Lexical Analysis

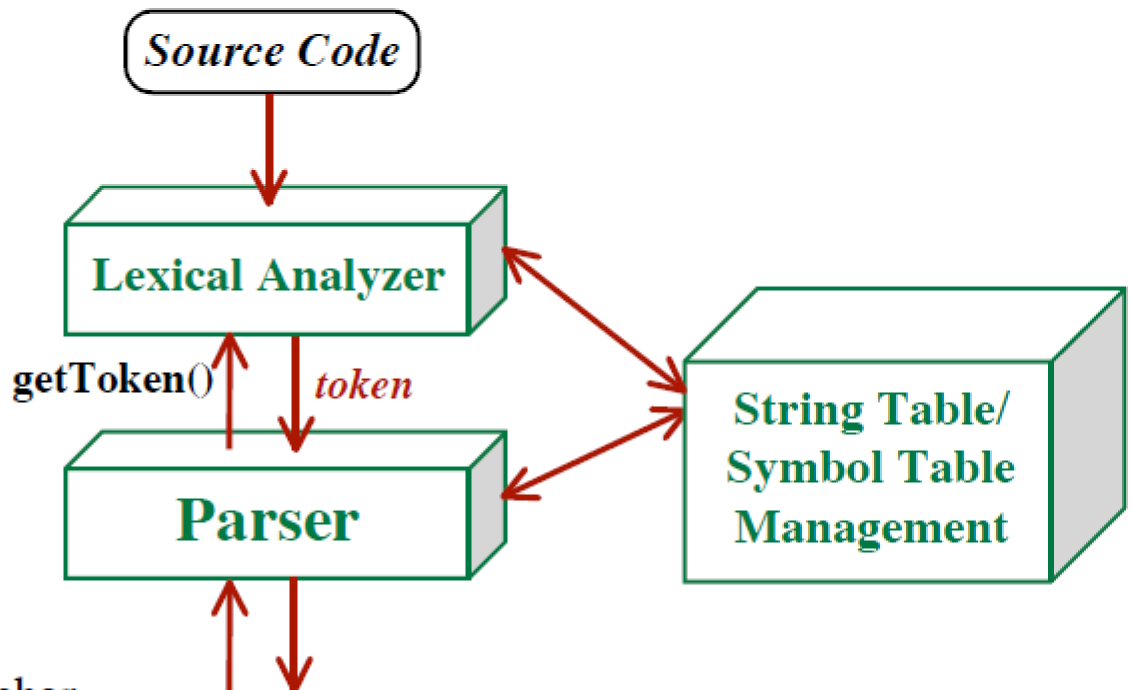
Chapter 3

Slide Credits

- Adapted from slides created by

  Harry H. Porter III, Ph.D., Portland State University
  - http://web.cecs.pdx.edu/~harry/compilers/syllabus.html

Lexical Analysis



also called
Lexer or scanner

- Must be efficient
- Looks at every input char
- Textbook, Chapter 3

Tokens

Token Type

Examples: ID, NUM, IF, EQUALS, ...

Lexeme

The characters actually matched.

Example:

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... if x == -12.30 then ...
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How to describe/specify tokens?

Formal:

Regular Expressions

```
Letter ( Letter | Digit )*
```

Informal:

"// through end of line"

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Formal:

Regular Expressions

`Letter ( Letter | Digit )*`

Informal:

"// through end of line"

Tokens will appear as TERMINALS in the grammar.

Stmt → `while` Expr `do` StmtList `endWhile`

→ ID "=" Expr ";"

→ ...

Lexical Errors

Most errors tend to be "typos"

Not noticed by the programmer

```
return 1.23;
retunn 1,23;
```

... Still results in sequence of legal tokens

```
<ID,"retunn">  <INT,1>  <COMMA>  <INT,23>  <SEMICOLON>
```

No lexical error, but problems during parsing!

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Errors caught by lexer:

- EOF within a String / missing "
- Invalid ASCII character in file
- String / ID exceeds maximum length
- Numerical overflow

etc...

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etc...

Lexer must keep going!

Always return a valid token.

Skip characters, if necessary. Other possibilities: insert, replace, transpose

May confuse the parser

The parser will detect syntax errors and get straightened out (hopefully!)

Managing Input Buffers

Option 1: Read one char from OS at a time.

Option 2: Read N characters per system call

e.g., N = 4096

Manage input buffers in Lexer

More efficient

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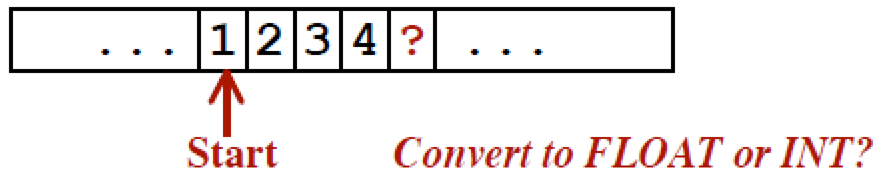

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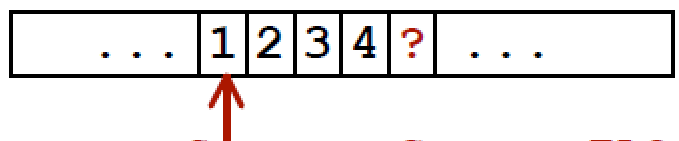

Start *Convert to FLOAT or INT?*

Token could overlap / span buffer boundaries.

⇒ need 2 buffers

Code:

```
if (ptr at end of buffer1) or (ptr at end of buffer2) then ...
```

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Often, we need to look ahead

...	1	2	3	4	?	...

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Code:

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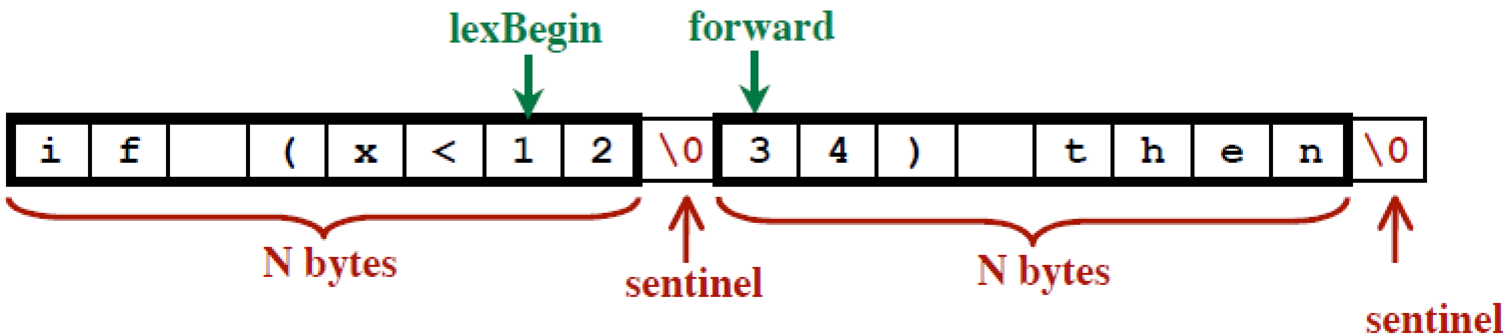
Technique: Use "Sentinels" to reduce testing

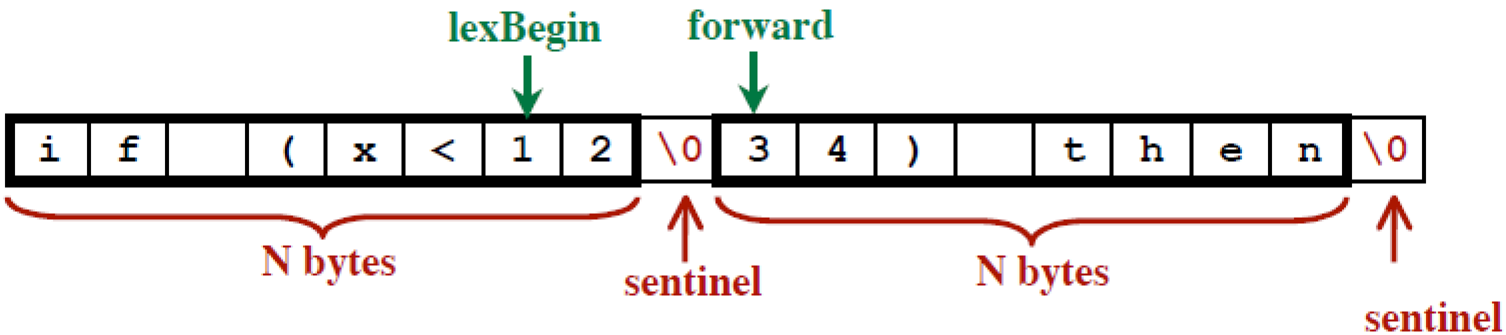
Choose some character that occurs rarely in most inputs

`'\0'`



Goal: Advance **`forward`** pointer to next character

...and reload buffer if necessary.



Goal: Advance **`forward`** pointer to next character
...and reload buffer if necessary.

One fast test
...which usually fails

Code :

```
forward++;
if *forward == '\0' then
  if forward at end of buffer #1 then
    Read next N bytes into buffer #2;
    forward = address of first char of buffer #2;
  elseIf forward at end of buffer #2 then
    Read next N bytes into buffer #1;
    forward = address of first char of buffer #1;
  else
    // do nothing; a real \0 occurs in the input
  endIf
endIf
```

"Alphabet" (Σ)

A set of symbols ("characters")

Examples: $\Sigma = \{$ `a, b, c, d` $\}$

Σ = ASCII character set

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Sequence of symbols

Finite in length

Example: `abbadc` Length of s = $|s|$

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It is a string

$|\varepsilon| = 0$

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"Language"

A set of strings

Examples:

$L_1 = \{$ `a, baa, bccb` $\}$

$L_2 = \{ \}$

$L_3 = \{ \varepsilon \}$

$L_4 = \{$ ε, `ab, abab, ababab, abababab, ...` $\}$

$L_5 = \{$ s | s can be interpreted as an English sentence making a true statement about mathematics$\}$

Each string is finite in length, but the set may have an infinite number of elements.

"Prefix" ...of string s

s = **`hello`**

Prefixes: **`he`**
`hello`
ε

"Prefix" ...of string s

s = `hello`

Prefixes: `he`
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ε

"Suffix" ...of string s

s = `hello`

Suffixes: `llo`
ε
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Suffixes: `llo`
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"Substring" ...of string s

Remove a prefix and a suffix

s = `hello`

Substrings: `ell`
`hello`
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"Proper" prefix / suffix / substring ... of s

≠ s and ≠ ε

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`hello`
ε

"Suffix" ...of string s

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ε
`hello`

"Substring" ...of string s
Remove a prefix and a suffix

s = `hello` *Substrings:* `ell`
`hello`
ε

"Proper" prefix / suffix / substring ... of s
≠ s and ≠ ε

"Subsequence" ...of string s,

s = `compilers` *Subsequences:* `opilr`
`cors`
`compilers`
ε

"Concatenation"

Strings: x, y

Concatenation: xy

Example:

x = `abb`

y = `cdc`

xy = `abbcdc`

yx = `cdcabb`

Other notations: **x || y**

x + y

x ++ y

x · y

"Concatenation"

Strings: x, y

Concatenation: xy

Example:

x = abb

y = cdc

xy = abbcdc

yx = cdcabb

Other notations: **x || y**

x + y

x ++ y

x · y

What is the "identity" for concatenation?

$\varepsilon x = x\varepsilon = x$

Multiplication ⇔ Concatenation

Exponentiation ⇔ ?

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Multiplication ⇔ Concatenation

Exponentiation ⇔ ?

Define $s^{\mathbf{0}} = \varepsilon$

$s^{\mathbf{N}} = s^{\mathbf{N-1}}s$

Example $x =$ ab

$x^{\mathbf{0}} = \varepsilon$

$x^{\mathbf{1}} = x =$ ab

$x^{\mathbf{2}} = xx =$ abab

$x^{\mathbf{3}} = xxx =$ ababab

...etc...

"Concatenation"

Strings: x, y

Concatenation: xy

Example:

$x = abb$

$y = cdc$

$xy = abbcdc$

$yx = cdcabb$

Other notations: $x \| y$

$x + y$

$x ++ y$

$x \cdot y$

What is the "identity" for concatenation?

$\varepsilon x = x\varepsilon = x$

Multiplication $\Leftrightarrow$ Concatenation

Exponentiation $\Leftrightarrow$?

Define $s^0 = \varepsilon$

$s^N = s^{N-1}s$

Example $x = ab$

$x^0 = \varepsilon$

$x^1 = x = ab$

$x^2 = xx = abab$

$x^3 = xxx = ababab$

...etc...

$x^* = x^\infty = abababababab...$

Infinite sequence of symbols!

Technically, this is not a "string"

"Language"

A set of strings

$L = \{ \dots \}$

$M = \{ \dots \}$

Generally, these are infinite sets.

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A set of strings

$L = \{ \ldots \}$

$M = \{ \ldots \}$

Generally, these are infinite sets.

"Union" of two languages

$L \cup M = \{ s \mid s \text{ is in } L \text{ or is in } M \}$

Example:

$L = \{$ `a`, `ab` $\}$

$M = \{$ `c`, `dd` $\}$

$L \cup M = \{$ `a`, `ab`, `c`, `dd` $\}$

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"Concatenation" of two languages

$L\,M = \{ st \mid s \in L \text{ and } t \in M \}$

Example:

$L = \{$ `a`, `ab` $\}$

$M = \{$ `c`, `dd` $\}$

$L\,M = \{$ `ac`, `add`, `abc`, `abdd` $\}$

Repeated Concatenation

Let: $L = \{ \texttt{a}, \texttt{bc} \}$

Example: $L^0 = \{ \varepsilon \}$

$L^1 = L = \{ \texttt{a}, \texttt{bc} \}$

$L^2 = LL = \{ \texttt{aa}, \texttt{abc}, \texttt{bca}, \texttt{bcbc} \}$

$L^3 = LLL = \{ \texttt{aaa}, \texttt{aabc}, \texttt{abca}, \texttt{abcbc}, \texttt{bcaa}, \texttt{bcabc}, \texttt{bcbca}, \texttt{bcbcbc} \}$

...etc...

$L^N = L^{N-1}L = LL^{N-1}$

Kleene Closure

Let: $L = \{ \texttt{a}, \texttt{bc} \}$

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...etc...

$$L^N = L^{N-1}L = LL^{N-1}$$

The "Kleene Closure" of a language:

$$\sum_{i=0}^{\infty} a^i = a^0 \cup a^1 \cup a^2 \cup \ldots$$

$$L^* = \bigcup_{i=0}^{\infty} L^i = L^0 \cup L^1 \cup L^2 \cup L^3 \cup \ldots$$

Example:

$$L^* = \{ \underbrace{\varepsilon,}_{L^0} \underbrace{\texttt{a}, \texttt{bc},}_{L^1} \underbrace{\texttt{aa}, \texttt{abc}, \texttt{bca}, \texttt{bcbc},}_{L^2} \underbrace{\texttt{aaa}, \texttt{aabc}, \texttt{abca}, \texttt{abcbc}, \ldots}_{L^3} \}$$

Positive Closure

Let: $L = \{ \texttt{a}, \texttt{bc} \}$

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The "Positive Closure" of a language:

$$L^+ = \bigcup_{i=1}^{\infty} L^i = L^1 \cup L^2 \cup L^3 \cup \ldots$$

Example:

$$L^+ = \{ \underbrace{\texttt{a}, \texttt{bc},}_{L^1} \underbrace{\texttt{aa}, \texttt{abc}, \texttt{bca}, \texttt{bcbc},}_{L^2} \underbrace{\texttt{aaa}, \texttt{aabc}, \texttt{abca}, \texttt{abcbc}, \ldots}_{L^3} \}$$

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Note that ε is not included UNLESS it is in L to start with

Examples

Let: $L = \{ \texttt{a}, \texttt{b}, \texttt{c}, \ldots, \texttt{z} \}$

$D = \{ \texttt{0}, \texttt{1}, \texttt{2}, \ldots, \texttt{9} \}$

$D^{+} =$

Examples

Let: $L = \{$ `a`, `b`, `c`, ..., `z` $\}$
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$D^+ =$
"The set of strings with one or more digits"

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$(L \cup D)^{*} =$

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$(L \cup D)^{*} =$
"Sequences of zero or more letters and digits"

$L((L \cup D)^{*}) =$
"Set of strings that start with a letter, followed by zero or more letters and and digits."

Regular Expressions

Assume the alphabet is given... *e.g.,* $\Sigma = \{$ `a, b, c, ... z` $\}$

Example: `a ( b | c ) d`* `e`

A regular expression describes a language.

Notation:

r = regular expression

L(r) = the corresponding language

Example:

r = `a ( b | c ) d`* `e`

L(r) = { `abe`,
`abde`,
`abdde`,
`abddde`,
`. . .`,
`ace`,
`acde`,
`acdde`,
`acddde`,
`. . .`}

Meta Symbols:

`( )`

`|`

`*`

ε

How to "Parse" Regular Expressions

* has highest precedence.

Concatenation as middle precedence.

| has lowest precedence.

Use parentheses to override these rules.

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Concatenation and | are associative.

`(a b) c = a (b c) = a b c`

`(a | b) | c = a | (b | c) = a | b | c`

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Example:

`b d | e f * | g a  =   (b d) | (e (f *))  | (g a)`

How to "Parse" Regular Expressions

* has highest precedence.
Concatenation as middle precedence.
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Use parentheses to override these rules.

Examples:

$a\ b^* = a\ (b^*)$

If you want $(a\ b)^*$ you must use parentheses.

$a \mid b\ c = a \mid (b\ c)$

If you want $(a \mid b)\ c$ you must use parentheses.

Concatenation and | are associative.

$(a\ b)\ c = a\ (b\ c) = a\ b\ c$

$(a \mid b) \mid c = a \mid (b \mid c) = a \mid b \mid c$

Example:

$b\ d \mid e\ f^* \mid g\ a = ((b\ d) \mid (e\ (f^*))) \mid (g\ a)$

Fully parenthesized

Definition: Regular Expressions

(Over alphabet Σ)

- ε is a regular expression.
- If `a` is a symbol (i.e., if $a \in \Sigma$), then `a` is a regular expression.
- If `R` and `S` are regular expressions, then `R|S` is a regular expression.
- If `R` and `S` are regular expressions, then `RS` is a regular expression.
- If `R` is a regular expression, then `R*` is a regular expression.
- If `R` is a regular expression, then `(R)` is a regular expression.

Definition: Regular Expressions

(Over alphabet Σ)

And, given a regular expression `R`, what is L(`R`) ?

- ε is a regular expression.

- If `a` is a symbol (i.e., if `a`$\in\Sigma$), then `a` is a regular expression.

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(Over alphabet Σ)

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  $L(\,(R)\,) = L(R)$

Regular Languages

Definition: "Regular Language" (or "Regular Set")

... A language that can be described by a regular expression.

- Any finite language (i.e., finite set of strings) is a regular language.
- Regular languages are (usually) infinite.
- Regular languages are, in some sense, simple languages.

Regular Langauges $\subset$ Context-Free Languages

Examples:

`a	b	cab`	{a, b, cab}
`b*`	{ε, b, bb, bbb, ...}
`a	b*`	{a, ε, b, bb, bbb, ...}
`(a | b)*`	{ε, a, b, aa, ab, ba, bb, aaa, ...}
	"Set of all strings of a's and b's, including ε."

Equality v. Equivalence

Are these regular expressions equal?

```
R = a a* (b | c)
S = a* a (c | b)
```

... No!

Yet, they describe the same language.

L(R) = L(S)

"Equivalence" of regular expressions

If L(R) = L(S) then we say R ≈ S

"R is equivalent to S"

"Syntactic" equality versus "deeper" equality...

Algebra:

Does... $x(3+b) = 3x+bx$?

From now on, we'll just say R = S to mean R ≈ S

Notation:

Equality	Equivalence
=	=
	==
	≈
	≡
	⇔

Algebraic Laws of Regular Expressions

Let R, S, T be regular expressions...

| is commutative

$$R \mid S = S \mid R$$

| is associative

$$R \mid (S \mid T) = (R \mid S) \mid T = \overbrace{R \mid S \mid T}^{\textit{Preferred}}$$

Concatenation is associative

$$R\,(S\,T) = (R\,S)\,T = \underbrace{R\,S\,T}_{\textit{Preferred}}$$

Concatenation distributes over |

$$R\,(S \mid T) = RS \mid RT$$
$$(R \mid S)\,T = RT \mid ST$$

ε is the identity for concatenation

$$\varepsilon R = R \varepsilon = R$$

* is idempotent

$$(R^*)^* = R^*$$

Relation between * and ε

$$R^* = (R \mid \varepsilon)^*$$

Regular Definitions

Letter	=	`a	b	c	...	z`
Digit	=	`0	1	2	...	9`
ID	=	Letter (Letter | Digit)*

Regular Definitions

```
Letter  = a | b | c | ... | z
Digit   = 0 | 1 | 2 | ... | 9
ID      = Letter ( Letter | Digit )*
```

Names (e.g., Letter) are underlined to distinguish from a sequence of symbols.

```
          Letter ( Letter | Digit )*
        = {"Letter", "LetterLetter", "LetterDigit", ... }
```

Regular Definitions

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ID	=	Letter `(` Letter `|` Digit `)*`

Names (e.g., Letter) are underlined to distinguish from a sequence of symbols.

`Letter ( Letter | Digit )*`

= {"`Letter`", "`LetterLetter`", "`LetterDigit`", ... }

Each definition may only use names *previously* defined.

⇒ No recursion

Regular Sets = no recursion

CFG = recursion

Addition Notation / Shorthand

One-or-more: $^{+}$

$X^{+} = X\,(X^{*})$

Digit^{+} = Digit Digit* = Digits

Addition Notation / Shorthand

One-or-more: $+$

$X^+ = X\,(X^*)$

Digit$^+$ = Digit Digit* = Digits

Optional (zero-or-one): $?$

$X? = (X \mid \varepsilon)$

Num = Digit$^+$ (. Digit$^+$) ?

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Optional (zero-or-one): ?

$X? = (X \mid \varepsilon)$

Num = Digit^{+} (. Digit^{+}) ?

Character Classes: [*FirstChar*-*LastChar*]

Assumption: The underlying alphabet is known ...and is ordered.

Digit = `[0-9]`

Letter = `[a-zA-Z]` = `[A-Za-z]`

Many sets of strings are not regular.

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(() (()))

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Strings with repeated substrings

{ X`c`X | X is a string of `a`'s and `b`'s }

$\underbrace{\texttt{abbbab}}\texttt{c}\underbrace{\texttt{abbbab}}$

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Strings with repeated substrings

{ X`c`X | X is a string of `a`'s and `b`'s }

`abbbabcabbbab`

CFG is not even powerful enough.

The Problem?

In order to recognize a string,

these languages require *memory*!

Problem: How to describe tokens?

Solution: Regular Expressions

Problem: How to recognize tokens?

Approaches:

- Hand-coded routines

  Examples: E-Language, PCAT-Lexer

- Finite State Automata
- Scanner Generators (Java: JLex, C: Lex)

Scanner Generators

Input: Sequence of regular definitions

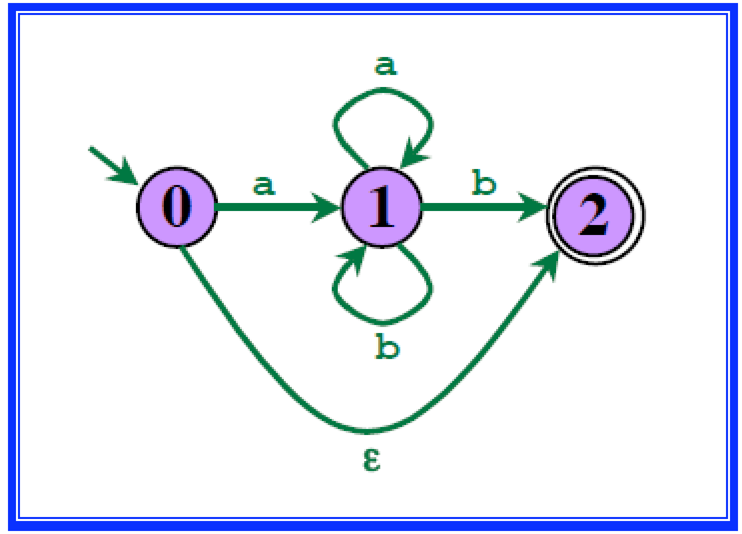
Output: A lexer (e.g., a program in Java or "C")

Approach:

- Read in regular expressions
- Convert into a Finite State Automaton (FSA)
- Optimize the FSA
- Represent the FSA with tables / arrays
- Generate a table-driven lexer (Combine "canned" code with tables.)

Finite State Automata (FSAs)

("Finite State Machines","Finite Automata", "FA")



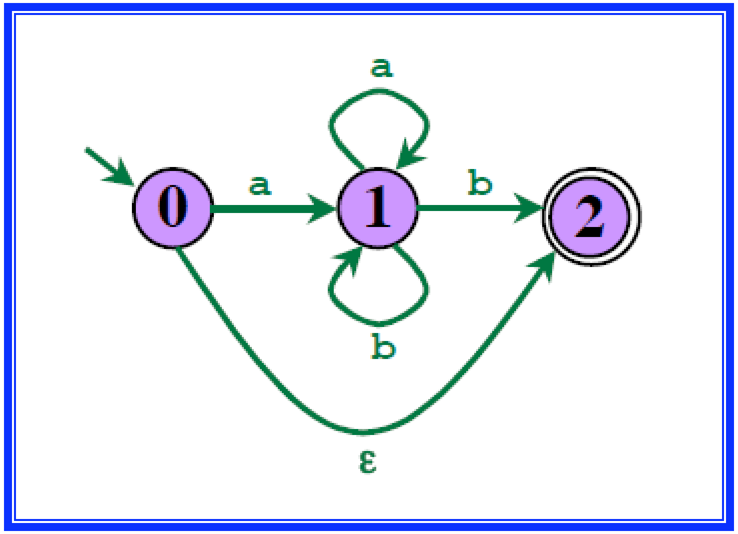
- One start state
- Many final states
- Each state is labeled with a state name
- Directed edges, labeled with symbols
  - Deterministic (DFA)

    No **ε**-edges

    Each outgoing edge has different symbol
  - Non-deterministic (NFA)

Finite State Automata (FSAs)

Formalism: $\langle S, \Sigma, \delta, S_0, S_F \rangle$



S = Set of states

$S = \{s_0, s_1, \ldots, s_N\}$

Σ = Input Alphabet

Σ = ASCII Characters

δ = Transition Function

$S \times \Sigma \rightarrow$ States (deterministic)

$S \times \Sigma \rightarrow$ Sets of States (non-deterministic)

s_0 = Start State

"Initial state"

$s_0 \in S$

S_F = Set of final states

"accepting states"

$S_F \subseteq S$

Finite State Automata (FSAs)

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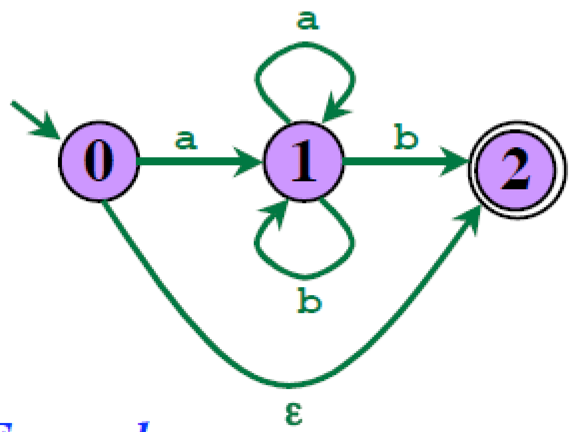


Example:

$S = \{0, 1, 2\}$

$\Sigma = \{a, b\}$

$s_0 = 0$

$S_F = \{ 2 \}$

δ =

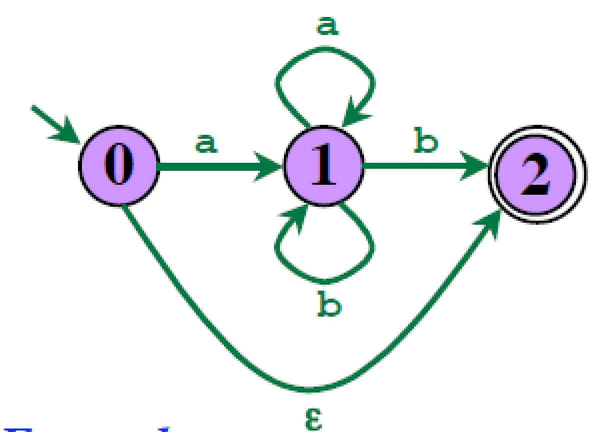
States \ Input Symbols	a	b	ε
0	{1}	{}	{2}
1	{1}	{1,2}	{}
2	{}	{}	{}

Finite State Automata (FSAs)

A string is "**accepted**"...
(a string is "**recognized**"...)

by a FSA if there is a path
from Start to any accepting state
where edge labels match the string.

Example:
This FSA accepts:
ε
aaab
abbb



Example:

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$\delta =$

Input Symbols

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1	{1}	{1,2}	{}
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Deterministic Finite Automata (DFAs)

No ε-moves

The transition function returns a single state

$\delta: S \times \Sigma \rightarrow S$

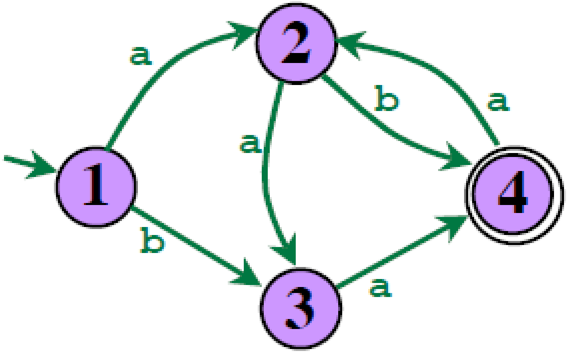
```
function Move (s:State, a:Symbol) returns State
```

$\delta =$

Input Symbols

States

	a	b
1	2	3
2	3	4
3	4	---
4	2	---



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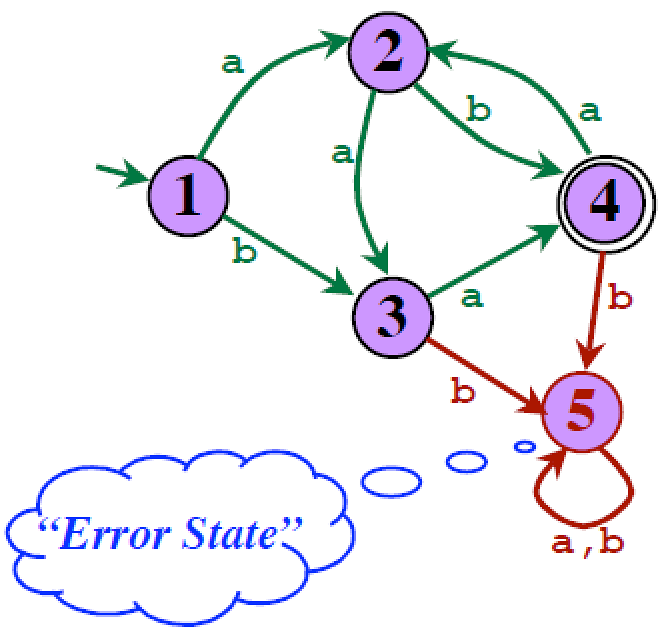
5	5	5



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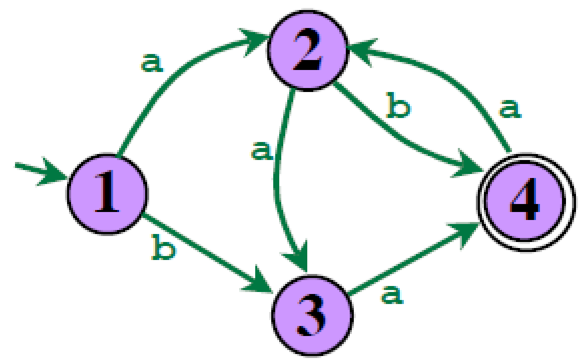


Non-Deterministic Finite Automata (NFAs)

Allow **ε**-moves

The transition function returns a *set of* states

$\delta: S \times \Sigma \rightarrow \text{Powerset}(S)$

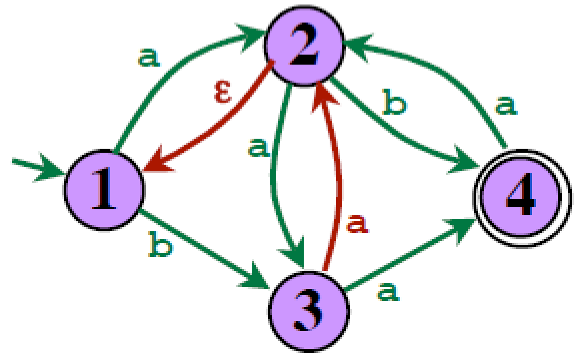
$\delta: S \times \Sigma \rightarrow P\,(S)$

```
function Move (s:State, a:Symbol) returns set of State
```

$\delta =$

Input Symbols

States	a	b	ε
1	{2}	{3}	{}
2	{3}	{4}	{1}
3	{4,2}	{}	{}
4	{2}	{}	{}



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- DFAs, NFAs, and Regular Expressions all have the same "power".
  They describe "Regular Sets" ("Regular Languages")
- The DFA may have a lot more states than the NFA.
  (May have exponentially as many states, but...)