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Maths BA exam pre-session

Math problems

1- Elementary Algebra.

problem 1.1 $\frac{x^{n+2}}{x^{n-2}} = x^n \cdot x^2 \cdot x^{-n} \cdot x^2 = x^4$

problem 1.2 $x^{-1} \& 8 = 2$ $\frac{8}{x} = 2$ $8 = 2x$ $4 = x$

Problem 1.3 $a=5$, $b=10$ $(ab)^0 = (5^{10})^0 = 1$ ∵ anything power zero = 1 $\forall a > 0$

Problem 1.4 $\frac{\sqrt{4x}}{\sqrt{x}} = \frac{2\sqrt{x}}{\sqrt{x}} = 2$

Problem 1.6 $2^x > 1024$ $2^x > 2^{10}$ $x \geq 10$

Problem 1.5 $x^2 + (x+1)^2 = (x+2)^2$

$x^2 + x^2 + 2x + 1 = x^2 + 4x + 4$

$x^2 - x^2 + x^2 + 2x - 4x + 1 - 4 = 0$

$x^2 - 2x - 3 = 0$

$(x+1)(x-3) = 0$

$x_1 = -1$ $x_2 = 3$

2. Functions of one variable

Problem 2.1.

$F = C(a) + b$

$32 = 0(a) + b$

$\boxed{32 = b}$

$212 = 100(a) + b$

$212 = 100(a) + 32$

$\frac{212 - 32}{100} = \boxed{a = 1.8}$

$F = 1.8C + 32$

$C = \frac{F - 32}{1.8}$

$F = C$

$F = 1.8F + 32$

$32 = -0.8F$

$\boxed{F = -40 = C}$

Problem 2.2

$f(x) = 5x + 4$

$y = 5(3) + 4$

$y = 19$

Problem 2.3

$x^2 - 4x + 3 = 0$

$x^2 - 3x - x + 3 = 0$

$x(x-3) - 1(x-3) = 0$

$(x-1)(x-3) = 0$

$x_1 = 3 \quad x_2 = 1$

Problem 2.4

$10(1 + 0.02)^{90} = 59.4313$ HUF after 90 years.

Problem 2.5

$e^{\ln 5} = 5$

3. Calculus

Problem 3.1

$$\sum_{i=1}^{\infty} \frac{12}{6^i} \qquad a = 12 \quad b = 1/6$$

$$= \frac{ab}{1-b}.$$

$$\sum_{i=1}^{\infty} 12 \cdot \frac{1}{6^i} = \frac{12 \cdot 1/6}{1 - 1/6}$$

$$\frac{2}{5/6} \quad \frac{6 \times 2}{5} = \frac{12}{5}$$

Problem 3.2

* multiply by 6^x

$$\lim_{x \to 1} \frac{6^{1-x}}{x}$$

$$\lim_{x \to 1} \frac{6^{1-x} \cdot 6^x}{x \cdot 6^x} = \frac{6}{x \cdot 6^x} \approx \frac{6}{1 \cdot 6^1} \approx 1.$$

Problem 3.3

$f(x) = x^5 - 8$ at $x = -3$

$f'(x) = 5x^4$

$f'(-3) = 5(-3)^4 = 5 \times 81 = 405$

Problem 3.4

$$\frac{d}{dx} \frac{x^3 + 2x - 1}{x - 2}$$

$a = x^3 + 2x - 1$, $a' = 3x^2 + 2$

$b = x - 2$, $b' = 1$

$$f'(x) = \frac{a'b - b'a}{b^2}$$

$$= \frac{(3x^2+2)(x-2) - (x^3+2x-1)(1)}{(x-2)^2}$$

$$= \frac{3x^3 + 2x - 6x^2 - 4 - x^3 - 2x + 1}{x^2 - 4x + 4}$$

$$= \frac{2x^3 - 6x^2 - 3}{x^2 - 4x + 4}$$

Problem 3.5

$$\frac{d^2}{dx^2} \; 4x^4 + 4x^2$$

$d' = 16x^3 + 8x$

$d'' = 48x^2 + 8$

Problem 3.6

$a = \ln x$, $a' = 1/x$; $b = e^x$, $b' = e^x$

$$\frac{d}{dx} \frac{\ln x}{e^x} = \frac{1/x \cdot e^x - \ln x \cdot e^x}{(e^x)^2}$$

$$= \frac{1/x - \ln x}{e^x}$$

Problem 3.7 $f(x) = 3x^2 + 5x + 2$

$f'(x) = 6x - 5$

$0 = 6x - 5$

$\frac{5}{6} = x$ stationary point

$f''(x) = 6$ → local min

since $f''(x) > 0$ it is convex

since $f'(x) > 0$ function is increasing.

	$< 5/6$	$5/6$	6	> 6
$f(x)$	+	+	+	+
$f'(x)$	−	stationary point	+	+
$f''(x)$	+	+	local min	+
shape	↘	↘	local min	↗

Problem 3.8 $f(x,y) = x^2 + y^3$

$f(2,3) = 2^2 + 3^3$

$= 4 + 27$

$= 31$

Problem 3.9 $f(x,y) = \ln(x-y)$ $x - y > 0$ $x > y$.

Problem 3.10 $\frac{d}{dx}\, x^5 + xy^3 = 5x^4 + y^3$

Problem 3.11 $f(x,y) = x^2y^2 + 10$

$f_x'(x,y) = 2xy^2$ $\quad f_y'(x,y) = 2x^2y$

$f_x''(x,y) = 2y^2$ $\quad f_y''(x,y) = 2x^2$

$0 = 2y^2$ $\quad 0 = 2x^2$.

$x = 0, y = 0$ → local min for every $x = 0$, $y = 0$ coordinates

Problem 3.12 max x^2y^2 st $x + y - 10 = 0$

$L = x^2y^2 - \lambda(x + y - 10)$

$\frac{\partial L}{\partial x} = 2xy^2 - \lambda$ $\quad \frac{\partial L}{\partial y} = 2x^2y - \lambda$ $\quad 2xy^2 = 2yx^2$

$x = y$

$x + y - 10 = 0$

$x + x - 10 = 0$

$2x = 10$

$x = 5 = y$

4. Linear Algebra

Problem 1.4. $A = \begin{bmatrix} 2 & 6 \\ 5 & 1 \\ 1 & 9 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 1 & 7 \\ 2 & 8 & 2 \end{bmatrix}$

$$A \cdot B = \begin{bmatrix} 1\cdot2+6\cdot2 & 1\cdot2+6\cdot8 & 2\cdot7+6\cdot2 \\ 1\cdot5+1\cdot2 & 1\cdot5+1\cdot8 & 5\cdot7+1\cdot2 \\ 1\cdot1+9\cdot2 & 1\cdot1+9\cdot8 & 7\cdot1+9\cdot2 \end{bmatrix}$$

$$A \cdot B = \begin{pmatrix} 14 & 50 & 26 \\ 7 & 13 & 37 \\ 19 & 73 & 25 \end{pmatrix}$$

Problem 4.2 $A = \begin{bmatrix} 2 & 2 \\ 4 & 6 \\ 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 9 & 1 \\ 2 & 1 & 2 \end{bmatrix}$

$$B \cdot A = \begin{bmatrix} 1\cdot2+4\cdot9+1\cdot1 & 2\cdot1+6\cdot9+3\cdot1 \\ 2\cdot2+4\cdot1+1\cdot2 & 2\cdot2+6\cdot1+3\cdot2 \end{bmatrix}$$

$$B \cdot A = \begin{pmatrix} 39 & 59 \\ 10 & 16 \end{pmatrix}$$

Problem 4.3 transpose of $\begin{bmatrix} 7.1 & 9.1 & 4.7 \\ 2 & 7.8 & 1.1 \\ 4 & 4.44 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 7.1 & 2 & 4 \\ 9.1 & 7.8 & 4.44 \\ 4.7 & 1.11 & 0 \end{bmatrix}$$

Problem 4.4 $\det(A)$ of $\begin{vmatrix} 1 & 9 \\ 2 & 8 \end{vmatrix}$ $\det = 1.8 - 9.2 = -10$

5. Probability Theory

Problem 5.1

d1 \ d2	1	2	3	4	5	6
1	11	21	31	41	51	61
2	12	22	32	42	52	62
3	13	23	33	43	53	63
4	14	24	34	44	54	64
5	15	25	35	45	55	65
6	16	26	36	46	56	66

Problem 5.2

Drug user %

		test +	test −
+	(0.01)	0.99	0.01
−	(0.99)	0.005	0.995

$P = 0.01 \cdot 0.99 + 0.99 \cdot 0.005 = 1.485\%$

Problem 5.3

$$P(B|A) = \frac{0.01 \cdot 0.99}{0.01 \cdot 0.99 + 0.99 \cdot 0.005} = 0.667$$

66.7%