Prediction Markets: A Mechanism for Information Aggregation

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Background

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Markets compress our collective information about the world into prices:

For instance, if the weather in Florida worsens then the price of oranges will rise. Prediction markets attempt to leverage this property to make forecasts (about e.g. the weather).

What Is Information Aggregation?

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Theorem

Probabilistic Arrow's Trilemma: If C preserves independence and only depends on the static probabilities assigned to each event, i.e.:

$$C(P_1, P_2 \ldots P_n)(A) = G(P_1(A), P_2(A) \ldots P_n(A))$$

then C is necessarily a dictatorship.

Aumann's Agreement Theorem

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Theorem

We Can't Disagree Forever: Suppose two agents who have identical priors about an event. Each of them receive different pieces information about the event, causing their beliefs diverge. If both agents proceed to truthfully report their beliefs back and forth, then they will eventually come to an agreement.

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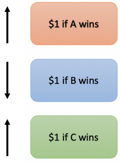

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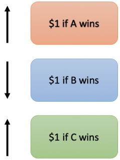

At what prices would you no longer buy nor sell?

Prediction Markets

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Contract prices will correspond to a logically consistent probability measure across markets since inconsistencies will be arbitraged away!

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In case $\neg X \wedge \neg Y$ the payoff is $p_x - p_y$.
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Arbitrage will increase the price of contract Y and decrease the price of contract X until $p_x \leq p_y$ as required by the Kolmogorov axioms.

Automated Market Makers

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If $\{1, 2...N\}$ is a disjoint partition of events and q_k net shares of contract k have been sold, then a Boltzmann AMM prices contract i as:

$$p_i(\vec{q}) = \frac{e^{q_i/b}}{\sum_{k=1}^{N} e^{q_k/b}}$$

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The market maker ALWAYS agrees to buy and sell shares at these prices.

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We can prove the worst case loss of the Boltzmann AMM is $b \cdot \ln(N)$. If the market prices are initially $\vec{p_1}$, a trader who changes the prices to $\vec{p_2}$ to reflect their beliefs expects to profit:

$$b \cdot KL(\vec{p_2} \mid\mid \vec{p_1})$$

Rational Inattention

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If R's outcome space is $\{1, 2...N\}$ and we create a market over this disjoint partition of events, we can show a trader's expected benefit from the experiment X is:

$$b \cdot I(R; X)$$

Rational Inattention

Weighing benefits against costs, our DM chooses to pay attention to X iff:

$$b \cdot I(R;X) - c \cdot I(R;X) > 0,$$

i.e.:

$$b > c$$

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Given K traders with information costs $c_1, c_2 ... c_K$, a designer can empirically calculate the c_k and set:

$$b > \max_{k \in [1,K]} c_k$$

so all participants incorporate outside information.

Questions/Future Work

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One idea is to increase b very slowly and see at what points the market's prices move/re-equilibrate.

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They could also lie to trick others into making worse reports.

Is there a way to modify our AMM to overcome these reticence and bluffing problems?

Maybe we should give traders the option to withdraw reports anytime before the market is settled.

The End

If you are interested in these problems or have any suggestions please let me know. Thanks for paying attention!