C for Science - 2011 - Practical Exercise #4

The three roots of the *reduced* cubic equation with real coefficients a_2, a_1 and a_0:

$$x^3 + a_2 x^2 + a_1 x + a_0 = 0 \tag{1}$$

can either all be real, or can consist of a complex conjugate pair of roots and one real root. To determine which case our cubic belongs to we first compute the quantities q and r:

$$q = \frac{a_2^2 - 3a_1}{9}, \quad r = \frac{2a_2^3 - 9a_1 a_2 + 27a_0}{54}.$$

- If $q^3 - r^2 < 0$ then two of the roots are complex. The real root in this case is given by:

$$r_{\text{real}} = -\frac{|r|}{r}\left[\left(\sqrt{r^2 - q^3} + |r|\right)^{\frac{1}{3}} + \frac{q}{\left(\sqrt{r^2 - q^3} + |r|\right)^{\frac{1}{3}}}\right] - \frac{a_2}{3}.$$

- Conversely, if $q^3 - r^2 \geq 0$ then all three roots are real. One of the roots is given by:

$$r_{\text{real}} = -2\sqrt{q}\cos\left(\frac{1}{3}\cos^{-1}(rq^{-\frac{3}{2}})\right) - \frac{a_2}{3}.$$

1. Write a C function to find the roots of a reduced cubic polynomial that
   (a) Determines whether there are 1 or 3 real roots,
   (b) Finds a single real root using the appropriate formula given above,
   (c) Calculates the three coefficients of the quadratic obtained from dividing (1) by $(x - r_{\text{real}})$. *Hint*: the quadratic is of the form:

$$x^2 + (a_2 + r_{\text{real}})x - \frac{a_0}{r_{\text{real}}} = 0. \tag{2}$$

   (d) Calculate the two remaining roots by solving (2) with the `quad_sol` function you developed in a previous exercise. (You will need to ensure that all the cases are dealt with correctly!).
2. Write a C `main` function to:
   (a) Prompt for the three coefficients of the reduced cubic equation.
   (b) Call the cubic polynomial solver you have written.
   (c) Print out all the roots and their properties.
3. Test your program on the following cubic equations:
   (a) $x^3 - 6x^2 + 11x - 6 = 0$
   (b) $x^3 - 4x^2 - 13x - 56 = 0$
   (c) $x^3 + 4x^2 + 8x + 8 = 0$

(You will need to use `cos(x)` and `acos(x)` ($=\cos^{-1}(x)$) from `<math.h>`).