Register No.: 1 7 4 0 2 5

Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

ODD Semester (2018- 2019) Examination: MID SEM
Course Code: MCA611 Course Name: Computer Oriented Statistical Methods
Date: 24/09/2018 Time: 1.30 PM to 3.00 PM Maximum Marks: 50

INSTRUCTIONS:
1. Answer ALL questions. 2. Rough work should NOT be done anywhere on the Question Paper.

Q.1. (a) Define Random variable on a sample space. In the experiment of rolling a fair die, let $\mathcal{A}$ be the $\sigma-$Algebra associated with the sample space Ω and is given by $\mathcal{A} = \{\emptyset, \Omega, \{1,3,5\}, \{2,4,6\}\}$. Define $X : \Omega \longrightarrow \mathbb{R}$ such that [5]

$$X(\omega) = \begin{cases} 1 & \text{if } \omega \text{ is divisible by } 3 \\ 0 & \text{otherwise} \end{cases}$$

Show that X is not a random variable.

(b) A factory production line is manufacturing bolts using three machines A, B, and C. Of the total output, machine A is responsible for, 25%, machine B is for 35% and machine C for the rest. It is known from previous experience with the machine that 5% of the output from machine A, 4% from machine B, 2% form machine C are defective. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from [5]

(i) machine A
(ii) machine B
(iii) machine C.

Q.2. (a) If a discrete random variable takes on four values $-1, 0, 3, 4$ with probabilities $\frac{1}{6}, k, \frac{1}{4}, 1-6k$ respectively, where k is a constant, then find out the value of k. [5]

(b) A point is chosen at random on a line of length L. What is the probability that the ratio of the shorter to the longer segment is less that $\frac{1}{4}$. [5]

Q.3. (a) Define Cumulative Distribution Function (CDF) of a random variable. A fair coin is tossed for 3 times. Define the random variable X such that [5]

$$X(\omega) = \text{ number of heads occur in } \omega \in \Omega$$

where Ω is the associated sample space. Find and sketch the CDF, F_X of X. Pdf

(b) An information source generates symbols at random from four-letter alphabet $\{a, b, c, d\}$ with probabilities $P(a) = \frac{1}{2}$, $P(b) = \frac{1}{4}$ and $P(c) = P(d) = \frac{1}{8}$. A coding scheme encodes these symbols into binary codes as follows: [5]

$$a = 0, \quad b = 10, \quad c = 110, \quad d = 111$$

Let X be the random variable denoting the length of the code, that is, the number of binary symbols (bits)

(i) Find the Probability mass function (PMF) and CDF of X and sketch both of these.
(ii) Find the values of $P(X \leq 1)$, $P(\cancel{a}\ X \leq 2)$, $P(X > 1)$, $P(1 \leq X \leq 2)$

Q.4. (a) Suppose that X is uniformly distributed over $(0, 1)$. Find the PDF of the following random variables (i) $Y = X^2 + 1$ (ii) $Z = 1/(X + 1)$.

(b) A lab network consisting of 20 computers was attacked by a computer virus. This virus enters each computer with probability 0.4, independently of other computers

(i) Find the probability that the virus enters at least 10 computers.

(ii) A computer manager checks the lab computer, one after another, to see if they were infected by the virus. What is the probability that she has to test at least 6 computers to find the first infected one

Q.5. (a) Define memoryless property of a random variable. Show that a random variable with geometric distribution satisfies this property.

(b) If X is a Poisson variate such that $P(X = 2) = 9P(X = 4) + 90P(X = 6)$ then find the mean of the distribution.

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ODD Semester (2018- 2019)
Course Code: MCA611
Date: 16/11/2018

Examination: END SEM
Course Name: Computer Oriented Statistical Methods
Time: 9.00 AM to 12.00 PM **Maximum Marks: 100**

INSTRUCTIONS:

1. Answer ALL questions. 2. Rough work should NOT be done anywhere on the Question Paper.

Q.1. (a) A lot of 100 semiconductor chips contains 20 that are defective. Two chips are selected at random, without replacement, from the lot. [5]

(i) What is the probability that the first one selected is defective?

(ii) What is the probability that the second one selected is is defective given that the first one was defective?

(iii) What is the probability that both are defective?

(b) Let A and B be two independent events ia a sample space Ω. Show that [5]

(i) A and $\overline{B}$ are independent

(ii) $\overline{A}$ and $\overline{B}$ are independent

Q.2. (a) Suppose the number of hits a website receives in any interval is a Poisson random variable. A particular site gets on average 5 hits per seconds. [5]

(i) What is the probability that there will be none hits in an interval of two seconds?

(ii) What is the probability that there is at least one hit in an interval of one second?

(b) Consider a random variable X with possible outcomes: 0, 1, 2, ... Suppose that $P(X = j) = (1-a)a^j;\ \ j = 0, 1, 2, ..$ [5]

(i) For what values of a, the above does represent a legitimate probability distribution.

(ii) Show that for any two positive integers s and t, $P(X > s+t|X > s) = P(X \geq t)$

Q.3. When a current I flows through a resistance R, the power generated is given by $W = I^2R$. Suppose that I and R are independent random variables with PDFs respectively [10]

$$f_I(i) = \begin{cases} 6i(1-i) & \text{for } 0 \leq i \leq 1 \\ 0 & \text{otherwise} \end{cases} \quad \text{and } f_R(r) = \begin{cases} 2r & \text{for } 0 \leq r \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Determine the PDF of W.

Q.4. Suppose the joint PMF of a random vector (X, Y) is given by [10]

$$p(x_i, y_j) = \begin{cases} \frac{1}{3} & \text{if } (x_i, y_j) \in \{(0,1), (1,0), (2,1)\} \\ 0 & \text{otherwise} \end{cases}$$

(i) Are X and Y independent? Justify.

(ii) Are X and Y uncorrelated? Justify.

Q.5. (a) The CDF of the continuous random variable X is [5]

$$F_X(x) = \begin{cases} 0 & \text{for } x < -5 \\ \frac{(x+5)^2}{144} & \text{for } -5 \leq x \leq 7 \\ 1 & \text{for } x \geq 7 \end{cases}$$

(i) Find out $P(X > 4)$

(ii) Find out $P(-3 < X \leq 0)$

(iii) What is the value of a such that $P(X > a) = 2/3$

(b) The random variable X has the PDF

$$f_X(x) = \begin{cases} cx & \text{for } 0 \leq x \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

(i) Find out the value of c

(ii) Find out $P(-0 < X \leq 1)$

~~(iii) Find out $P(-0 < X \leq 1)$~~

(iv) Find out the CDF $F_X(x)$

Q.6. (a) Let X be a random variable that follows binomial distribution with $\mu = 7$ and $\sigma^2 = 6$ then find the value of p, the probability of success. [5]

(b) The average number of chocolates a nine year old child eats per month is uniformly distributed from 0.5 to 4 chocolates, inclusive. [5]

(i)Find the PDF (Probability Density Function) and CDF of the above distribution.

(ii)Find the probability that a randomly selected nine-year old child eats an average of more than two chocolates.

(iii)Find the probability that the child eats an average of more than two chocolates given that his or her amount is more than 1.5 chocolates.

Q.7. (a) The peak temperature T, as as measured in degree Fahreheit on a July day in Mangaluru is $N(85, 100)$ random variable. Find out the value of $P(T > 100)$, $P(T < 60)$, and $P(70 < T < 100)$. [5]

(b) X is a normal random variable with zero mean but unknown variance. It is also given that $P(|X| \leq 10) = 0.1$. Find the standard deviation of X. [5]

Q.8. Consider a binary communication channel. Let (X, Y) be a 2-dimensional random vector, where X is the input to the channel and Y is the output of the channel. Let $P(X = 0) = 0.5$, $P(Y = 1|X = 0) = 0.1$ and $P(Y = 0|X = 1) = 0.2$. [10]

(i) Find the joint PMF of (X, Y)

ii(i) Find the marginal PMF's of X and Y

iii(i) Are X and Y independent? Justify.

Q.9. Suppose a sample of 50 is taken from a population with standard deviation 27 and that the sample mean is 86 [10]

(i) Establish an interval estimate for the population mean that is 95.5% certain to include the true population mean.

(ii) Suppose, instead, that the sample size was 50,00. Estiblish an interval for the population mean that is 95.5% certain to include the true population mean.

(iii) which of the above would be more preferred? Justify.

Q.10. (a) A sample of 100 dry battery cells tested to find their length of life. It is found that this data are normally distributed with mean 12 hours and variance 9 hours. What percentage of battery cells are expected to have a life of [5]

(i) at least 15 hours.

(ii) at most 6 hours.

(iii) more than 10 hours and less than 14 hours.

(b) In an electric generating plant, the manager wanted to estimate the coal needed for the current year, and he took a sample by measuring coal usage for 10 weeks. The sample data for sample size 10 weeks are the sample mean $\overline{X} = 11,400$ tons and the sample standard deviation $S = 700$ tons. Find out the confidence interval of 95% confidence level of this statistics. [5]

Standard Normal Tables

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998