# Position: Don’t Use the CLT in LLM Evals With Fewer Than a Few Hundred Datapoints

Sam Bowyer, Laurence Aitchison, and Desi R. Ivanova



May 27, 2025

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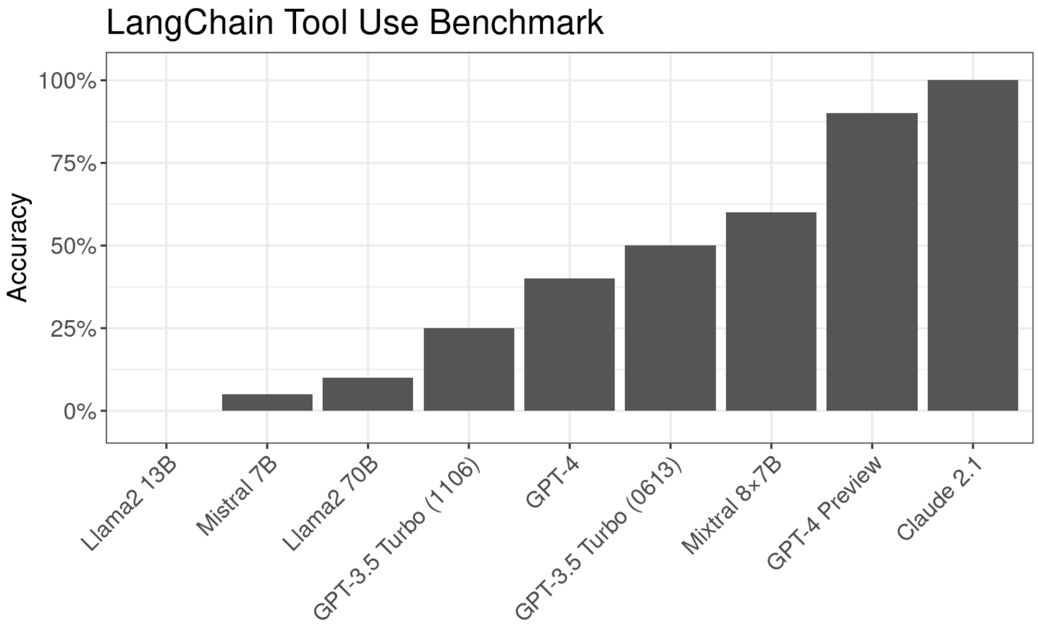
- Error bars are important for evals



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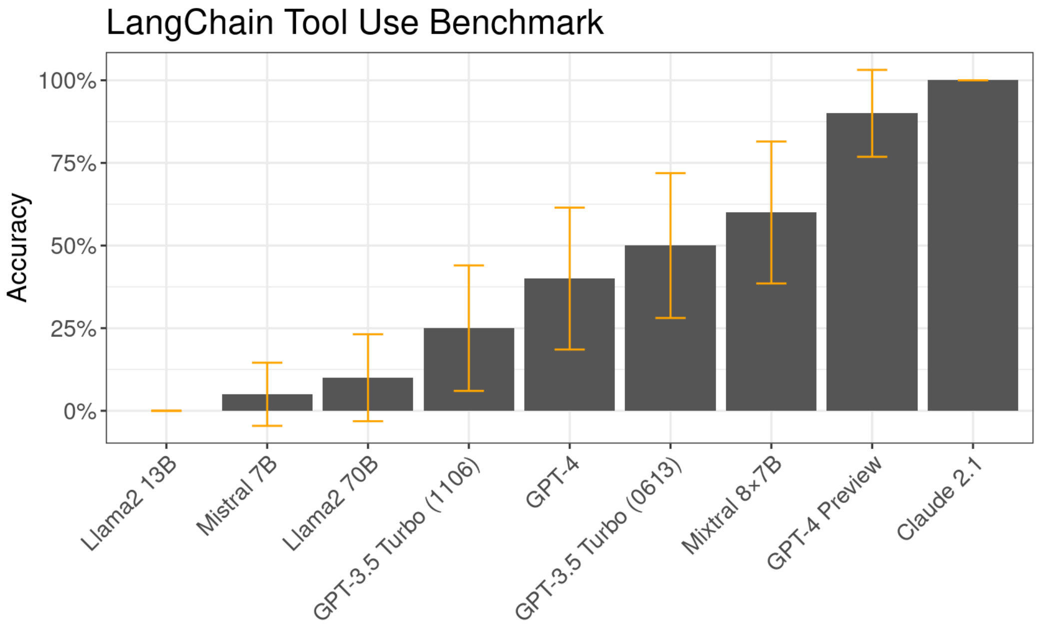

# TL;DR

- Error bars are important for evals
- CLT-based methods are (increasingly) unwise
- We can do a lot better, very easily

```python
# y is a length N binary "eval" vector
from scipy.stats import binomtest, beta

S, N = y.sum(), len(y) # total successes & questions
result = binomtest(k=S, n=N)

# 95% Wilson score and Clopper-Pearson intervals
wilson_ci = result.proportion_ci("wilson", 0.95)
cp_ci = result.proportion_ci("exact", 0.95)

# Bayesian Credible interval
posterior = beta(1+S, 1+(N-S))
bayes_ci = posterior.interval(confidence=0.95)
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# Central Limit Theorem (CLT)

If $X_1, \ldots, X_N$ are IID r.v.s with mean $\mu \in \mathbb{R}$ and finite variance $\sigma^2$, then

$$\sqrt{N}(\hat{\mu} - \mu) \xrightarrow{d} \mathcal{N}\left(0, \sigma^2\right) \text{ as } \boxed{N \to \infty},$$

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# Central Limit Theorem (CLT) - Confidence Intervals

We construct CLT-based confidence intervals at confidence level $1 - \alpha \in [0, 1]$ as

$$\mathrm{CI}_{1-\alpha}(\mu) = \hat{\mu} \pm z_{\alpha/2}\mathrm{SE}(\hat{\mu}),$$

where $z_{\alpha/2}$ is the $100(1 - \alpha/2)$-th percentile of the standard normal distribution and

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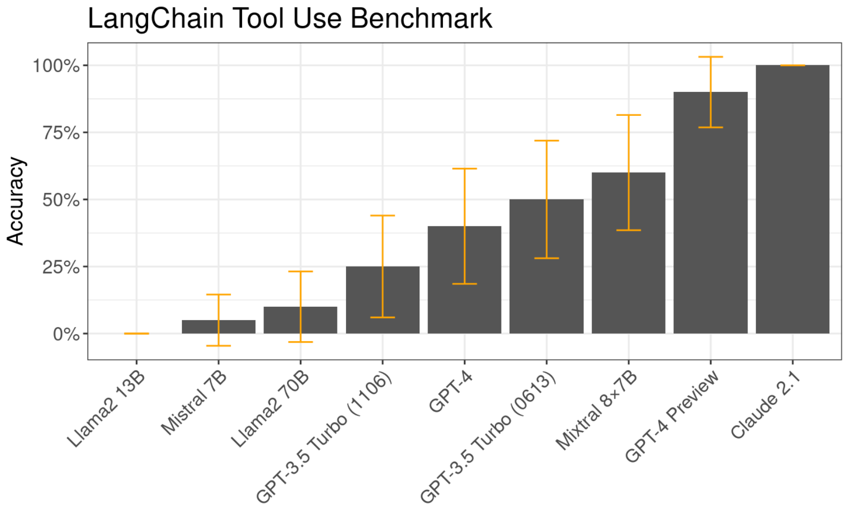
For binary data (e.g. correct/incorrect), $X_i \sim \mathrm{Bernoulli}(\theta)$, we can use the Bernoulli variance formula:

# Real-world failures

As models get better (and more expensive), benchmarks get harder and smaller, posing problems for the CLT. (E.g. Math Arena's AIME II 2025 Benchmark has N=15 competition maths problems.)

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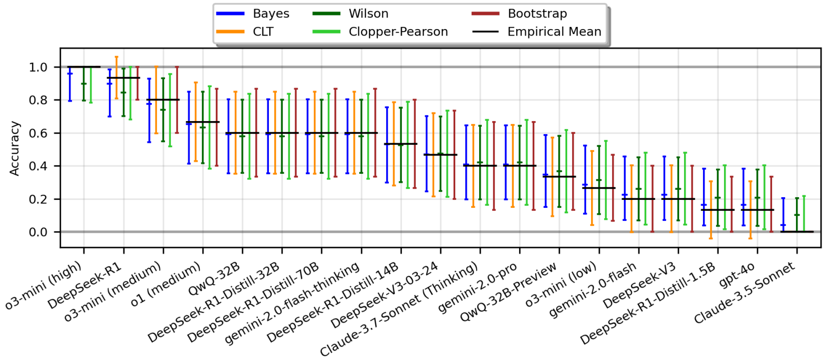


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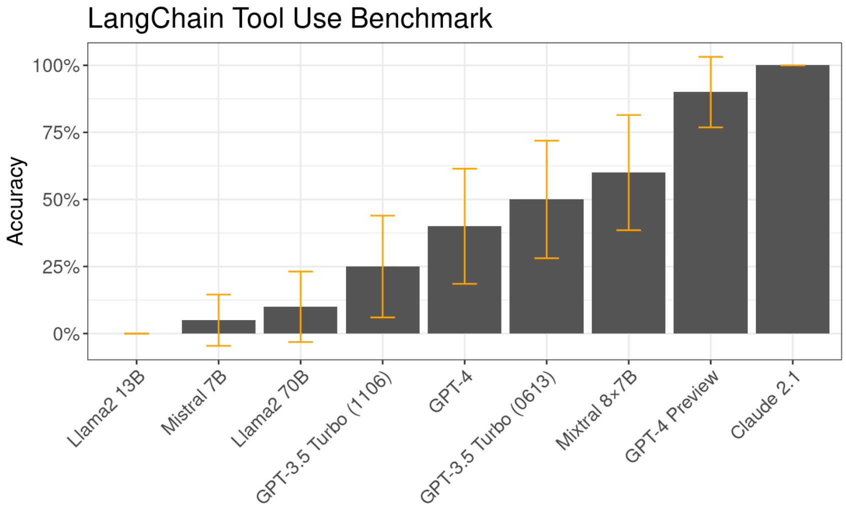


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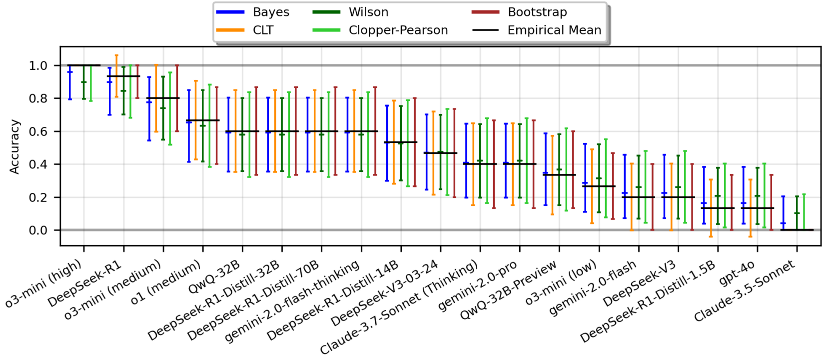


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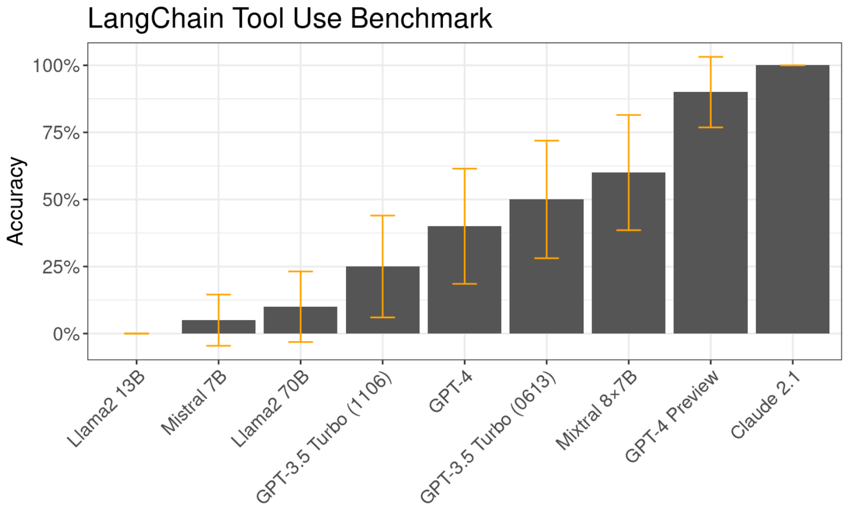
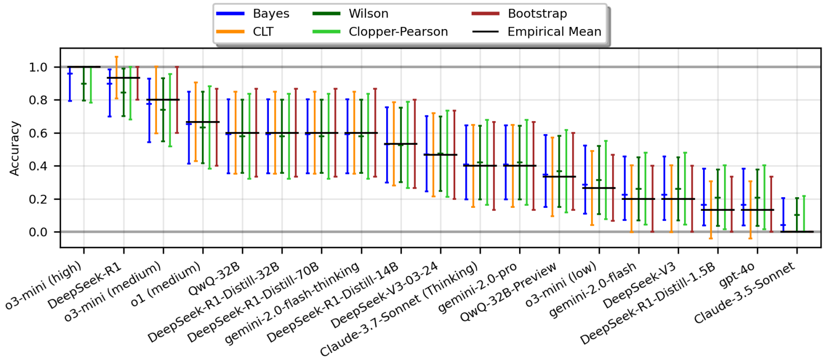
- Error bars can extend past [0,1].

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Treat the data as IID Bernoulli with a uniform prior on the parameter $\theta$.

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$$y_i \sim \mathrm{Bernoulli}(\theta) \text{ for } i = 1, \ldots N$$

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- Bayesian *credible interval:*
  - The parameter is random, we infer the posterior distribution of the parameter given the data.
  - "*There is a* $100 \times (1 - \alpha)\%$ *probability that the interval contains the true parameter. (Under some modelling assumptions.)*"

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  - Ideally, our intervals would be as tight as possible.

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- Compute the coverage and average width of the intervals.

# IID Questions Setting - Bayes vs. CLT

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$$\mathrm{CI}_{1-\alpha,\mathrm{Wilson}}(\theta) = \frac{\hat{\theta} + \frac{z_{\alpha/2}^2}{2N}}{1 + \frac{z_{\alpha/2}^2}{N}} \pm \frac{\frac{z_{\alpha/2}}{2N}}{1 + \frac{z_{\alpha/2}^2}{N}} \sqrt{4N\hat{\theta}(1-\hat{\theta}) + z_{\alpha/2}^2}$$

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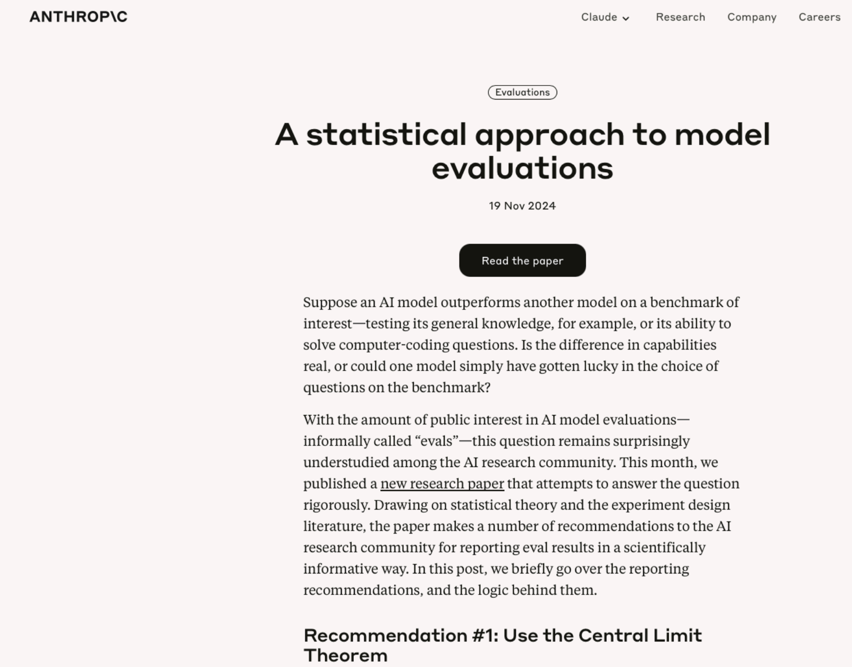
# IID Questions Setting

# Recommendation

Use Bayes or Wilson Score Intervals, not the CLT.



ANTHROP\C

Claude Research Company Careers

Evaluations

## A statistical approach to model evaluations

19 Nov 2024

Read the paper

Suppose an AI model outperforms another model on a benchmark of interest—testing its general knowledge, for example, or its ability to solve computer-coding questions. Is the difference in capabilities real, or could one model simply have gotten lucky in the choice of questions on the benchmark?

With the amount of public interest in AI model evaluations—informally called “evals”—this question remains surprisingly understudied among the AI research community. This month, we published a new research paper that attempts to answer the question rigorously. Drawing on statistical theory and the experiment design literature, the paper makes a number of recommendations to the AI research community for reporting eval results in a scientifically informative way. In this post, we briefly go over the reporting recommendations, and the logic behind them.

### Recommendation #1: Use the Central Limit Theorem

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## Metrics that aren't simple averages of binary results (e.g. F1 score).

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## Bayesian Inference

The number of successes per task is Beta-Binomial distributed:

$$\sum_{i=1}^{N_t} y_{i,t} = Y_t \sim \mathrm{BetaBinomial}(N_t, d\theta, d(1-\theta))$$

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## Bayesian Inference

The number of successes per task is Beta-Binomial distributed:

$$\sum_{i=1}^{N_t} y_{i,t} = Y_t \sim \text{BetaBinomial}(N_t, d\theta, d(1-\theta))$$

Get an importance-weighted posterior for $\theta$: draw prior samples $\{(\theta^{(k)}, d^{(k)})\}_{k=1}^{K}$, then compute weights

$$w^{(k)} = \prod_{t=1}^{T} \text{BetaBinomial}(Y_t; N_t, d^{(k)}\theta^{(k)}, d^{(k)}(1-\theta^{(k)}))$$

# Clustered Questions Setting

(Instead of $N$ IID questions, we have $T$ tasks, each with $N_t$ IID questions.)

## Generative Model

- 'Dispersion' parameter $d$ controls the range of difficulty of the questions.
- We ensure that the mean difficulty of the questions across tasks is $\theta$ (that is, $\mathbb{E}[\theta_t] = \theta$).

$$d \sim \text{Gamma}(1,1), \quad \theta \sim \text{Beta}(1,1), \quad \theta_t \quad \sim \text{Beta}(d\theta, d(1-\theta)), \quad y_{i,t} \sim \text{Bernoulli}(\theta_t)$$

## Clustered Standard Error (CLT-based Approach)

Update the standard error to account for the clustering:

$$\text{SE}_{\text{clust.}} = \sqrt{\text{SE}_{\text{CLT}}^2 + \frac{1}{N^2} \sum_{t=1}^{T} \sum_{i=1}^{N_t} \sum_{j \neq i} (y_{i,t} - \bar{y})(y_{j,t} - \bar{y})}$$

# Clustered Questions Setting

# Model Comparison (Unpaired)

Compare $\theta_A$ and $\theta_B$ for two different models, with access *only* to $N = N_A = N_B$, $\hat{\theta}_A$, and $\hat{\theta}_B$.

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## Bayesian Approach

Obtain a posterior for model A and a posterior for model B, using the earlier Beta-Binomial model.

```
# y_A and y_B are vectors of evals for two models
import numpy as np

S_A, S_B = y_A.sum(), y_B.sum()
# draw posterior samples (ps)
ps_A = np.random.beta(1 + S_A, 1 + (N - S_A), size=2000)
ps_B = np.random.beta(1 + S_B, 1 + (N - S_B), size=2000)
# posterior difference and 95% QBI
ps_diff = ps_A - ps_B
bayes_diff = np.percentile(ps_diff, [2.5, 97.5])
# posterior odds ratio and 95% QBI
```

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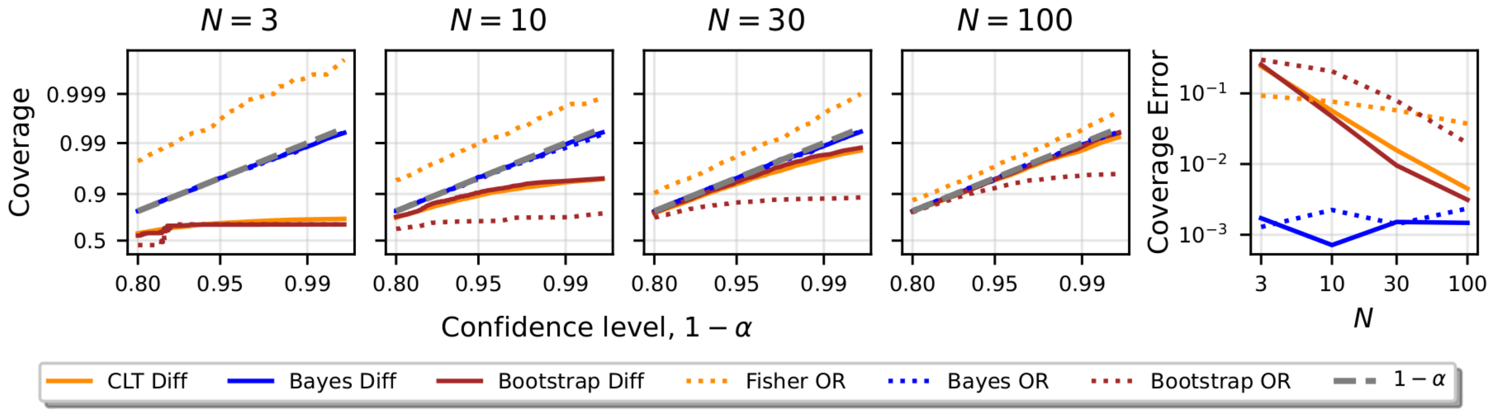
## Frequentist Approach

- Use the CLT for the **difference** and add $A$ and $B$'s squared standard errors:

$$\mathrm{CI}_{1-\alpha}(\theta_A - \theta_B) = (\hat{\theta}_A - \hat{\theta}_B) \pm z_{\alpha/2}\sqrt{S_A^2/N_A + S_B^2/N_B}.$$

# Model Comparison (Unpaired)



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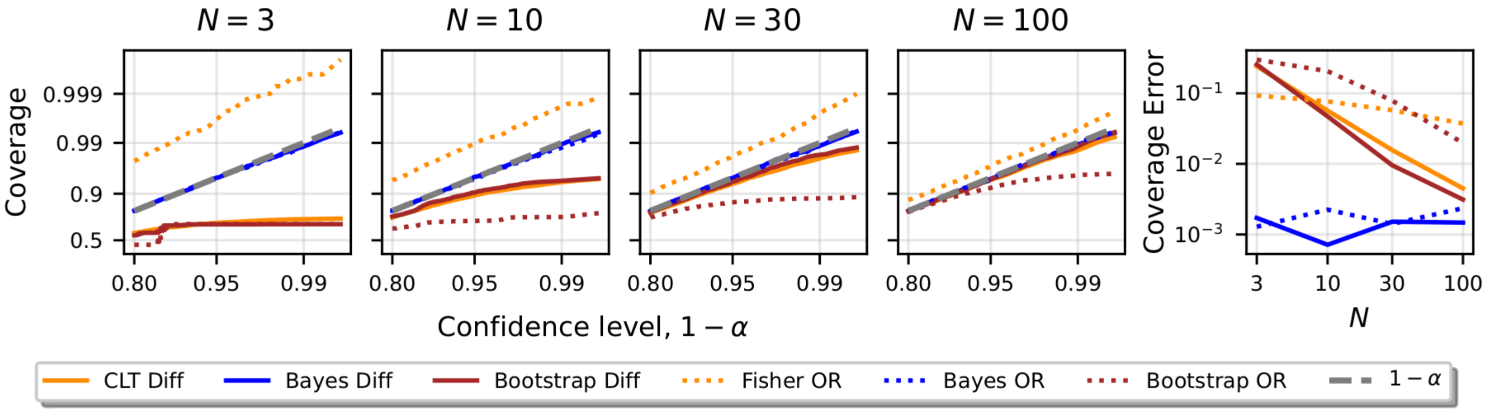


**Bayesian Bonus:** we can easily compute probabilities of one model being better than the other:

$$\mathbb{P}(\theta_A > \theta_B | y_{A;1:N}, y_{B;1:N}) = \frac{1}{K}\sum_{k=1}^{K} 1[\theta_A^{(k)} > \theta_B^{(k)}],$$

# Model Comparison (Paired)

Compute intervals over the difference $\theta_A - \theta_B$, where we have access to the same $N$ (IID) questions for both models: $\{y_{A;i}\}_{i=1}^N$ and $\{y_{B;i}\}_{i=1}^N$.

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## Frequentist Approach

Use the CLT directly for the difference $D_i = y_{A;i} - y_{B;i}$:

$$D_i \sim \mathrm{Bernoulli}(\theta_A - \theta_B),$$

$$\hat{\theta}_D = \frac{1}{N}\sum_{i=1}^{N} D_i,$$

$$\mathrm{CI}_{1-\alpha}(\theta_A - \theta_B) = \hat{\theta}_D \pm z_{\alpha/2}\,\mathrm{SE}(\hat{\theta}_D).$$

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- Bayesian methods still generally outperform CLT-based approaches when the underlying prior is different.

$$\text{e.g.} \quad \mathrm{Beta}(100, 20), \quad \mathbb{E}[\theta] = 0.83, \quad \mathrm{Var}[\theta] = 0.0011$$

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# Conclusion

- Use Bayes (or Wilson), it's not hard (`scipy` or `bayes_evals`), it's safer, and it's still cheap for large $N$.
- Plus you get the flexibility of Bayes!
    - Computing probabilities $\mathbb{P}(\theta_A > \theta_B)$.
    - Intervals on nonlinear functions of parameters e.g. F1 score (harmonic mean of precision and recall).

# Thanks for listening!

**Paper**

https://arxiv.org/pdf/2503.01747

**bayes_evals** package

https://github.com/sambowyer/bayes_evals

# Summary Table

Table 1: **Overview of methods**. *Coverage* describes whether the method provides the desired nominal coverage in small-sample settings. *Efficiency* describes how tight (and precise) the resulting confidence/credible intervals are given the nominal coverage (e.g., CLT-based intervals can be invalid or too wide). Although the *computational cost* of these methods is negligible compared to the cost of evaluating LLMs, we indicate their relative costs for comparison among the methods.

| | Coverage small $N$ | Efficiency small $N$ | Computational cost | Easy to implement |
|---:|:---:|:---:|:---:|:---:|
| CLT | ✗ | ✗ | Very low | Yes |
| CLT-based variants (e.g. Delta method) | ✗ | ✗ | Very low | Moderate |
| Custom frequentist (e.g. Wilson) | ✓ | ✓ | Very low | Moderate |
| Bootstrap | ✗ | ✗ | Low | Moderate |
| Bayes (conjugate) | ✓ | ✓ | Very low | Yes |
| Bayes (importance sampling) | ✓ | ✓ | Low | Moderate |

# Appendix – Clustered Importance Sampling Code

```
# S_t, N_t: np.arrays of length T with total successes & questions per task
# set number of samples, K
K = 10_000

# get K samples from the prior (with extra dimension for broadcasting over tasks)
thetas = np.random.beta(1,1, size=(K,1))
ds = np.random.gamma(1,1, size=(K,1))

# obtain weights via the likelihood (sum the per-task log-probs)
log_weights = scipy.stats.betabinom(N_t, (ds*thetas), (ds*(1-thetas))).logpmf(S_t).sum(-1)

# normalise the weights
weights = np.exp(log_weights - log_weights.max())
weights /= weights.sum()

# obtain samples from the posterior
posterior = thetas[np.random.choice(K, size=K, replace=True, p=weights)]

# Bayesian credible interval
bayes_ci = np.percentile(posterior, [2.5, 97.5])
```

# Appendix – Paired Importance Sampling Code

```
# y_A, y_B: length N binary "eval" vectors
from binorm import binorm_cdf # 2D Gaussian CDF, defined elsewhere
K = 10_000
# get K samples from the prior
theta_As, theta_Bs, rhos = np.random.beta(1,1, size=K), np.random.beta(1,1,size=K), 2*np.random.beta(4,2, size=K) - 1
# 2x2 contingency table (flattened)
S = (y_A * y_B).sum(-1)             # S = A correct,   B correct
T = (y_A * (1 - y_B)).sum(-1)       # T = A correct,   B incorrect
U = ((1 - y_A) * y_B).sum(-1)       # U = A incorrect, B correct
V = ((1 - y_A) * (1 - y_B)).sum(-1) # V = A incorrect, B incorrect
# calculate the bivariate normal mean
mu_As, mu_Bs = scipy.stats.norm(0,1).ppf(theta_As), scipy.stats.norm(0,1).ppf(theta_Bs)
# Calculate probabilities of each cell in the 2x2 table
theta_V = binorm_cdf(x1=0, x2=0, mu1=mu_As, mu2=mu_Bs, sigma1=1, sigma2=1, rho=rhos)
theta_S = theta_As + theta_Bs + theta_V - 1
theta_T = 1 - theta_Bs - theta_V
theta_U = 1 - theta_As - theta_V
# (probabilities may be very small and negative instead of 0)
valid_idx = (theta_S > 0) & (theta_T > 0) & (theta_U > 0) & (theta_V > 0)
log_weights = S*np.log(theta_S[valid_idx]) + T*np.log(theta_T[valid_idx]) + \
              U*np.log(theta_U[valid_idx]) + V*np.log(theta_V[valid_idx])
# normalise the weights and obtain samples from the posterior
weights = np.zeros(K)
weights[valid_idx] = np.exp(log_weights - log_weights.max())
posterior = (theta_As - theta_Bs)[np.random.choice(K, size=K, replace=True, p=weights/weights.sum())]
bayes_ci = np.percentile(posterior, [2.5, 97.5])
```