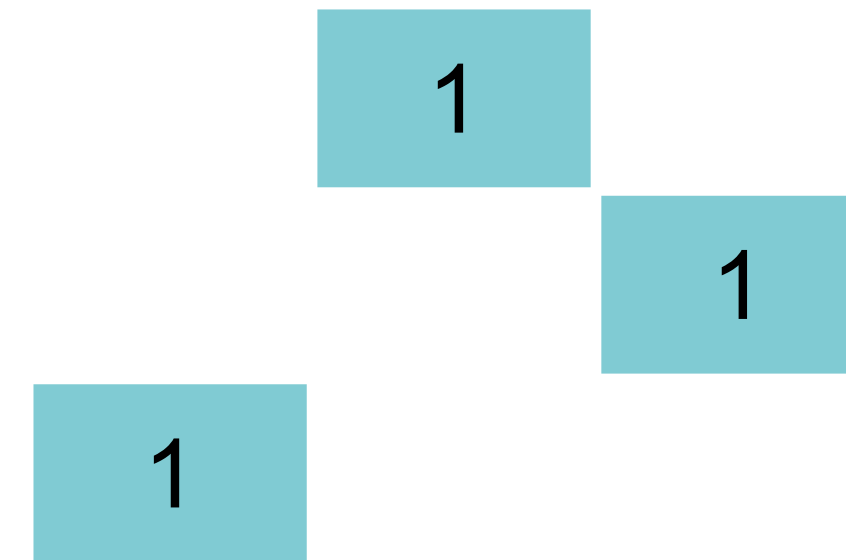


# The determinant

1	1	1
1	2	1
1	1	2



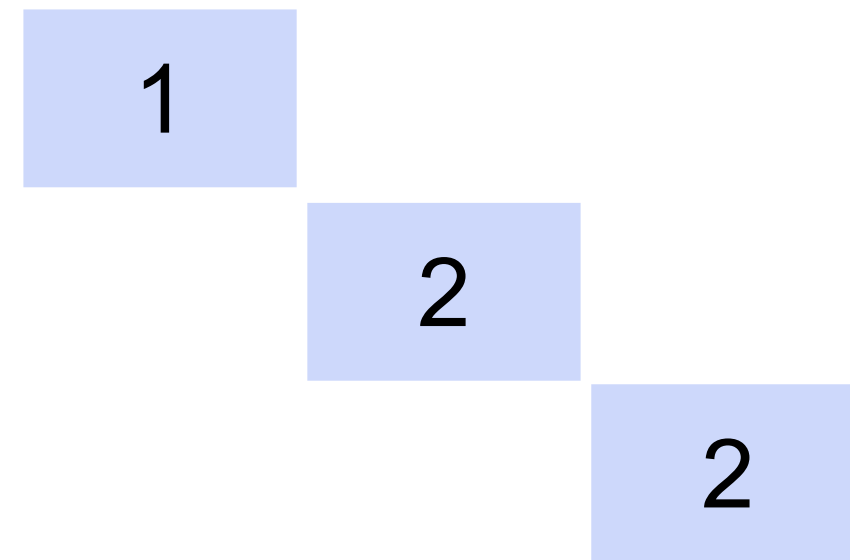
$$+ 1 \cdot 2 \cdot 2$$



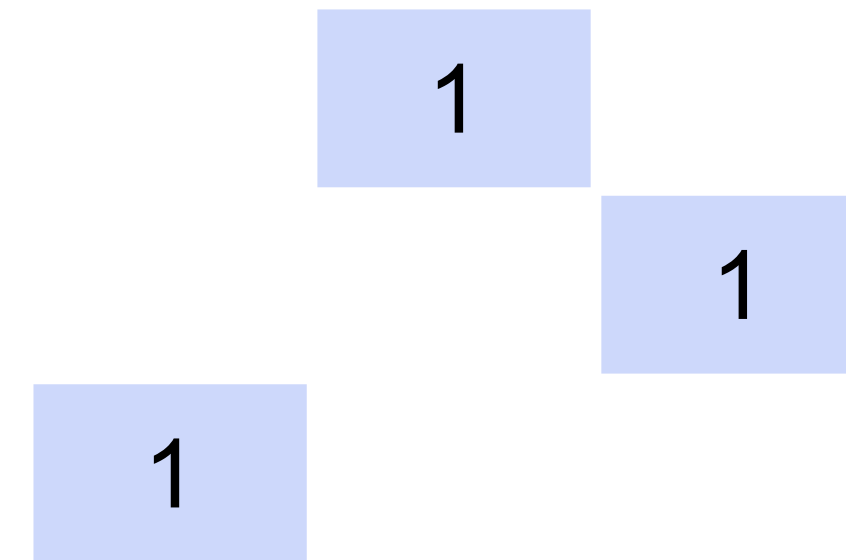
$$+ 1 \cdot 1 \cdot 1$$

# The determinant

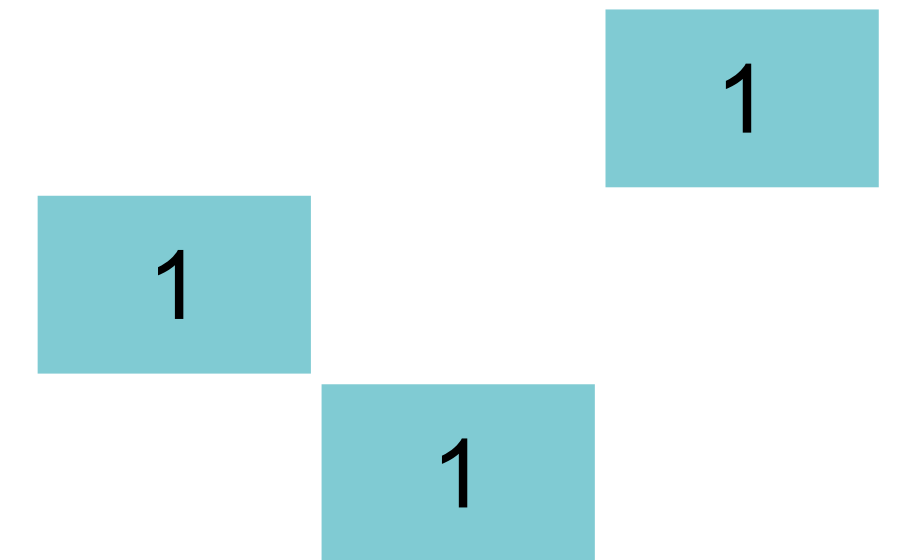
1	1	1
1	2	1
1	1	2



$$+ 1 \cdot 2 \cdot 2$$



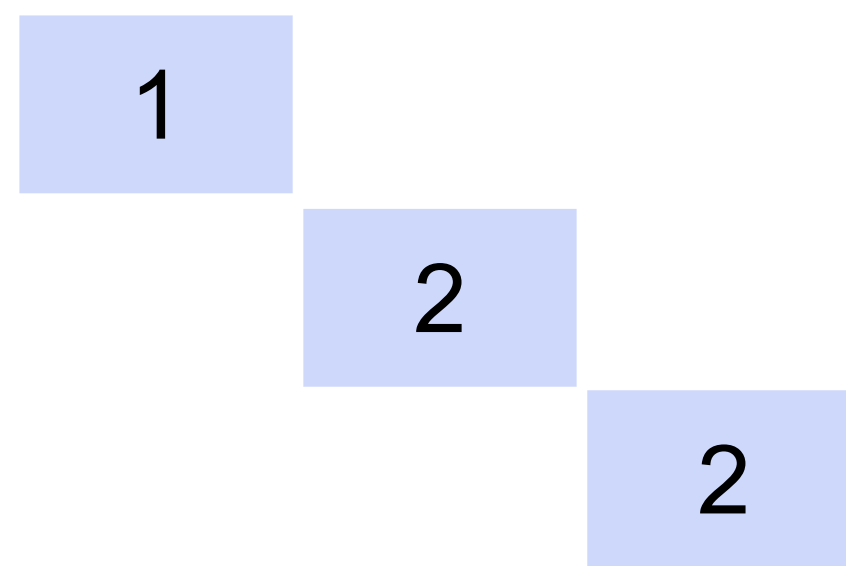
$$+ 1 \cdot 1 \cdot 1$$



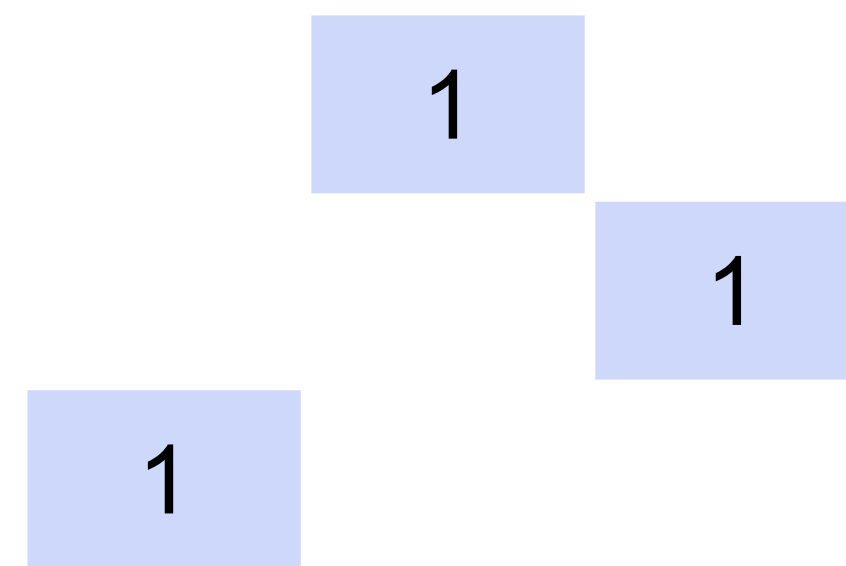
$$+ 1 \cdot 1 \cdot 1$$

# The determinant

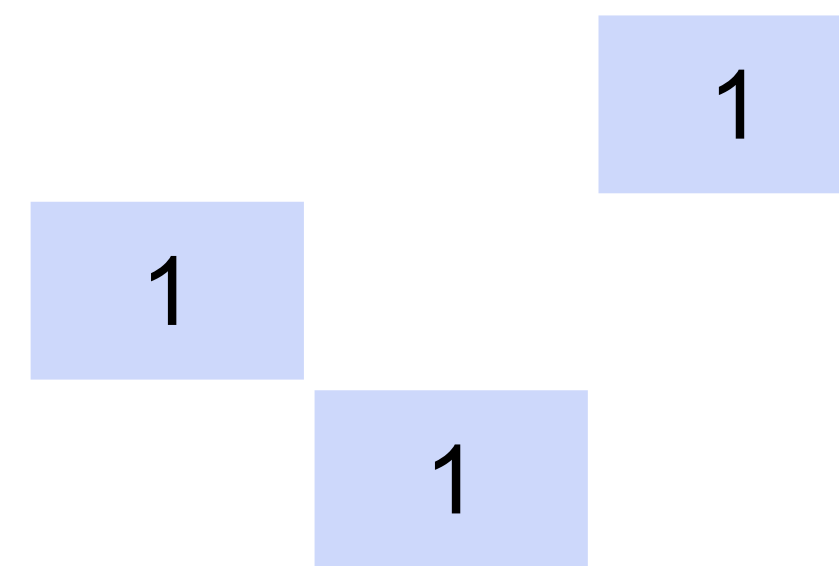
1	1	1
1	2	1
1	1	2



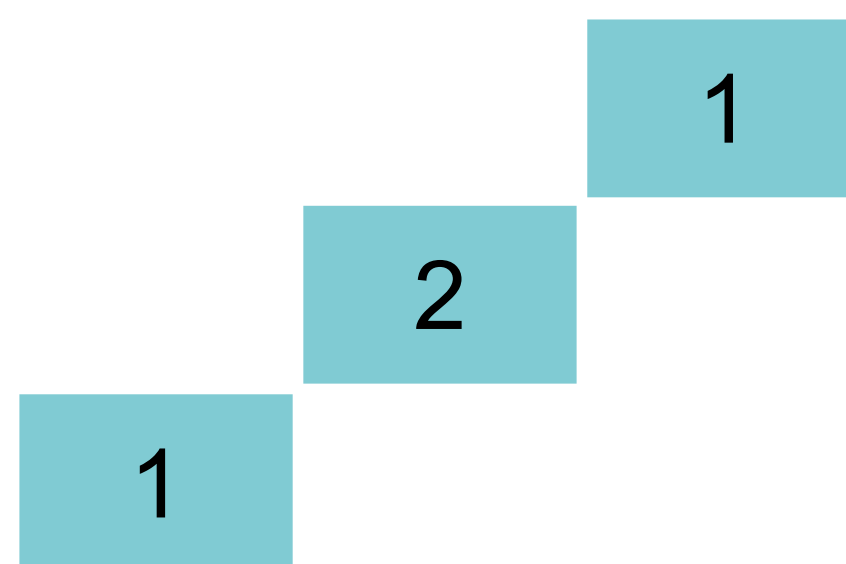
$$+ 1 \cdot 2 \cdot 2$$



$$+ 1 \cdot 1 \cdot 1$$



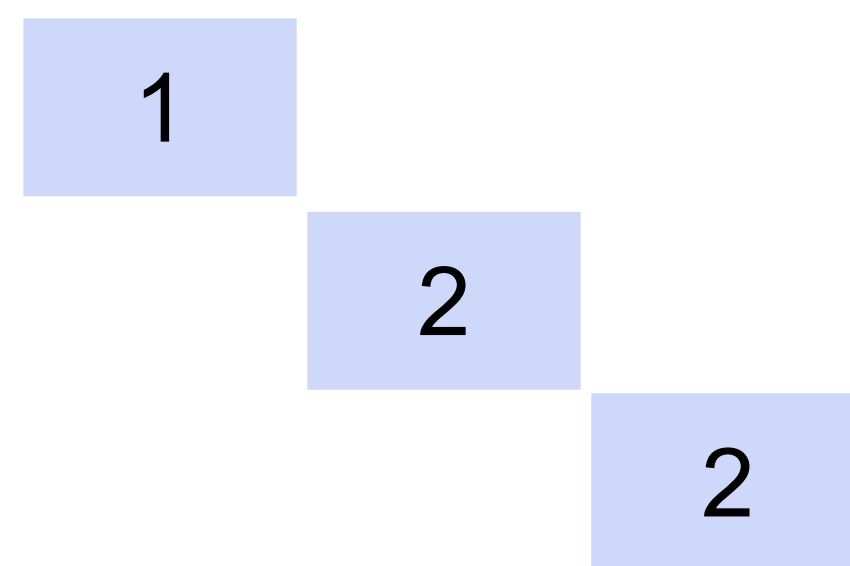
$$+ 1 \cdot 1 \cdot 1$$



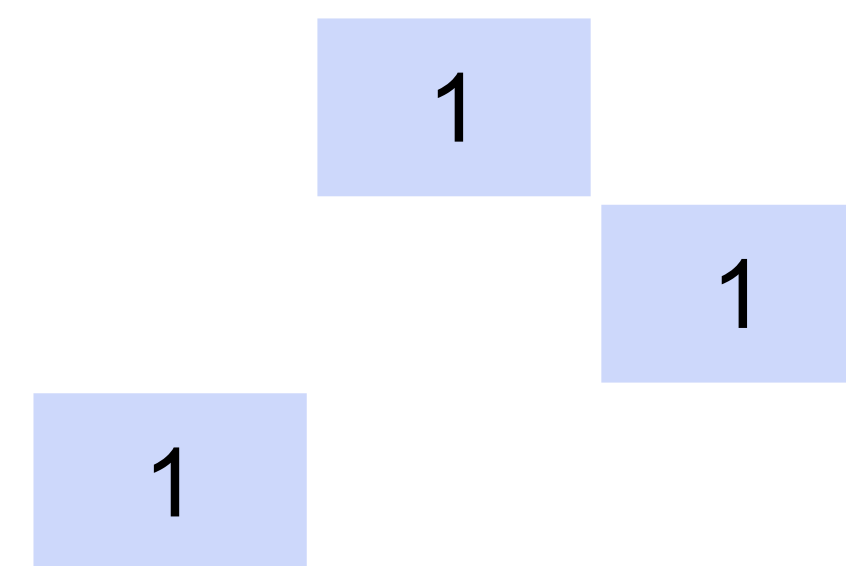
$$- 1 \cdot 2 \cdot 1$$

# The determinant

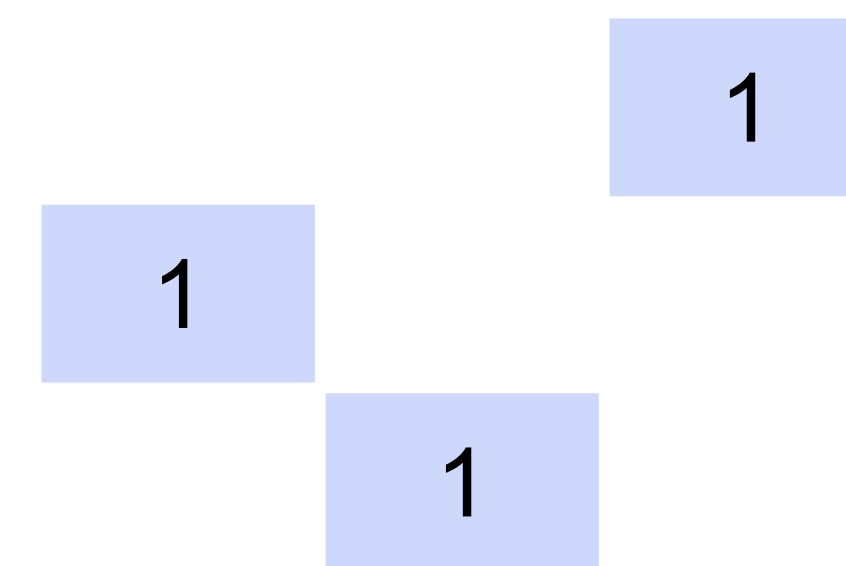
1	1	1
1	2	1
1	1	2



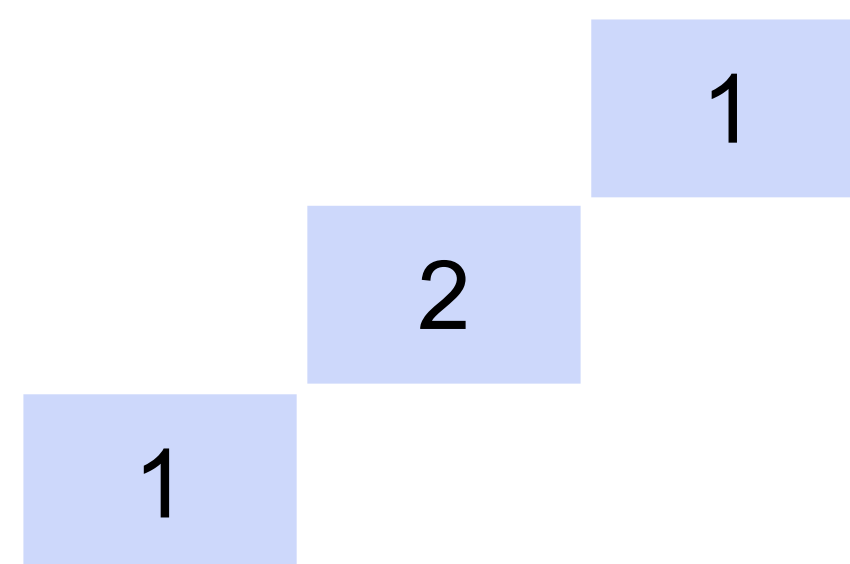
$$+ 1 \cdot 2 \cdot 2$$



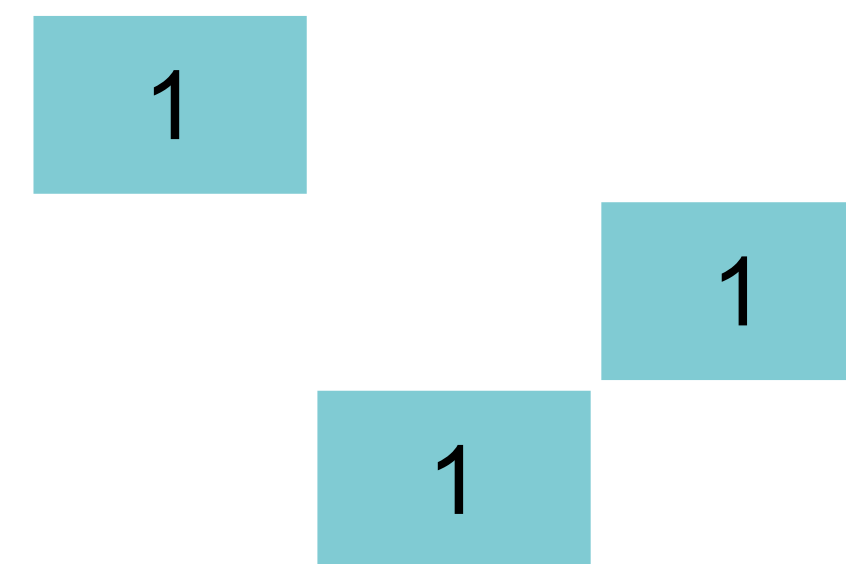
$$+ 1 \cdot 1 \cdot 1$$



$$+ 1 \cdot 1 \cdot 1$$



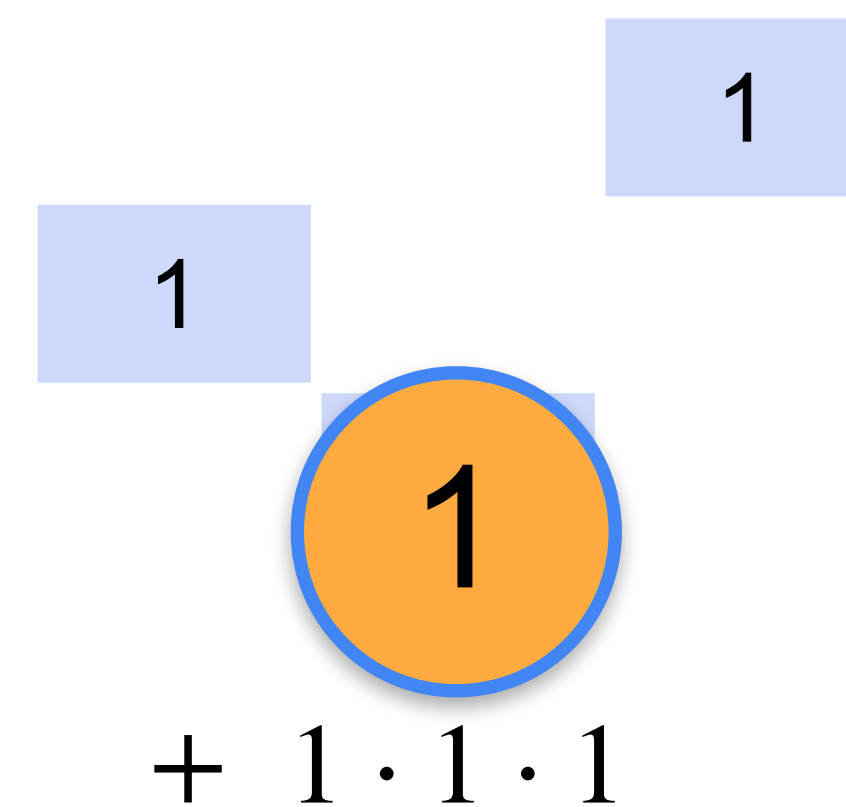
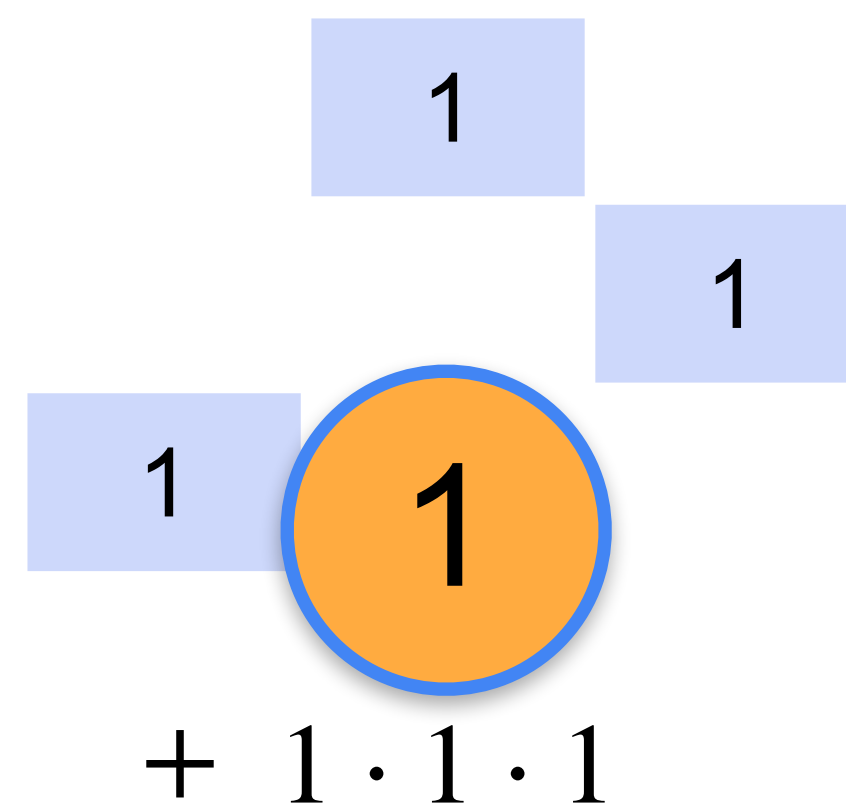
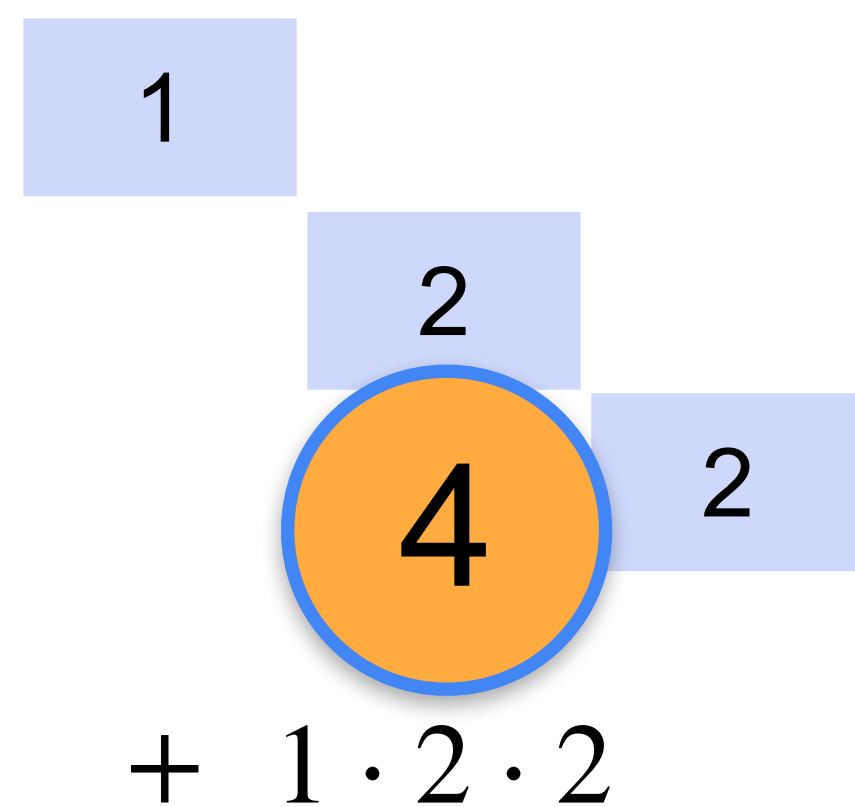
$$- 1 \cdot 2 \cdot 1$$



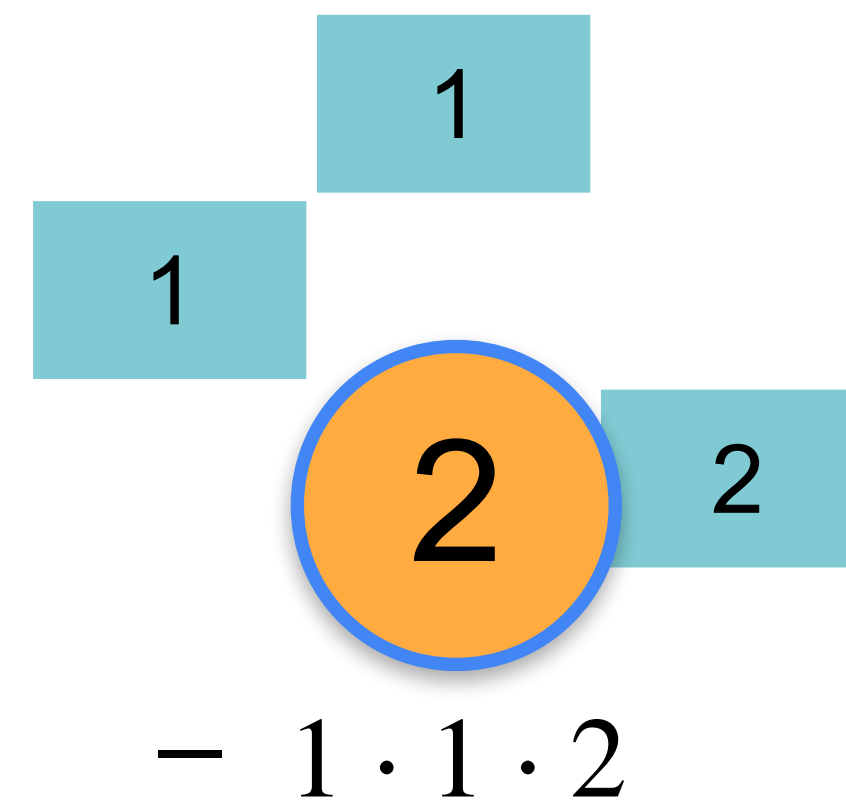
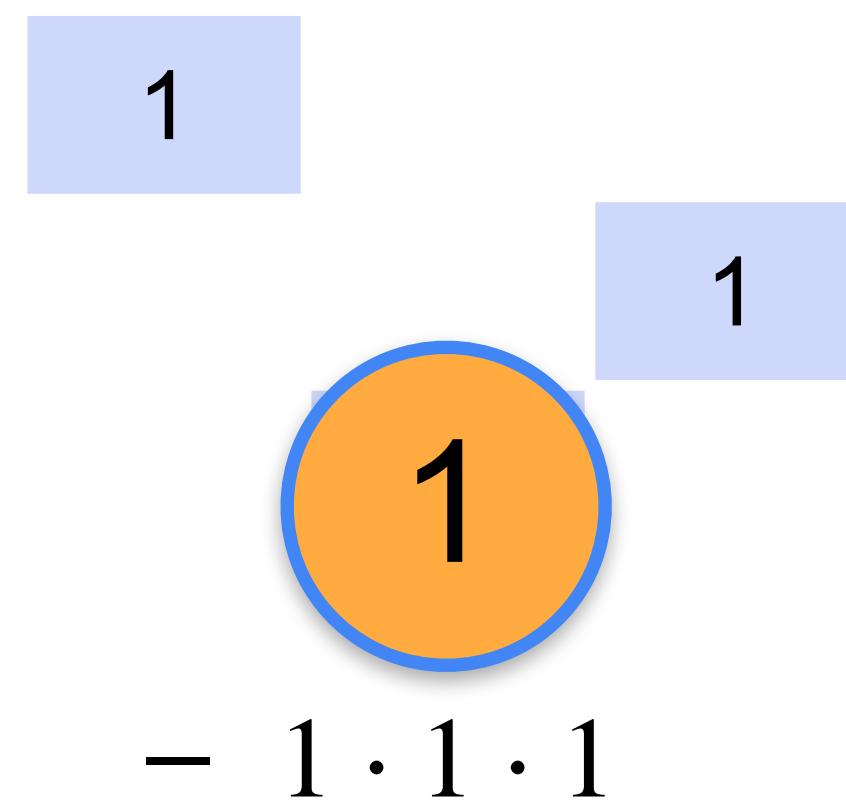
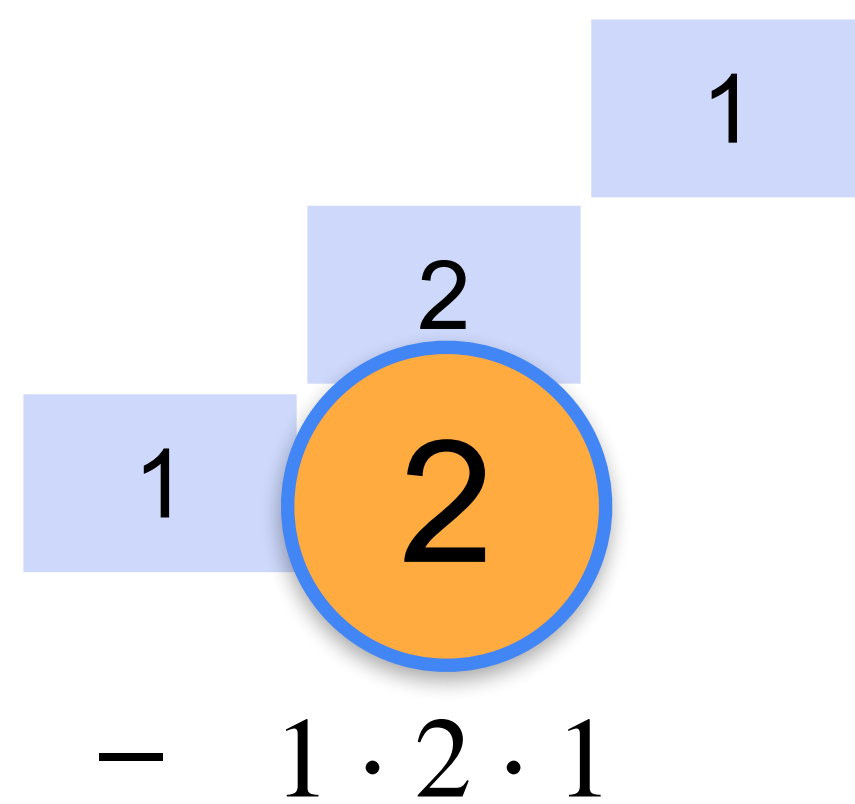
$$- 1 \cdot 1 \cdot 1$$

# The determinant

1	1	1
1	2	1
1	1	2



$$\begin{aligned} \text{Det} &= 4 + 1 + 1 \\ &\quad - 2 - 1 - 2 \\ &= 1 \end{aligned}$$



# Quiz: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

# Solution: Determinants

**Problem:** Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

**Singular**

1	1	1
1	1	2
0	0	-1

Determinant = 0

**Singular**

1	1	1
0	2	2
0	0	3

Determinant = 6

**Non-singular**

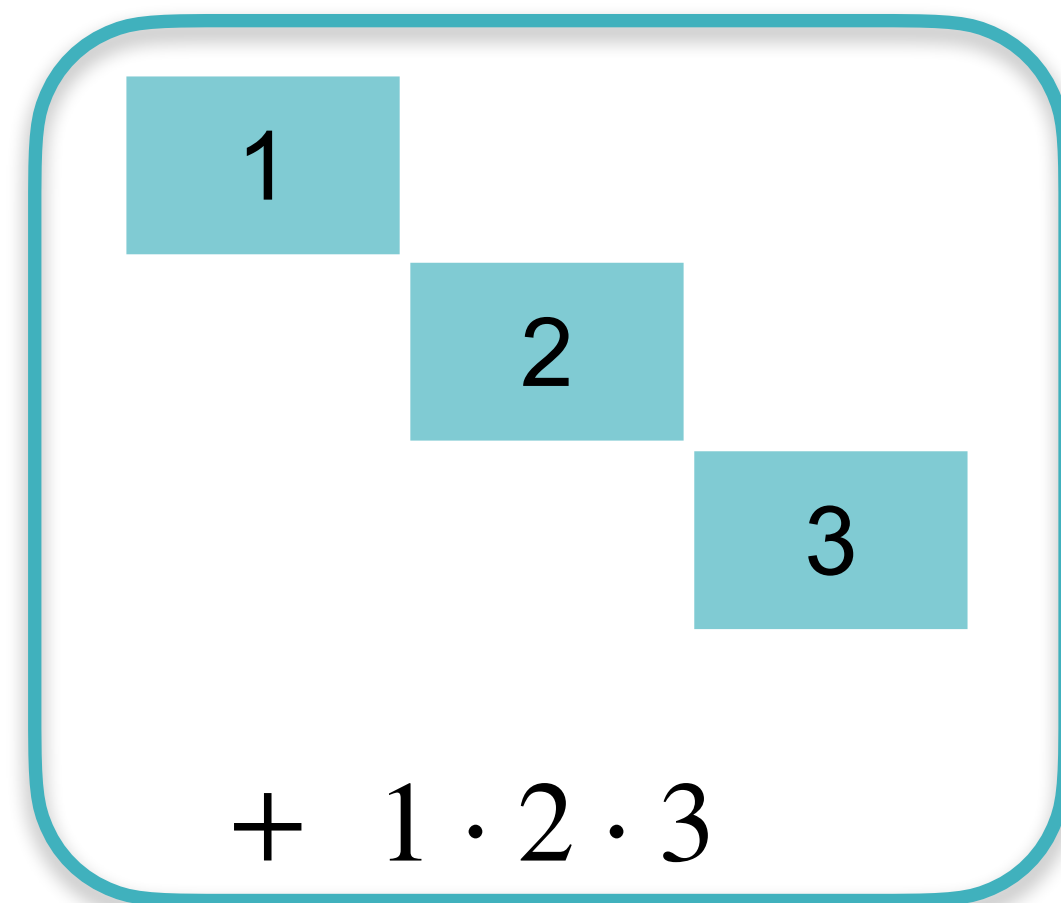
1	2	5
0	3	-2
2	4	10

Determinant = 0

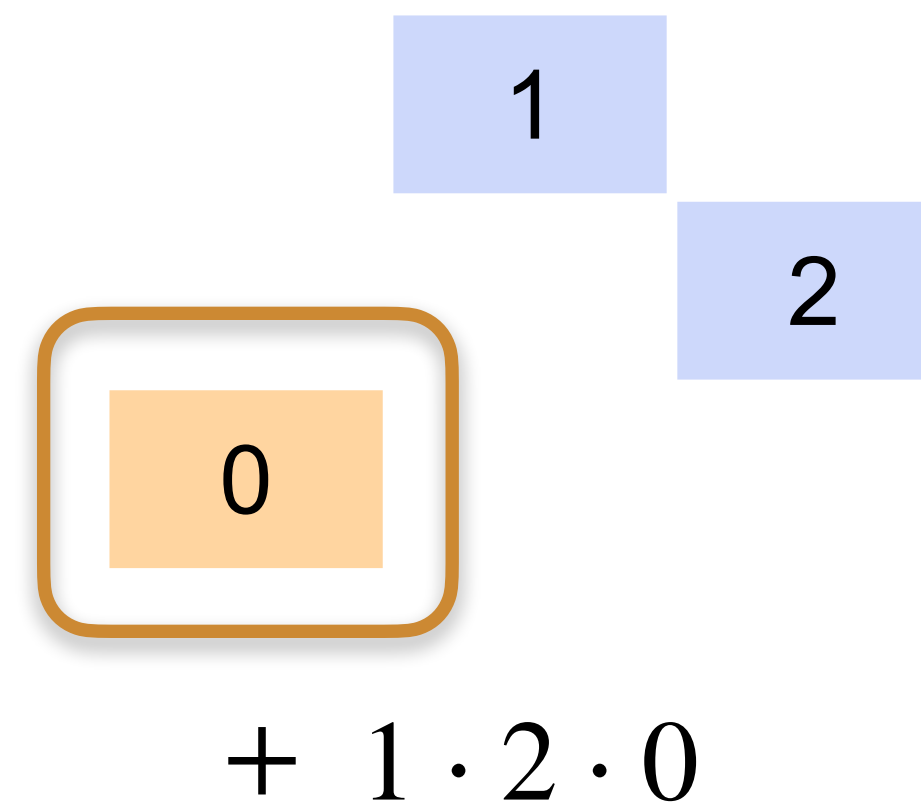
**Singular**

# The determinant

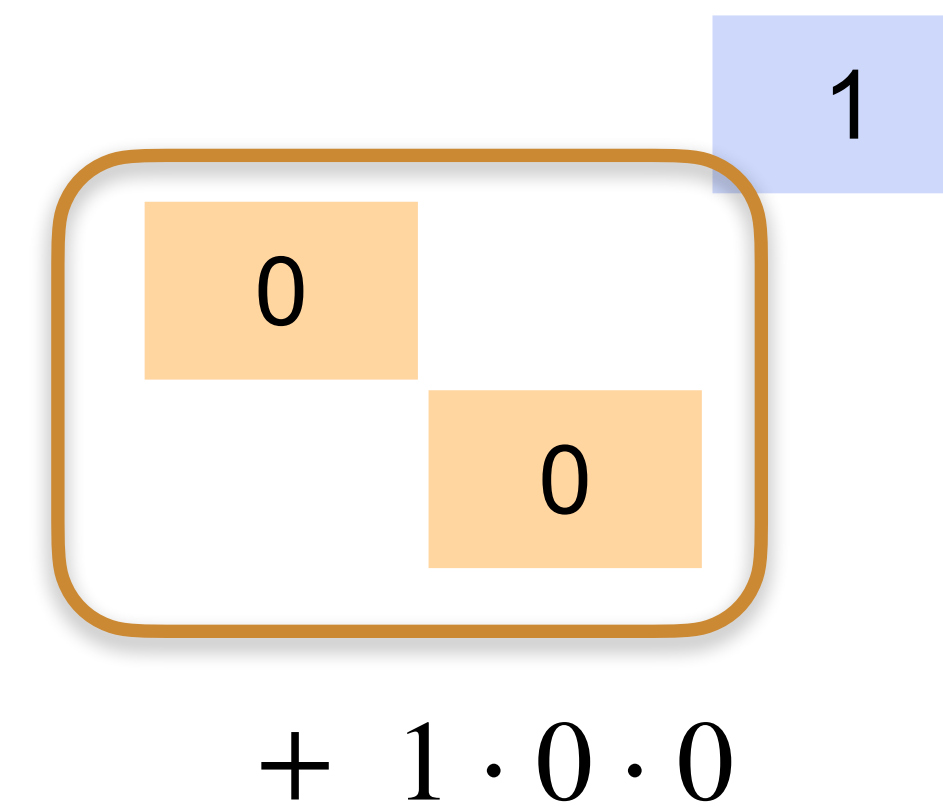
1	1	1
0	2	2
0	0	3



$$+ 1 \cdot 2 \cdot 3$$



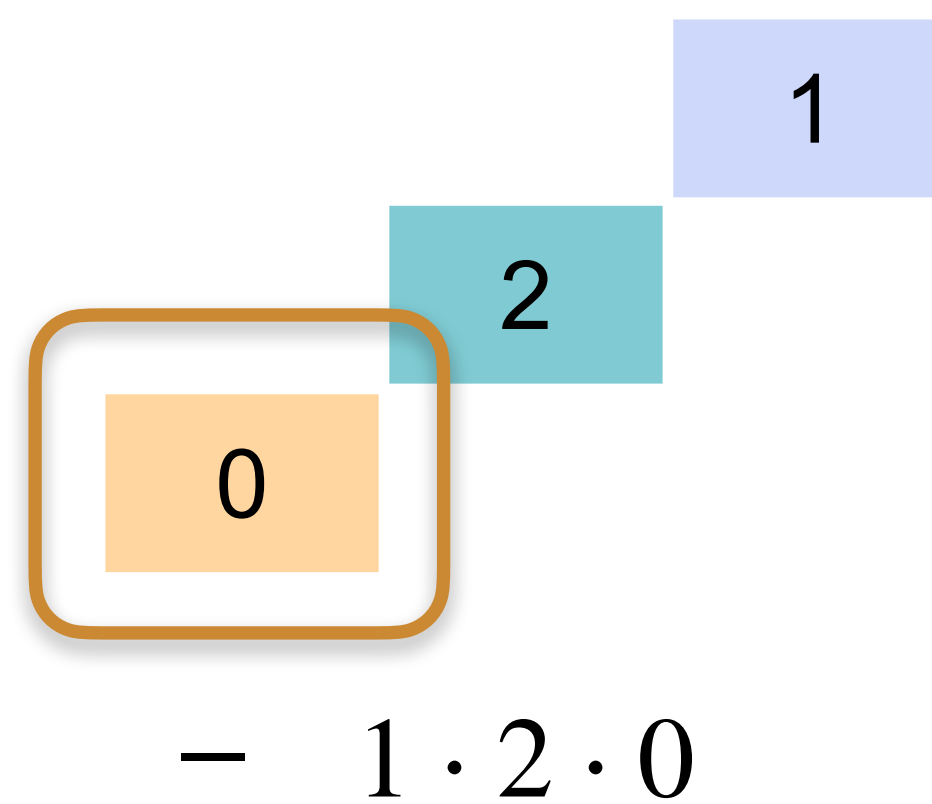
$$+ 1 \cdot 2 \cdot 0$$



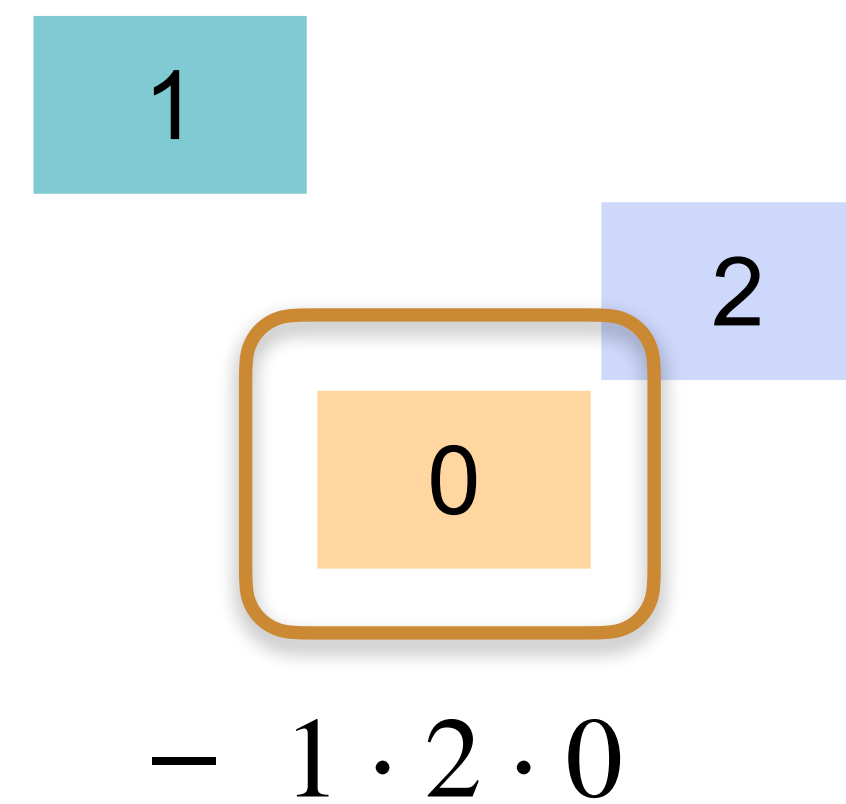
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

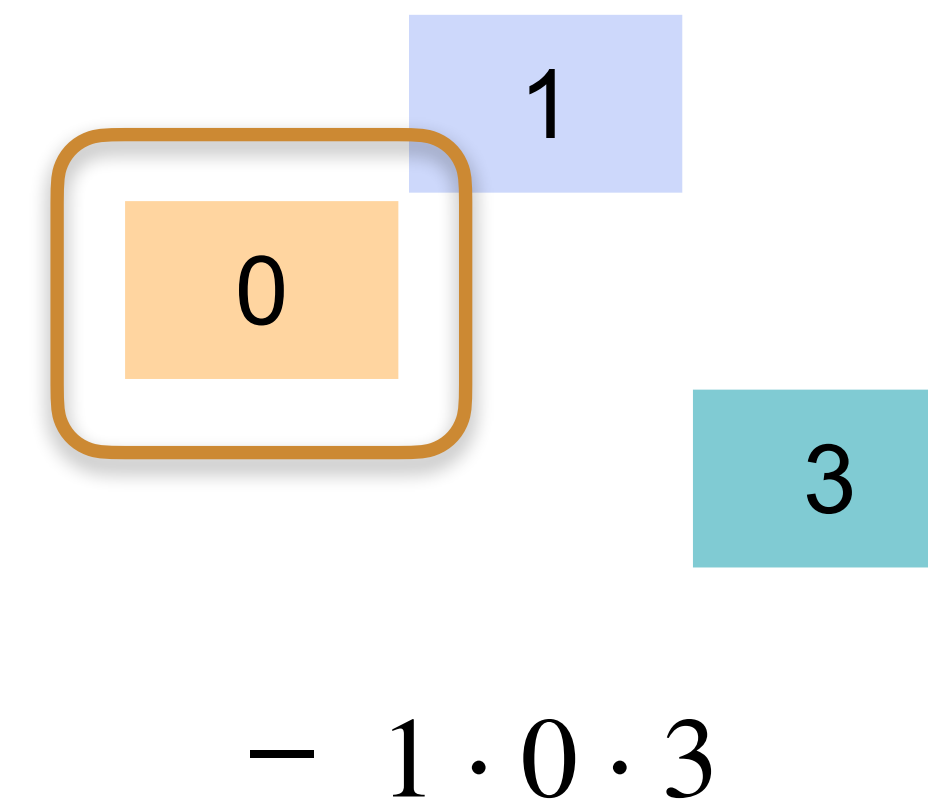
$$= 6$$



$$- 1 \cdot 2 \cdot 0$$



$$- 1 \cdot 2 \cdot 0$$



$$- 1 \cdot 0 \cdot 3$$



# The determinant

1	1	1
0	2	2
0	0	0

$$+ 1 \cdot 2 \cdot 0$$

$$\text{Det} = 0+0+0-0-0-0$$

$$= 0$$

$$- 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 2 \cdot 0$$

$$- 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 0 \cdot 0$$

$$- 1 \cdot 0 \cdot 0$$



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# System of Linear Equations

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## Conclusion