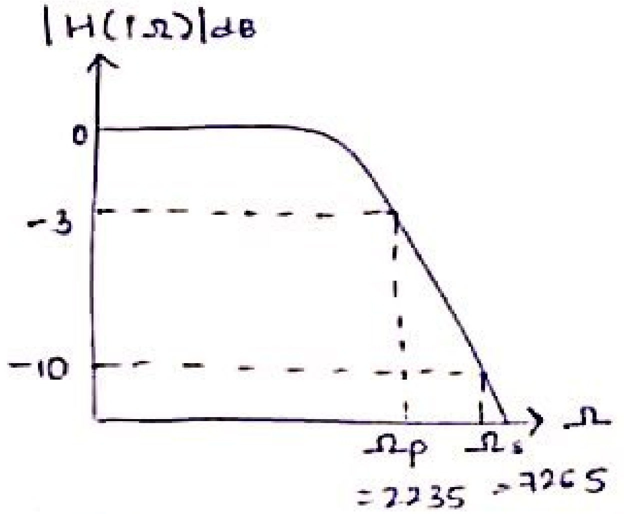
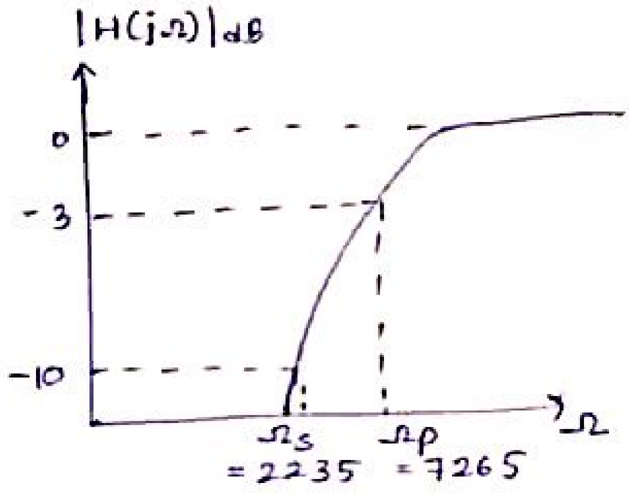
Solution.

Given $\alpha_p = 3\,dB$; $\omega_c = \omega_p = 2\times\pi\times 1000 = 2000\pi$ rad/sec

$\alpha_s = 10\,dB$; $\omega_s = 2\times\pi\times 350 = 700\pi$ rad/sec

$T = \frac{1}{f} = \frac{1}{5000} = 2\times 10^{-4}$ sec





Prewarping the digital frequencies we have

$$\Omega_p = \frac{2}{T}\tan\frac{\omega_p T}{2} = \frac{2}{2\times 10^{-4}}\tan\frac{(2000\pi\times 2\times 10^{-4})}{2}$$

$$= 10^4 \tan(0.2\pi) = 7265 \text{ rad/sec}$$

$$\Omega_s = \frac{2}{T}\tan\frac{\omega_s T}{2} = \frac{2}{2\times 10^{-4}}\tan\frac{(700\pi\times 2\times 10^{-4})}{2}$$

$$= 10^4 \tan(0.07\pi) = 2235 \text{ rad/sec}$$

The order of the filter

$$N = \frac{\log\sqrt{\frac{10^{0.1\alpha_s}-1}{10^{0.1\alpha_p}-1}}}{\log\frac{\Omega_s}{\Omega_p}} = \frac{\log\sqrt{\frac{10^{0.1(10)}-1}{10^{0.1(3)}-1}}}{\log\frac{7265}{2235}} = \frac{\log 3}{\log 3.25} = \frac{0.4771}{0.5118} = 0.932$$

$\therefore$ we take $N = 1$. The 1st order butterworth filter for $\Omega_c = 1$ rad/sec is $H(s) = \frac{1}{1+s}$

$\Omega_c = \Omega_p = 7265$ rad/sec

$$s \rightarrow \frac{\Omega_c}{s} \quad \text{i.e} \quad s \rightarrow \frac{(7265)}{s}$$

The transfer function of highpass filter

$$H(s) = \frac{1}{s+1}\bigg|_{s = \frac{7265}{s}}$$

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Using bilinear transformation,

$$H(z) = H(s)\Big|_{s=\frac{2}{T}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{s}{s+7265}\Bigg|_{s=\frac{2}{2*10^{-4}}\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}$$

$$= \frac{10000\left[\frac{1-z^{-1}}{1+z^{-1}}\right]}{1000\left[\frac{1-z^{-1}}{1+z^{-1}}\right]+7265}$$

$$= \frac{0.5792(1-z^{-1})}{1-0.1584z^{-1}}$$