Scheduling Resources for Executing a Partial Set of Jobs

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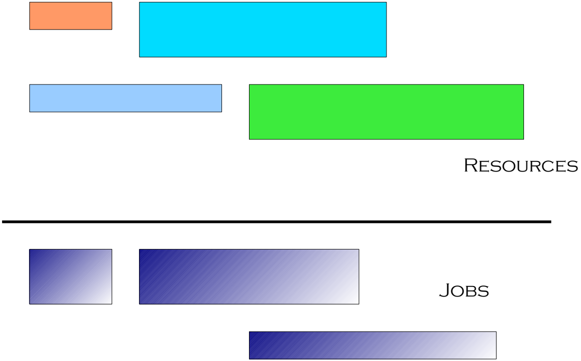
FSTTCS 2012

A Typical Scheduling Scenario

- We are given a collection of jobs. Each job is an interval, with attributes: *starting time*, *ending time* and *capacity requirement*.
- We are also given a collection of resources; resources are also intervals. A resource interval has attributes *starting time*, *ending time*, *capacity* and also an associated *cost*.

An example scenario



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- When does a resource *cover* a job? If it
  - (a) spans the length of the job, and
  - (b) the capacity of the resource is sufficient to fulfill the capacity requirement of the job.
- Generalize this notion to *collection of resources*, and *collection of jobs*. A collection of resources *covers* a collection of jobs, if at any point of time, the total capacity of resources available is sufficient to fulfill the capacity requirements of the jobs active at that time point.
- **Overall Objective:** To fulfill (i.e. complete) the jobs using a *minimum cost* collection of resources.
- Call this the `ResAll` problem.

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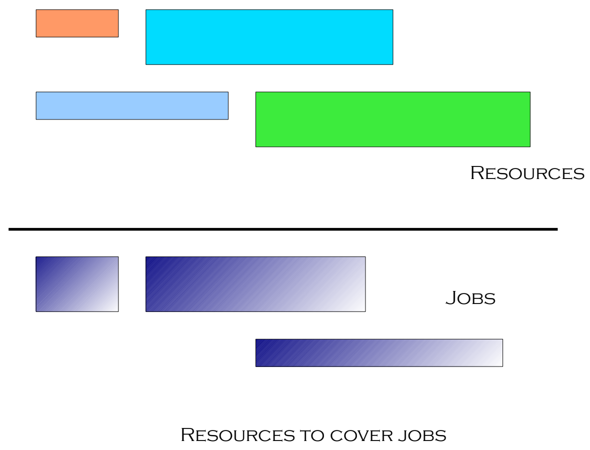

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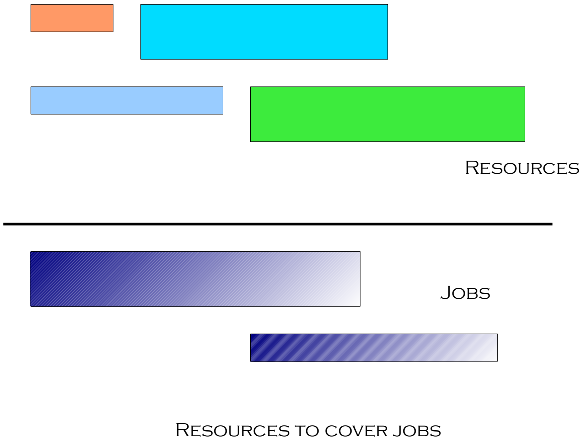

Two Variants of `ResAll`

- Resources may be picked at most once (the $\{0, 1\}$ version) or multiple times (the multiplicity version).
- Both variants have a 4-factor approximation [BarNoy et al.01].
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A Real Life Scenario

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Problem Definition

- Formally, we are given a collection $\mathcal{R}$ of $m \in \mathbb{N}$ *resource intervals*. Each resource type has a cost per unit of the resource.
- Thus, a resource interval i is equipped with a "height" h_i (the *proficiency or capacity* of the resource) cost c_i, starting time s_i and ending time e_i.
- We are also given a collection $\mathcal{J}$ of $n \in \mathbb{N}$ *job intervals*. Each job interval j has a starting time s_j, ending time e_j and a height h_j (i.e. capacity requirement).

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The SLA requirement

- We are also given a number k ($1 \leq k \leq n$), that corresponds to the SLA requirement: the number of *jobs* that need to be fulfilled.
- The **objective** is to select a minimum cost collection of resource intervals from $\mathcal{R}$ so that at least k of the jobs in $\mathcal{J}$ are satisfied.
- Call this problem the `PartialResAll` problem (it is NP-hard – Minimum Knapsack Cover is a special case).
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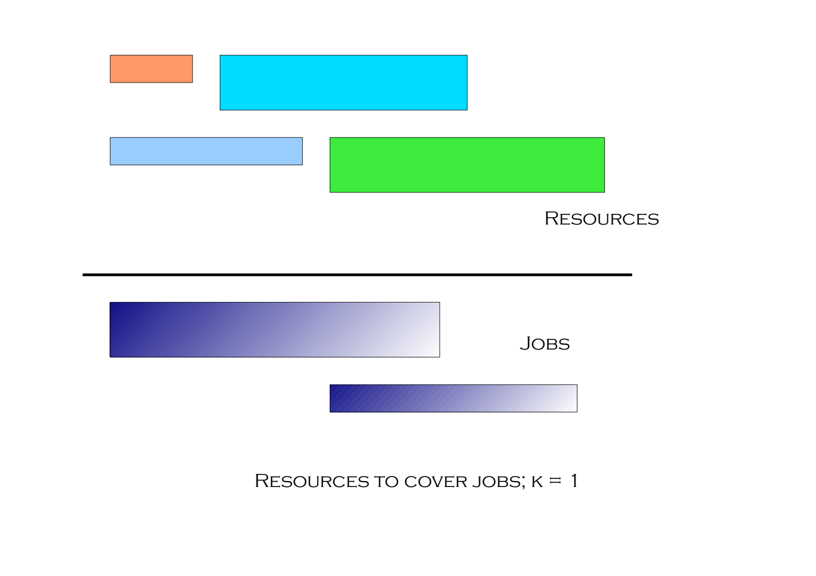

Ambient space of our problem

- Instances of covering problems abound: Set Cover, Vertex Cover, Minimum Spanning Tree, Steiner Network etc.
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History of Partial Covering problems

- 2-factor approximations for the Partial Vertex Cover problem [Bar-Yehuda99, Gandhi-Khuller-Srinivasan01, Mestre07].
- The k-Median problem has constant factor approximations; methods are based on Lagrangian Relaxations [cf. JainVazirani99]
- 2-factor approximations for the k-MST problem [Garg05]
- etc. . .

Holy Grail

- The most general problem in this direction is to prove a constant factor approximation for the $\{0, 1\}$ version of the `PartialResAll` problem.

Main Result

- Here we consider the following *restriction* of the `PartialResAll` problem:
  - The **multiplicity** version: resources are allowed to be picked multiple times.
  - Jobs have **unit** height.
- Call this problem the restricted `PartialResAll` problem.

Theorem

There is a polynomial time $O(\log \ell_{\max}/\ell_{\min})$-factor approximation for the restricted `PartialResAll` problem.

Here, $\ell_{\max}$ and $\ell_{\min}$ stands for the longest and shortest lengths of the **jobs**. Note that this can be easily seen to be bounded by n, the *number* of jobs.

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Prior Work

- Recall that for the `ResAll` problem ($\{0, 1\}$ or multiplicity versions), a 4-factor approximation was known [BarNoy et al.01].
- Only special cases of the `PartialResAll` problem have been considered in the literature. The case where resources can be picked multiple times, and where jobs have arbitrary height, but unit length was considered in [Chakravarthy et al.11]. They show that in this special case, the `PartialResAll` problem has a 16-factor approximation.
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Warmup

- From now on, all jobs will have unit height, and resources can be picked multiple times.
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The overall theme

- We would like to simplify the structure of jobs.
- We can do a decomposition according to job lengths to reduce the general problem to the following special case, where the jobs form a so-called **mountain range**.
- This happens at a loss of $O(\log \ell_{\max}/\ell_{\min})$-factor.
- This is similar to the work of [Bansal et al.09] – they gave a logarithmic approximation for the UFP problem on a line.
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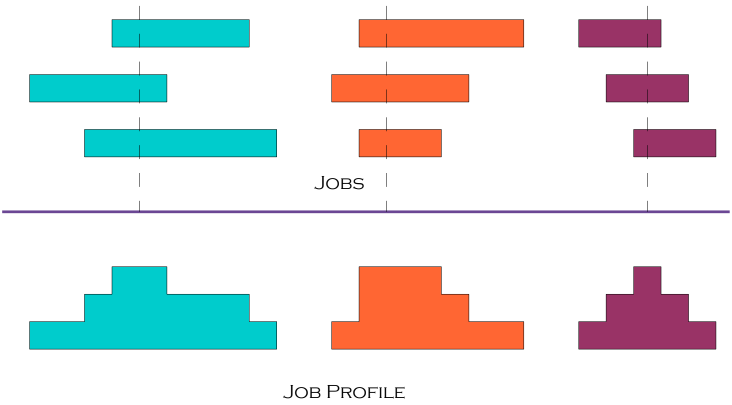
Example Mountain Range



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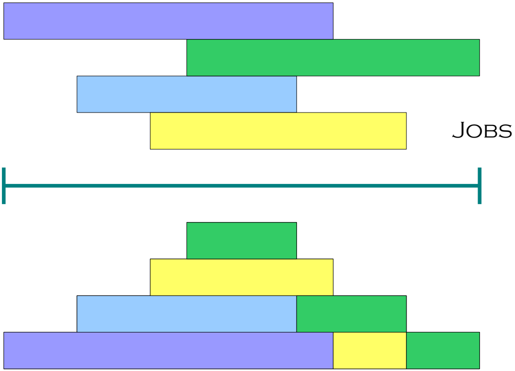
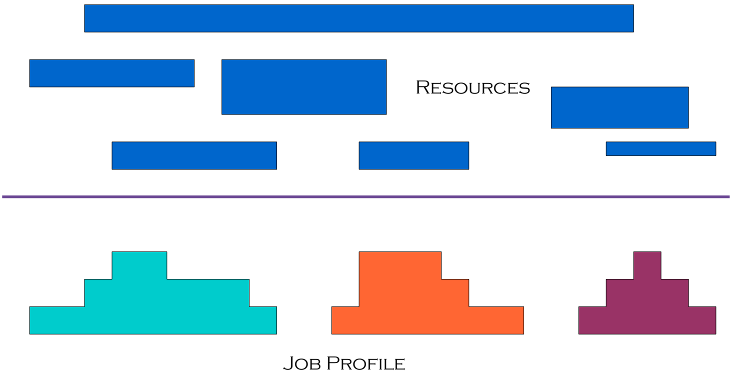


A SINGLE MOUNTAIN

A Natural Algorithm for a Single Mountain

- Recall that all jobs have the same height. A natural idea is to *drop* jobs that extend farthest to the right (or to the left) while still observing the partiality constraint.
- This actually works: this yields a 8-factor approximation for the case of a single mountain.

Back to a mountain range



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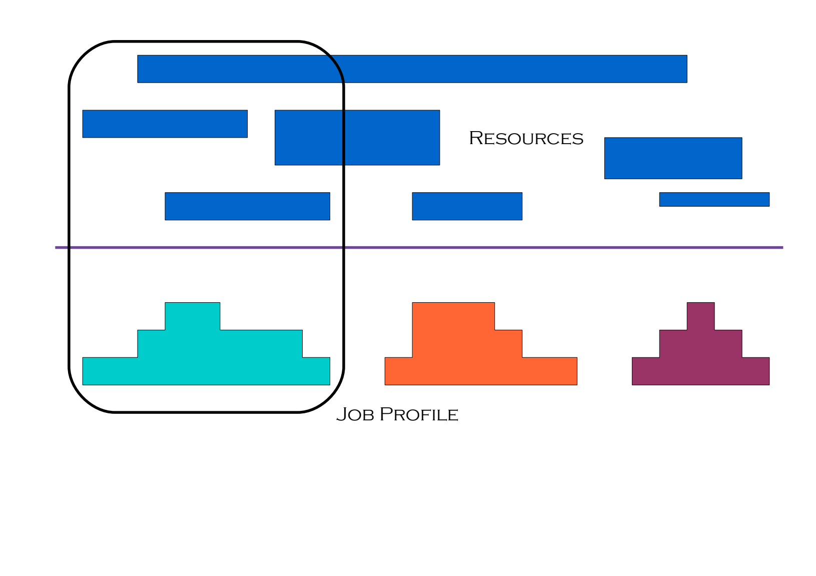

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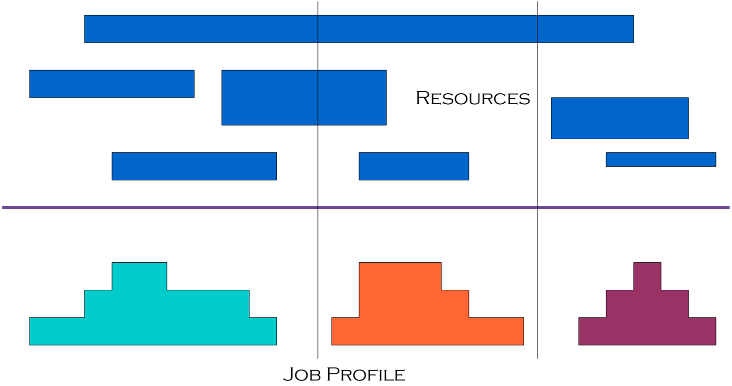

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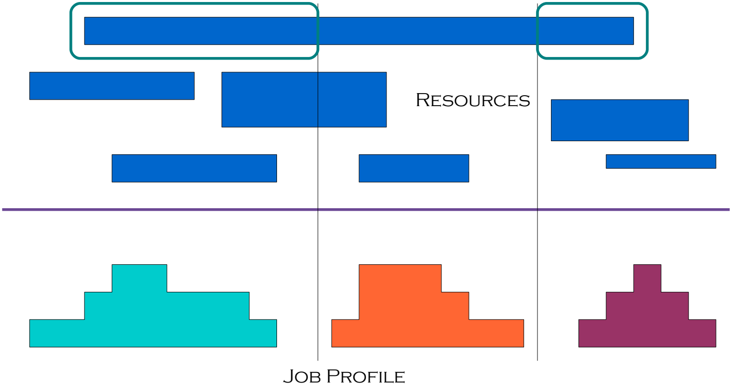

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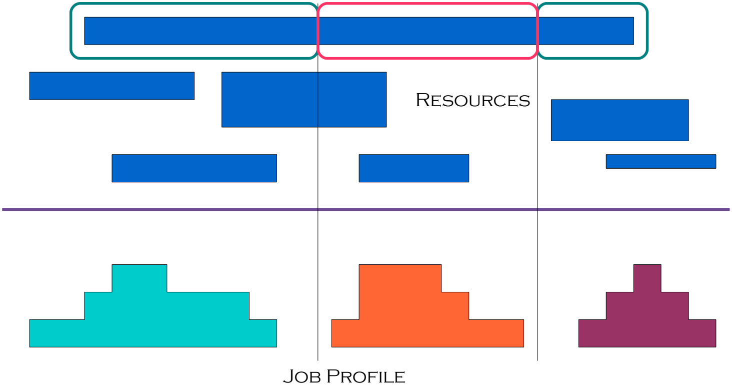

Resource Decomposition

- We decompose resources so that a resource fully spans a collection of (consecutive) single mountains (a long resource), or that a resource is contained in the span of only a single mountain (a short resource).
- This happens at a factor 3 loss.
- We can now “shrink" the mountains in a mountain range to timeslots, and thus we can modify our problem so that all the jobs are now one timeslot long.

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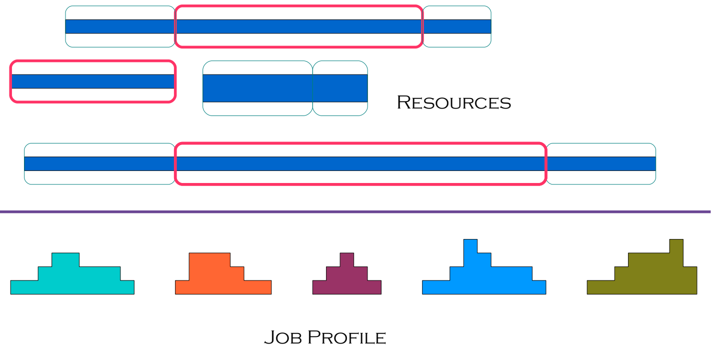

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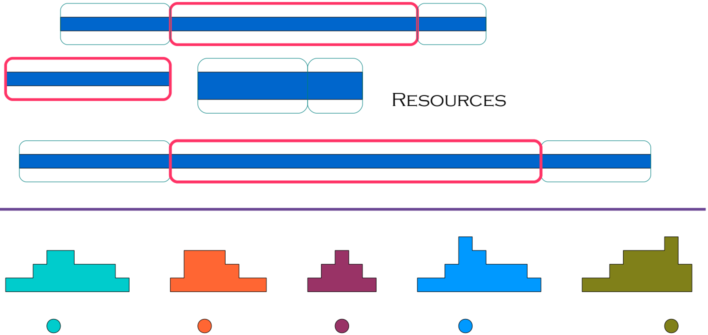

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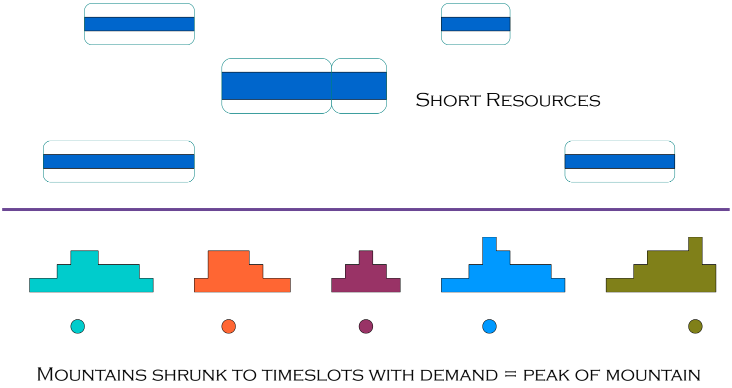

What happens to the short resources?

- The short resources along with the corresponding mountain form a “single mountain" `PartialResAll` problem.
- Solve this `PartialResAll` problem for various values of the partiality parameter k.
- For each (approximate) solution, place a resource of unit length on the corresponding timeslot of cost equal to that of the solution, and capacity equal to k.

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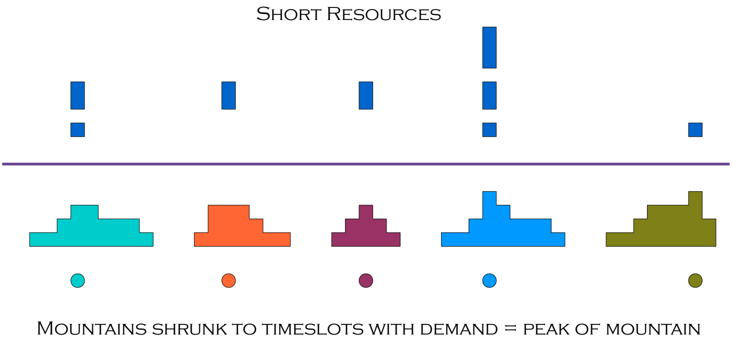

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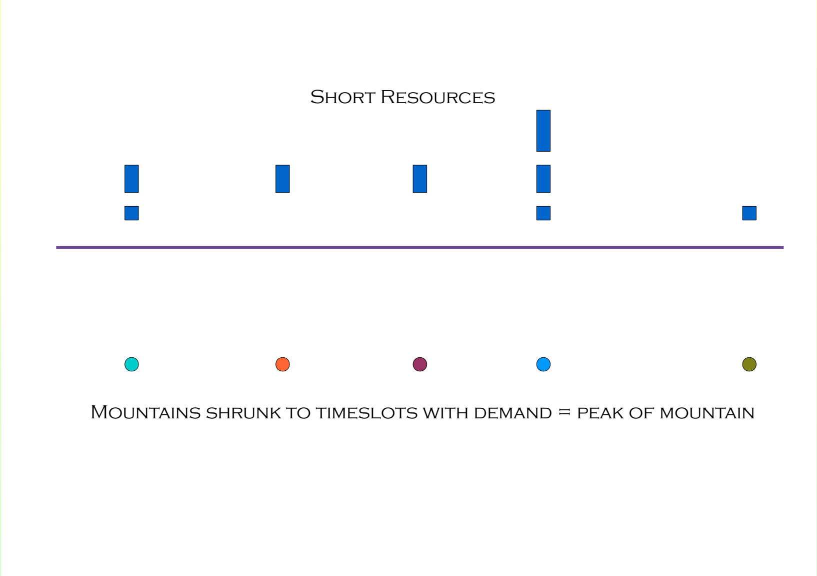

The long and short of it. . .

MOUNTAINS SHRUNK TO TIMESLOTS WITH DEMAND = PEAK OF MOUNTAIN

Upshot

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- Long resources are (parts) of original resources, so can be picked multiple times.
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Ruminations

- This is reminiscent of the $\{0, 1\}$ `PartialResAll` problem even when jobs are unit timeslot long.
- However, here the only resources restricted to be picked up at most once are the *short* resources.
- If only the long resources were present, we get a 16-factor approximation algorithm, via [Chakravarthy et al.11]. Why?
- If only the short resources are present, a simple DP gives the optimal solution.
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Main Result

- This completes a proof sketch of the $\log(\ell_{\max}/\ell_{\min})$-factor approximation for the restricted case of `PartialResAll`.

Auxiliary Result

- One can also define the “Prize Collecting" version of `ResAll`. We prove a 4-factor approximation for the Prize Collecting version.
- However, this approximation algorithm does not have the so-called LMP (Lagrangian Multiplier Preserving) property (as in the framework of [JainVazirani99]).
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Open Problems

- To prove a constant factor approximation for the $\{0, 1\}$ version of the `PartialResAll` problem.
- As a first step, prove this for the case where jobs are timeslots with demands (i.e. jobs have unit lengths).
- Remove the restriction of *unit heights* in the result discussed.
- Prove better hardness results. Strong NP-hardness?

THANK YOU.