Cohere Research Talk: Massively Parallel Inference & Bayesian Evals

Sam Bowyer

November 2025

Outline

1. About Me
2. Alan: Massively Parallel Probabilistic Programming
3. Bayesian Evals: Uncertainty Quantification for LLM Evals

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- Currently working on discrete diffusion models (training an 'auxilliary' model with VI to suggest the order in which to decode tokens).
- Two projects I'll be talking about today: Alan (massively parallel probabilistic programming) & Bayesian Evals.

Alan: A Massively Parallel Probabilistic Programming Language

  

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- Dual goals:
  - Develop ‘massively parallel’ Bayesian inference algorithms: fast, accurate, and scalable; desinged for GPU acceleration.
  - Implement these algorithms in a probabilistic programming language in pytorch (`alan`), allowing users to specify general probabilistic models.

Regular Bayesian Inference

- **Bayesian inference**: Prior $P(z)$ and likelihood $P(x|z)$ for latent variables z and data x.

$$P(z|x) = \frac{P(x|z)P(z)}{\int_{\mathcal{Z}} P(x, z')dz'}$$

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  $$r_k(z) = \frac{P(x, z^k)}{Q(z^k)}.$$
  3. Approximate the normalising constant using the 'global' estimator:
  $$\mathcal{P}_{\text{global}}(z) = \frac{1}{K}\sum_{k=1}^{K} r_k(z) \quad \text{such that} \quad \mathbb{E}_{z\sim Q}[\mathcal{P}_{\text{global}}(z)] = P(x).$$

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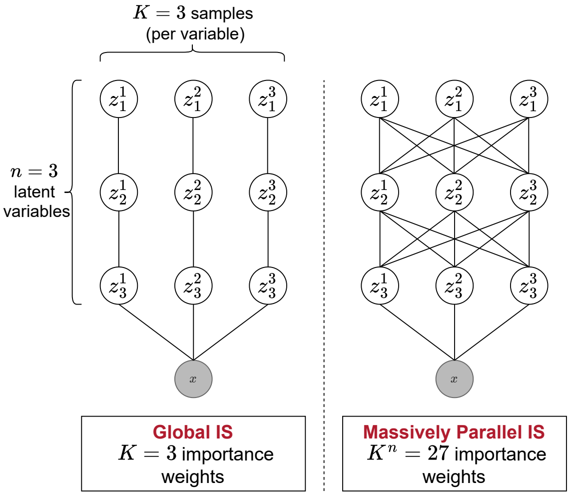
- Solution: **Massively Parallel Importance Sampling (MP-IS)**
  - Reason about all K^n possible joint samples at once.

Massively Parallel Importance Sampling (MP-IS)



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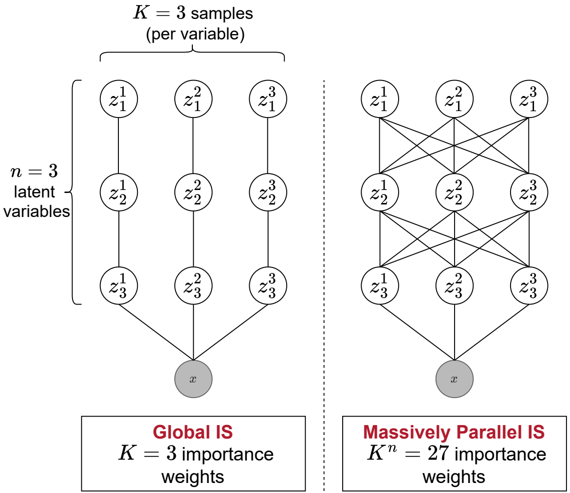

- Suppose each latent sample $z^k = (z_1^k, \ldots, z_n^k) \sim Q(z)$ is comprised of n variables.
- We can construct K^n different samples from the full joint space

$$(z_1^{k_1}, \ldots, z_n^{k_n}) \in \mathcal{Z}$$

where $\mathbf{k} = (k_1, \ldots, k_n) \in [K]^n$ is the indexing vector for each latent variable.

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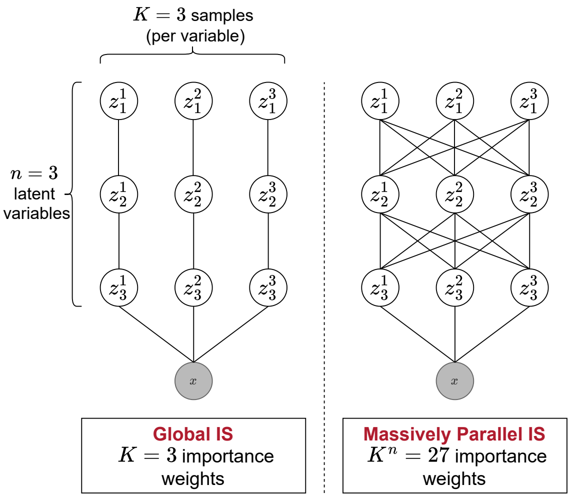

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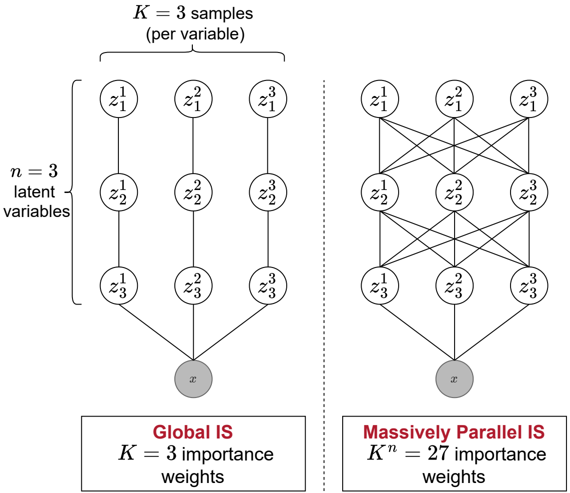

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- ...we can use the MP-IS estimator

$$\mathcal{P}_{\text{MP}}(z) = \frac{1}{K^n}\sum_{\mathbf{k}\in[K]^n}\frac{P(x, z^{\mathbf{k}})}{Q_{\text{MP}}(z^{\mathbf{k}}, \mathbf{k})}.$$

(Which is still unbiased.)

MP-IS: Some Complications...

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- Also, at first glance, this thing doesn't look all that nice to compute...
- But we can exploit the conditional independencies in the model to render it tractable.

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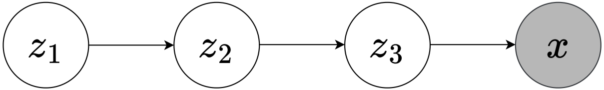

- E.g. with the model from before with $n = 3$, $P(x, z) = P(z_1)P(z_2|z_1)P(z_3|z_2)P(x|z_3)$, we can move the sums inside the product and get a bunch of tensor products:

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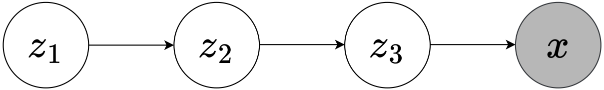

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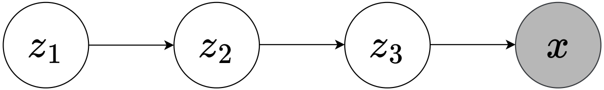

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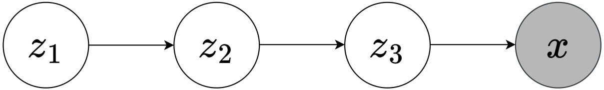

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$$= \frac{1}{K^3} \sum_{k_1 \in [K]} \underbrace{\frac{P(z_1^{k_1})}{Q(z_1^{k_1})}}_{\text{Vector of size } K} \sum_{k_2 \in [K]} \underbrace{\frac{P(z_2^{k_2}|z_1^{k_1})}{Q(z_2^{k_2})}}_{\text{Matrix of size } K \times K} \sum_{k_3 \in [K]} \underbrace{\frac{P(z_3^{k_3}|z_2^{k_2})}{Q(z_3^{k_3})}}_{\text{Matrix of size } K \times K} \underbrace{P(x|z_3^{k_3})}_{\text{Vector of size } K}$$

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- Yes! We do this with `alan`.
- User specifies the model with P and Q as pytorch modules, and we handle the massively parallel inference for them.

Alan: A Probabilistic Programming Language

```python
from alan import Normal, Bernoulli, Plate, BoundPlate, OptParam, Data, Problem
import torch as t

# Set up the model
d_z = 10

P = Plate(
    mu_z = Normal(t.zeros((d_z,)), t.ones((d_z,))),
    psi_z = Normal(t.zeros((d_z,)), t.ones((d_z,))),
    plate_1 = Plate(
        z = Normal("mu_z", lambda psi_z: psi_z.exp()),
        plate_2 = Plate(
            obs = Bernoulli(logits = lambda z, x: z @ x),
        )
    ),
)

Q = Plate(
    mu_z = Normal(OptParam(t.zeros((d_z,))), OptParam(t.zeros((d_z,)), transformation=t.exp)),
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P = BoundPlate(P, platesizes={'plate_1': num_users, 'plate_2': num_movies}, inputs = {'x': x})
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prob = Problem(P, Q)
```

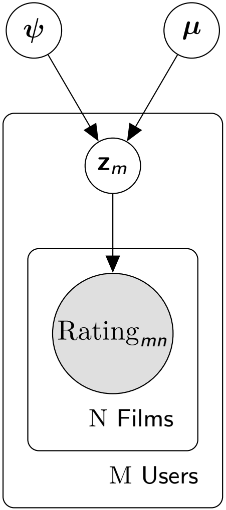

MP-VI

- Using $\mathcal{P}_{\text{MP}}(z)$, we can do variational inference (VI) by maximising the ELBO:

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- Aitchison (2019) showed that MP-VI is a tighter bound than the global VI objective (IWAE):

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prob = Problem(P, Q)
opt = t.optim.Adam(prob.Q.parameters(), lr=lr)

# Train Q with VI
for i in range(num_iterations):
    opt.zero_grad()
    elbo = prob.sample(K=K).elbo_vi()
    elbo.backward()
    opt.step()
```

MP Algorithms

- We can obtain unbiased posterior moment estimates via autodiff (Bowyer et al. (2024)).

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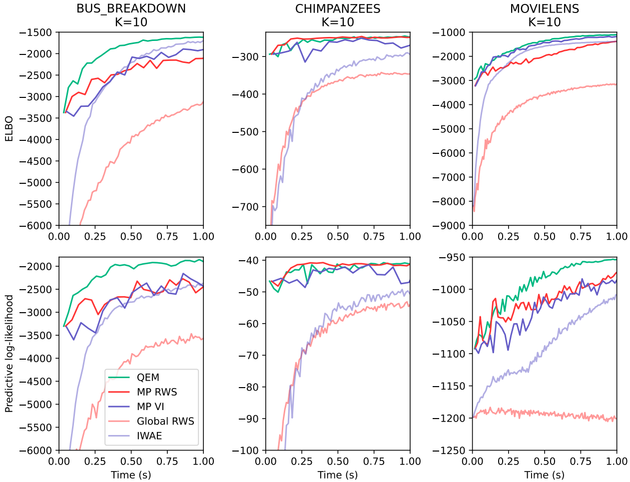
- By similar arguments: $J \in \mathbb{R}^K$ gives us marginal importance weights; $J \in \mathbb{R}^{K^{1+|pa(i)|}}$ gives us importance samples for z_i, given its parents $pa(i)$.

QEM: An Adaptive Importance Sampling Algorithm

QEM (Heap et al. (2025))

1. Start with an initial approximate posterior Q_0.
2. Compute posterior moment estimates $m_{\text{MP}}(z)$ using MP-IS.
3. Update the approximate posterior Q_{t+1} using the moment estimates.

Can be seen as an EM-like algorithm for adaptive imortance sampling.



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- This project was a great (if, at times, complicated) way to learn about Bayesian inference and numerical and probabilistic programming.
- The results were pretty promising, but but there are some drawbacks to massively parallel methods:
  - The algorithms are complex to implement (hence wrapping them in a PPL).
  - Not all models have lots of conditional independencies to exploit.
  - Although it's slower and harder to tune, HMC is often hard to beat in terms of quality of inference.

Bayesian Evals: Uncertainty Quantification for LLM Evals



Work done with Laurence Aitchison and Desi R. Ivanova.

- Two directions:

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- Two directions:
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- The former direction led to an ICML spotlight position paper: *'Position: Don't Use the CLT in LLM Evals With Fewer Than a Few Hundred Datapoints'*.
- The latter fell by the wayside, but is something I'd like to come back to at some point.

Motivation

Central Limit Theorem (CLT)

If $X_1, \ldots, X_N$ are IID r.v.s with mean $\mu \in \mathbb{R}$ and finite variance σ^2, then

$$\sqrt{N}(\hat{\mu} - \mu) \xrightarrow{d} \mathcal{N}(0, \sigma^2) \text{ as } N \to \infty,$$

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- The CLT-based confidence interval is:

$$\mathrm{CI}_{1-\alpha}(\mu) = \hat{\mu} \pm z_{\alpha/2} \mathrm{SE}(\hat{\mu})$$

where $z_{\alpha/2}$ is the $100(1-\alpha/2)$-th percentile of $\mathcal{N}(0,1)$ and $\mathrm{SE}(\hat{\mu}) = \sqrt{\frac{\hat{\sigma}^2}{N}}$ is the standard error of the sample mean.

Motivation

Central Limit Theorem (CLT)

If $X_1, \ldots, X_N$ are IID r.v.s with mean $\mu \in \mathbb{R}$ and finite variance σ^2, then

$$\sqrt{N}(\hat{\mu} - \mu) \xrightarrow{d} \mathcal{N}(0, \sigma^2) \text{ as } N \to \infty,$$

where $\hat{\mu} = \frac{1}{N}\sum_{i=1}^{N} X_i$ is the sample mean.

- The CLT-based confidence interval is:

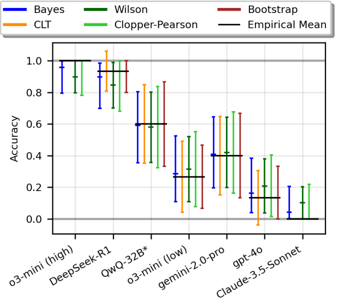
$$\mathrm{CI}_{1-\alpha}(\mu) = \hat{\mu} \pm z_{\alpha/2}\mathrm{SE}(\hat{\mu})$$

  where $z_{\alpha/2}$ is the $100(1-\alpha/2)$-th percentile of $\mathcal{N}(0,1)$ and $\mathrm{SE}(\hat{\mu}) = \sqrt{\frac{\hat{\sigma}^2}{N}}$ is the standard error of the sample mean.
- In the case of binary data $X_i \in \{0, 1\}$, this becomes:

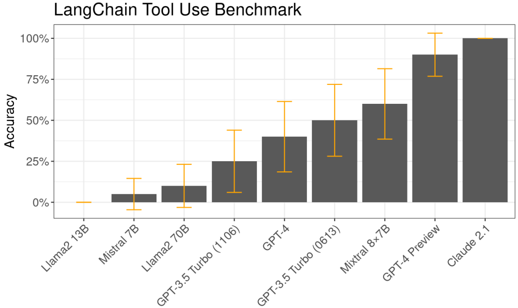
$$\mathrm{CI}_{1-\alpha}(\theta) = \bar{X} \pm z_{\alpha/2}\sqrt{\bar{X}(1-\bar{X})/N}.$$

Real-World Failures of the CLT

- If N is too small, CLT-based error bars can collapse to zero-width or extend past $[0, 1]$.



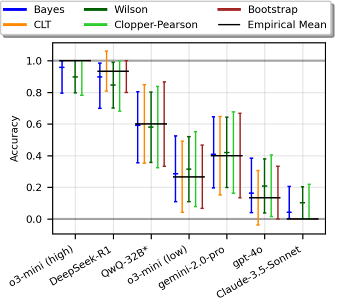
Math Arena's AIME II 2025 Benchmark (N=15). Various 95% interval types shown.



Langchain Typewriter Tool Use Benchmark (N=20). CLT-based 95% intervals only.

Real-World Failures of the CLT

- If N is too small, CLT-based error bars can collapse to zero-width or extend past $[0, 1]$.
- Smaller, more intricate, and expensive LLM benchmarks are becoming increasingly common, so we need to find alternatives for the few-data regime.



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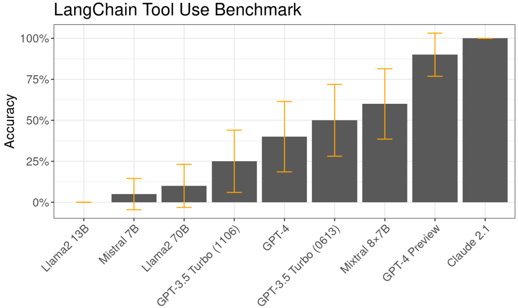

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Bayesian Alternative: Beta-Binomial Model

- Treat the data as IID Bernoulli with a uniform prior on the parameter θ.

$$\theta \sim \text{Beta}(1, 1) = \text{Uniform}[0, 1]$$
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- Obtain quantile-based Bayesian credible intervals for θ from the closed form posterior.

$$p(\theta | y_{1:N}) = \text{Beta}\left(1 + \sum_{i=1}^{N} y_i, 1 + \sum_{i=1}^{N} (1 - y_i)\right)$$

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Beta-Bernoulli Bayesian Credible Interval

```
posterior = scipy.stats.beta(1 + sum(y), 1 + N - sum(y))
bayes_ci  = posterior.interval(confidence=0.95)
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Wilson & Clopper-Pearson Confidence Interval

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result = scipy.stats.binomtest(k=sum(y), n=N)
wilson_ci = result.proportion_ci("wilson", 0.95)
clop_ci   = result.proportion_ci("exact",  0.95)
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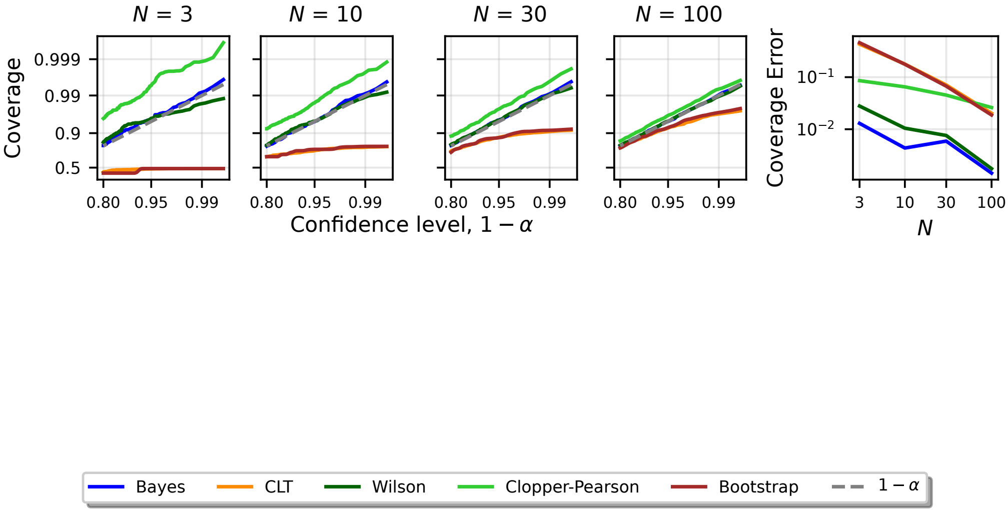
  - Ideally, coverage $= 1 - \alpha$.

IID Questions Setting: Results



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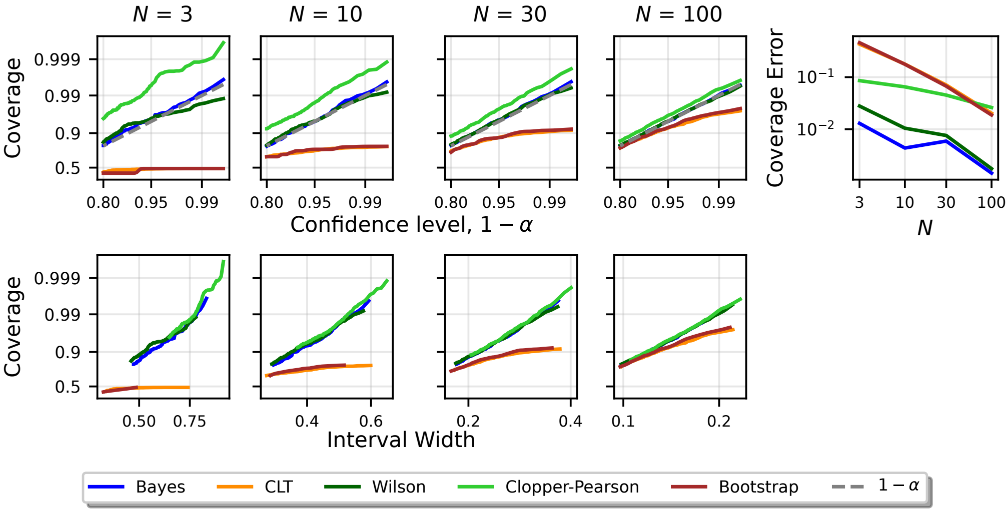

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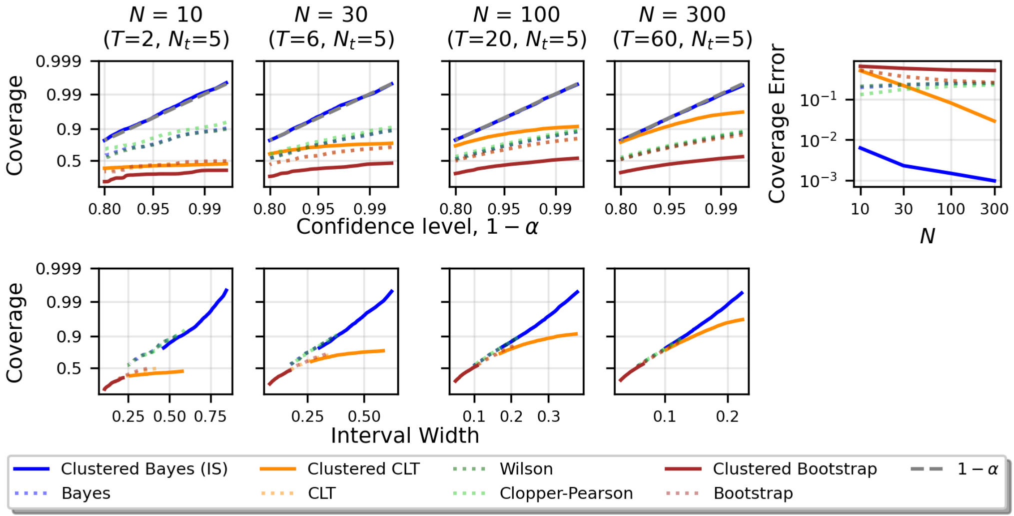

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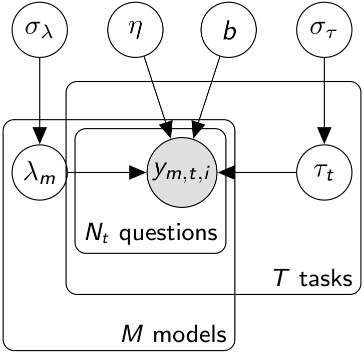

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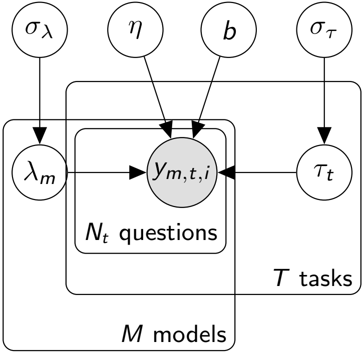

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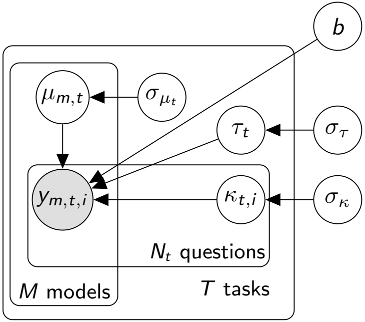
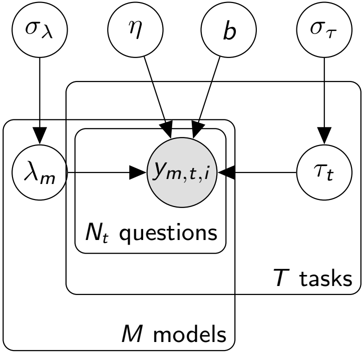


Question-task difficulty model.

Bayesian Evals for Interpretability

- Apply hierarchical Bayesian modelling to evals: infer latent variables for benchmark difficulty and model performance.



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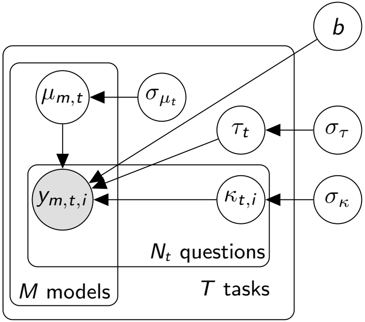

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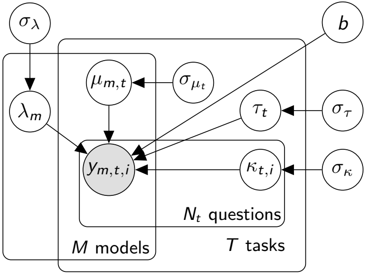
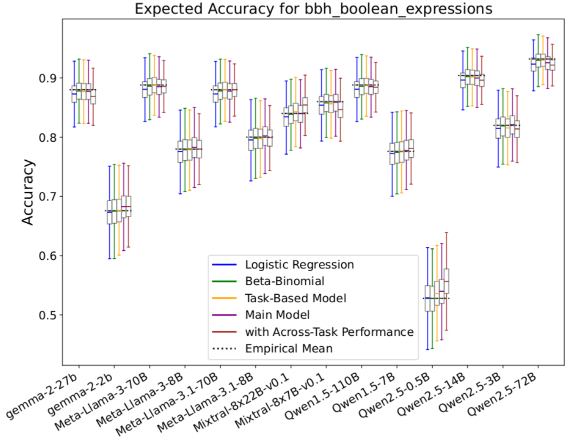
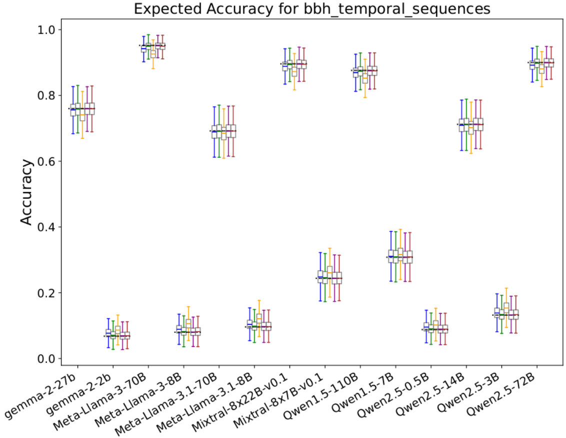


+ Across-task performance latent variable.

Graphical models of the proposed hierarchical models. Way too complicated!

Bayesian Evals for Interpretability





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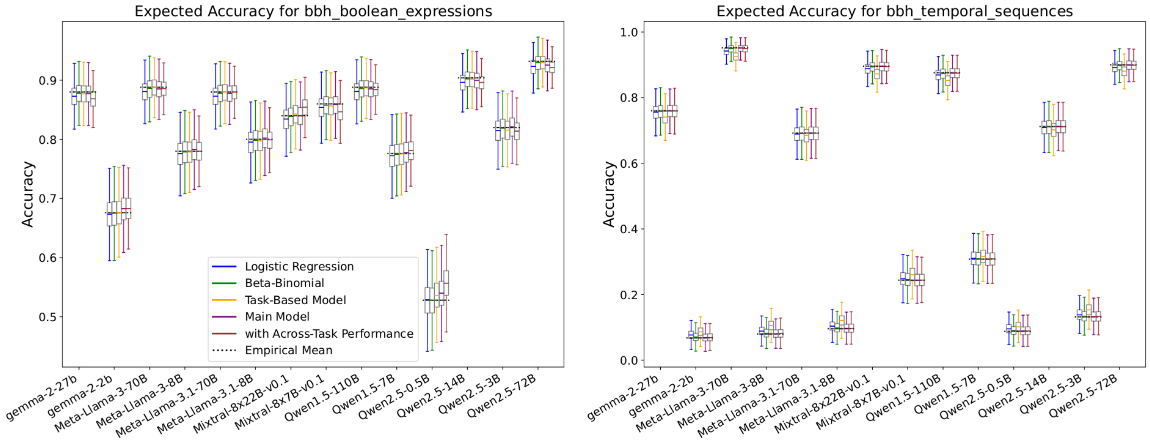

So just use Beta-Binomial!

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- Perhaps these techniques could be used more successfully for predicting eval performance based on the mix of pretraining data?
- Or predicting the effectiveness of post-training on downstream evals?

Thank You

- Any questions?

References I

Bowyer, Sam, Laurence Aitchison, and Desi R. Ivanova (Oct. 2025). "Position: Don't Use the CLT in LLM Evals With Fewer Than a Few Hundred Datapoints". en. In: *Proceedings of the 42nd International Conference on Machine Learning*. ISSN: 2640-3498. PMLR, pp. 81143–81184. URL: https://proceedings.mlr.press/v267/bowyer25a.html (visited on 11/09/2025).

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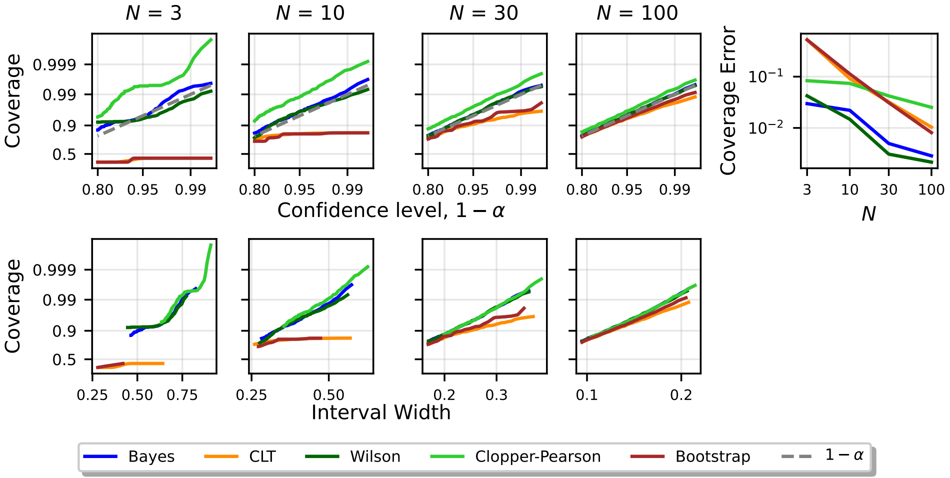
IID Questions Setting: Prior Mismatch

Use $\theta \sim \text{Uniform}[0,1]$ as the prior, but $\theta \sim \text{Beta}(100,20)$ as the true data distribution. ($\mathbb{E}[\theta] = 0.83$ and $\text{Var}(\theta) = 0.034^2$.)



Bayesian Evals for Interpretability

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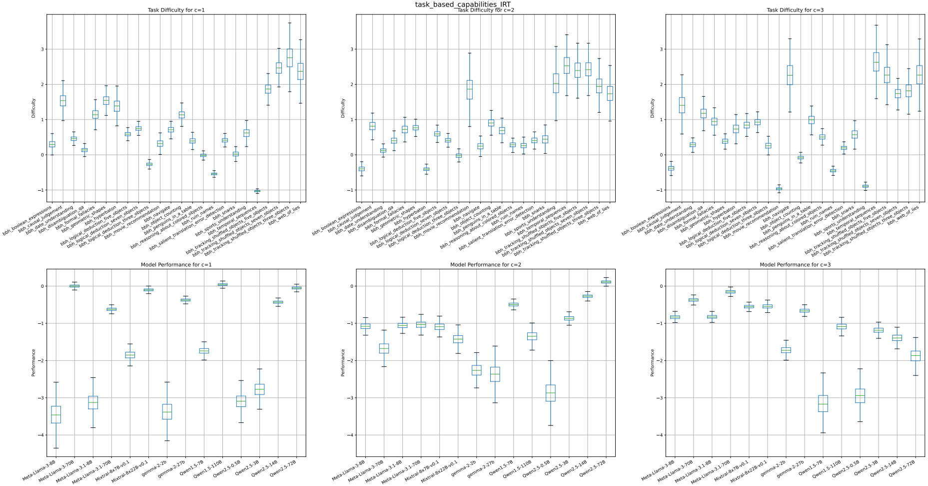

Clustered Questions Setting: Bayesian Implementation

Snippet 4: Bayesian analysis for clustered evals

```python
# S_t, N_t: np.arrays of length T with total
# sucesses & questions per task
import numpy as np
from scipy.stats import betabinom

# set number of samples, K
K = 10_000

# get K samples from the prior (with extra dimension for broadcasting over tasks)
thetas = np.random.beta(1,1, size=(K,1))
ds = np.random.gamma(1,1, size=(K,1))

# obtain weights via the likelihood (sum the per-task log-probs)
log_weights = betabinom(N_t, (ds*thetas), (ds*(1-thetas))).logpmf(S_t).sum(-1)

# normalise the weights
weights = np.exp(log_weights - log_weights.max())
weights /= weights.sum()

# obtain samples from the posterior
posterior = thetas[np.random.choice(K, size=K, replace=True, p=weights)]

# Bayesian credible interval
bayes_ci = np.percentile(posterior, [2.5, 97.5])
```