

Preuve

April 11, 2017

Abstract

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$$\begin{aligned} [\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j &= [\nabla_x \frac{1}{2} (Ax - b) \cdot (Ax - b)]_j \\ [\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j &= [\nabla_x \frac{1}{2} (\sum_{i=1}^n (\sum_{k=1}^p A_{i,k} x_k)^2 - 2b_i (\sum_{k=1}^p A_{i,k} x_k) + b_i^2)]_j \\ [\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j &= \frac{\partial \frac{1}{2} (\sum_{i=1}^n (\sum_{k=1}^p A_{i,k} x_k)^2 - 2b_i (\sum_{k=1}^p A_{i,k} x_k) + b_i^2)}{\partial x_j} \\ [\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j &= \frac{1}{2} (\sum_{i=1}^n 2 (\sum_{k=1}^p A_{i,k} x_k) A_{i,j} - 2b_i A_{i,j}) \\ [\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j &= (\sum_{i=1}^n (\sum_{k=1}^p A_{i,k} x_k) A_{i,j} - b_i A_{i,j}) \\ [\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j &= (\sum_{i=1}^n A_{i,j} (\sum_{k=1}^p A_{i,k} x_k) - b_i) \\ [\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j &= \sum_{i=1}^n A_{j,i}^T [Ax - b]_i \\ [\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j &= [A^T (Ax - b)]_j \end{aligned}$$

$$\begin{aligned} [\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j &= [\nabla_x \frac{1}{2} (Ax - b) \cdot (Ax - b)]_j \\ [\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j &= [\nabla_x \frac{1}{2} (\sum_{i=1}^n (\sum_{k=1}^p A_{i,k} x_k)^2 - 2b_i (\sum_{k=1}^p A_{i,k} x_k) + b_i^2)]_j \\ [\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j &= \frac{\partial \frac{1}{2} (\sum_{i=1}^n (\sum_{k=1}^p A_{i,k} x_k)^2 - 2b_i (\sum_{k=1}^p A_{i,k} x_k) + b_i^2)}{\partial x_j} \\ [\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j &= \frac{1}{2} (\sum_{i=1}^n 2 (\sum_{k=1}^p A_{i,k} x_k) A_{i,j} - 2b_i A_{i,j}) \end{aligned}$$

$$[\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j = (\sum_{i=1}^n (\sum_{k=1}^p A_{i,k} x_k) A_{i,j} - b_i A_{i,j})$$

$$[\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j = (\sum_{i=1}^n A_{i,j} (\sum_{k=1}^p A_{i,k} x_k) - b_i)$$

$$[\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j = \sum_{i=1}^n A_{j,i}^T [Ax - b]_i$$

$$[\nabla_x \frac{1}{2} \|Ax - b\|_2^2]_j = [A^T (Ax - b)]_j$$