Risk- and Variance-Aware Electricity Pricing

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***Abstract*—The roll-out of stochastic renewable energy sources (RES) undermines the efficiency of power system and market operations. This paper proposes an approach to derive electricity prices that internalize RES stochasticity. We leverage a chance-constrained AC Optimal Power Flow (CC AC-OPF) model, which is robust against RES uncertainty and is also aware of the resulting variability (variance) of the system state variables. Using conic duality theory, we derive and analyze energy and balancing reserve prices that internalize the risk of system limit violations and the variance of system state variables. We compare the risk- and variance-aware prices on the IEEE 118-node testbed.**

I. Introduction

Power systems and electricity markets struggle to accommodate the massive roll-out of renewable energy sources (RES), which are stochastic in nature and impose additional risks on the system operations and market-clearing decisions. The current industry practice to mitigate these risks is based on procuring additional reserves, which are selected based on exogenous and often ad-hoc policies (e.g., 95-percentile rule in ERCOT, [1], or (5+7) rule in CAISO, [2]).

Alternatively, such risk assessments can be carried out endogenously, i.e. while optimizing operational and market-clearing decisions, using high-fidelity prediction and historical data parameterizing the RES stochasticity. Bienstock et al. [3] proposed a *risk-aware* approach to solving an Optimal Power Flow (OPF) problem that uses chance constraints (CC) to internalize the RES stochasticity and risk tolerance of the system operator to violating system constraints. Since [3], the CC-OPF has been shown to scale efficiently for large networks [4], accommodate various assumptions on the RES stochasticity (e.g. parametric distributions and distributional robustness) [4]–[6], as well as to accurately account for AC power flow physics, [7], [8]. However, this framework has primarily been applied to risk-aware operational planning in a vertically integrated environment, neglecting market considerations. From a market design perspective, RES stochasticity has been primarily dealt with using scenario-based stochastic programming, e.g. [9]–[11], which is more computationally demanding than chance constraints, [3]. With the exception of our recent work in [12], [13], chance constraints have so far been overlooked in electricity pricing applications. The chance-constrained market design proposed in [13] leads to a stable robust equilibrium that, unlike scenario-based approaches in [9]–[11], guarantees welfare maximization, revenue adequacy and cost recovery under various assumptions on the RES stochasticity. However, [13] neglects network constraints, an important modeling feature for real-life market applications.

This paper uses a chance-constrained AC OPF (CC AC-OPF) from [7] to derive network-aware electricity prices that internalize the RES stochasticity with the intention to produce more accurate signals to market participants. This convex formulation allows the use of duality theory to derive risk-aware marginal-cost-based prices, which are similar to traditional deterministic locational marginal prices (LMPs) based on linear duality, [14]. Furthermore, the CC AC-OPF can explicitly consider reactive power and voltage support and analyze their role in the deliverabilty of active power, thus supporting the design of a more "complete"[1] electricity market, [15], [16]. We also extend the CC AC-OPF to follow a *variance-aware* dispatch paradigm, introduced in [17], to compute variance-aware prices and analyze the relationship between the system cost, risk and variance.

II. Model Formulation

A. Preliminaries

Consider a transmission network with set of nodes $\mathcal{N}$, set of lines $\mathcal{L}$, set of generators $\mathcal{G}$ and set of renewable generators $\mathcal{U}$ (e.g. wind or commercial solar farms). For simplicity of notation, we assume that each node hosts one conventional and one renewable generator, such that $\mathcal{G} = \mathcal{U} = \mathcal{N}$. We denote the set of PQ and PV nodes as $\mathcal{N}^{PQ}, \mathcal{N}^{PV} \subset \mathcal{N}$ and index reference (θV) node as $i = ref$. Let vectors p_G indexed as $p_{G,i}$, p_D indexed as $p_{D,i}$, and p_U indexed as $p_{U,i}$, denote the total active power output of conventional generators, the total active power demand and the active power injections from renewable generation at every node. The corresponding reactive power injections are denoted q_G, q_D, q_U and the resulting vectors of net active and reactive power injections are thus given by:

$$p = p_G - p_D + p_U, \tag{1a}$$

$$q = q_G - q_D + q_U. \tag{1b}$$

In the following, we assume that there is no curtailment of renewable generation and that that all loads p_D are fixed. We denote v and θ, indexed as v_i and θ_i, as the vectors of voltage magnitudes and voltage angles. The range of feasible voltage magnitudes is given as $v \in [v^{min}, v^{max}]$. Each line in $\mathcal{L}$ is a tuple ij denoting its connected nodes $i, j \in \mathcal{N}$. For simplicity, we assume a single line between two nodes. Vectors f^p and f^q indexed as f^p_{ij} and f^q_{ij} denote the active and reactive power flows from node i to node j. The vector of apparent power flow limits is denoted as s^{max}, indexed by s^{max}_{ij}. We summarize the physical relationship between p, q, f^p, f^q, v and θ as

$$F(p, q, v, \theta) = 0, \tag{2}$$

where $F(p, q, v, \theta)$ are the non-linear, non-convex AC power flow equations, [7, Eq. (2)].

[1] I.e. allowing all assets and services (reactive power, transmission and voltage support) to be transacted, [15].

B. Uncertain Power Injections

We model the real-time deviations from the forecasted renewable active power generation p_U by the random vector $\boldsymbol{\omega}$, indexed by $\boldsymbol{\omega}_i$, so that the real-time injection from uncertain renewable sources is given by $p_U(\boldsymbol{\omega}) = p_U + \boldsymbol{\omega}$. The expected value and covariance matrix of $\boldsymbol{\omega}$ are given by $\mathbb{E}[\boldsymbol{\omega}] = 0$ and $\mathrm{Var}[\boldsymbol{\omega}] = \Sigma$ and we write $\boldsymbol{\Omega} = e^\top \boldsymbol{\omega}$ and $S^2 = e^\top \Sigma e$, where e is the vector of ones. The corresponding uncertain reactive power $q_U(\boldsymbol{\omega})$ is linked to the active power generation through a constant power factor $\cos\phi_i$, i.e. $q_{U,i}(\boldsymbol{\omega}) = q_{U,i} + \gamma_i \boldsymbol{\omega}_i$, where $\gamma_i := \sqrt{1-\cos^2\phi_i}/\cos\phi_i$ can either be optimized or fixed in advance. Vector γ collects all $\gamma_i, i \in \mathcal{U}$.

C. System Response

To mitigate the effects of $\boldsymbol{\omega}$, the controllable generators adjust their output $p_G(\boldsymbol{\omega})$ and $q_G(\boldsymbol{\omega})$ to maintain the active and reactive power balance. Subsequently, system state variables $v(\boldsymbol{\omega})$, $\theta(\boldsymbol{\omega})$, $f^p(\boldsymbol{\omega})$, $f^q(\boldsymbol{\omega})$ will respond to those changes based on the the system controls and their physical relationship $F(p(\boldsymbol{\omega}), q(\boldsymbol{\omega}), v(\boldsymbol{\omega}), \theta(\boldsymbol{\omega})) = 0$.

As in [3], [7], [18] the response of each generator is given by participation factors $0 \leq \alpha_i \leq 1$ that represent the relative amount of the system-wide forecast error ($\boldsymbol{\Omega}$) that the generator at node i has to compensate for. Therefore, the real-time active power output of each generator is:

$$p_{G,i}(\boldsymbol{\omega}) = p_{G,i} - \alpha_i \boldsymbol{\Omega}, \tag{3}$$

and we require $\sum_{i\in\mathcal{G}} \alpha_i = 1$ to balance the system. Vector α collects all $\alpha_i, i \in \mathcal{G}$. The response of reactive power generation $q_{G,i}(\boldsymbol{\omega})$, voltage magnitudes $v_i(\boldsymbol{\omega})$ and voltage angles $\theta_i(\boldsymbol{\omega})$ is determined by the the type of node i. At PV nodes $v_i(\boldsymbol{\omega}) = v_i, \forall i \in \mathcal{N}^{PV}$, is controlled and $q_{G,i}(\boldsymbol{\omega}), \theta_i(\boldsymbol{\omega}), \forall i \in \mathcal{N}^{PV}$, are implicitly determined by power flow equations $F(p, q, v, \theta)$. Similarly, at PQ nodes $q_{G,i}(\boldsymbol{\omega}) = q_{G,i}, \forall i \in \mathcal{N}^{PQ}$, is controlled and $v_i(\boldsymbol{\omega}), \theta_i(\boldsymbol{\omega}), \forall i \in \mathcal{N}^{PQ}$, are implicitly determined by power flow equations $F(p, q, v, \theta)$. Finally, at the θV node $v_{ref}(\boldsymbol{\omega}) = v_{ref}$ and $\theta_{ref}(\boldsymbol{\omega}) = 0$. Thus, active and reactive power response at the θV node is also determined implicitly by power flow equations $F(p, q, v, \theta)$. The resulting active and reactive power flows are implicitly given by $f^p_{ij}(\boldsymbol{\omega}) = f^p_{ij}(v(\boldsymbol{\omega}), \theta(\boldsymbol{\omega}))$ and $f^q_{ij}(\boldsymbol{\omega}) = f^q_{ij}(v(\boldsymbol{\omega}), \theta(\boldsymbol{\omega}))$.

D. Production Cost

The production cost of each generator is approximated by a quadratic function, [19]:

$$c_i(p_{G,i}) = c_{2,i}(p_{G,i})^2 + c_{1,i} p_{G,i} + c_{0,i} \tag{4}$$

and, for the compactness of derivations, we denote $c_{2,i} = 1/2b_i$, $c_{1,i} = a_i/b_i$ and $c_{0,i} = a_i^2/2b_i$. Given uncertainty $\boldsymbol{\omega}$ and the response in (3), the expected operating cost is:

$$\mathbb{E}[c_i(g_i^P(\boldsymbol{\omega}))] = c_i(p_{G,i}) + \frac{\alpha_i^2}{2b_i} S^2. \tag{5}$$

E. Linearization of AC Power Flow Equations

As discussed in Section II-C, some system state variables are determined implicitly by the non-linear, non-convex AC power flow equations in (2), which do not permit a direct solution. Therefore, we linearize $F(p, q, v, \theta) = 0$ around a given (forecast) operating point using Taylor's theorem as in [7]. Let $(\bar{p}, \bar{q}, \bar{f}^p, \bar{f}^q, \bar{v}, \bar{\theta})$ be the linearization result, then the nodal power injections and line flows are:

$$p_i = \bar{p}_i + J_i^{p,v}(\bar{v}, \bar{\theta})v + J_i^{p,\theta}(\bar{v}, \bar{\theta})\theta \tag{6}$$

$$q_i = \bar{q}_i + J_i^{q,v}(\bar{v}, \bar{\theta})v + J_i^{q,\theta}(\bar{v}, \bar{\theta})\theta \tag{7}$$

$$f^p_{ij} = \bar{f}^p_{ij} + J_{ij}^{f^p,v}(\bar{v}, \bar{\theta})v + J_{ij}^{fp,\theta}(\bar{v}, \bar{\theta})\theta \tag{8}$$

$$f^q_{ij} = \bar{f}^q_{ij} + J_{ij}^{f^q,v}(\bar{v}, \bar{\theta})v + J_{ij}^{fq,\theta}(\bar{v}, \bar{\theta})\theta, \tag{9}$$

where $J_i^{p,v}, J_i^{p,\theta}, J_i^{q,v}, J_i^{q,\theta}, J_{ij}^{f^p}, J_{ij}^{fp,\theta}, J_{ij}^{f^q,v}, J_{ij}^{fq,\theta}$ are row-vectors of sensitivity factors describing the change of active and reactive nodal injections as functions of v and θ derived from the AC power flow linearization. Similarly, the response of voltages, flows and reactive power outputs to $\boldsymbol{\omega}$ can be modeled as (see Appendix A):

$$q_{G,i}(\boldsymbol{\omega}) = q_{G,i} + [R_i^q(I - \alpha e^\top) + X_i^q \operatorname{diag}(\gamma)]\boldsymbol{\omega} \tag{10}$$

$$v_i(\boldsymbol{\omega}) = v_i + [R_i^v(I - \alpha e^\top) + X_i^v \operatorname{diag}(\gamma)]\boldsymbol{\omega} \tag{11}$$

$$f^p_{ij}(\boldsymbol{\omega}) = f^p_{ij} + [R_{ij}^{f^p}(I - \alpha e^\top) + X_{ij}^{f^p} \operatorname{diag}(\gamma)]\boldsymbol{\omega} \tag{12}$$

$$f^q_{ij}(\boldsymbol{\omega}) = f^q_{ij} + [R_{ij}^{f^q}(I - \alpha e^\top) + X_{ij}^{f^q} \operatorname{diag}(\gamma)]\boldsymbol{\omega}, \tag{13}$$

where row-vectors R_i^q, R_i^v, $R_{ij}^{f^p}$, $R_{ij}^{f^q}$ map adjustments of the respective variables to active power changes, row-vectors X_i^q, X_i^v, $X_{ij}^{f^p}$, $X_{ij}^{f^q}$ map adjustments of the respective variables to reactive power changes and I is the identity matrix. Note that sensitivity vectors $R_i^q, X_i^q, R_i^v, X_i^v, R_{ij}^{f^p}, X_{ij}^{f^p}, R_{ij}^{f^q}, X_{ij}^{f^q}$ can be zero, if i is a PV or PQ node, and depend on a chosen linearization point.

F. Chance Constrained Optimal Power Flow

For a given operating point $(p_G, q_G, v, \theta, \gamma, \alpha)$ the system will respond to any realization of $\boldsymbol{\omega}$ according to (3), (6)–(13). To ensure that this system response does not violate the physical system limits with a high probability, we formulate the following chance constraints:

$$\mathbb{P}(p_{G,i}^{min} \leq p_{G,i}(\boldsymbol{\omega}) \leq p_{G,i}^{max}) \geq 1 - 2\epsilon_p \qquad i \in \mathcal{G} \tag{14}$$

$$\mathbb{P}(q_{G,i}^{min} \leq q_{G,i}(\boldsymbol{\omega}) \leq q_{G,i}^{max}) \geq 1 - 2\epsilon_q \qquad i \in \mathcal{G} \tag{15}$$

$$\mathbb{P}(v_i^{min} \leq v_i(\boldsymbol{\omega}) \leq v_i^{max}) \geq 1 - 2\epsilon_v, \qquad i \in \mathcal{N} \tag{16}$$

$$\mathbb{P}((f^p_{ij}(\boldsymbol{\omega}))^2 + (f^q_{ij}(\boldsymbol{\omega}))^2 \leq (s_{ij}^{max})^2) \geq 1 - \epsilon_f \qquad ij \in \mathcal{L}, \tag{17}$$

where ϵ_p, ϵ_q, ϵ_v, $\epsilon_f < 1/2$ can be chosen to tune the risk level associated with the individual chance constraints. Using (10)–(13), we can obtain computationally tractable reformulations of chance constraints (14)–(17), [3], [7], [18], and formulate the deterministic equivalent of the CC AC-OPF:

$$\text{EQV-CC}: \qquad \min_{\substack{p_G, q_G \\ v, \alpha, \theta}} \sum_{i\in\mathcal{G}} c_i(p_{G,i}) + \sum_{i\in\mathcal{G}} \frac{\alpha_i^2}{2b_i} S^2 \tag{18a}$$

s.t.

$$(\lambda_i^p, \lambda_i^q): \qquad (6), (7) \tag{18b}$$

$$(\beta_{ij}^p, \beta_{ij}^q): \qquad (8), (9) \tag{18c}$$

$$(\chi): \qquad \sum_{i\in\mathcal{G}} \alpha_i = 1 \tag{18d}$$

$$(\delta_i^{p,+}): \qquad p_{G,i} + \alpha_i z_{\epsilon_p} S \leq p_{G,i}^{max} \qquad i \in \mathcal{G} \tag{18e}$$

$$(\delta_i^{p,-}): \qquad -p_{G,i} + \alpha_i z_{\epsilon_p} S \leq -p_{G,i}^{min} \qquad i \in \mathcal{G} \tag{18f}$$

$$(\delta_i^{q,+}): \qquad q_{G,i} + z_{\epsilon_q} t_i^q \leq q_{G,i}^{max} \qquad i \in \mathcal{G} \tag{18g}$$

$$(\delta_i^{q,-}): \qquad -q_{G,i} + z_{\epsilon_q} t_i^q \leq -q_{G,i}^{min} \qquad i \in \mathcal{G} \tag{18h}$$

$$(\zeta_i^q): \quad \left\|\left(R_i^q - \rho_i^q e^\top + X_i^q \operatorname{diag}(\gamma)\right)\Sigma^{1/2}\right\|_2 \le t_i^q \quad i \in \mathcal{G} \tag{18i}$$

$$(\nu_i^q): \quad R_i^q \alpha = \rho_i^q \quad i \in \mathcal{G} \tag{18j}$$

$$(\mu_i^+): \quad v_i + z_{\epsilon_v} t_i^v \le v_i^{max} \quad i \in \mathcal{N} \tag{18k}$$

$$(\mu_i^-): \quad -v_i + z_{\epsilon_v} t_i^v \le -v_i^{min} \quad i \in \mathcal{N} \tag{18l}$$

$$(\zeta_i^v): \quad \left\|\left(R_i^v - \rho_i^v e^\top + X_i^v \operatorname{diag}(\gamma)\right)\Sigma^{1/2}\right\|_2 \le t_i^v \quad i \in \mathcal{G} \tag{18m}$$

$$(\nu_i^v): \quad R_i^v \alpha = \rho_i^v \quad i \in \mathcal{N} \tag{18n}$$

$$(\eta_{ij}): \quad (a_{ij}^{f^p})^2 + (a_{ij}^{f^q})^2 \le (s_{ij}^{max})^, \quad ij \in \mathcal{L} \tag{18o}$$

$$(\xi_{ij}^{f^p,+}): \quad -a_{ij}^{f^p} + z_{\epsilon_f/2.5} t_{ij}^{f^p} \le f_{ij}^p \quad ij \in \mathcal{L} \tag{18p}$$

$$(\xi_{ij}^{f^p,-}): \quad -a_{ij}^{f^p} + z_{\epsilon_f/2.5} t_{ij}^{f^p} \le -f_{ij}^p \quad ij \in \mathcal{L} \tag{18q}$$

$$(\xi_{ij}^{f^p,0}): \quad z_{\epsilon/5} t_{ij}^{f^p} \le a_{ij}^{f^p} \quad ij \in \mathcal{L} \tag{18r}$$

$$(\xi_{ij}^{f^q,+}): \quad -a_{ij}^{f^q} + z_{\epsilon_f/2.5} t_{ij}^{f^q} \le f_{ij}^q, \quad ij \in \mathcal{L} \tag{18s}$$

$$(\xi_{ij}^{f^q,-}): \quad -a_{ij}^{f^q} + z_{\epsilon_f/2.5} t_{ij}^{f^q} \le -f_{ij}^q, \quad ij \in \mathcal{L} \tag{18t}$$

$$(\xi_{ij}^{f^q,0}): \quad z_{\epsilon_f/5} t_{ij}^{f^q} \le a_{ij}^{f^q} \quad ij \in \mathcal{L} \tag{18u}$$

$$(\zeta_{ij}^\diamond): \quad \left\|\left(R_i^\diamond - \rho_i^\diamond e^\top + X_i^\diamond \operatorname{diag}(\gamma)\right)\Sigma^{1/2}\right\|_2 \le t_i^\diamond \quad ij \in \mathcal{L}, \diamond = f^p, f^q \tag{18v}$$

$$(\nu_{ij}^\diamond): \quad R_{ij}^\diamond \alpha = \rho_{ij}^\diamond \quad ij \in \mathcal{L}, \diamond = f^p, f^q, \tag{18w}$$

where Greek letters in parentheses in (18b)–(18w) denote dual multipliers of constraints. Objective (18a) minimizes the expected cost as in (5). Eqs. (18b)–(18c) are the active and reactive power balances and flows based on the linearized AC power flow equations. Eq. (18d) is the balancing reserve adequacy constraint and (18e)–(18w) are the deterministic reformulation of chance constraints (14)–(17), [7]. Constraints (18e)–(18f) limit the active power production $p_{G,i}$ and the amount of reserve $\alpha_i z_{\epsilon_p} S$ provided by each generator, [13], [20]. Risk parameters are given by $z_\epsilon = \Phi^{-1}(1-\epsilon)$, where $\Phi^{-1}(1-\epsilon)$ is the $(1-\epsilon)$-quantile of the standard normal distribution, if $\boldsymbol{\omega}$ follows a normal distribution, and Φ^{-1} can be chosen as described in [21]. The standard deviation of reactive power outputs, voltage levels and flows resulting from the the uncertainty and the system response is given by the SOC constraints (18i), (18m) and (18v). Given the convexity of the SOC constraints, auxiliary variables t_i^q, t_i^v, $t_{ij}^{f^p}$, $t_{ij}^{f^q}$ relate these standard deviations to the reactive output limits (18g)–(18h), voltage bounds (18k)–(18l) and flow limits (18p)–(18u). Due to its quadratic dependency on the uncertain variable, the chance constraint in (17) requires a more complex reformulation than (14)–(16). To accommodate this reformulation, we follow [7] and introduce auxiliary variables $a_{ij}^{f^p}$, $a_{ij}^{f^q}$ and risk parameters $\epsilon_f/2.5$ and $\epsilon_f/5$ (i.e. ϵ_f divided by 2.5 and 5, respectively, see [7]). Auxiliary variables ρ_i^v, $\rho_{ij}^{f^p}$, $\rho_{ij}^{f^q}$ and constraints (18j), (18n) and (18w) have been introduced to simplify subsequent derivations.

III. Risk-Aware Pricing

The EQV-CC endogenously trades off the expected operating point $(p_G, q_G, v, \theta, \gamma, \alpha)$ and the risk of system limit violations defined by the choice of parameters $z_{\epsilon_g}, z_{\epsilon_q}, z_{\epsilon_v}, z_{\epsilon_f/2.5}, z_{\epsilon_f/5}$. Since the EQV-CC is a convex program, we can use its dual form to compute the marginal prices for active and reactive power, and balancing reserve that internalize this trade-off.

A. Prices with Chance Constraints on Generation

First, we consider a modification of the EQV-CC given as:

$$\text{GEN-CC}: \quad \min_{\substack{p_G, q_G \\ v, \alpha, \theta}} \sum_{i \in \mathcal{N}} c_i(p_{G,i}) + \sum_{i \in \mathcal{N}} \frac{\alpha_i^2}{2b_i} S^2 \tag{19a}$$

$$\text{s.t.} \quad \text{(18b)–(18f)}$$

$$(\delta_i^{q,+}, \delta_i^{q,-}): \quad q_{G,i}^{min} \le q_{G,i} \le q_{G,i}^{max} \tag{19b}$$

$$(\mu_i^-, \mu_i^+): \quad v_i^{min} \le v_i \le v_i^{max} \tag{19c}$$

$$(\eta_{ij}): \quad (f_{ij}^p)^2 + (f_{ij}^q)^2 \le (s_{ij}^{max})^2, \tag{19d}$$

where, relative to the EQV-CC in (18), chance constraints are only enforced on active power generation limits and reactive power, voltage and power flow constraints are enforced deterministically by (19b)–(19d). In other words, the GEN-CC determines the optimal balancing participation of each generator and, thus, the optimal amount and allocation of committed reserve given by $\alpha_i z_{\epsilon_g} S$. Therefore, the GEN-CC replicates a traditional deterministic OPF that allocates the reserve requirement ($\sum_{i \in \mathcal{G}} \alpha_i z_{\epsilon_g} S = z_{\epsilon_g} S$) among individual generators, see [7].

Using the GEN-CC, we compute the following prices:

Proposition 1. Consider the GEN-CC in (19). Let λ_i^p, λ_i^q be dual multipliers of the nodal active and reactive power balance at node i in (18b). Then λ_i^p and λ_i^q are given as:

$$\lambda_i^p = \frac{p_{G,i} + a_i}{b_i} + \delta_i^{p,-} - \delta_i^{p,+} \tag{20}$$

$$\lambda_i^q = \delta_i^{q,-} - \delta_i^{q,+}. \tag{21}$$

Proof. The first order optimality conditions of (19) for $p_{G,i}$, $q_{G,i}$, α_i, f_{ij}^p, f_{ij}^q are:

$$(p_{G,i}): \quad \lambda_i^p + (\delta_i^{p,+} - \delta_i^{p,-}) = \frac{p_{G,i} + a_i}{b_i} \quad i \in \mathcal{G} \tag{22a}$$

$$(q_{G,i}): \quad \lambda_i^q + (\delta_i^{q,+} - \delta_i^{q,-}) = 0 \quad i \in \mathcal{G} \tag{22b}$$

$$(\alpha_i): \quad z_{\epsilon_p} S(\delta_i^{p,+} + \delta_i^{p,-}) + \chi = \frac{\alpha_i}{b_i} S^2 \quad i \in \mathcal{G} \tag{22c}$$

$$(f_{ij}^p): \quad 2 f_{ij}^p \eta_{ij} + \beta_{ij}^{f^p} = 0 \quad ij \in \mathcal{L} \tag{22d}$$

$$(f_{ij}^q): \quad 2 f_{ij}^q \eta_{ij} + \beta_{ij}^{f^q} = 0 \quad ij \in \mathcal{L}. \tag{22e}$$

Eqs. (20)–(21) follow directly from (22a)–(22b). $\square$

Dual multiplier λ_i^p of the active power balance, itemized in (20), is interpreted as the real power LMP at node i and a function of production cost coefficients a_i, b_i and scarcity rent $\delta_i^{p,+}$, $\delta_i^{p,-}$ related to active generation limits. Dual multiplier λ_i^q of the reactive power balance, itemized in (21), is interpreted as the reactive power LMP given by scarcity rent $\delta_i^{q,+}$, $\delta_i^{q,-}$ related to reactive generation limits. Although there is no explicit production cost for reactive power in (18a), providing reactive power can have a non-zero value, if at least one reactive power limit is binding. Further, Proposition 1 shows that both λ_i^p and λ_i^q in (20)–(21) do not explicitly depend on uncertainty and risk parameters.

In contrast, the price for balancing reserve explicitly depends on the uncertainty and set risk levels:

Proposition 2. Consider the GEN-CC in (19). Let χ be the dual multiplier of the balancing adequacy constraint in (18d). Then χ is given as:

$$\chi = \frac{1}{\sum_{i\in\mathcal{G}} b_i}\Big(S^2 + z_{\epsilon_p} S \sum_{i\in\mathcal{G}} b_i(\delta_i^{p,+} + \delta_i^{p,-})\Big). \tag{23}$$

Proof. Using (18d) to eliminate α_i in (22c) yields (23). □

Dual multiplier χ of (18d) is interpreted as the price for balancing reserve, because it enforces sufficiency of the system-wide reserve. As per (23), χ is an explicit function of the uncertainty $S^2 = e^\top \Sigma e$ and risk parameter z_{ϵ_g}. Notably, the balancing reserve price is always non-zero, if there is uncertainty in the system (i.e. $S > 0$), even if all constraints (18e)–(18f) are inactive, i.e. $\delta_i^{p,+} = \delta_i^{p,-} 0, \forall i \in \mathcal{G}$. In this case, χ is independent of the risk parameters and is determined by the total uncertainty S^2 weighted by the total marginal generator cost $\sum_{i\in\mathcal{G}} b_i$ of all generators, $i \in \mathcal{G}$, including those generators that do not provide any balancing reserve, i.e. $\alpha_i = 0$.

B. Prices with Complete Chance Constraints

We now consider the complete EQV-CC in (18), i.e. including chance constraints on reactive power generation, voltages and flows, and prove the following proposition:

Proposition 3. Consider the EQV-CC in (18). Let λ_i^p, λ_i^q be dual multipliers of the nodal active and reactive power balances at node i as in (18b). Further, let χ be the dual multiplier of the balancing adequacy constraint in (18d). Then (i) λ_i^p and λ_i^q are given as (20)–(21) and (ii) χ is given as:

$$\chi = \frac{1}{\sum_{i\in\mathcal{G}} b_i}\Big(\overbrace{S^2 + z_\epsilon S \sum_{i\in\mathcal{G}} b_i(\delta_i^+ + \delta_i^-)}^{\text{Influenced by generator decisions}} + \overbrace{\sum_{i\in\mathcal{G}} b_i(y_i^q + y_i^v + y^{f^p} + y^{f^q})}^{\text{Influenced by system decisions}}\Big), \tag{24}$$

where:

$$y_i^q = z_{\epsilon_q} \sum_{j\in\mathcal{G}} [R_j^q]_i \delta_j^q \frac{(R_j^q + X_j^q \operatorname{diag}(\gamma))\Sigma e - R_j^q \alpha S^2}{\sigma_{q_{G,j}}(\alpha,\gamma)} \tag{25}$$

$$y_i^v = z_{\epsilon_v} \sum_{j\in\mathcal{N}} [R_j^v]_i \mu_j \frac{(R_j^v + X_j^v \operatorname{diag}(\gamma))\Sigma e - R_j^v \alpha S^2}{\sigma_{v_j}(\alpha,\gamma)} \tag{26}$$

$$y_i^\diamond = 2 \sum_{jk\in\mathcal{L}} [R_{jk}^\diamond]_i \zeta_{ij}^\diamond \frac{(R_{jk}^\diamond + X_{jk}^\diamond \operatorname{diag}(\gamma))\Sigma e - R_{jk}^\diamond \alpha S^2}{\sigma_{\diamond_{jk}}(\alpha,\gamma)}, \tag{27}$$

where $\diamond = f^p, f^q$ and $\delta_j^q = \delta_j^{q,+} + \delta_j^{q,-}$, $\mu_j = \mu_j^+ + \mu_j^-$, and $\zeta_{ij}^\diamond = z_{\epsilon_f/2.5}(\xi_{ij}^{\diamond,+} + \xi_{ij}^{\diamond,-}) + z_{\epsilon_f/5}\xi_{ij}^{\diamond,0}$. Terms $\sigma_{q_{G,j}}(\alpha,\gamma), \sigma_{v_j}(\alpha,\gamma)$, $\sigma_{f^p_{jk}}(\alpha,\gamma)$, $\sigma_{f^q_{jk}}(\alpha,\gamma)$ denote the standard deviations of reactive power at node j, voltage at node j, active power flow on line jk and reactive power flow on line jk, respectively, and $[\cdot]_i$ denotes the i-th element of a vector.

Proof. The first order optimality conditions of (18) for $p_{G,i}$, $q_{G,i}$, α_i, f_{ij}^p, f_{ij}^q and auxiliary variables are:

(22a), (22b), (28b) and (28c)

$$(\alpha_i): \quad \chi + z_{\epsilon_p} S(\delta_i^{p,+} + \delta_i^{p,-}) + \sum_{j\in\mathcal{G}} \nu_j^q [R_j^q]_i + \sum_{j\in\mathcal{N}} \nu_j^v [R_j^v]_i + \sum_{jk\in\mathcal{L}} \nu_{jk}^{f^p}[R_{jk}^{f^p}]_i + \sum_{jk\in\mathcal{L}} \nu_{jk}^{f^q}[R_{jk}^{f^q}]_i = \frac{\alpha_i}{b_i} S^2 \qquad i \in \mathcal{G} \tag{28a}$$

$$(t_i^q): \quad z_{\epsilon_p}(\delta_i^{q,+} + \delta_i^{q,-}) - \zeta_i^q = 0 \qquad i \in \mathcal{G} \tag{28b}$$

$$(\rho_i^q): \quad \zeta_i^q \frac{(R_i^q - \rho_i^q e^\top + X_i^q \operatorname{diag}(\gamma))\Sigma e}{\left\|(R_i^q - \rho_i^q e^\top + X_i^q \operatorname{diag}(\gamma))\,\Sigma^{1/2}\right\|_2} - \nu_i^q = 0 \qquad i \in \mathcal{G} \tag{28c}$$

$$(\rho_i^v): \quad \zeta_i^v \frac{(R_i^v - \rho_i^v e^\top + X_i^v \operatorname{diag}(\gamma))\Sigma e}{\left\|(R_i^v - \rho_i^v e^\top + X_i^v \operatorname{diag}(\gamma))\,\Sigma^{1/2}\right\|_2} - \nu_i^v = 0 \qquad i \in \mathcal{N} \tag{28d}$$

$$(t_i^v): \quad z_{\epsilon_v}(\mu_i^+ + \mu_i^-) - \zeta_i^v = 0 \qquad i \in \mathcal{N} \tag{28e}$$

$$(f_{ij}^p): \quad \beta_{ij}^{f^p} - \xi_{ij}^{f^p,+} + \xi_{ij}^{p,-} = 0 \qquad ij \in \mathcal{L} \tag{28f}$$

$$(f_{ij}^q): \quad \beta_{ij}^q - \xi_{ij}^{f^q,+} + \xi_{ij}^{f^q,-} = 0 \qquad ij \in \mathcal{L} \tag{28g}$$

$$(\rho_{ij}^\diamond): \quad \zeta_i^v \frac{(R_{ij}^\diamond - \rho_{ij}^\diamond e^\top + X_{ij}^\diamond \operatorname{diag}(\gamma))\Sigma e}{\left\|(R_{ij}^\diamond - \rho_i^v e^\top + X_{ij}^\diamond \operatorname{diag}(\gamma))\,\Sigma^{1/2}\right\|_2} - \nu_{ij}^\diamond = 0 \qquad ij \in \mathcal{L}, \diamond = f^p, f^q \tag{28h}$$

$$(a_{ij}^\diamond): \quad 2\eta_{ij} a_{ij}^\diamond - (\xi_{ij}^{\diamond,+} + \xi_{ij}^{\diamond,-}) - \xi_{ij}^{\diamond,0} = 0 \qquad ij \in \mathcal{L}, \diamond = f^p, f^q \tag{28i}$$

$$(t_{ij}^\diamond): \quad z_{\epsilon_f/2.5}(\xi_{ij}^{\diamond,+} + \xi_{ij}^{\diamond,-}) + z_{\epsilon_f/5}\xi_{ij}^{\diamond,0} - \zeta_{ij}^\diamond = 0 \qquad ij \in \mathcal{L}, \diamond = f^p, f^q \tag{28j}$$

The result (i) follows directly from the proof of Proposition 1. The result (ii) follows from (28a) by eliminating α_i using (18d). Note that terms ν_i^q, ν_i^v, $\nu_{ij}^{f^p}$, $\nu_{ij}^{f^q}$ are given by (28c), (28d) and (28h). Further, $t_i^q = \sigma_{q_{G,i}}(\alpha,\gamma)$, if $\zeta_i^q > 0$ as per (18i), $t_i^v = \sigma_{v_j}(\alpha,\gamma)$, if $\zeta_i^v > 0$ as per (18m) and $t_{ij}^\diamond = \sigma_{\diamond_{jk}}(\alpha,\gamma)$, if $\zeta_{ij}^\diamond > 0$ as per (18v) for $\diamond = f^p, f^q$. Thus, for any $\zeta_i^q, \zeta_i^v, \zeta_{ij}^{f^p}, \zeta_{ij}^{f^q} = 0$ the dependency on the standard deviation would disappear. Finally, terms $\zeta_i^q, \zeta_i^v, \zeta_{ij}^{f^p}, \zeta_{ij}^{f^q}$ are given by (28b), (28e) and (28j). □

Similar to the result of Proposition 1, prices λ_i^p and λ_i^q do not explicitly depend on uncertainty and risk parameters. On the other hand, relative to (23), balancing reserve price χ depends on additional terms y_i^q, y_i^v, $y_i^{f^p}$, $y_i^{f^q}$, see (24), that relate the balancing reserve provided by each generator at node i to the risk of reactive power and voltage limits violation at every node $j \in \mathcal{N}$ and to the risk of power flow violations on every line $jk \in \mathcal{L}$. This risk awareness is not part of the generator decisions, which are only driven by its own production limits and cost, as indicated in (24).

IV. Variance-Aware Pricing

The risk-aware results of the EQV-CC in (18) yield solutions with a high variability (variance) of system state variables, which has been shown to complicate real-time operations, [17], [22]. The variances of reactive power generation, voltage magnitudes, and active and reactive flows can directly be computed from the standard deviations related to t_i^q, t_i^v, $t_{ij}^{f^p}$, $t_{ij}^{f^q}$, respectively. We introduce the metric $V(t_i^q, t_i^v, t_i^{f^p}, t_i^{f^q})$ that models a connection between the variances and system cost in the following variance-aware formulation:

$$\text{VA-CC}: \quad \min_{\substack{p_G, q_G\\ v, \alpha, \theta}} \sum_{i\in\mathcal{N}} c_i(p_{G,i}) + \sum_{i\in\mathcal{N}} \frac{\alpha_i^2}{b_i} S^2 + V(t_i^q, t_i^v, t_{ij}^{f^p}, t_{ij}^{f^q})$$
$$\text{s.t.} \quad \text{(18b)–(18w)}. \tag{29}$$

Specifically, metric $V(\cdot)$ penalizes the variance of state variables, thus imposing soft constraints on the variances as discussed in [17], and is defined as:

$$V(t_i^q, t_i^v, t_{ij}^{f^p}, t_{ij}^{f^q}) = \sum_{i\in\mathcal{G}}(\Psi_i^q(t_i^q)^2) + \sum_{i\in\mathcal{N}}\Psi_i^v(t_i^v)^2 + \sum_{ij\in\mathcal{L}}(\Psi_{ij}^{f^p}(t_{ij}^{f^p})^2 + \Psi_i^{f^q}(t_{ij}^{f^q})^2), \tag{30}$$

where Ψ_i^q, Ψ_i^v, $\Psi_{ij}^{f^p}$, $\Psi_{ij}^{f^q}$ are variance penalty factors in the units of [\$/MVAr²], [\$/V²], [\$/MW²] and [\$/MVAr²], respectively.

Proposition 4. Consider the VA-CC in (29). Let λ_i^p, λ_i^q be dual multipliers of the nodal active and reactive power balance at node i as in (18b). Further, let χ be the dual multiplier of the the balancing adequacy constraint in (18d). Then (i) λ_i^p and λ_i^q are given by (20)–(21) and (ii) χ is given as:

$$\chi=\frac{1}{\sum_{i\in\mathcal{G}}b_i}\Big(S^2+z_\epsilon S\sum_{i\in\mathcal{G}} b_i(\delta_i^+ + \delta_i^-) + \sum_{i\in\mathcal{G}} b_i(y_i^q + y_i^v + y^{f^p} + y^{f^q})\Big), \tag{31}$$

where:

$$y_i^q = \sum_{j\in\mathcal{G}}[R_j^q]_i\zeta_j^q\frac{(R_j^q+X_j^q\operatorname{diag}(\gamma))\Sigma e - R_j^q\alpha S^2}{\sigma_{q_{G,j}}(\alpha,\gamma)} \tag{32}$$

$$y_i^v = \sum_{j\in\mathcal{N}}[R_j^v]_i\zeta_j^v\frac{(R_j^v+X_j^v\operatorname{diag}(\gamma))\Sigma e - R_j^v\alpha S^2}{\sigma_{v_j}(\alpha,\gamma)} \tag{33}$$

$$y_i^\diamond = 2\sum_{jk\in\mathcal{L}}[R_{jk}^\diamond]_i\zeta_{ij}^\diamond\frac{(R_{jk}^\diamond+X_{jk}^\diamond\operatorname{diag}(\gamma))\Sigma e - R_{jk}^\diamond\alpha S^2}{\sigma_{\diamond_{jk}}(\alpha,\gamma)} \tag{34}$$

$$\zeta_j^q = z_{\epsilon_q}(\delta_j^{q,+}+\delta_j^{q,-}) - 2\sigma_{q_{G_j}}(\alpha,\gamma)\Psi_j^q \tag{35}$$

$$\zeta_j^v = z_{\epsilon_v}(\mu_j^+ + \mu_j^-) - 2\sigma_{v_j}(\alpha,\gamma)\Psi_j^v \tag{36}$$

$$\zeta_{jk}^\diamond = z_{\epsilon_{f/2.5}}(\xi_{ij}^{\diamond,+}+\xi_{ij}^{\diamond,-}) + z_{\epsilon_{f/5}}\xi_{ij}^{\diamond,0} - 2\sigma_{\diamond_{jk}}(\alpha,\gamma)\Psi_j^\diamond \tag{37}$$

where $\diamond = f^p, f^q$.

Proof. The first-order optimality conditions of (29) for $p_{G,i}$, $q_{G,i}$, α_i, f_{ij}^p, f_{ij}^q and auxiliary variables are:

(22a), (22b), (28c), (28d) and (28f)–(28i)

$$(\alpha_i): \quad z_{\epsilon_p}S(\delta_i^{p,+}+\delta_i^{p,-}) + \chi + \sum_{j\in\mathcal{G}}\nu_j^q[R_j^q]_i + \sum_{j\in\mathcal{N}}\nu_j^v[R_j^v]_i + \sum_{jk\in\mathcal{L}}\nu_{jk}^\diamond[R_{jk}^\diamond]_i = (\frac{1}{b_i}+2\Psi_i^p)\alpha_i S^2 \quad i\in\mathcal{G}, \diamond = f^p, f^q \tag{38a}$$

$$(t_i^q): \quad z_{\epsilon_p}(\delta_i^{q,+}+\delta_i^{q,-}) - \zeta_i^q = 2t_i^q\Psi_i^q \qquad i\in\mathcal{G} \tag{38b}$$

$$(t_i^v): \quad z_{\epsilon_v}(\mu_i^+ + \mu_i^-) - \zeta_i^v = 2t_i^v\Psi_i^v \qquad i\in\mathcal{N} \tag{38c}$$

$$(t_{ij}^\diamond): \quad z_{\epsilon_{f/2.5}}(\xi_{ij}^{\diamond,+}+\xi_{ij}^{\diamond,-}) + z_{\epsilon_{f/5}}\xi_{ij}^{\diamond,0} - \zeta_{ij}^\diamond = 2t_{ij}^\diamond\Psi_{ij}^\diamond \quad ij\in\mathcal{L}, \diamond = f^p, f^q \tag{38d}$$

The result (i) follows directly from the proof of Proposition 1. The result (ii) follows from re-arranging (38a) using (18d) to eliminate α_i. Note that terms ν_i^q, ν_i^v, $\nu_{ij}^{f^p}$, $\nu_{ij}^{f^q}$ are given by (28c), (28d) and (28h) and terms (35)–(37) follow from (38b)–(38d). Similarly to the proof of Proposition 3, $t_i^v = \sigma_{v_j}(\alpha,\gamma)$, if $\zeta_i^v > 0$ as per (18m), $t_{ij}^{f^p} = \sigma_{f^p{}_{jk}}(\alpha,\gamma)$, if $\zeta_{ij}^{f^p} > 0$ as per (18m), and $t_{ij}^{f^q} = \sigma_{f^p{}_{jk}}(\alpha,\gamma)$, if $\zeta_{ij}^{f^q} > 0$ as per (18v). □

Relative to the results of Proposition 3, terms y_i^q, y_i^v, $y_i^{f^p}$, $y_i^{f^q}$ now include an inherent trade-off between the risk of limit violation and the absolute standard deviations weighted by penalty factors Ψ_i^p, Ψ_i^q, Ψ_i^v, $\Psi_{ij}^{f^p}$, $\Psi_{ij}^{f^q}$, see (35)–(37). Since dual multipliers $\zeta_j^q, \zeta_j^v, \zeta_{jk}^{f^p}, \zeta_{jk}^{f^q}$ must be non-negative by definition, the scarcity rents of reactive power $\delta_j^{q,+}, \delta_j^{q,-}$, voltage magnitude μ_j^+, μ_j^-, active power flows $\xi_{ij}^{f^p,+}, \xi_{ij}^{p,-}, \xi_{ij}^{f^p,0}$ and reactive power flows $\xi_{ij}^{f^q,+}, \xi_{ij}^{f^q,-}, \xi_{ij}^{f^q,0}$ and risk parameters $z_{\epsilon_g}, z_{\epsilon_v}, z_{\epsilon_f}$ set an upper bound to the standard deviations $\sigma_{p_{G,j}}, \sigma_{v_j}, \sigma_{f^p_{jk}}, \sigma_{f^q_{jk}}$ weighted by the penalty factors.

V. Case Study

We conduct numerical experiments using the modified 118-node IEEE test system from [7], which includes 11 wind farms with the total forecast power output of 1196 MW ($\approx 28.2\,\%$ of the total active power demand). As in [4], [7], the wind power forecast error is zero-mean with the standard deviation of $\sigma_{p_{U,i}} = 0.125 p_{U,i}, \forall i \in \mathcal{U}$. In addition to the GEN-CC, EQV-CC and VA-CC, we solve a deterministic AC OPF (reference) case using the forecast renewable generation and $\alpha_i = 0, \forall i \in \mathcal{G}$. All calculations have been performed for risk levels $\epsilon = 0.1$ and $\epsilon = 0.01$ assuming that $\epsilon_p = \epsilon_q = \epsilon_v = \epsilon_f = \epsilon$. Additionally, the VA-CC has been computed for various values of $\Psi = \{0.1, 1, 10, 100, 1000\}$ assuming that $\Psi_i^p = \Psi_i^q = \Psi_i^v = \Psi, \forall i \in \mathcal{N}$ and $\Psi_{ij}^{f^p} = \Psi_{ij}^{f^q} = \Psi, \forall ij \in \mathcal{L}$. All models are implemented in Julia using JuMP [23] and the code and input data are reported in [24]. The linearization point (see Section II-E) has been obtained as described in [7] using the IPOPT solver, [25], and the chance-constrained models have been solved using the MOSEK solver, [26].

A. Cost and Price Analysis

Table I compares the results of the deterministic, GEN-CC, EQV-CC and VA-CC cases for different values of ϵ and Ψ. As expected, the objective value and expected generation cost increase as we introduce additional chance constraints and increase the value of Ψ, thus internalizing the cost of re-dispatch to ensure larger security margins and lower variance of state variables. Similarly to the results in [17], which uses DC power flow assumptions, increasing variance penalty factor Ψ does not significantly raise the expected generation cost. Also, increasing conservatism of the model increases system-wide balancing reserve price χ for both values of ϵ. For example, in the GEN-CC, the value of χ is only driven by chance constraints on power output limits of generators, as per Proposition 2, while the EQV-CC and VA-CC introduce additional components (e.g. ractive power, voltage and flow variances) to price χ as per Propositions 3 and 4. Location-specific prices λ_i^p and λ_i^q for all network nodes are displayed in Fig. 1a), while Figs. 1b)–c) map the relative difference between λ_i^p for the VA-CC case with $\Psi = 100$ and $\epsilon = 0.01$ and the deterministic case. At the majority of nodes, prices λ_i^p (indicated by the box-plots in Fig. 1a) remain within 32–38 \$/MWh. Note that unlike χ, which significantly increases for more conservative models, prices for λ_i^p and λ_i^q do not vary as much as conservatism increases. This corresponds to our findings in Propositions 1–4, which show that active and reactive power prices do not explicitly depend on the

TABLE I: Optimal Solutions of the deterministic, GEN-CC, EQV-CC and VA-CC cases.

Risk Level	Model	Det	GEN-CC	EQV-CC	VA-CC ($\Psi = \Psi_i^p = \Psi_i^q = \Psi_i^v = \Psi_{ij}^{f^p} = \Psi_{ij}^{f^q}, \forall i, \forall ij$)				
	Ψ	–	–	–	0.1	1	10	100	1000
$\epsilon_p = \epsilon_q = \epsilon_v = \epsilon_f = 0.1$	Objective [\$]	91103.22	91107.33	92237.67	92237.74	92238.30	92243.86	92296.91	92764.30
	Exp. Gen. Cost [\$]	91103.22	91107.33	92237.67	92237.68	92237.68	92237.72	92239.70	92260.83
	Δ rel. to EQV-CC	98.770%	98.774%	100.000%	100.000%	100.000%	100.000%	100.002%	100.025%
	χ [\$]	–	8.72	28.10	28.11	28.23	29.40	40.35	125.54
	$\Delta \sum_i \sigma^2_{q_{G,i}}$ [%]	–	–	100.0%	0.132%	0.103%	0.090%	0.087%	0.064%
	$\Delta \sum_i \sigma^2_{v_i}$ [%]	–	–	100.0%	3.459%	1.215%	0.349%	0.269%	0.225%
	$\Delta \sum_{ij} \sigma^2_{f^p_{ij}}$ [%]	–	–	100.0%	61.071%	60.458%	60.537%	59.798%	59.614%
	$\Delta \sum_{ij} \sigma^2_{f^q_{ij}}$ [%]	–	–	100.0%	55.808%	54.793%	54.925%	54.584%	54.313%
$\epsilon_p = \epsilon_q = \epsilon_v = \epsilon_f = 0.01$	Objective [\$]	91103.22	91107.71	93744.95	93745.01	93745.57	93751.17	93805.19	94281.35
	Exp. Gen. Cost [\$]	91103.22	91107.71	93744.95	93744.95	93744.94	93744.96	93747.04	93772.27
	Δ rel. to EQV-CC	97.182%	97.187%	100.000%	100.000%	100.000%	100.000%	100.002%	100.029%
	χ [\$]	–	9.74	25.93	25.94	26.03	26.95	37.47	126.42
	$\Delta \sum_i \sigma^2_{q_{G,i}}$ [%]	–	–	100.0%	0.194%	0.188%	0.187%	0.163%	0.149%
	$\Delta \sum_i \sigma^2_{v_i}$ [%]	–	–	100.0%	25.384%	4.570%	1.073%	0.752%	0.650%
	$\Delta \sum_{ij} \sigma^2_{f^p_{ij}}$ [%]	–	–	100.0%	64.291%	64.526%	64.404%	62.879%	62.103%
	$\Delta \sum_{ij} \sigma^2_{f^q_{ij}}$ [%]	–	–	100.0%	54.022%	54.241%	54.193%	52.940%	52.626%

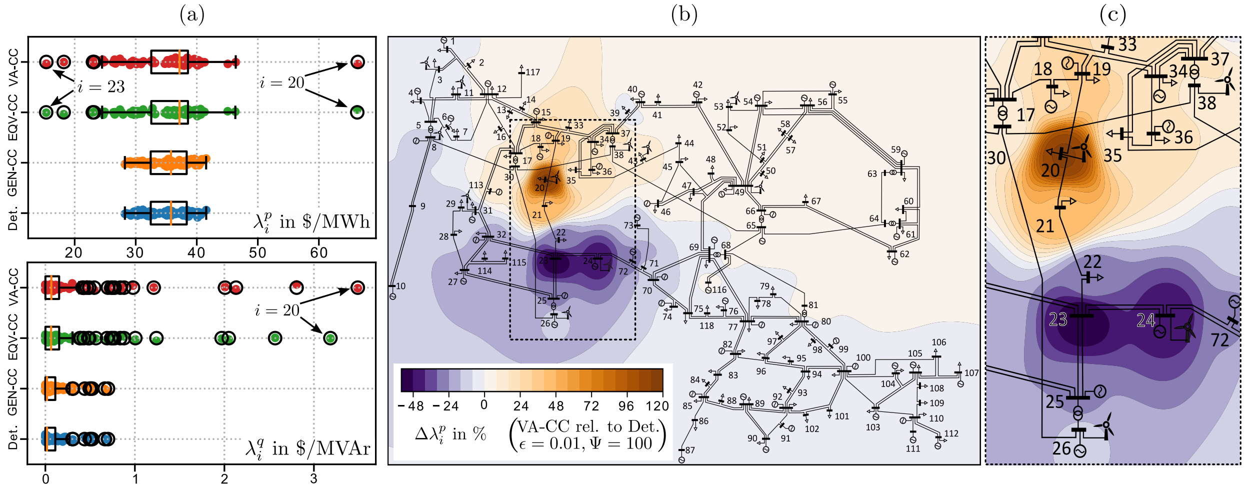

Fig. 1: (a) Active and reactive power prices λ_i^p and λ_i^q for the deterministic, GEN-CC and EQV-CC cases and VA-CC with $\Psi = 100$ for risk level $\epsilon = 0.01$. The orange line within the blue box represents the median value, the left and right edges of the box represent the first and third quartiles and the outliers are plotted as circles. (b) Difference of active power prices λ_i^p in the VA-CC ($\Psi = 100$) relative to the deterministic case (in %). (c) Magnification of the area indicated by the doted rectangle in (b).

uncertainty and risk parameters. However, at some nodes, prices λ_i^p and λ_i^q in the GEN-CC and VA-CC cases exhibit larger deviations, e.g. see λ_i^p at nodes 20 and 23, which are also in proximity of wind farms, as shown in Fig. 1c). A resulting high flow variance on the line between nodes 19 and 23 causes price differentiation at nodes 19, 20, 21 and 23, 24, 25.

B. Analysis of Variance of State Variables

Table I shows how the aggregated variance of state variables $\sum_i \sigma^2_{q_{G,i}}$, $\sum_i \sigma^2_{v_i}$, $\sum_i \sigma^2_{f^p_{ij}}$, $\sum_i \sigma^2_{f^q_{ij}}$ change relative to the EQV-CC case as penalty Ψ increases. Even if Ψ is set to a small value, the variance of state variables reduce significantly, without a large increase in the objective function, expected generation cost, and prices λ_i^p and λ_i^q. Furthermore, as the value of ϵ increases, the relative reduction in variances of all state variables slightly reduces. The effect of variance penalty Ψ on prices is two-fold. First, it does not affect prices λ_i^p and λ_i^q relative to the EQV-CC case. Second, system-wide balancing price χ, which internalizes the variance penalties as per Proposition 4, increases with penalty Ψ.

VI. Conclusion

This paper described an approach to internalize RES stochasticity in electricity prices. Using SOC duality, these risk- and variance-aware prices are derived from a chance-constrained AC-OPF and are itemized in terms of active and reactive power, voltage support and power flow components. We proved that active and reactive power prices do not explicitly depend on uncertainty and risk parameters, while expressions for balancing reserve prices explicitly include these parameters. Further, introducing variance penalties on the system state variables has been shown to internalize the trade-off between variance, risk and system cost at a modest increase in the expected operating cost. The results have been demonstrated and analyzed on the modified IEEE 118-node testbed.

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Appendix A

Rewrite (6)–(7) in the following form:

$$\begin{bmatrix} p(\boldsymbol{\omega}) \\ q(\boldsymbol{\omega}) \end{bmatrix} - \begin{bmatrix} \bar{p} \\ \bar{q} \end{bmatrix} = \begin{bmatrix} J^{p,v} & J^{p,\theta} \\ J^{q,v} & J^{q,\theta} \end{bmatrix} \begin{bmatrix} v(\boldsymbol{\omega}) \\ \theta(\boldsymbol{\omega}) \end{bmatrix} = J \begin{bmatrix} v(\boldsymbol{\omega}) \\ \theta(\boldsymbol{\omega}) \end{bmatrix}, \quad \text{(A.1)}$$

where the rows of matrices $J^{\diamond}$ are equal to sensitivity vectors $J_i^{\diamond}$ for $i \in \mathcal{N}$ and $\diamond = \{(p,v);(p,\theta);(q,v);(q,\theta)\}$. First, we sort the rows of the terms in (A.1) by node types and introduce superscripts PQ, PV, θV to indicate the node type:

$$\begin{bmatrix} \begin{bmatrix} p^{PQ}(\boldsymbol{\omega}) \\ p^{PV}(\boldsymbol{\omega}) \\ q^{PQ}(\boldsymbol{\omega}) \end{bmatrix} \\ \begin{bmatrix} p^{\theta V}(\boldsymbol{\omega}) \\ q^{PV}(\boldsymbol{\omega}) \\ q^{\theta V}(\boldsymbol{\omega}) \end{bmatrix} \end{bmatrix} - \begin{bmatrix} \begin{bmatrix} \bar{p}^{PQ} \\ \bar{p}^{PV} \\ \bar{q}^{PQ} \end{bmatrix} \\ \begin{bmatrix} \bar{p}^{\theta V} \\ \bar{q}^{PV} \\ \bar{q}^{\theta V} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} J^A & J^B \\ J^C & J^D \end{bmatrix} \begin{bmatrix} \begin{bmatrix} v^{PQ}(\boldsymbol{\omega}) \\ \theta^{PQ}(\boldsymbol{\omega}) \\ \theta^{PV}(\boldsymbol{\omega}) \end{bmatrix} \\ \begin{bmatrix} v^{PV}(\boldsymbol{\omega}) \\ v^{\theta V}(\boldsymbol{\omega}) \\ \theta^{\theta V}(\boldsymbol{\omega}) \end{bmatrix} \end{bmatrix}, \quad \text{(A.2)}$$

where J^{A-D} denote the blocks of re-arranged matrix J from (A.1). Quantities $p^{PQ}(\boldsymbol{\omega}), p^{PV}(\boldsymbol{\omega}), q^{PQ}(\boldsymbol{\omega})$ are explicitly given by the uncertain generation and the respective system responses such that:

$$\begin{bmatrix} p^{PQ}(\boldsymbol{\omega}) \\ p^{PV}(\boldsymbol{\omega}) \\ q^{PQ}(\boldsymbol{\omega}) \end{bmatrix} - \begin{bmatrix} \bar{p}^{PQ} \\ \bar{p}^{PV} \\ \bar{q}^{PQ} \end{bmatrix} = \begin{bmatrix} p_G^{PQ} \\ p_G^{PV} \\ q_G^{PQ} \end{bmatrix} + \begin{bmatrix} (\boldsymbol{\omega} + \alpha\boldsymbol{\Omega})^{PQ} \\ (\boldsymbol{\omega} + \alpha\boldsymbol{\Omega})^{PV} \\ (\mathrm{diag}(\gamma)\boldsymbol{\omega})^{PQ} \end{bmatrix}. \quad \text{(A.3)}$$

Notably, p_U and p_D are not part of the right-hand side of (A.3) because they are fixed parameters. Further, $v^{PV}(\boldsymbol{\omega}) = v^{PV}$, $v^{\theta V}(\boldsymbol{\omega}) = v^{PV}$, and $\theta^{\theta V}(\boldsymbol{\omega}) = \theta^{\theta V}$ as discussed in Section II-C. We use this relationship and (A.2)–(A.3) to compute the reactions of the uncontrolled variables to uncertainty $\boldsymbol{\omega}$:

$$\begin{bmatrix} v^{PQ}(\boldsymbol{\omega}) \\ \theta^{PQ}(\boldsymbol{\omega}) \\ \theta^{PV}(\boldsymbol{\omega}) \end{bmatrix} - \begin{bmatrix} v^{PQ} \\ \theta^{PQ} \\ \theta^{PV} \end{bmatrix} = (J^A)^{-1} \begin{bmatrix} (\boldsymbol{\omega} + \alpha\boldsymbol{\Omega})^{PQ} \\ (\boldsymbol{\omega} + \alpha\boldsymbol{\Omega})^{PV} \\ (\mathrm{diag}(\gamma)\boldsymbol{\omega})^{PQ} \end{bmatrix}. \quad \text{(A.4)}$$

Note that although $v^{PQ}, \theta^{PQ}, \theta^{PV}$ implicitly depend on the AC power flow equations, these variables are endogenous to the model and not subject to uncertainty. Similarly, we get:

$$\begin{bmatrix} p^{\theta V}(\boldsymbol{\omega}) \\ q^{PV}(\boldsymbol{\omega}) \\ q^{\theta V}(\boldsymbol{\omega}) \end{bmatrix} - \begin{bmatrix} p^{\theta V} \\ q^{PV} \\ q^{\theta V} \end{bmatrix} - \begin{bmatrix} \bar{p}^{\theta V} \\ \bar{p}^{PV} \\ \bar{q}^{\theta V} \end{bmatrix} = J^C (J^A)^{-1} \begin{bmatrix} (\boldsymbol{\omega} + \alpha\boldsymbol{\Omega})^{PQ} \\ (\boldsymbol{\omega} + \alpha\boldsymbol{\Omega})^{PV} \\ (\mathrm{diag}(\gamma)\boldsymbol{\omega})^{PQ} \end{bmatrix}. \quad \text{(A.5)}$$

Using (A.4), we immediately obtain (11) by separating matrix $(J^A)^{-1}$. Similarly, we obtain (10) from separating matrix $J^C(J^A)^{-1}$. In analogy, (12)–(13) can be obtained by noting that $p_i = \sum_{j:ij\in\mathcal{L}} f_{ij}^p$ and $q_i = \sum_{j:ij\in\mathcal{L}} f_{ij}^q$ and combining the sensitivity factors respectively.