C for Science - Practical Exercise #4

1. Starting with the code provided in the lecture for an implementation of the Sieve of Eratosthenes, create a program to count the number of prime numbers below a given value. You will need to:

   (a) Start a project with the given functions (careful of the '~' symbol when copy/pasting).

   (b) Add a `main` function to prompt for the max value and call `findPrimes` and `printPrimes`.

   (c) Thoroughly read the given bit-wise manipulations of data to understand the algorithm.

   (d) Add a new function to count and return the number of primes that have been found.

   Count the prime numbers between 2 and $1,000,000,000$.

2. By using the `time.h` C library, one can time how long it takes to perform a calculation. Adjust your `main` function for the previous question in the following ways:

   (a) Add to your `.c` file:
   ```
   #include <time.h>
   ```

   (b) Declare the variable:
   ```
   float ticks;
   ```

   (c) Before `findPrimes` is called, insert:
   ```
   ticks = clock();
   ```

   (d) After `findPrimes` is called, use the following as the number of seconds elapsed (of type `float`):
   ```
   (clock() - ticks)/CLOCKS_PER_SEC
   ```

   Time how long it takes to calculate the primes between 2 and $1,000,000,000$, in "Debug"(default) and "Release"(Optimised) modes. Is there a difference?

[P.T.O.]

3. The three roots of the *reduced* cubic equation with real coefficients a_2, a_1 and a_0:

$$x^3 + a_2 x^2 + a_1 x + a_0 = 0 \tag{1}$$

can either all be real, or can consist of a complex conjugate pair of roots and one real root. To determine which case our cubic belongs to we first compute the quantities q and r:

$$q = \frac{a_2^2 - 3a_1}{9}, \qquad r = \frac{2a_2^3 - 9a_1 a_2 + 27a_0}{54}.$$

- If $q^3 - r^2 < 0$ then two of the roots are complex. The real root in this case is given by:

$$r_{\text{real}} = -\frac{|r|}{r}\left[\left(\sqrt{r^2 - q^3} + |r|\right)^{\frac{1}{3}} + \frac{q}{\left(\sqrt{r^2 - q^3} + |r|\right)^{\frac{1}{3}}}\right] - \frac{a_2}{3}.$$

- Conversely, if $q^3 - r^2 \geq 0$ then all three roots are real. One of the roots is given by:

$$r_{\text{real}} = -2\sqrt{q}\cos\left(\frac{1}{3}\cos^{-1}(rq^{-\frac{3}{2}})\right) - \frac{a_2}{3}.$$

Write a C function to find the roots of a reduced cubic polynomial that

(a) Determines whether there are 1 or 3 real roots,

(b) Finds a single real root using the appropriate formula given above,

(c) Calculates the three coefficients of the quadratic obtained from dividing (1) by $(x - r_{\text{real}})$. Rather than programming a synthetic division algorithm for the polynomial (unless you want to), you can use the quadratic:

$$x^2 + (a_2 + r_{\text{real}})x - \frac{a_0}{r_{\text{real}}} = 0. \tag{2}$$

(d) Calculate the two remaining roots by solving (2) with the `quad_sol` function you developed in a previous exercise. (You will need to ensure that all the cases are dealt with correctly!).

4. Write a C `main` function to:

(a) Prompt for the three coefficients of the reduced cubic equation.

(b) Call the cubic polynomial solver you have written.

(c) Print out all the roots and their properties.

5. Test your program on the following cubic equations:

(a) $x^3 - 6x^2 + 11x - 6 = 0$

(b) $x^3 - 4x^2 - 13x - 56 = 0$

(c) $x^3 + 4x^2 + 8x + 8 = 0$

(d) $x^3 + 6x^2 + 12x + 8 = 0$

(You will need to use `cos(x)` and `acos(x)` (=$\cos^{-1}(x)$) from `<math.h>`).