PRE-CALCULUS 11

CHAPTER 6: RATIONAL EXPRESSIONS AND EQUATIONS

LESSON 6.1: SIMPLIFYING RATIONAL EXPRESSIONS

A rational number is said to be reduced or simplified if the numerator and denominator have no common factors.

$\frac{12}{18} =$ ______________ $=$ $\qquad\qquad$ $\frac{15a^2b^3}{25ab^2} =$ ______________ $=$

A rational expression is in simplest form or lowest terms when the numerator and denominator have no common factors other than 1.

Because division by 0 is not defined, the values of variables in the ______________________________ of rational expressions may be restricted.

▶ ***Example 1.1:*** Express in simplest form and determine all non-permissible values.

(a) $\frac{27a^3}{12a}$

(b) $\frac{3y^2+5y}{2y}$

(c) $\frac{2y^2-y-15}{4y^2-13y+3}$

(d) $\frac{16-m^2}{2m^2-5m-12}$

(e) $\frac{16x^2-9y^2}{8x-6y}$

LESSON 6.2: MULTIPLYING AND DIVIDING RATIONAL EXPRESSIONS

Rational expressions can be multiplied and divided in the same way that simple fractions are multiplied and divided.

$$\frac{5}{18}\times\frac{9}{10}=\underline{\qquad\qquad\qquad}=$$

$$\frac{3a^3}{2b^2}\times\frac{10b^3}{9a^2}=\underline{\qquad\qquad\qquad}=$$

$$\frac{2}{9}\div\frac{4}{27}=\underline{\qquad}\times\underline{\qquad}=$$

$$\frac{15a^2b}{4c}\div\frac{8abc}{-3}=\underline{\qquad}\times\underline{\qquad}=$$

When multiplying some rational expressions, it may be easier to factor the numerator and denominator first.

▶ ***Example 2.1:*** Multiply. State the restrictions on the variable.

(a) $\dfrac{6m^3(n+1)}{15m(n-1)}\times\dfrac{2(n-1)}{mn}$

(b) $\dfrac{x^2+5x+6}{x^2+x-2}\times\dfrac{x+1}{x+3}$

▶ ***Example 2.2:*** Divide. State the restrictions on the variable.

(a) $\dfrac{12m^2-3}{2m^2n-2mn^2}\div\dfrac{2m+1}{5mn-5n^2}$

(b) $\dfrac{x^2+x-6}{x^2+2x-15}\times\dfrac{x-3}{x-2}$

Note: When dividing rational expression of the form $\dfrac{a}{b}\div\dfrac{c}{d}$, it may be necessary to restrict variables in expressions b, c and d.

Complex Fractions

A rational expression that contains a fraction in both the numerator and denominator is called a complex fraction. A complex fraction can be simplified by either rewriting it as a multiplication or by multiplying the numerator and denominator by the lowest common denominator.

▶ ***Example 2.3:*** Simplify using either method.

(a) $\dfrac{\frac{2}{3}-a}{\frac{1}{4}+a}$

(b) $\dfrac{3+\frac{1}{b}}{2-\frac{1}{b}}$

(c) $\dfrac{\frac{3x^2y^3}{-6xy^2}}{\frac{4x^3y}{2x^2y^2}}$

(d) $\dfrac{\frac{a^2-4}{a+3}}{\frac{2a-4}{a^2+2a-3}}$

LESSON 6.3: ADDING AND SUBTRACTING RATIONAL EXPRESSIONS

Rational expressions can be added and divided in the same way that simple fractions are added and subtracted.

$\frac{3}{5}+\frac{2}{5}-\frac{4}{5}=$

$\frac{5}{y}+\frac{4}{y}-\frac{3}{y}=$

Adding and Subtracting with common denominators

▶ ***Example 3.1:*** Simplify.

(c) $\frac{3}{x^2}+\frac{5}{x^2}-\frac{2}{x^2}=$

(d) $\frac{4x-1}{x+2}-\frac{x+3}{x+2}=$

Adding and Subtracting with whole-number denominators

To add or subtract fractions with different denominators, rewrite the fractions as equivalent fractions with a common denominator.

▶ ***Example 3.2:*** Simplify.

$\frac{3x+2}{4}+\frac{x-4}{8}-\frac{2x-1}{6}=$

Adding and Subtracting with Monomial Denominators.

▶ ***Example 3.3:*** Simplify.

(a) $\dfrac{7}{10y}-\dfrac{1}{15y^2}=$

(b) $\dfrac{2m}{n}+\dfrac{3n}{m^2}-\dfrac{2n-3}{5m}=$

Adding and Subtracting with binomial factors.

▶ ***Example 3.4:*** Simplify.

(a) $\dfrac{m}{2m-4}-\dfrac{3}{3m-6}+1=$

(b) $\dfrac{x}{6x+6}+\dfrac{5}{4x-12}=$

Adding and Subtracting with trinomial factors.

▶ ***Example 3.5:*** Simplify using either method.

$$\frac{4}{y^2+5y+6}-\frac{5}{y^2-y-12}=$$

LESSON 6.4: SOLVING RATIONAL EQUATIONS

Equations that involve rational expressions are known as ________________________ equations. They can be solved by performing the same operations on both sides.

Some equations such as $\frac{4}{x-1}=\frac{2}{x+1}$, have the variable in the denominator of one or more expressions. The value of the variable must be restricted in these cases, because division by zero is not defined. In $\frac{4}{x-1}=\frac{2}{x+1}$, x cannot equal _______ or _______.

▶ ***Example 4.1:*** Solve the following equation and check your answer. State the restrictions on the variable.

$$\frac{4}{x-1}=\frac{2}{x+1}$$

▶ ***Example 4.2:*** Solve the following equation and check your answer. State the restrictions on the variable.

$$\frac{2}{z^2-4}+\frac{10}{6z+12}=\frac{1}{z-2}$$

LESSON 6.5: EQUATIONS WITH LITERAL COEFFICIENTS

Sometimes, the numerical coefficients in an equation are replaced with letters. The result is a formula where one variable is written in terms of other variables.

▶ ***Example 5.1:*** Solve each formula for the variable indicated.

(a) $A=\frac{1}{2}bh$, b

(b) $y=mx+b$, b

(c) $ax+by=z$, y

(d) $v=u+at$, t

(e) $ax+ay=cz$, a

(f) $\frac{1}{a}+\frac{1}{b}=\frac{1}{c}$, b

LESSON 6.6: SOLVING PROBLEMS WITH EQUATIONS

Solving problems involving equations involves the following steps

1. **Determine what you are asked to find.**
2. **Determine what you are given.**
3. **Assign a variable to the unknown quantity.**
4. **Write an equation that relates the unknown quantity to any other information you are given.**
5. **Solve the equation.**
6. **Check the solution.**
7. **Write a concluding statement.**

▶ ***Example 6.1:*** Three-fifths of the students in class are boys. If there are 21 boys, how many students are in the class?

▶ ***Example 6.2:*** The sum of the reciprocals of two consecutive integers is $\frac{11}{30}$. What are the integers?

▶ ***Example 6.3:*** Two hoses together fill a pool in 2 h. If only hose A is used, the pool fills in 3 h. How long would it take to fill the pool if only hose B were used?

	Time to fill pool	Fraction filled in 1 h.
Hose A		
Hose B		
Both Hoses		

▶ ***Example 6.4:*** A boat takes 1 hr longer to go 36 km up a river than to go down the river. If the boat travels 15 km/h in still water, what is the speed of the current?

Trip	Distance (km)	Speed (km/h)	Time (h)
Upstream			
Downstream			