**Situation 1: There are already random addresses in the tree. What score will a new random IP address get?**

*Note: We assume completely random addresses. That means it can happen with a certain probability that some of the random addresses are identical.*

First level: With probability 0.5, the new address goes to the 0-branch and the probability that branch contains more than of the entries is . Also with probability 0.5, the new address goes to the 1-branch and the probability that branch contains more than of the entries is Since the binomial distribution is symmetric, . The average score for the first level is:

This will introduce an error if n is not an integer and multiple of two. The best seems to be to take the average of the two approximations:

We assume that the levels are independent and that we will have, on average, entries in each subtree of level . Using the above equation on each level gives

**Situation 2: There are already addresses in the tree, however are random and are identical. What score will a new random IP address get?**

*Note: We assume completely random addresses. That means it can happen with a certain probability that one of the random addresses is identical to another random address or even to the identical ones.*

Let’s first define the probability that at least out of fair coin tosses are head:

Again, errors will be introduced if is not a natural number. Better results are obtained with the approximation in equation (\*) from situation 1.

First level (“level 0”): With probability 0.5, the new IP address is in the same branch as the identical addresses. The probability that more than half of the entries are in this branch knowing that the identical are in this branch is:

With probability 0.5, the new IP address is in the branch that only contains random addresses. The probability to have the majority of the entries in this branch knowing that are definitely not in this branch is:

In total, the score for the first level is

On the second level, we have four branches, of which one contains with certainty the identical addresses:

And so on for the other levels. In total, the average score is:

**Situation 3: There are already addresses in the tree, however are random and are identical (also random). What score will an IP address get that is identical to the identical entries?**

The situation is similar to situation 2. However, we know that we always stay on the branch with the identical entries. The score is:

**Situation 4: There are already addresses in the tree, however are random and there are “groups” of entries with identical addresses. Those addresses are distributed perfectly over the tree. What score will a new random IP address get?**

Since the addresses are distributed perfectly over the tree, the new random address will see exactly of them in its branch at the first level, in its branch at the second level etc. After level , the subtrees start to behave like in situation 2.

The score for the level is

After level , we can apply the score of situation 2 to each subtree:

**Situation 5: Like situation 4 ( addresses), but the new address is not random; it is one of the addresses.**

The score for levels 1 to is the same as in situation 4. However, for the levels , the “new” address will always stay on a branch that contains the entries with the same address. For those levels, we can use the result from situation 3.