THE MAGNITUDE-PHASE REPRESENTATION OF THE FOURIER TRANSFORM

STUDY MATERIAL

- -The magnitude-phase representation of the continuous-time Fourier transform X(jw) is:

$$X(jw) = |X(jw)e^{j<X(jw)}|$$

- -Similarly,the magnitude-phase representation of the discrete-time Fourier transformX(e^{jw}) is:

$$X(e^{jw}) = |X(e^{jw})|e^{j<X(e^{jw})}$$

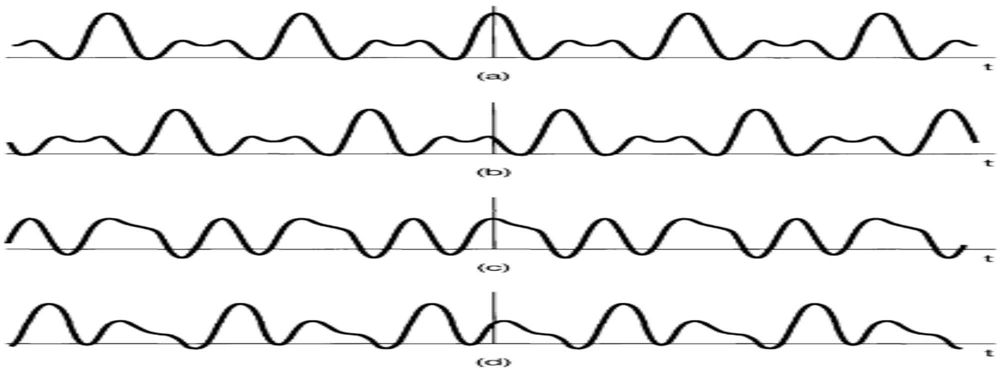
-We can think of X(jw) as providing us with a decomposition of the signal x(t) into a "sum" of complex exponentials at different frequencies.$|X(jw)|^2$may be interpreted as the energy-density spectrum of x(t). That is, $|X(jw)|^2$dw/2*Pie can be thought of as the amount of energy in the signal x(t) that lies in the infinitesimal frequency band between w and w + dw. Thus, the magnitude IX(jw)I describes the basic frequency content of a signal-i.e., IX(jw)I provides us with the information about the relative magnitudes of the complex exponentials that make up x(t). For example, if IX(jw)I = 0 outside of a small band of frequencies centered at zero, then x(t) will display only relatively low-frequency oscillations.

-The phase angle <X(jw), on the other hand, does not affect the amplitudes of the individual frequency components, but instead provides us with information concerning the relative phases of these exponentials. The phase relationships captured by <X(jw) have a significant effect on the nature of the signal x(t) and thus typically contain a substantial amount of information about the signal. In particular, depending upon what this phase function is, we can obtain very different-looking signals, even if the magnitude function remains unchanged.

For Example:

A ship encounters the superposition of three wave trains, each of which can be modeled as a sinusoidal signal. With fixed magnitudes for these sinusoids, the amplitude of their sum may be quite small or very large, depending on the relative phases. The implications of phase for the ship, therefore, are quite significant. As another illustration of the effect of phase, consider the signal

$$X(t) = 1 + 1/2 * cos(2 * pie * t + phie1) + cos(4 * pie * t + phie2) + 2/3 * cos(6 * pie * t + phie3)$$



(a) phie1 =phie2 = phie3 = 0
(b)phie1 = 4 rad, phie2 = 8 rad, phie3 = 12 rad
(c) phie1 = 6 rad, phie2 = -2.7 rad,phie3 =0.93 rad;
(d) phie1= 1.2 rad, phie2 = 4.1 rad, phie3 = -7.02 rad.

CHANGES IN THE PHASE FUNCTION:

In general, changes in the phase function of X(jw) lead to changes in the time domain characteristics of the signal x(t). In some instances phase distortion may be important, whereas in others it is not.

Example-1, a well-known property of the auditory system is a relative insensitivity to phase. Specifically, if the Fourier transform of a spoken sound (e.g., a vowel) is subjected to a distortion such that the phase is changed but the magnitude is unchanged, the effect can be perceptually negligible, although the waveform in the time domain may look considerably different. While mild phase distortions such as those affecting individual sounds do not lead to a loss of intelligibility, more severe phase distortions of speech certainly do. As an extreme illustration, if x(t) is a tape recording of a sentence, then the signal x(-t) represents the sentence played backward.

$$F\{x(-t)\} - X(-jw) = |X(jw)|e^{-j<X(jw)}$$

Clearly, this phase change has a significant impact on the intelligibility of the recording.

Example-2:A second example illustrating the effect and importance of phase is found in examining images.

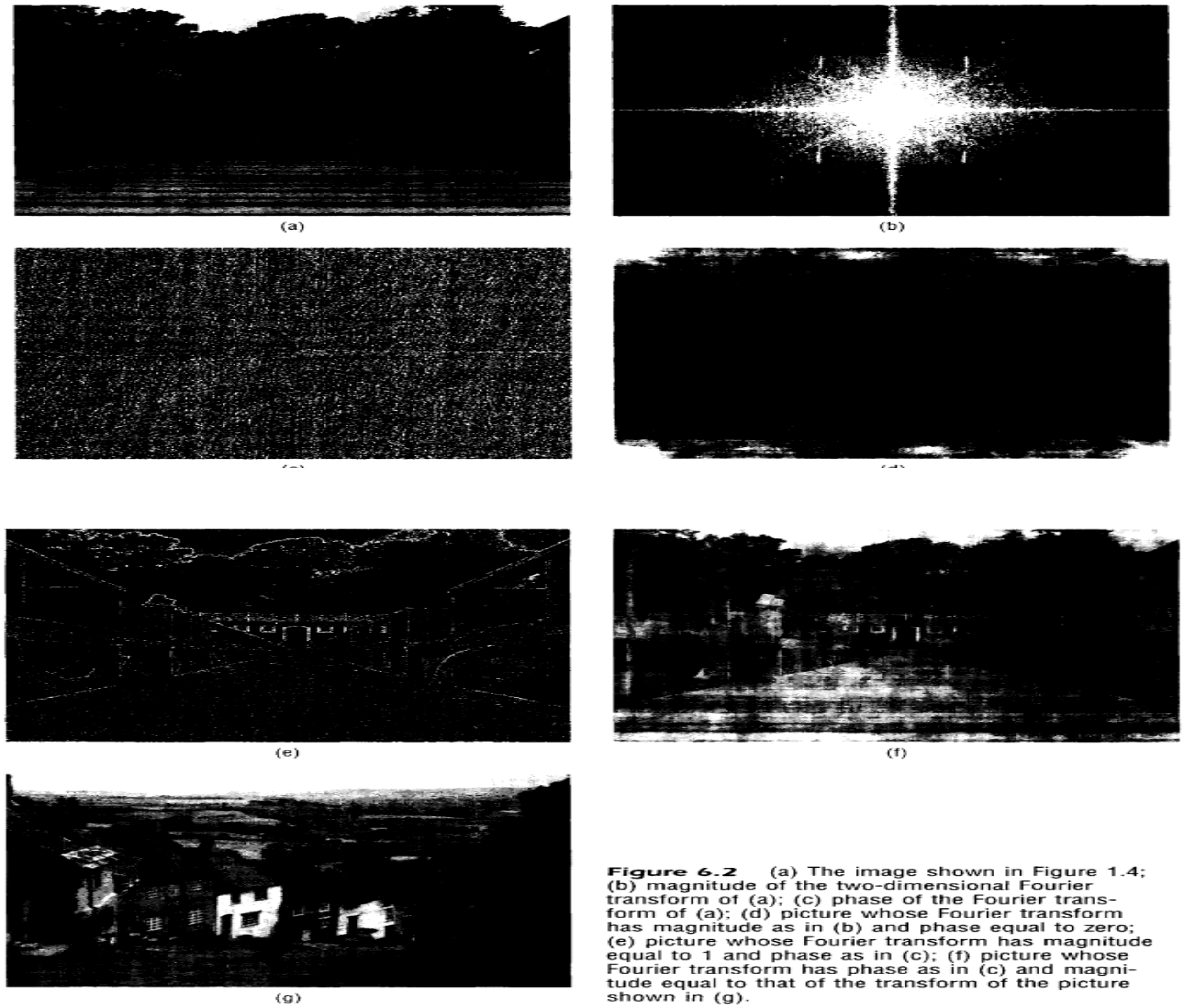


Figure 6.2 (a) The image shown in Figure 1.4; (b) magnitude of the two-dimensional Fourier transform of (a); (c) phase of the Fourier transform of (a); (d) picture whose Fourier transform has magnitude as in (b) and phase equal to zero; (e) picture whose Fourier transform has magnitude equal to 1 and phase as in (c); (f) picture whose Fourier transform has phase as in (c) and magnitude equal to that of the transform of the picture shown in (g).

In viewing a picture, some of the most important visual information is contained in the edges and regions of high contrast. Intuitively, regions of maximum and minimum intensity in a picture are places at which complex exponentials at different frequencies are in phase. Therefore, it seems plausible to expect the phase of the Fourier transform of a picture to contain much of the information in the picture, and in particular, the phase should capture the information about the edges.

1. Figure 6.2(a)we have repeated the picture shown in Figure 1.4.
2. Figure 6.2(b) we have depicted the magnitude of the two-dimensional Fourier transform of the image in Figure 6.2(a), where in this image the horizontal axis is w 1, the vertical is w 2, and the brightness of the image at the point (w 1, w2) is proportional to the magnitude of the transform X(jw 1, jw2) of the image in Figure 6.2(a). Similarly, the phase of this transform is depicted in Figure 6.2(c).
3. Figure 6.2(d) is the result of setting the phase [Figure 6.2(c)] of X(jw 1, jw2) to zero (with-out changing its magnitude) and inverse transforming. In Figure 6.2(e) the magnitude of X(jw 1, jw2) was set equal to 1, but the phase was kept unchanged from what it was in Figure 6.2(c). Finally, in Figure 6.2(f) we have depicted the image obtained

by inverse transforming the function obtained by using the phase in Figure 6.2(c) and the magnitude of the transform of a completely different image-the picture shown in Figure 6.2(g) These figures clearly illustrate the importance of phase in representing images.

THE MAGNITUDE-PHASE REPRESENTATION OF THE FREQUENCY RESPONSE OF LTI SYSTEMS

-The effect that an LTI system has on the input is to change the complex amplitude of each of the frequency components of the signal.The effect that an LTI system has on the input is to change the complex amplitude of each of the frequency components of the signal.

$$|y(jw)| = |H(jw)||X(jw)|$$
$$< Y(jw) =< H(jw) +< X(jw)$$

-The effect an LTI system has on the magnitude of the Fourier transform of the signal is to scale it by the magnitude of the frequency response.For this reason, |H(jw)| (or|H(e^{jw})|) is commonly referred to as the gain of the system.The phase shift of the system can change the relative phase relationships among the components of the input, possibly resulting in significant modifications to the time domain characteristics of the input even when the gain of the system is constant for all frequencies.

LINEAR & NON LINEAR PHASE

-When the phase shift at the frequency w is a linear function of w, there is a particularly straightforward interpretation of the effect in the time domain. Consider the continuous time LTI system with frequency response:

$$H(jw) = e^{-jwto}$$

so that the system has unit gain and linear phase

$$|H(jw)| = 1 \; \& \; < H(jw) =- wto$$

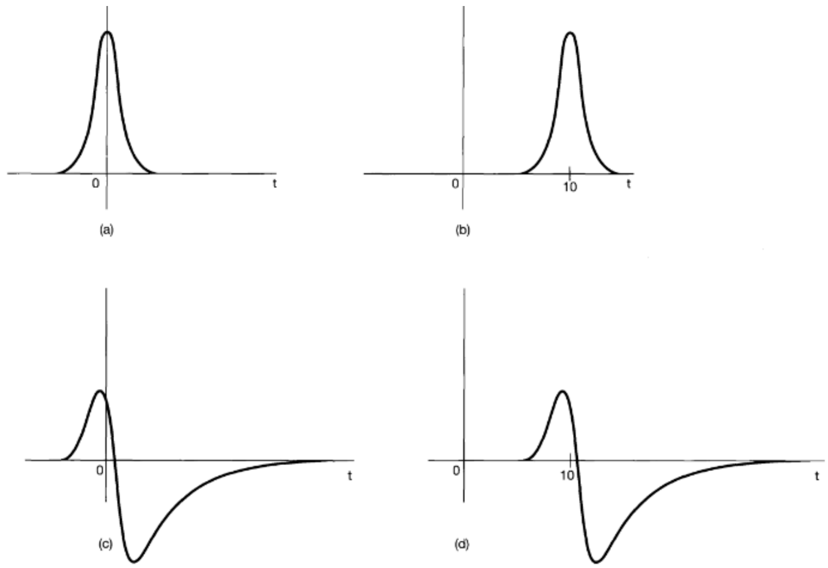


Figure 6.3 (a) Continuous-time signal that is applied as the input to several systems for which the frequency response has unity magnitude; (b) response for a system with linear phase; (c) response for a system with nonlinear phase; and (d) response for a system with phase equal to the nonlinear phase of the system in part (c) plus a linear phase term.

1. Figure 6.3(a), we depict a signal that is applied as the input to three different systems.
2. Figure 6.3(b) shows the output when the signal is applied as input to a system with frequency response H1(jw) = e^{-jwto}, resulting in an output that equals the input delayed by two seconds.
3. Figure 6.3(c), we display the output when the signal is applied to a system with unity gain and nonlinear phase function-i.e., $H2(jw)$=$e^{j<H2(jw)}$,where <H2(jw) is a nonlinear function of w.
4. Figure 6.3(d) shows the output from another system with nonlinear phase.

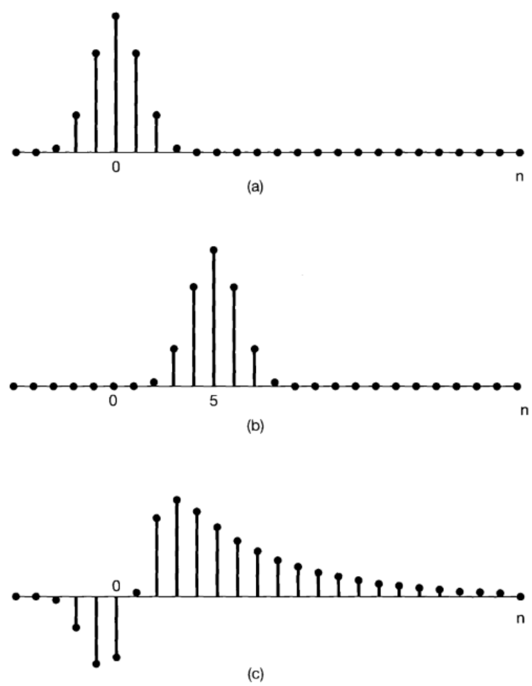
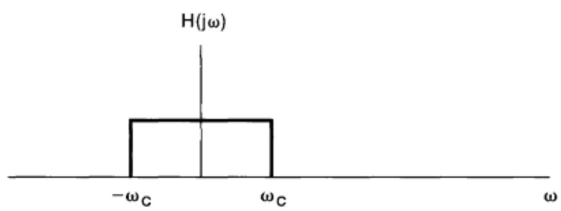


Figure 6.4, we illustrate the effect of both linear and nonlinear phase in the discrete-time case.
(a) Discrete-time signal that is applied as input to several systems for which the frequency response has unity magnitude.
(b) Response for a system with linear phase with slope of -5.
(c) Response for a system with nonlinear phase.

TIME-DOMAIN PROPERTIES OF IDEAL FREQUENCY-SELECTIVE FILTERS

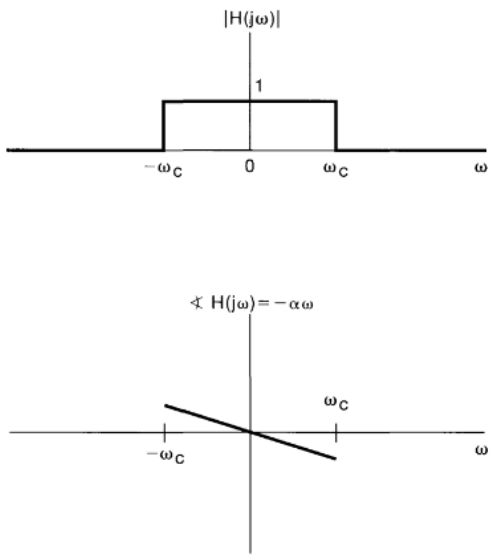
-We introduced the class of frequency-selective filters, i.e LTI systems with frequency responses chosen so as to pass one or several bands of frequencies with little or no attenuation and to stop or significantly attenuate frequencies outside those bands.In this section, we take another look at such filters and their properties. We focus our attention here on lowpass filters, although very similar concepts and results hold for other types of frequency-selective filters such as highpass or bandpass filters.

-A continuous-time ideal low pass filter has a frequency response of the form



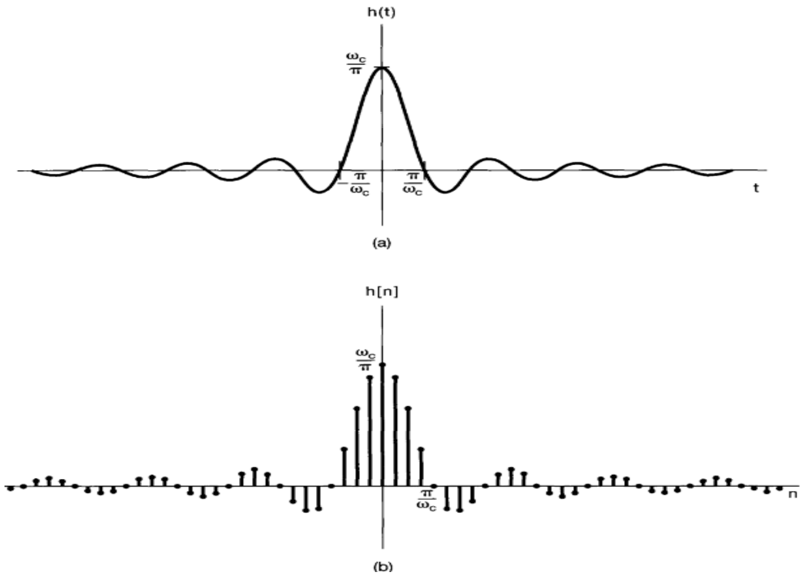
H(jw)=1,|W|<=Wc & H(jw)=0,wc<|w|<=pie
Ideal low pass filters have perfect frequency selectivity.That is, they pass without attenuation all frequencies at or lower than the cutoff frequency wc and completely stop all frequencies in the stopband (i.e., higher than wc). Moreover,these filters have zero phase characteristics, so they introduce no phase distortion.

-Nonlinear phase characteristics can lead to significant changes in the time-domain characteristics of a signal even when the magnitude of its spectrum is not changed by the system, and thus, a filter with a magnitude characteristic but with nonlinear phase, might produce undesirable effects in some applications. On the other hand, an ideal filter with linear phase over the passband, introduces only a simple time shift relative to the response of the ideal low pass filter with zero phase characteristic.

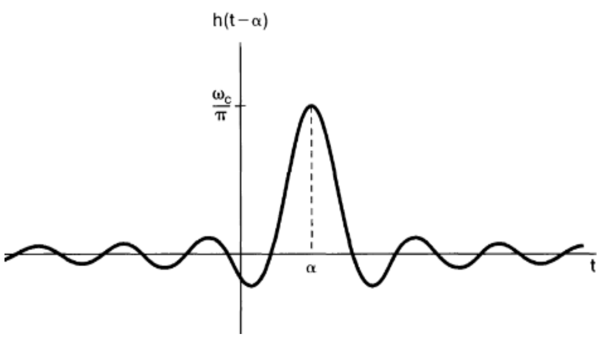


1. Figure 6. 11 Continuous-time ideal lowpass filter with linear phase characteristic.

-If either of the ideal frequency responses is augmented with a linear phase characteristic, the impulse response is simply delayed by an amount equal to the negative of the slope of this phase function,for the continuous-time impulse response. Note that in both continuous and discrete time, the width of the filter passband is proportional to we, while the width of the main lobe of the impulse is proportional to 1/we. As the bandwidth of the filter increases, the impulse response becomes narrower, and vice versa, consistent with the inverse relationship between time and frequency.

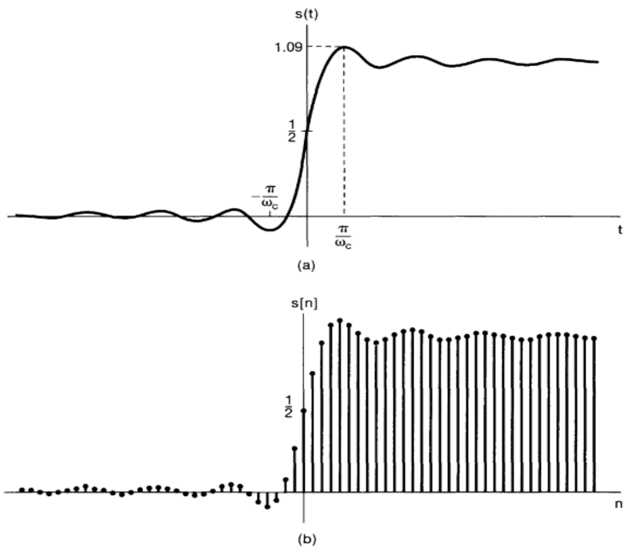


1. The impulse response of the continuous-time ideal lowpass filter
2. the impulse response of the discrete-time ideal lowpass filter of with we = 7r/4.



1. Impulse response of an ideal lowpass filter with magnitude and phase shown in Figure 6.11.

-The step responses s(t) and s[n] of the ideal low pass filters in continuous time and discrete time are displayed and In both cases, we note that the step responses exhibit several characteristics that may not be desirable. In particular, for these filters, the step responses overshoot their long-term final values and exhibit oscillatory behavior, frequently referred to as ringing.



1. a:> Step response of a continuous time ideal low pass filter,
2. b:> Step response of a discrete time ideal low pass filter