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W1 Lesson 1

Introduction to Probability

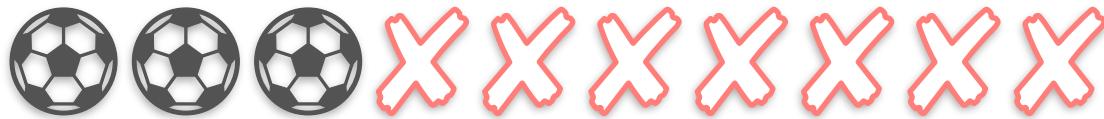


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Introduction to probability

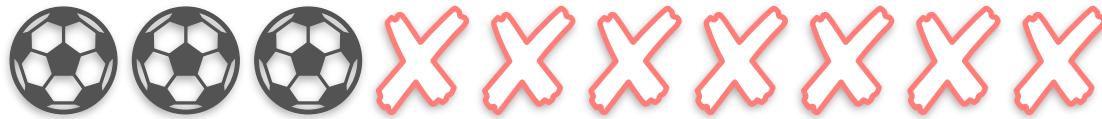
What is Probability?

Introduction to Probability



Find the probability that a child picked at random plays soccer.

Introduction to Probability



Find the probability that a child picked at random plays soccer.

The probability that a child picked at random plays soccer.

$$P(\text{soccer})$$

A teal curved arrow points from the text above down to the mathematical expression below.

Introduction to Probability

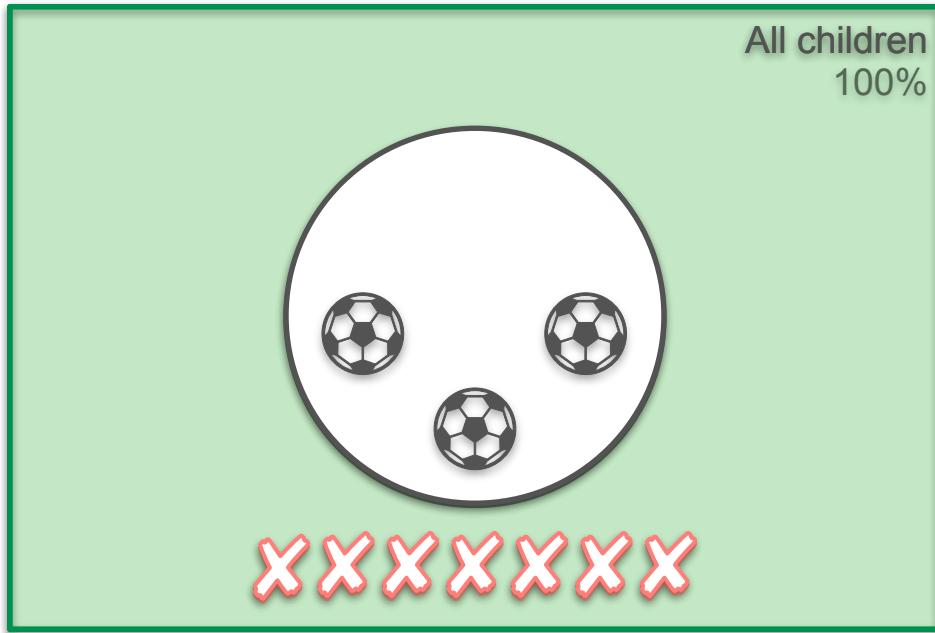


Find the probability that a child picked at random plays soccer.

$$P(\text{soccer}) = \frac{\text{soccer}}{\text{total}} = \frac{\text{Event}}{\text{Sample space}} = \frac{3}{10} = 0.3$$

The diagram illustrates the calculation of probability. A horizontal bar represents the 'Sample space' containing 10 items: three soccer balls and seven 'X' marks. Above this bar, a smaller horizontal bar represents the 'Event' containing the 3 soccer balls. Arrows point from the labels 'Event' and 'Sample space' to their respective bars.

Introduction to Probability: Venn Diagram



Introduction to Probability: Coin Example 1

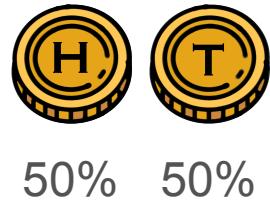


Experiment

Probability of landing on heads

$$P(\text{heads})$$

Introduction to Probability: Coin Example 1



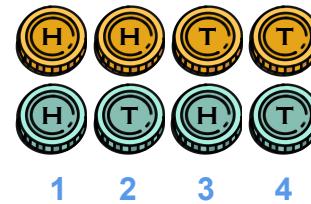
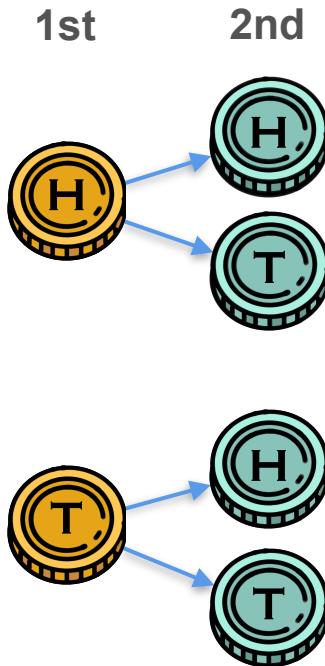
$$P(\text{heads}) = \frac{\text{Number of heads}}{\text{Total number of outcomes}} = \frac{1}{2} = 0.5$$
A horizontal fraction bar is used to represent the probability formula. The numerator is a single gold coin showing 'H' (heads). The denominator consists of two gold coins, one showing 'H' (heads) and the other showing 'T' (tails).

Introduction to Probability: Coin Example 2



50% 50%

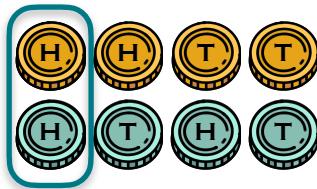
What is the probability of landing on heads twice?



Introduction to Probability: Coin Example 2



50% 50%

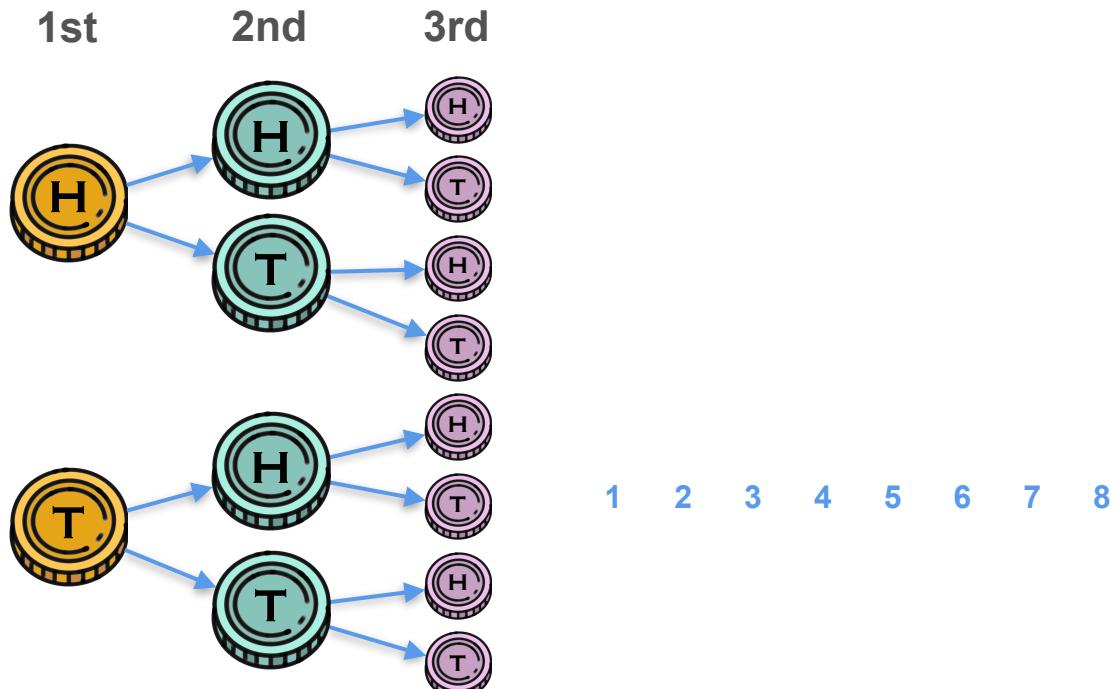


$$P(HH) = \frac{1}{4} = 0.25$$

Introduction to Probability: Coin Example 3



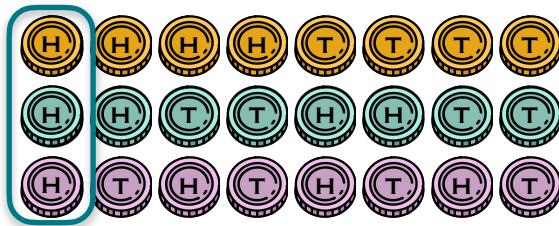
What is the probability of landing on heads 3 times?



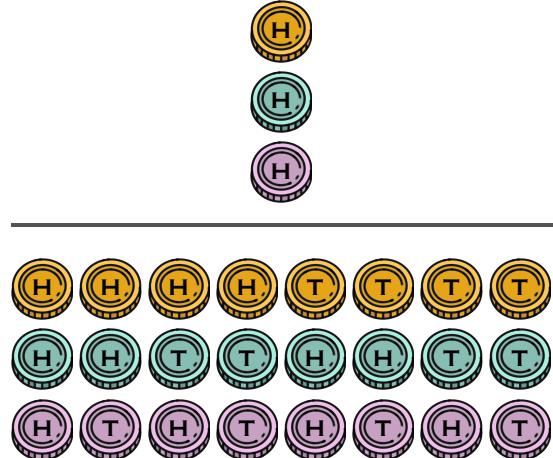
Introduction to Probability: Coin Example 3



50% 50%



$$P(HHH) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}}$$



$$= \frac{1}{8} = 0.125$$

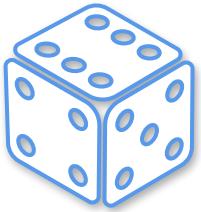


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Introduction to Probability

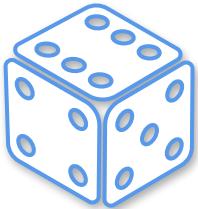
What is Probability? - Dice Example

Introduction to Probability: Dice Example 1

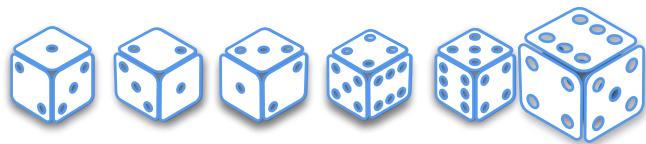


What is the probability of obtaining 6?

Introduction to Probability: Dice Example 1

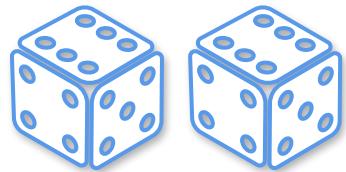


What is the probability of obtaining 6?



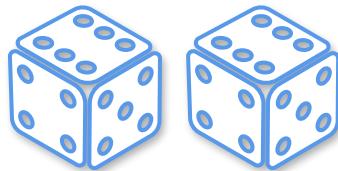
$$P(6) = \text{_____} = \frac{1}{6}$$

Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

Introduction to Probability: Dice Example 2



What is the probability of obtaining 6,6?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(6,6) = \frac{1}{36} = \frac{1}{36}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

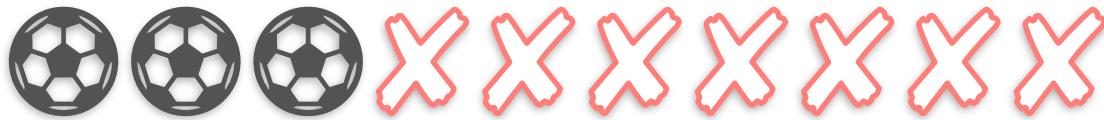


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Introduction to probability

Complement of Probability

Complement of Probability



30%

What is the probability of a child NOT playing soccer?

Complement of Probability

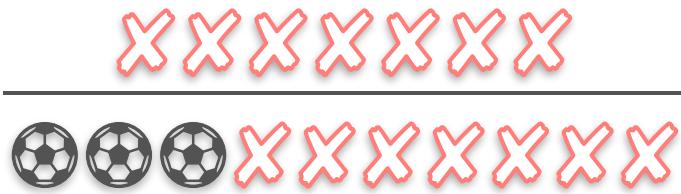


30%

What is the probability of a child NOT playing soccer?

$$P(\text{not soccer}) = \frac{\text{not soccer}}{\text{total}} = \frac{\text{XXXXXXX}}{\text{Soccer Balls XXXXXXXX}} = \frac{7}{10} = 0.7$$

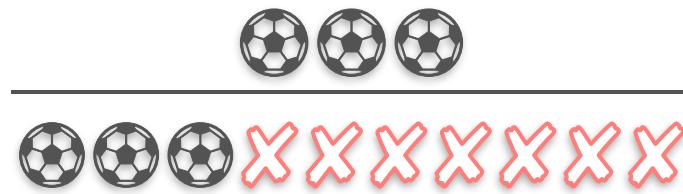
Complement of Probability



$P(\text{not soccer})$

0.7

+

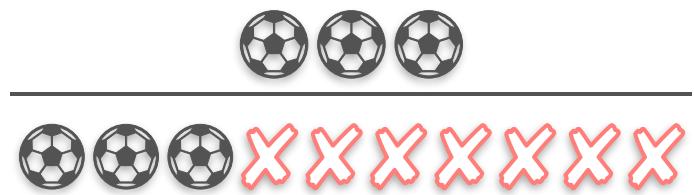
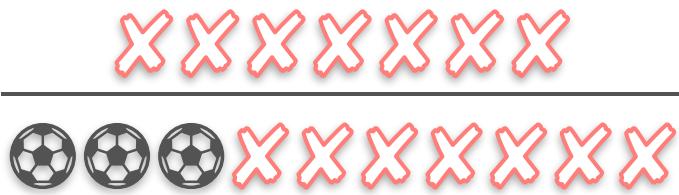


$P(\text{soccer})$

0.3

= 1

Complement of Probability



$P(\text{not soccer})$

$P(\text{soccer})$

0.7

= 1 —

0.3

Complement Rule

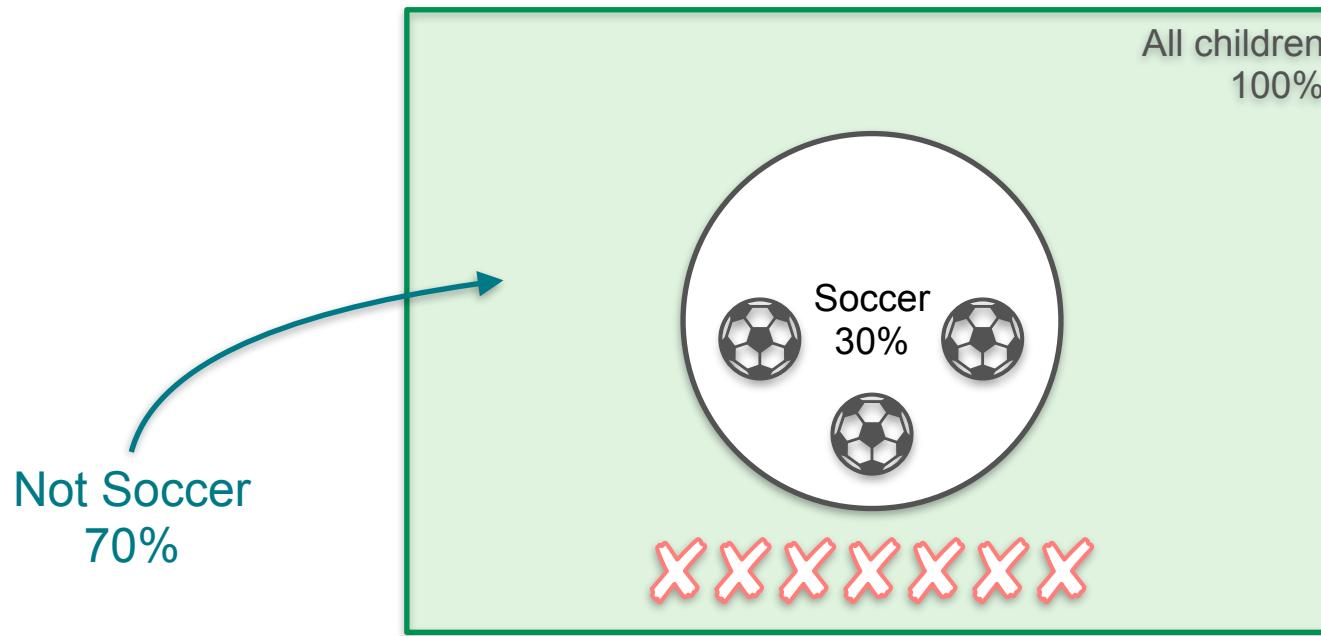
Complement of Probability

$$P(\text{not soccer}) = 1 - P(\text{soccer})$$

$$P(A') = 1 - P(A)$$

Complement Rule

Complement of Probability: Venn Diagram



Complement of Probability: Coin Example 1



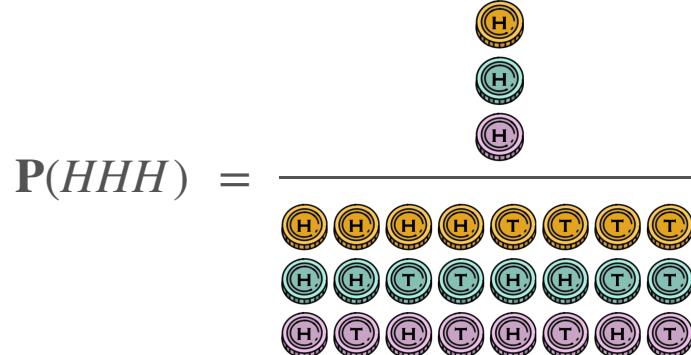
What is the probability of not landing on heads 3 times?

Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$



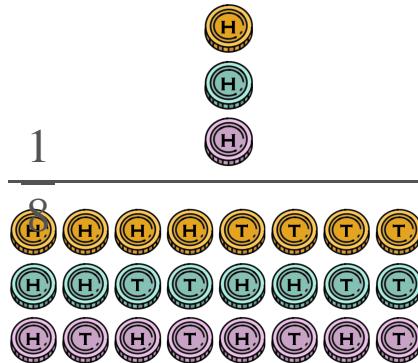
Complement of Probability: Coin Example 1



What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$P(\text{not } HHH) = 1 - \frac{1}{8}$$



Complement of Probability: Coin Example 1

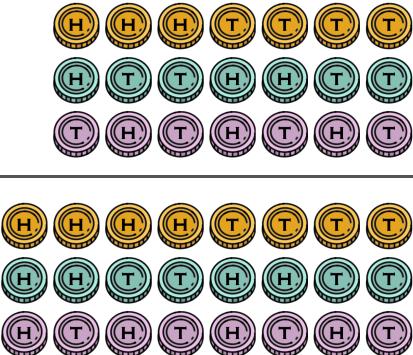


What is the probability of not landing on heads 3 times?

$$P(\text{not } HHH) = 1 - P(HHH)$$

$$P(\text{not } HHH) = 1 - \frac{1}{8}$$

$$= \frac{7}{8}$$



Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?

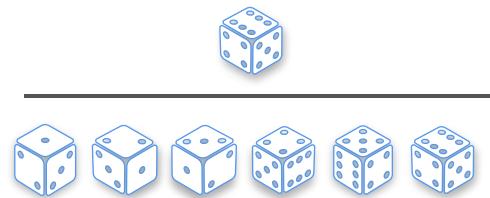
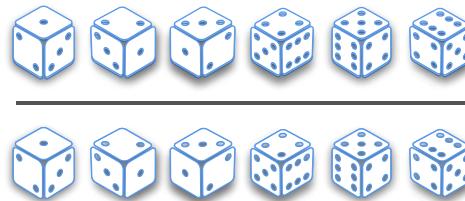
Complement of Probability: Dice Example 1



What is the probability of obtaining anything other than 6?



$$P(\text{not } 6) =$$



$$= \underline{\hspace{2cm}}$$

$$= \frac{5}{6}$$

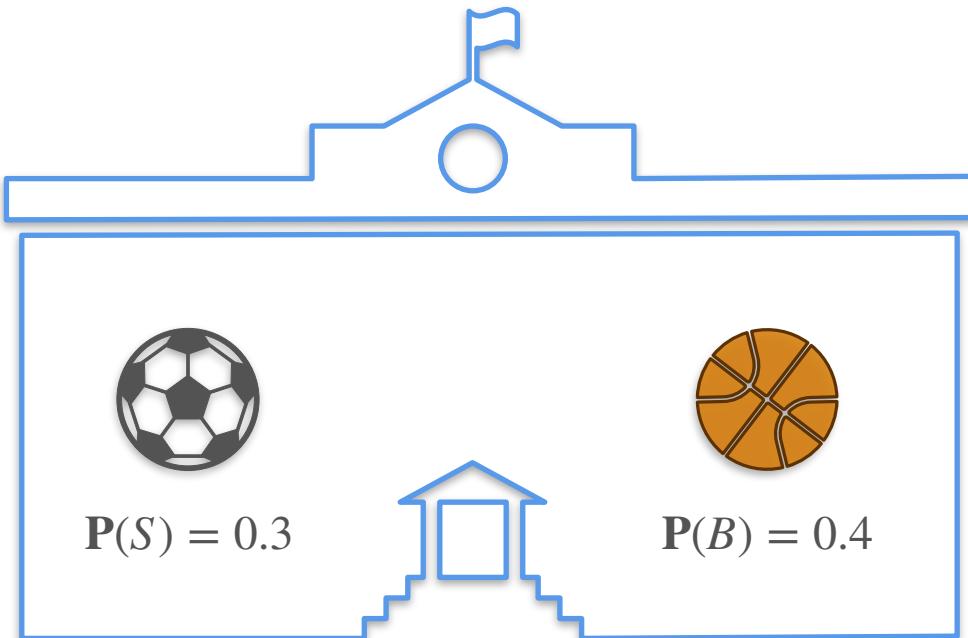


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Introduction to probability

**Sum of Probabilities
(Disjoint Events)**

Sum of Probabilities: Quiz 1

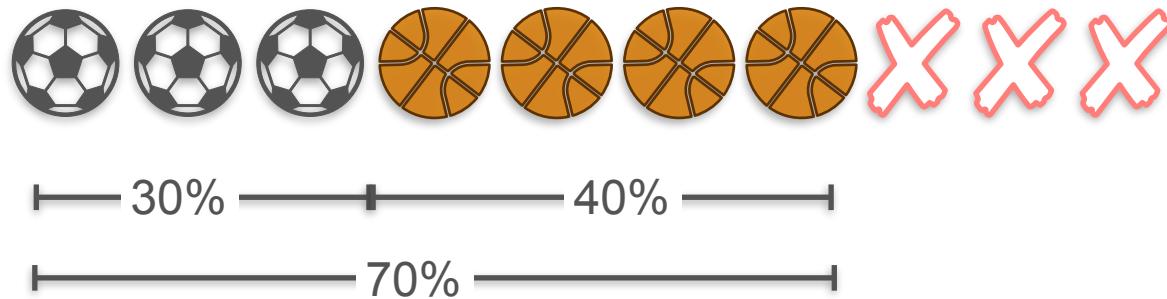


At a school, kids can only play one sport.

What is the probability that a kid plays soccer or basketball?

Hint: What if there were only 10 kids?

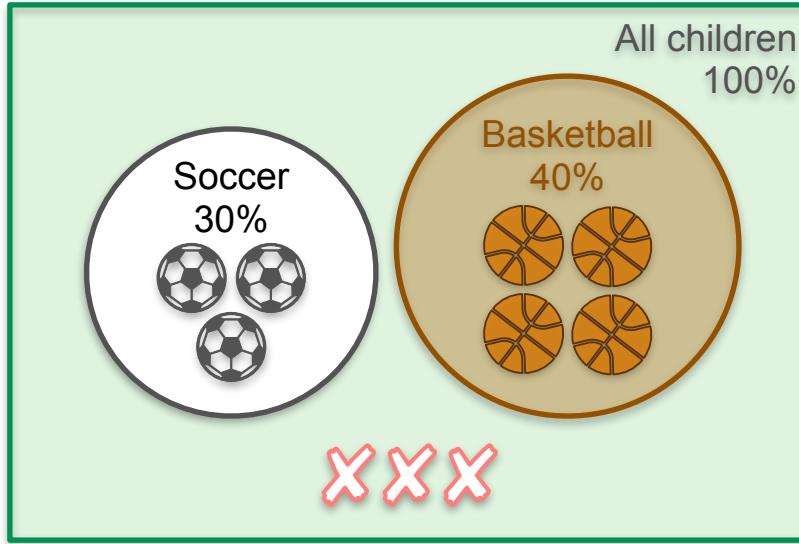
Sum of Probabilities: Quiz 1 Solution



$$P(\text{soccer or basketball}) = \frac{\text{soccer or basketball}}{\text{total}} = \frac{3 + 4}{10} = 0.7$$

$$P(\text{soccer or basketball}) = P(\text{soccer}) + P(\text{basketball})$$

Sum of Probabilities: Quiz 1 Solution



$$P(\text{soccer} \cup \text{basketball}) = P(\text{soccer}) + P(\text{basketball})$$

$$P(A \cup B) = P(A) + P(B)$$

Sum of Probabilities: Dice Example 1



What is the probability of obtaining
an even number or a 5?

Sum of Probabilities: Dice Example 1



What is the probability of obtaining an even number or a 5?

A



_____ + _____

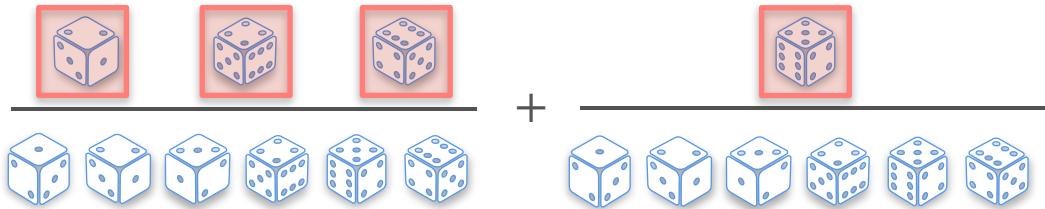
B



Sum of Probabilities: Dice Example 1

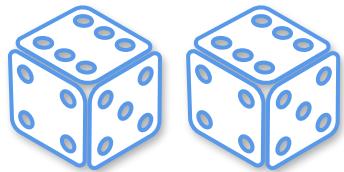


What is the probability of obtaining an even number or a 5?



$$P(\text{even number or } 5) = P(\text{even number}) + P(5) = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$

Sum of Probabilities: Dice Example 2



What is the probability of obtaining a sum of 7 or a sum of 10?

Sum of Probabilities: Dice Example 2

A

sum of 7

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

B

sum of 10

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

A or *B*

sum of 7 or sum of 10

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 2

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

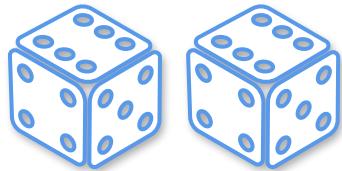
P(sum of 7 or sum of 10)

$$= \boxed{P(\text{sum of 7})} + \boxed{P(\text{sum of 10})}$$

$$= \frac{6}{36} + \frac{3}{36}$$

$$= \frac{9}{36} = \frac{1}{4}$$

Sum of Probabilities: Dice Example 3



What is the probability of obtaining
a difference of 2 or a difference of 1?

Sum of Probabilities: Dice Example 3

A

diff = 2

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities: Dice Example 3

A or *B*

diff = 2 or diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Dice icons are placed along the top row and left column of the grid.

Sum of Probabilities: Dice Example 3

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$P(\text{diff} = 2 \text{ or } \text{diff} = 1)$

$$= P(\text{diff} = 2) + P(\text{diff} = 1)$$

$$= \frac{8}{36} + \frac{10}{36}$$

$$= \frac{18}{36}$$

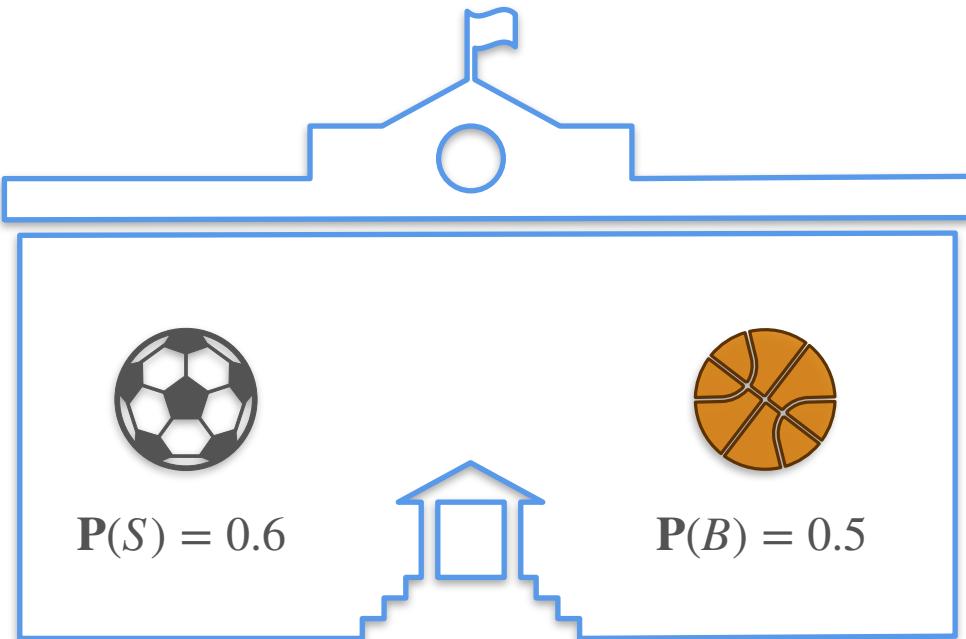


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Introduction to probability

**Sum of Probabilities
(Joint Events)**

Sum of Probabilities (Joint Events): Quiz 1

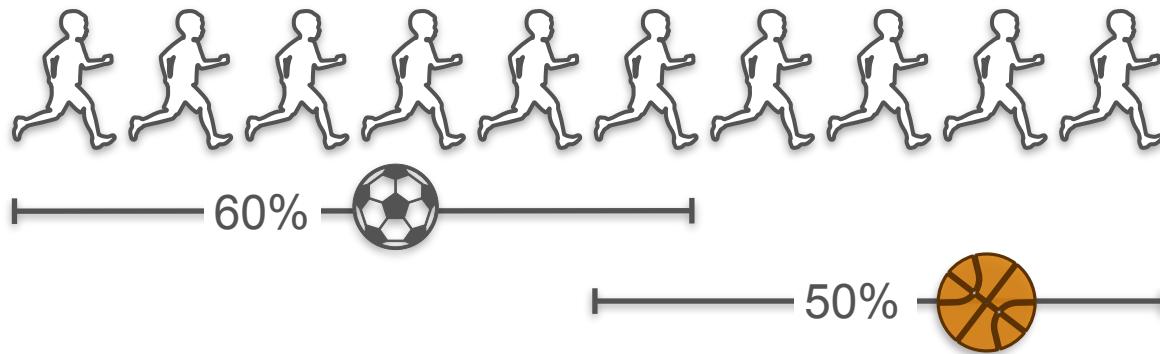


At a school, kids can play as many sports as they want.

What is the probability that a kid plays soccer or basketball?

Hint: What if there were only 10 kids?

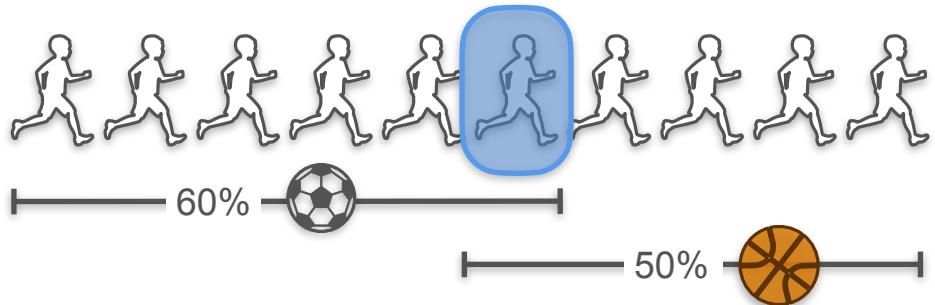
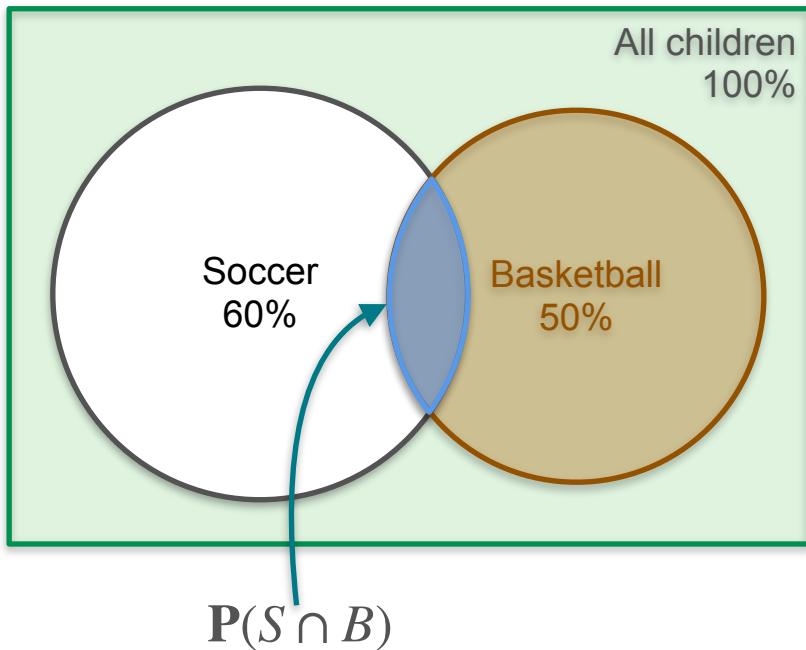
Sum of Probabilities (Joint Events): Quiz 1 Solution



$$P(\text{soccer} \cup \text{basketball}) = ?$$

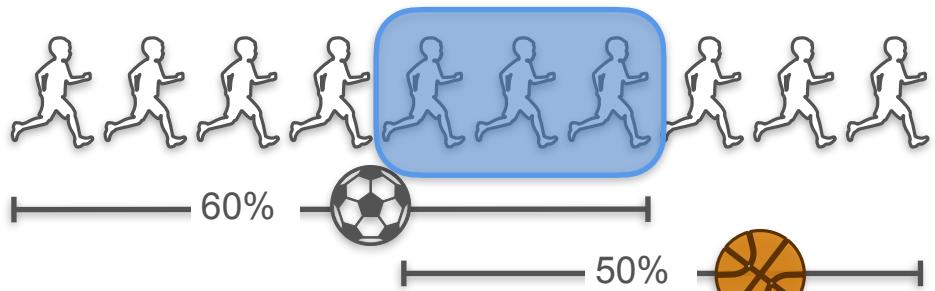
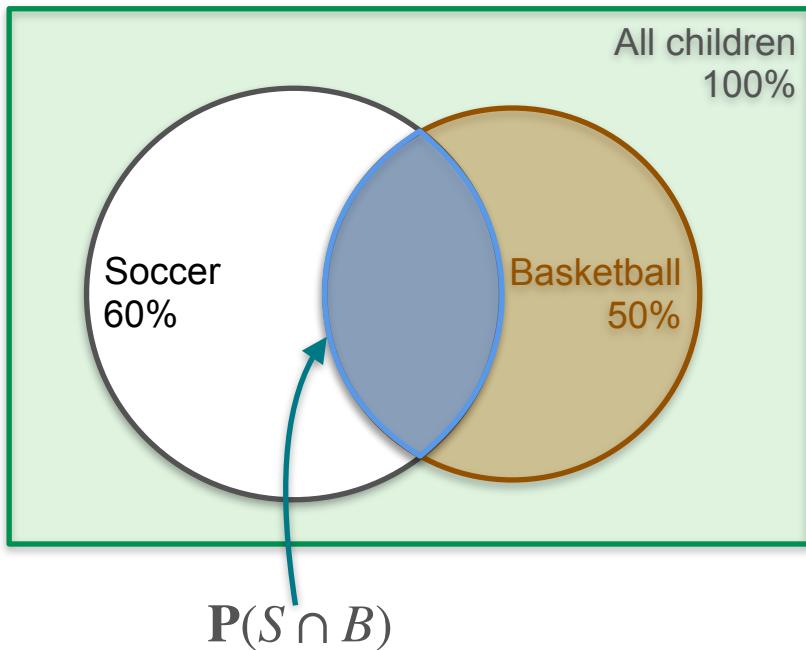
We don't know how many children play multiple sports

Sum of Probabilities (Joint Events): Quiz 1 Solution



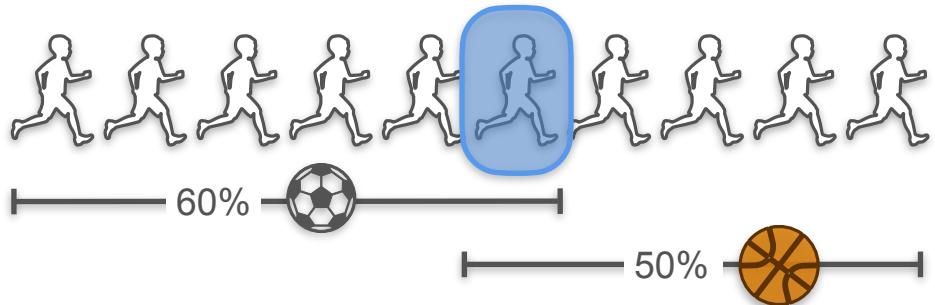
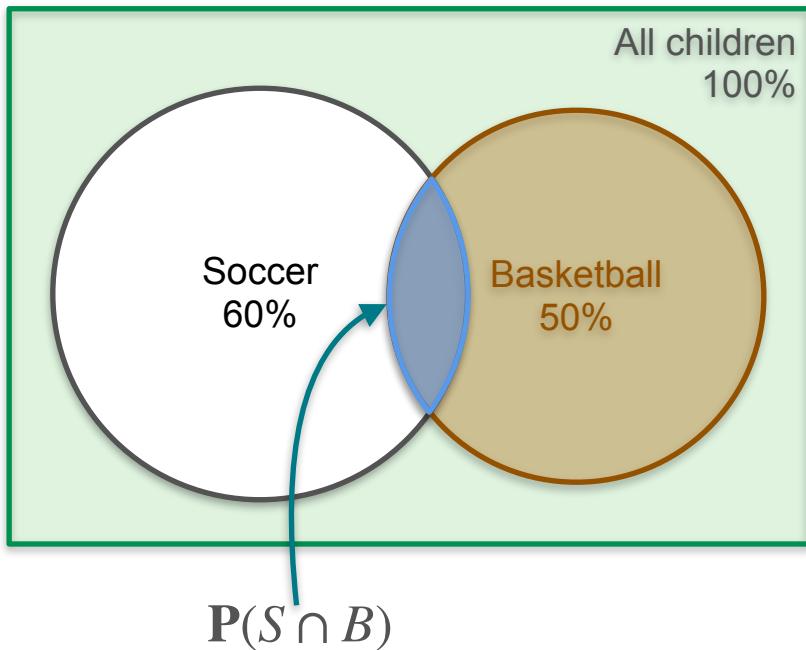
$$P(S \cup B) = P(S) + P(B)$$

Sum of Probabilities (Joint Events): Quiz 1 Solution



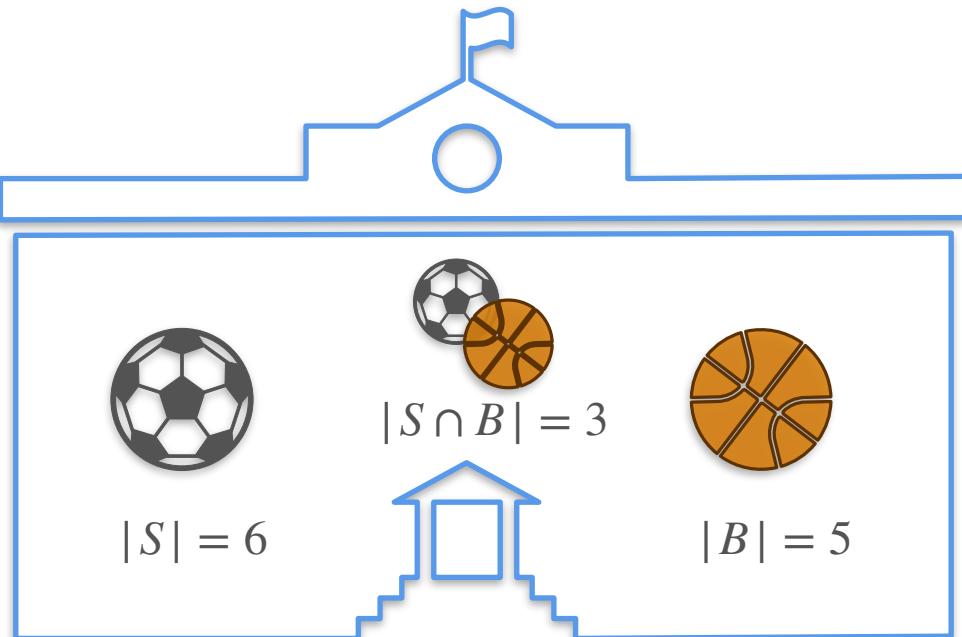
$$P(S \cup B) = P(S) + P(B)$$

Sum of Probabilities (Joint Events): Quiz 1 Solution



$$P(S \cup B) = P(S) + P(B)$$

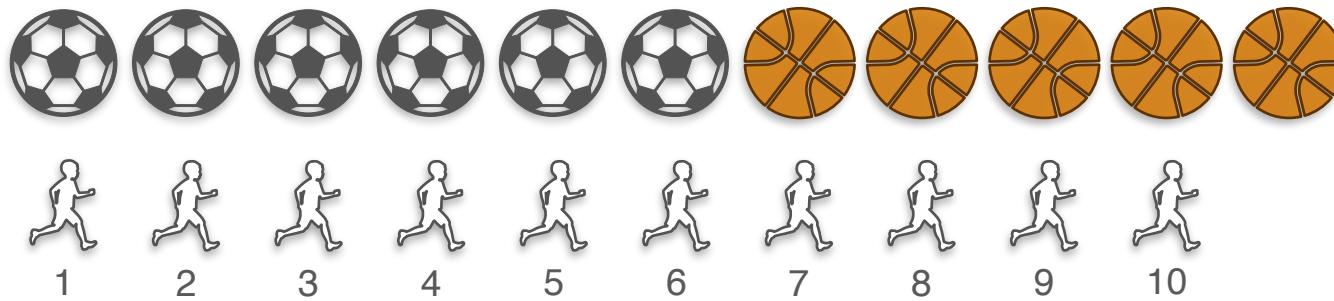
Sum of Probabilities (Joint Events): Quiz 2



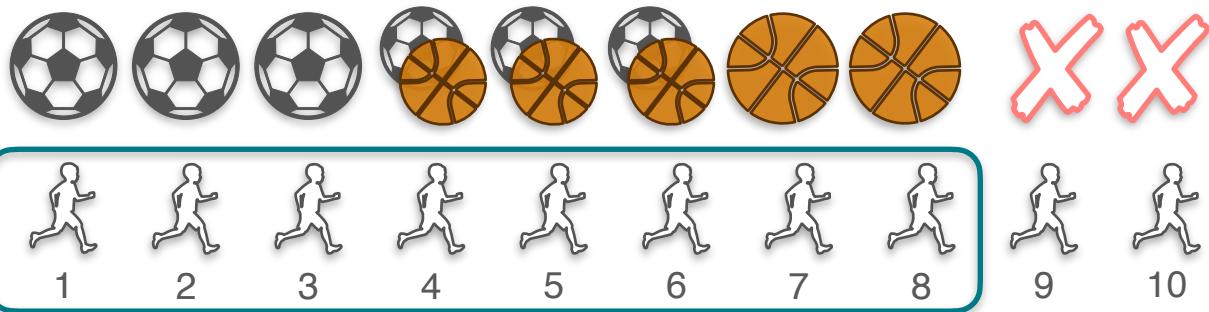
How many kids play soccer or basketball?

Hint: What if there were only 10 kids?

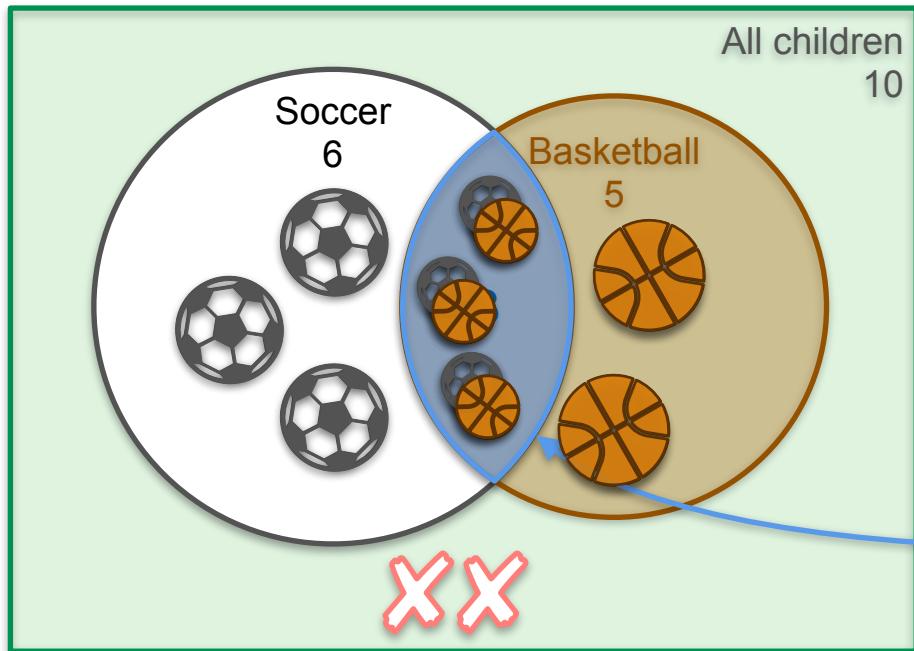
Sum of Probabilities (Joint Events): Quiz 2 Solution



Sum of Probabilities (Joint Events): Quiz 2 Solution



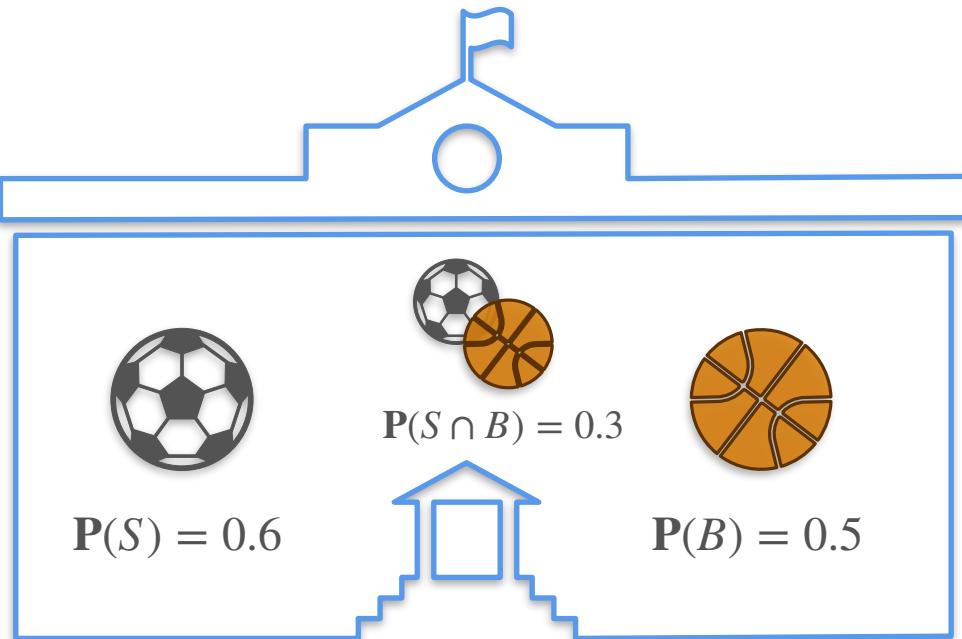
Sum of Probabilities (Joint Events): Venn Diagram



$$\begin{aligned}|S \cup B| &= |S| + |B| - |S \cap B| \\&= 6 + 5 - 3 \\&= 8\end{aligned}$$

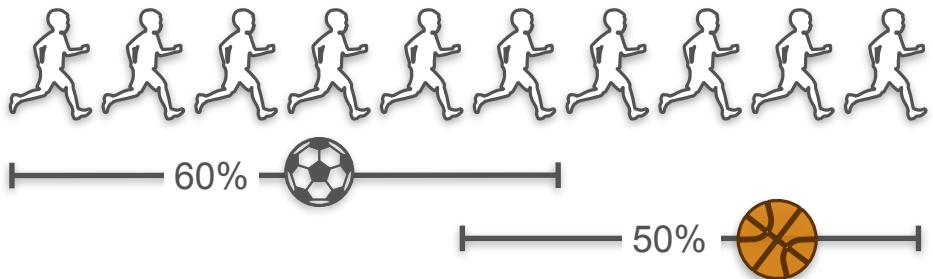
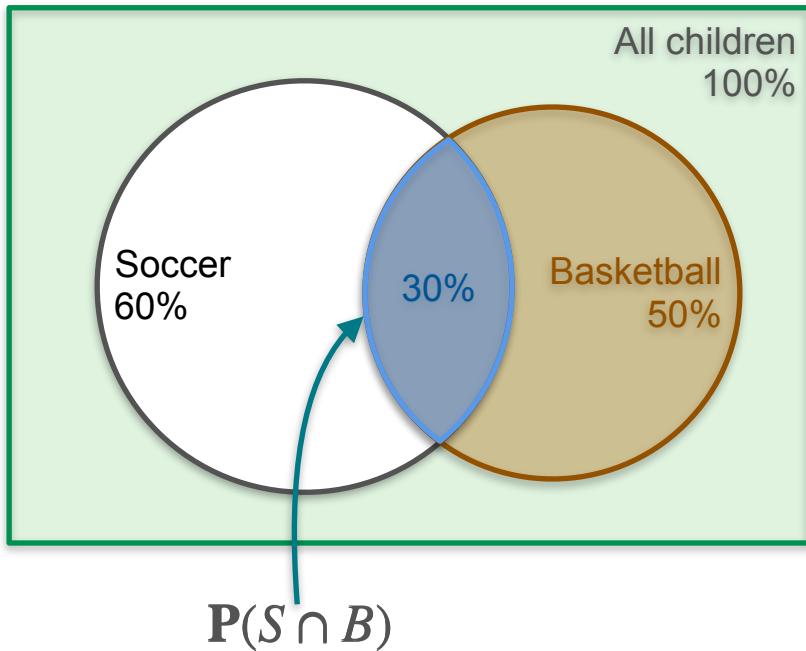
Soccer and Basketball

Sum of Probabilities (Joint Events): Quiz 3



What is the probability that a child plays soccer or basketball?

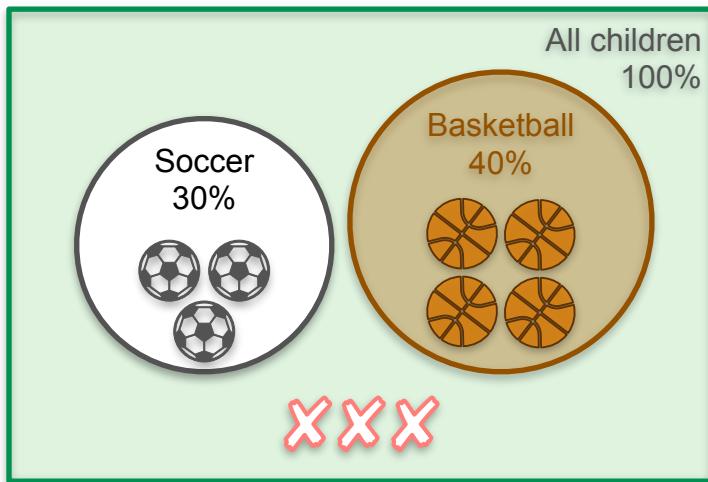
Sum of Probabilities (Joint Events): Quiz 3 Solution



$$\begin{aligned}\mathbf{P}(S \cup B) &= \mathbf{P}(S) + \mathbf{P}(B) - \mathbf{P}(S \cap B) \\ &= 0.6 + 0.5 - 0.3 \\ &= 0.8\end{aligned}$$

Disjoint Events vs Joint Events

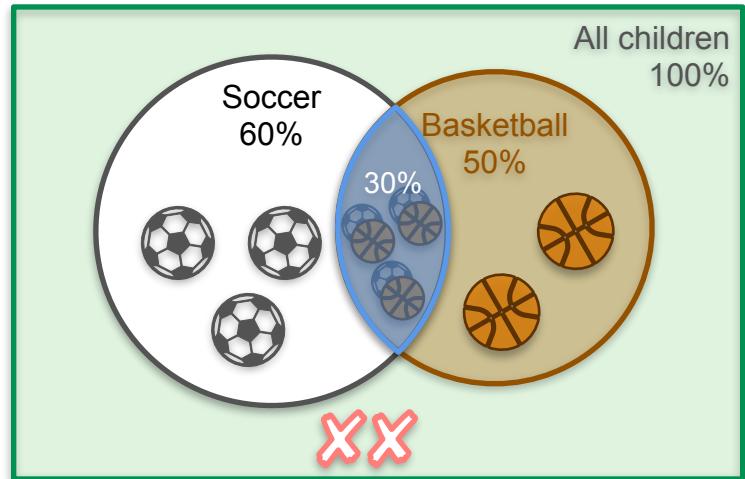
Disjoint



Mutually exclusive

$$P(S \cup B) = P(S) + P(B)$$

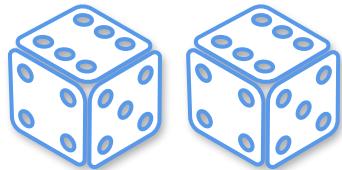
Joint



Non-mutually exclusive

$$P(S \cup B) = P(S) + P(B) - P(S \cap B)$$

Sum of Probabilities (Joint Events): Dice Example 1



What is the probability of obtaining a sum of 7 or a difference of 1?

Sum of Probabilities (Joint Events): Dice Example 1

A

sum = 7

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

B

diff = 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

Sum of Probabilities (Joint Events): Dice Example 1

A or B

sum = 7 or diff = 1



sum = 7 and diff = 1

Sum of Probabilities (Joint Events): Dice Example 1



1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$P(\text{sum} = 7 \text{ or } \text{diff} = 1)$

$$= P(\text{sum} = 7) + P(\text{diff} = 1) - P(\text{sum} = 7 \cap \text{diff} = 1)$$

$$= \frac{6}{36} + \frac{10}{36} - \frac{2}{36}$$

$$= \frac{14}{36}$$

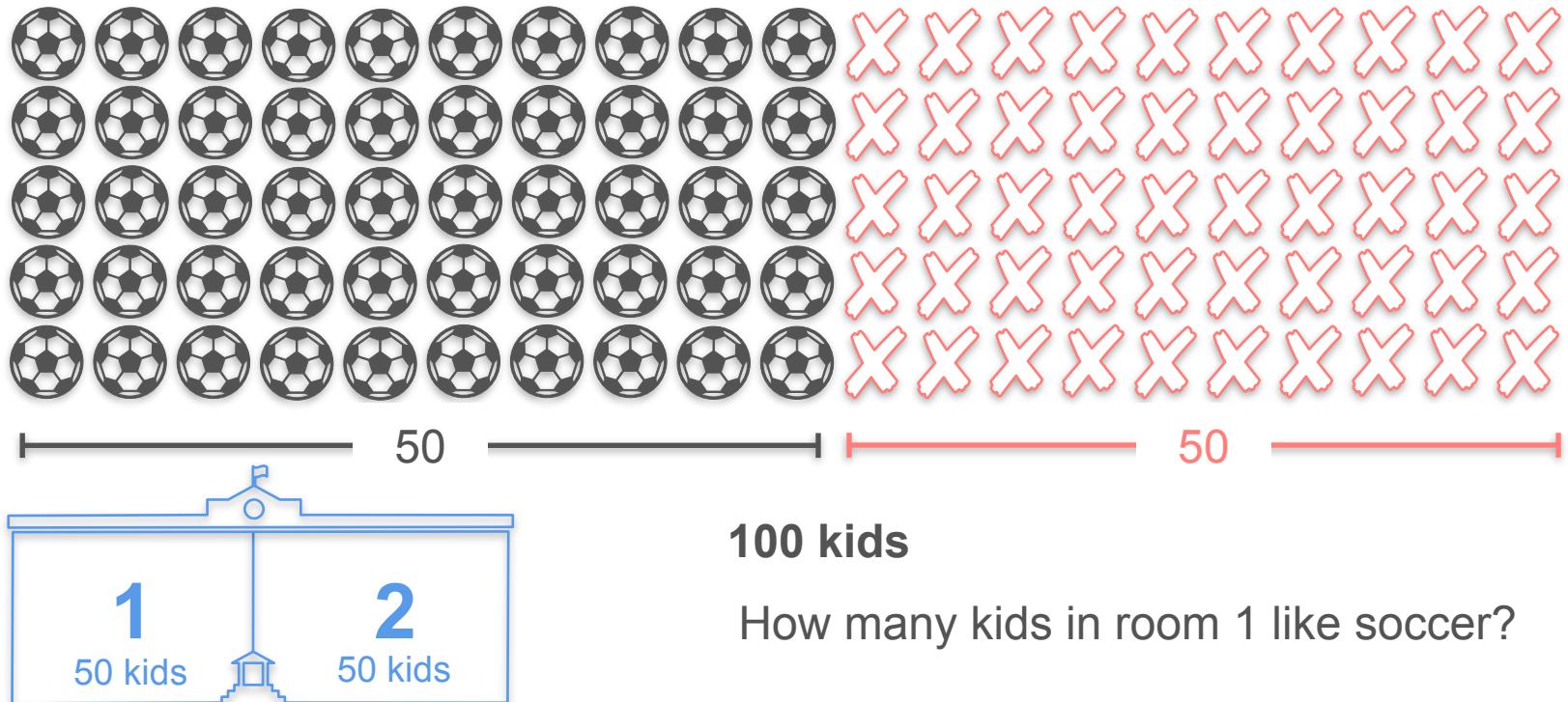


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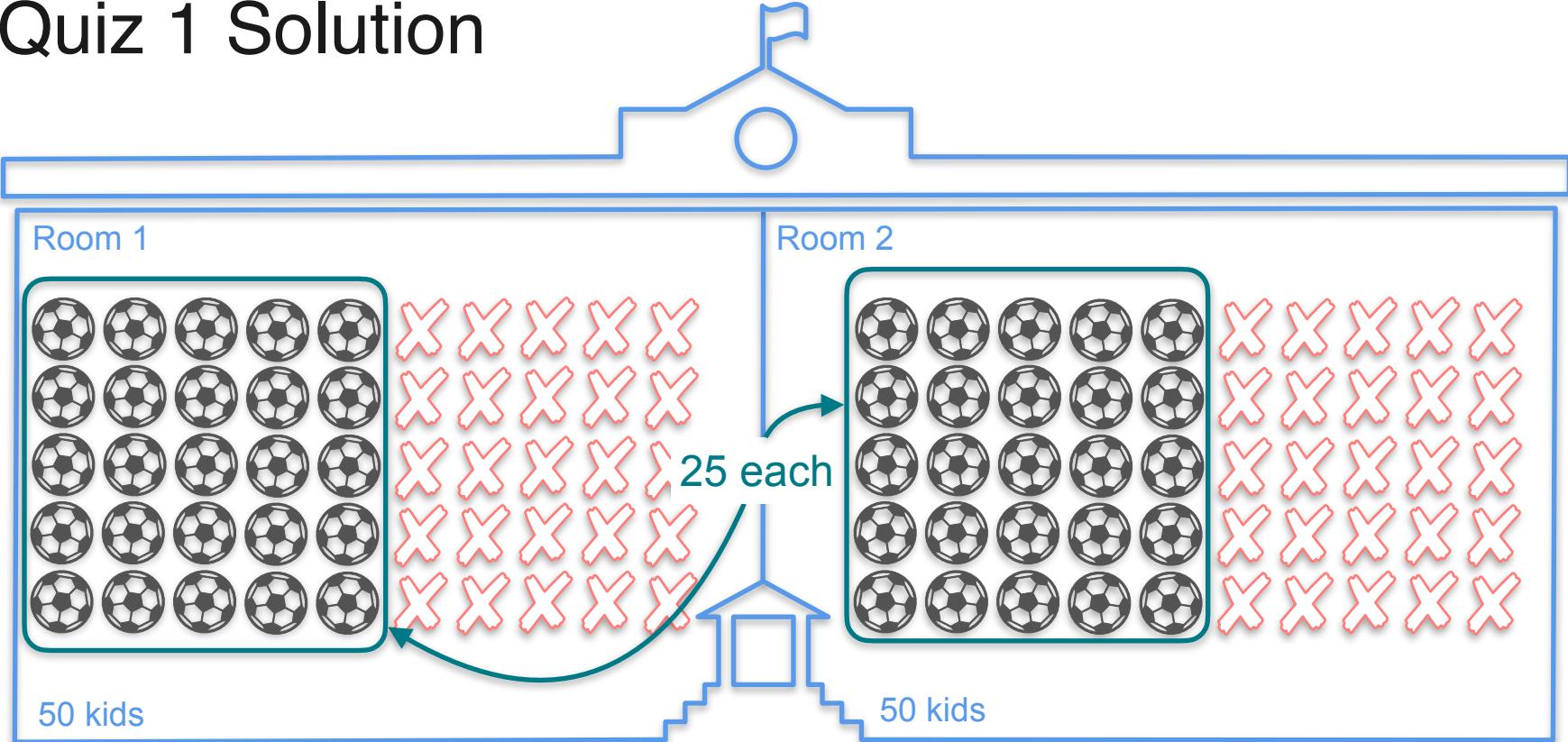
Introduction to probability

Independence

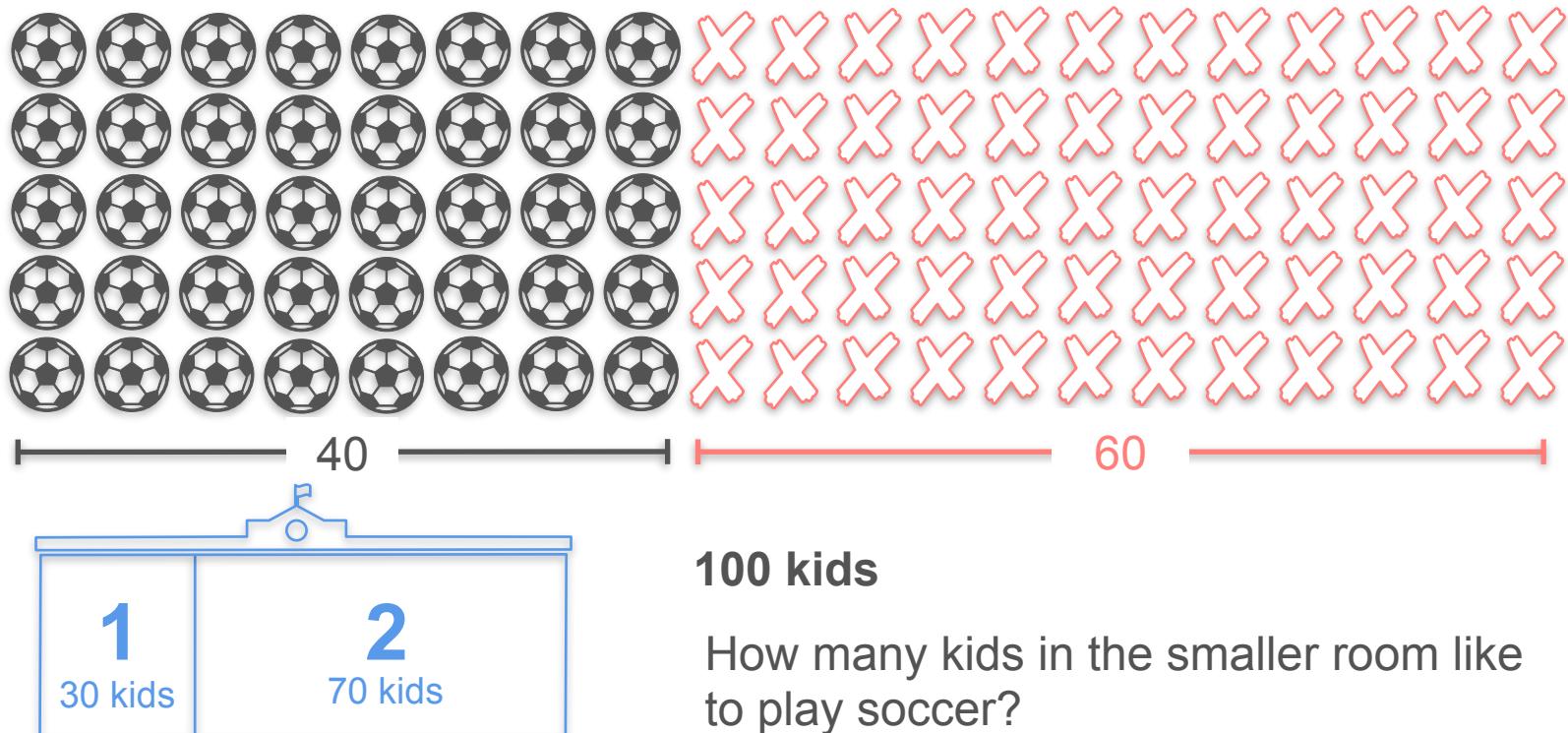
Independence: Quiz 1



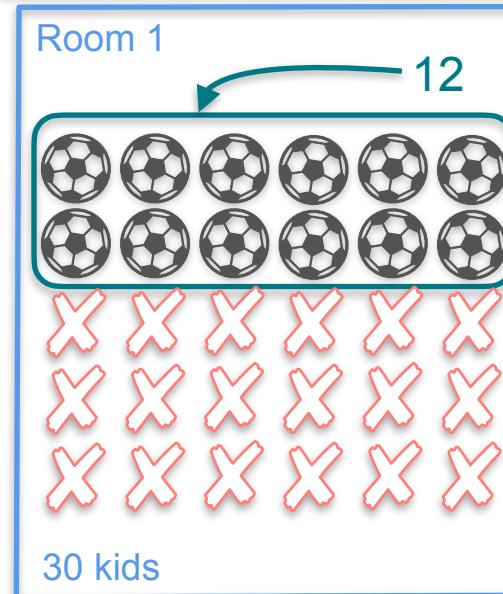
Quiz 1 Solution



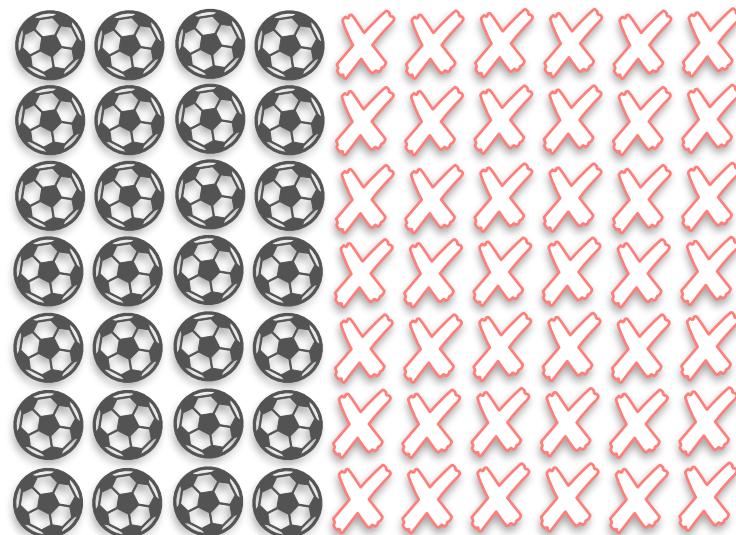
Independence: Quiz 2



Quiz 2 Solution



Room 2



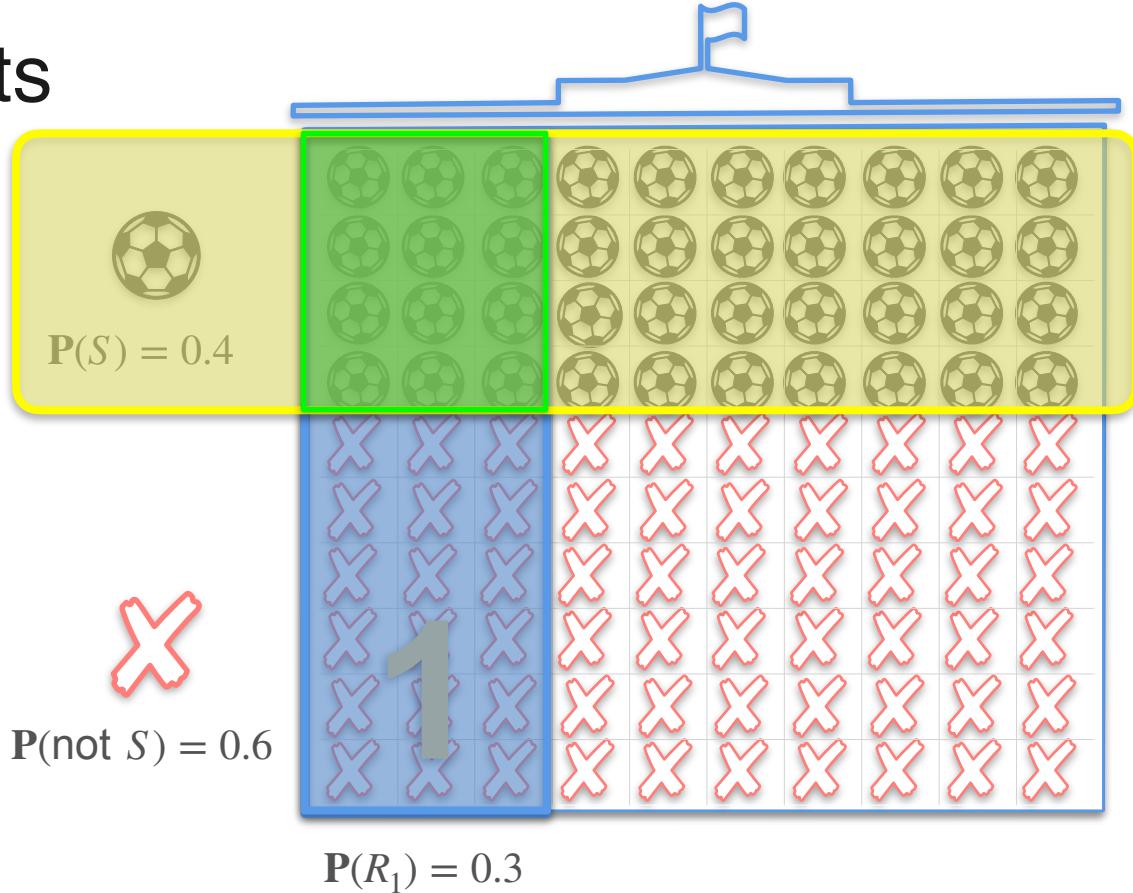
Independent Events

$P(\text{Soccer and Room 1})$

$$P(S \cap R_1) = P(S) \bullet P(R_1)$$

$$= 0.4 \bullet 0.3$$

$$= 0.12$$



Product Rule (for Independent Events)

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

Independent Events: Coin Example 1



What is the probability of landing on heads five times?



Independent Events: Coin Example 1



50% 50%

What is the probability of landing on heads five times?

$$P(5 \text{ heads}) =$$

$$\frac{\text{Diagram}}{\text{Probability}} \cdot \frac{\text{Diagram}}{\text{Probability}} \cdot \frac{\text{Diagram}}{\text{Probability}} \cdot \frac{\text{Diagram}}{\text{Probability}} \cdot \frac{\text{Diagram}}{\text{Probability}}$$

The diagram illustrates the calculation of the probability of getting 5 heads in a row. It shows five separate coin tosses, each with two possible outcomes: heads (H) or tails (T). The top row shows the outcome of each toss, and the bottom row shows the probability of each outcome, which is $\frac{1}{2}$.

Diagram:

- Toss 1: H (top), H (bottom)
- Toss 2: H (top), T (bottom)
- Toss 3: H (top), T (bottom)
- Toss 4: H (top), T (bottom)
- Toss 5: H (top), T (bottom)

Probability:

- Toss 1: $\frac{1}{2}$
- Toss 2: $\frac{1}{2}$
- Toss 3: $\frac{1}{2}$
- Toss 4: $\frac{1}{2}$
- Toss 5: $\frac{1}{2}$

Independent Events: Coin Example 2



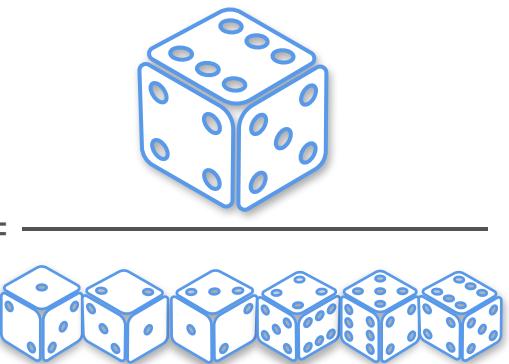
What is the probability of landing 5n heads five times?

$$P(5 \text{ heads}) = \left(\frac{\text{Diagram of 5 heads}}{\text{Diagram of 5 tails}} \right)$$

The numerator shows three coins stacked vertically, all showing heads (H). The denominator shows three coins stacked vertically, with the top coin showing heads (H) and the bottom two showing tails (T).

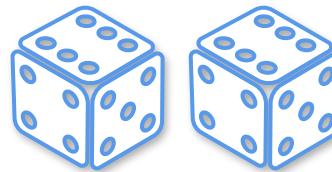
$$\left(\frac{1}{2} \right)^5 = \frac{1}{32}$$

Independent Events: Dice Example 1

$$P(6) = \frac{1}{6}$$


Independent Events: Dice Example 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6



2 dice

$$P(6,6) = \text{_____} = \frac{1}{36}$$

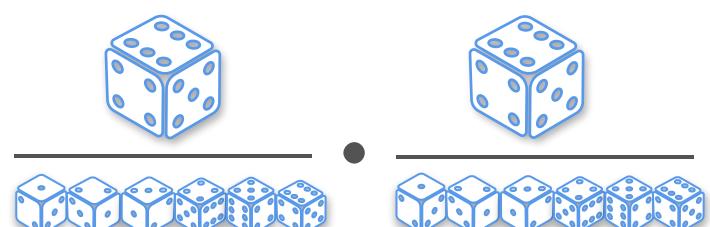
Independent Events: Dice Example 1

						$\frac{1}{6}$
	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
$\frac{1}{6}$	6,1	6,2	6,3	6,4	6,5	6,6

2 dice

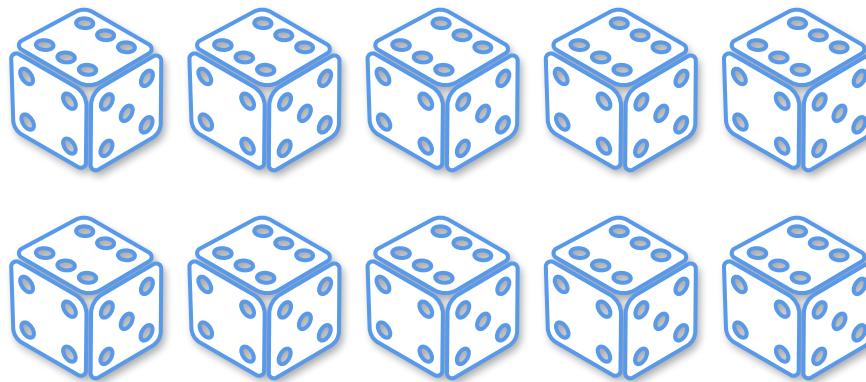
$$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

$P(6,6) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{6} \cdot \frac{1}{6} = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$



Independent Events: Dice Example 2

What is the probability of getting 10 sixes?



Independent Events: Dice Example 2

What is the probability of getting 10 sixes?

$$\begin{aligned} P(10 \text{ sixes}) &= \left(\frac{\text{one die showing 6}}{\text{one die showing 6}} \right)^{10} \\ &= \left(\frac{1}{6} \right)^{10} \end{aligned}$$



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Introduction to probability

Birthday problem

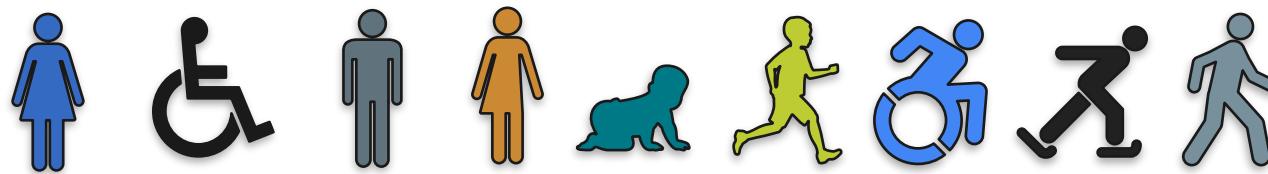
Quiz

- You have 30 friends at a party. What do you think is more likely:
 - That there exist two people with the same birthday
 - That no two of them have the same birthday
- (Assume the year has 365 days, nobody has a birthday on Feb 29).

Quiz

- Answer: It's more likely that 2 people have the same birthday.
- In fact, the probability of no two people having the same birthday is around 0.3.

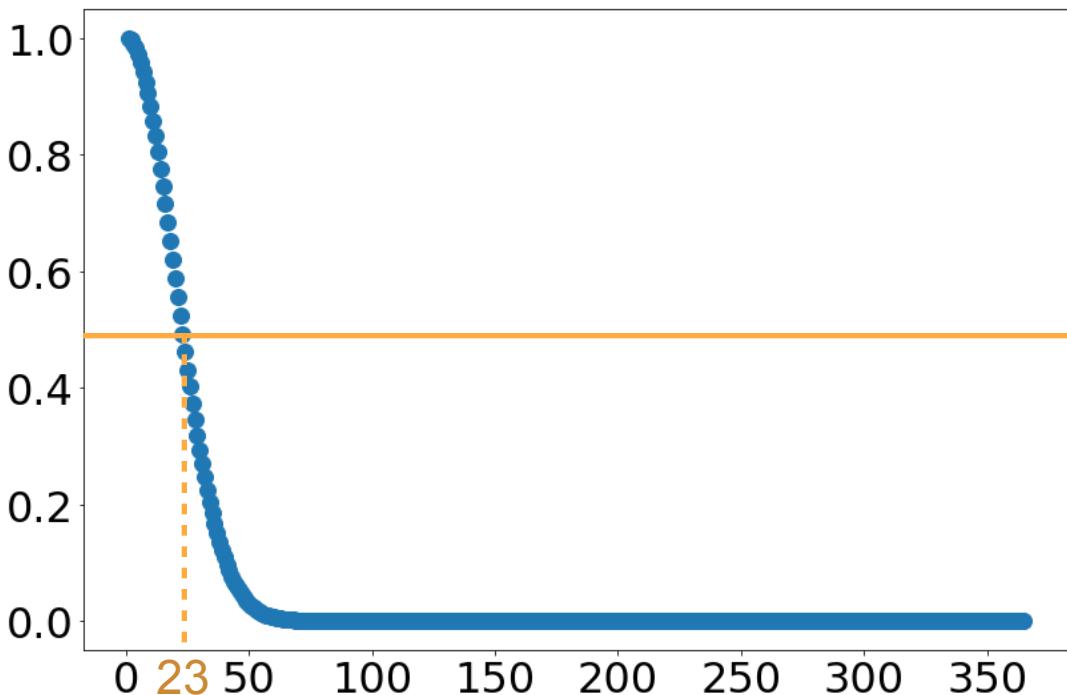
Probability That Everyone Has a Different Birthday



$$\frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \frac{361}{365} \cdot \frac{360}{365} \cdot \frac{359}{365} \cdot \frac{358}{365} \cdot \frac{357}{365} = 0.905$$

Probability That no Two People Have the Same Birthday

1 person: 1
2 people: 0.997
3 people: 0.992
4 people: 0.984
5 people: 0.973
10 people: 0.883
20 people: 0.589
23 people: 0.493
30 people: 0.294
50 people: 0.030
100 people: 0.0000003
365 people: 0





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Introduction to Probability

Conditional probability

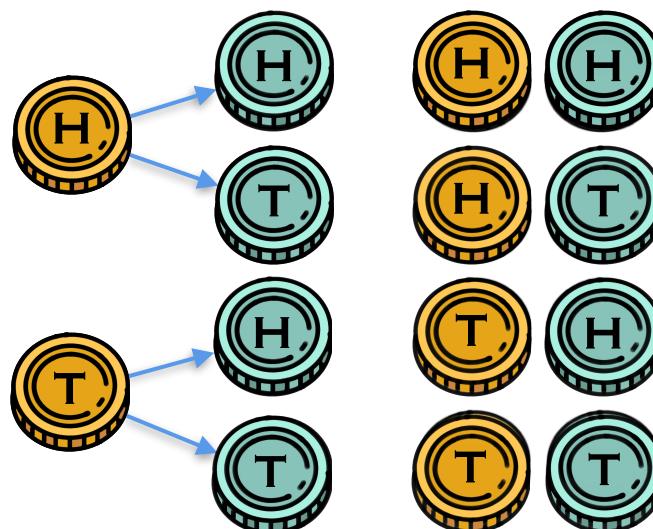
Conditional Probability: Coin Example 1



50% 50%

What is the probability of landing on heads twice?

1st 2nd



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$

The equation shows the probability of getting heads on both the 1st and 2nd coin tosses given that the first one is heads. The numerator is 1, representing the favorable outcome (HH), and the denominator is 4, representing the total number of outcomes in the sample space (HH, HT, TH, TT).

Conditional Probability: Coin Example 1

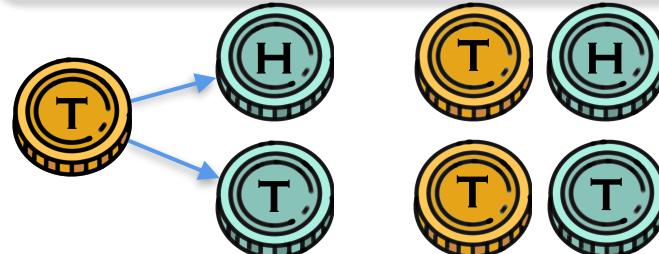
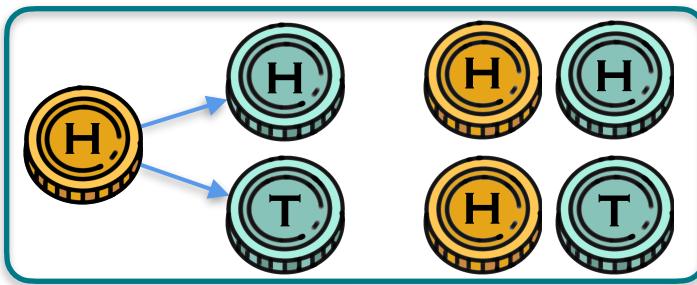


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is heads



$$P(HH) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{1}{4}$$

Conditional Probability: Coin Example 1

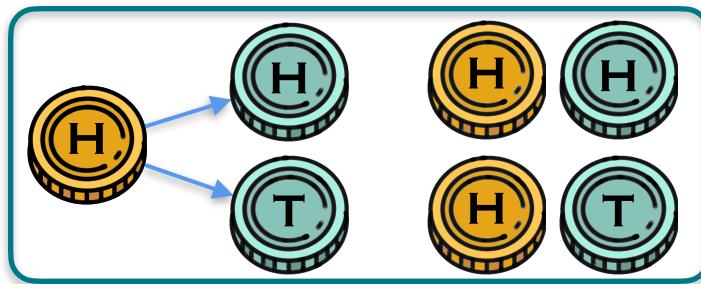


50% 50%

What is the probability of landing on heads twice?

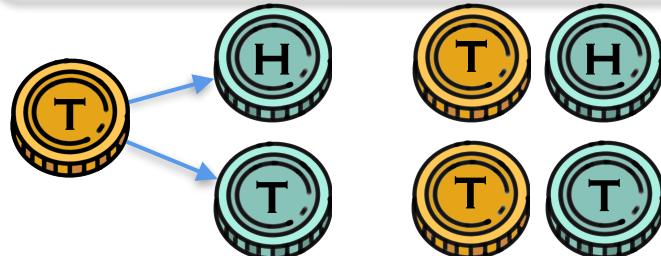
1st 2nd

GIVEN that the first one is heads



$$P(HH \mid 1\text{st is } H) =$$

$$= \frac{1}{2}$$



Conditional Probability: Coin Example 1

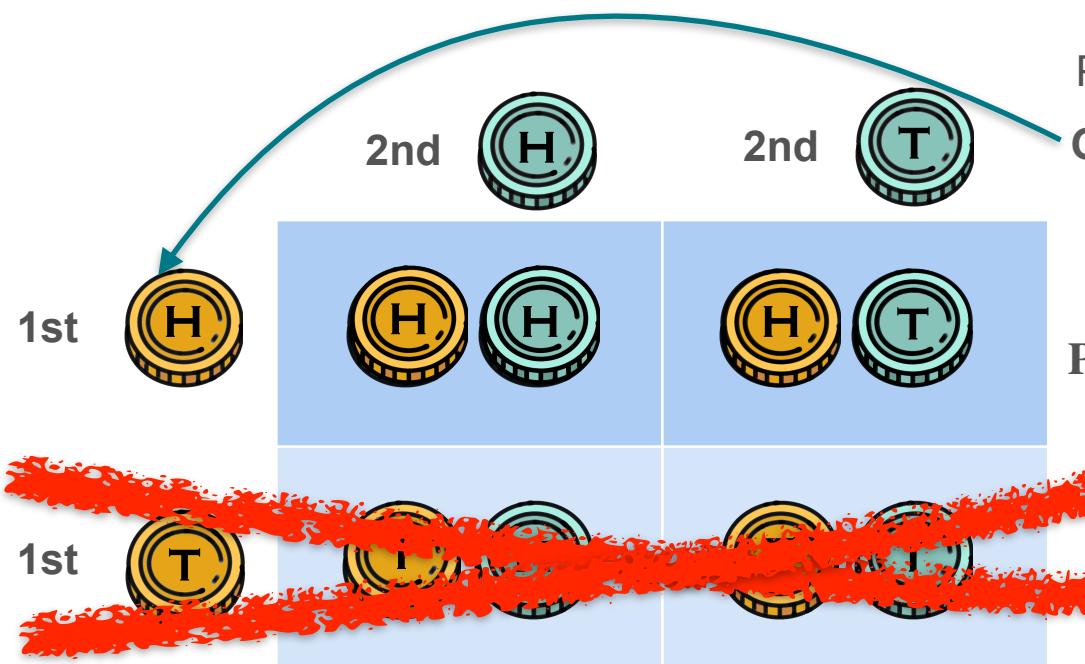
What is the probability of landing on heads twice?

GIVEN that the first one is heads

$$P(HH | \text{1st is } H)$$



Conditional Probability: Coin Example 1



Probability of landing on heads twice
GIVEN that the first one is heads

$$P(HH | \text{1st is } H) = \frac{1}{2}$$



Conditional Probability: Coin Example 2

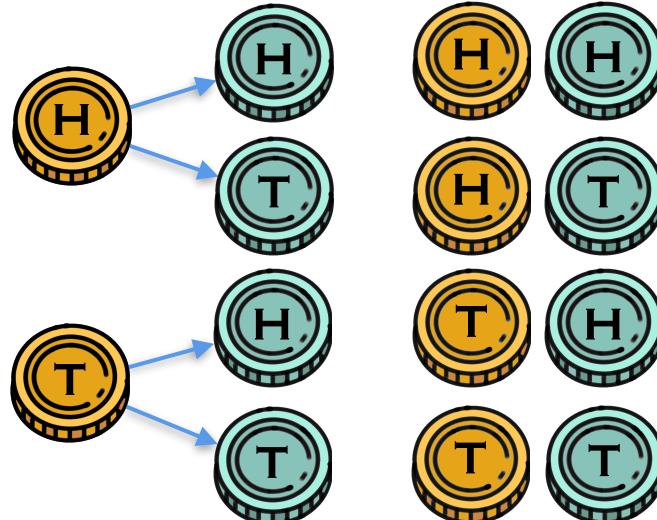


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \frac{1}{4} = \frac{1}{4}$$



Conditional Probability: Coin Example 2

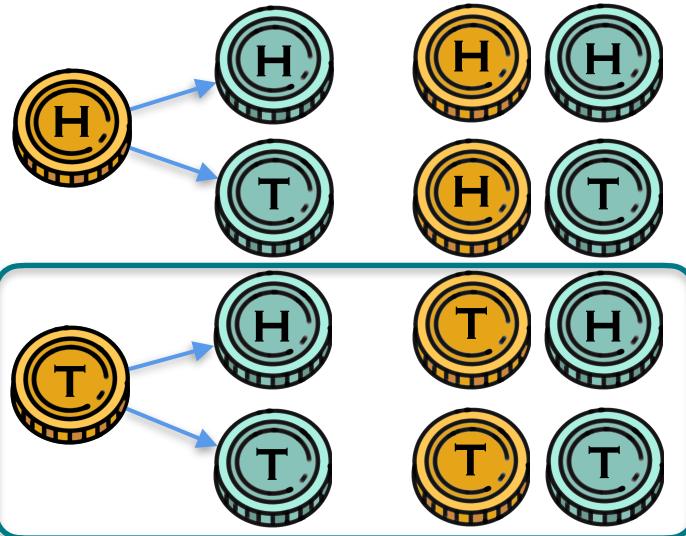


50% 50%

What is the probability of landing on heads twice?

1st 2nd

GIVEN that the first one is tails



$$P(HH) = \frac{\text{Number of HH outcomes}}{\text{Total number of outcomes given first is tails}} = \frac{1}{4}$$



Conditional Probability: Coin Example 2

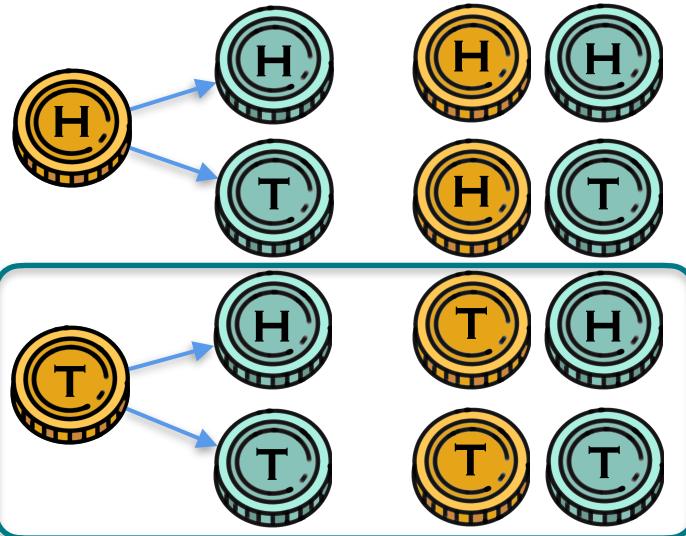


50% 50%

What is the probability of landing on heads twice?

1st 2nd

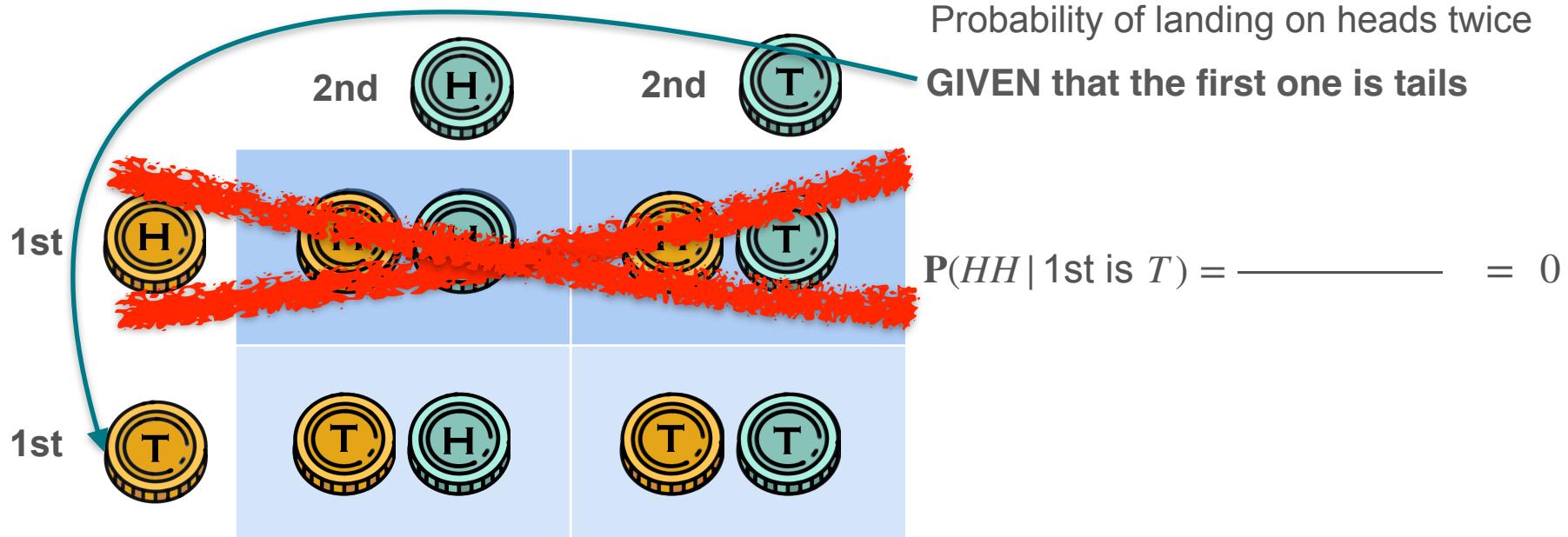
GIVEN that the first one is tails



$$P(HH \mid \text{1st is } T) = \frac{0}{\text{number of outcomes}} = 0$$



Conditional Probability: Coin Example 2

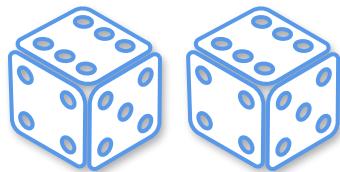


Product Rule (for Independent Events)

When A and B independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

Conditional Probability: Dice Example 3



What is the probability that
the first is 6 **AND** the sum = 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{1st is 6} \cap \text{sum} = 10) =$$

$$= \frac{1}{36}$$

Conditional Probability: Dice Example 3

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(1\text{st is } 6 \cap \text{sum} = 10) =$$

$$P(1\text{st is } 6)$$



$$P(\text{sum} = 10 | 1\text{st } 6)$$

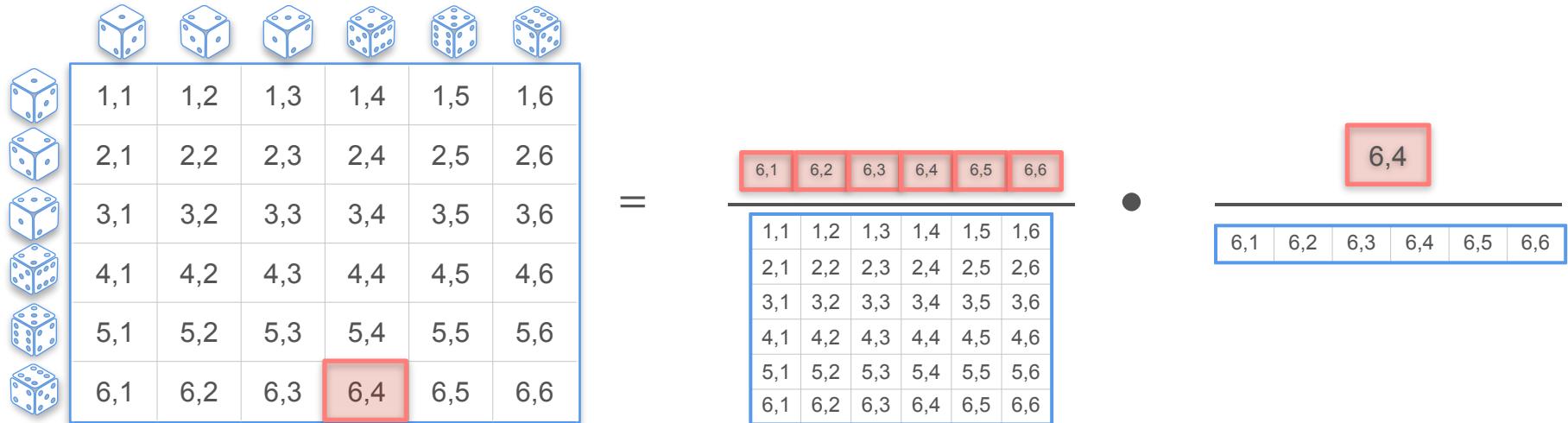
$$\frac{6}{36}$$



$$\frac{1}{6}$$

$$= \frac{1}{36}$$

Conditional Probability: Dice Example 3



$$P(A \cap B) = P(A) \cdot P(B | A)$$

The General Product Rule

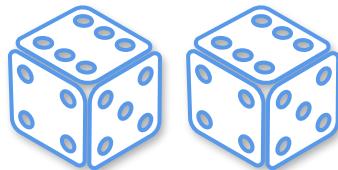
$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B | A)$$



When independent

$$\mathbf{P}(B | A) = \mathbf{P}(B)$$

Conditional Probability: Dice Example 1

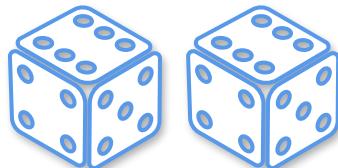


What is the probability that the sum is 10?

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{3}{36} = \frac{1}{12}$$

Conditional Probability: Dice Example 1



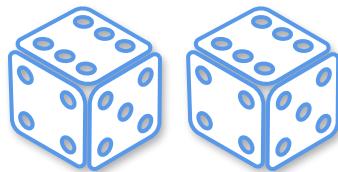
What is the probability that the sum is 10?

GIVEN that the first one is 6

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 1



What is the probability that the sum is 10?

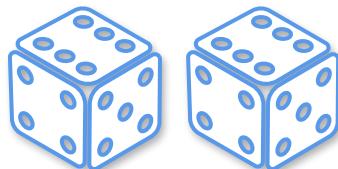
GIVEN that the first one is 6

	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6	
3,1	3,2	3,3	3,4	3,5	3,6	
4,1	4,2	4,3	4,4	4,5	4,6	
5,1	5,2	5,3	5,4	5,5	5,6	
6,1	6,2	6,3	6,4	6,5	6,6	

$$P(\text{sum} = 10 \mid \text{1st is } 6) = \underline{\hspace{10cm}}$$

$$= \frac{1}{6}$$

Conditional Probability: Dice Example 2



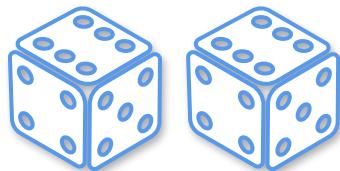
What is the probability that the sum is 10?

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{\text{Number of favorable outcomes}}{\text{Total number of outcomes}} = \frac{3}{36}$$

1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,2	6,3	6,4	6,5	6,6

Conditional Probability: Dice Example 2



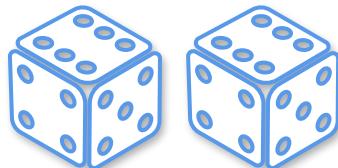
What is the probability that the sum is 10?

GIVEN that the first one is 1

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	2,1	2,2	2,3	2,4	2,5	2,6
2,1	3,1	3,2	3,3	3,4	3,5	3,6
3,1	4,1	4,2	4,3	4,4	4,5	4,6
4,1	5,1	5,2	5,3	5,4	5,5	5,6
5,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10cm}}$$

Conditional Probability: Dice Example 2



What is the probability that the sum is 10?

GIVEN that the first one is 1

	1,1	1,2	1,3	1,4	1,5	1,6
	2,1	2,2	2,3	2,4	2,5	2,6
	3,1	3,2	3,3	3,4	3,5	3,6
	4,1	4,2	4,3	4,4	4,5	4,6
	5,1	5,2	5,3	5,4	5,5	5,6
	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10 \mid \text{1st is } 1) = \underline{\hspace{10em}}^0$$

$$= 0$$

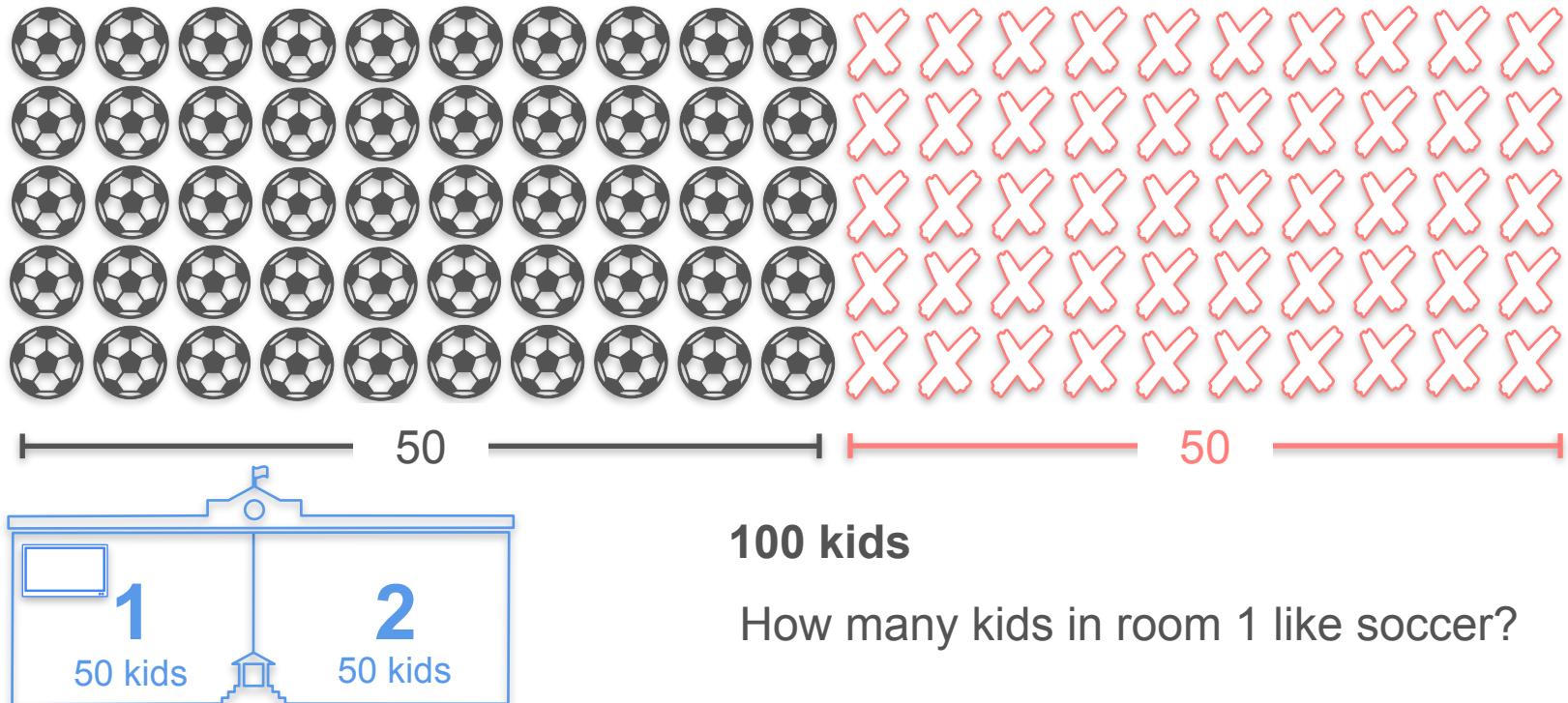


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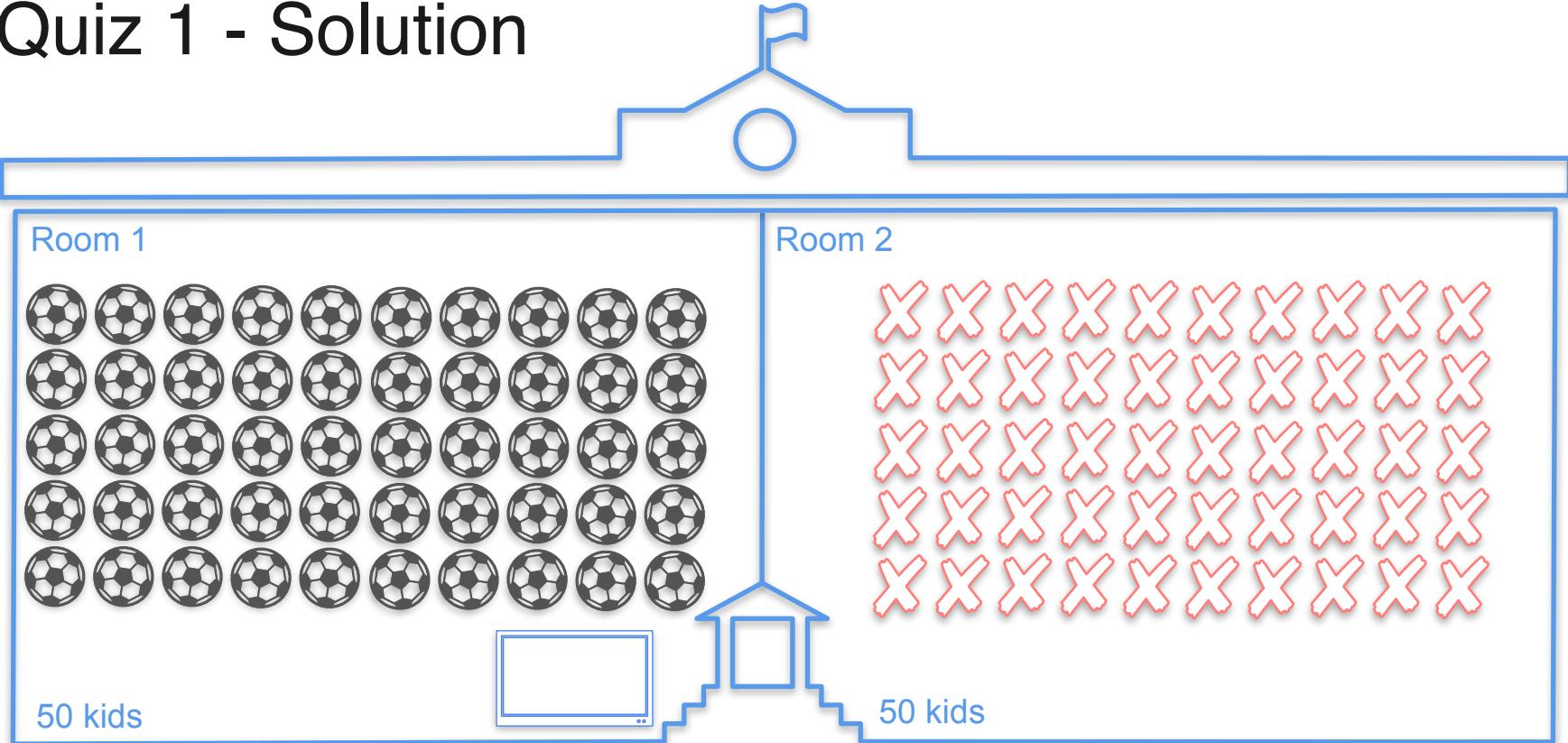
Introduction to Probability

Conditional probability - Part 2

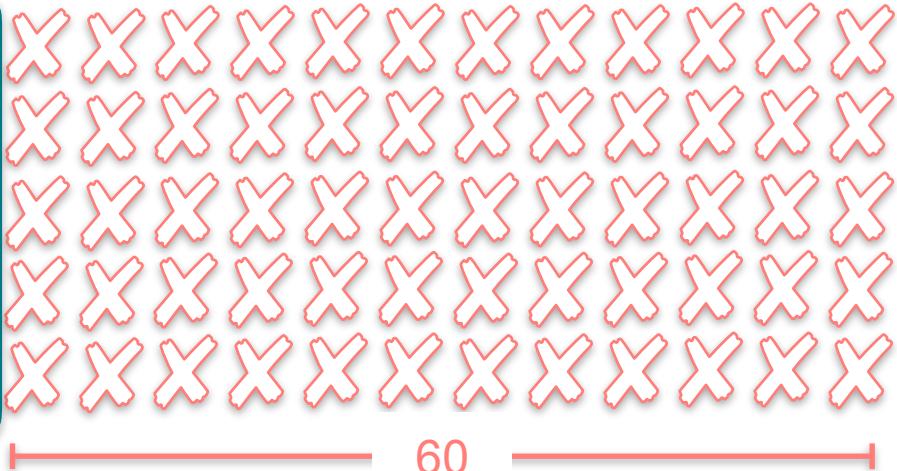
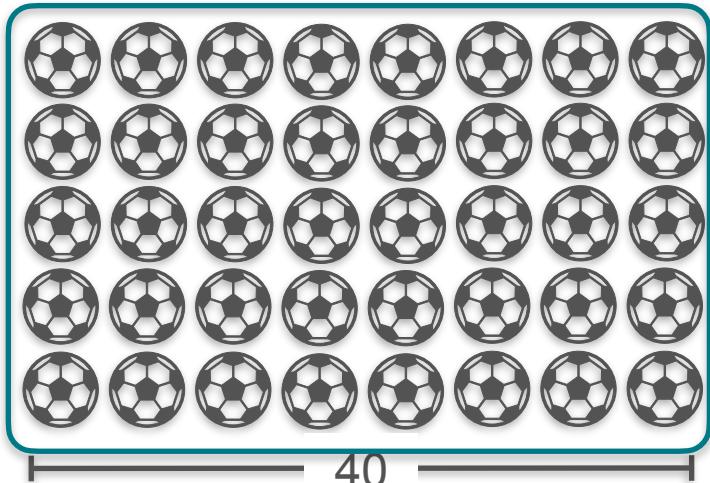
Quiz 1



Quiz 1 - Solution



Quiz 2

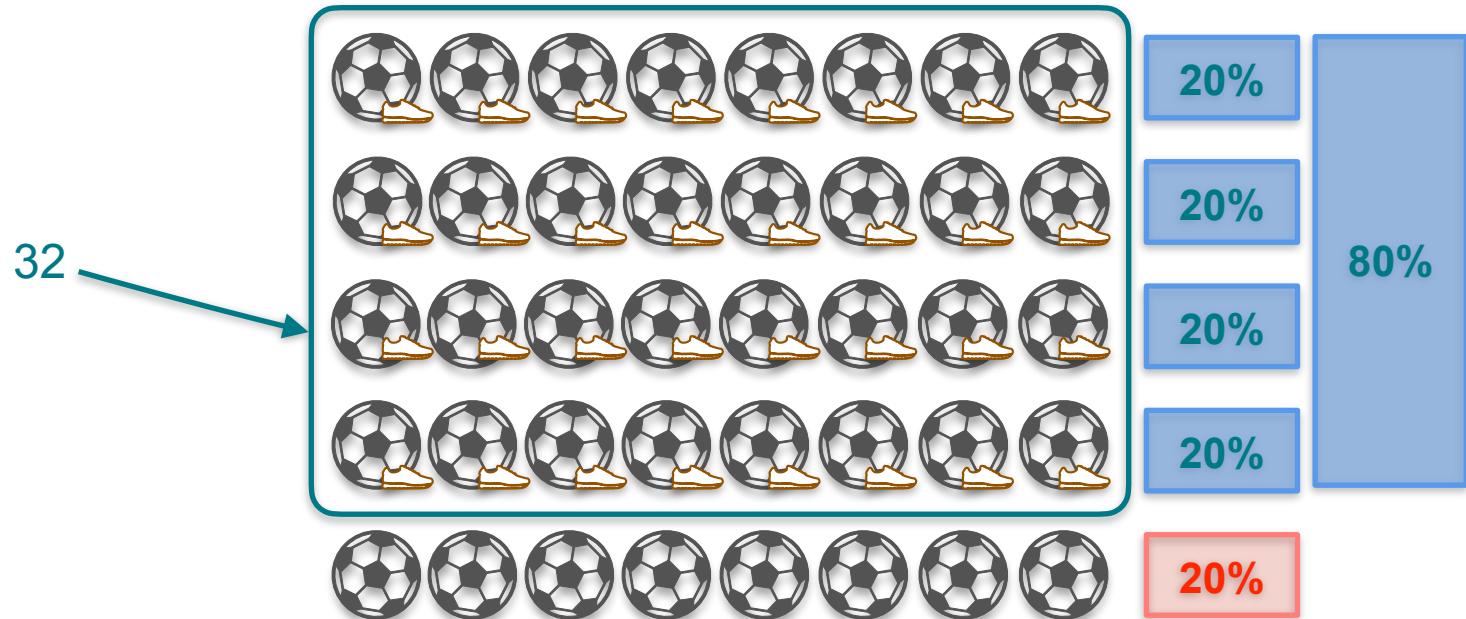


80%

100 kids

How many kids play soccer and wear running shoes

Quiz 2 - Solution



Conditional Probability

$P(\text{Soccer and Running shoes})$

$$P(S \cap R) = P(S) \bullet P(R|S)$$

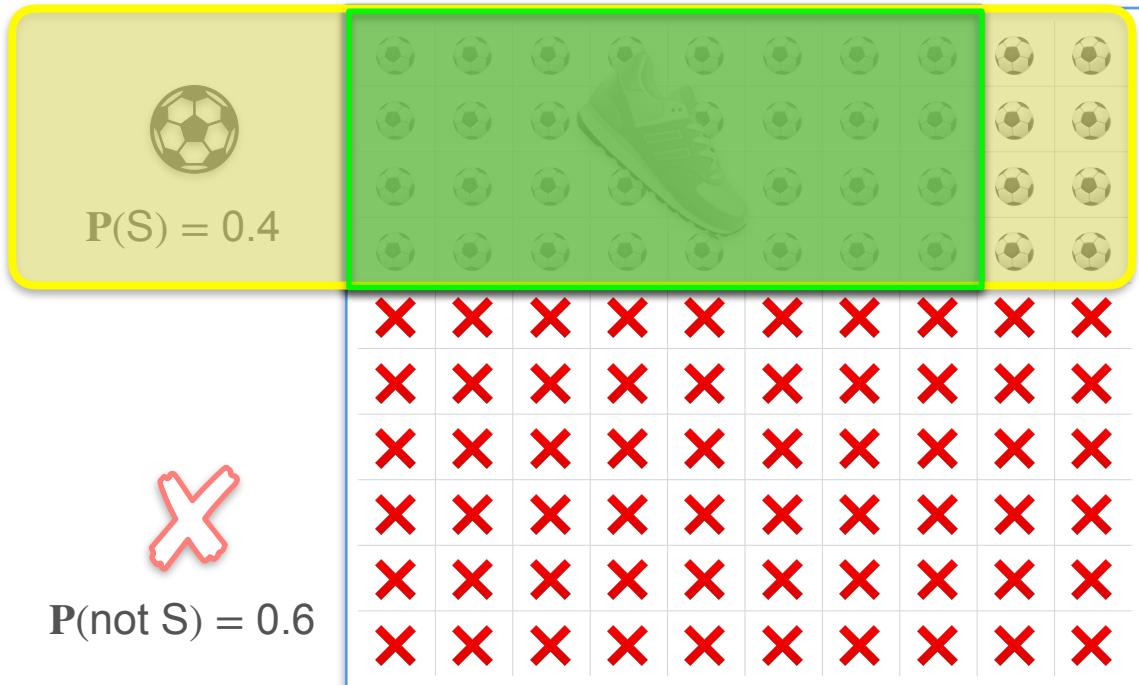
$$= 0.4 \bullet 0.8$$

$$= 0.32$$



$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



Conditional Probability

$P(\text{Soccer and Running shoes})$



$$P(S \cap R) = 0.32$$

$$P(S) = 0.4$$

$P(\text{not Soccer and Running shoes})$

$$P(\text{not } S \cap R) = P(\text{not } S) \bullet P(R | \text{not } S)$$

$$= 0.6 \bullet 0.5$$

$$= 0.3$$



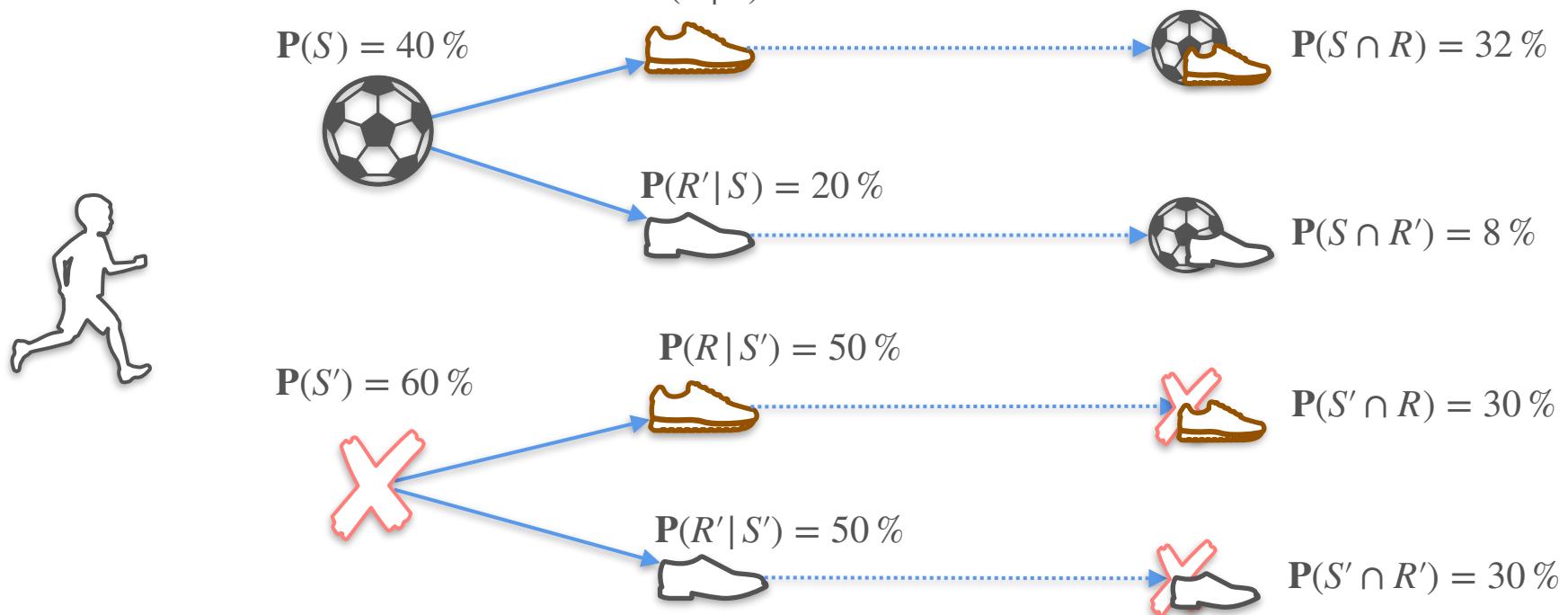
$$P(\text{not } S) = 0.6$$

$$P(R | S) = 0.8$$



$$P(R | \text{not } S) = 0.5$$

Conditional Probability

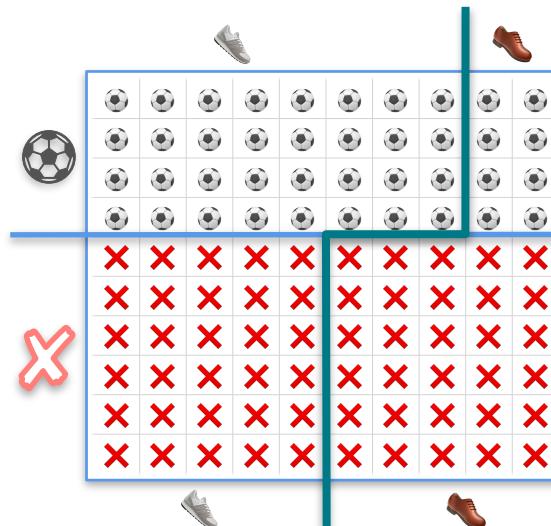


Independent vs Dependent Events

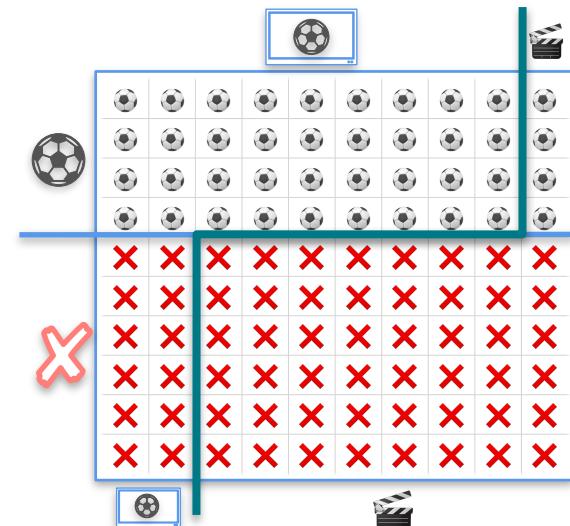
Independent



Dependent



Dependent





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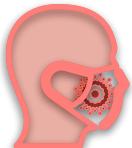
Introduction to probability

Bayes Theorem - Intuition

Bayes Theorem: Intuition



1,000,000 people



1 / 10,000 people



100 people



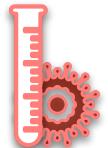
99



1



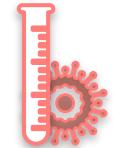
99% Effective



Tested Sick



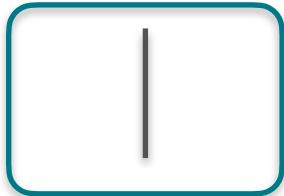
100 people



1

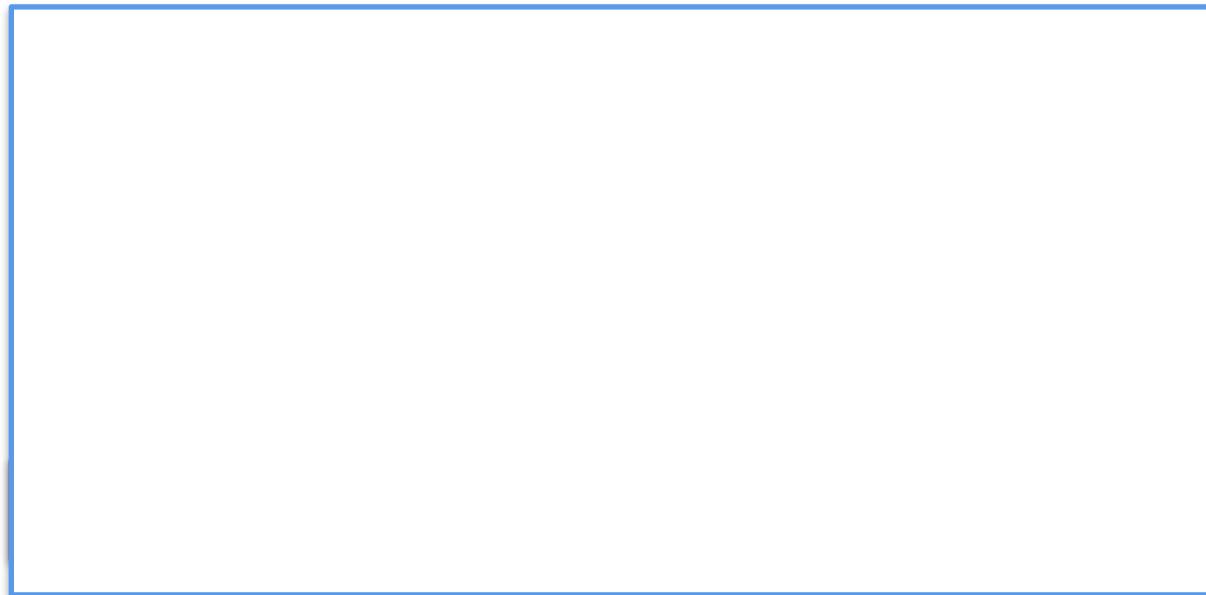


99



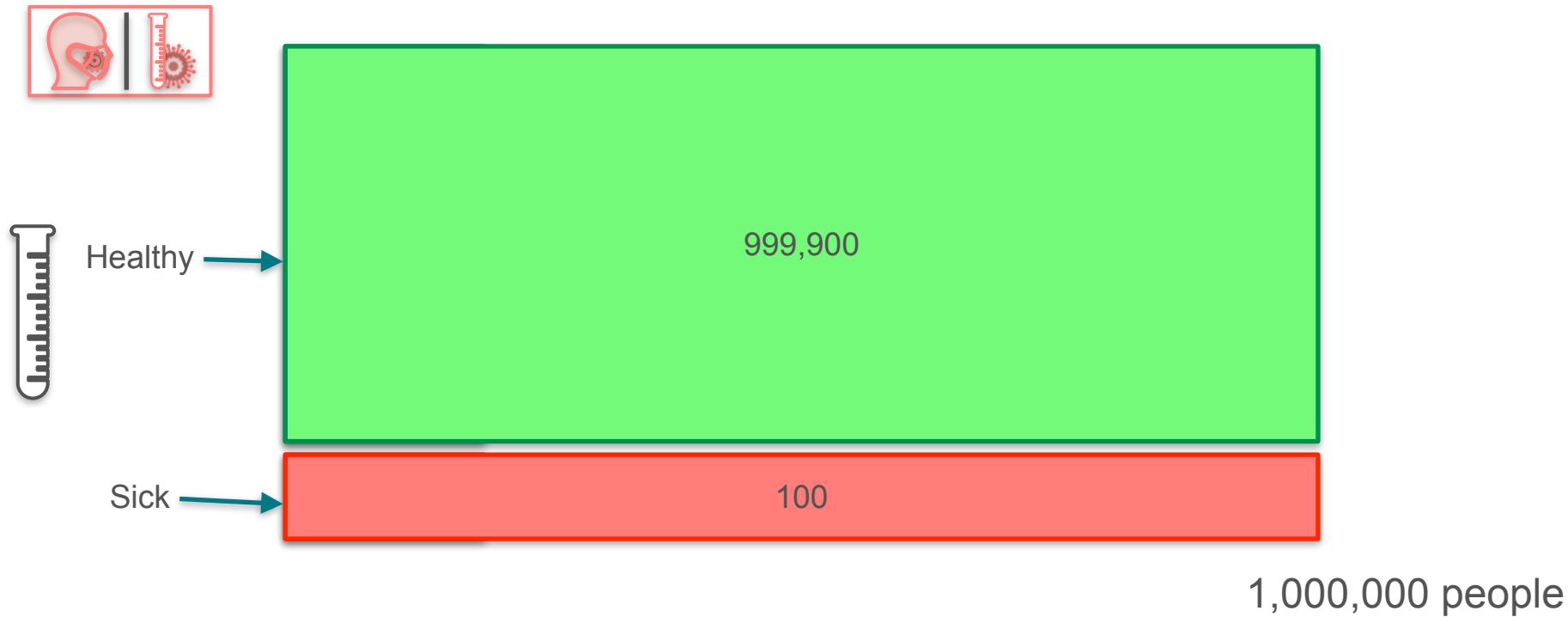
What's the probability that **you are sick**
GIVEN that you tested sick?

Bayes Theorem: Intuition

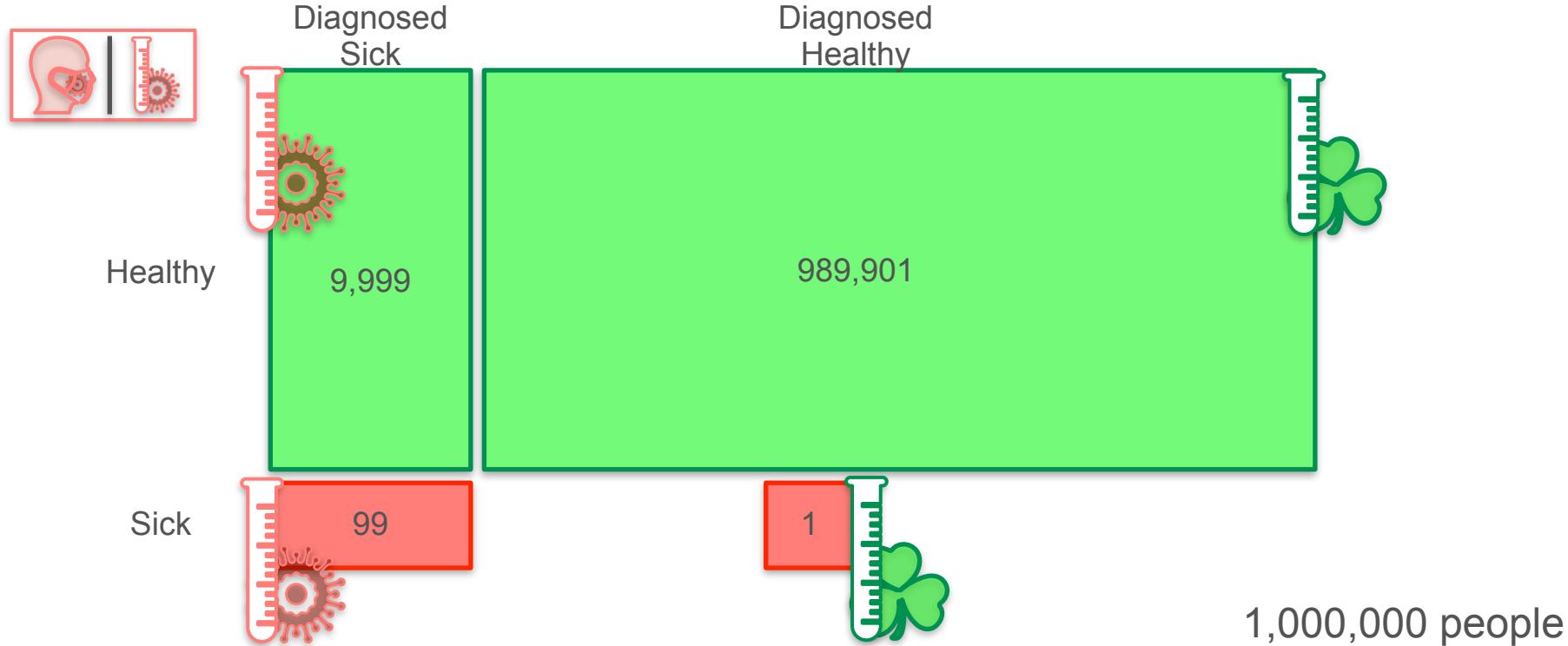


1,000,000 people

Bayes Theorem: Intuition



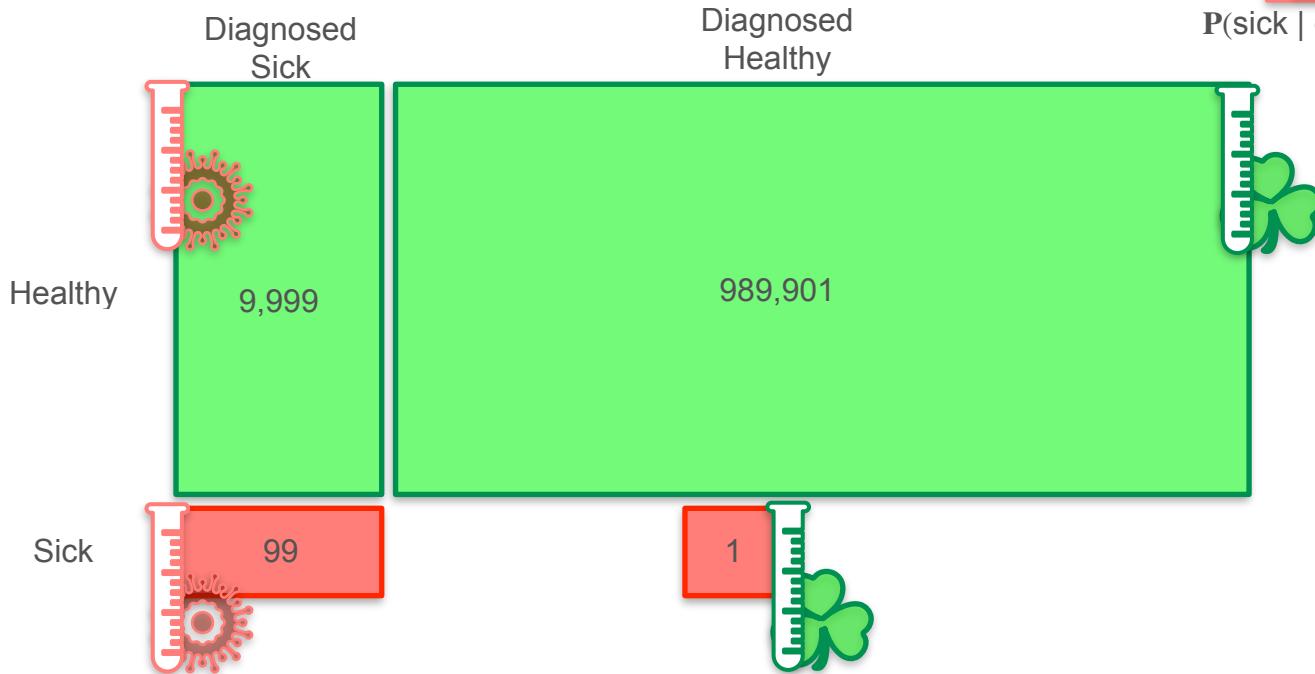
Bayes Theorem: Intuition



Bayes Theorem: Intuition



$P(\text{sick} | \text{diagnosed sick}) = \underline{\hspace{2cm}}$

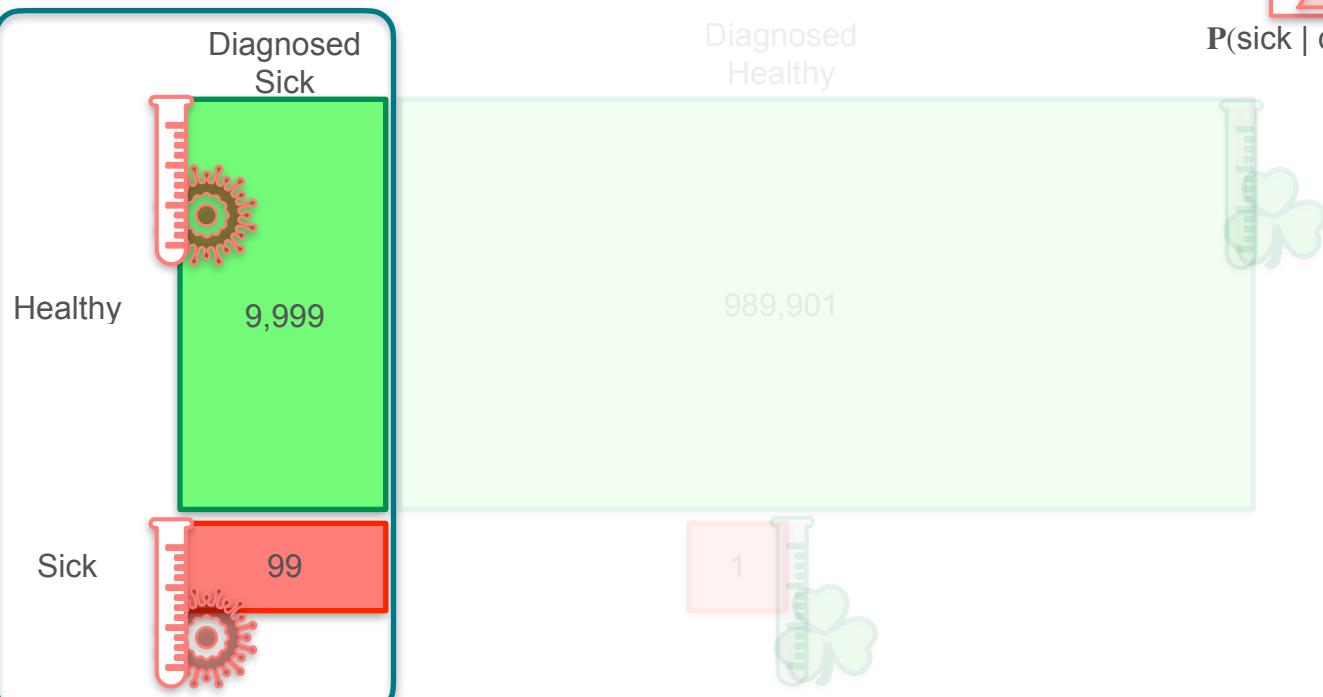


1,000,000 people

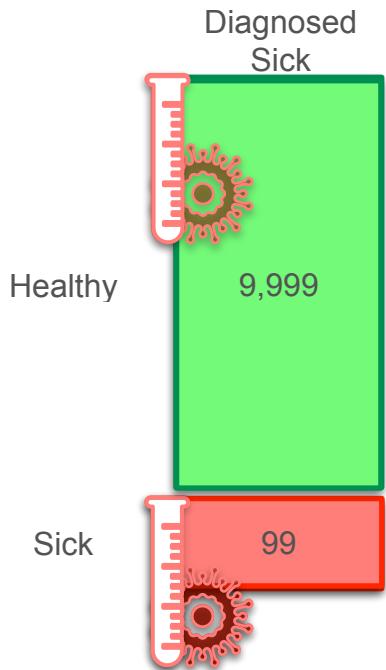
Bayes Theorem: Intuition



$P(\text{sick} | \text{diagnosed sick}) = \underline{\hspace{2cm}}$

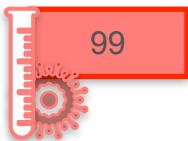


Bayes Theorem: Intuition

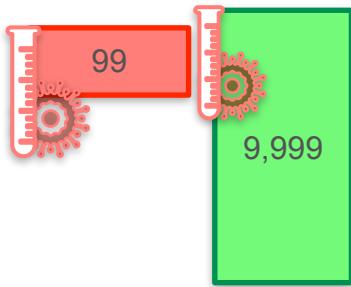


$$\begin{aligned} P(\text{sick} | \text{diagnosed sick}) &= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}} \\ &= \frac{99}{99 + 9999} \\ &= \frac{99}{10098} = 0.0098 \end{aligned}$$

Bayes Theorem: Intuition



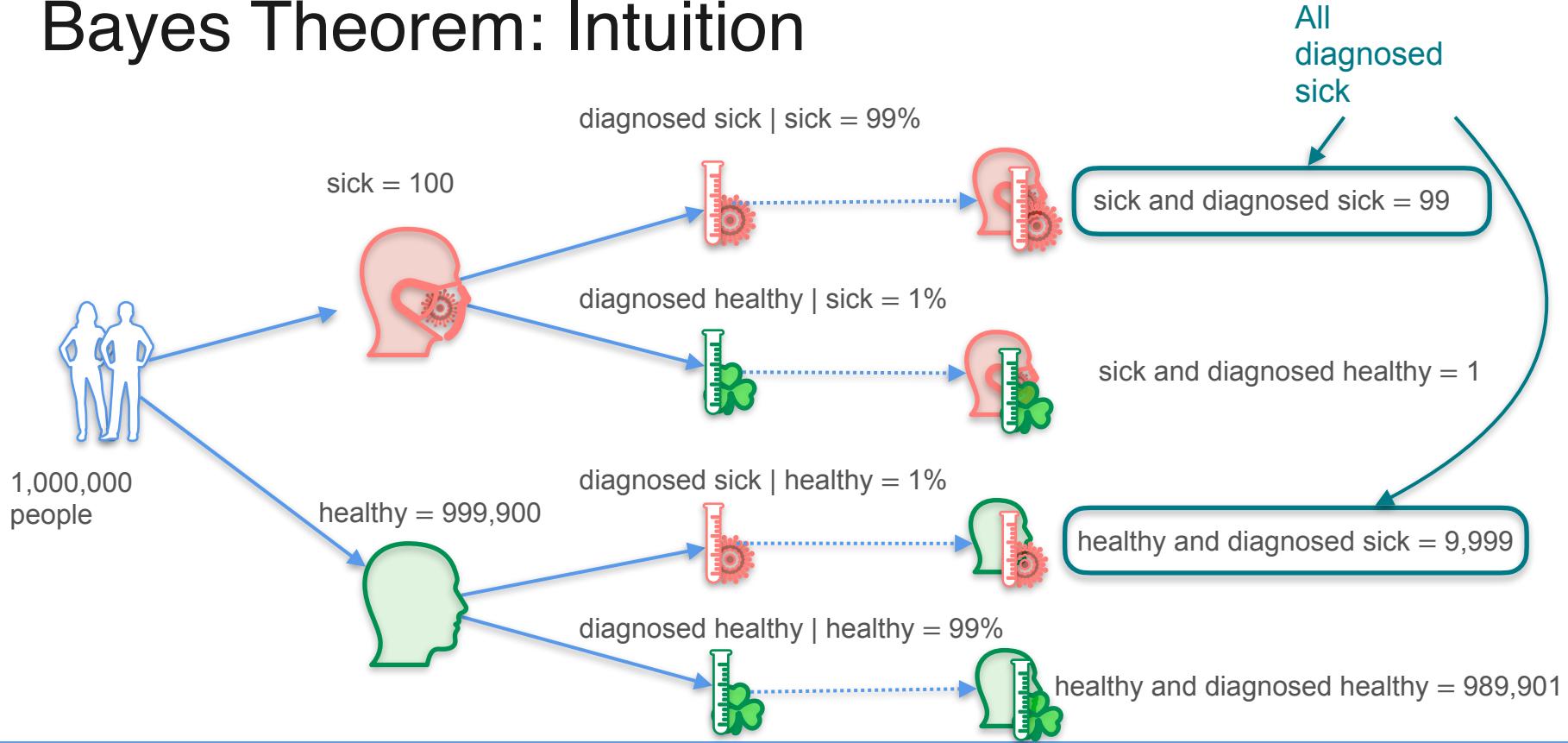
$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$



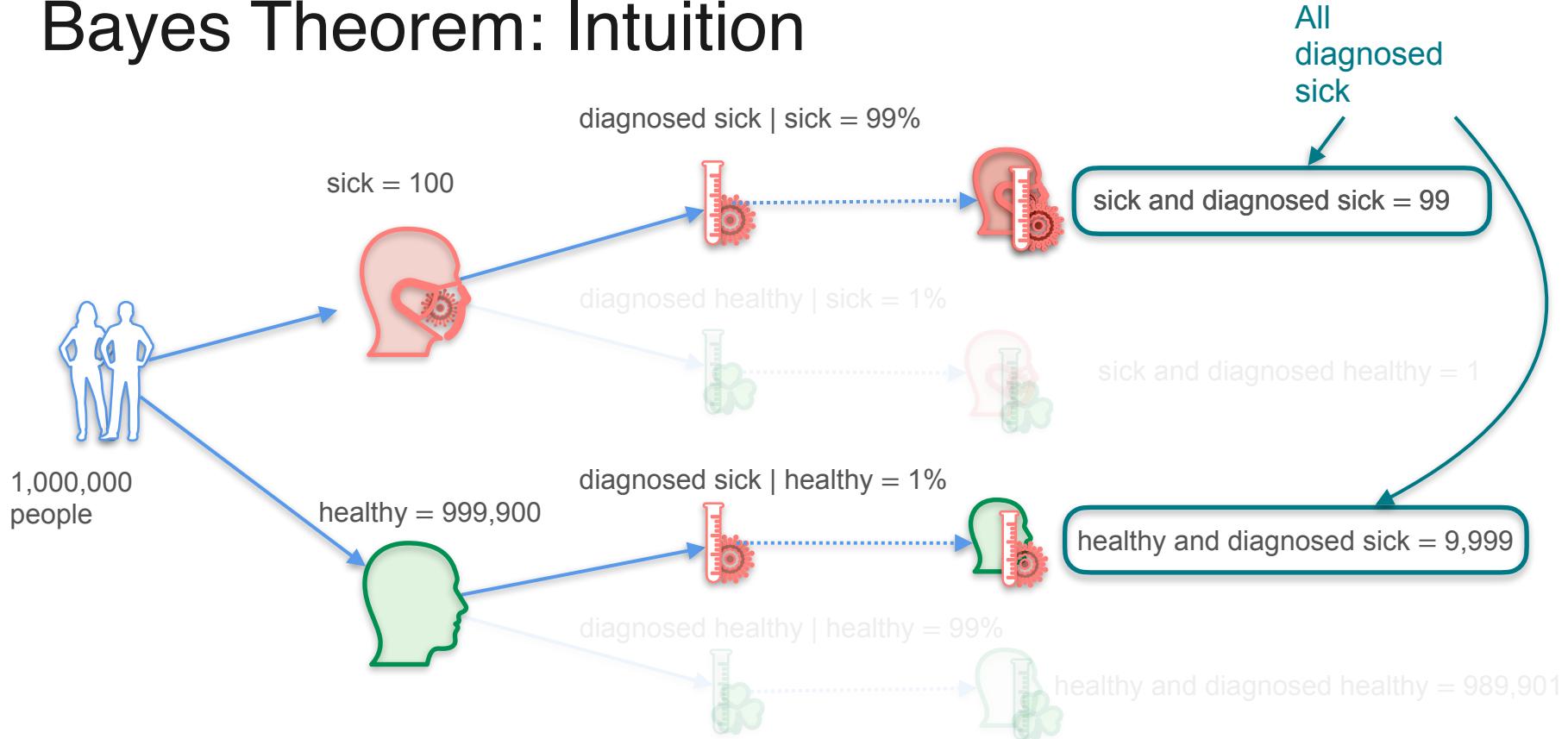
$$= \frac{\text{sick and diagnosed sick}}{\text{sick and diagnosed sick} + \text{healthy and diagnosed sick}}$$

$$= \frac{\text{sick and diagnosed sick}}{\text{everyone diagnosed sick}}$$

Bayes Theorem: Intuition

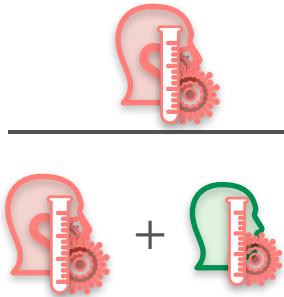


Bayes Theorem: Intuition



Bayes Theorem: Intuition

$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{ }}{\text{ }}$$



$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{\text{sick and diagnosed sick} = 99}{\text{healthy and diagnosed sick} = 9,999 + \text{sick and diagnosed sick} = 99}$$

$$P(\text{sick} \mid \text{diagnosed sick}) = \frac{99}{10098} = 0.0098$$

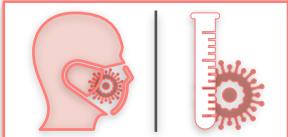
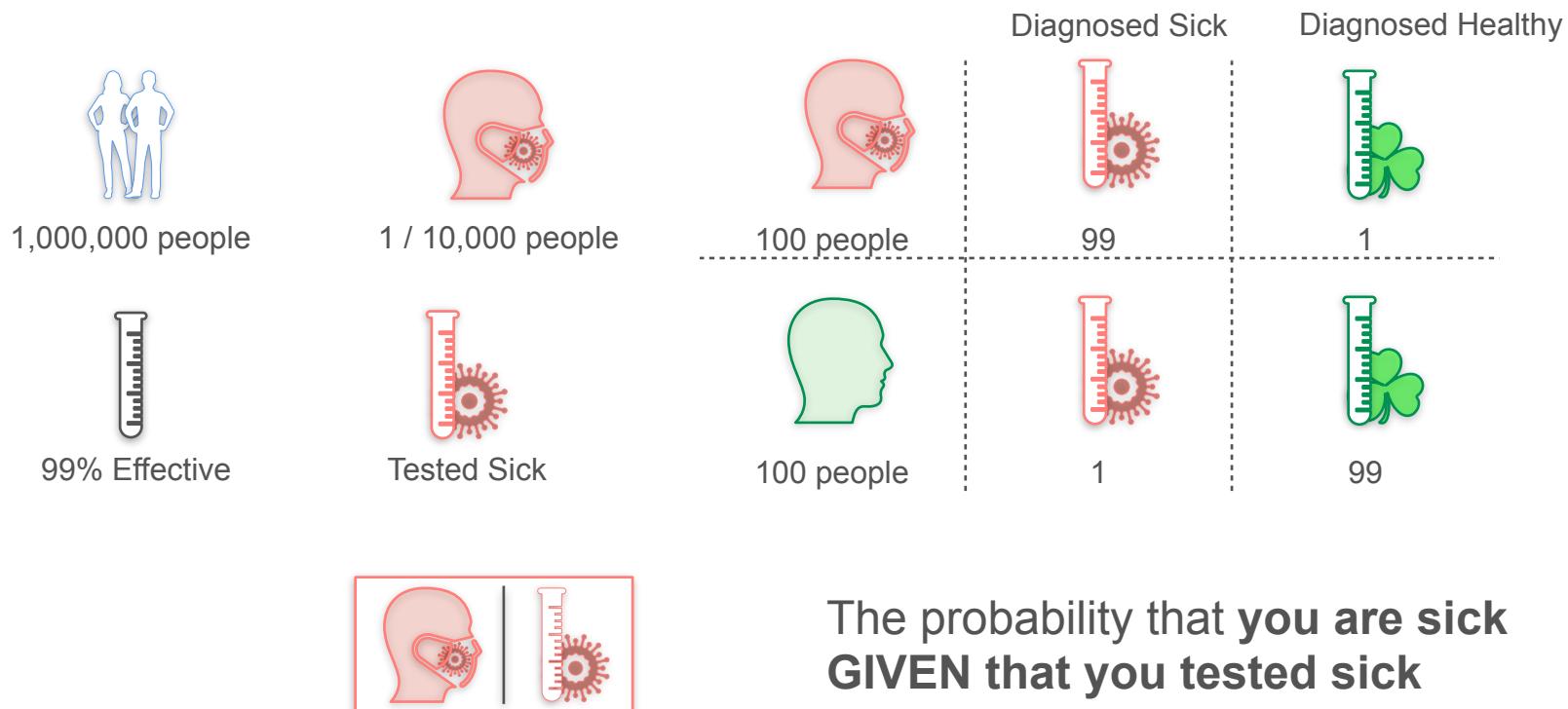


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Introduction to probability

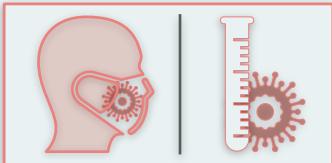
**Bayes Theorem -
Mathematical Formula**

Bayes Theorem: Formula



The probability that **you are sick**
GIVEN that you tested sick

Bayes Theorem: Formula



$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

The probability that **you are sick**
GIVEN that you tested sick



1,000,000

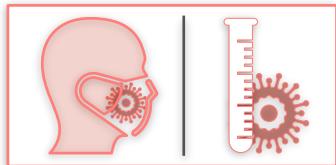


1 / 10,000



99% Effective

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(A \mid B) = ?$$

$$P(\text{sick}) = 0.01\%$$

From Conditional Probability

$$P(\text{not sick}) = 99.99\%$$

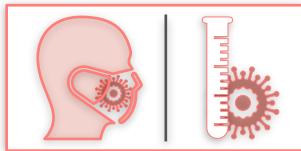
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

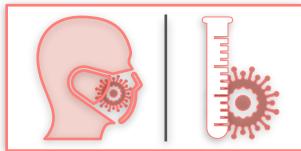
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})}$$

$$P(\text{sick and diagnosed sick}) = ?$$

$$P(\text{diagnosed sick}) = ?$$

BAYES THEOREM FORMULA CAN HELP

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$P(\text{sick} \mid \text{diagnosed sick}) = ?$

$P(\text{sick}) = 0.01\%$

$P(\text{not sick}) = 99.99\%$

$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$

$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

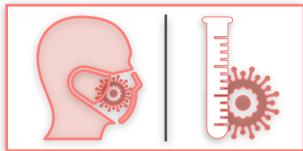
$$P(\text{sick and diagnosed sick}) = ?$$
$$P(A \mid B) = \frac{P(\text{sick and diagnosed sick})}{P(\text{diagnosed sick})} = ?$$

From Conditional Probability

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$P(\text{sick and diagnosed sick}) = P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

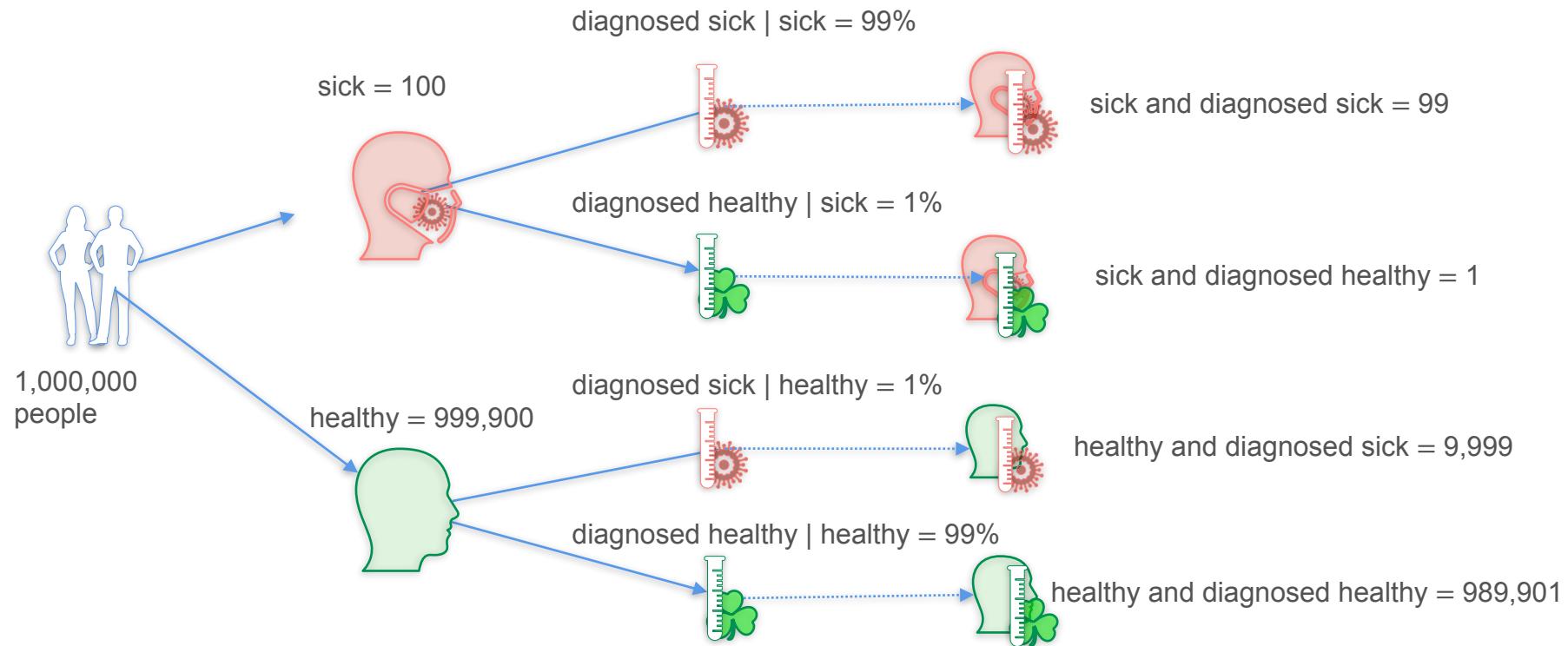
$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

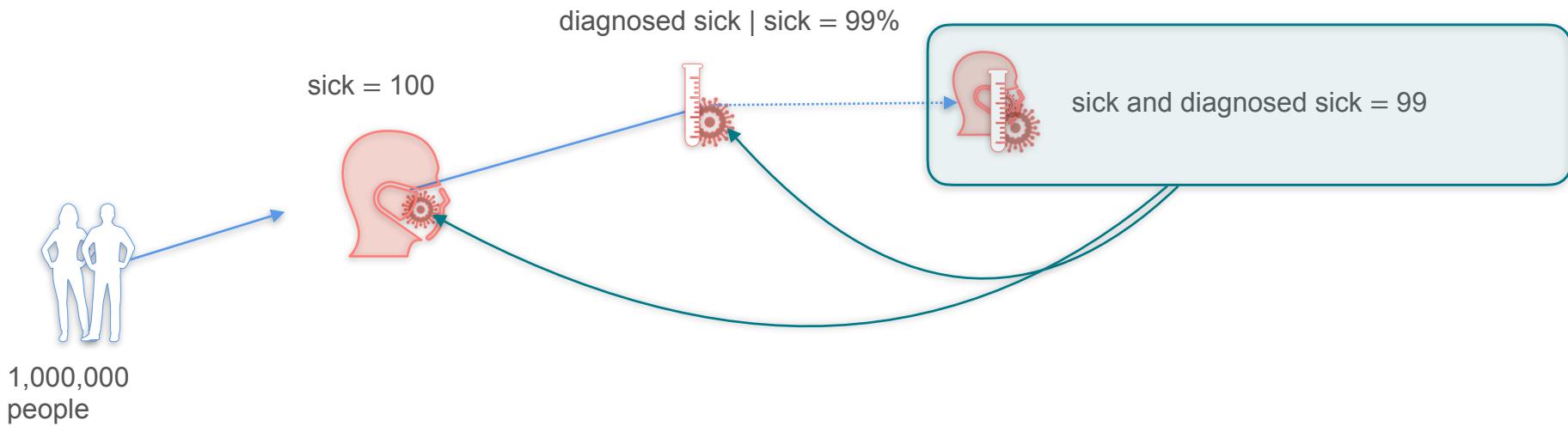
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})}{P(\text{diagnosed sick})} = ?$$

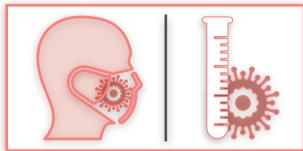
Bayes Theorem: Formula



Bayes Theorem: Formula



Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(\text{sick} \mid \text{diagnosed sick}) = ?$$

$$P(\text{sick}) = 0.01\%$$

$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} \mid \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} \mid \text{not sick}) = 1\%$$

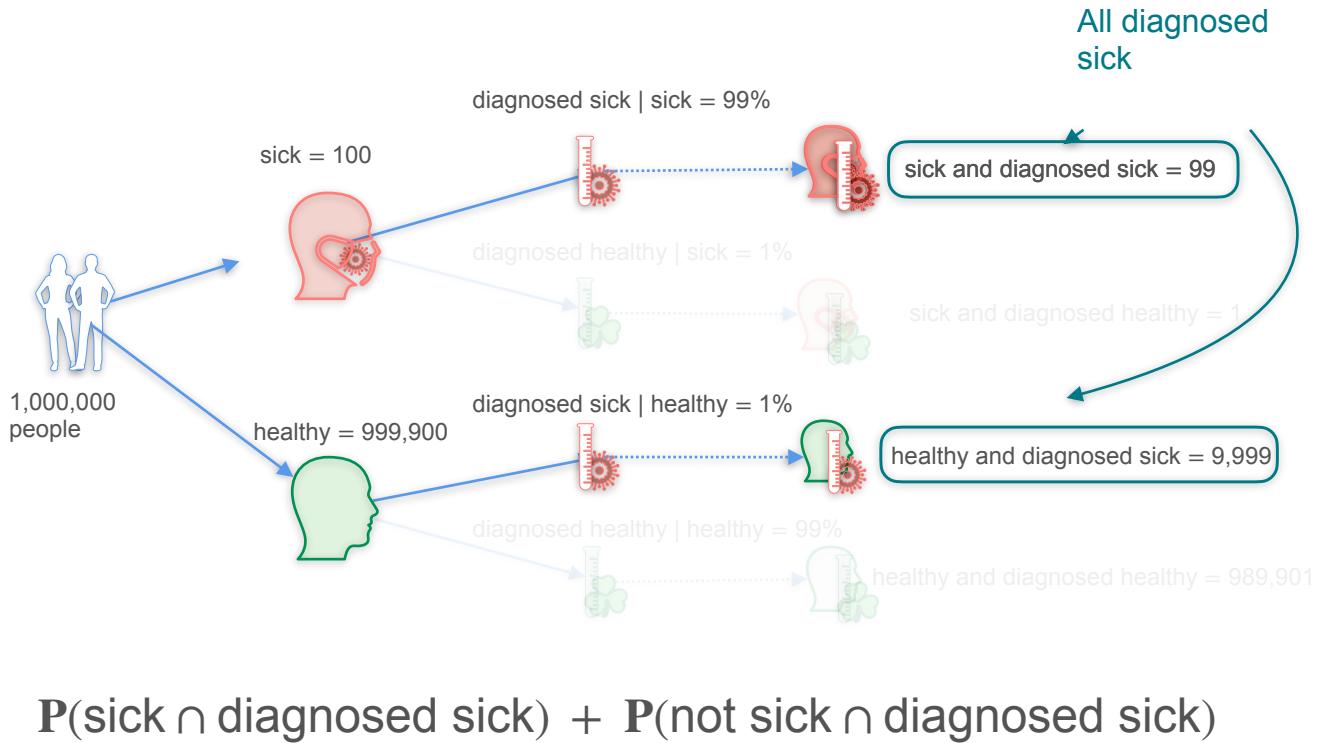
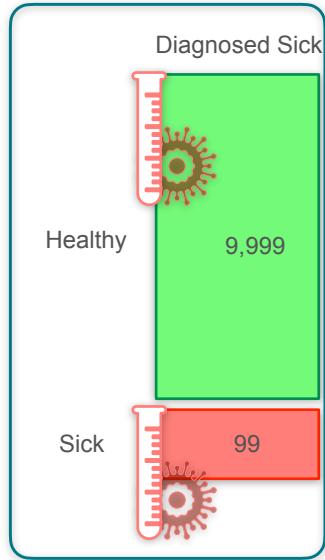
$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \mid B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} \mid \text{sick})}{P(\text{diagnosed sick})} = ?$$

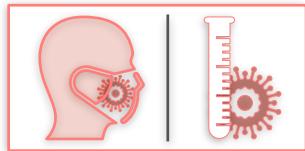
$$P(\text{diagnosed sick}) =$$

Bayes Theorem: Formula

$P(\text{diagnosed sick}) =$



Bayes Theorem: Formula



A : sick

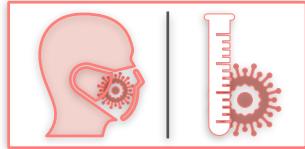
B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{diagnosed sick}) = ?}$$

$$P(\text{diagnosed sick}) = P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})$$

Bayes Theorem: Formula



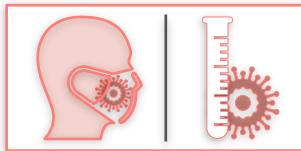
A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

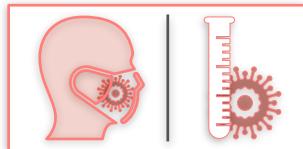
$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick} \cap \text{diagnosed sick}) + P(\text{not sick} \cap \text{diagnosed sick})}$$

$$\begin{aligned} P(\text{sick} \cap \text{diagnosed sick}) &= P(A \cap B) \\ &= P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) \end{aligned}$$

$$\begin{aligned} P(\text{not sick} \cap \text{diagnosed sick}) &= P(A' \cap B) \\ &= P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick}) \end{aligned}$$

Bayes Theorem: Formula



A: sick

B: diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

?

$$P(A | B) = \frac{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick})}{P(\text{sick}) \cdot P(\text{diagnosed sick} | \text{sick}) + P(\text{not sick}) \cdot P(\text{diagnosed sick} | \text{not sick})}$$

$$P(\text{sick}) = 0.01\%$$

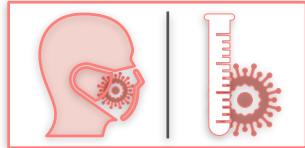
$$P(\text{not sick}) = 99.99\%$$

$$P(\text{diagnosed sick} | \text{sick}) = 99\%$$

$$P(\text{diagnosed sick} | \text{not sick}) = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(B | A')}$$

$$P(A) = 0.01\%$$

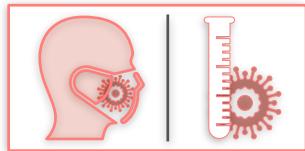
$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

**BAYES THEOREM
FORMULA**

Bayes Theorem: Formula



A : sick

B : diagnosed sick

$$P(A | B) = \frac{P(A \cap B)}{P(B)}$$

$$P(A | B) = \frac{P(A) \cdot P(B | A)}{P(A) \cdot P(B | A) + P(A') \cdot P(B | A')}$$

$$P(A) = 0.01\%$$

$$P(A') = 99.99\%$$

$$P(B | A) = 99\%$$

$$P(B | A') = 1\%$$

$$P(A | B) = \frac{0.0001 \times 0.99}{(0.0001 \times 0.99) + (0.9999 \times 0.01)}$$

$$P(A | B) = 0.0098$$

**BAYES THEOREM
FORMULA**



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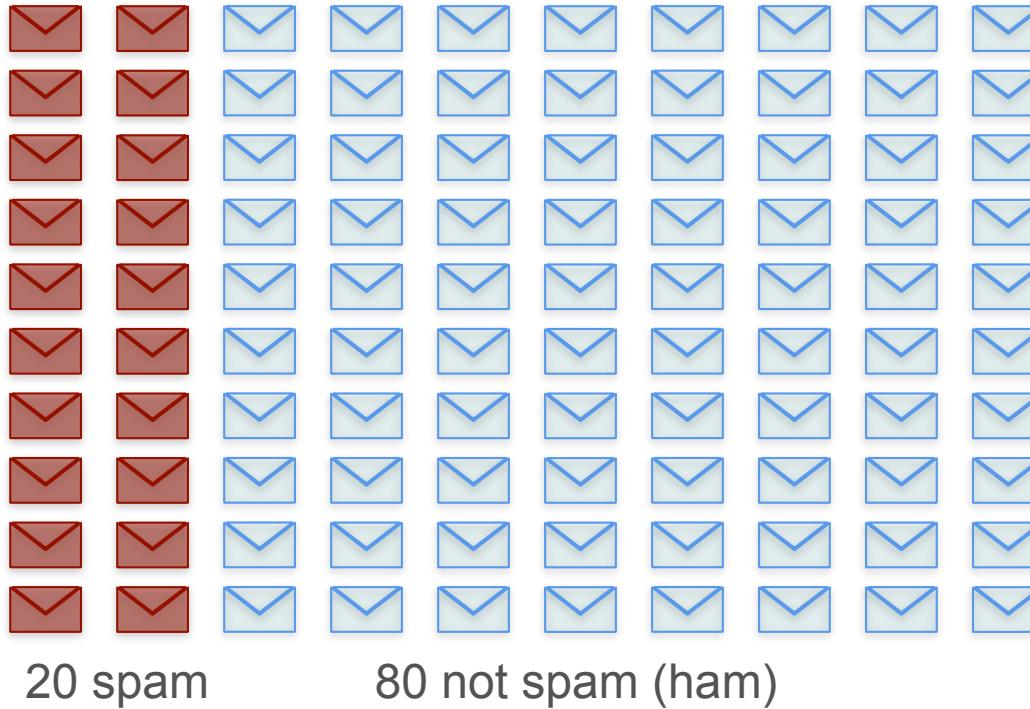
Introduction to probability

**Bayes Theorem -
Spam Example**

Bayes Theorem: Spam Example



Bayes Theorem: Spam Example



“lottery”

Bayes Theorem: Spam Example

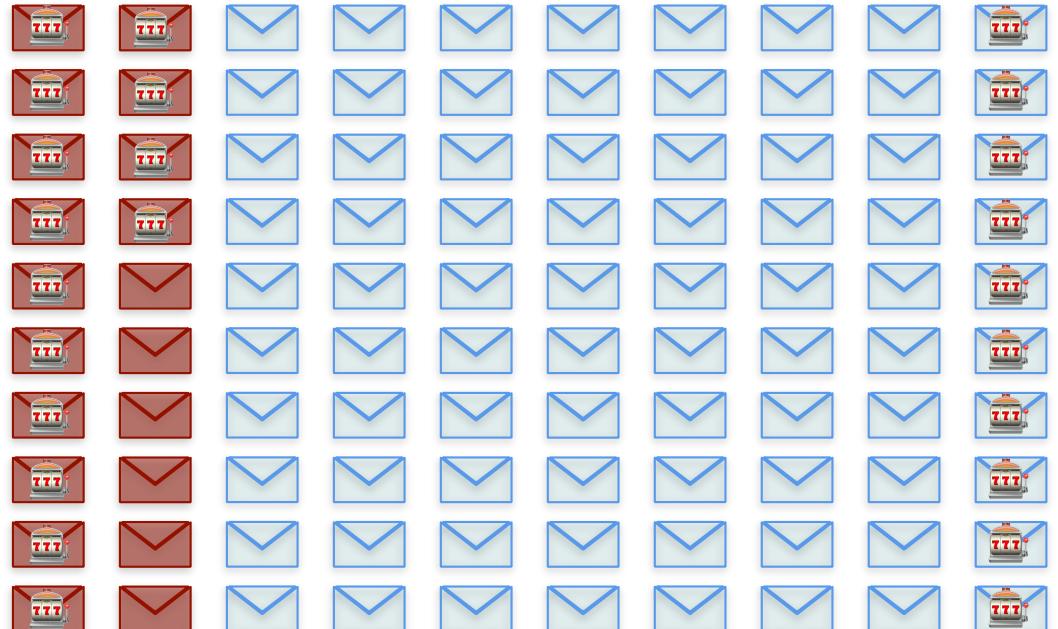
10



What is the probability that an email containing lottery is a spam?

$P(\text{spam} \mid \text{lottery})$

14



20 spam

80 not spam (ham)

Bayes Theorem: Spam Example (Intuition Solution)



$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$



14

24 emails
containing lottery

$$P(\text{spam} \mid \text{lottery}) = \frac{\text{spam and lottery}}{\text{all lottery}}$$

$$= \frac{14}{24}$$

$$= \frac{7}{12} = 0.583$$

10



Bayes Theorem: Spam Example (Formula Solution)

$P(\text{spam} \mid \text{lottery})$

$$P(A \mid B) = \frac{P(A) \cdot P(B \mid A)}{P(A) \cdot P(B \mid A) + P(A') \cdot P(B \mid A')}$$

A : Email is spam B : Email contains lottery

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

Bayes Theorem: Spam Example (Formula Solution)

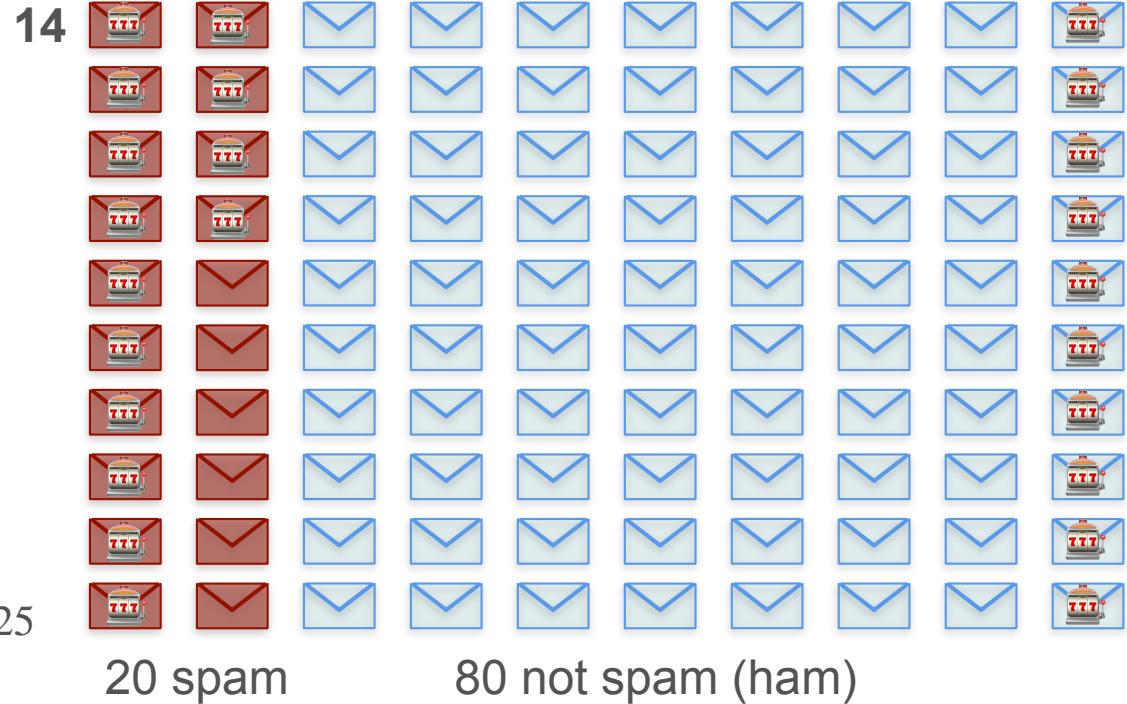
10

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{not spam}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} \mid \text{spam}) = \frac{14}{20} = 0.7$$

$$P(\text{lottery} \mid \text{not spam}) = \frac{10}{80} = 0.125$$



Bayes Theorem: Spam Example (Formula Solution)

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{not spam}) = 0.125$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) + P(\text{not spam}) \cdot P(\text{lottery} \mid \text{not spam})}$$

$$P(\text{spam} \mid \text{lottery}) = \frac{0.2 \times 0.7}{(0.2 \times 0.7) + (0.8 \times 0.125)} = 0.583$$



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Introduction to probability

**Bayes Theorem -
Prior and Posterior**

Bayes Theorem

PRIOR

$$\mathbf{P}(A)$$

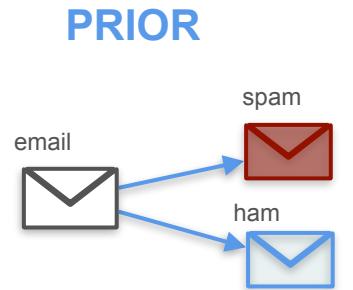
EVENT

$$E$$

POSTERIOR

$$\mathbf{P}(A | E)$$

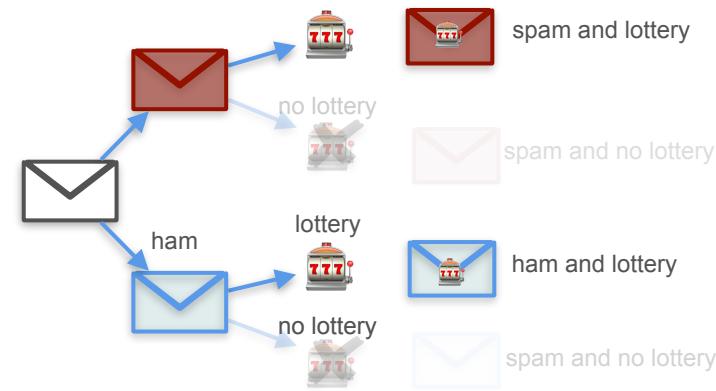
Prior and Posterior



EVENT



Email contains lottery



$$P(\text{spam}) = \frac{\text{spam}}{\text{spam} + \text{ham}}$$

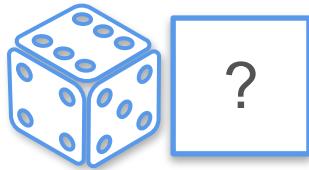
$$P(\text{spam} | \text{lottery}) = \frac{\text{spam and lottery}}{\text{spam and lottery} + \text{ham and lottery}}$$

Prior and Posterior

PRIOR

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

EVENT



1st dice is 6

POSTERIOR

	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

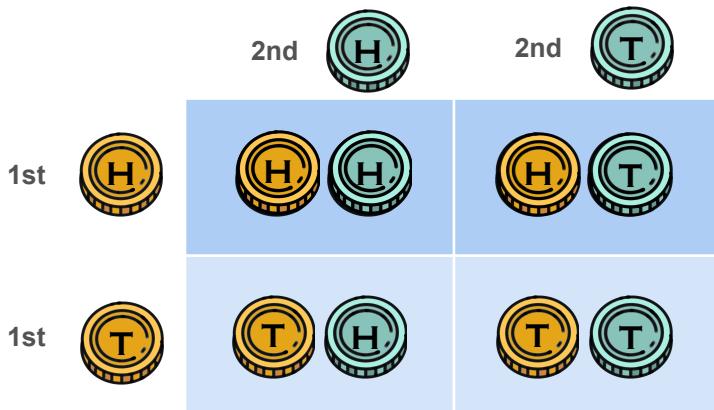
	1,1	1,2	1,3	1,4	1,5	1,6
1,1	1,1	1,2	1,3	1,4	1,5	1,6
2,1	2,1	2,2	2,3	2,4	2,5	2,6
3,1	3,1	3,2	3,3	3,4	3,5	3,6
4,1	4,1	4,2	4,3	4,4	4,5	4,6
5,1	5,1	5,2	5,3	5,4	5,5	5,6
6,1	6,1	6,2	6,3	6,4	6,5	6,6

$$P(\text{sum} = 10) = \frac{3}{36}$$

$$P(\text{sum} = 10 | \text{1st is } 6) = \frac{1}{6}$$

Prior and Posterior

PRIOR



$$P(HH) = \frac{1}{4}$$

EVENT



POSTERIOR



$$P(HH | \text{1st is } H) = \frac{1}{2}$$



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Introduction to probability

**Bayes Theorem -
The Naive Bayes Model**

What About 2 Events?

PRIOR

$$P(\text{spam}) = \frac{\text{Red Envelope}}{\text{Red Envelope} + \text{Blue Envelope}}$$

EVENT



Email contains 'lottery'



Email contains 'winning'

POSTERIOR

$$P(\text{spam} | \text{lottery}) = \frac{\text{Red Envelope with lottery icon}}{\text{Red Envelope with lottery icon} + \text{Blue Envelope with lottery icon}}$$

$$P(\text{spam} | \text{winning}) = \frac{\text{Red Envelope with winning icon}}{\text{Red Envelope with winning icon} + \text{Blue Envelope with winning icon}}$$

?



Email contains 'lottery' and 'winning'

What About 2 Events?

EVENT



Email contains 'lottery' and 'winning'

POSTERIOR



$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{\text{# Spam emails with 'lottery' and 'winning'}}{\text{# Spam emails}}$$

?

$$P(\text{spam} | \text{lottery} \& \text{winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \& \text{winning} | \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \& \text{winning} | \text{ham})}$$

What About More Than 2 Events?

EVENT

POSTERIOR

Email contains w_1, w_2, \dots, w_{100}

$$\frac{\# \text{ Spam emails with } w_1, \dots, w_{100}}{\# \text{ Spam emails}} = 0$$

?

$$P(\text{spam} | w_1, \dots, w_{100}) = \frac{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam})}{P(\text{spam}) \cdot P(w_1, \dots, w_{100} | \text{spam}) + P(\text{ham}) \cdot P(w_1, \dots, w_{100} | \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption



The appearances of 'lottery' and 'winning' are independent

$$P(\text{spam} \mid \text{lottery \& winning}) = \frac{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery \& winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery \& winning} \mid \text{ham})}$$

$P(A \cap B) = P(A) \cdot P(B)$

↓

The terms $P(\text{lottery \& winning} \mid \text{spam})$ and $P(\text{lottery \& winning} \mid \text{ham})$ are circled in red.

Is There a Quicker Way To Estimate the Probability?

Naive assumption



The appearances of ‘lottery’ and ‘winning’ are independent

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B)$$

$$\mathbf{P}(\text{spam} \mid \text{lottery} \& \text{winning}) = \frac{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam})}{\mathbf{P}(\text{spam}) \cdot \mathbf{P}(\text{lottery} \mid \text{spam}) \cdot \mathbf{P}(\text{winning} \mid \text{spam}) + \mathbf{P}(\text{ham}) \cdot \mathbf{P}(\text{lottery} \mid \text{ham}) \cdot \mathbf{P}(\text{winning} \mid \text{ham})}$$

Is There a Quicker Way To Estimate the Probability?

Naive assumption

The appearances of the words w_1, w_2, \dots, w_n are independent

$$P(\text{spam} \mid w_1, \dots, w_n) = \frac{P(\text{spam}) \cdot P(w_1 \mid \text{spam}) \cdots P(w_n \mid \text{spam})}{P(\text{spam}) \cdot P(w_1 \mid \text{spam}) \cdots P(w_n \mid \text{spam}) + P(\text{ham}) \cdot P(w_1 \mid \text{ham}) \cdots P(w_n \mid \text{ham})}$$

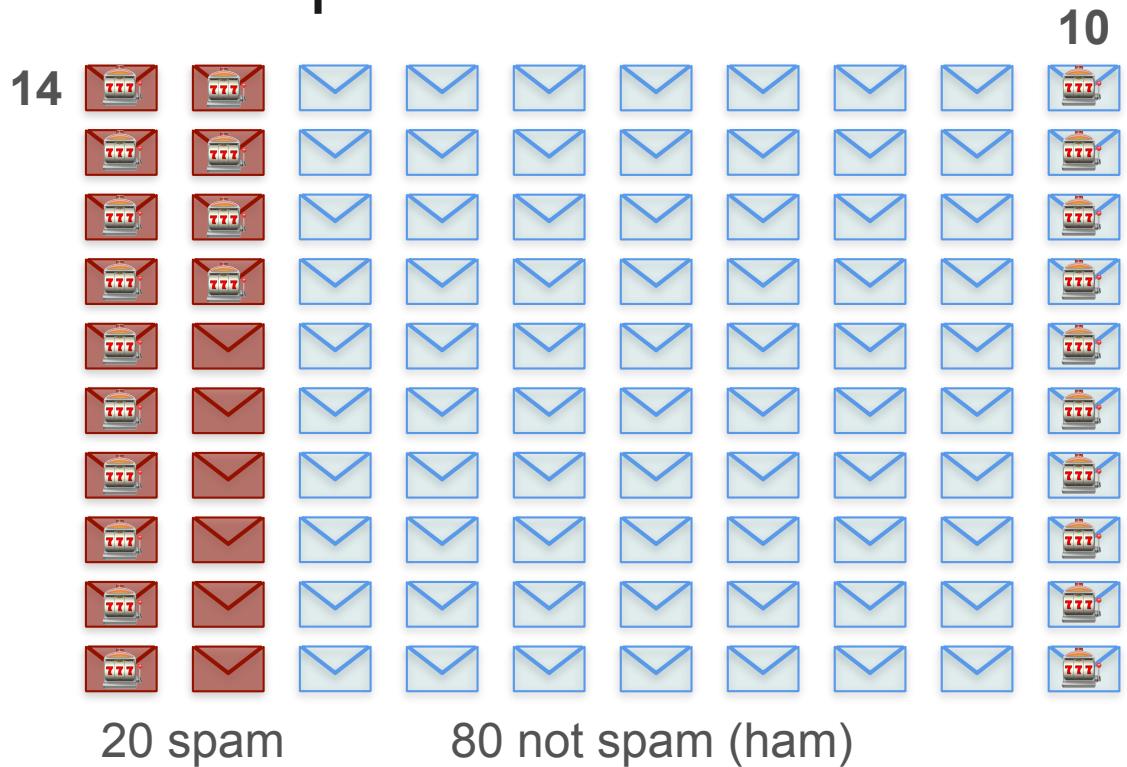
Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{lottery} | \text{spam}) = \frac{14}{20} = 0.7$$

$$P(\text{lottery} | \text{ham}) = \frac{10}{80} = 0.125$$



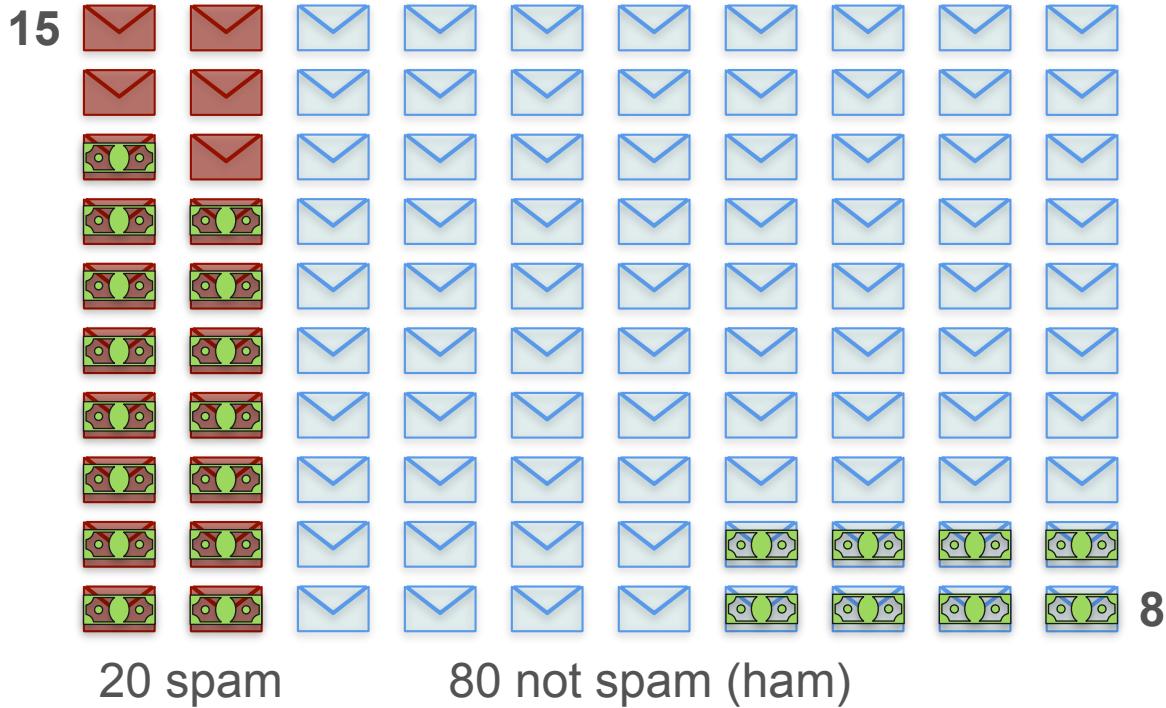
Naive Bayes: Spam Example

$$P(\text{spam}) = \frac{20}{100} = 0.2$$

$$P(\text{ham}) = \frac{80}{100} = 0.8$$

$$P(\text{winning} \mid \text{spam}) = \frac{15}{20} = 0.75$$

$$P(\text{winning} \mid \text{ham}) = \frac{8}{80} = 0.1$$



Naive Bayes: Spam Example

$$P(\text{spam}) = 0.2$$

$$P(\text{lottery} \mid \text{spam}) = 0.7$$

$$P(\text{winning} \mid \text{spam}) = 0.75$$

$$P(\text{not spam}) = 0.8$$

$$P(\text{lottery} \mid \text{ham}) = 0.125$$

$$P(\text{winning} \mid \text{ham}) = 0.1$$

$$P(\text{spam} \mid \text{lottery}) = \frac{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam})}{P(\text{spam}) \cdot P(\text{lottery} \mid \text{spam}) \cdot P(\text{winning} \mid \text{spam}) + P(\text{ham}) \cdot P(\text{lottery} \mid \text{ham}) \cdot P(\text{winning} \mid \text{ham})}$$

$$P(\text{spam} \mid \text{lottery} \& \text{winning}) = \frac{0.2 \times 0.7 \times 0.75}{(0.2 \times 0.7 \times 0.75) + (0.8 \times 0.125 \times 0.1)} = 0.913$$



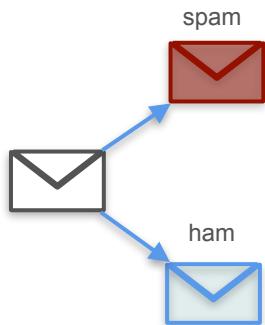
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Introduction to probability

Probability in Machine Learning

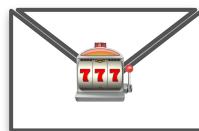
Bayes Theorem

PRIOR

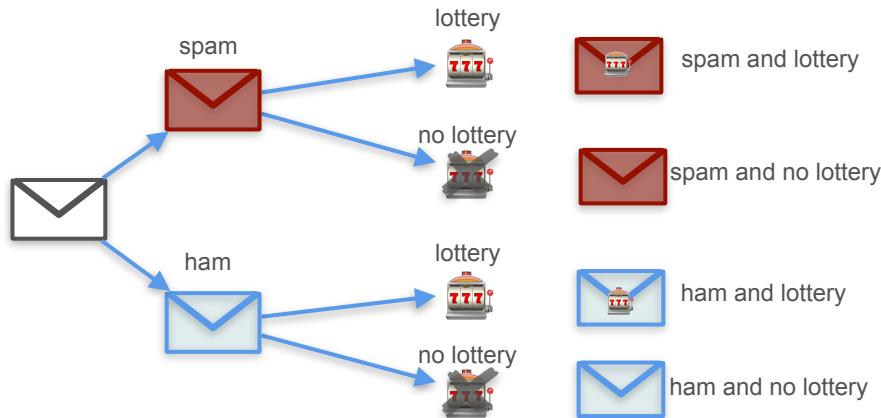


$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

EVENT



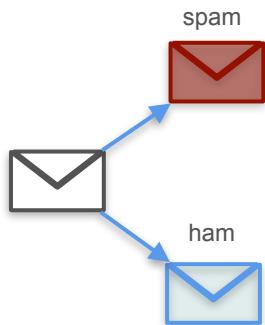
POSTERIOR



$$P(\text{spam} | \text{lottery}) =$$

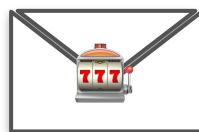
Bayes Theorem

PRIOR

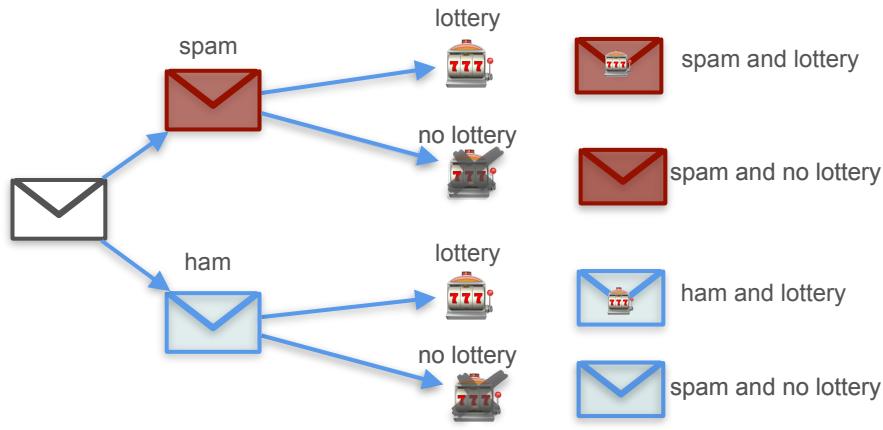


$$P(\text{spam}) = \frac{\text{red envelope}}{\text{red envelope} + \text{blue envelope}}$$

EVENT



POSTERIOR



$$P(\text{spam} | \text{lottery}) = \frac{\text{red envelope and lottery}}{\text{red envelope and lottery} + \text{blue envelope and lottery}}$$

Example Problem

Image recognition

- What is the probability that there is a cat in the image
- $P(\text{cat} \mid \text{image}) = P(\text{cat} \mid \text{pixel}_1, \text{pixel}_2, \dots, \text{pixel}_n)$



Example Problem

Classification

Patient 1		
A	Age	29
G	Gender	Female
H	Height	169 cm
W	Weight	62 kg
S	Smoker	No
...
B	Heart rate	63
B	Blood pressure	120 90

- Is this patient healthy?
- Calculate $P(\text{healthy} \mid \text{symptoms and history})$

Example Problem

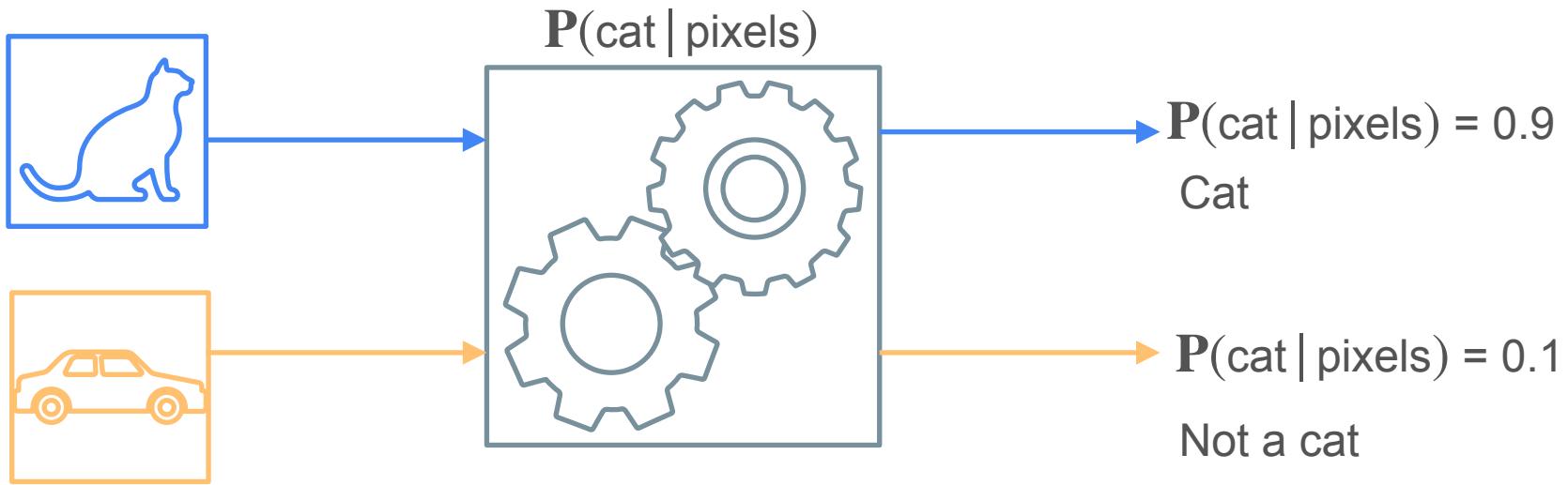
Sentiment analysis

the first cold shower
even the monkey seems to want
a little coat of straw

Matsuo Bashō

- Is this a happy sentence?
- Calculate $P(\text{happy} \mid \text{words in the sentence})$

Example Solution



Example Problem: Generative Models

Face generation

- Generate a group of pixels such that the resulting image looks like a human face.
- Goal: generate images such that $P(\text{face} \mid \text{pixels})$ is high.



Image generated by a StyleGAN

W1 Lesson 2

Probability Distributions

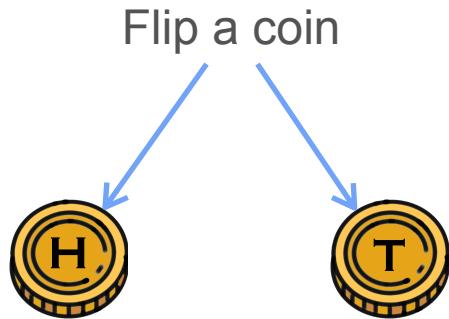


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Probability Distributions

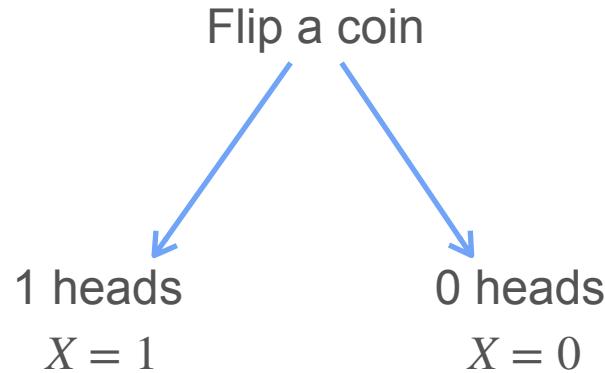
Random Variables

From Events to Random Variables



$$\mathbf{P}(H) = 0.5 \quad \mathbf{P}(T) = 0.5$$

X = Number of heads



$$\mathbf{P}(X = 1) = 0.5 \quad \mathbf{P}(X = 0) = 0.5$$

X is a random variable

From Events to Random Variables

X = Number of heads in 10 coin tosses

$$\mathbf{P}(H) = 0.5$$



$$X = 10$$

$$0.5^{10}$$

$$\mathbf{P}(X = 0)?$$



$$X = 9$$

$$0.5^9 0.5$$

$$\mathbf{P}(X = 1)?$$

...



$$X = 9$$

$$0.5^9 0.5$$

$$\mathbf{P}(X = 10)?$$

From Events to Random Variables

X = Number of heads in 10 coin tosses



Possible outcomes:

0	4	8
1	5	9
2	6	10
3	7	

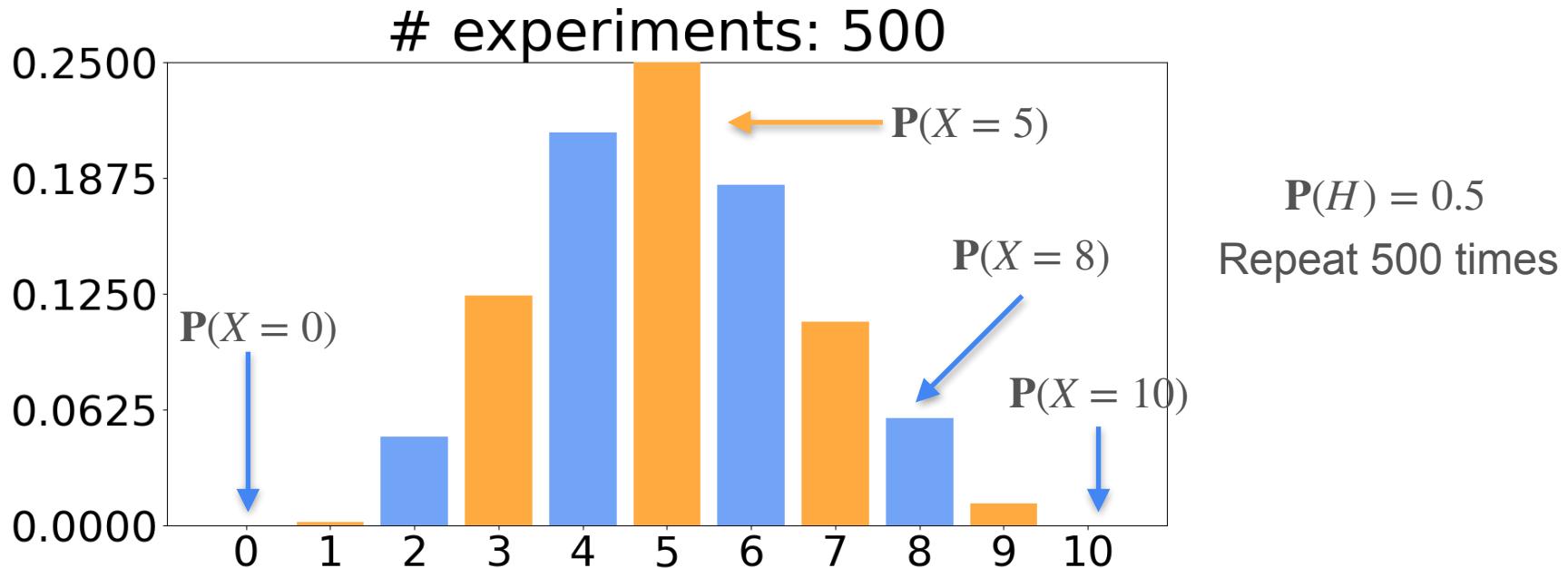


⋮

$$P(H) = 0.5$$

Repeat 500 times

Flipping a Fair Coin 500 Times



Why Random Variables?

- Random variables allow you to model the whole experiment at once



X = Number of heads



X = Number of 1's



X = Number of sick patients

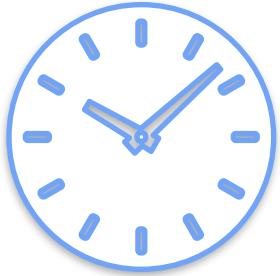


$$\mathbf{P}(X = 1) = 0.5$$

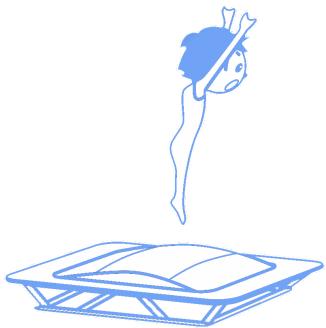
$$\mathbf{P}(X = -7) = 0.2$$

$$\mathbf{P}(X = 3.14159) = 0.3$$

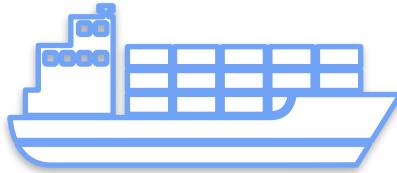
Other Random Variables



Wait time until the
next bus arrives



Height of an
gymnast's jump



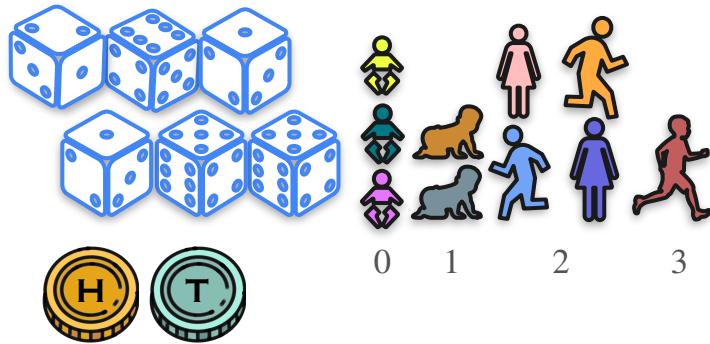
Number of
defective products
in a shipment



mm. of rain in
November

Discrete and Continuous Random Variables

Discrete random variables



~~Finite number of values~~

(Could be infinite too)

Can take only a **countable** number of values

Continuous random variables



Infinite number of values

Takes values on an interval

Random Variable Vs. Deterministic Variable

Deterministic

$$x = 2, f(x) = x^2$$

Fixed outcome

Random

X = number of defective items in
a shipment

Uncertain outcome



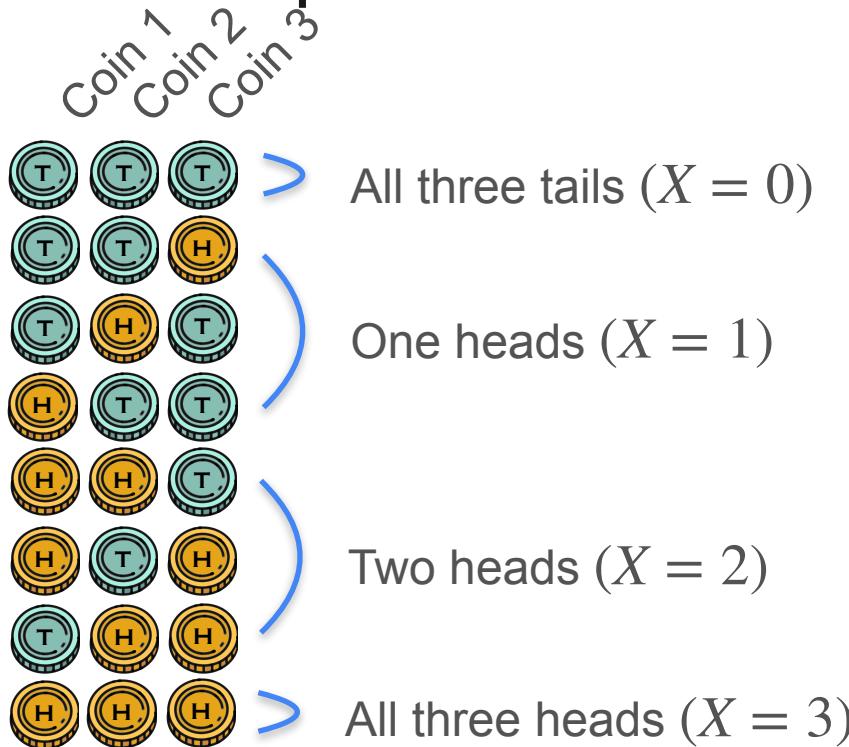
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Probability Distributions

Probability Distributions (Discrete)

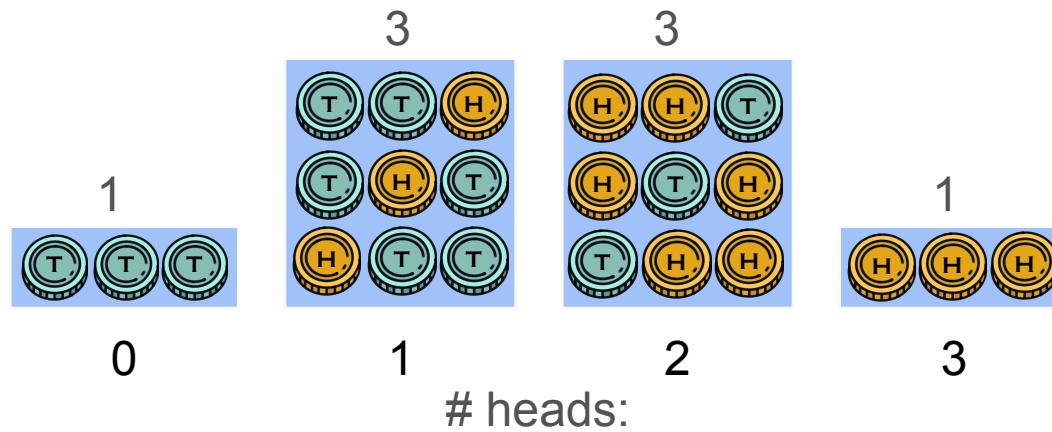
Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3 coin tosses



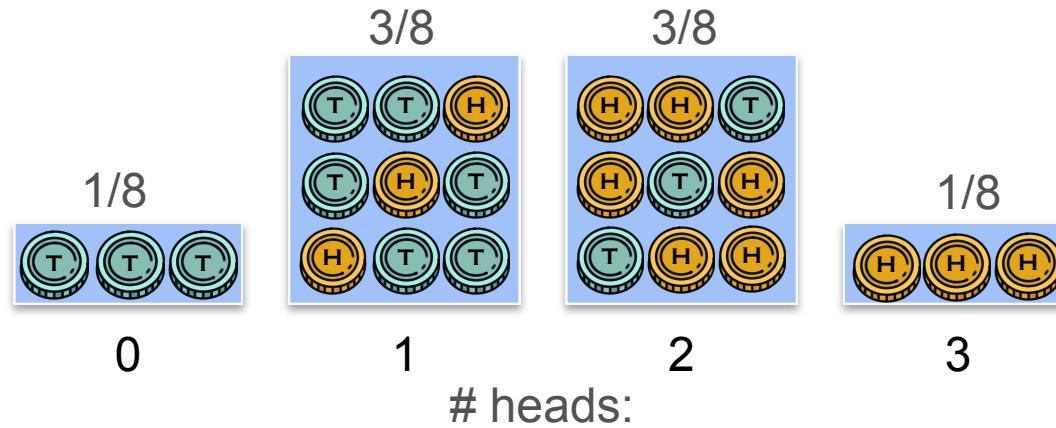
Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3 coin tosses



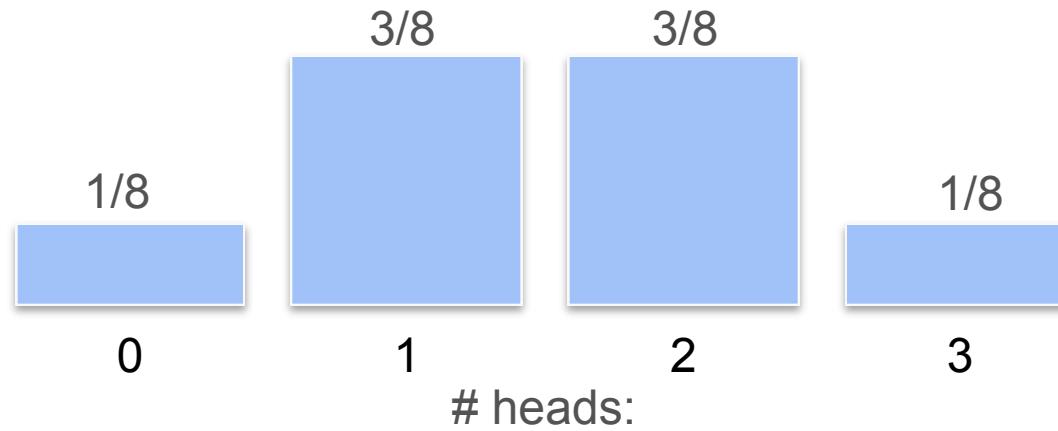
Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3 coin tosses



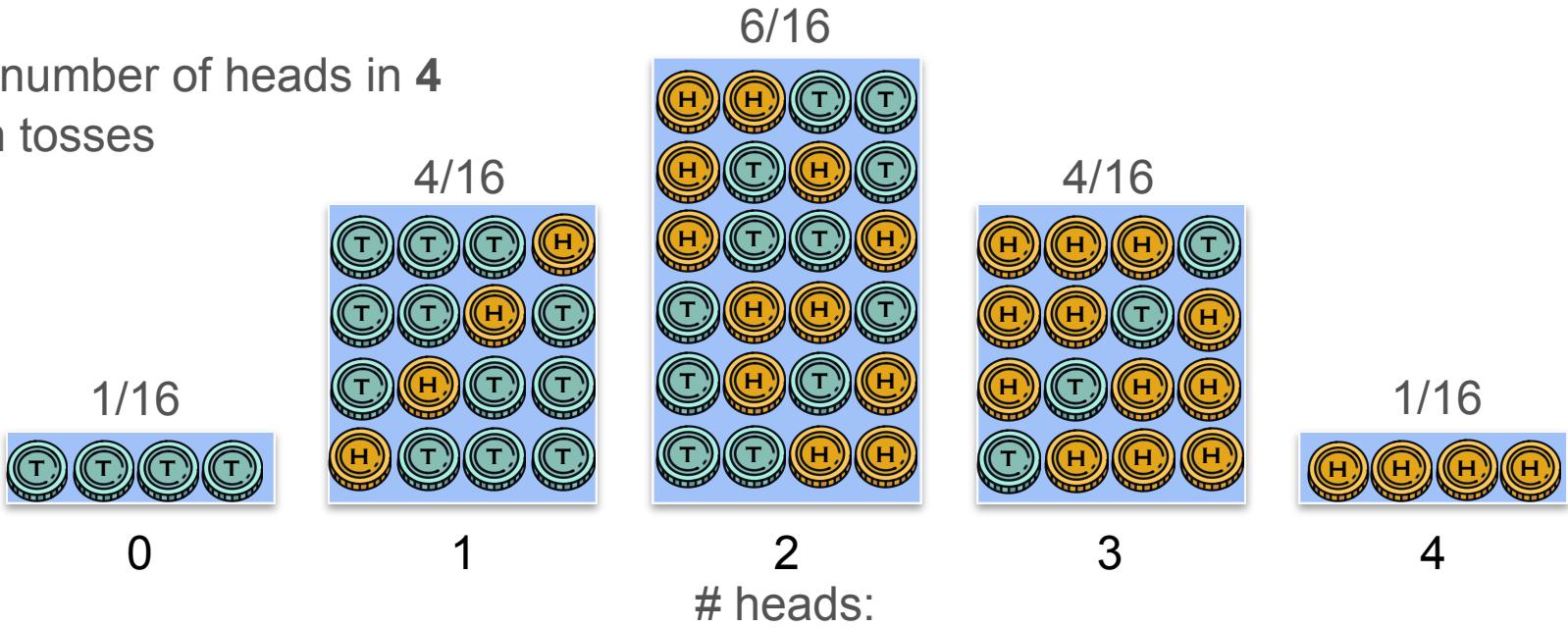
Discrete Distributions: Flip Three Coins

X_1 : number of heads in 3 coin tosses



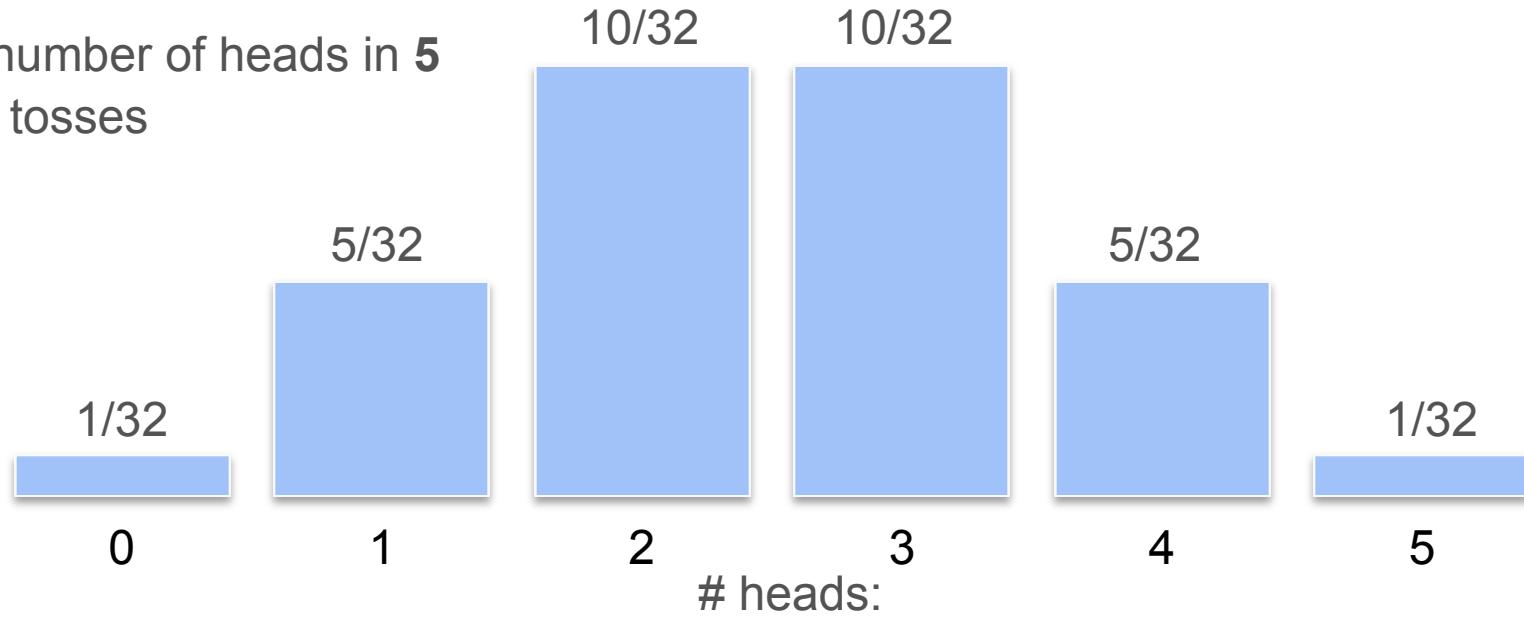
Discrete Distributions: Flip Four Coins

X_2 : number of heads in 4 coin tosses



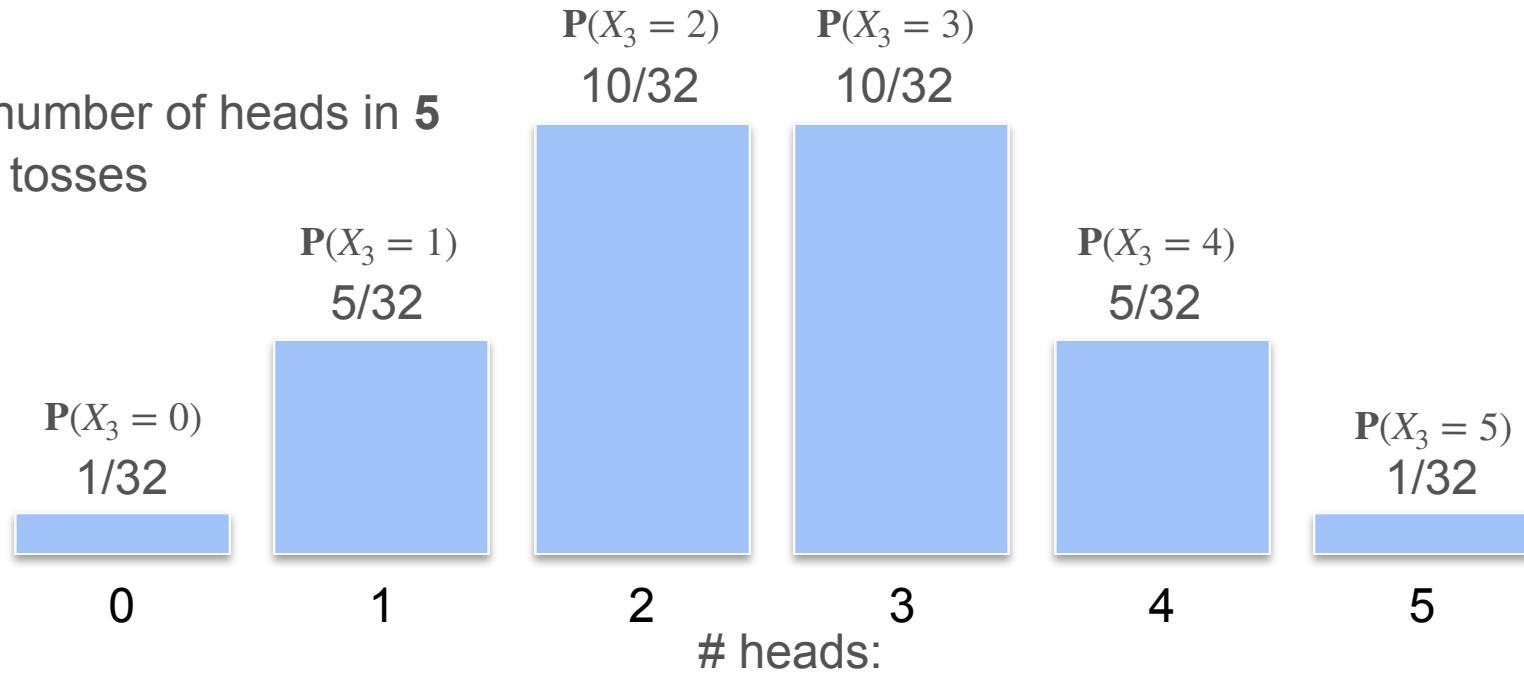
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



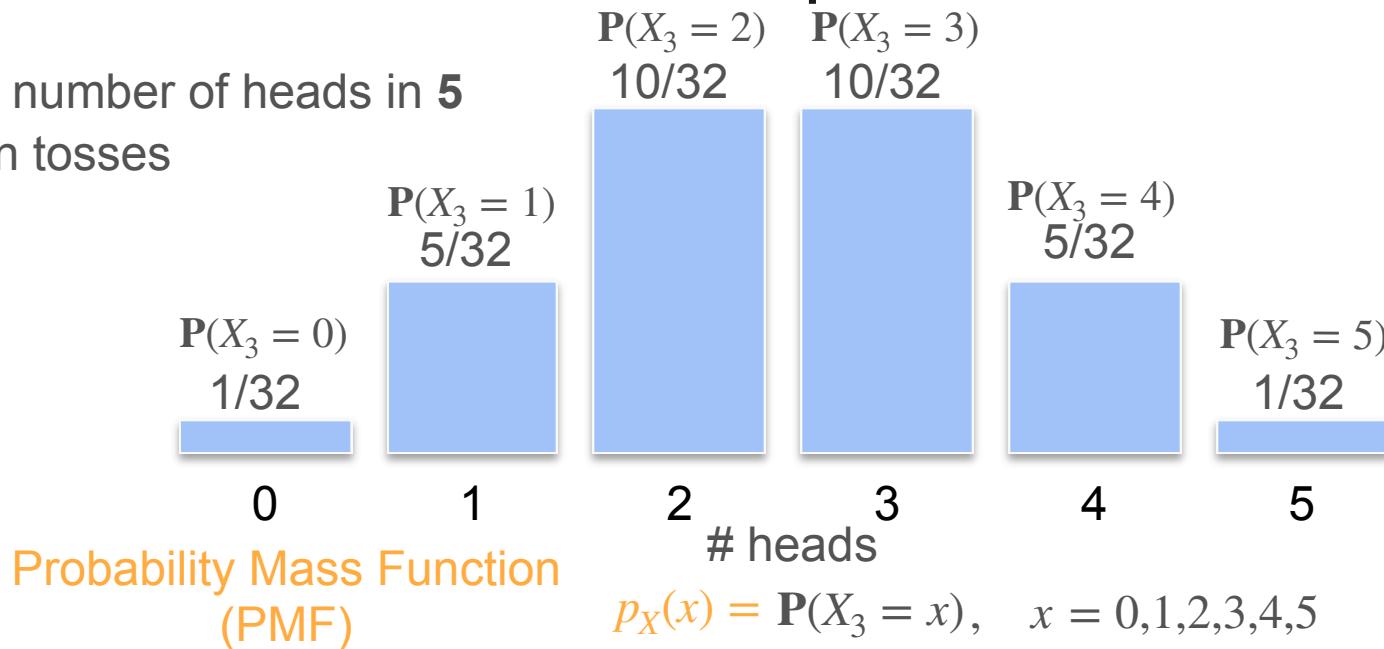
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



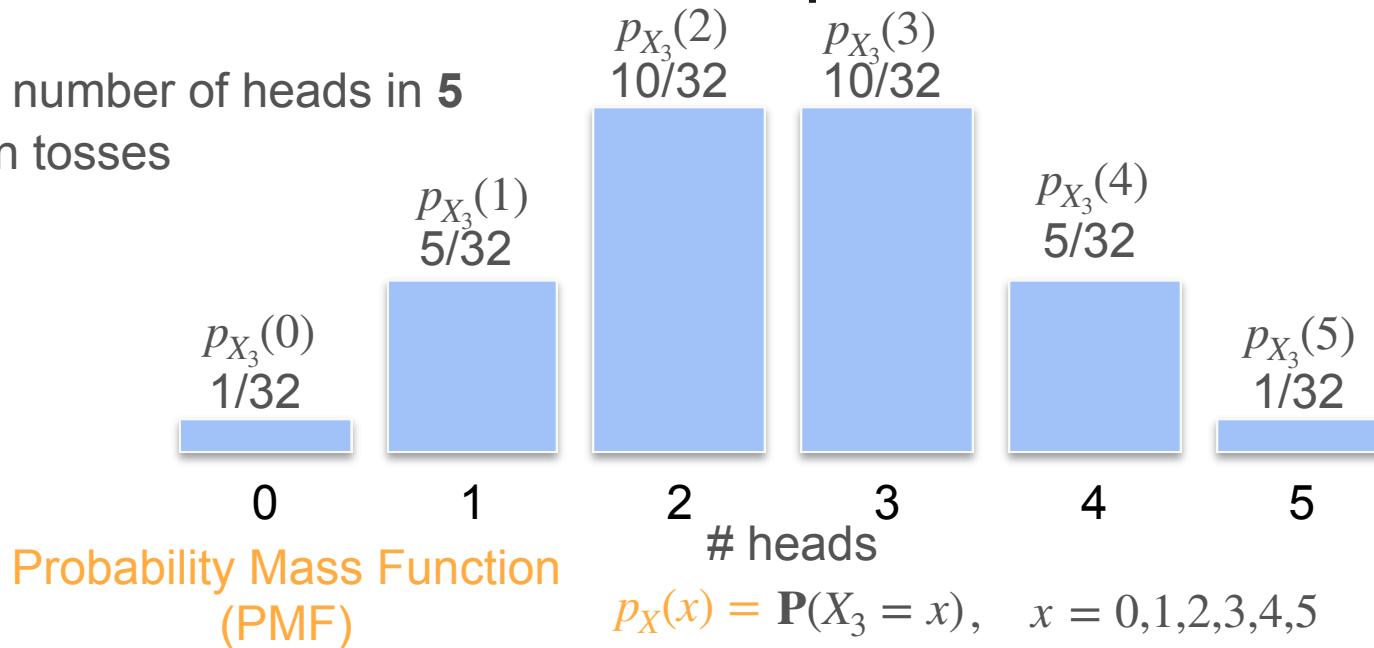
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



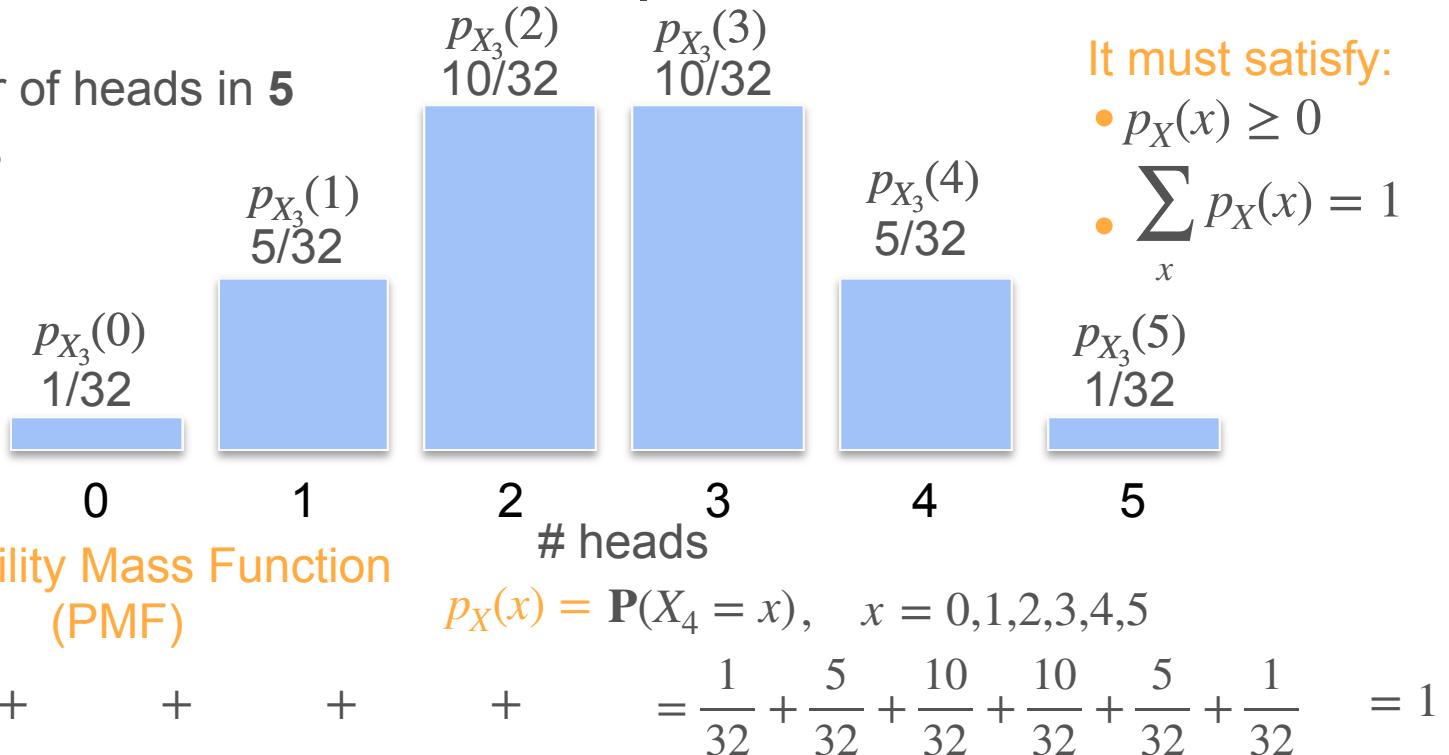
Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



Discrete Distributions: Flip Five Coins

X_3 : number of heads in 5 coin tosses



Can You See a Pattern?

X_1, X_2, X_3, X_4 are very similar

They all represent **number of heads in n experiments**

The way the probability distributes along the possible outcomes seems to have a similar pattern

Could there be a **single model** to represent all this variables?



Binomial distribution



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Probability Distributions

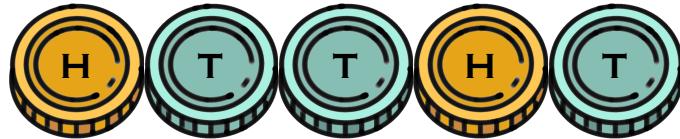
Binomial Distribution

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



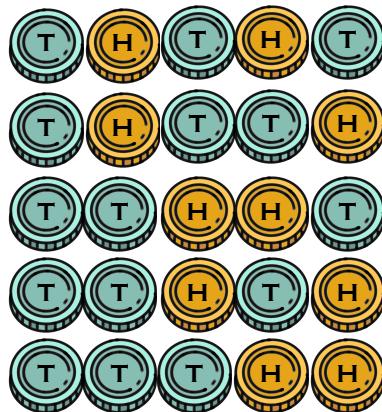
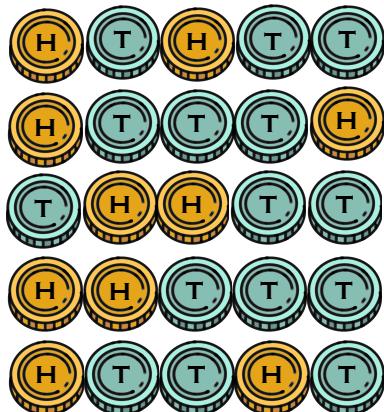
$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$



$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 = \frac{1}{32}$$

Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{2!}$$

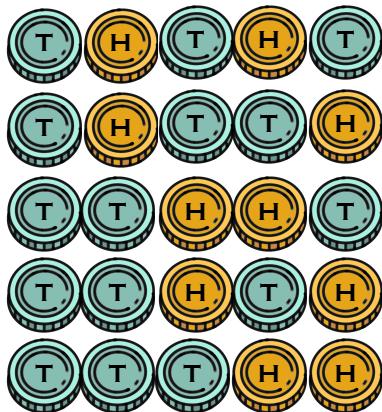
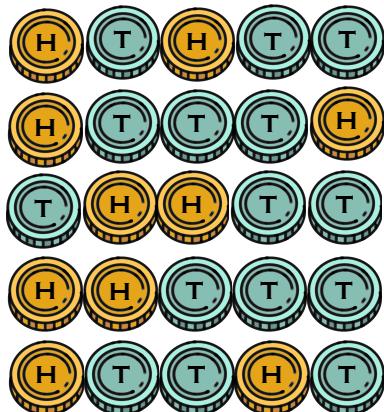
Number of ways you can order 5 coins

Number of H



Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{2!(5-2)!}$$

Number of ways you can order 5 coins

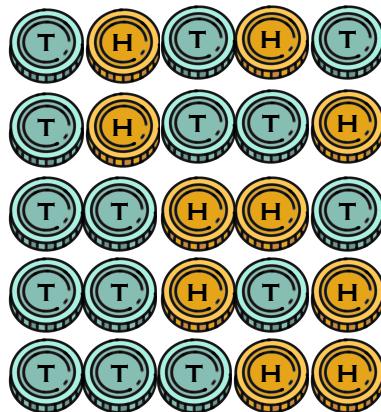
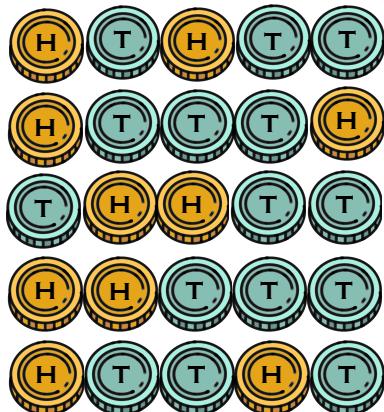
Number of H Number of T

The equation $10 = \frac{5!}{2!(5-2)!}$ represents the number of ways to order 5 coins, where 5! is the total number of permutations of 5 items, and $2!(5-2)!$ is the number of ways to choose 2 positions for heads (or tails) from 5.



Binomial Distribution: Example

What is the probability that if I flip 5 coins, 2 of them land in heads?



$$10 = \frac{5!}{2!(5-2)!} = \binom{5}{2}$$

Binomial coefficient

Number of ways you can get 2 heads in 5 coin tosses

Binomial Distribution: Binomial Coefficient

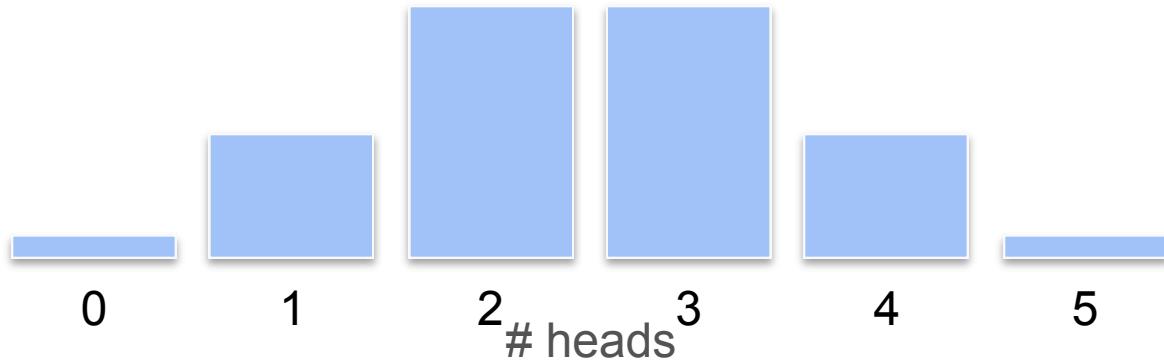
In general:

$\binom{n}{k}$ counts all the combinations for landing k heads in n coin tosses

Property:

The PMF with a fair coin is symmetrical

$$\binom{n}{k} = \binom{n}{n-k}$$



Binomial Distribution: Binomial Coefficient

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$\binom{5}{x} p^x (1 - p)^{5-x}$$

All the possible orders → $\binom{5}{x}$

Probability of seeing x heads ↑ p^x

Probability of seeing $5 - x$ tails $(1 - p)^{5-x}$

Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

X follows a binomial distribution

$X \sim \text{Binomial}(5, p)$

Number of flips $\mathbf{P}(H)$

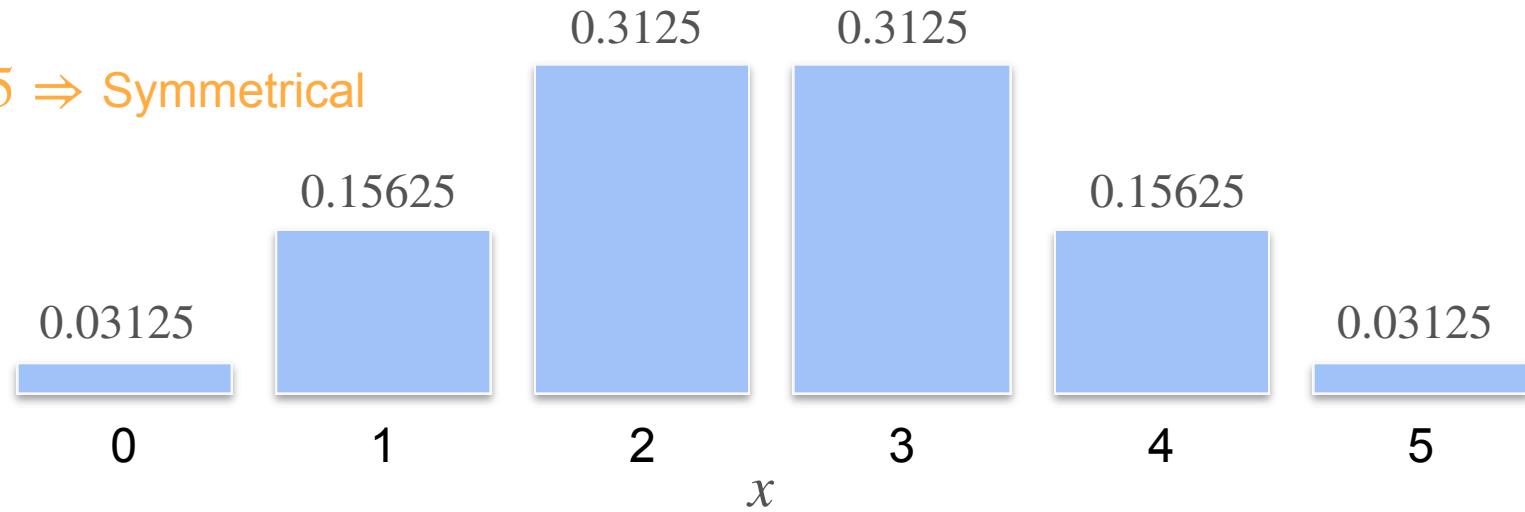
```
graph LR; A["p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, x = 0,1,2,3,4,5"] --> B["X ~ Binomial(5, p)"]; C["Number of flips"] --> D["5"]
```

Binomial Distribution

$$\begin{array}{|l|} \hline n = 5 \\ p = 0.5 \\ \hline \end{array}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.5^k 0.5^{5-k}$$

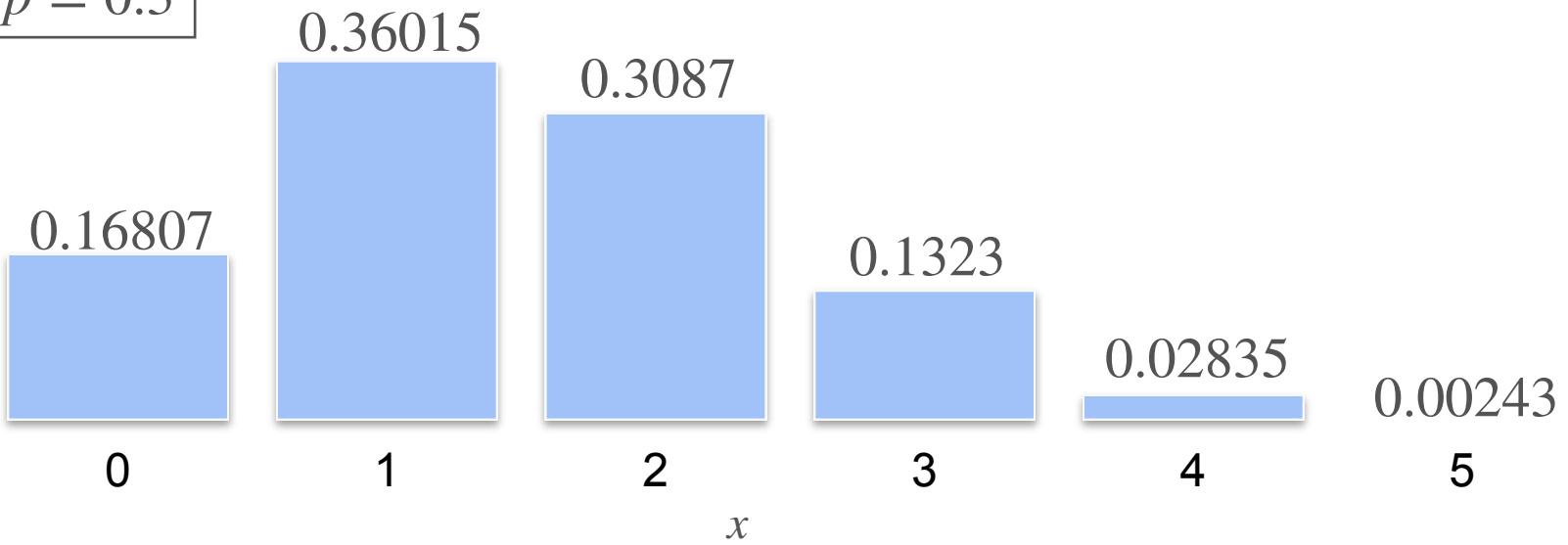
$p = 0.5 \Rightarrow$ Symmetrical



Binomial Distribution

$$\begin{aligned} n &= 5 \\ p &= 0.3 \end{aligned}$$

$$p_X(x) = \mathbf{P}(X = x) = \binom{5}{k} 0.3^k 0.7^{5-k}$$



Binomial Distribution

General PMF for X : number of heads in 5 coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in 5 tosses

$$p_X(x) = \binom{5}{x} p^x (1-p)^{5-x}, \quad x = 0,1,2,3,4,5$$

$$X \sim \text{Binomial}(5,p)$$

Binomial Distribution

General PMF for X : number of heads in n coin tosses?

Your coin has $\mathbf{P}(H) = p$

Event: $X = x$: x heads in n tosses

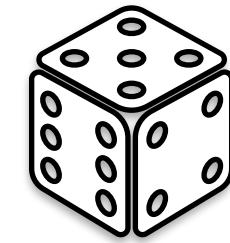
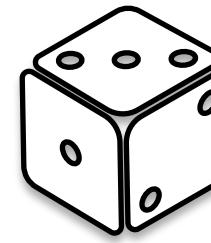
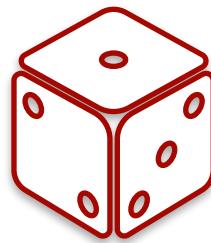
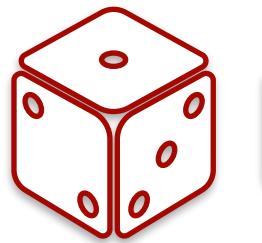
$$p_X(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n$$

$X \sim \text{Binomial}(n, p)$

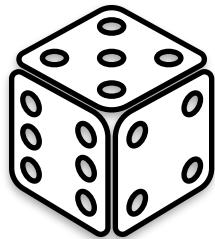
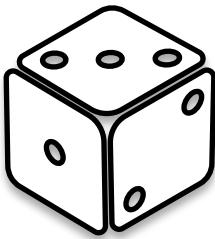
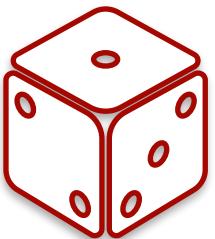
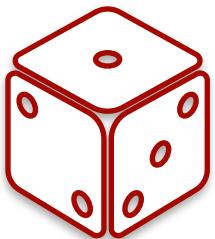
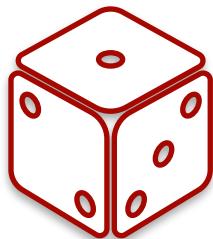
n and p are called the **parameters** of the binomial distribution

Binomial Distribution: Quiz

What is the probability of getting three ones when rolling a dice five times (no matter on which dice)?



Binomial Distribution: Dice Is a Biased Coin!



one
 $p = \frac{1}{6}$



heads
 $p = \frac{1}{6}$

one
 $p = \frac{1}{6}$



heads
 $p = \frac{1}{6}$

one
 $p = \frac{1}{6}$



heads
 $p = \frac{1}{6}$

not one
 $p = \frac{5}{6}$



not heads
 $p = \frac{5}{6}$

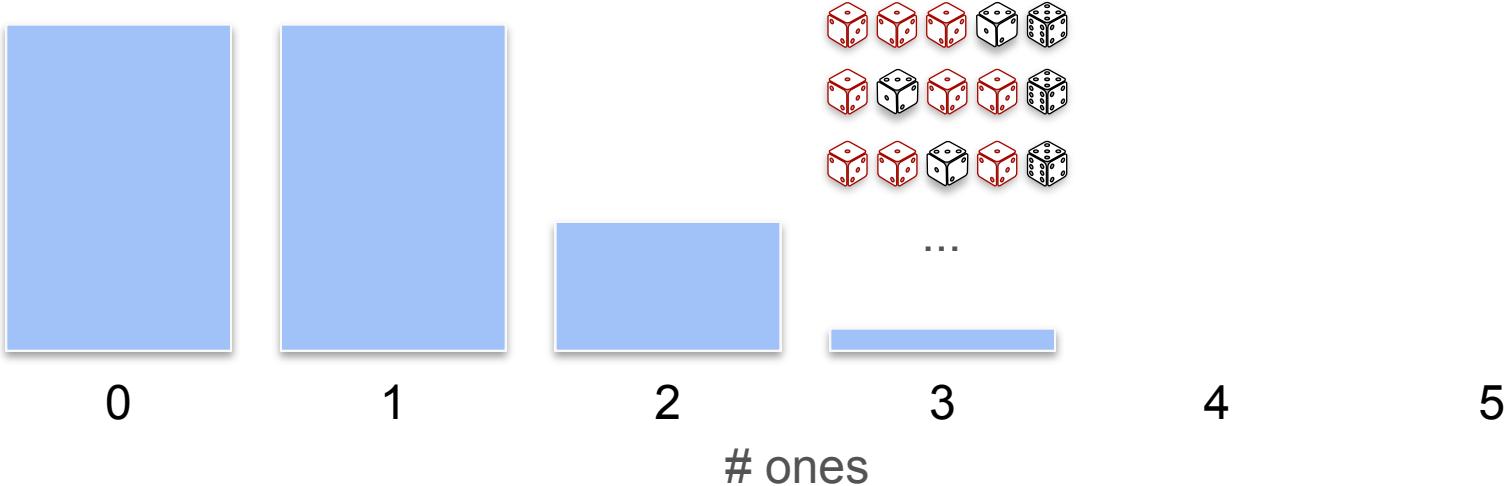
not one
 $p = \frac{5}{6}$



not heads
 $p = \frac{5}{6}$

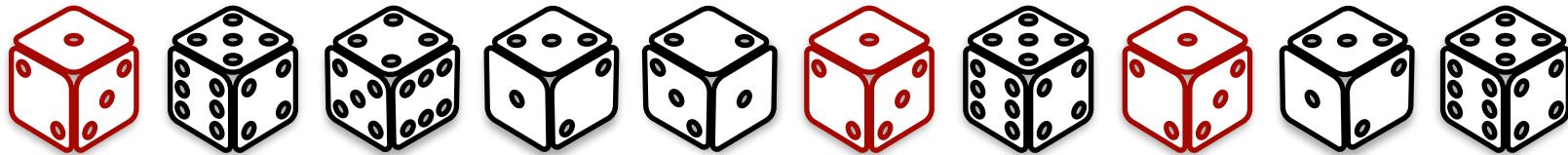
Binomial Distribution: Dice Is a Biased Coin!

$n = 5$
 $p = 0.1666$



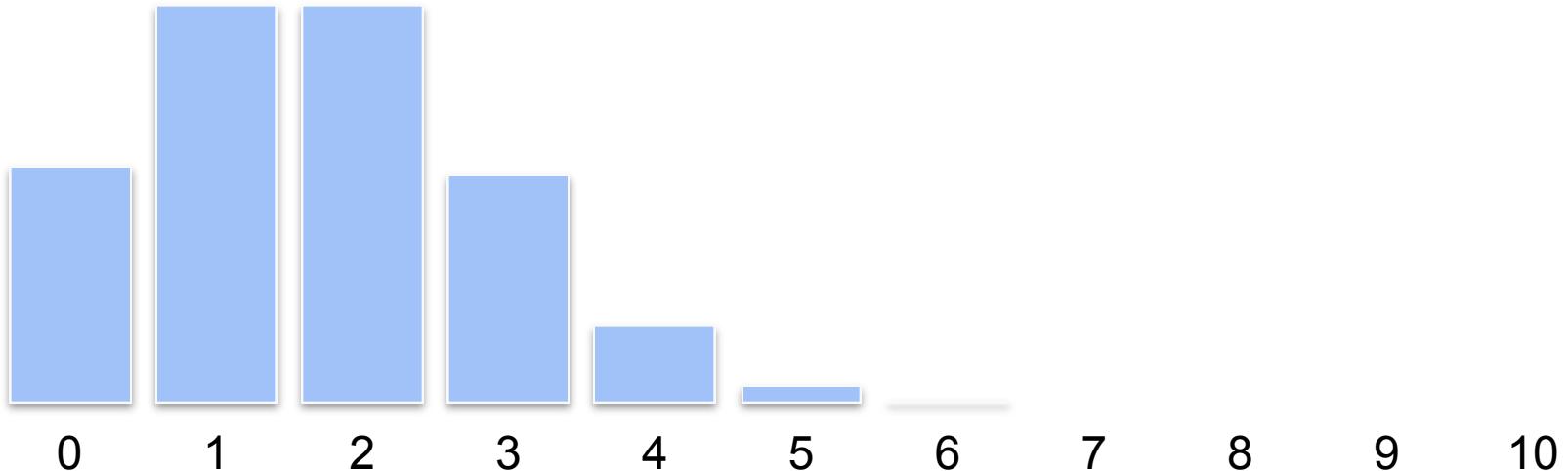
Binomial Distribution: Quiz

- Quiz: What are the parameters for the following binomial distribution:
 - I roll 10 dice
 - I want to record the number of times I obtain the number 1



Binomial Distribution: Quiz

$$\begin{aligned}n &= 10 \\p &= 0.1666\end{aligned}$$



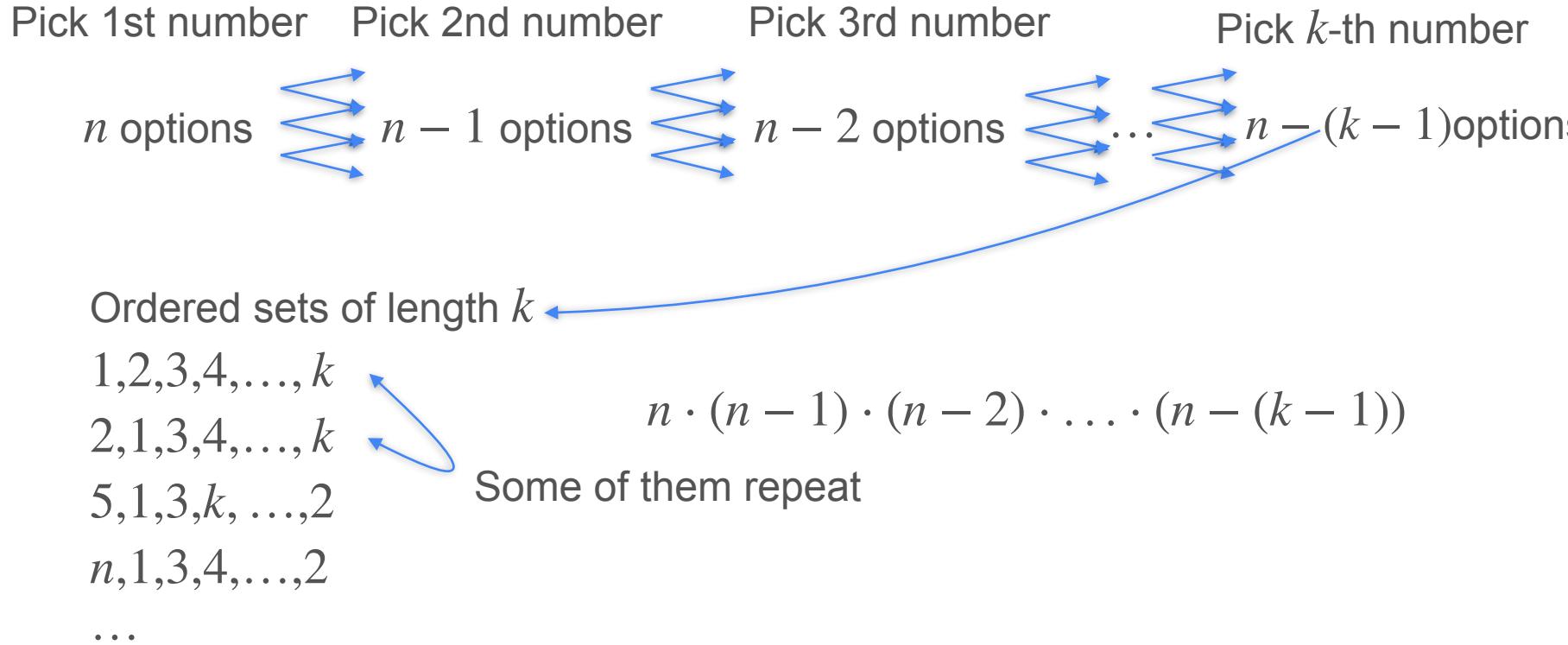


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Probability Distributions

(Optional)
Binomial Coefficient

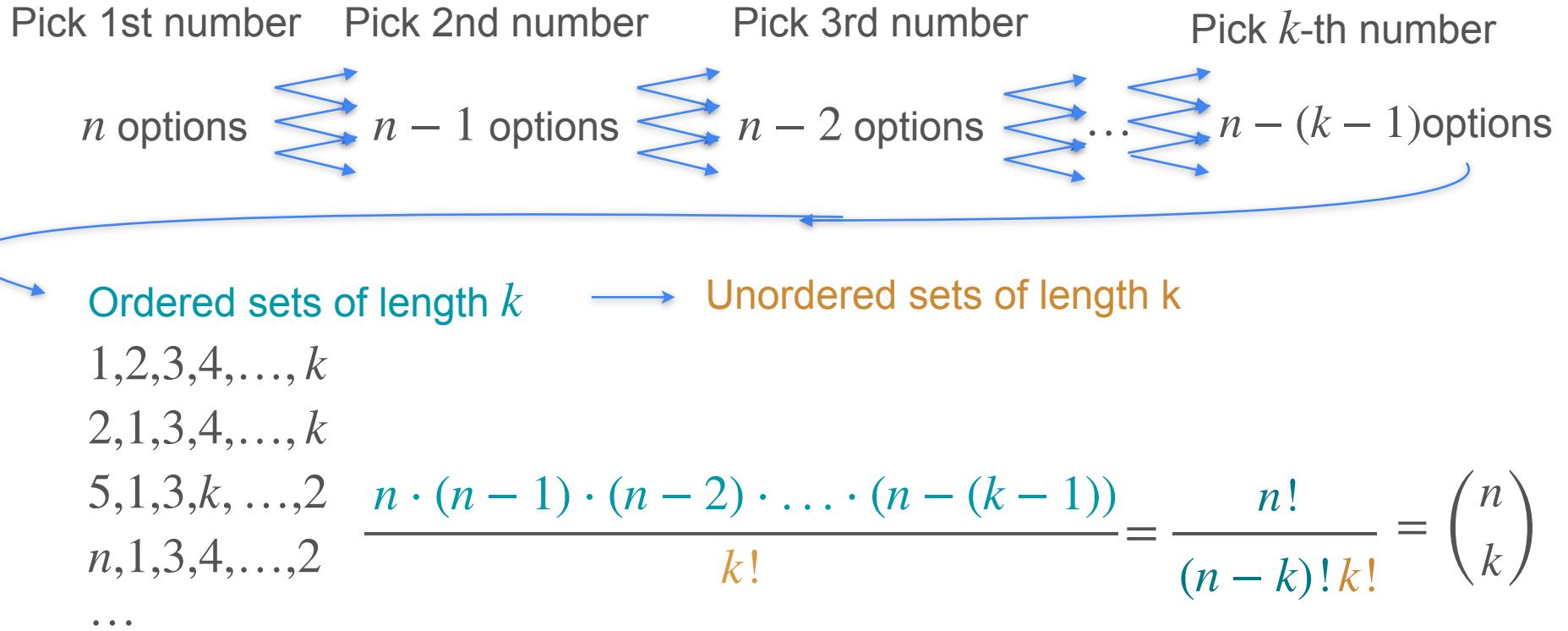
Binomial Coefficient



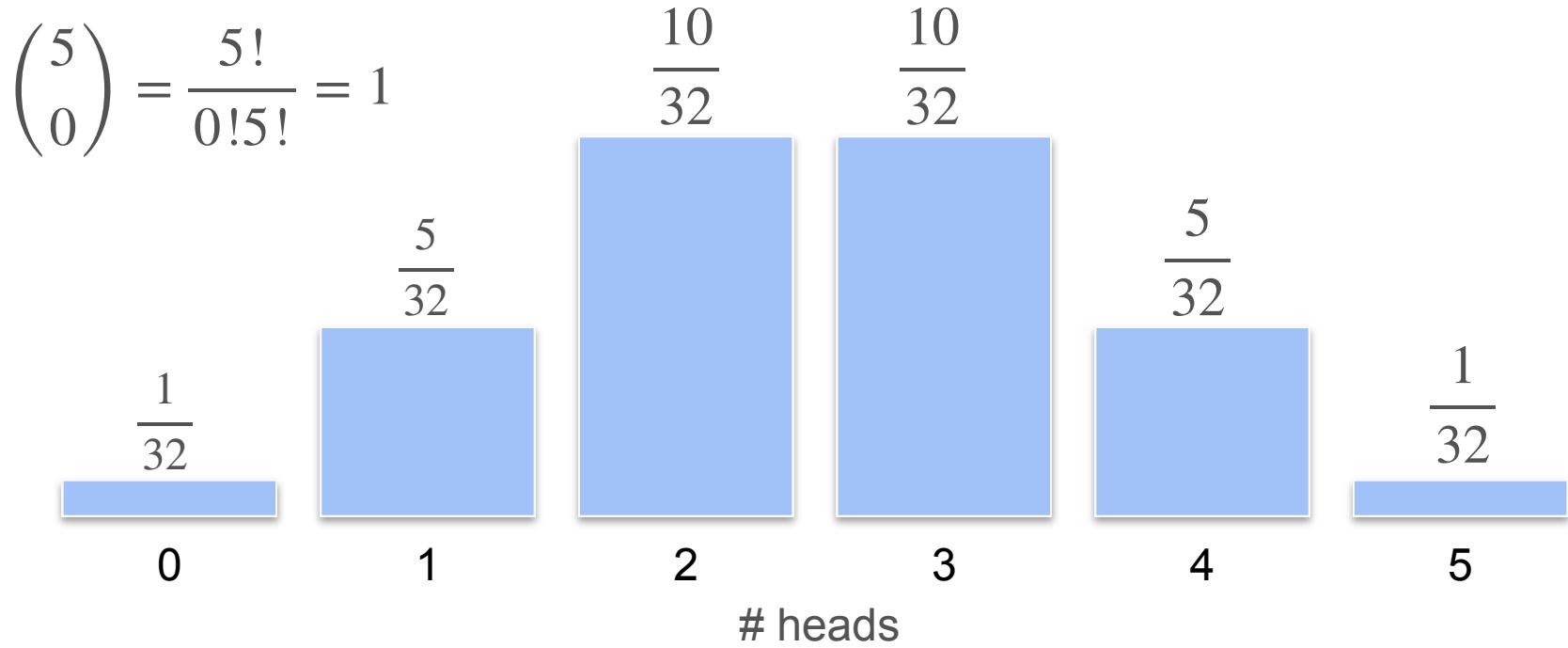
Binomial Coefficient

	Pick 1st	Pick 2nd	Pick 3rd	Pick 4th
1,2,3,4	4 options	3 options	2 options	1 option
1,2,4,3				
1,3,2,4		$4 \cdot 3 \cdot 2 \cdot 1 = 4!$		
1,3,4,2		For five numbers:		
		$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5!$		
...				
4,3,2,1		General solution:	$k!$	

Binomial Coefficient



Binomial Distribution: Fair Coins



Binomial Distribution: Fair Coins



50 %



50 %



$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5$$

$$= \frac{1}{32}$$



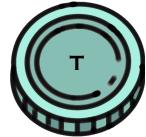
$$0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5 \cdot 0.5$$

$$= \frac{1}{32}$$

Binomial Distribution: Biased Coins



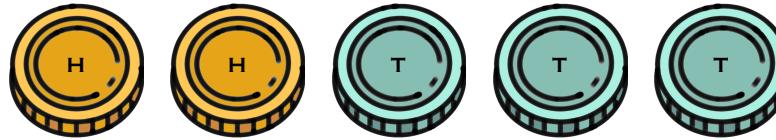
30 %



70 %



$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.00243$$



$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.01323$$

Binomial Distribution: Biased Coins


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 = 0.3^5 \cdot 0.7^0$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 = 0.3^4 \cdot 0.7^1$$


$$0.3 \cdot 0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^3 \cdot 0.7^2$$


$$0.3 \cdot 0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^2 \cdot 0.7^3$$


$$0.3 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^1 \cdot 0.7^4$$


$$0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 \cdot 0.7 = 0.3^0 \cdot 0.7^5$$

$$= 0.3^k \cdot 0.7^{n-k}$$

Binomial Distribution: Biased Coins

	$0.3^5 \cdot 0.7^0$
	$0.3^4 \cdot 0.7^1$
	$0.3^3 \cdot 0.7^2$
	$0.3^2 \cdot 0.7^3$
	$0.3^1 \cdot 0.7^4$
	$0.3^0 \cdot 0.7^5$

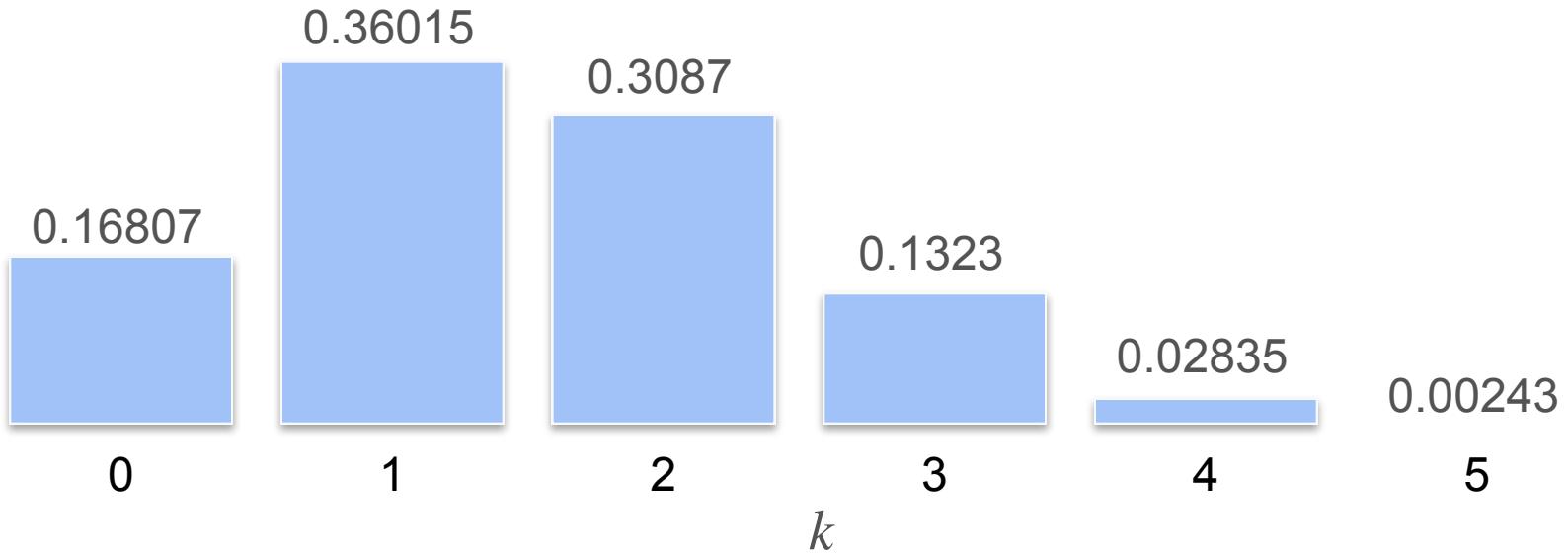
$$= 0.3^k \cdot 0.7^{n-k} \rightarrow \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

Account for all possible orders of heads and tails

Binomial Distribution: Biased Coins

$$= \binom{n}{k} 0.3^k \cdot 0.7^{n-k}$$

n = 5
p = 0.3





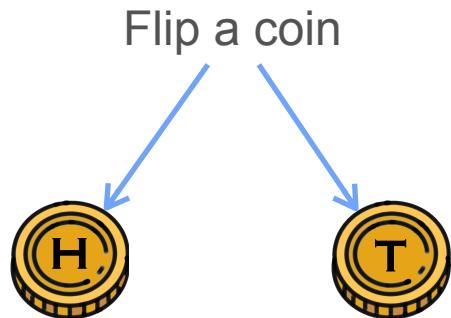
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Probability Distributions

Bernoulli Distribution

Bernoulli Distribution

X = Number of heads



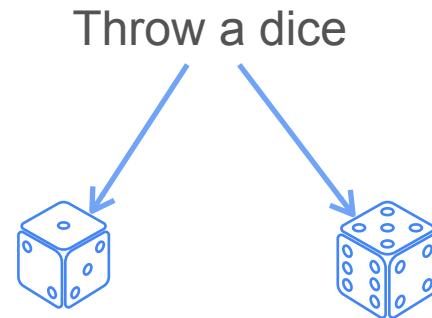
$$P(X = 1) = 0.5$$

Success

$$P(X = 0) = 0.5$$

Failure

X = Number of 1's



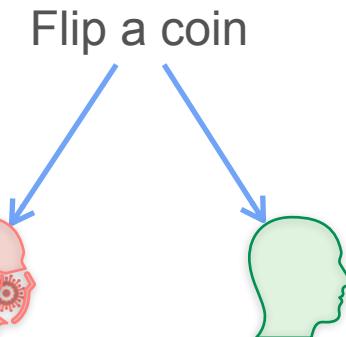
$$P(X = 1) = \frac{1}{6}$$

Success

$$P(X = 0) = \frac{5}{6}$$

Failure

X = Number of sick patients



$$P(X = 1) = p$$

Success

$$P(X = 0) = 1 - p$$

Failure

$$X \sim \text{Bernoulli}(p)$$

p is the parameter of the Bernoulli distribution

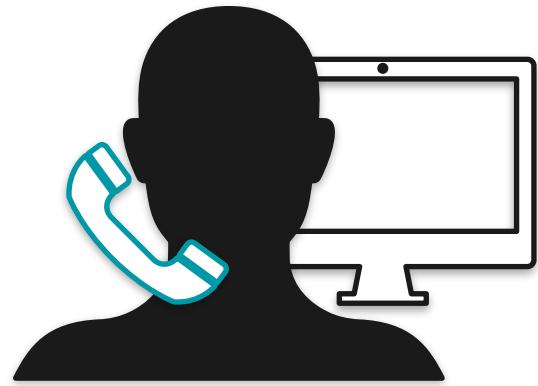


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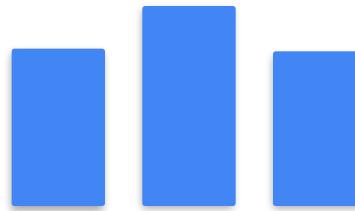
Probability Distributions

Probability Distributions (Continuous)

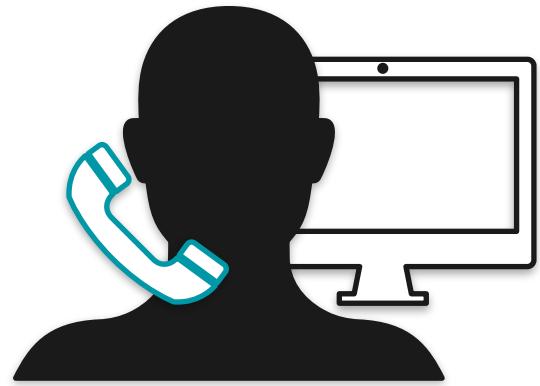
From Discrete to Continuous Distributions



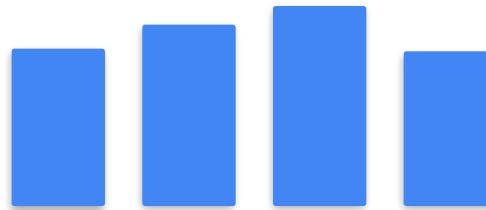
Waiting time: 1 2 3 (min)



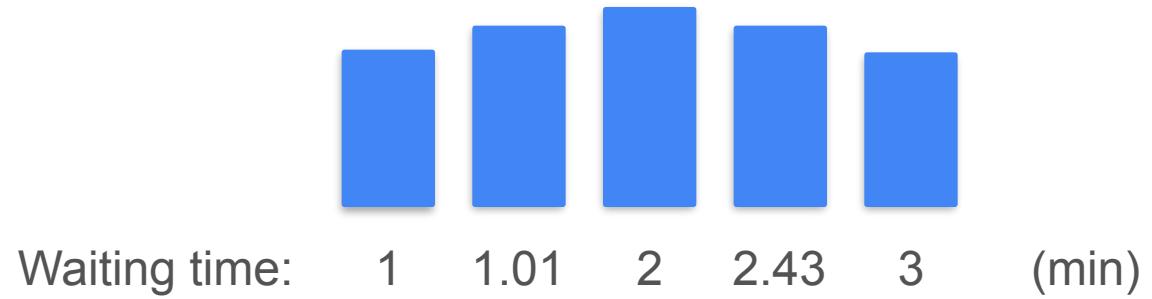
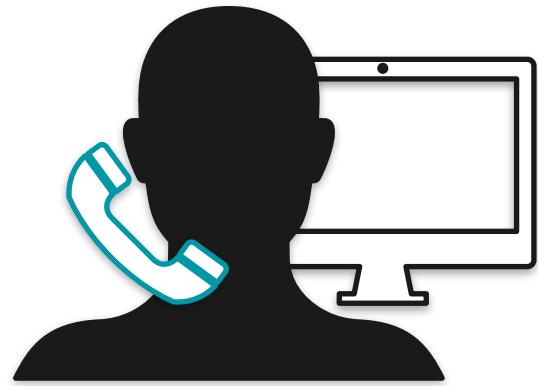
From Discrete to Continuous Distributions



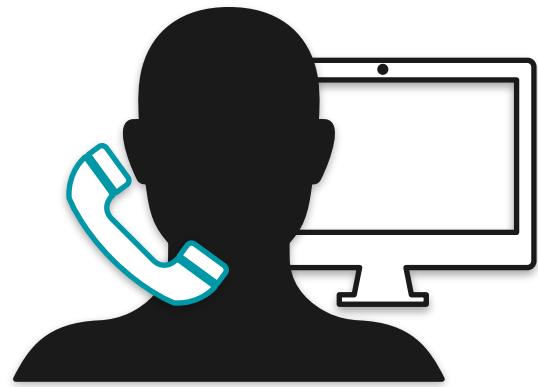
Waiting time: 1 1.01 2 3 (min)



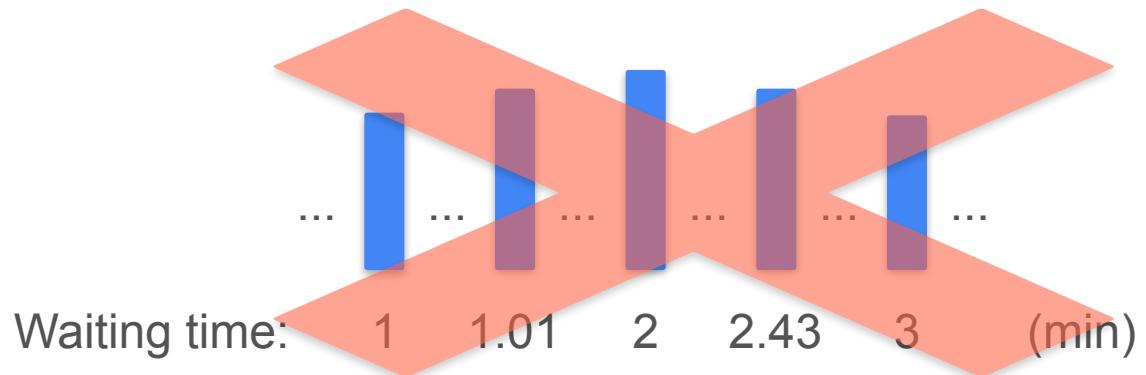
From Discrete to Continuous Distributions



From Discrete to Continuous Distributions



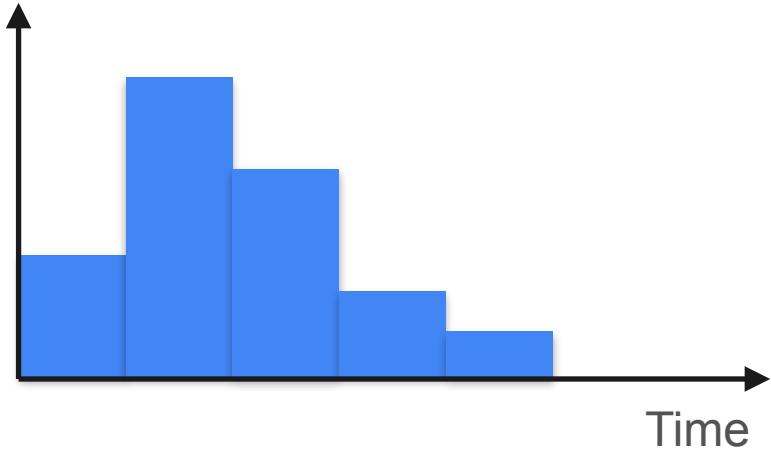
This is a continuous distribution!



What is the probability that you will wait EXACTLY one minute for the call?

Answer: ZERO

From Discrete to Continuous Distributions



$P(\text{between 0 and 1 mins})$

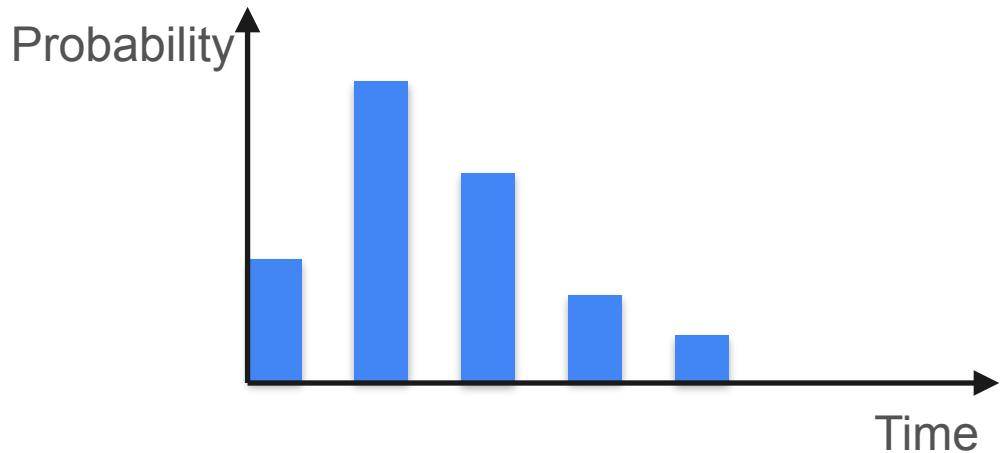
$P(\text{between 1 and 2 mins})$

$P(\text{between 2 and 3 mins})$

$P(\text{between 3 and 4 mins})$

$P(\text{between 4 and 5 mins})$

From Discrete to Continuous Distributions



$P(\text{between 0 and 1 mins})$

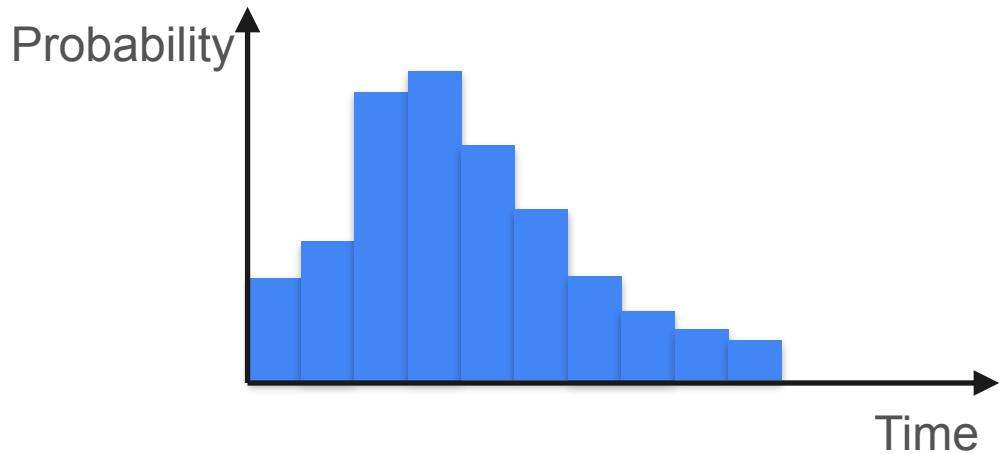
$P(\text{between 1 and 2 mins})$

$P(\text{between 2 and 3 mins})$

$P(\text{between 3 and 4 mins})$

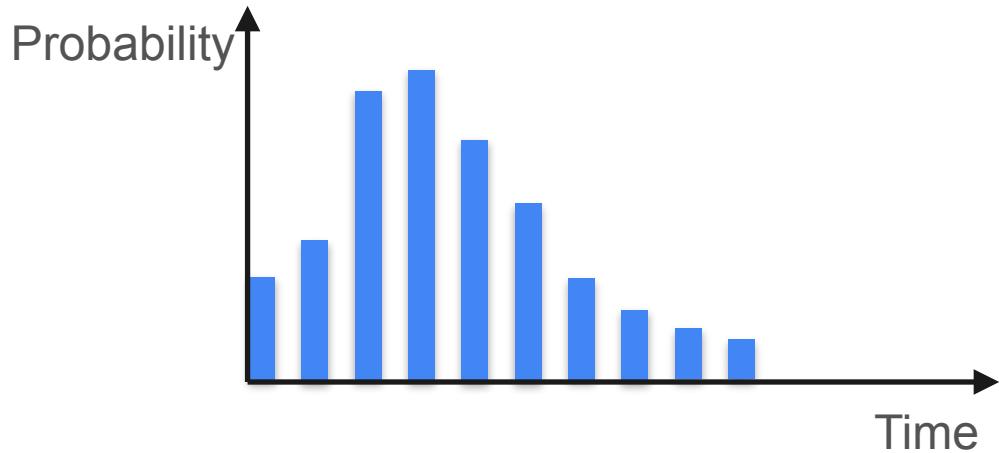
$P(\text{between 4 and 5 mins})$

From Discrete to Continuous Distributions



$P(\text{between } 0 \text{ and } 0.5 \text{ mins})$
 $P(\text{between } 0.5 \text{ and } 1 \text{ mins})$
 $P(\text{between } 1 \text{ and } 1.5 \text{ mins})$
⋮
 $P(\text{between } 3.5 \text{ and } 4 \text{ mins})$
 $P(\text{between } 4 \text{ and } 4.5 \text{ mins})$
 $P(\text{between } 4.5 \text{ and } 5 \text{ mins})$

From Discrete to Continuous Distributions



$P(\text{between } 0 \text{ and } 0.5 \text{ mins})$

$P(\text{between } 0.5 \text{ and } 1 \text{ mins})$

$P(\text{between } 1 \text{ and } 1.5 \text{ mins})$

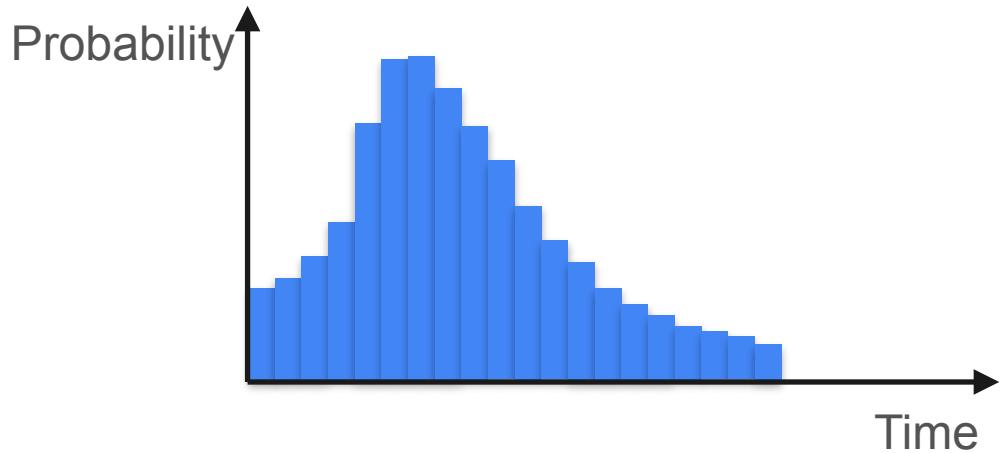
⋮

$P(\text{between } 3.5 \text{ and } 4 \text{ mins})$

$P(\text{between } 4 \text{ and } 4.5 \text{ mins})$

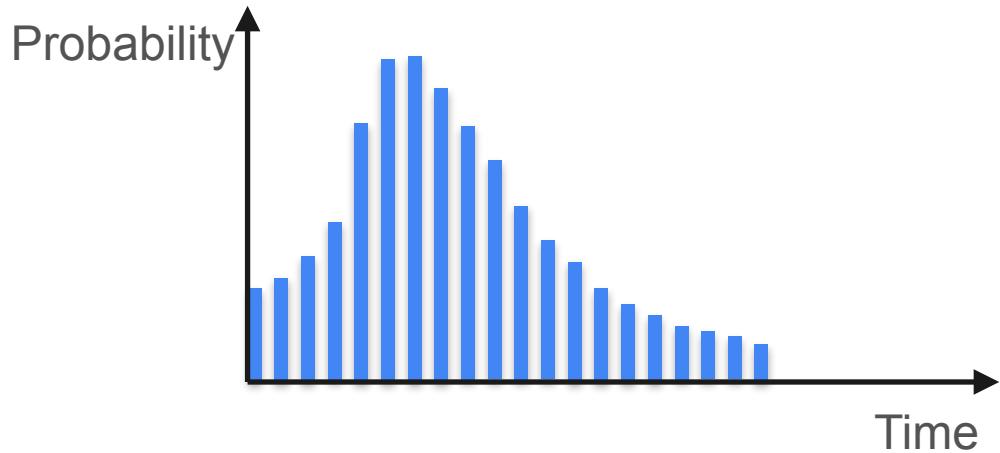
$P(\text{between } 4.5 \text{ and } 5 \text{ mins})$

From Discrete to Continuous Distributions



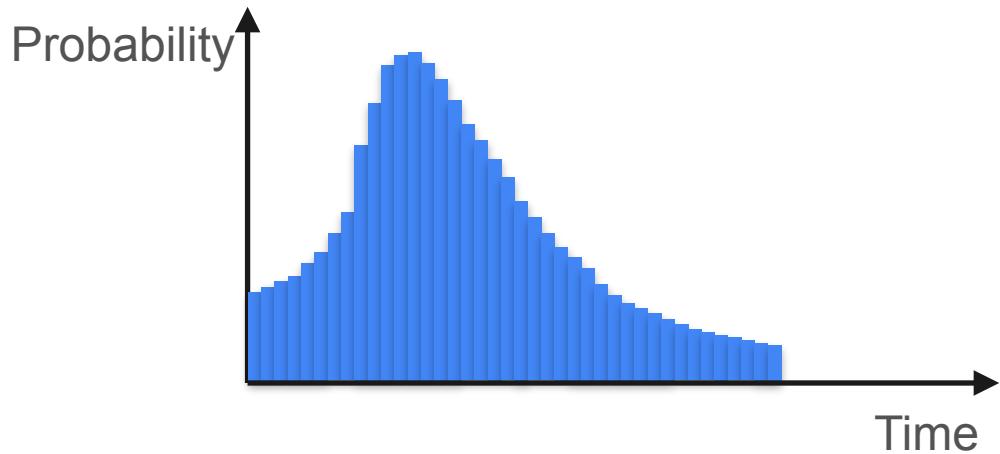
$P(\text{between } 0 \text{ and } 0.25 \text{ mins})$
 $P(\text{between } 0.25 \text{ and } 0.5 \text{ mins})$
 $P(\text{between } 0.5 \text{ and } 0.75 \text{ mins})$
⋮
 $P(\text{between } 4.25 \text{ and } 4.5 \text{ mins})$
 $P(\text{between } 4.5 \text{ and } 4.75 \text{ mins})$
 $P(\text{between } 4.75 \text{ and } 5 \text{ mins})$

From Discrete to Continuous Distributions



$P(\text{between } 0 \text{ and } 0.25 \text{ mins})$
 $P(\text{between } 0.25 \text{ and } 0.5 \text{ mins})$
 $P(\text{between } 0.5 \text{ and } 0.75 \text{ mins})$
⋮
 $P(\text{between } 4.25 \text{ and } 4.5 \text{ mins})$
 $P(\text{between } 4.5 \text{ and } 4.75 \text{ mins})$
 $P(\text{between } 4.75 \text{ and } 5 \text{ mins})$

From Discrete to Continuous Distributions

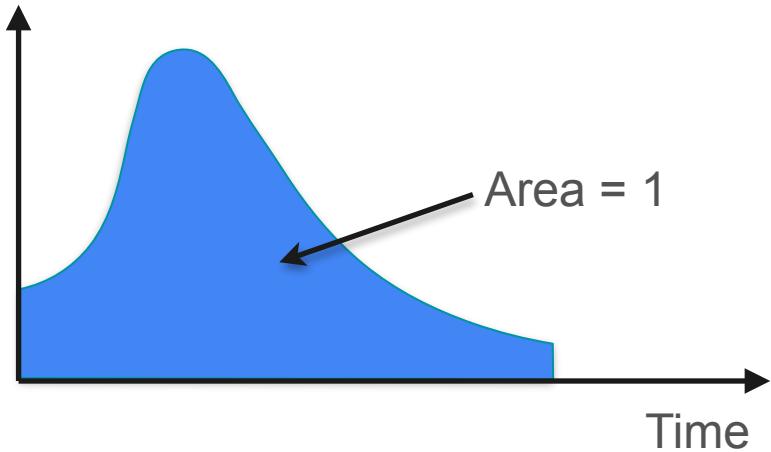


$P(\text{between } 0 \text{ and } 0.125 \text{ mins})$

⋮

$P(\text{between } 4.875 \text{ and } 5 \text{ mins})$

From Discrete to Continuous Distributions



- Discrete:
 - Sum of heights equals 1
- Continuous:
 - Area under the curve equals 1

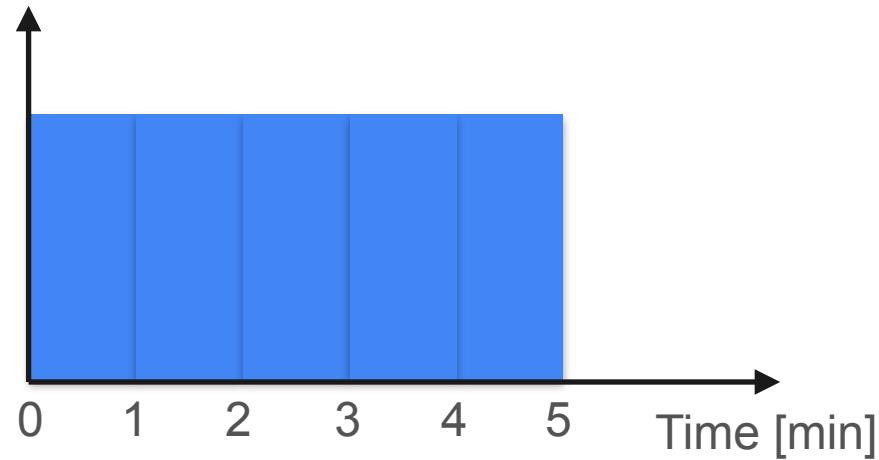


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Probability Distributions

Probability density function

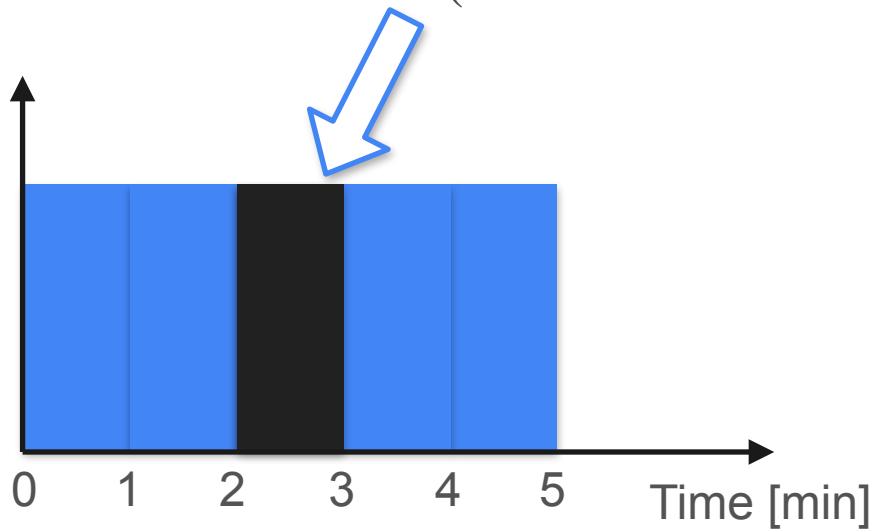
Probability Density Function



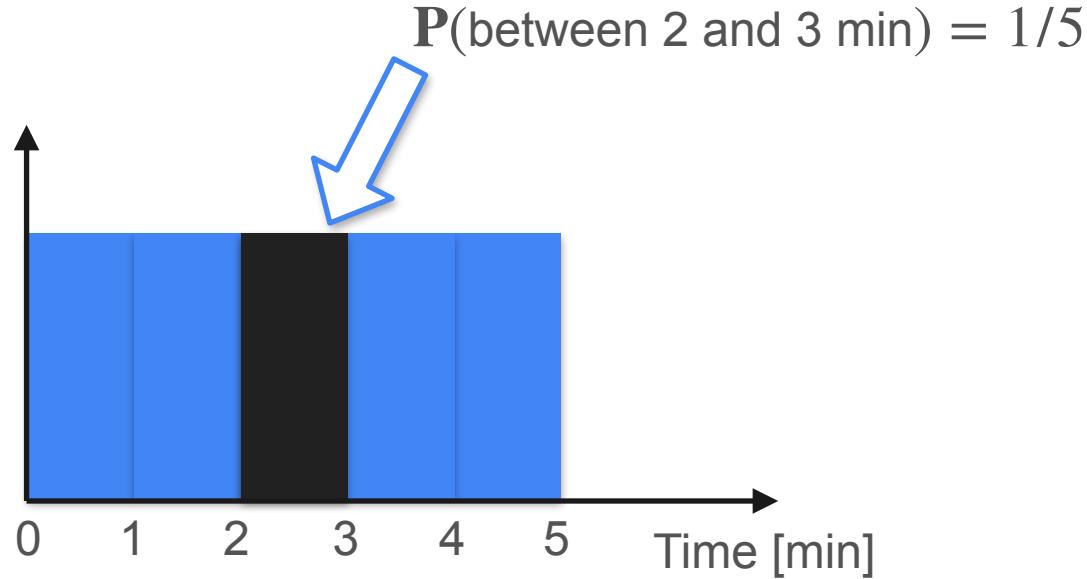
Probability Density Function



$P(\text{between 2 and 3 min}) = ?$



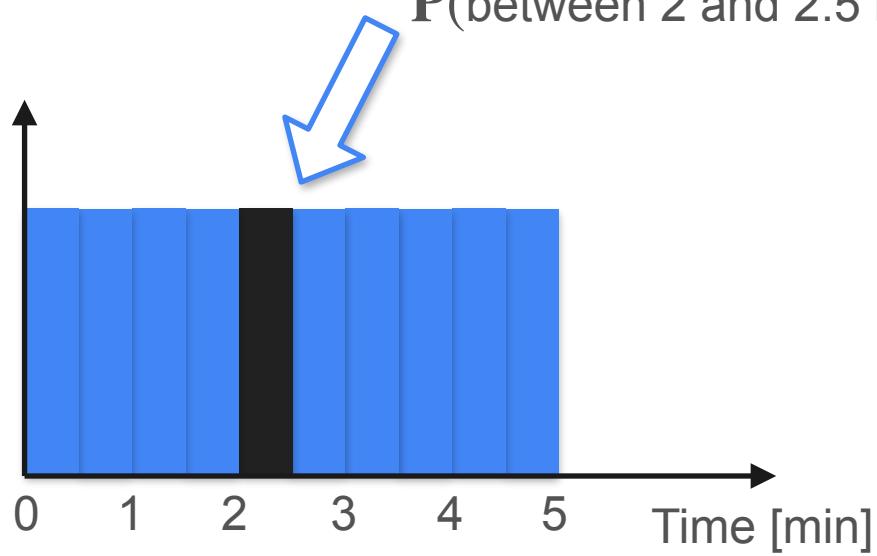
Probability Density Function



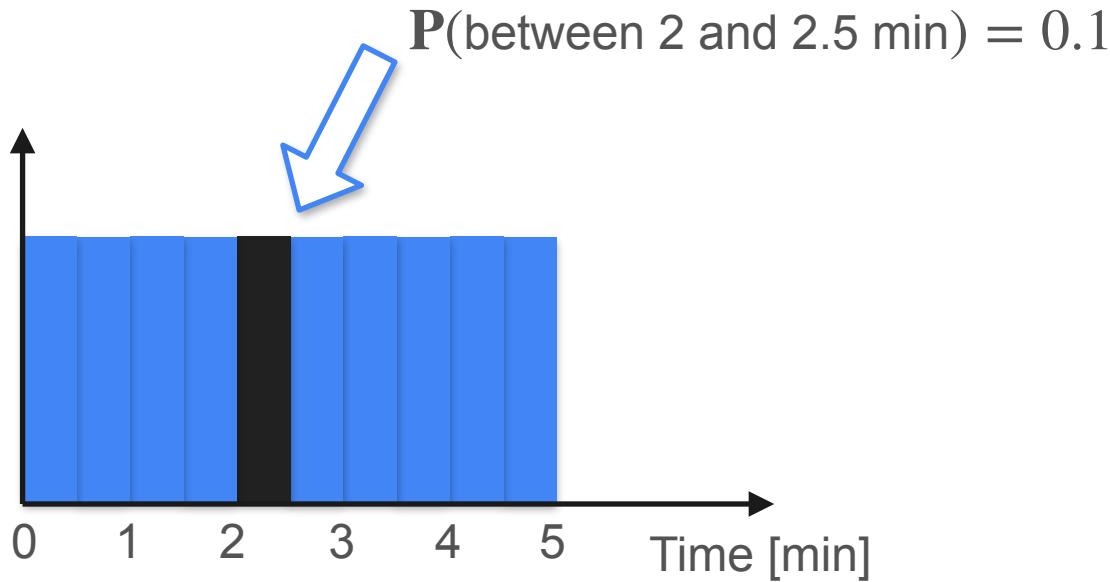
Probability Density Function



$P(\text{between } 2 \text{ and } 2.5 \text{ min}) = ?$



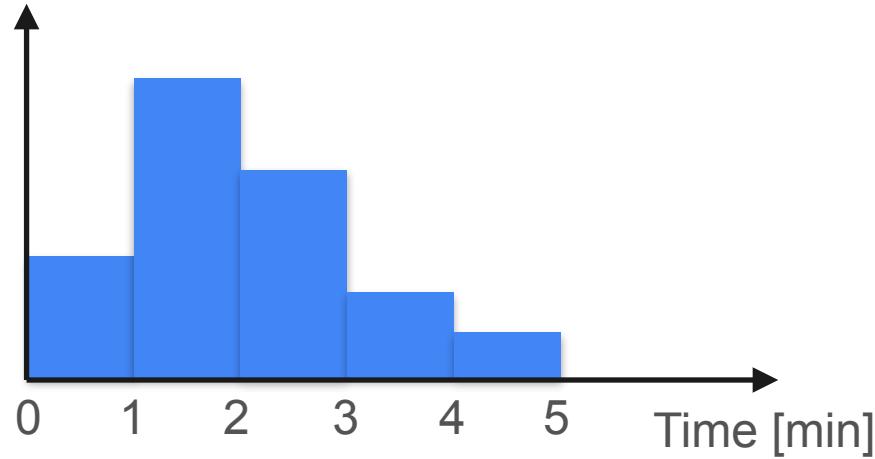
Probability Density Function



Probability Density Function



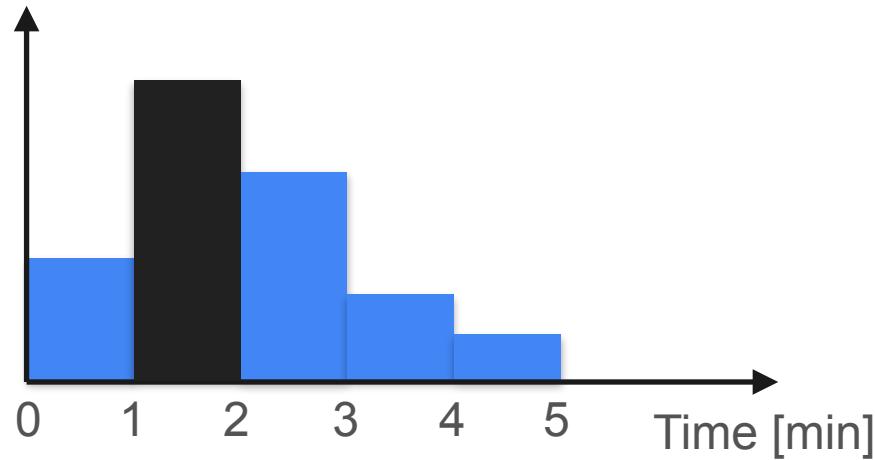
$P(\text{between 1 and 2 min})$



Probability Density Function



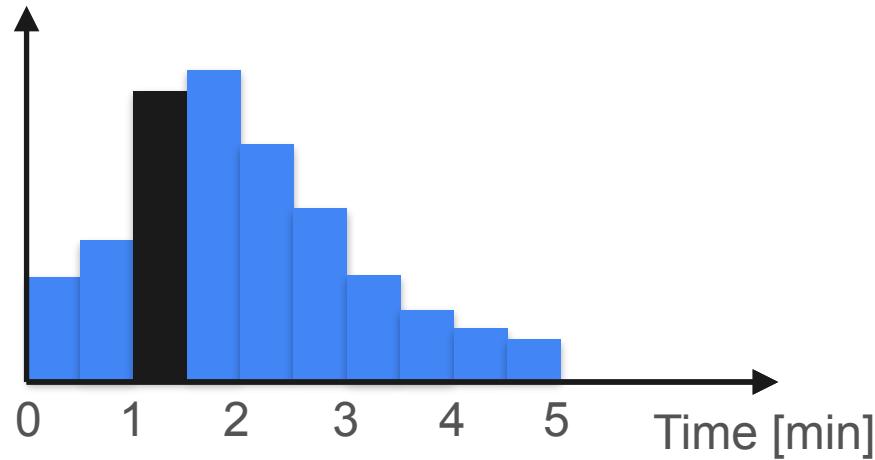
$P(\text{between 1 and 2 min})$



Probability Density Function



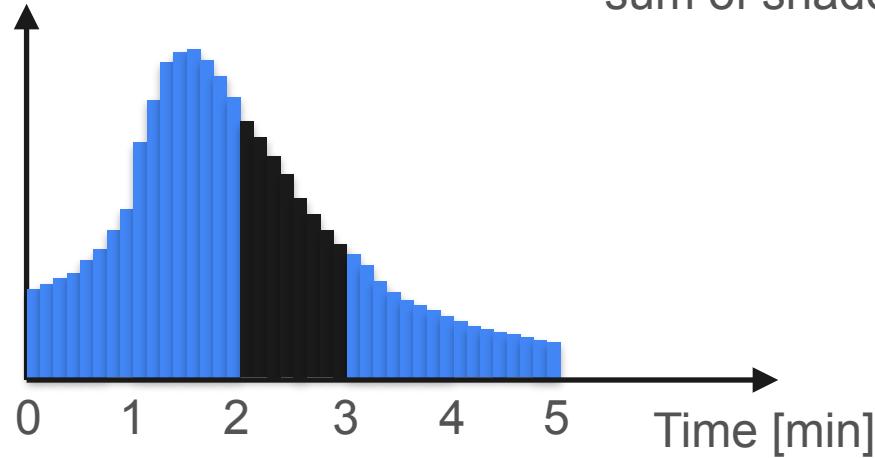
$P(\text{between 1 and 1:30})$



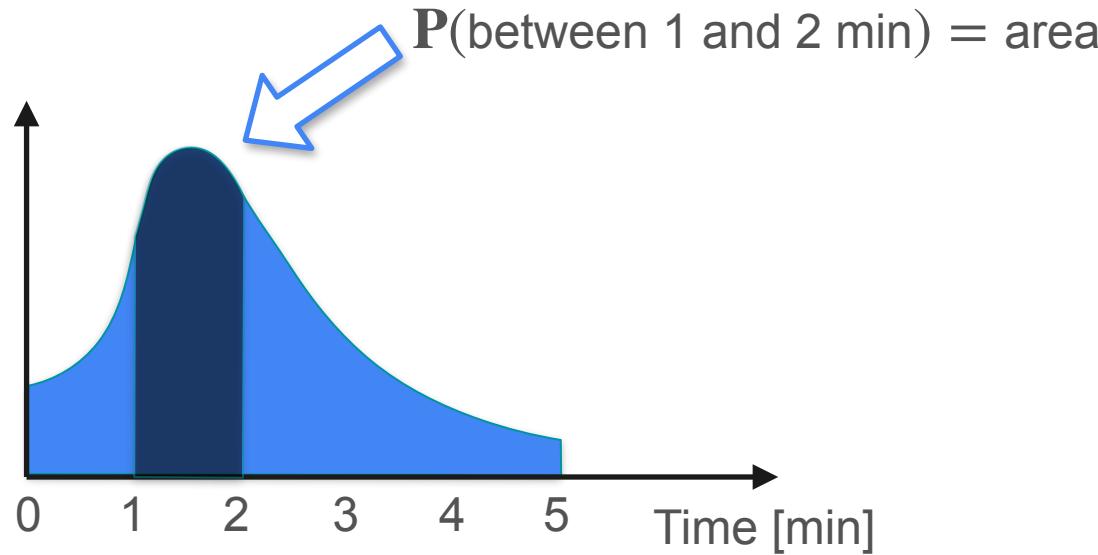
Probability Density Function



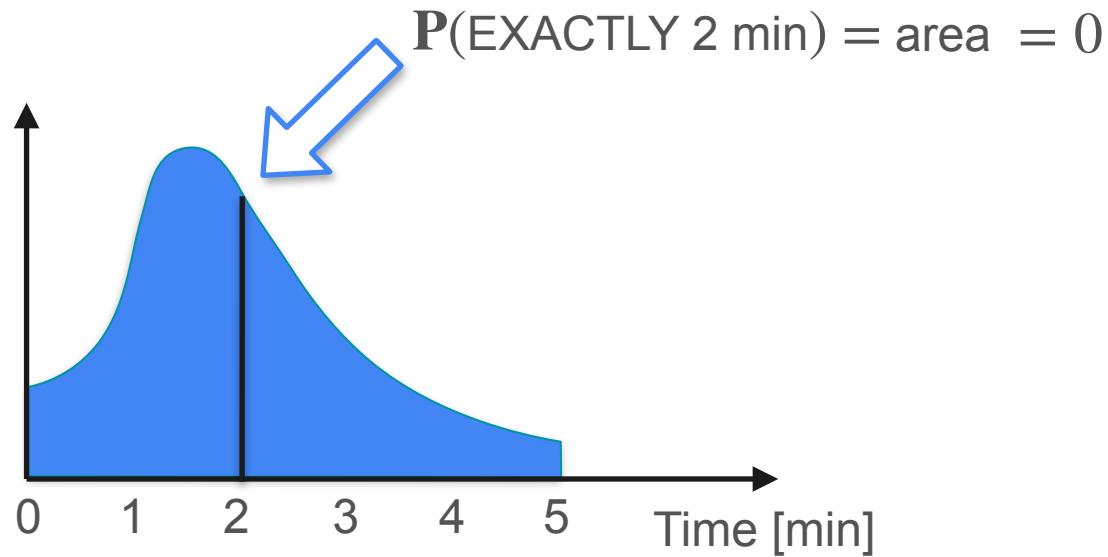
$P(\text{between 2 and 3 min}) =$
sum of shaded areas



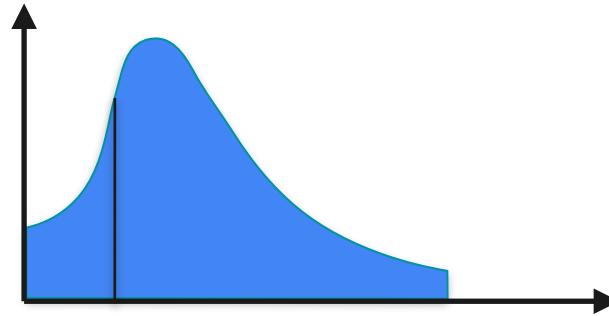
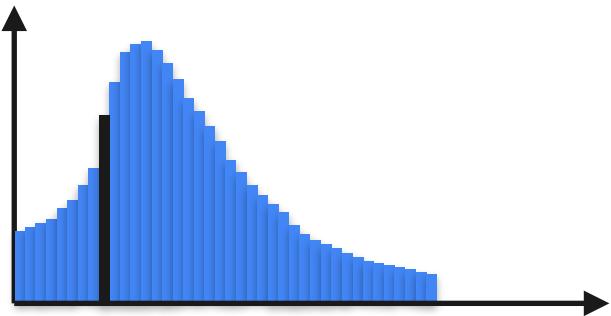
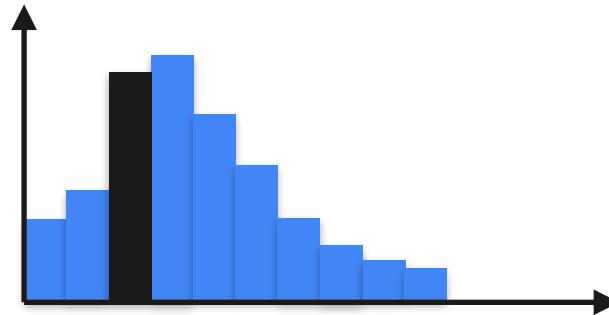
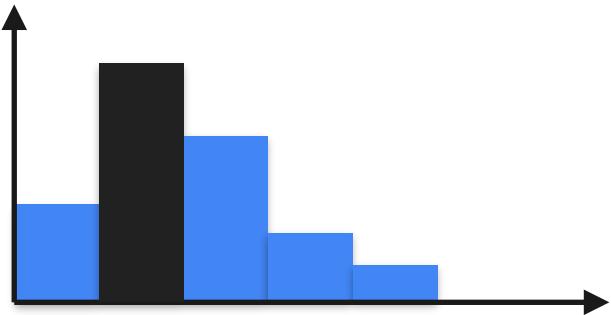
Probability Density Function



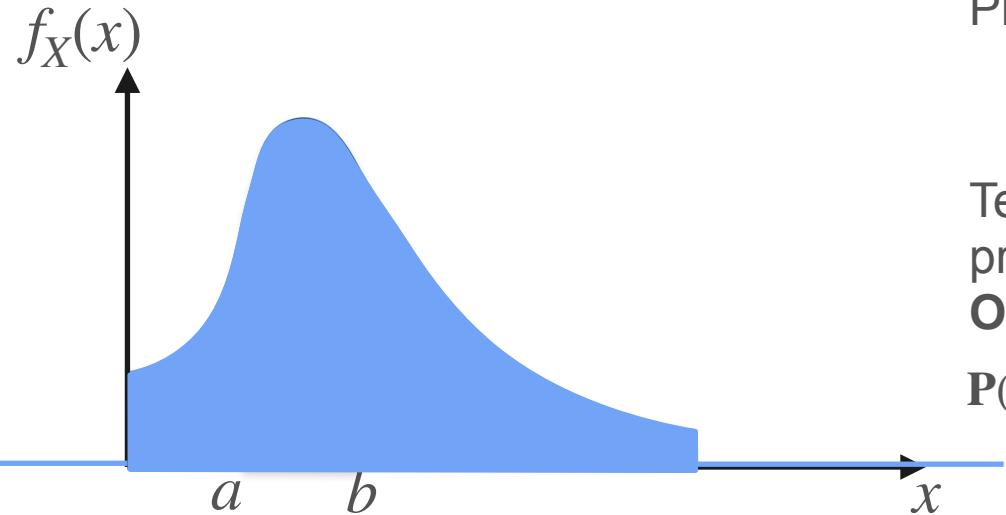
Probability Density Function



Probability Density Function



Probability Density Function: Formal Definition



Probability Density Function (PDF)

$$f_X(x)$$

Tells you the rate you accumulate probability around each point.

Only defined for continuous variables!

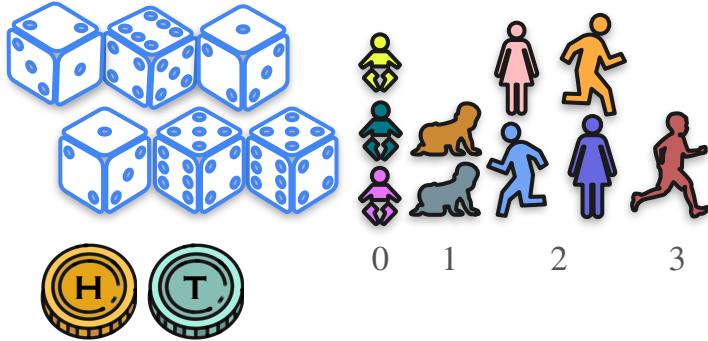
$P(a < X < b) = \text{area under } f_X(x)$

$f_X(x)$ needs to satisfy:

- It is defined for all numbers
- $f_X(x) \geq 0$
- Area under $f_X(x) = 1$

Discrete and Continuous Random Variables

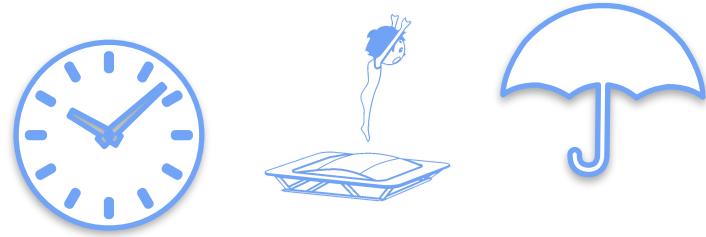
Discrete random variables



Can take only a **finite** or at most countable number of values

$$\text{PMF: } p_X(x) = \mathbf{P}(X = x)$$

Continuous random variables



Takes values on an interval
(infinite possibilities!)

$$\begin{aligned} \text{PDF: } & f_X(x) \\ \mathbf{P}(X = x) &= 0 \quad \forall x \end{aligned}$$

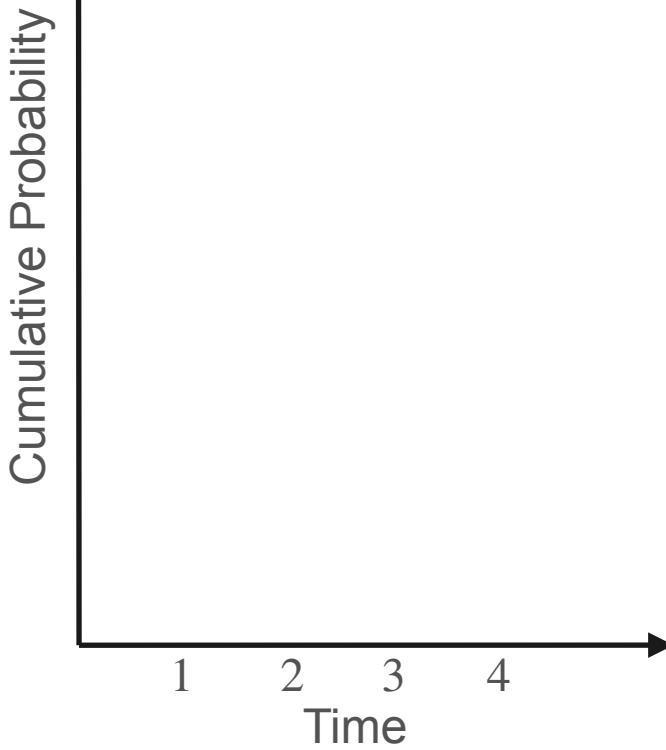
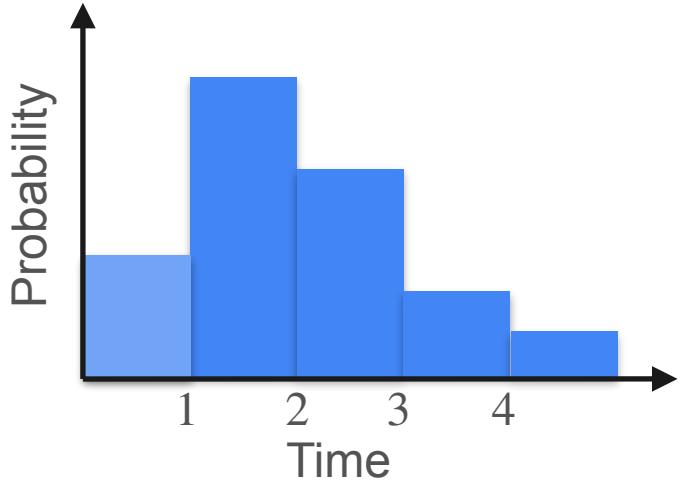


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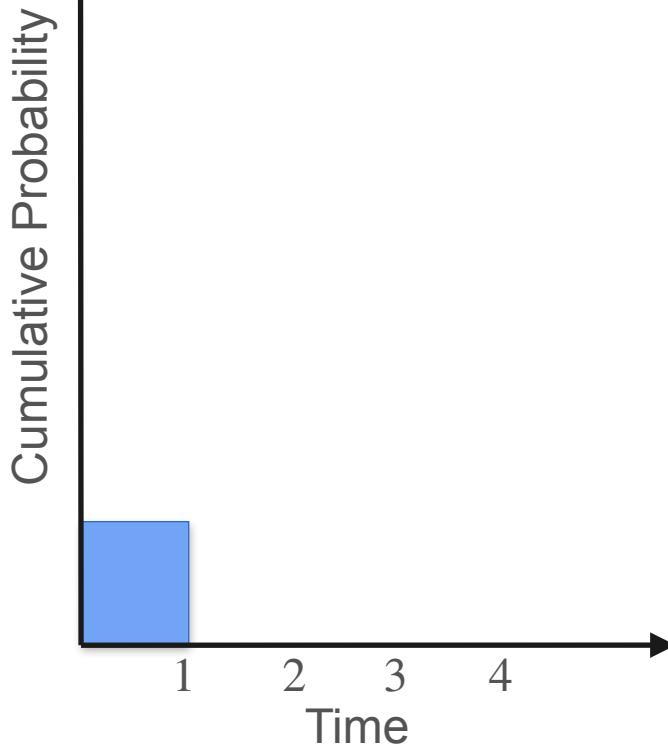
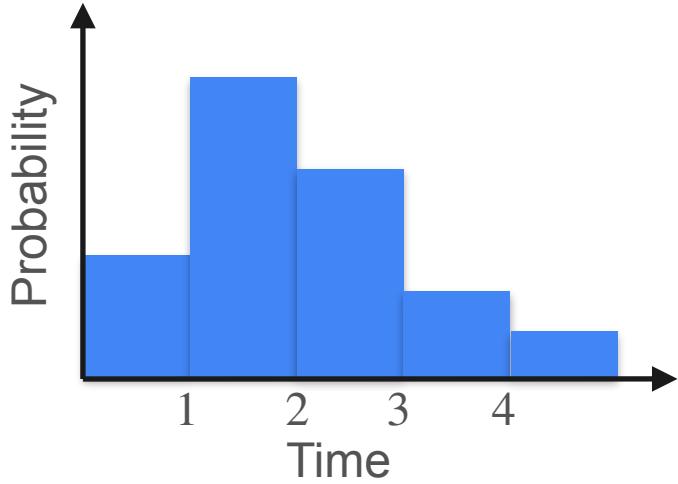
Probability Distributions

Cumulative Distribution Function

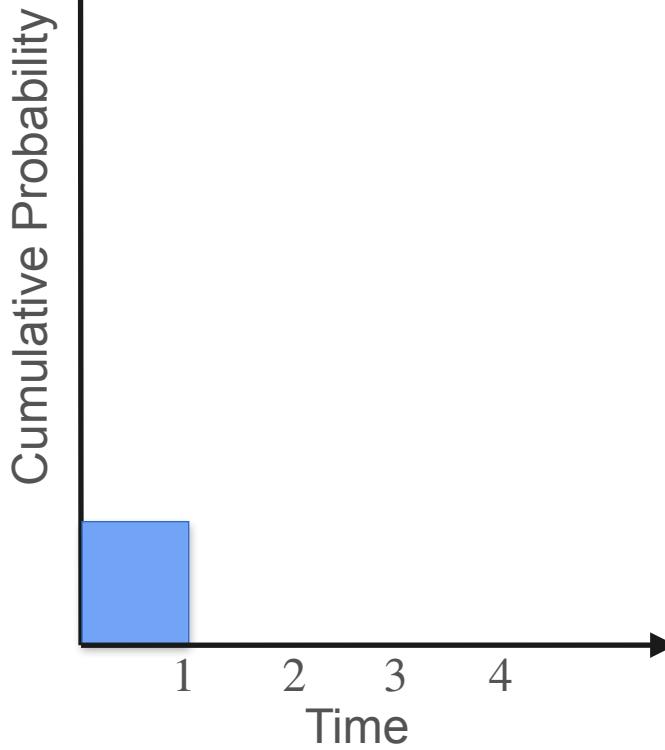
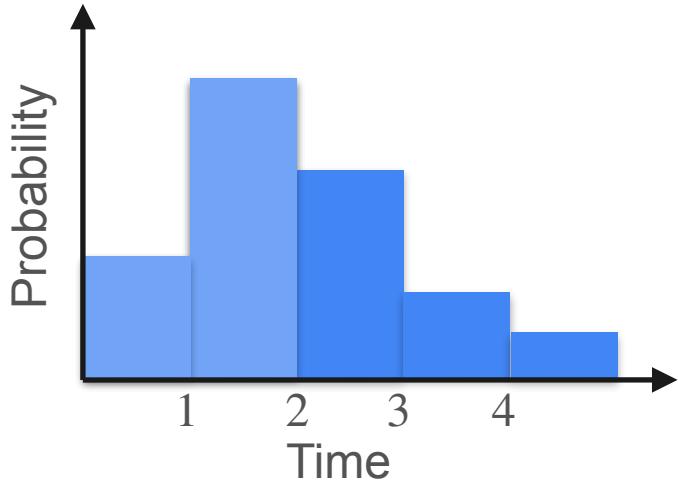
Cumulative Distribution



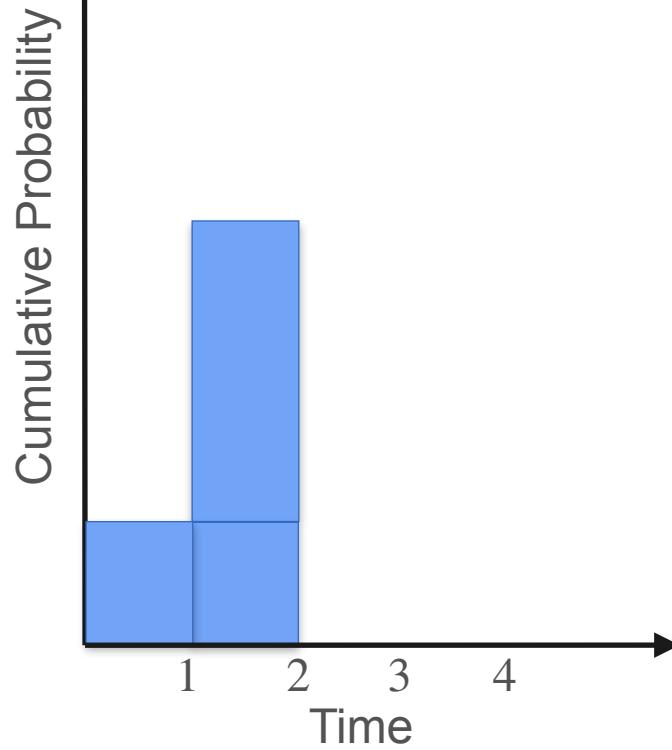
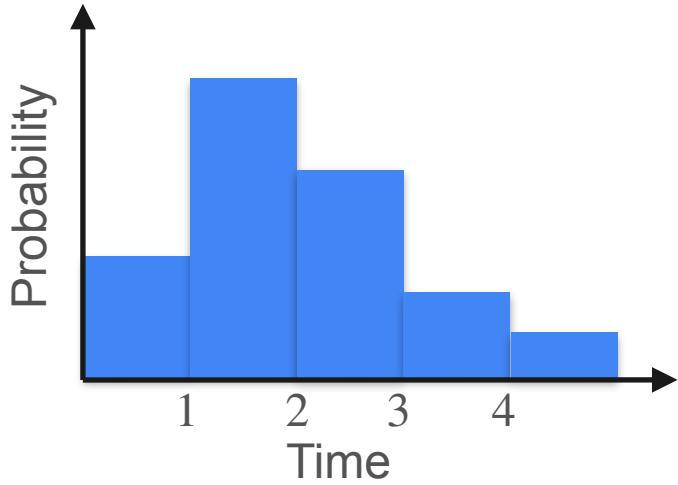
Cumulative Distribution



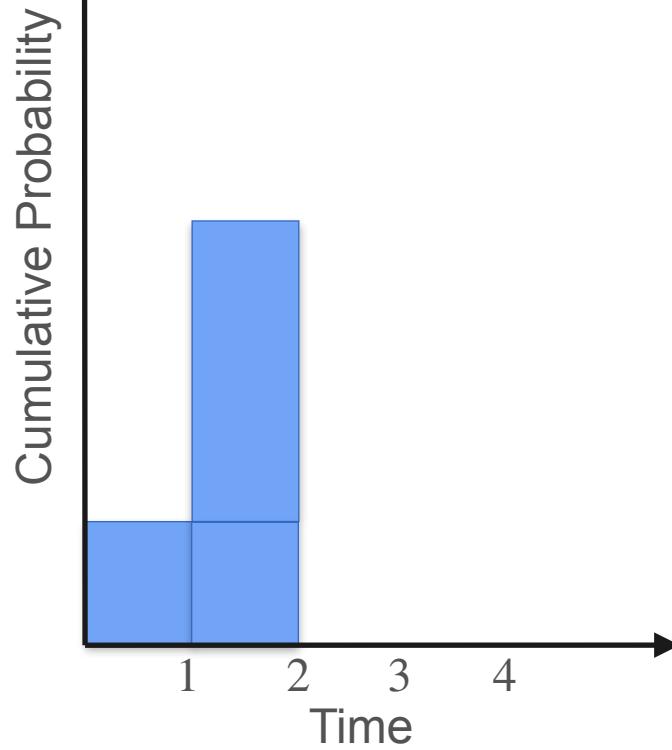
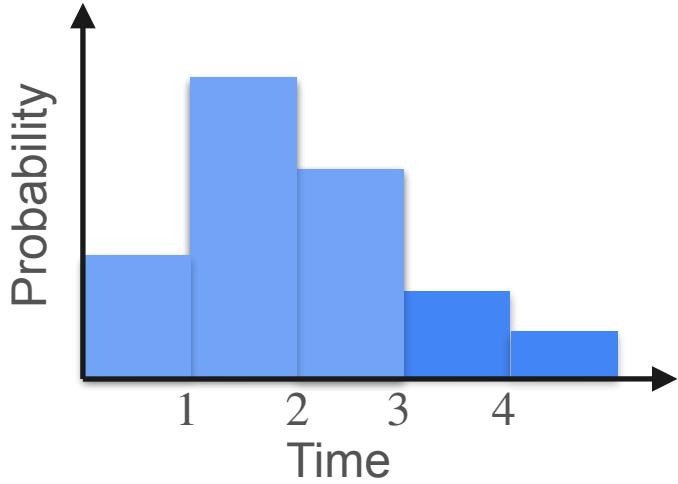
Cumulative Distribution



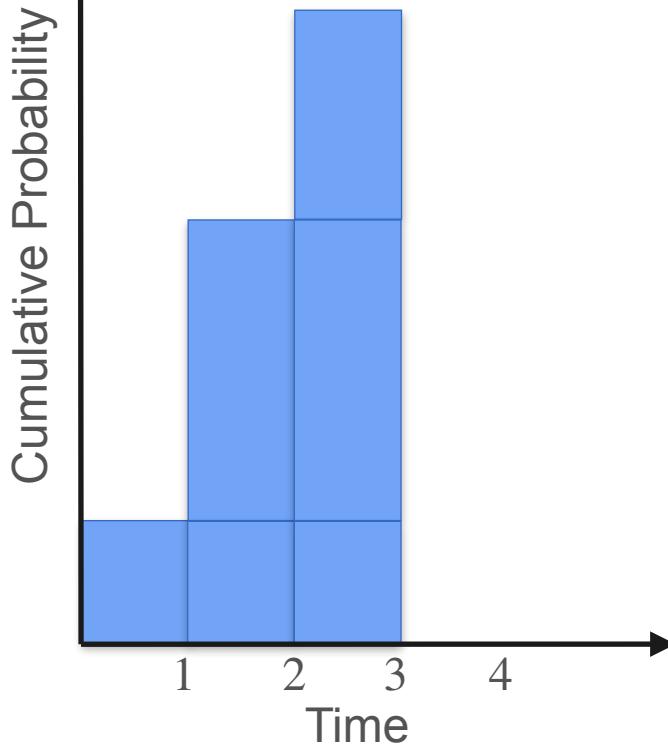
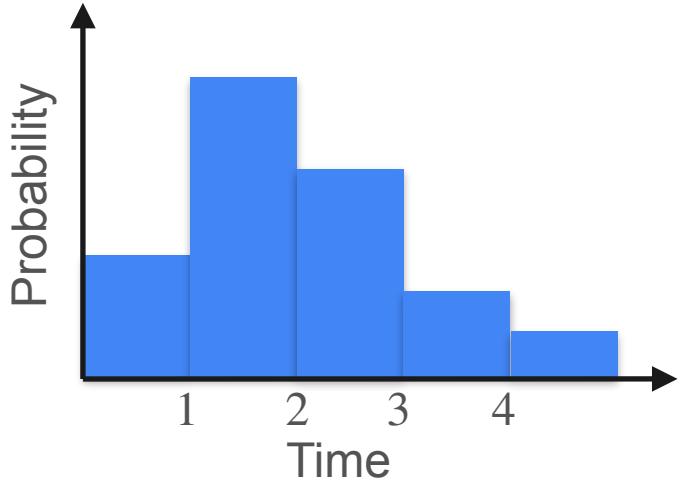
Cumulative Distribution



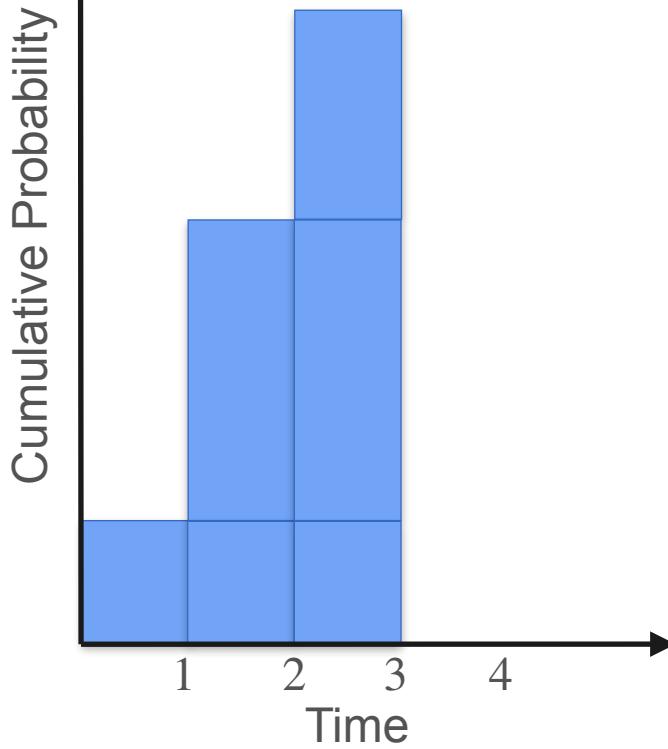
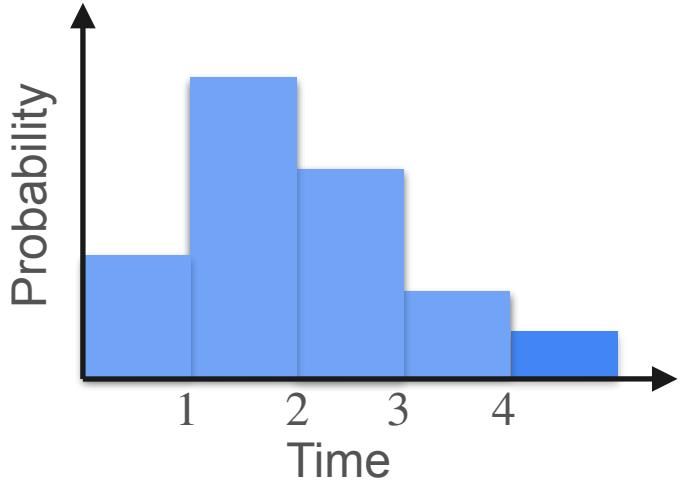
Cumulative Distribution



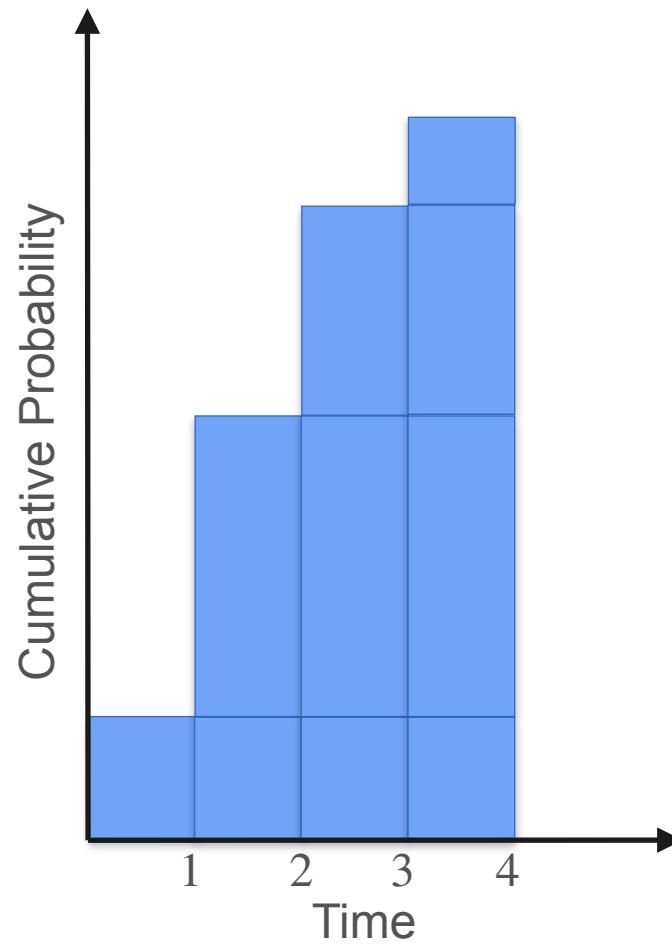
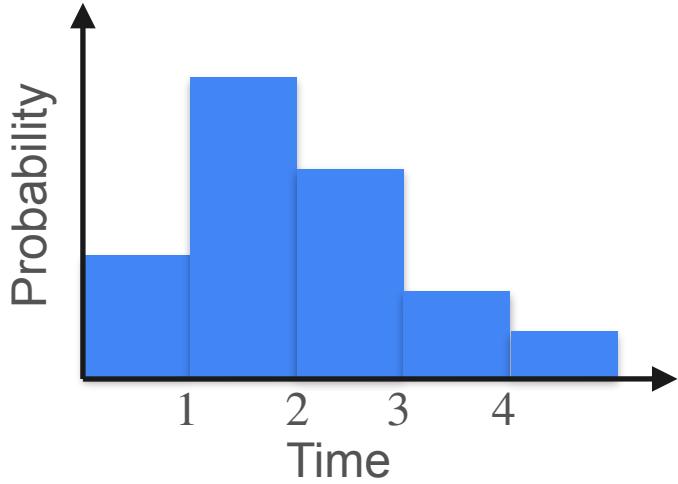
Cumulative Distribution



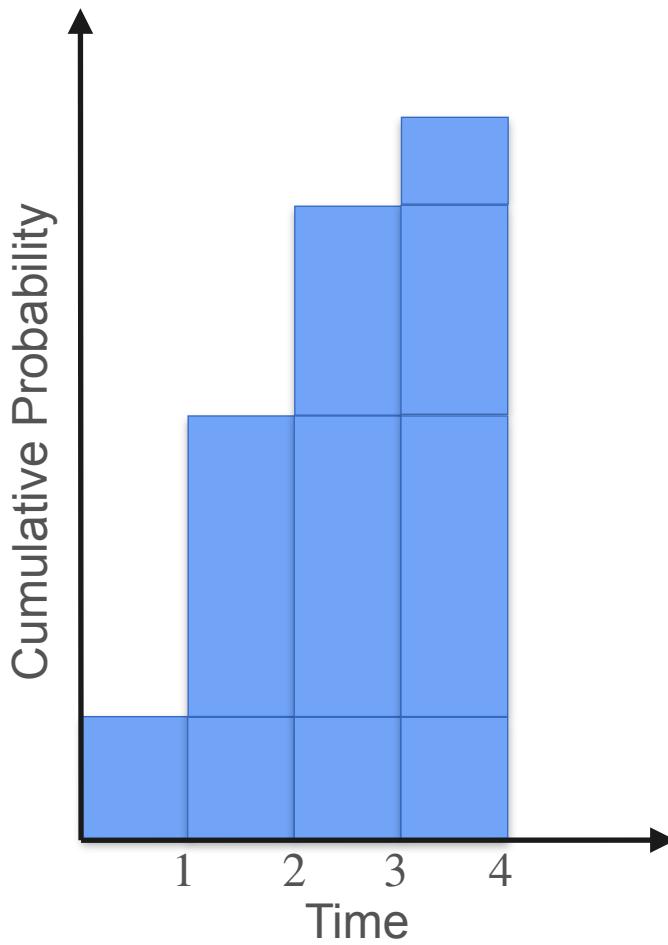
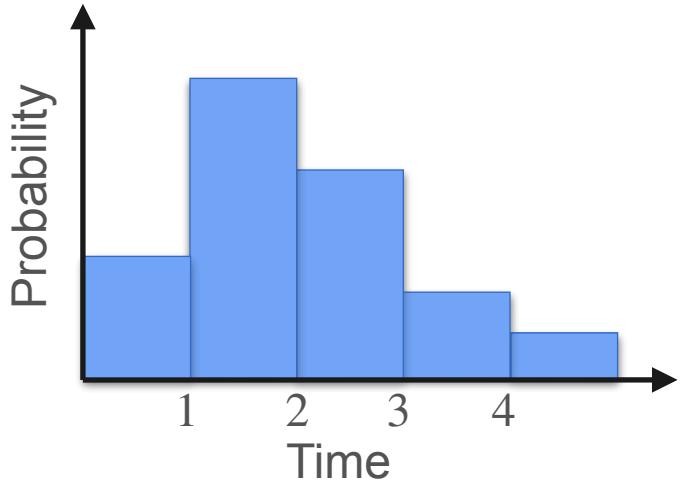
Cumulative Distribution



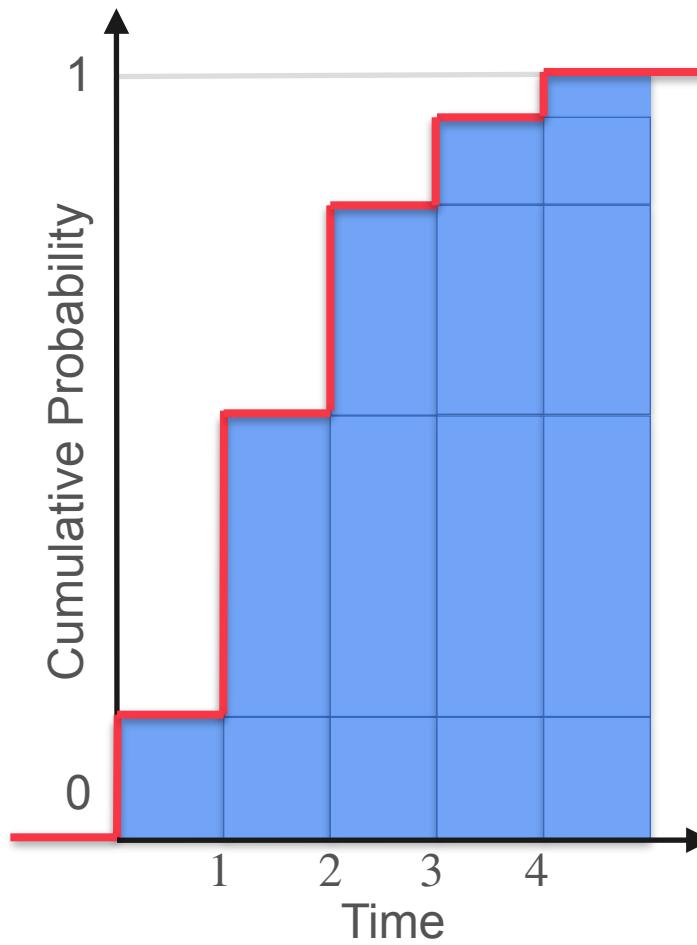
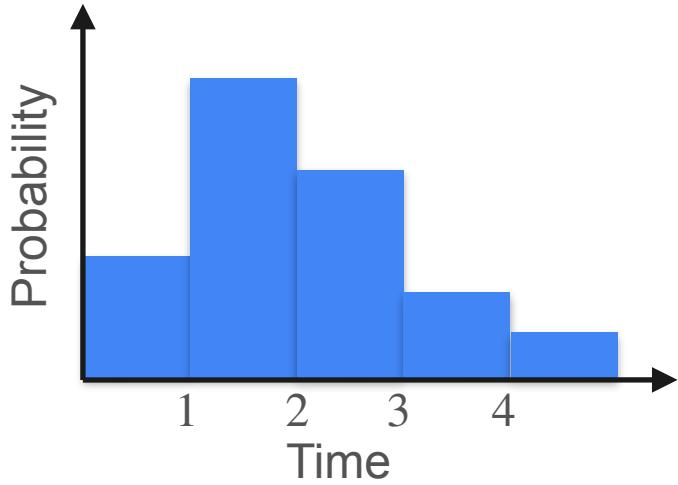
Cumulative Distribution



Cumulative Distribution

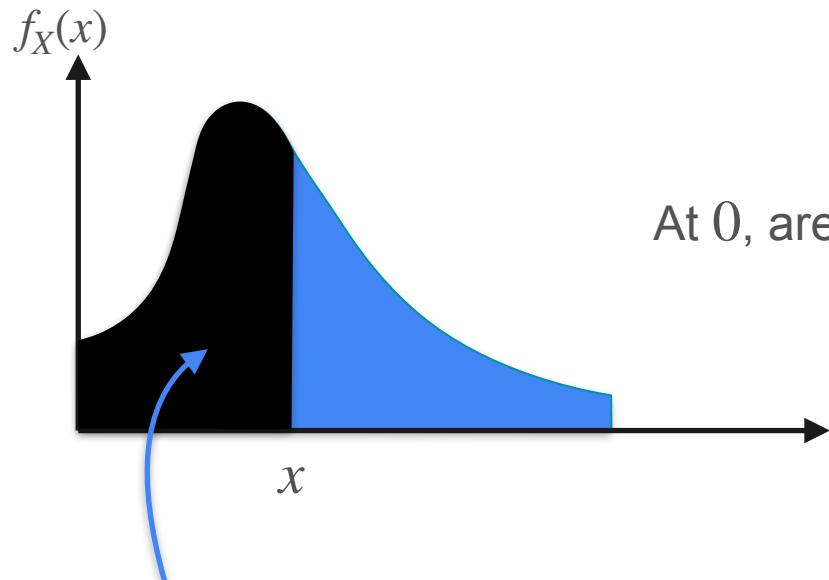


Cumulative Distribution



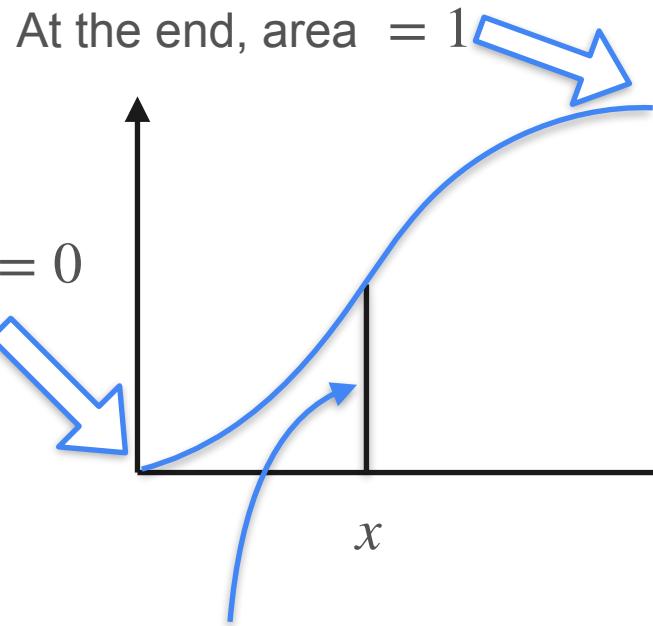
Cumulative Distribution

CDF: Cumulative distribution function



At 0, area = 0

$P(\text{less than or equal to 2 minutes}) = 0.5$



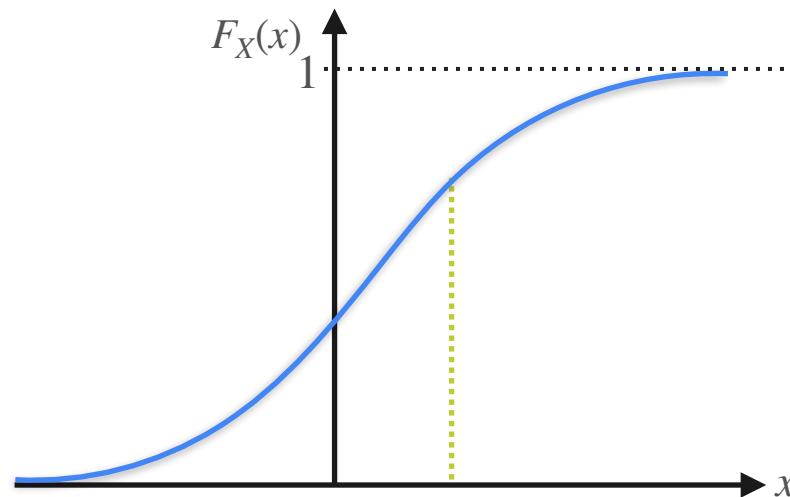
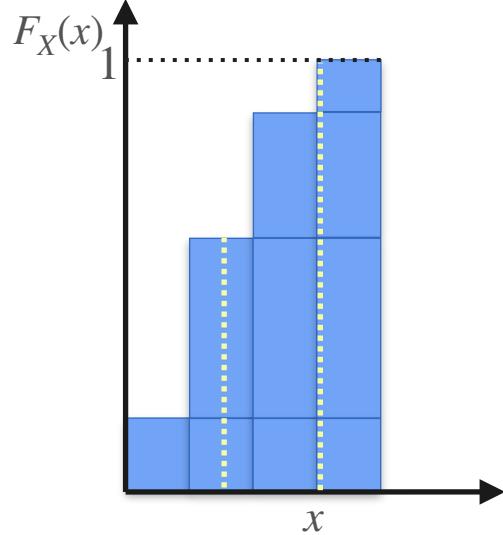
$P(\text{less than or equal to 2 minutes}) = 0.5$

Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

That means that

$$F_X(x) = \mathbf{P}(X \leq x) \quad \text{It is defined for every real number}$$

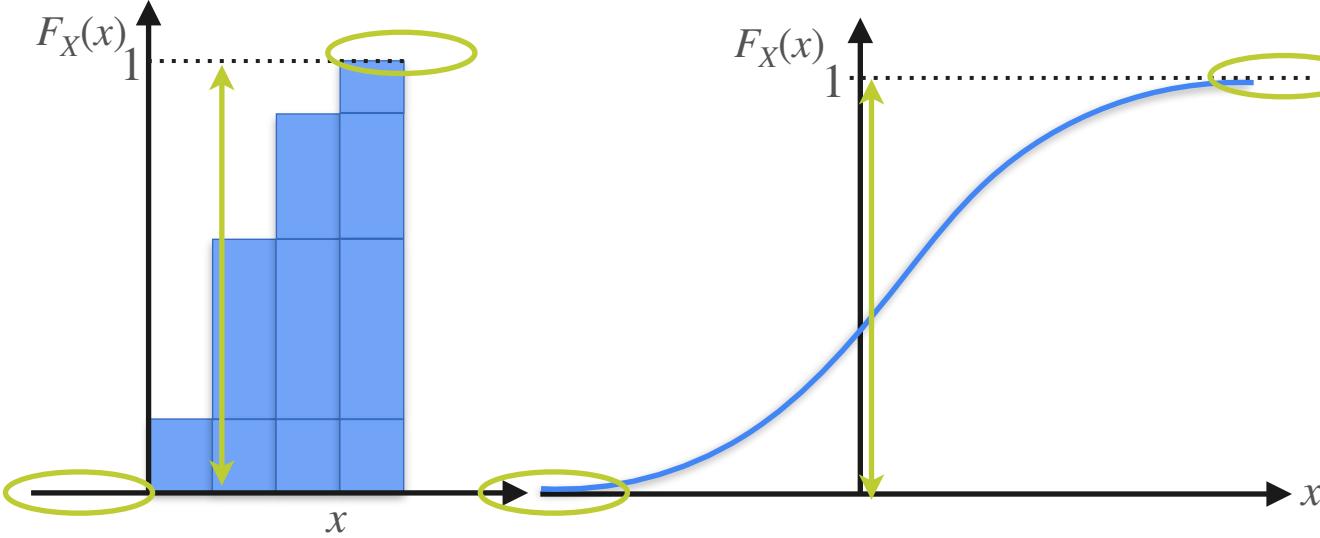


Cumulative Distribution Function: Formal Definition

The CDF shows how much probability the variable has accumulated until a certain value

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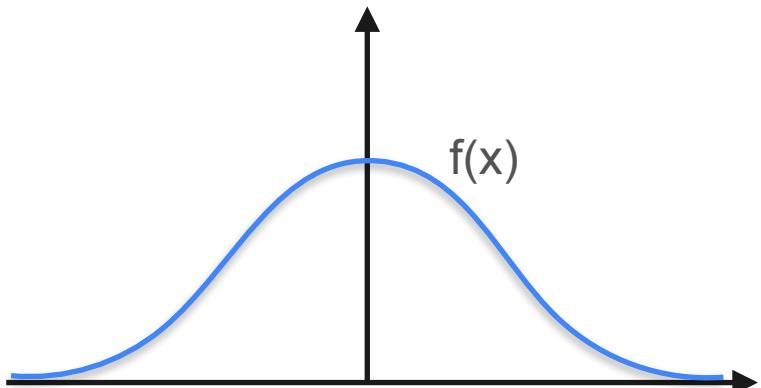


Properties

- $0 \leq F_X(x) \leq 1$
- Left “endpoint” is 0
- Right “endpoint” is 1
- Never decreases

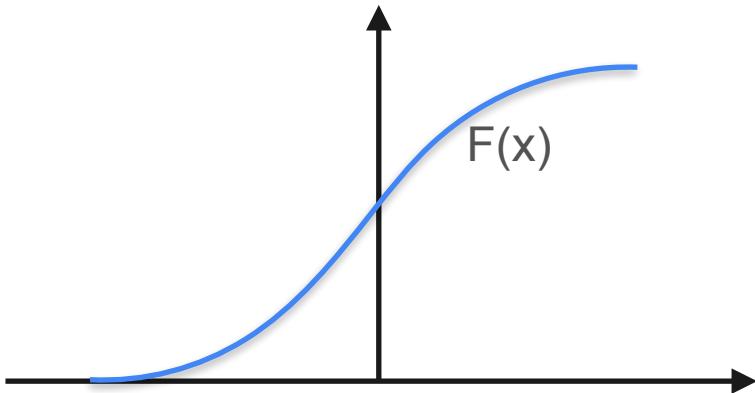
PDF and CDF Summary

PDF



- area = 1
- Always positive

CDF



- left “endpoint” is 0
- right “endpoint” is 1
- (endpoints can be at infinity)
- Always positive and increasing

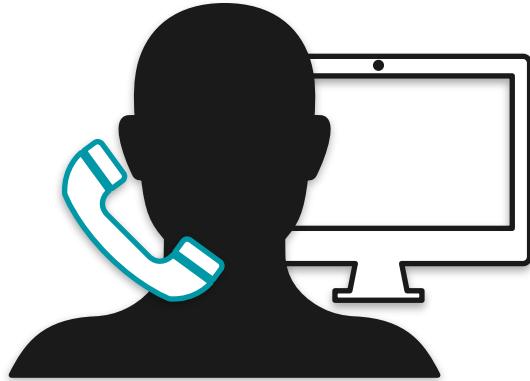


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Probability Distributions

Uniform Distribution

Uniform Distribution: Motivation

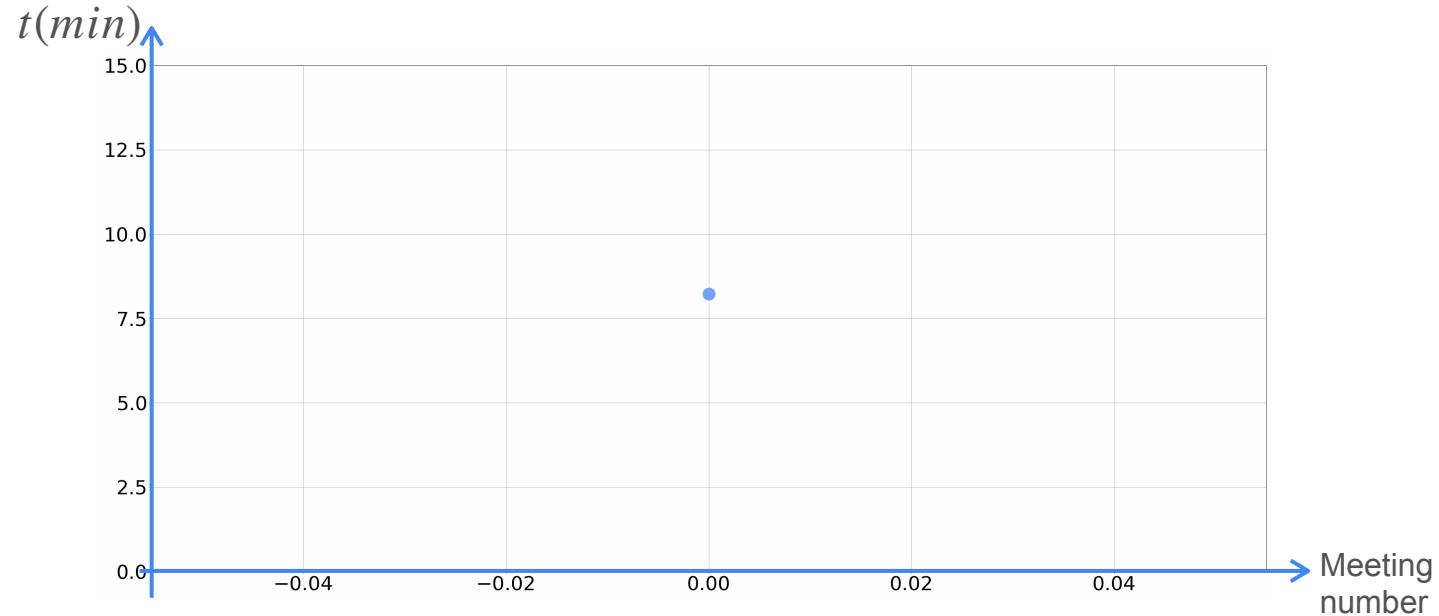


You're calling a tech support line. They can answer any time between zero and 15 minutes and if they don't answer in this time, the line is disconnected.

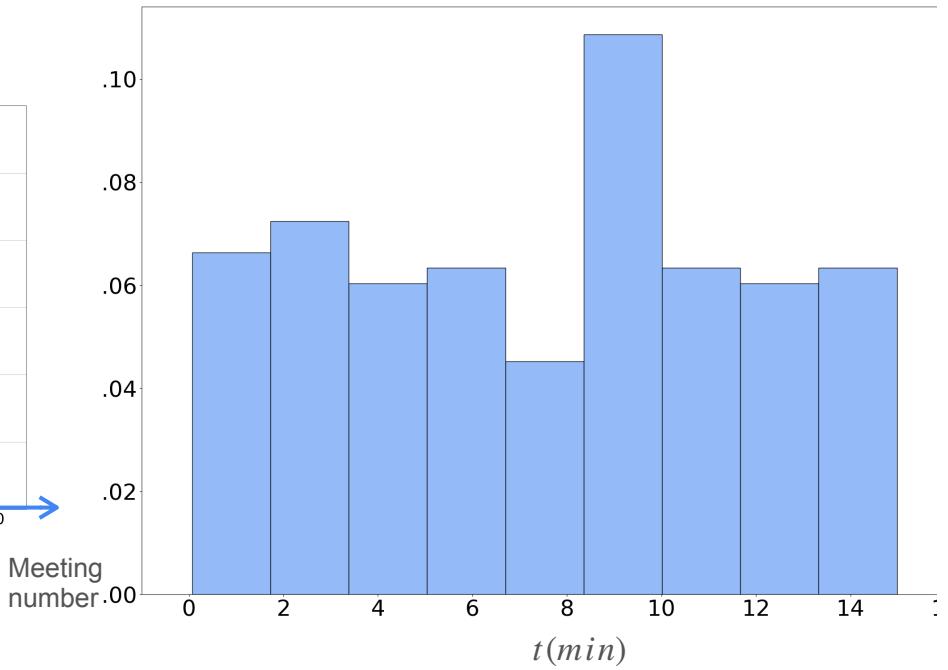
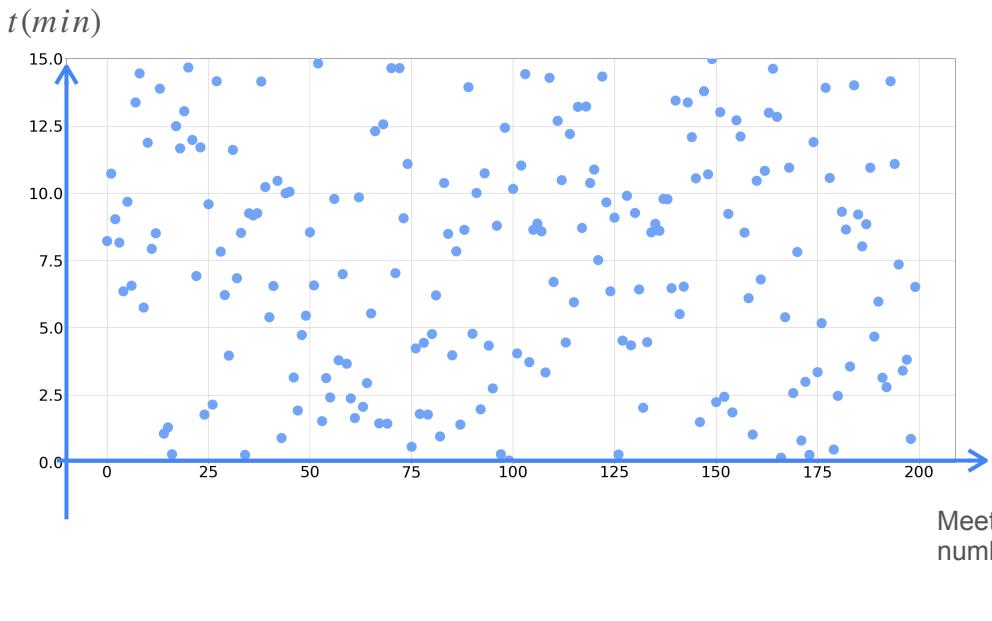
Uniform Distribution: Motivation



Last 200 times you called them, you took down notes of how long they took to respond



Uniform Distribution: Motivation



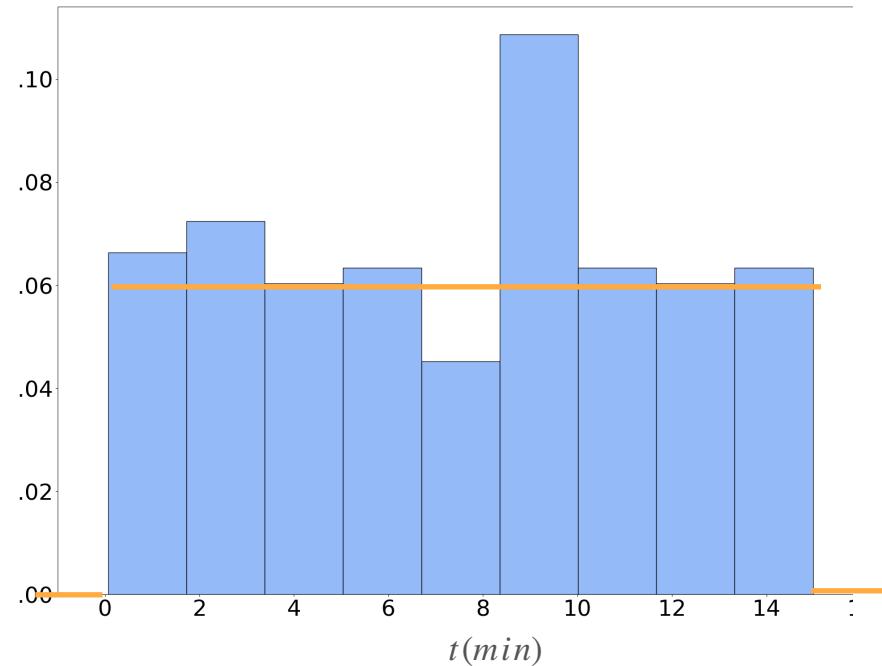
Uniform Distribution: Motivation

T: time (in minutes) you have to wait

Any value between 0 and 15 minutes must have the same frequency of occurrence.

The pdf must be constant for all values in the interval (0,15)

Which constant? $\rightarrow 15 \times h = 1 \rightarrow h = \frac{1}{15} = 0.06$



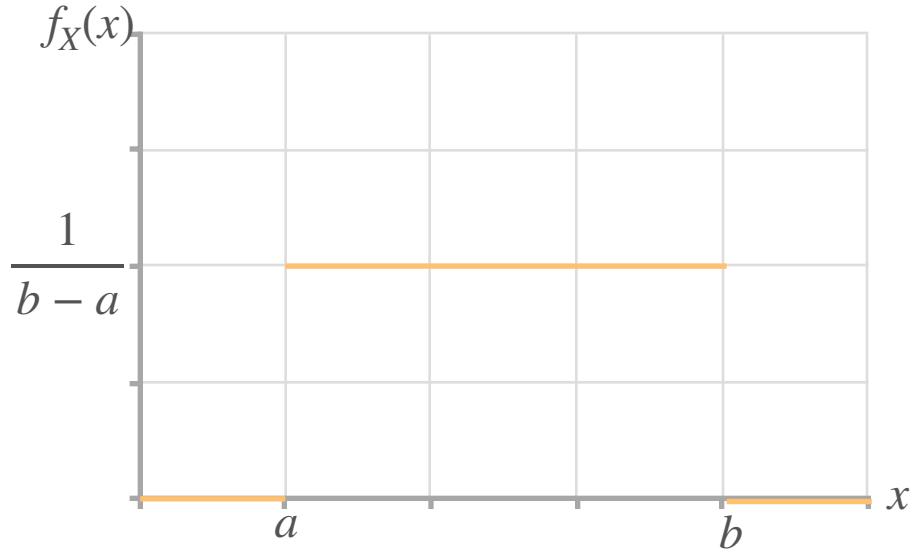
Uniform Distribution: Model

A continuous random variable can be modeled with a **uniform** distribution if all possible values lie in an interval and have the **same frequency** of occurrence

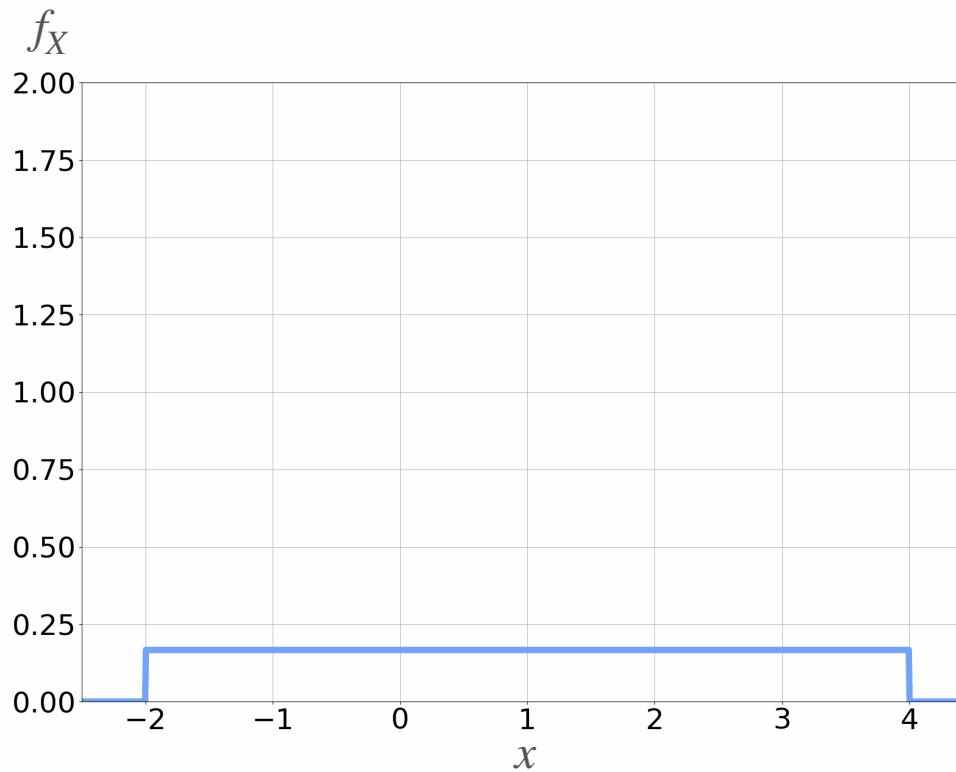
Parameters:

- a : beginning of the interval
- b : end of the interval

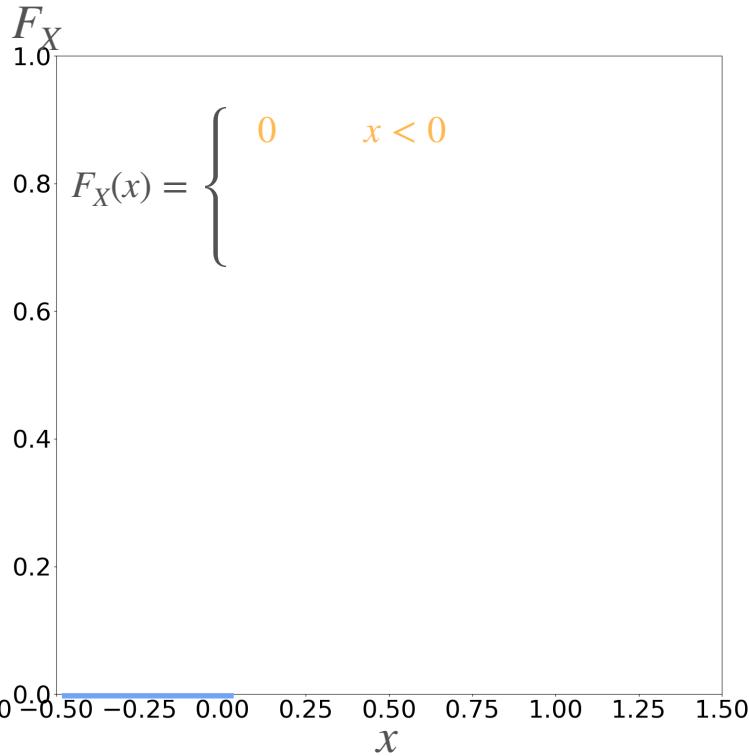
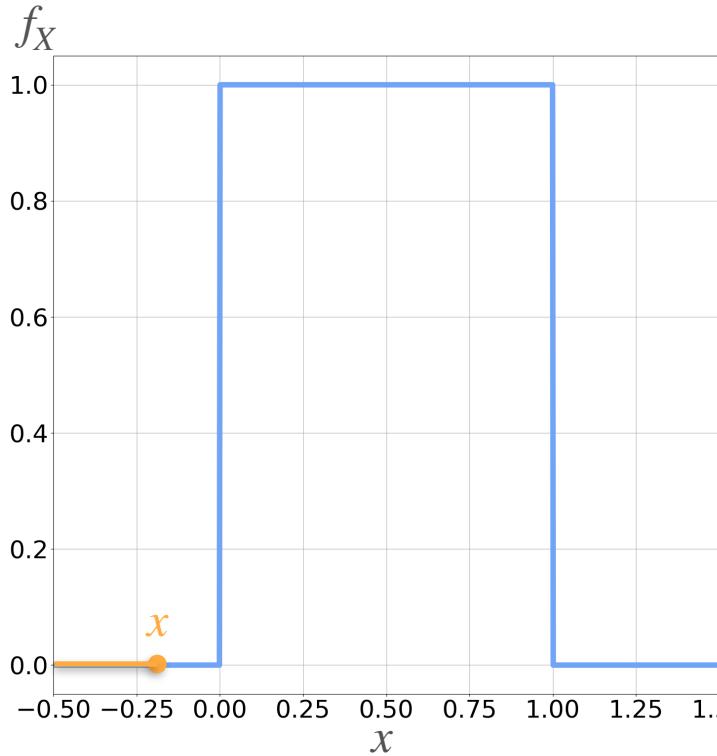
$$f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & x \notin (a, b) \end{cases}$$



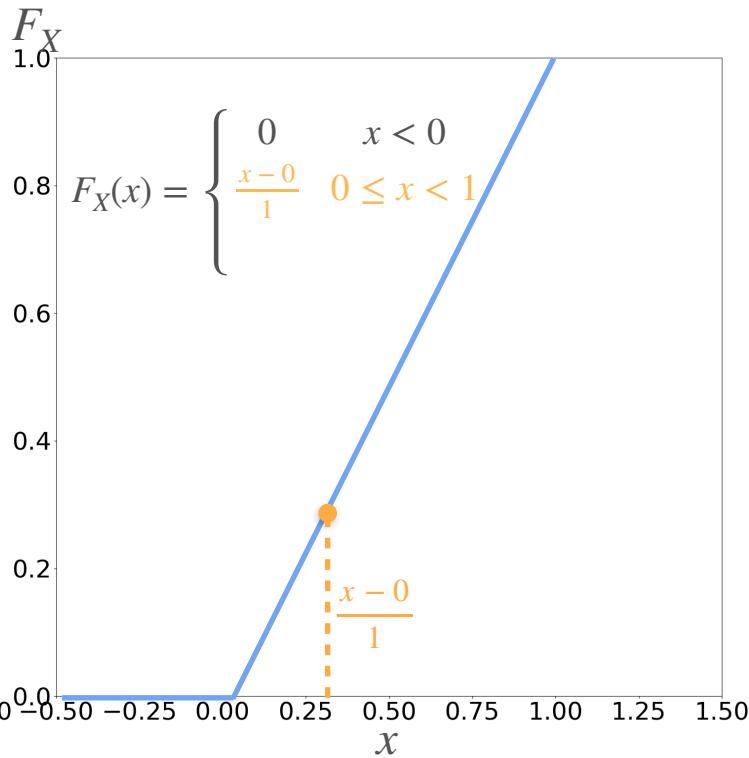
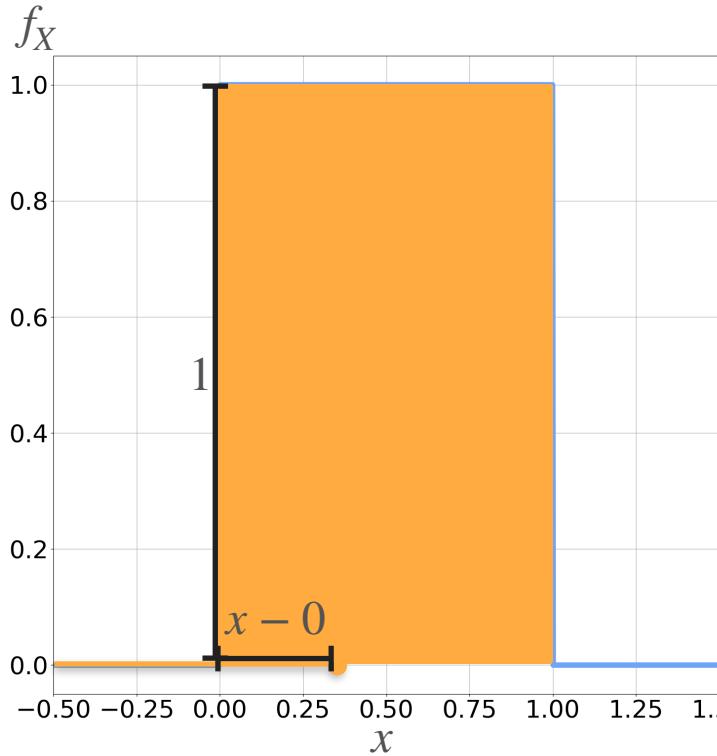
Uniform Distribution: PDF



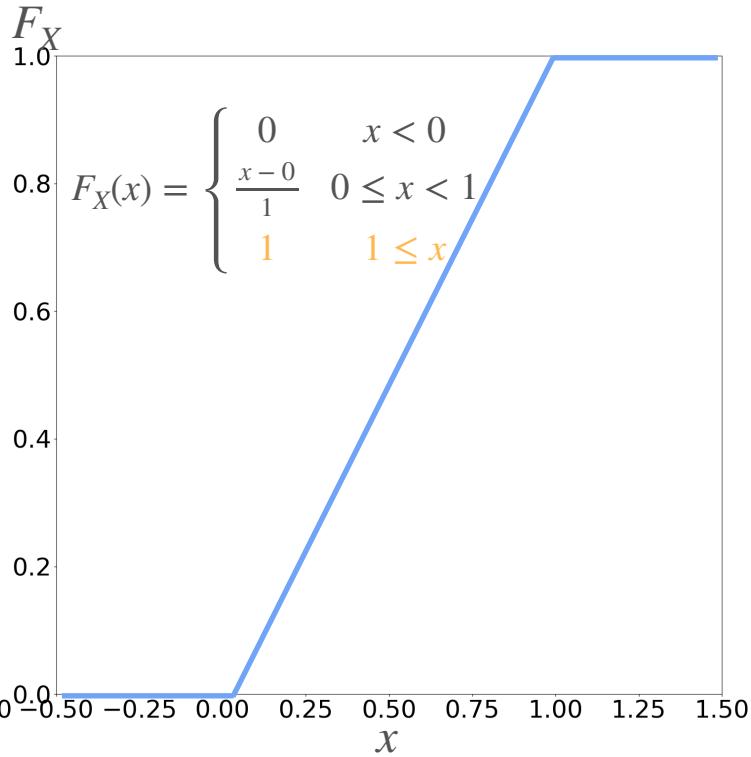
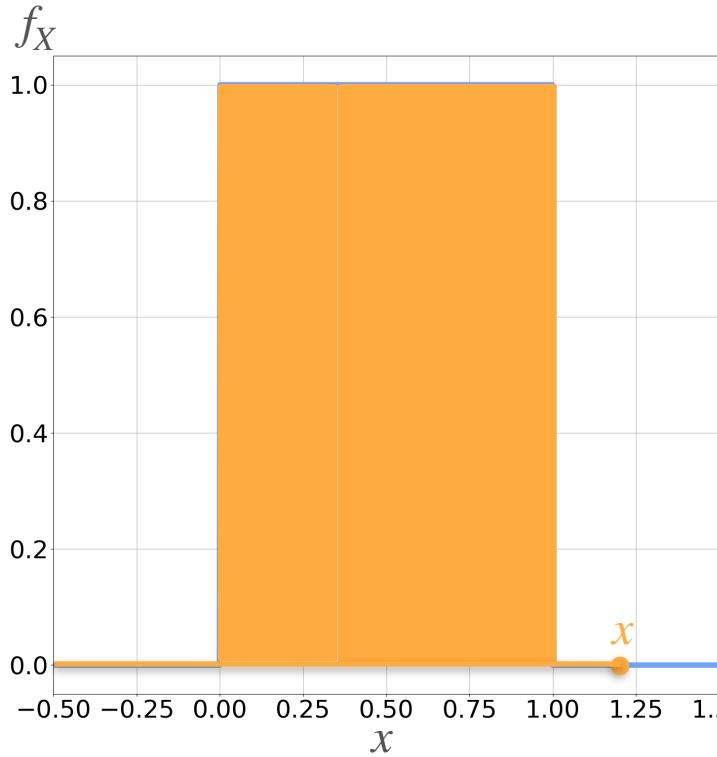
Uniform Distribution: CDF



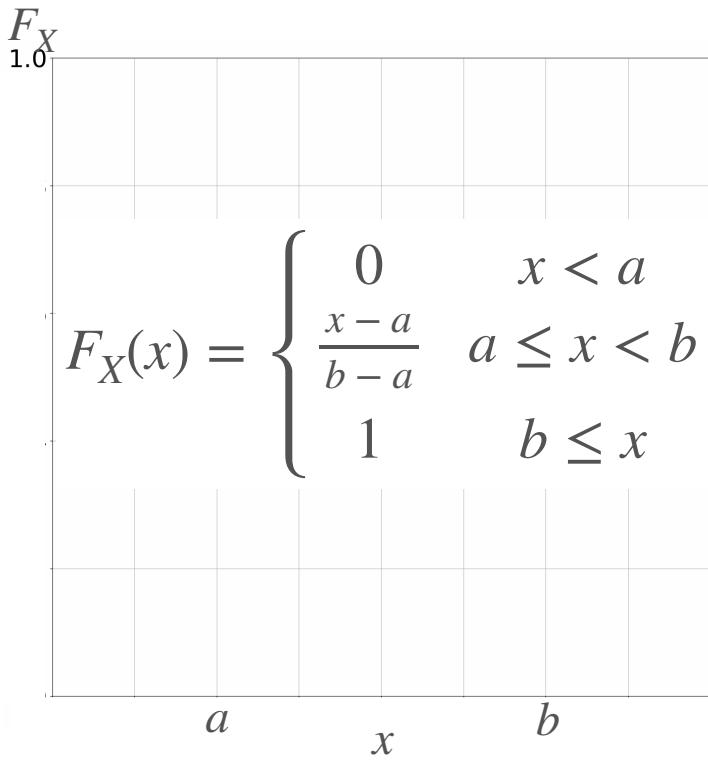
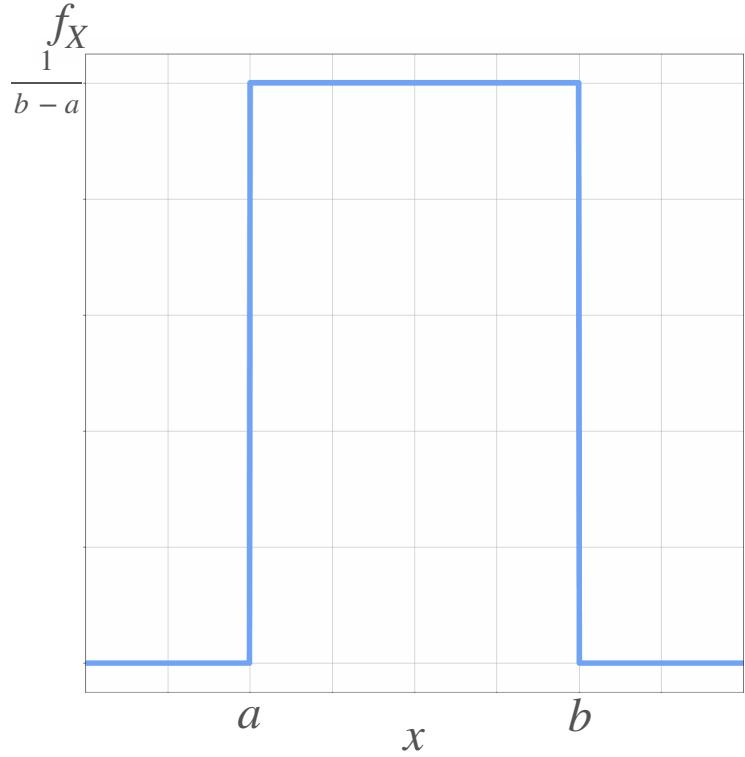
Uniform Distribution: CDF



Uniform Distribution: CDF



Uniform Distribution: CDF



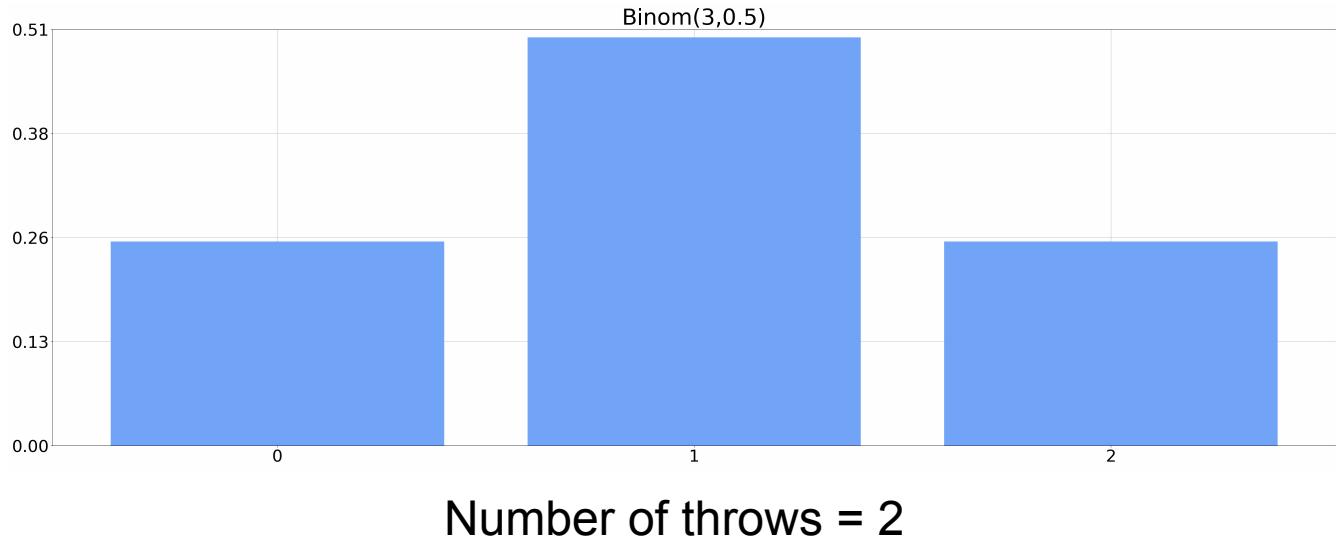


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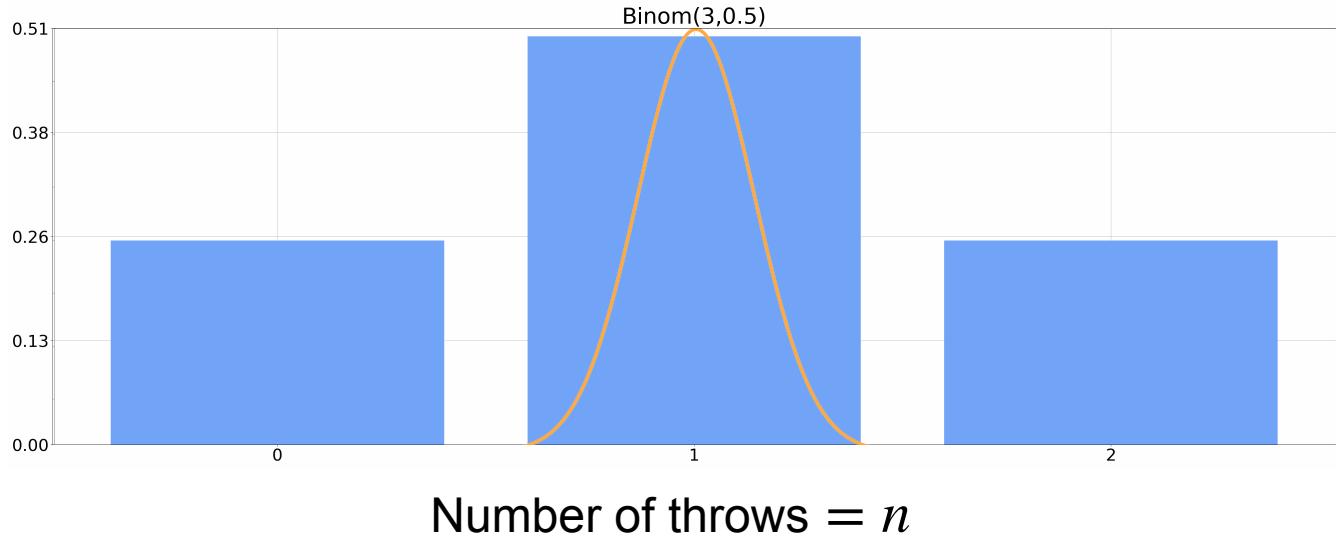
Probability Distributions

Normal distribution

Binomial Distribution With Very Large n

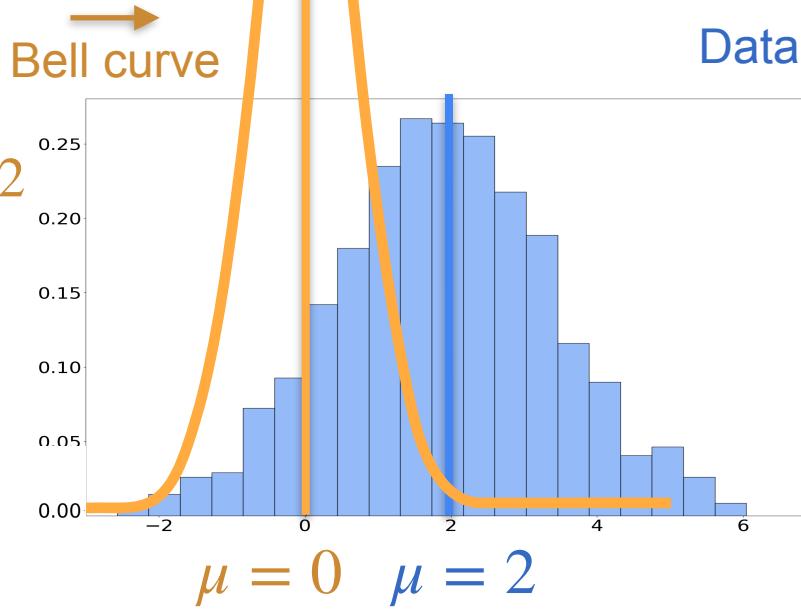


Binomial Distribution With Very Large n

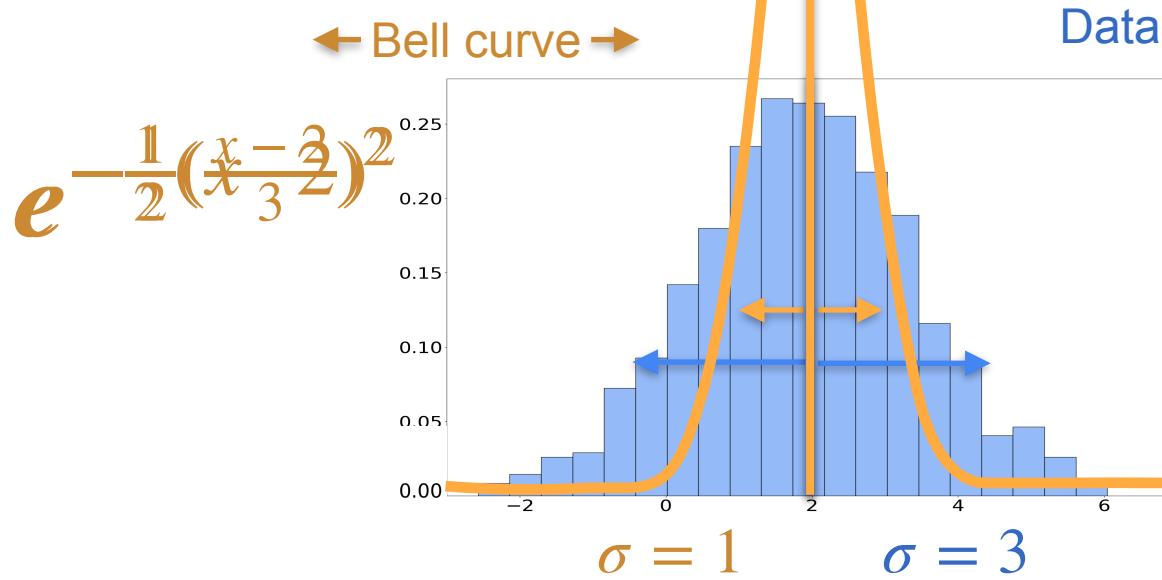


Bell Shaped Data

$$e^{-\frac{x^2}{2}(x-2)^2}$$

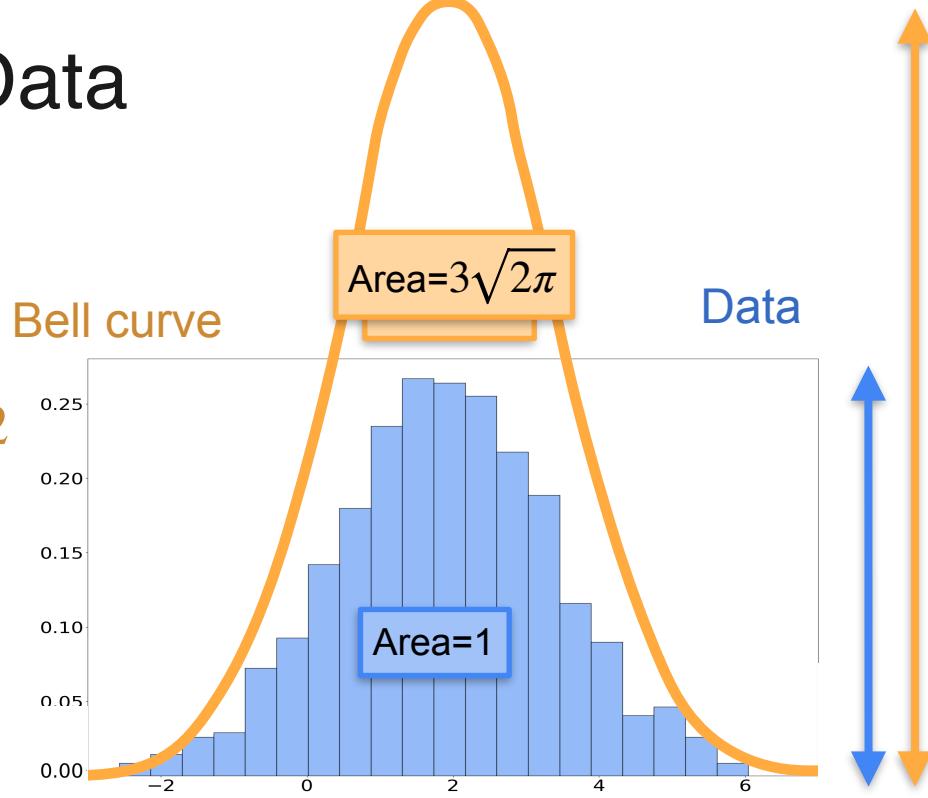


Bell Shaped Data



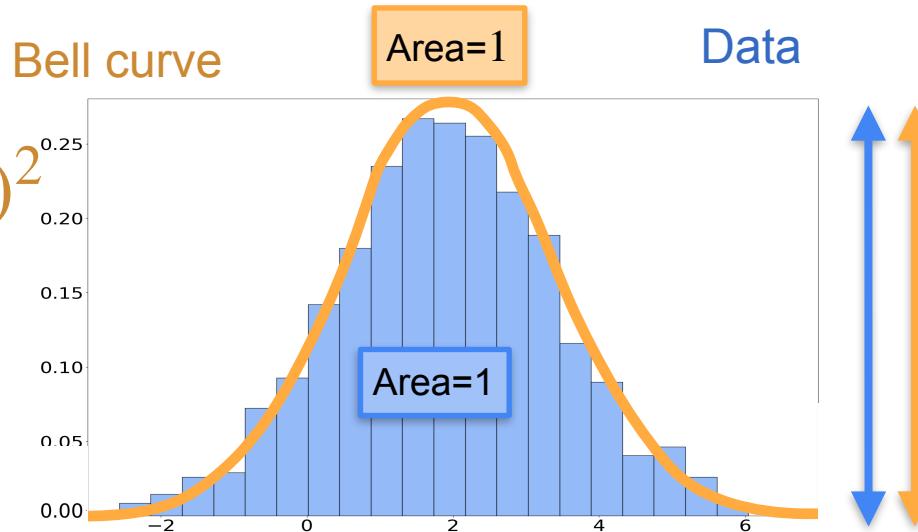
Bell Shaped Data

$$\frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-2}{3})^2}$$



Bell Shaped Data

$$\frac{1}{3\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-2}{3}\right)^2}$$

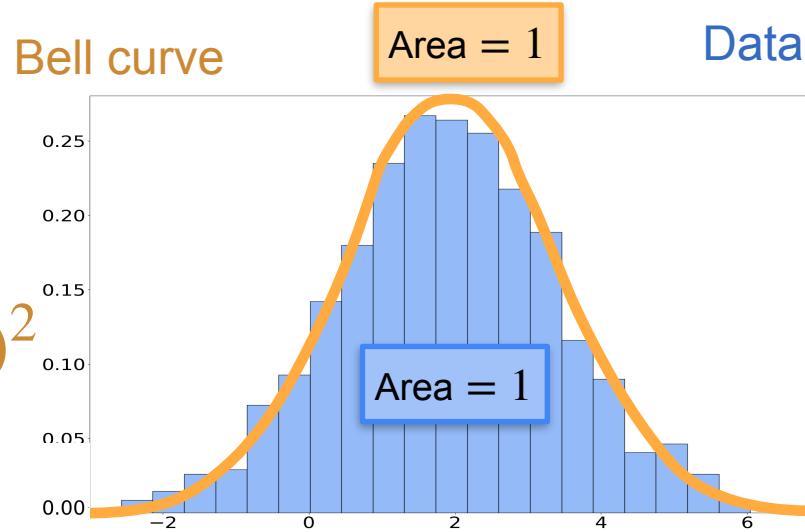


Bell Shaped Data

Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

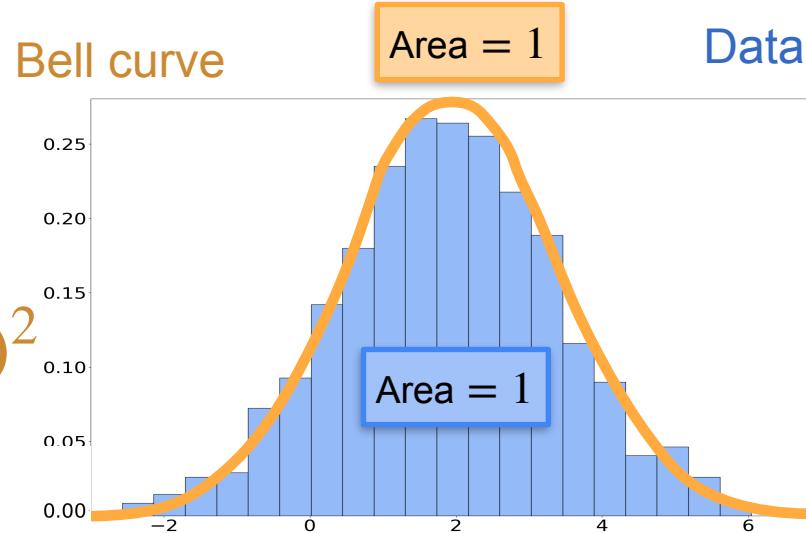


Bell Shaped Data

Bell curve
Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

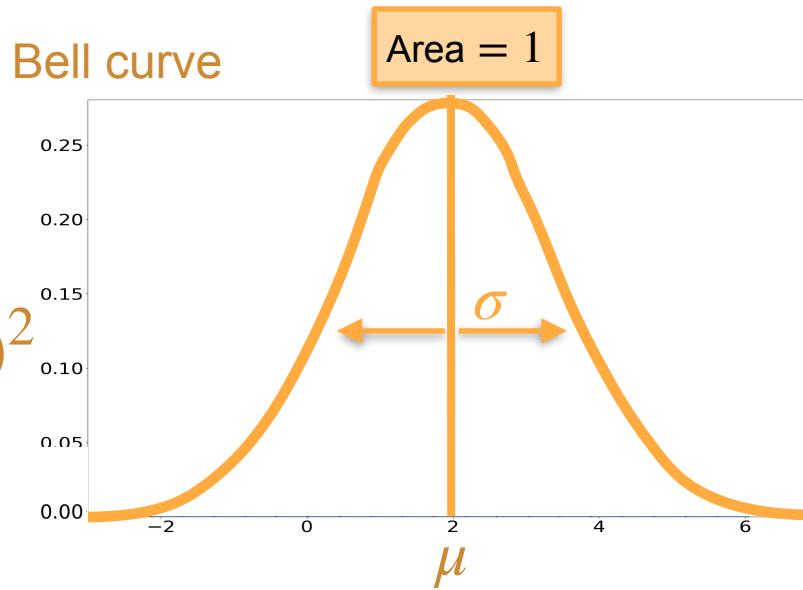


Normal Distribution

Mean = μ

Standard deviation = σ

$$\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



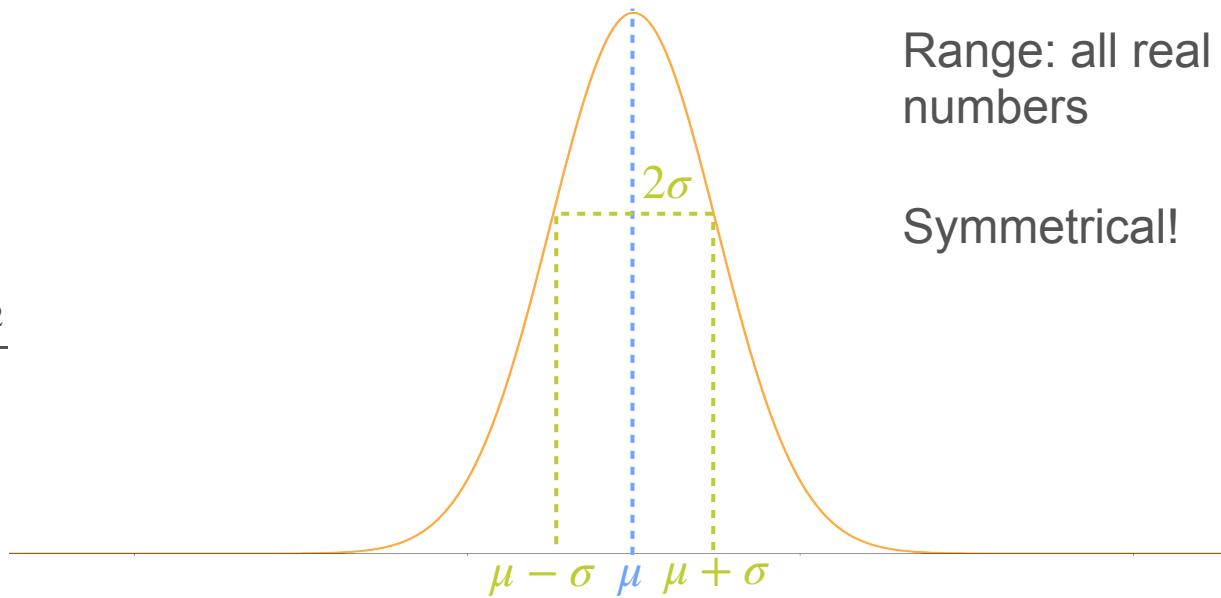
Normal Distribution

Parameters:

- μ : center of the bell
- σ : spread of the bell

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

Scaling constant



Range: all real numbers

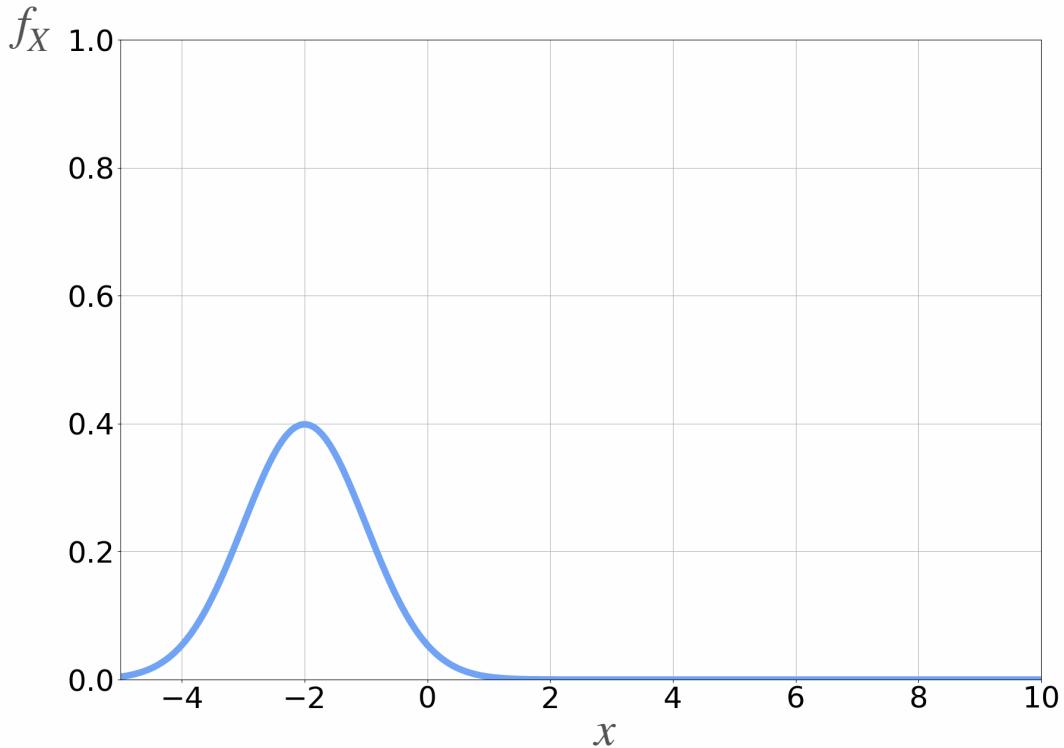
Symmetrical!

Normal Distribution

Parameters:

- μ : center of the bell
- σ : spread of the bell

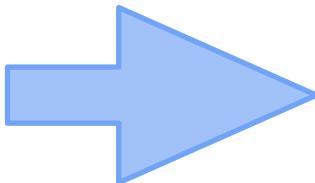
$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$



Normal Distribution - Notation

Parameters:

- μ : center of the bell
- σ : spread of the bell



$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}}$$

$$X \sim \mathcal{N}(\mu, \sigma^2)$$

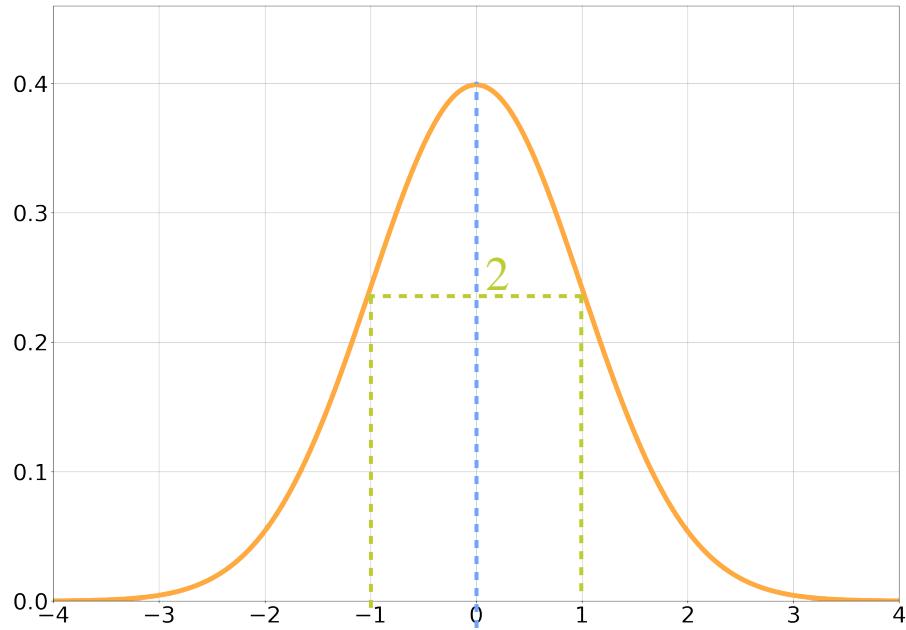
Standard Normal Distribution

Parameters:

- μ : 0
- σ : 1

$$X \sim \mathcal{N}(0, 1^2)$$

$$\begin{aligned}f_X(x) &= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\frac{(x-0)^2}{1^2}} \\&= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\end{aligned}$$



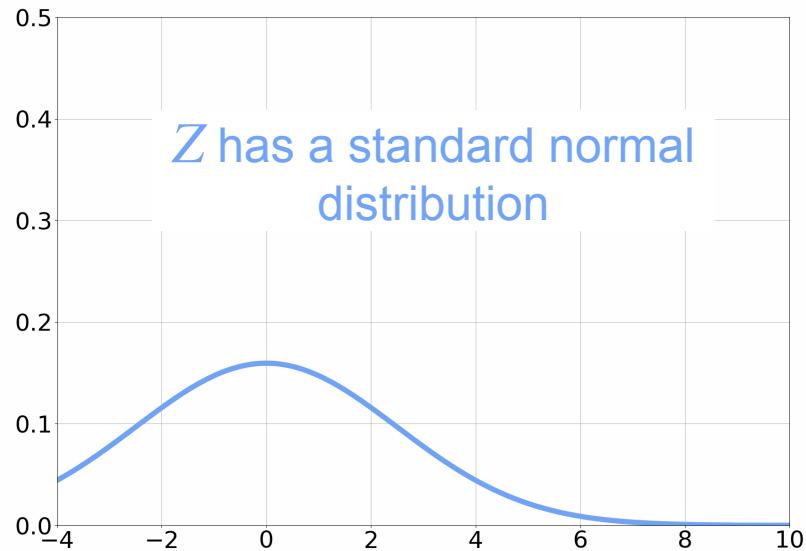
Standardization

There's a really easy way to convert any normal distribution to the standard one!

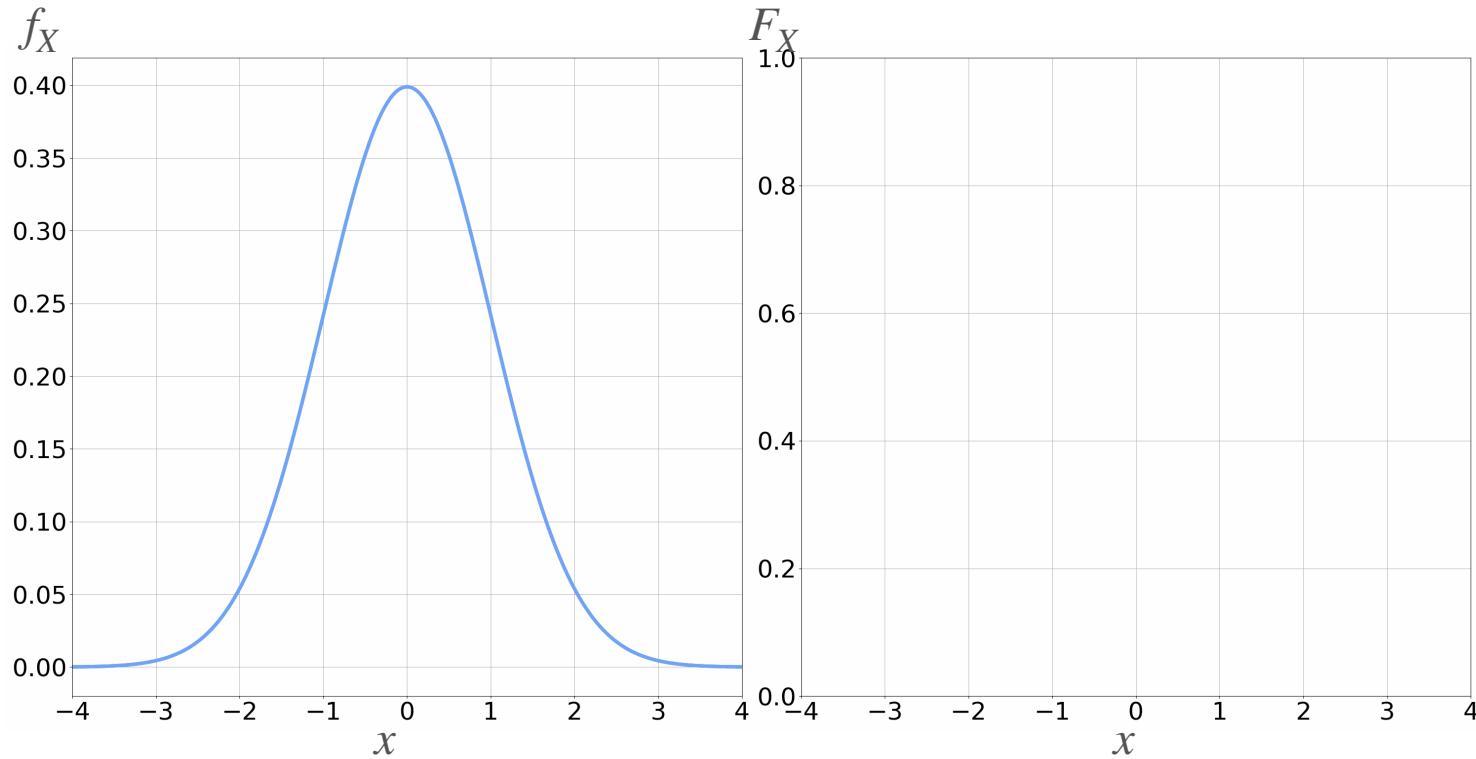
X distributes normally with
 $\mu = 2, \sigma = 2.5$

$$Z = \frac{X - \mu}{\sigma}$$

Standardization is crucial to compare variables of different magnitudes!

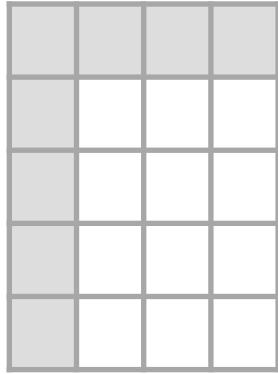


What Does the CDF Look Like?

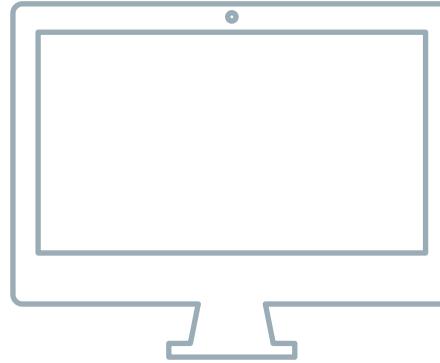


Computing Probabilities From the PDF

This math can't be done by hand

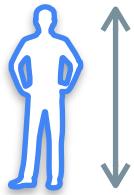


In the old days, people used tables of data



Now, you can use the help of some software to do the approximate area under the curve for you!

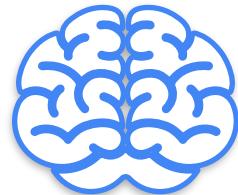
Normal Distribution: Applications



Height



Weight



IQ



Noise in a
communication channel

In general, characteristics that are the sum of many independent processes

Many models in ML are designed under the assumption that the variables follow a normal distribution



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Probability Distributions

(Optional)
Chi-squared distribution

Chi-Square Distribution: Motivation



Communication channel

Noise

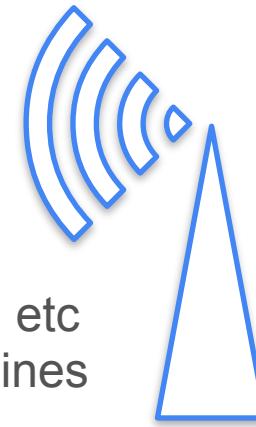
Interference from other devices

Obstructions like walls, trees, etc.

Atmospheric conditions: rain, humidity, etc

Electrical interference, i.e. from power lines

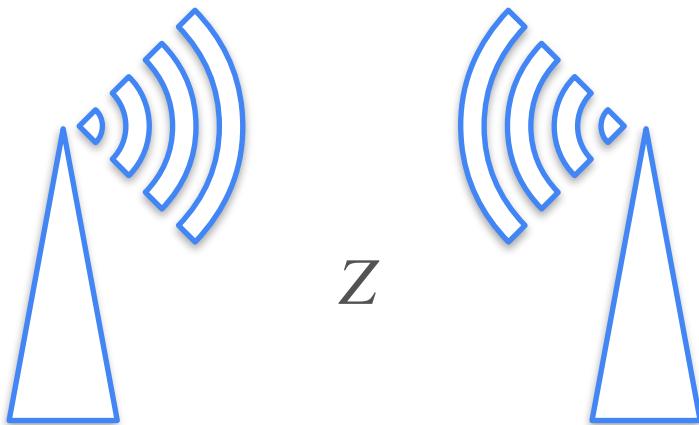
Others



Message sent: 10010

Message received: 10010 +Z

Chi-Square Distribution: Motivation



The communication channel has noise with a standard normal distribution

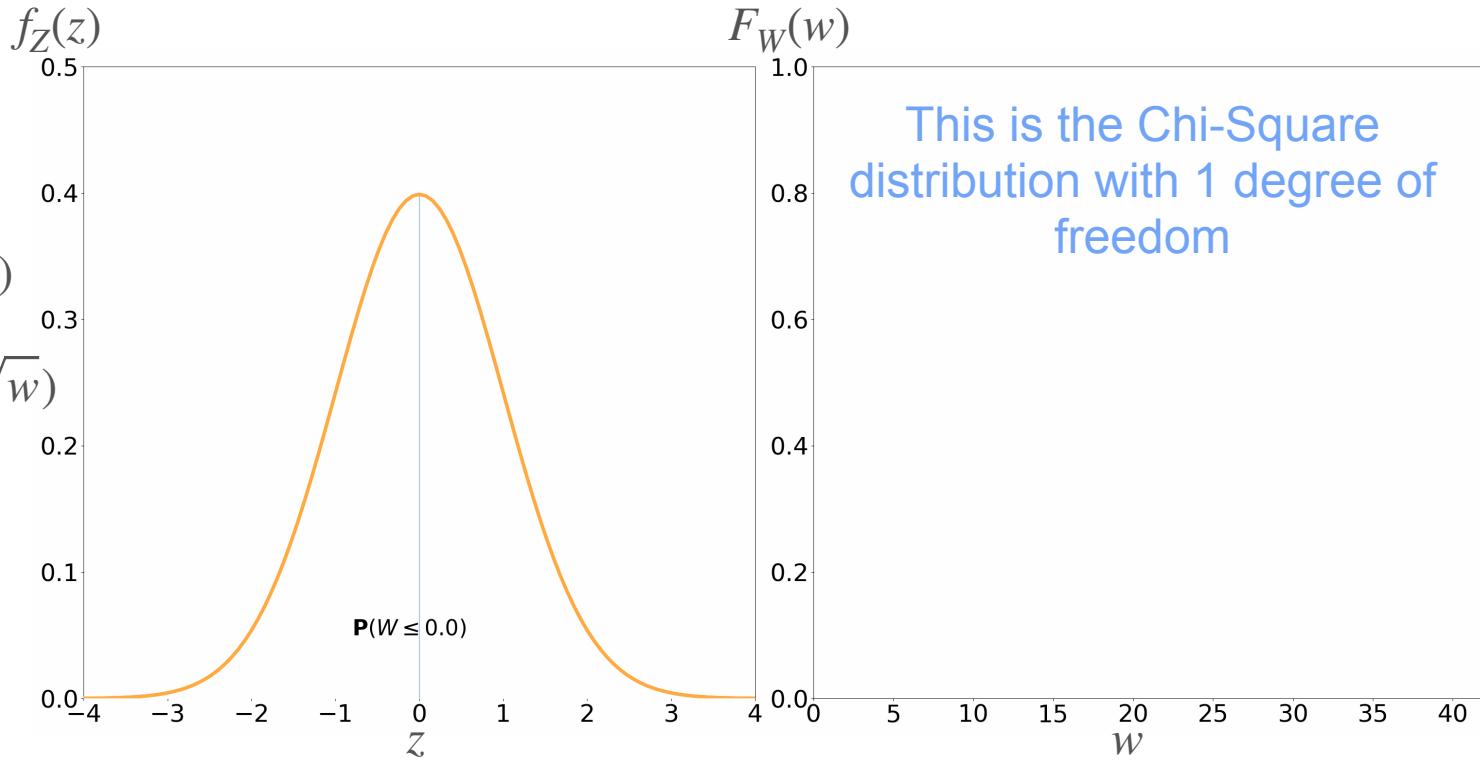
What is the **power** of the noise in the channel?

$$W = Z^2$$

What is the distribution of W ?

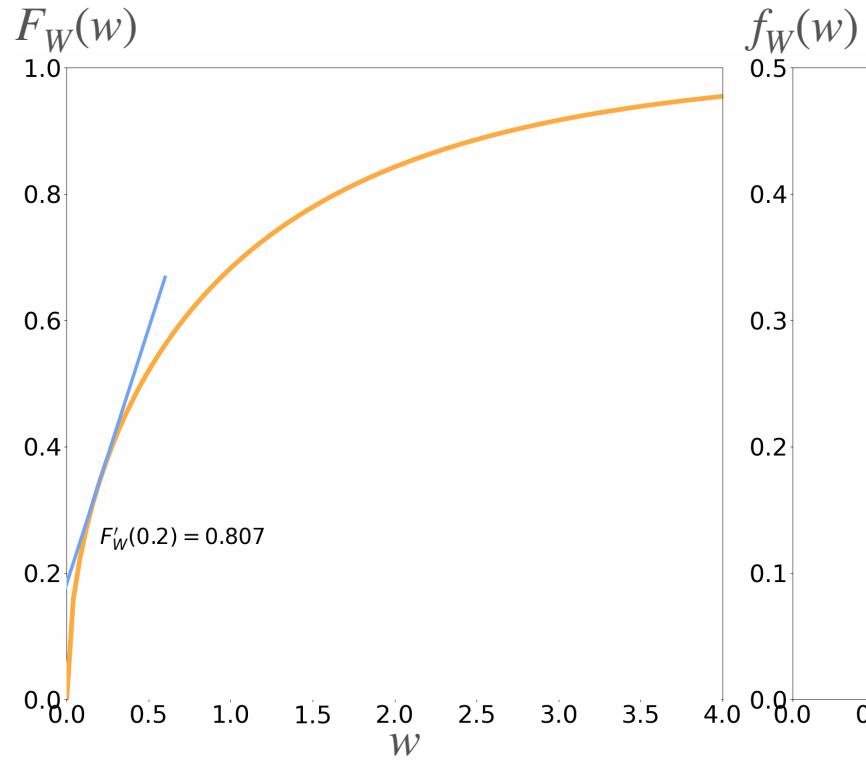
Chi Square Distribution

$$\begin{aligned}F_W(w) &= \mathbf{P}(W \leq w) \\&= \mathbf{P}(Z^2 \leq w) \\&= \mathbf{P}(|Z| \leq \sqrt{w}) \\&= \mathbf{P}(-\sqrt{w} \leq Z \leq \sqrt{w})\end{aligned}$$



Chi Square Distribution

$$f_W(w) = F'_W(w)$$



Chi-Square Distribution

Accumulated power over 2 transmissions?

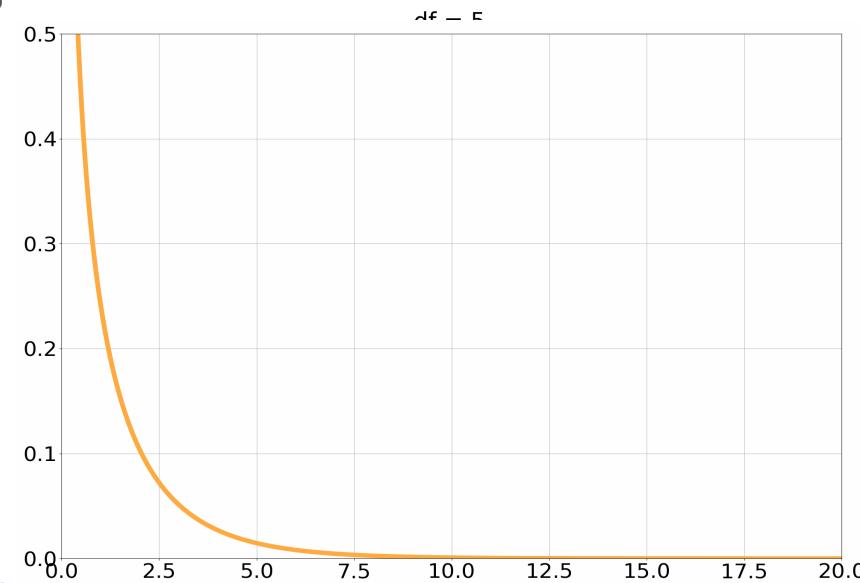
$$W_2 = Z_1^2 + Z_2^2 \quad \text{Chi-Square with 2 df}$$

Accumulated power over 5 transmissions?

$$W_5 = Z_1^2 + Z_2^2 + Z_3^2 + Z_4^2 + Z_5^2 \quad \text{Chi-Square with 5 df}$$

Accumulated power over k transmissions?

$$W_k = \sum_{i=1}^k Z_i^2 \quad \text{Chi-Square with } k \text{ df}$$



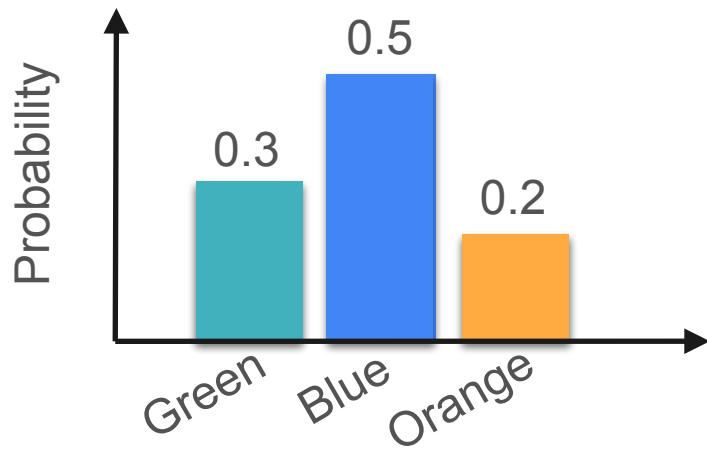


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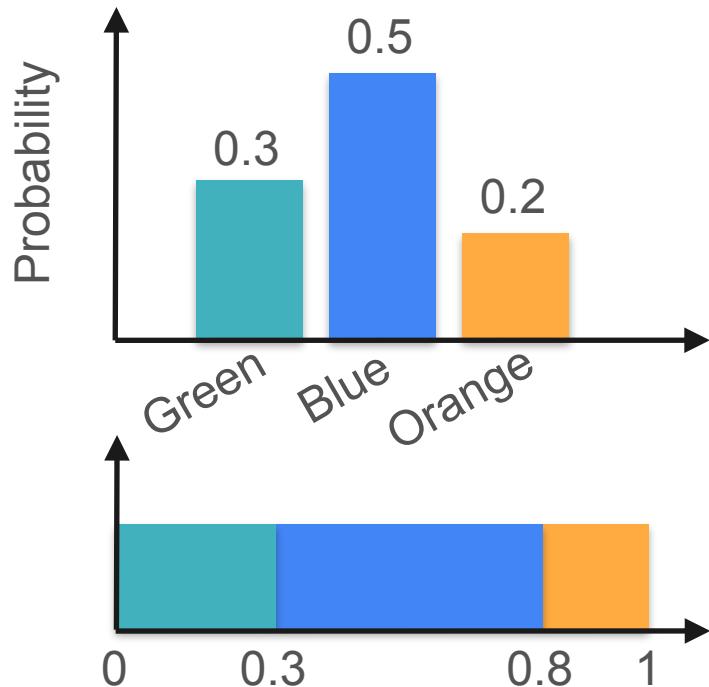
Probability Distributions

Sampling from a Distribution

Sampling From a Distribution

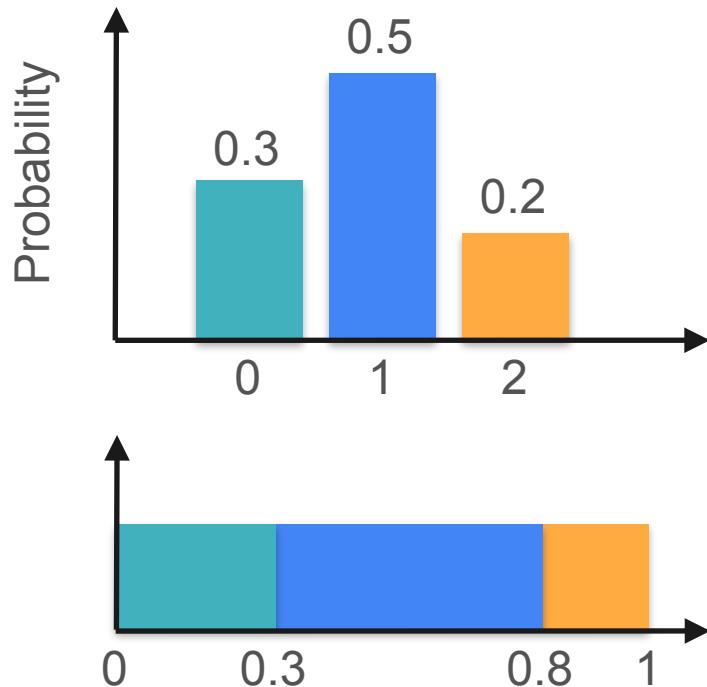


Sampling From a Distribution



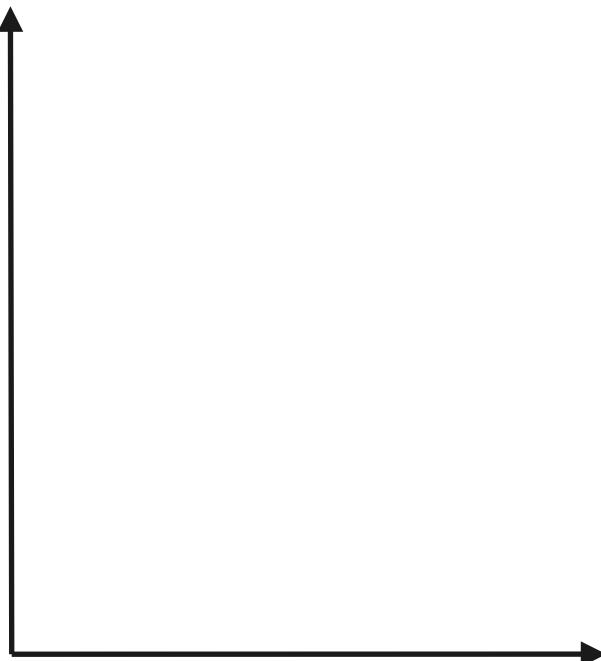
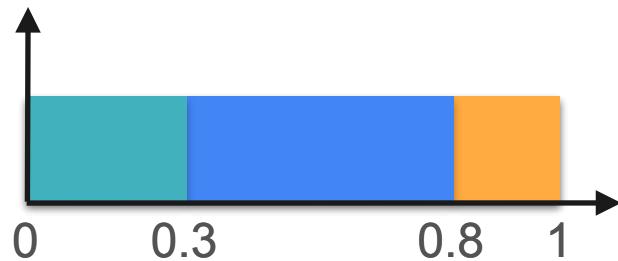
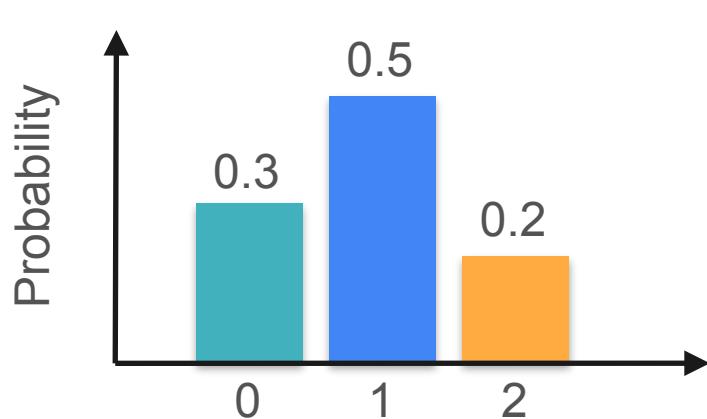
- **Step 1:** generate a random number between 0 and 1
- **Step 2:** find out which interval the number belongs to
 - $[0, 0.3)$
 - $[0.3, 0.8)$
 - $[0.8, 1]$
- **Step 3:** Assign an outcome based on the interval

Sampling From a Distribution

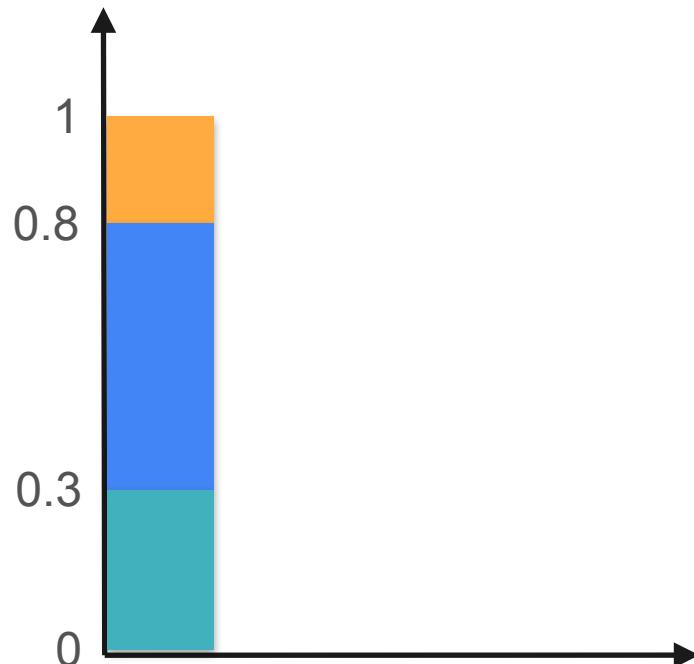
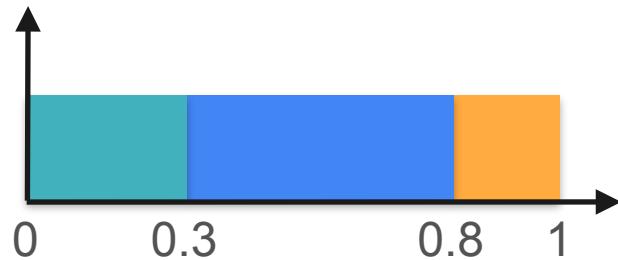
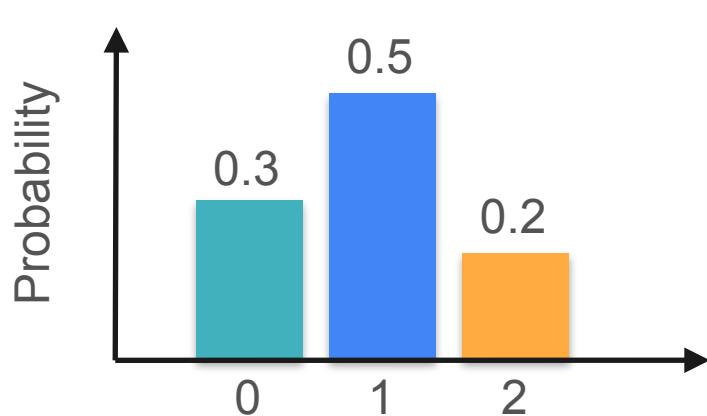


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- **Step 3:** Assign an outcome based on the interval

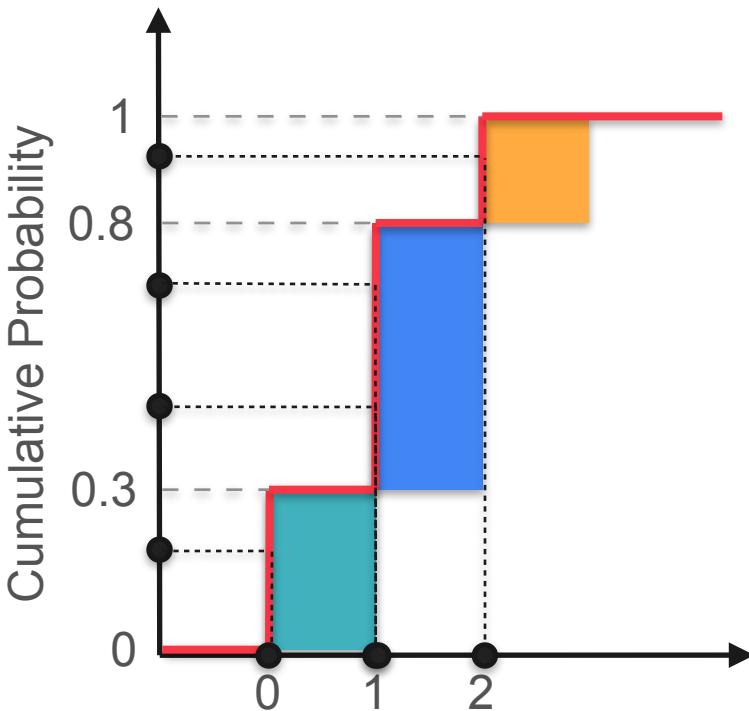
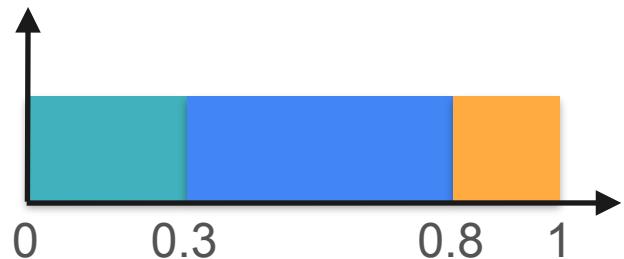
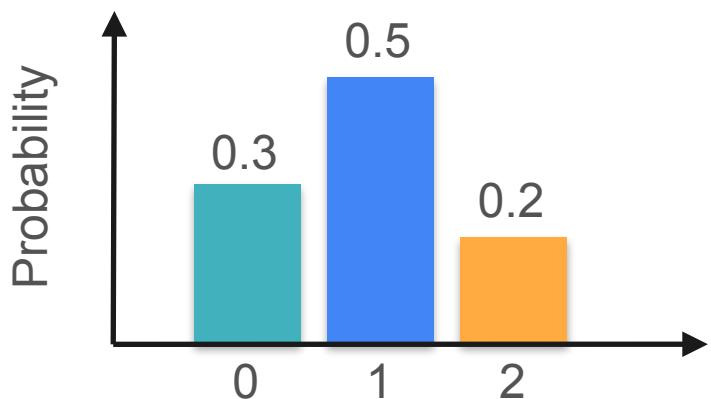
Sampling From a Distribution



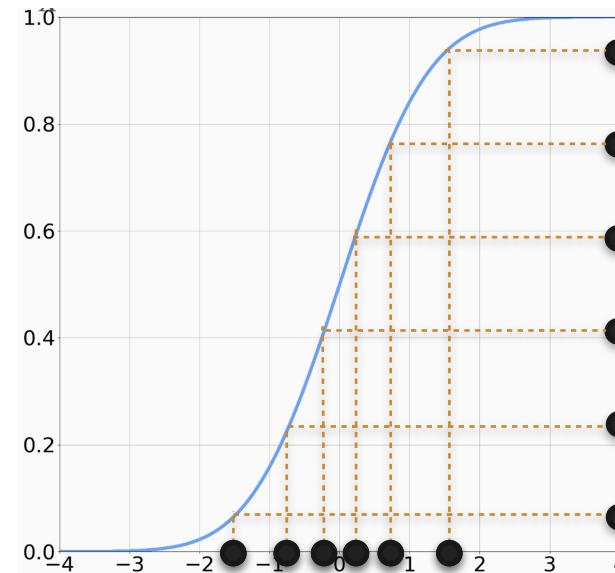
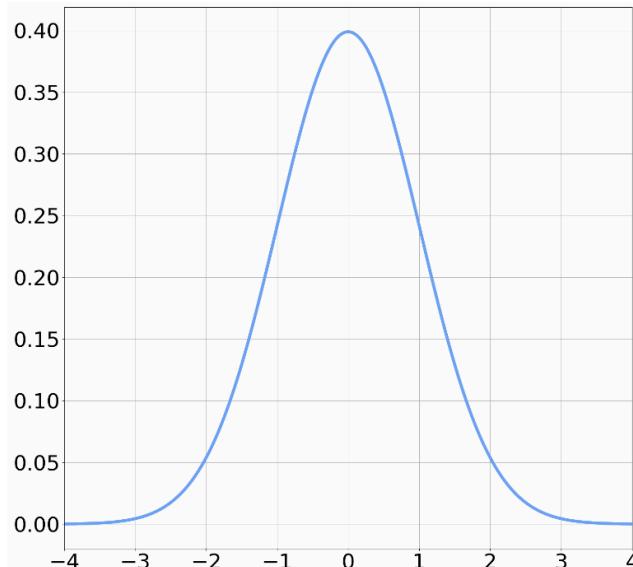
Sampling From a Distribution



Sampling From a Distribution



Sampling From a Normal Distribution





DeepLearning.AI

Probability Distributions

Conclusion

Week 1 - Conclusions

Talking head