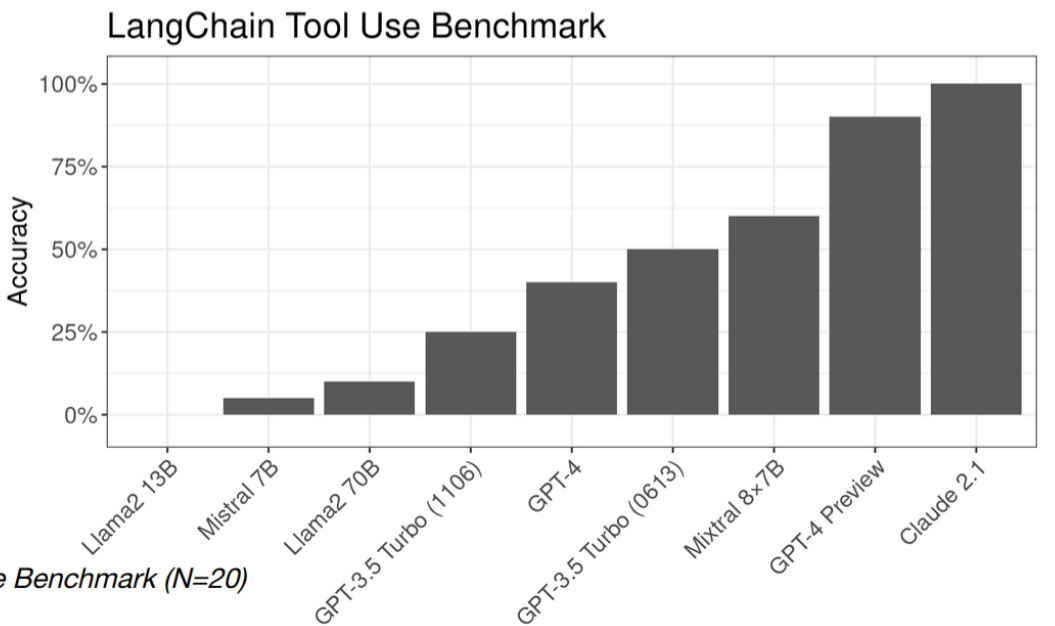
Position: Don’t Use the CLT in LLM Evals With Fewer Than a Few Hundred Datapoints

Sam Bowyer, Laurence Aitchison, and Desi R. Ivanova



ICML 2025 Spotlight Position Paper

Motivation



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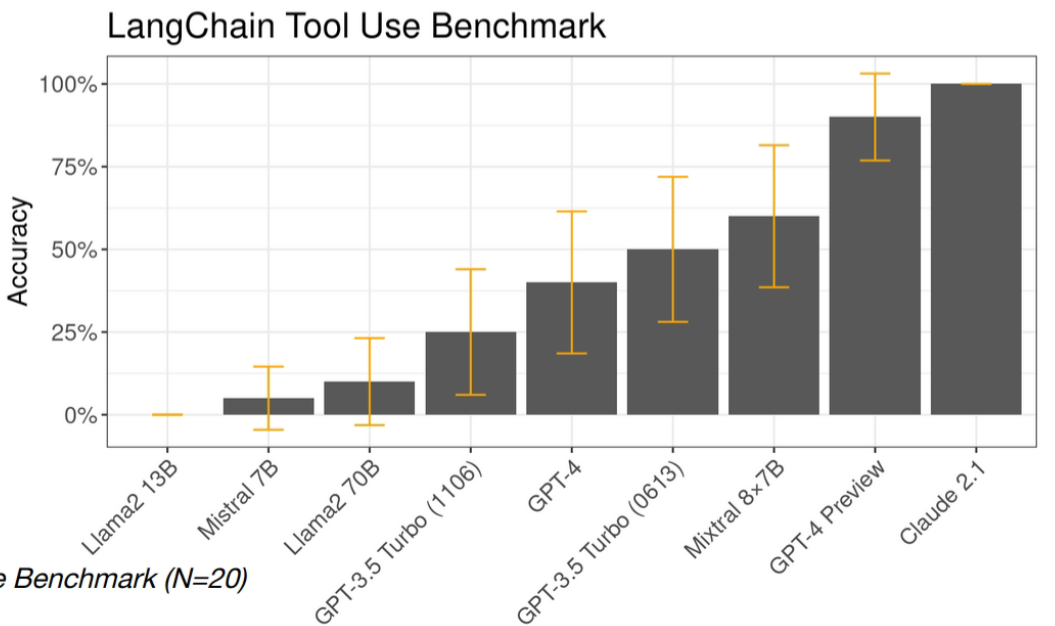

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- Error bars are important for interpreting evals.
- The CLT is the most common method for computing error bars, but it's often unwise.

CLT-based CI at confidence level $1-\alpha$ for binary data $X_i \sim \text{Bernoulli}(\theta)$:

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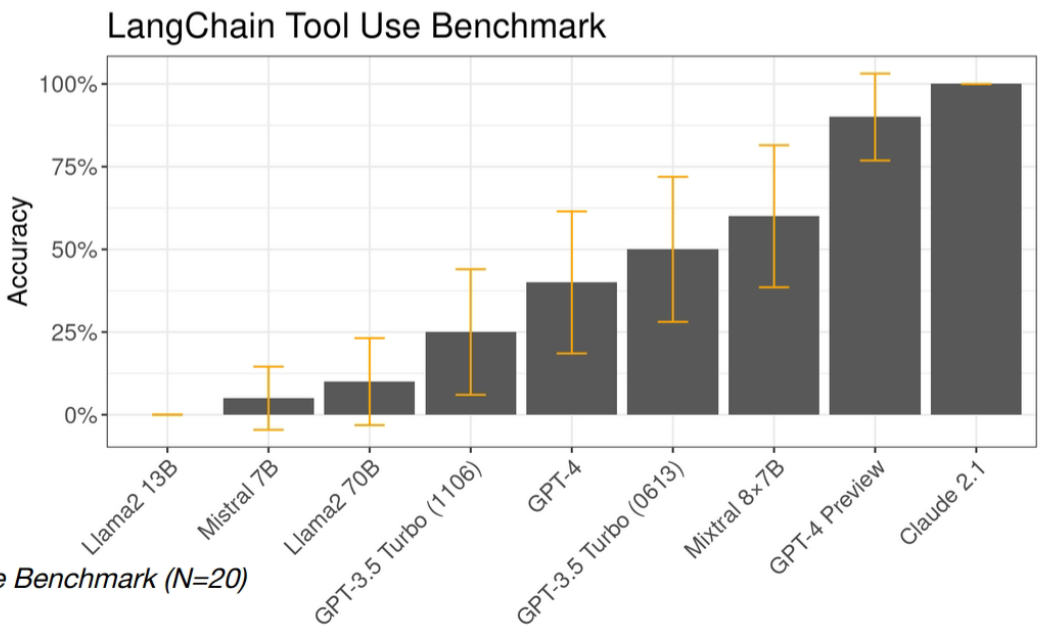

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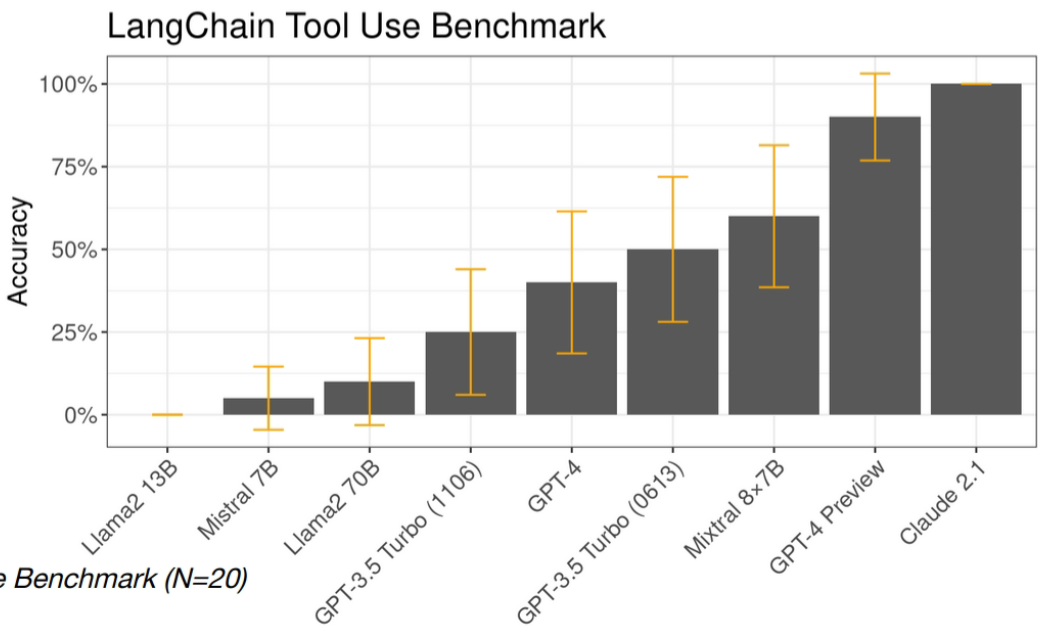

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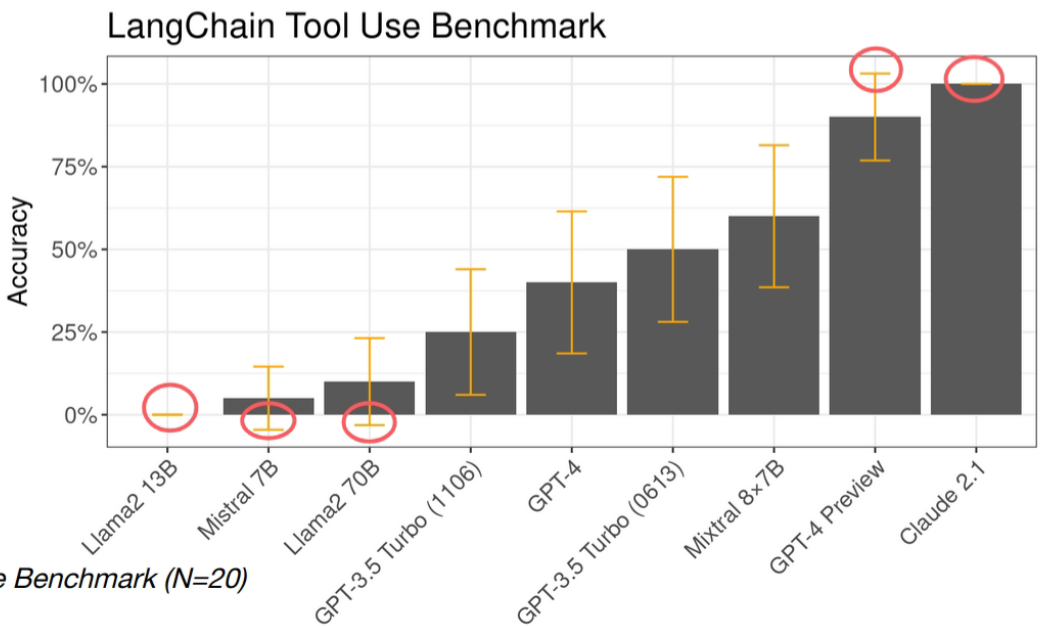

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Central Limit Theorem (CLT)

If $X_1, \ldots, X_N$ are IID r.v.s with mean $\mu \in \mathbb{R}$ and finite variance σ^2, then

$$\sqrt{N}(\hat{\mu} - \mu) \xrightarrow{d} \mathcal{N}\left(0, \sigma^2\right) \text{ as } N \to \infty,$$

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- As LLM capabilitites improve, constructing and running benchmarks is becoming more time-intensive.
- Researchers are increasingly using benchmarks with smaller N.
- We need alternative methods for computing error bars that work for both large *and* small N.

Alternative #1 – Beta-Binomial Model

Treat the data as IID Bernoulli with a uniform prior on the parameter θ.

$$\theta \sim \text{Beta}(1, 1) = \text{Uniform}[0, 1]$$
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```
# y is a length N binary "eval" vector
S, N = y.sum(), len(y) # total successes & questions

# Bayesian Credible interval
posterior = scipy.stats.beta(1+S, 1+(N-S))
bayes_ci = posterior.interval(confidence=0.95)
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```
# y is a length N binary "eval" vector
S, N = y.sum(), len(y) # total successes & questions
result = scipy.stats.binomtest(k=S, n=N)

# 95% Wilson score interval and Clopper-Pearson exact interval
wilson_ci = result.proportion_ci("wilson", 0.95)
cp_ci = result.proportion_ci("exact", 0.95)
```

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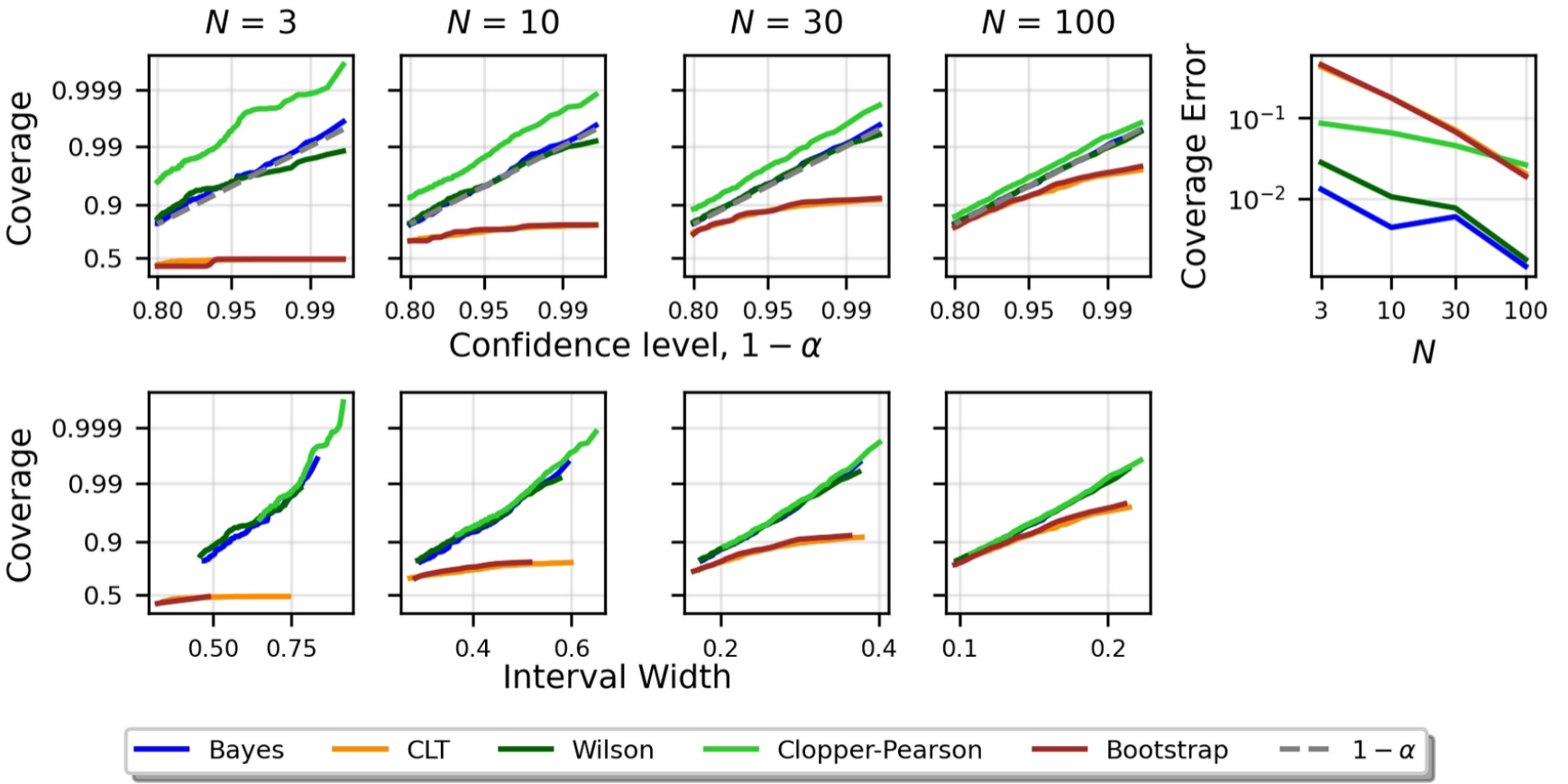
- Repeat many times and calculate the true coverage and width of the intervals.

IID Questions Setting



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Metrics that aren't simple averages of binary results (e.g. F1 score).

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Paper

https://arxiv.org/pdf/2503.01747

bayes_evals package

https://github.com/sambowyer/bayes_evals