Robust Uncertainty Quantification for LLM Evaluations

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ICML 2025 Spotlight Position Paper

Position: Don't Use the CLT in LLM Evals With Fewer Than a Few Hundred Datapoints

Motivation

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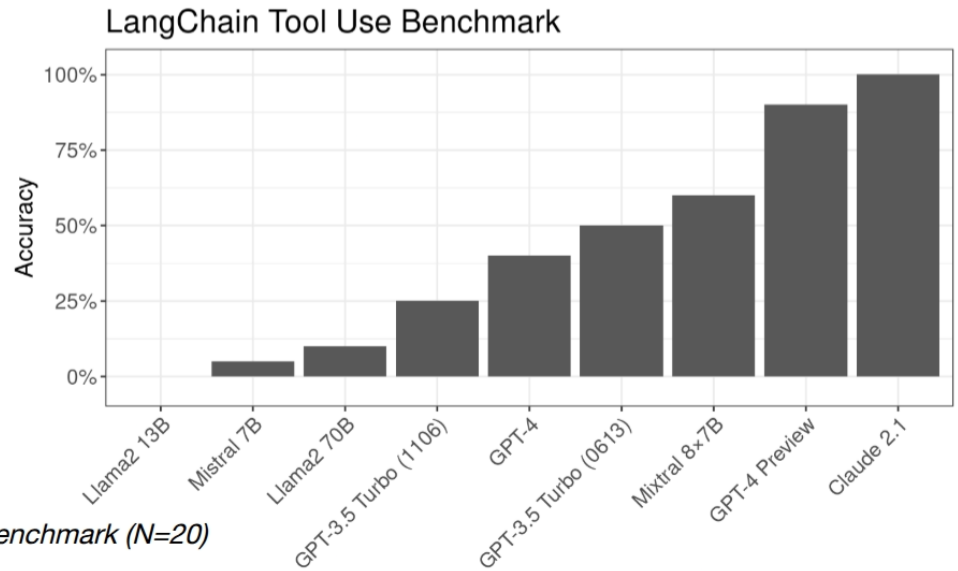
- Error bars are important for interpreting evals.



Langchain Typewriter Tool Use Benchmark (N=20)

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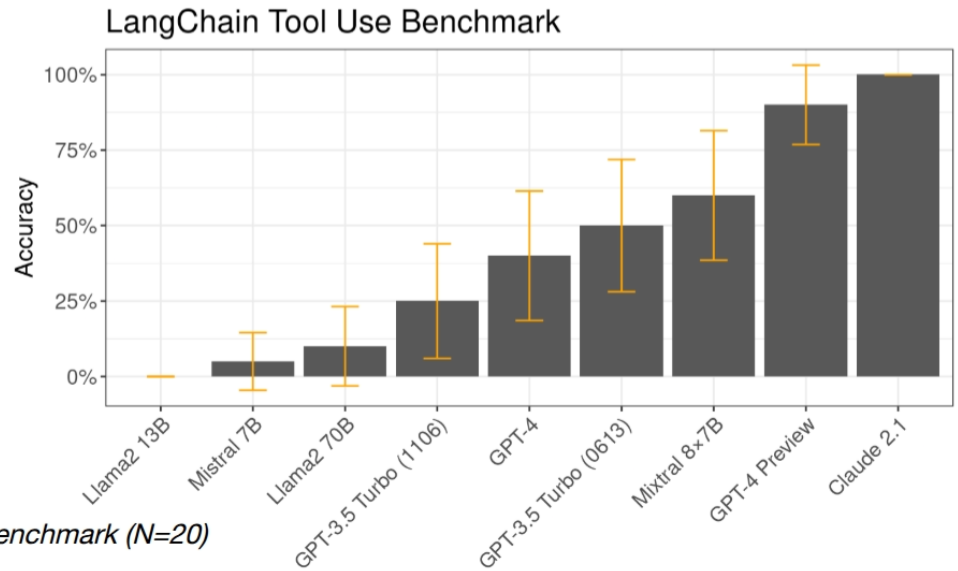
- The CLT is the most common method for computing error bars, but it's often unwise (assumes large N).

CLT-based CI at confidence level $1-\alpha$ for binary data $X_i \sim \text{Bernoulli}(\theta)$:

$$\text{CI}_{1-\alpha}(\theta) = \bar{X} \pm z_{\alpha/2}\sqrt{\frac{\bar{X}(1-\bar{X})}{N}}$$



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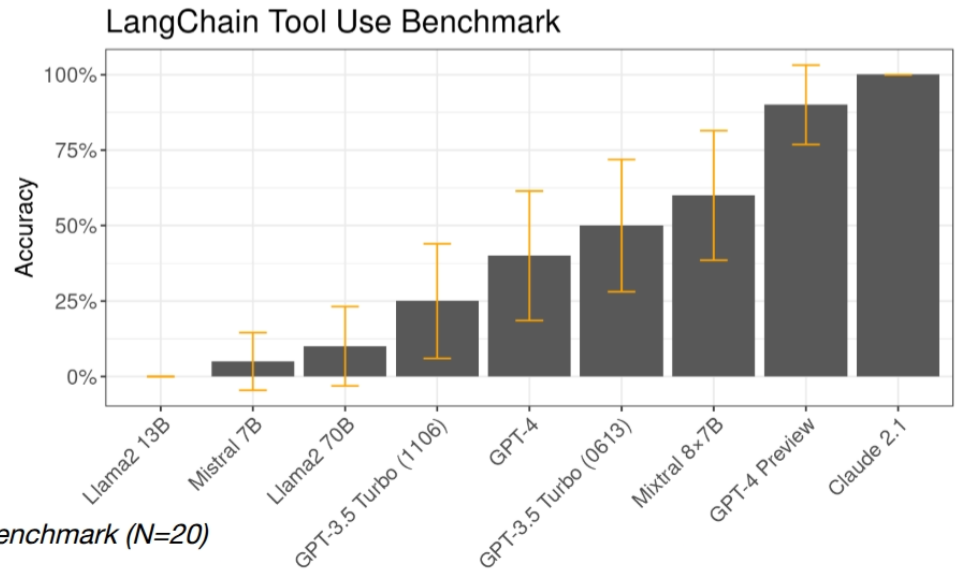

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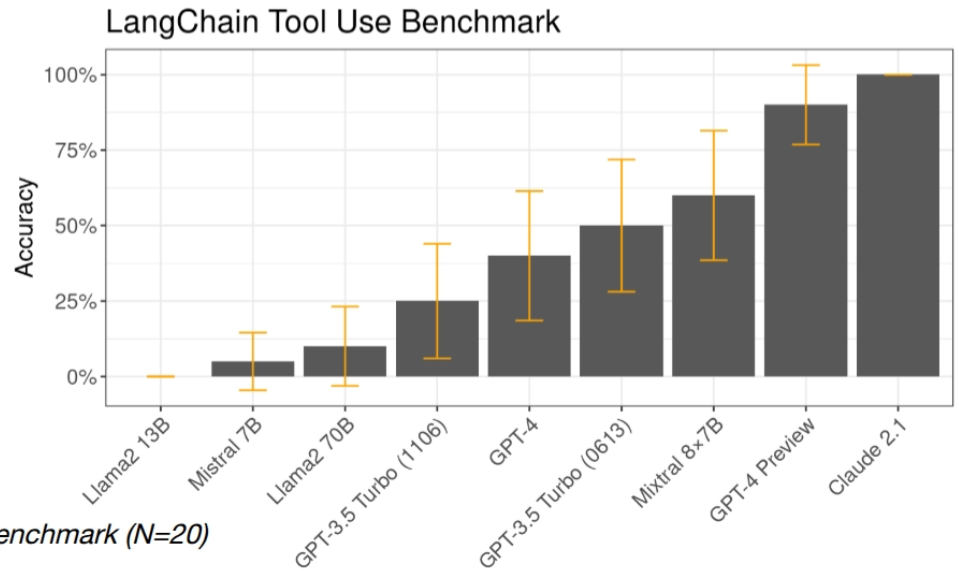
- Error bars can collapse to zero-width or extend past $[0, 1]$.
- Smaller, more intricate and expensive LLM benchmarks are becoming increasingly common.



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Bayesian Alternative – Beta-Bernoulli Model

Treat the data as IID Bernoulli with a uniform prior on the parameter θ.

$$\theta \sim \text{Beta}(1, 1) = \text{Uniform}[0, 1]$$
$$y_i \sim \text{Bernoulli}(\theta) \text{ for } i = 1, \dots N$$

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```
# y is a length N binary "eval" vector
S, N = y.sum(), len(y) # total successes & questions

# Bayesian Credible interval
posterior = scipy.stats.beta(1+S, 1+(N-S))
bayes_ci = posterior.interval(confidence=0.95)
```

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```
# y is a length N binary "eval" vector
S, N = y.sum(), len(y) # total successes & questions
result = scipy.stats.binomtest(k=S, n=N)

# 95% Wilson score interval and Clopper-Pearson exact interval
wilson_ci = result.proportion_ci("wilson", 0.95)
cp_ci = result.proportion_ci("exact", 0.95)
```

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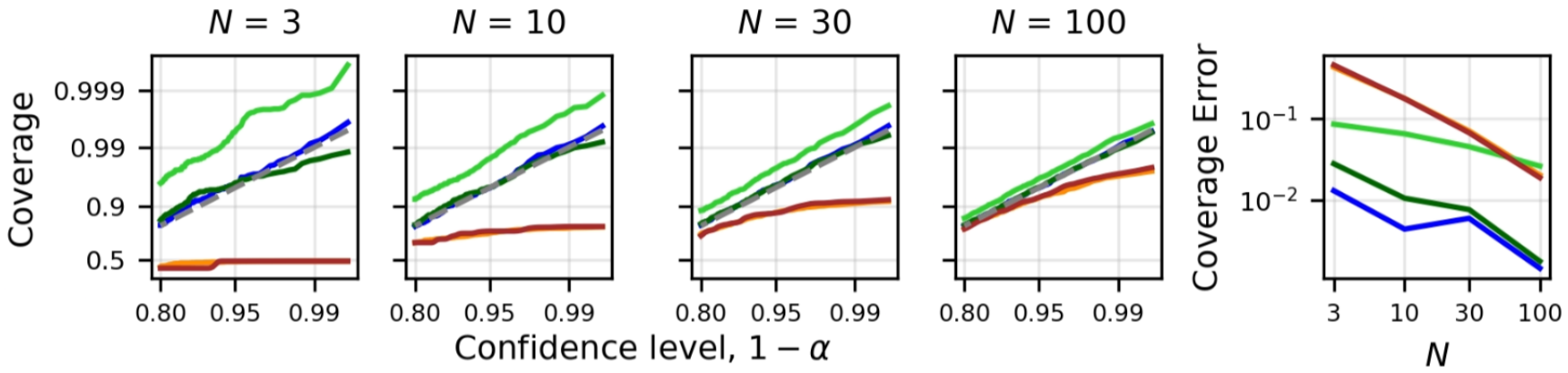
- Repeat many times and calculate the true coverage and width of the intervals.

IID Questions Setting



Bayes CLT Wilson Clopper-Pearson Bootstrap $1 - \alpha$

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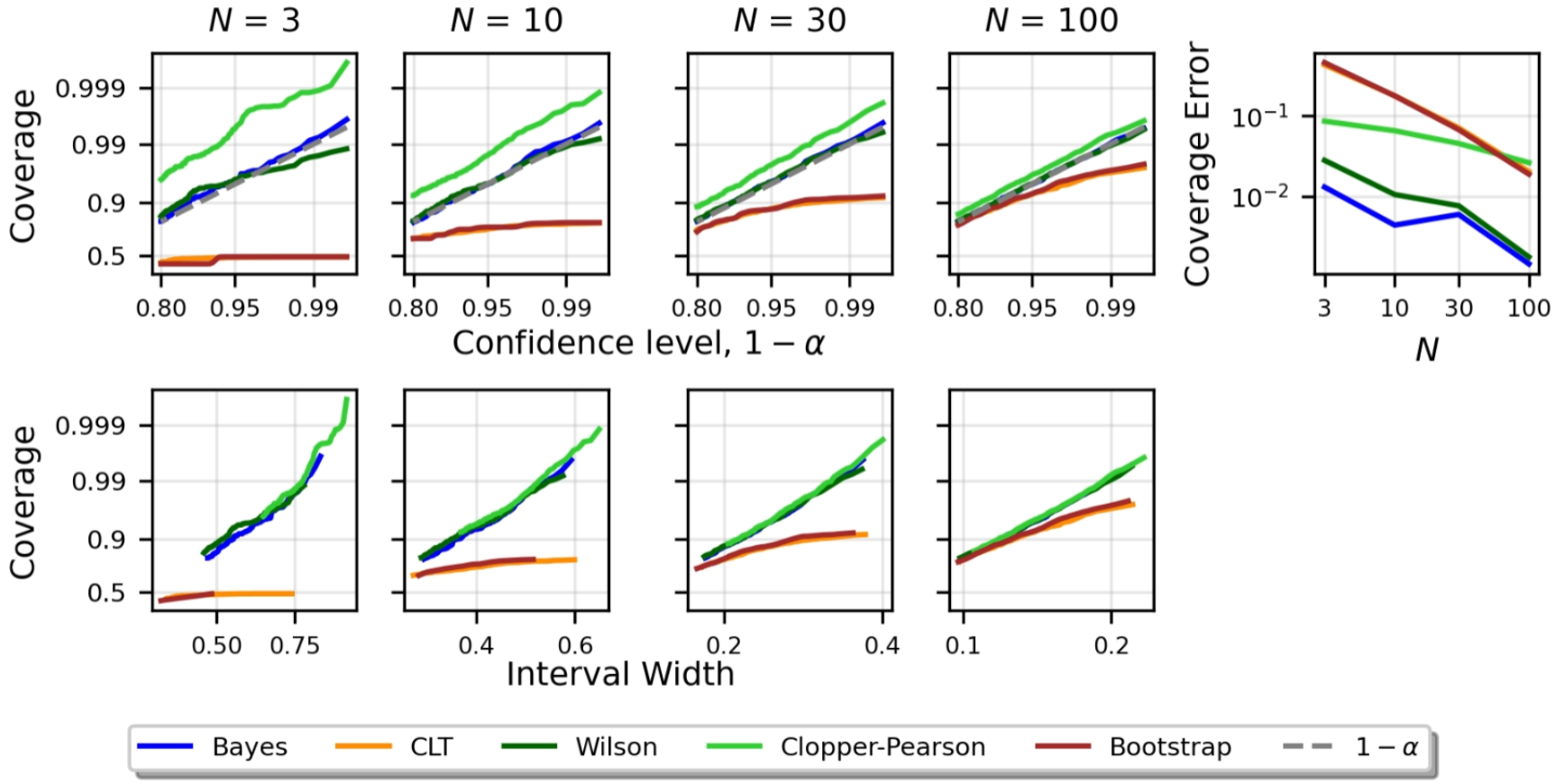

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- **Independent Comparisons**: Using $N_A, N_B, \bar{y}_A$, and $\bar{y}_B$.
- **Paired Comparisons**: Using $N_A = N_B, \{y_{A;i}\}_{i=1}^N$, and $\{y_{B;i}\}_{i=1}^N$.

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Also, what if the prior is mismatched? (i.e. $\theta \sim \mathrm{Uniform}[0, 1]$)

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Paper

https://arxiv.org/pdf/2503.01747

bayes_evals package

https://github.com/sambowyer/bayes_evals