# Position: Don't Use the CLT in LLM Evals With Fewer Than a Few Hundred Datapoints (...it's *really* easy to do *a lot* better!)

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## 1. Failures of the Central Limit Theorem

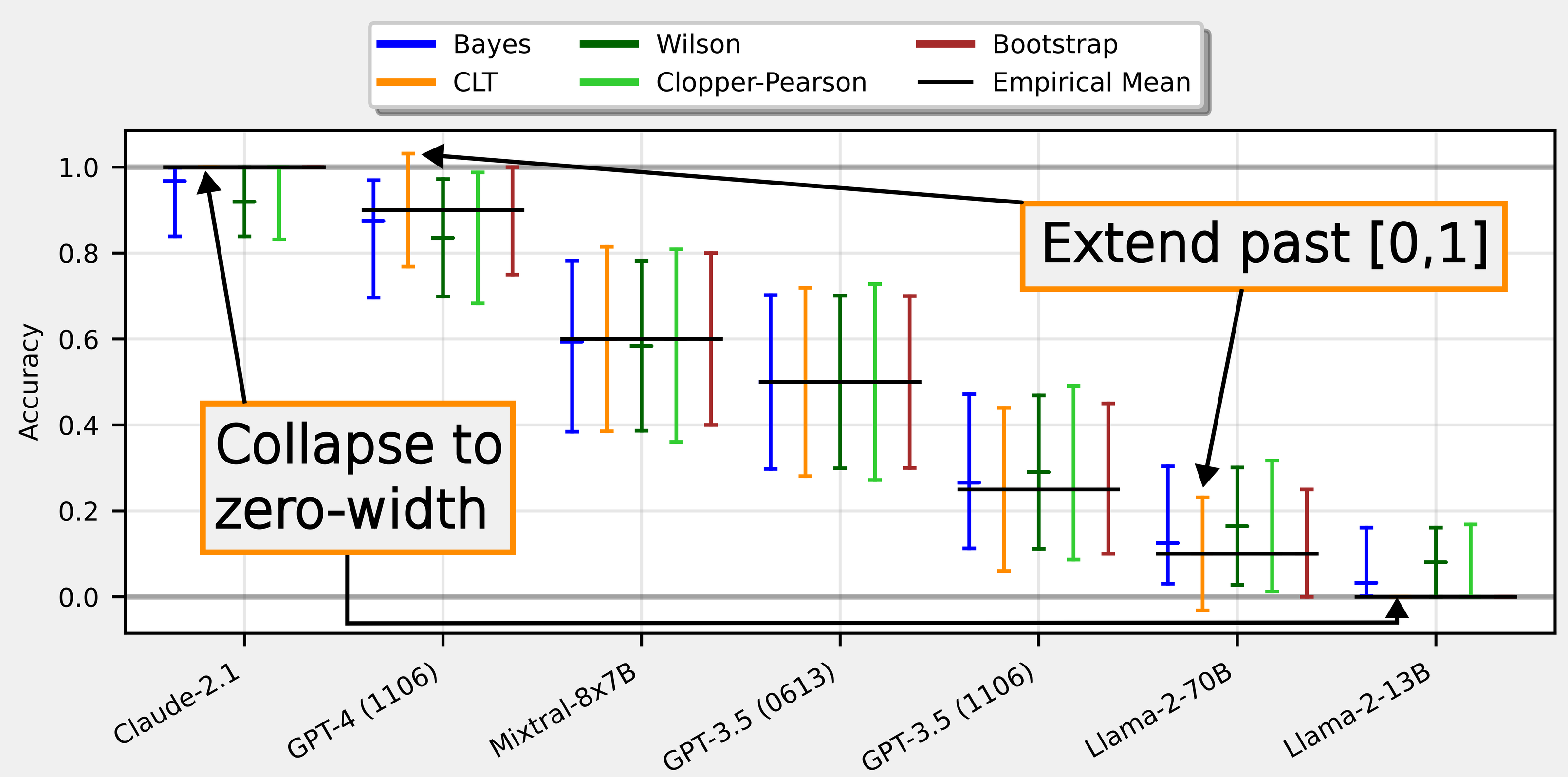
**Central Limit Theorem:** If $X_1, \dots, X_N$ are IID r.v.s with mean $\mu \in \mathbb{R}$ and finite variance $\sigma^2$, then with sample mean $\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} X_i$,

$$\sqrt{N}(\hat{\mu} - \mu) \xrightarrow{d} \mathcal{N}\left(0, \sigma^2\right) \text{ as } N \to \infty.$$

**CLT-based confidence interval** on binary data $y_i \in \{0, 1\}$ (incorrect/correct) for $i = 1, \dots, N$, with standard error $\sqrt{\bar{y}(1 - \bar{y})/N}$:

$$\mathrm{CI}_{1-\alpha} = \bar{y} \pm z_{\alpha/2}\mathrm{SE}(\bar{y}),$$

where $z_{\alpha/2}$ is the $(1 - \alpha/2)$-th quantile of $\mathcal{N}(0, 1)$.




As LLM capabilities improve, it's becoming more common to run small-$N$ benchmarks, e.g. the Langchain Typewriter tool-use benchmark shown above ($N = 20$).

## 2. Simulation Setup

1. **Synthetic datasets**: $N$ samples $y_i \sim \mathrm{Ber}(\theta)$, $\theta \sim \mathrm{Uniform}[0, 1]$.
2. **Construct intervals with various confidence levels** $1 - \alpha \in [0.8, 0.995]$.
3. **Repeat the above**, and compare different interval-construction methods via **coverage** (proportion of intervals that contain true value of $\theta$; should equal $1 - \alpha$) and **interval width**.

## 3. Alternative Interval Construction

**Bayesian Beta-Bernoulli credible interval** — uniform prior over probability of success $\theta$:

$$\theta \sim \mathrm{Beta}(1, 1) = \mathrm{Uniform}[0, 1]$$
$$y_i \sim \mathrm{Bernoulli}(\theta) \text{ for } i = 1, \dots N$$

Use quantiles of closed-form posterior to construct $1 - \alpha$ CIs:

$$\theta \mid y_{1:N} \sim \mathrm{Beta}\left(1 + \sum_{i=1}^{N} y_i, 1 + N - \sum_{i=1}^{N} y_i\right)$$

Beta-Bernoulli Bayesian Credible Interval

```
posterior = scipy.stats.beta(1 + sum(y), 1 + N - sum(y))
bayes_ci  = posterior.interval(confidence=0.95)
```

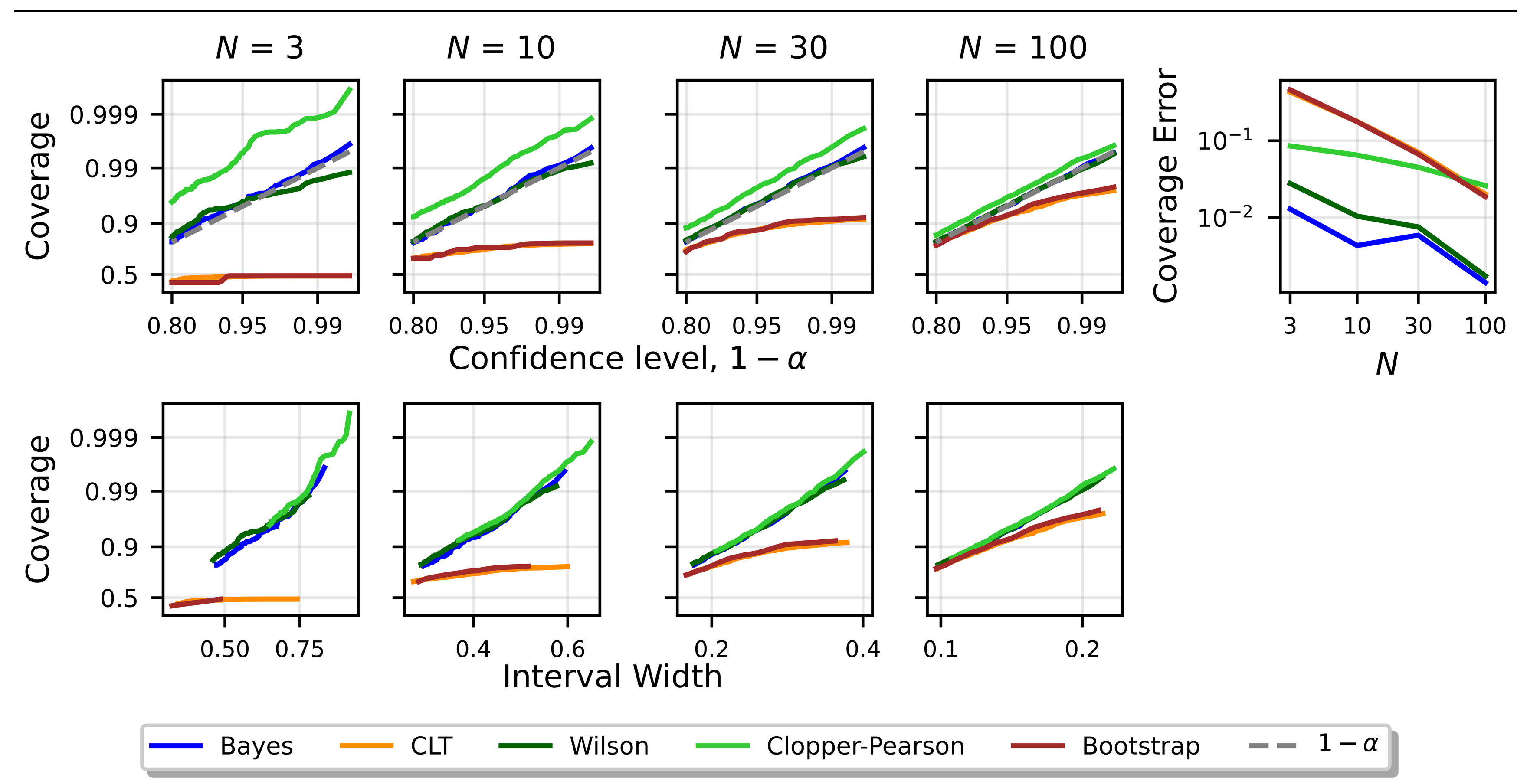
**Wilson Score confidence interval** — based on binomial distribution:

$$\mathrm{CI}_{1-\alpha,\mathrm{Wilson}} = \frac{\hat{\theta} + \frac{z^2_{\alpha/2}}{2N}}{1 + \frac{z^2_{\alpha/2}}{N}} \pm \frac{\frac{z_{\alpha/2}}{2N}}{1 + \frac{z^2_{\alpha/2}}{N}} \sqrt{4N\hat{\theta}(1 - \hat{\theta}) + z^2_{\alpha/2}}.$$

Wilson Score Confidence Interval

```
result = scipy.stats.binomtest(k=sum(y), n=N)
wilson_ci = result.proportion_ci("wilson", 0.95)
```

## 4. Results



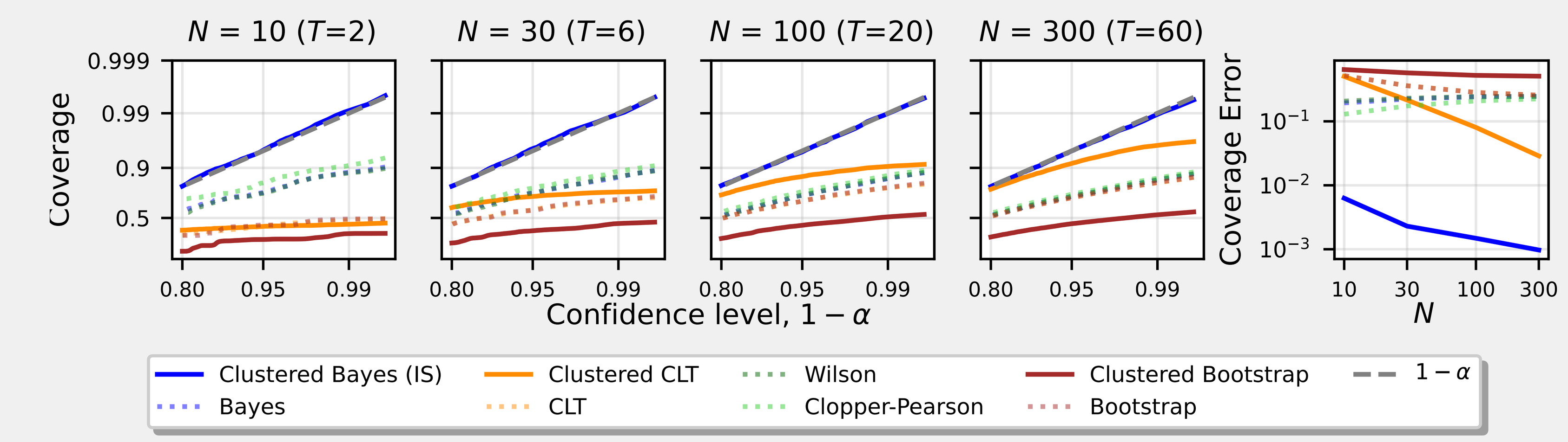

## 5. Other Settings

- **Clustered Questions** — $T$ tasks, each with $N_t$ IID questions. Clustered CLT uses a different standard error formulation:

$$\mathrm{SE}_{\text{clust.}} = \sqrt{\mathrm{SE}^2_{\mathrm{CLT}} + \frac{1}{N^2}\sum_{t=1}^{T}\sum_{i=1}^{N_t}\sum_{j \neq i}(y_{i,t} - \bar{y})(y_{j,t} - \bar{y})}.$$

Bayesian credible interval found via importance sampling:

$$\theta \sim \mathrm{Beta}(1, 1), \qquad d \sim \mathrm{Gamma}(1, 1),$$
$$\theta_t \sim \mathrm{Beta}(d\theta, d(1 - \theta)), \qquad y_{i,}\ t \sim \mathrm{Bernoulli}(\theta_t).$$




- **Independent Comparisons** — Compare $\theta_A$ and $\theta_B$ for two different models, with access *only* to $N_A, N_B, \bar{y}_A$, and $\bar{y}_B$.
- **Paired Comparisons** — Compare $\theta_A$ and $\theta_B$ for two different models, each with the same $N$ IID questions and access to question-level successes $\{y_{A;i}\}_{i=1}^{N}$ and $\{y_{B;i}\}_{i=1}^{N}$.
- **Metrics that aren't simple averages** — e.g. F1 score.

We construct Bayesian credible intervals that outperform CLT-based intervals in all settings (available in `bayes_evals`).

## 6. Recommendations

**IID setting:** use **Beta-Bernoulli** credible intervals or **Wilson score** confidence intervals via `scipy` or equivalent.

**Other settings:** use **Bayesian credible intervals** as implemented in our simple package `bayes_evals`.