

Look-Ahead SCOPF (LASCOF) for Tracking Demand Variation via Auxiliary Proximal Message Passing (APMP) Algorithm [☆]

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Comments: 1) make these into italics, and 2) I think that subsequent is a better word here than future. May need to make the change from future to subsequent in several places.

Abstract

In this paper, we will consider the Look-Ahead Security Constrained Optimal Power Flow (LASCOF) problem looking forward multiple dispatch intervals, in which the load demand varies ~~from one~~^{over} dispatch interval ~~to the other~~^{to} according to some forecast. We will consider the base-case and several contingency scenarios in the upcoming as well as in the ~~future~~^{subsequent} dispatch intervals. We will formulate and solve the problem in a Model Predictive Control (MPC) paradigm. We will present the *Auxiliary Proximal Message Passing (APMP)* algorithm to solve this problem, which is a bi-layered decomposition-coordination type distributed algorithm, consisting of an outer *Auxiliary Problem Principle (APP)* layer and an inner *Proximal Message Passing (PMP)* layer. The *APP* part of the algorithm distributes the computation across several dispatch intervals and the *PMP* part performs the distributed computation within each of the dispatch interval across different devices (i.e. generators, transmission lines, loads) and nodes or nets. We will demonstrate the effectiveness of our method with a series of numerical simulations.

Keywords: Alternating Direction Method of Multipliers (ADMM), Proximal Message Passing (PMP), Auxiliary Problem Principle (APP), contingency scenarios, Model Predictive Control (MPC), Electric Power Systems Planning and Operations

1. Introduction

In this paper, we will consider the $(N-1)$ *Security-Constrained Optimal Power Flow (SCOPF)* problem, in which devices (i.e. generators, transmission lines, loads etc.) are connected on

[☆]The objective of this paper is to present novel algorithmic and computational tools for solving the Look-Ahead Security Constrained Optimal Power Flow (LASCOF) accurately, provably, and reasonably fast. The authors were supported, in part, by the National Science Foundation (NSF) under grant ECCS-1406894.

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the power network and there exists a set of *scenarios* — each corresponding to the failure of a particular transmission line — over which we must ensure optimal feasible operation of the network. But, instead of solving the SCOPF for the upcoming dispatch interval only, at the start of each such interval, we will also look forward several ~~future~~^{subsequent} dispatch intervals, in which the load demands change according to a forecast. We will take into account the ramp rates of generators and operate the system such that the generation cost is minimized, subject to the constraints of satisfying the changing load demand, line power flow limit constraints, minimum and maximum generation constraint of each generator. ~~in each of the present and future dispatch intervals considered.~~^{enforced}

The goal is to minimize a composite cost function that includes the cost (and constraints) of nominal operation, as well as those associated with constraints on operation in any of the (adverse) scenarios, for multiple dispatch intervals. This results in a large optimization problem, since variables in the network, namely, real power injection and bus voltage phase angle, are repeated $|\mathcal{L}|$ times for each of the $|\Omega|$ dispatch intervals, where $|\mathcal{L}|$ is the number of contingencies. Therefore we present a distributed algorithm to solve this problem, whereby we break the problem into several smaller independent sub-problems and solve them in parallel ~~and~~^{such that} the only co-ordination required is local, through the Locational Marginal Prices (LMP) updates. Specifically, we use a suitably modified version of the message passing algorithm from [1] and [2] to solve this problem efficiently.

For simplicity, we consider only DC power flow in this paper. The extension to AC power flow, involves simply applying the AC-OPF model from [3], [4], [5], [6], [7], and [8] to each scenario and requiring that the phase angles of a given device are equal across all scenarios in the respective time periods.

In the previous works in [1], [9], and [2], Kraning *et al.*, Liu *et al.* and Chakrabarti *et al.*, respectively, applied the Proximal Message Passing Algorithm to solving the standard Static Optimal Power Flow (OPF) Problem and ~~solving~~^{to} the $(N - 1)$ Security Constrained OPF (SCOPF). In this paper, we extend this approach to solving a look-ahead dispatch problem, which considers the variation of load over a time horizon. There can be interesting and direct underpinnings of solving such problems, when it comes to design and allocation of Financial Transmission Rights (FTRs) as well [10]. Our formulation for the LASCOPF is based on the Model Predictive Control (MPC) or Receding Horizon Control (RHC) methodology [11, 12, 13, 14, 15]. The rest of the paper is organized as follows: In section 2 we present a brief literature survey, after which, in section 3 we introduce the system of notations and present the conventional formulation of look-ahead SCOPF in the “Angles Represented” version, where the different real power flows on the Transmission Lines are represented in terms of the real powers injected at the buses and the voltage phase angles at the buses. In section 4, we apply the coarse-grained APP decomposition to split the problem into different dispatch intervals. ~~We then reformulate all the preceding SCOPF problems (belonging to different dispatch intervals) into a different framework, the \mathcal{DTN} (Devices-Terminals-Nets) formulation, which is particularly suitable for the ADMM (Alternating Direction Method of Multipliers) [16] based Proximal Message Passing algorithm to be applied to the problems, in section 5.~~^{In section 5.} ~~applying~~ In section 6, we present the Proximal Message Passing fine-grained decomposition for the present problem. In section 7, we present the results of some simulation studies

conducted on the IEEE test systems and we draw the concluding remarks and point to future research in section 8.

2. Literature Survey and Related work

The Optimal Power Flow Problem is at the heart of Power Systems planning and operations. ~~Started~~ by Carpentier in [17], it has been studied for more than half a century, and has extensively been applied in the industry as well [18]. Happ in [19] and Chowdhury & Rahman in [20] trace the historical development of OPF upto the 90s. A recent reference that provides a good summary of the historical development of the problem is Cain, O'Neill and Castillo [21]. The references cited there also provide good insights into formulation and modeling particularly, of the ACOPF. There has been recently a surge in the interest in ACOPF following the works of Bai *et al.* in [3] and Lavaei *et al.* in [4], [5], [6], [7], where the authors showed that the ACOPF problem can be solved accurately by a convex relaxation of the original problem into a Semidefinite Programming problem.

The pioneering work on the Security Constrained OPF (SCOPF) was done by Stott *et al* in [22]. Some of the recent references include works by Chiang *et al.* [23], Phan *et al.* [24], Chakrabarti *et al.* [2] etc. The last two deal with application of distributed algorithms to the SCOPF problem. In [8] and in [9], the authors have applied ADMM to solve the ACOPF and SCOPF, but not to a look-ahead problem. Significant early works on ADMM method during the 70s and 80s were [25], [26], [27] etc. followed by work during the 90's which include [28], [29]. Combining these two fields gives rise to the Distributed Computational methods for OPF [30], [31]. The last reference provides a good comparison of the distributed methods till the end of 90s. Good references on Model Predictive Control (MPC) or Receding Horizon Control (RHC) are [11, 12, 13, 14, 15]. In our present work, we will be solving the Look-Ahead Security Constrained Optimal Power Flow (LASCOPF) problem, formulated in a MPC/RHC format, using a combination of APP and ADMM-PMP algorithms, which we will call the APMP algorithm. As in all MPC/RHC formulation, the implicit assumption is ~~will be that, this problem, for several look-ahead dispatch intervals is solved at the beginning of each dispatch interval, updating the forecasts,~~ ^{is solved at the beginning of each dispatch interval with updated forecasts} thereby reaching the best accuracy and ability to track the time-varying load demand.

3. Conventional Look-Ahead SCOPF Formulation

In this section, we will formulate the models for the look-ahead SCOPF in the conventional framework. First we will introduce the notations and symbols to be used for the rest of the paper, which are same as those introduced in [2] with a few additions.

3.1. Notations and Conventions

We have categorized the entities used in the subsequent formulations into five different groups: Sets, Elements, Index, Parameters, and Variables.

- *Sets*

\mathcal{D} : Set of Devices

\mathcal{T} : Set of Terminals

\mathcal{N} : Set of Nets or Buses or Nodes. The next three sets form partitions of the set of devices:

$G \subseteq \mathcal{D}$: Set of Generators

$T \subseteq \mathcal{D}$: Set of Transmission Lines

$L \subseteq \mathcal{D}$: Set of Loads

Move up to just after definition of devices

$J(N_i)$: Set of buses/nodes/nets directly connected to node N_i $\in \{\mathcal{N}\}$

$\mathcal{L} = \{0, 1, 2, \dots, |\mathcal{L}|\}$: Set of possible $(N - 1)$ Contingencies. The element 0 indicates the base case

$\Omega = \{0, 1, 2, \dots, |\Omega|\}$: Set of Dispatch intervals or, the net Dispatch Horizon under consideration. 0 indicates the ~~first~~ ^{upcoming} dispatch interval under consideration. ~~It is to be noted that the first dispatch interval under consideration, for which the calculation is done is actually the forthcoming (upcoming) one.~~ Hence, dispatch interval -1 (which is not in this set) is the current running one.

\dagger will be used to denote the transpose of a vector or matrix.

- *Elements*

for which a dispatch decision will definitively be implemented. The rest, $\{1, 2, \dots, |\mathcal{L}|\}$, are the subsequent intervals in the future.

t : Elements of \mathcal{T}

g : Elements of G

D : Elements of L

T : Elements of T

N : Elements of \mathcal{N}

- *Indices*

i, j : Nets

k : Terminals

q : Generators

r : Transmission Lines

d : Loads

c : Contingencies

τ : Dispatch Intervals

ν : Iteration count for ADMM/PMP algorithm

μ_{APP} : Iteration count for APP algorithm

- *Parameters*

$R_{T_r}, X_{T_r}, Z_{T_r} = R_{T_r} + (\sqrt{-1})X_{T_r}$: Resistance, Reactance and Impedance of the r^{th} Transmission Line, respectively

β, γ : APP tuning parameters, ρ : ADMM-PMP tuning parameter

$\alpha_{g_q}, \beta_{g_q}, \gamma_{g_q}$: Quadratic, Linear, and Constant Cost Co-efficients of the q^{th} Generator

$C_{g_q}(\cdot), f_{dev}(\cdot)$ will be used to denote the cost function of the g_q^{th} generator and that of a generic device, respectively, throughout. We will introduce the other cost functions in the appropriate sections

$I_{\leq}(\cdot), I_{\geq}(\cdot), I_{=}(.)$: Indicator functions such that $I_{\leq}(x) = 0$, if $0 \leq x$, and $= \infty$ otherwise and similarly for I_{\geq} and $I_{=}$.

$\bar{P}_{g_q}, \underline{P}_{g_q}, \bar{R}_{g_q}, \underline{R}_{g_q} (= -\bar{R}_{g_q}, \text{usually}), \bar{L}_{T_r}$ denote the maximum and minimum generating limits of generators, maximum ramp-up and ramp-down limits of generators, and power carrying capacity of transmission lines respectively

- *Variables*

P : The real power or, MW associated with different devices

θ : The bus voltage phase angles associated with different nodes, as well as the devices

The following is the convention we follow in order to identify the associations of any particular variable to the sets: $(P_{N_t(c_1)(\tau_1)}^{(c_2)(\tau_2)})^{(\nu)}$. The foregoing notation refers to a variable (P) associated with a particular terminal indexed by the t of either a net or a device for the contingency scenario indexed by c_2 during τ_2 , as estimated by a computing unit linked to τ_1 and contingency scenario, c_1 . Sometimes we will use the net number instead of the terminal number in the above convention, when we want to indicate several devices connected to a particular net. If it is part of an iterative algorithm, then the outermost superscript ν indicates the iteration count. Whenever a variable is boldface, one or more of the indices will be missing and that means the boldface variable is a vector each of whose components will have all or some of the missing indices (the components themselves can be vectors or scalars). When the variable is not bold-face and still some of the indices are missing, that means it is a scalar and the missing indices are either irrelevant or their values are implied from the context. Also it is to be observed that since generators and loads are single terminal devices, it is not necessary to specify the terminals for these, unless absolutely required.

3.2. Look-Ahead Dispatch: Generalized Case of Demand Variation for Multi-Bus Case

In this paper, we build on the static

~~Based on the material presented in our paper on the SCOPF problem in its static form [2].~~
~~in this paper,~~ ^W we will develop the mathematical model for a look-ahead dispatch calculation that considers several future dispatch intervals at the onset of each current dispatch interval and takes into account the possible variations of operating parameters across different scenarios represented in different future dispatch intervals, so that at each interval the entire system is secure. At the end of each current dispatch interval, the calculation “rolls forward”

an upcoming and several subsequent

and the whole look-ahead calculation is repeated with updated demand forecast, just as in a “Model Predictive Control (MPC)” or a “Receding Horizon Control (RHC).” The generalized version of this problem for a multi-time horizon, arbitrary network is as follows:

$$\min_{P_{gq}^{(\tau)}, \theta} \sum_{\tau \in \Omega} \sum_{gq \in G} C_{gq}(P_{gq}^{(\tau)}) \quad (1a)$$

Subject to : $\forall (c) \in \mathcal{L}, \forall \tau \in \Omega, \forall T_r \in T$

$$P_{gqN_i}^{(\tau)} - P_{DdN_i}^{(\tau)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(0)}(\theta_{N_i}^{(0)(\tau)} - \theta_{N_{\bar{i}}}^{(0)(\tau)}); \forall N_i \in \mathcal{N} \quad (1b)$$

$$P_{gqN_i}^{(\tau)} - P_{DdN_i}^{(\tau)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(c)}(\theta_{N_i}^{(c)(\tau)} - \theta_{N_{\bar{i}}}^{(c)(\tau)}); \forall N_i \in \mathcal{N} \quad (1c)$$

$$|B_{T_r}^{(0)}(\theta_{T_{rt_1}}^{(0)(\tau)} - \theta_{T_{rt_2}}^{(0)(\tau)})| \leq \bar{L}_{T_r}^{(0)}, \forall T_r \in T \quad (1d)$$

$$|B_{T_r}^{(c)}(\theta_{T_{rt_1}}^{(c)(\tau)} - \theta_{T_{rt_2}}^{(c)(\tau)})| \leq \bar{L}_{T_r}^{(c)}, \forall T_r \in T \quad (1e)$$

$$\underline{R}_{gq} \leq (P_{gq}^{(\tau+1)} - P_{gq}^{(\tau)}) \leq \bar{R}_{gq} \quad \forall gq \in G \quad (1f)$$

4. APMP Algorithm: The Coarse Grained Decomposition Component

In this section, we will provide the mathematical formulation of the Auxiliary Proximal Message Passing (APMP) algorithm, as applied to the LASCOPF problem. We will start ~~first~~ by describing the APP based coarse-grained component of the algorithm, which will be applied to the LASCOPF problems, followed by the PMP fine-grained component, which decomposes the SCOPF problems (and the OPF problems) of each dispatch interval into the device level computations. The basic idea behind the APP based coarse-grained distributed component of the algorithm is depicted in figure 1, where the two circles correspond to different dispatch intervals and/or scenarios, across which the computation is split, and the arrows indicate the exchange of messages between those. The messages correspond to the beliefs of each circle or coarse grain about the values of power generations and/or injections/line flows within itself and also those belonging to the neighboring circles.

4.1. Generalized Case of Demand Variation for Multi-Bus Case

Referring to the figure 2, the different overlapping circles here represent the different dispatch intervals, and with $\tau(-\tau-1) = 0$ and $\tau(-\tau+3) = 4$ (Assuming, $\tau = 1$ as the reference) representing, respectively, the latest dispatch interval for which the LASCOPF problem is already solved & the results are known, and the dispatch interval following the concerned time horizon. In the coarse-grained decomposition, when $\tau = 1$, $P_{g_2(\tau)}^{(\tau-1)}$ and $P_{g_1(\tau)}^{(\tau-1)}$ are replaced by $P_{g_2}^{(0)}$ and $P_{g_1}^{(0)}$, respectively, which are the (known) MW outputs from the current dispatch interval. When $\tau = 3$, $P_{g_2(\tau)}^{(\tau+1)}$ and $P_{g_1(\tau)}^{(\tau+1)}$ are replaced by $P_{g_2}^{(4)}$ and $P_{g_1}^{(4)}$, respectively, which are the last but one iterate values of the MW outputs from the last dispatch interval. We will now apply the Auxiliary Problem Principle (APP)[32], [33] to the above optimization problem to derive a set of expressions for the iterate updates. This

Comment: on page 4, interval 0 was the upcoming interval, with interval -1 the latest dispatch interval with problem solved. However, here it appears that interval 1 is the upcoming interval, and interval 0 is the latest dispatch interval with problem solved. Please make consistent. I think it would be better to make 1 the upcoming interval and 0 the latest dispatch with problem solved.

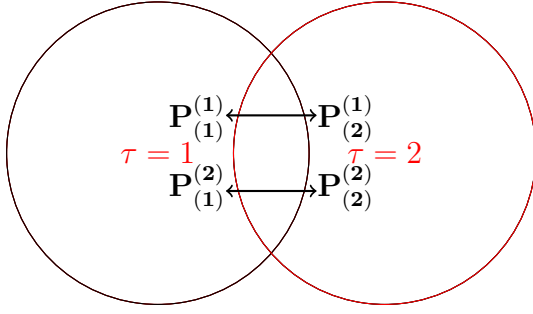


Figure 1: Schematic for APP Message Exchange.

reformulation is very similar in flavor to the ones presented previously in [30], [34], [35], [36] etc. We will state the coarse-grained APP based decomposition of the problem (the one for which the conventional formulation has been presented in equation (1)). We will distribute the SCOPF across each dispatch interval and then exchange messages regarding what a particular interval thinks about the optimal values of the decision variables for the intervals immediately preceding and succeeding ~~the current one~~^{it}, eventually attempting to achieve a consensus between those. In the figure, the values shown at the two sides of the arrows are those “beliefs” among which we are trying to achieve consensus. We follow the same convention as introduced in section 3.1. As before, notice that each of the subproblems is very similar to the classical $(N - 1)$ SCOPF problems, but with two important differences. First of all, each of these has ramp rate constraints, to account for the change in generator outputs over multiple intervals. Secondly, there are regularization terms added to the cost function for attaining consensus among different coarse grains about the values of the decision variables, which, in this case, are generator outputs. The second terms in the objective functions are the ones representing the proximity from previous iterates. The third and fourth terms in the objective functions of equations (2) and (4) & the third, fourth, fifth, and sixth terms in the objective function of equation (3) are the ones for attaining consensus among the coarse grains. The last terms in the objective functions of equations (2) and (4) & the last two terms in the objective function of equation (3) correspond to complementary slackness. Note that, in this case also, the real power generation variables are the only ones at which we wish to attain consensus through the application of APP. The bus voltage angles can be thought of as localized to the particular dispatch interval sub-problems, whose values at any iteration are determined by the power generation or injection profiles. We have split the LASCOPF problem into three types of sub-problems, each corresponding to a particular dispatch time interval.

First Dispatch Interval SCOPF:

$$\begin{aligned} \mathbf{P}_{(1)}^{(\mu_{APP}+1)} = \operatorname{argmin}_{\mathbf{P}_{(1)}, \theta} & \sum_{g_q \in G} C_{g_q}(P_{g_q(1)}^{(1)}) + \\ & \frac{\beta}{2} \|\mathbf{P}_{(1)} - \mathbf{P}_{(1)}^{(\mu_{APP})}\|_2^2 \\ & + \gamma [\mathbf{P}_{(1)}^{(1)\dagger} (\mathbf{P}_{(1)}^{(1)(\mu_{APP})} - \mathbf{P}_{(2)}^{(1)(\mu_{APP})}) + \\ & \mathbf{P}_{(1)}^{(2)\dagger} (\mathbf{P}_{(1)}^{(2)(\mu_{APP})} - \mathbf{P}_{(2)}^{(2)(\mu_{APP})})] \end{aligned}$$

Comment: I would avoid using &, just say "and"

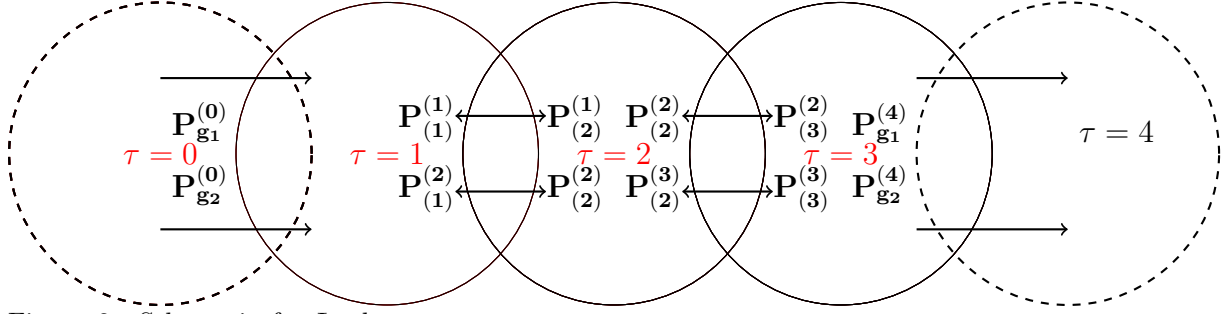


Figure 2: Schematic for Look-Ahead SCOPF for Demand Variation

$$+\lambda_1^{(\mu_{APP})^\dagger} \mathbf{P}_{(1)}^{(1)} + \lambda_2^{(\mu_{APP})^\dagger} \mathbf{P}_{(1)}^{(2)} \quad (2a)$$

Subject to : $\forall(c) \in \mathcal{L}, \forall T_r \in T$

Power-Balance Constraints (Base-Case & Contingency):

$$P_{gqN_i(1)}^{(1)} - P_{DdN_i}^{(1)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(0)} (\theta_{N_i}^{(0)(1)} - \theta_{N_{\bar{i}}}^{(0)(1)}); \forall N_i \in \mathcal{N} \quad (2b)$$

$$P_{gqN_i(1)}^{(1)} - P_{DdN_i}^{(1)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(c)} (\theta_{N_i}^{(c)(1)} - \theta_{N_{\bar{i}}}^{(c)(1)}); \forall N_i \in \mathcal{N} \quad (2c)$$

Flow Limit Constraints (Base-Case & Contingency):

$$|B_{T_r}^{(0)} (\theta_{T_{rt_1}}^{(0)(1)} - \theta_{T_{rt_2}}^{(0)(1)})| \leq \bar{L}_{T_r}^{(0)}, \forall T_r \in T \quad (2d)$$

$$|B_{T_r}^{(c)} (\theta_{T_{rt_1}}^{(c)(1)} - \theta_{T_{rt_2}}^{(c)(1)})| \leq \bar{L}_{T_r}^{(c)}, \forall T_r \in T \quad (2e)$$

Ramp-Rate Constraints:

$$\underline{R}_{gq} \leq P_{gq(1)}^{(2)} - P_{gq(1)}^{(1)} \leq \bar{R}_{gq}, \forall gq \in G \quad (2f)$$

$$\underline{R}_{gq} \leq P_{gq(1)}^{(1)} - P_{gq}^{(0)} \leq \bar{R}_{gq}, \forall gq \in G \quad (2g)$$

Intermediate Dispatch Interval $\tau \in \{2, 3, \dots, |\Omega| - 1\}$ SCOPF:

$$\begin{aligned} \mathbf{P}_{(\tau)}^{(\mu_{APP}+1)} = & \underset{\mathbf{P}_{(\tau)}, \theta}{\operatorname{argmin}} \sum_{gq \in G} C_{gq}(P_{gq(\tau)}^{(\tau)}) + \\ & \frac{\beta}{2} \|\mathbf{P}_{(\tau)} - \mathbf{P}_{(\tau)}^{(\mu_{APP})}\|_2^2 \\ & + \gamma [\mathbf{P}_{(\tau)}^{(\tau-1)\dagger} (\mathbf{P}_{(\tau)}^{(\tau-1)(\mu_{APP})} - \mathbf{P}_{(\tau-1)}^{(\tau-1)(\mu_{APP})}) + \\ & \mathbf{P}_{(\tau)}^{(\tau)\dagger} (\mathbf{P}_{(\tau)}^{(\tau)(\mu_{APP})} - \mathbf{P}_{(\tau-1)}^{(\tau)(\mu_{APP})}) \\ & + \mathbf{P}_{(\tau)}^{(\tau)\dagger} (\mathbf{P}_{(\tau)}^{(\tau)(\mu_{APP})} - \mathbf{P}_{(\tau+1)}^{(\tau)(\mu_{APP})}) + \\ & \mathbf{P}_{(\tau)}^{(\tau+1)\dagger} (\mathbf{P}_{(\tau)}^{(\tau+1)(\mu_{APP})} - \mathbf{P}_{(\tau+1)}^{(\tau+1)(\mu_{APP})})] \end{aligned}$$

$$\begin{aligned}
& +\lambda_{2\tau-1}^{(\mu_{APP})\dagger} \mathbf{P}_{(\tau)}^{(\tau)} + \lambda_{2\tau}^{(\mu_{APP})\dagger} \mathbf{P}_{(\tau)}^{(\tau+1)} \\
& -\lambda_{2\tau-3}^{(\mu_{APP})\dagger} \mathbf{P}_{(\tau)}^{(\tau-1)} - \lambda_{2\tau-2}^{(\mu_{APP})\dagger} \mathbf{P}_{(\tau)}^{(\tau)}
\end{aligned} \tag{3a}$$

Subject to : $\forall(c) \in \mathcal{L}, \forall T_r \in T$

Power-Balance Constraints (Base-Case & Contingency):

$$P_{gqN_i}^{(\tau)} - P_{DdN_i}^{(\tau)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(0)} (\theta_{N_i}^{(0)(\tau)} - \theta_{N_{\bar{i}}}^{(0)(\tau)}); \forall N_i \in \mathcal{N} \tag{3b}$$

$$P_{gqN_i}^{(\tau)} - P_{DdN_i}^{(\tau)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(c)} (\theta_{N_i}^{(c)(\tau)} - \theta_{N_{\bar{i}}}^{(c)(\tau)}); \forall N_i \in \mathcal{N} \tag{3c}$$

Flow Limit Constraints (Base-Case & Contingency):

$$|B_{T_r}^{(0)} (\theta_{T_{rt_1}}^{(0)(\tau)} - \theta_{T_{rt_2}}^{(0)(\tau)})| \leq \bar{L}_{T_r}^{(0)}, \forall T_r \in T \tag{3d}$$

$$|B_{T_r}^{(c)} (\theta_{T_{rt_1}}^{(c)(\tau)} - \theta_{T_{rt_2}}^{(c)(\tau)})| \leq \bar{L}_{T_r}^{(c)}, \forall T_r \in T \tag{3e}$$

Ramp-Rate Constraints:

$$\underline{R}_{gq} \leq P_{gq}^{(\tau+1)} - P_{gq}^{(\tau)} \leq \bar{R}_{gq}, \forall gq \in G \tag{3f}$$

$$\underline{R}_{gq} \leq P_{gq}^{(\tau)} - P_{gq}^{(\tau-1)} \leq \bar{R}_{gq}, \forall gq \in G \tag{3g}$$

Last Dispatch Interval, or Dispatch Interval $|\Omega|$ SCOPF:

$$\begin{aligned}
\mathbf{P}_{(|\Omega|)}^{(\mu_{APP}+1)} &= \underset{\mathbf{P}_{(|\Omega|)}, \theta}{\operatorname{argmin}} \sum_{gq \in G} C_{gq} (P_{gq}^{(|\Omega|)}) + \\
& \frac{\beta}{2} \|\mathbf{P}_{(|\Omega|)} - \mathbf{P}_{(|\Omega|)}^{(\mu_{APP})}\|_2^2 \\
& + \gamma [\mathbf{P}_{(|\Omega|)}^{(|\Omega|-1)\dagger} (\mathbf{P}_{(|\Omega|)}^{(|\Omega|-1)(\mu_{APP})} - \mathbf{P}_{(|\Omega|-1)}^{(\mu_{APP})}) + \\
& \mathbf{P}_{(|\Omega|)}^{(|\Omega|)\dagger} (\mathbf{P}_{(|\Omega|)}^{(\mu_{APP})} - \mathbf{P}_{(|\Omega|-1)}^{(\mu_{APP})})] \\
& - \lambda_{2|\Omega|-3}^{(\mu_{APP})\dagger} \mathbf{P}_{(|\Omega|)}^{(|\Omega|-1)} - \lambda_{2|\Omega|-2}^{(\mu_{APP})\dagger} \mathbf{P}_{(|\Omega|)}^{(|\Omega|)}
\end{aligned} \tag{4a}$$

Subject to : $\forall(c) \in \mathcal{L}, \forall T_r \in T$

Power-Balance Constraints (Base-Case & Contingency):

$$\begin{aligned}
P_{gqN_i}^{(|\Omega|)} - P_{DdN_i}^{(|\Omega|)} &= \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(0)} (\theta_{N_i}^{(0)(|\Omega|)} - \theta_{N_{\bar{i}}}^{(0)(|\Omega|)}); \\
& \forall N_i \in \mathcal{N}
\end{aligned} \tag{4b}$$

$$\begin{aligned}
P_{gqN_i}^{(|\Omega|)} - P_{DdN_i}^{(|\Omega|)} &= \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(c)} (\theta_{N_i}^{(c)(|\Omega|)} - \theta_{N_{\bar{i}}}^{(c)(|\Omega|)}); \\
& \forall N_i \in \mathcal{N}
\end{aligned} \tag{4c}$$

Flow Limit Constraints (Base-Case & Contingency):

$$|B_{T_r}^{(0)} (\theta_{T_{rt_1}}^{(0)(|\Omega|)} - \theta_{T_{rt_2}}^{(0)(|\Omega|)})| \leq \bar{L}_{T_r}^{(0)}, \forall T_r \in T \tag{4d}$$

$$|B_{T_r}^{(c)}(\theta_{T_{rt_1}}^{(c)(|\Omega|)} - \theta_{T_{rt_2}}^{(c)(|\Omega|)})| \leq \bar{L}_{T_r}^{(c)}, \forall T_r \in T \quad (4e)$$

Ramp-Rate Constraints:

$$\underline{R}_{g_q} \leq P_{g_q(|\Omega|)}^{(|\Omega|)(\mu_{APP})} - P_{g_q(|\Omega|)}^{(|\Omega|)} \leq \bar{R}_{g_q}, \forall g_q \in G \quad (4f)$$

$$\underline{R}_{g_q} \leq P_{g_q(|\Omega|)}^{(|\Omega|)} - P_{g_q(|\Omega|)}^{(|\Omega|-1)} \leq \bar{R}_{g_q}, \forall g_q \in G \quad (4g)$$

We have presented a coarse grained parallelization of the look-ahead SCOPF to cope with demand variation, where the entire problem is decomposed across the different dispatch time intervals. Subsequently, we will present the ADMM-Proximal Message Passing based fine grained decomposition and parallelization of the problem.

5. \mathcal{DTN} Reformulations of the SCOPF Problems

In this section we carry out the reformulation of the SCOPF models that we presented in the last section within each APP coarse grain, in order for us to be able to solve those problems by the Proximal Message Passing method. In order to simplify the presentation and also better clarify the meaning of each term, we will group the terms of the objective into four different categories. We will define them and write down the expressions pertaining to the SCOPF in the coarse grain corresponding to equation (3). These are:

1) Cost of Generation ($C(P)$): This term consists of the actual total cost of generating real power by the different generators as well as the indicator functions corresponding to the lower and upper generating limits of the different generators. For this term, the real power generated is always considered at the base case. This term is given as:

$$\begin{aligned} C(\mathbf{P}^{(0)(\tau)}) = & \sum_{t_k \in g_q \cap \mathcal{T}, q=1}^{|G|} \left(C_{g_q}(P_{g_{qt_k}}^{(0)(\tau)}) + I_{\leq}(\bar{P}_{g_q} - P_{g_{qt_k}}^{(0)(\tau)}) + I_{\leq}(P_{g_{qt_k}}^{(0)(\tau)} - \underline{P}_{g_q}) + \frac{\beta}{2} \left[(P_{g_{qt_k}}^{(0)(\tau)} - \right. \right. \\ & P_{g_{qt_k}}^{(0)(\tau)(\mu_{APP})})^2 + \sum_{s=-1,1} (P_{g_{qt_k}}^{(0)(\tau+s)} - P_{g_{qt_k}}^{(0)(\tau+s)(\mu_{APP})})^2 \left. \right] + \gamma \left[\sum_{s=-1,1} (P_{g_{qt_k}}^{(0)(\tau)} (P_{g_{qt_k}}^{(0)(\tau)(\mu_{APP})} - \right. \\ & P_{g_{qt_k}}^{(0)(\tau)(\mu_{APP})}) + P_{g_{qt_k}}^{(0)(\tau+s)} (P_{g_{qt_k}}^{(0)(\tau+s)(\mu_{APP})} - P_{g_{qt_k}}^{(0)(\tau+s)(\mu_{APP})}) \left. \right] + (\lambda_{g_q 2(\tau-1)}^{(\mu_{APP})} - \lambda_{g_q(2\tau-2)}^{(\mu_{APP})}) P_{g_{qt_k}}^{(0)(\tau)} + \\ & \left. \lambda_{g_q(2\tau)}^{(\mu_{APP})} P_{g_{qt_k}}^{(0)(\tau+1)} - \lambda_{g_q(2\tau-3)}^{(\mu_{APP})} P_{g_{qt_k}}^{(0)(\tau-1)} \right) \end{aligned}$$

For the sake of brevity, we are presenting here the terms corresponding to line flow limit constraint ($F(P^{(c)(\tau)})$), power-angle relation ($\chi(P^{(c)(\tau)}, \theta^{(c)(\tau)})$), and ramp constraint ($\Delta(P^{(0)(\tau)})$) in the most general and condensed forms. The details of these can be found in [2] and [37].

2) Line Flow Limit Constraint ($F(P)$): This term consists of the sum of the indicator functions corresponding to enforcing that real power flows on the lines are less than the maximum allowed, both at the base-case as well as ^{under} different contingencies. This term can be written as:

$$F(P^{(c)(\tau)}) = \sum_{(c) \in \mathcal{L}} \sum_{T_r \in T} \sum_{t_k \in T_r \cap \mathcal{T}} I_{\leq}(\bar{L}_{T_r}^{(c)} - |P_{T_{rt_k}}^{(c)(\tau)}|)$$

3) Power-Angle Relation ($\chi(P, \theta)$): This term consists of the sum of the indicator functions corresponding to the relation of the power flow at each end of the lines and the voltage phase angles at the two ends, both at the base-case and the contingencies. This term can be written as:

$$\chi(P^{(c)(\tau)}, \theta^{(c)(\tau)}) = \sum_{(c) \in \mathcal{L}} \sum_{T_r \in \mathcal{T}} \sum_{t_k, t_k' \in T_r \cap \mathcal{T}} I_{=}(P_{T_r t_k}^{(c)(\tau)} + \frac{\theta_{T_r t_k}^{(c)(\tau)} - \theta_{T_r t_k'}^{(c)(\tau)}}{X_{T_r}^{(c)}})$$

Comment: for cases where you have a "tall" expression in parentheses, use \left(and \right) to make the parentheses taller.

4) Ramp Constraint ($\Delta(P)$): This term corresponds to the change of power output of generator from one time period to another and the maximum rate at which it can go up or down. This term can be written as:

$$\Delta(P^{(0)(\tau)}) = \sum_{s=-1,1} \sum_{t_k \in g_q \cap \mathcal{T}, q=1}^{|G|} (I_{\leq}(\bar{R}_{g_q} - P_{g_q t_k}^{(0)(\tau+s)} + P_{g_q t_k}^{(0)(\tau)}) + I_{\leq}(P_{g_q t_k}^{(0)(\tau+s)} - P_{g_q t_k}^{(0)(\tau)} - \underline{R}_{g_q}))$$

The reformulated equations for the problem stated in equations (2)-(4) are as follows:

$$\min_{P_{t_k}^{(c)(\tau)}, \theta_{t_k}^{(c)(\tau)}} f(P) = C(P^{(0)(\tau)}) + F(P^{(c)(\tau)}) + \chi(P^{(c)(\tau)}, \theta^{(c)(\tau)}) + \Delta(P^{(0)(\tau)}) \quad (5a)$$

$$\begin{aligned} \text{Subject to: } & \hat{P}_{N_{it_k}}^{(c)(\tau)} = 0, \tilde{\theta}_{N_{it_k}}^{(c)(\tau)} = 0, \\ & \forall N_i \in \mathcal{N}, \forall t_k \in \mathcal{T}, \forall (c) \in \mathcal{L} \end{aligned} \quad (5b)$$

In the above set of equations, $\hat{P}_{N_{it_k}}^{(c)(\tau)}$ and $\tilde{\theta}_{N_{it_k}}^{(c)(\tau)}$ are respectively, the average node power injection and deviation of the voltage phase angle of each device connected to a particular node from the average node voltage phase angle. Readers are referred to [2], [1] for more information on these. Hence, constraints stated in equation (5b) represent the Kirchhoff's law for node power balance and angle consistency.

6. APMP Algorithm: The Fine Grained Decomposition Component

In this section we state the equations for the Proximal Message Passing algorithm based fine-grained decomposition of the above problem.

6.1. Proximal Message Passing for the Look-Ahead Dispatch: Generalized Case of Demand Variation for the Multi Bus-Case

From equation (5), the slightly reformulated \mathcal{DTN} equations are as follows:

$$\begin{aligned} \min_{P_{t_k}^{(c)}, \theta_{t_k}^{(c)}} & C(P^{(0)(\tau)}) + F(P^{(c)(\tau)}) + \chi(P^{(c)(\tau)}, \theta^{(c)(\tau)}) + \\ & \Delta(P^{(0)(\tau)}) + \sum_{(c) \in \mathcal{L}} \sum_{N_i \in \mathcal{N}} (\hat{I}(z_{N_i t_k}^{(c)(\tau)}) + \tilde{I}(\xi_{N_i t_k}^{(c)(\tau)})) \\ \text{Subject to: } & P_{t_k}^{(c)(\tau)} = z_{t_k}^{(c)(\tau)}, \theta_{t_k}^{(c)(\tau)} = \xi_{t_k}^{(c)(\tau)}, \\ & \forall N_i \in \mathcal{N}, \forall t_k \in \mathcal{T}, \forall (c) \in \mathcal{L} \end{aligned} \quad (6)$$

where $\hat{I}(z_{N_i t_k})$ and $\tilde{I}(\xi_{N_i t_k})$ are indicator functions of the sets $\{z_{t_k} | \hat{z}_{N_i t_k} = 0\}$ and $\{\xi_{t_k} | \tilde{\xi}_{N_i t_k} = 0\}$ respectively. Again, for the sake of brevity, we will skip writing down the iterates for the transmission lines and loads (which are simple projections), and make a mention that they are the same as the ones presented in references [2] and [37].

6.1.1. Iterates for Generators

The ~~iterates~~ ^{iterates for generators} consist of the update equations for the real power output and voltage-phase angles of the generator terminals for both the base case and the different $(N - 1)$ contingency scenarios and are as follows:

$$\begin{aligned}
& (P_{g_q t_k}^{(0)(\tau)(\nu+1)}, \theta_{g_q t_k}^{(c)(\tau)(\nu+1)}) \\
& = \operatorname{argmin}_{P_{g_q t_k}^{(0)(\tau)}, \theta_{g_q t_k}^{(c)(\tau)}} [C_{g_q}(P_{g_q t_k}^{(0)(\tau)}) + I_{\leq}(\bar{P}_{g_q} - P_{g_q t_k}^{(0)(\tau)}) \\
& + I_{\leq}(P_{g_q t_k}^{(0)(\tau)} - \underline{P}_{g_q}) + \frac{\beta}{2}[(P_{g_q t_k}^{(0)(\tau)} - P_{g_q t_k}^{(0)(\tau)(\mu_{APP})})^2 \\
& + \sum_{s=-1,1} (P_{g_q t_k}^{(0)(\tau+s)} - P_{g_q t_k}^{(0)(\tau+s)(\mu_{APP})})^2] + \\
& \gamma[\sum_{s=-1,1} (P_{g_q t_k}^{(0)(\tau)}(P_{g_q t_k}^{(0)(\tau)(\mu_{APP})} - P_{g_q t_k}^{(0)(\tau)(\mu_{APP})}) \\
& + P_{g_q t_k}^{(0)(\tau+s)}(P_{g_q t_k}^{(0)(\tau+s)(\mu_{APP})} - P_{g_q t_k}^{(0)(\tau+s)(\mu_{APP})}))] + \\
& (\lambda_{g_q 2(\tau-1)}^{\mu_{APP}} - \lambda_{g_q 2\tau-2}^{\mu_{APP}})P_{g_q t_k}^{(0)(\tau)} + \lambda_{g_q 2\tau}^{\mu_{APP}}P_{g_q t_k}^{(0)(\tau+1)} - \\
& \lambda_{g_q 2\tau-3}^{\mu_{APP}}P_{g_q t_k}^{(0)(\tau-1)} + \Delta(P_{g_q t_k}^{(0)(\tau)}) + \\
& \sum_{(c) \in \mathcal{L}} (\frac{\rho}{2})(\|P_{g_q t_k}^{(0)(\tau)} - z_{g_q t_k}^{(c)(\tau)(\nu)} + u_{g_q t_k}^{(c)(\tau)(\nu)}\|_2^2 \\
& + \|\theta_{g_q t_k}^{(c)(\tau)} - \xi_{g_q t_k}^{(c)(\tau)(\nu)} + v_{g_q t_k}^{(c)(\tau)(\nu)}\|_2^2), \\
& \forall g_q \in G, \tau \in \Omega, t_k \in \mathcal{T} \cap G
\end{aligned}$$

6.1.2. Iterates for Nets

We are writing here just the analytical forms already derived in [1].

$$\begin{aligned}
& \forall N_i \in \mathcal{N}, \forall t_k \in \mathcal{T} \cap N_i, \forall (c) \in \mathcal{L}, \tau \in \Omega \\
& z_{N_i t_k}^{(c)(\tau)(\nu+1)} \\
& = u_{N_i t_k}^{(c)(\tau)(\nu)} + P_{N_i t_k}^{(c)(\tau)(\nu+1)} - \hat{u}_{N_i t_k}^{(c)(\tau)(\nu)} - \hat{P}_{N_i t_k}^{(c)(\tau)(\nu+1)} \\
& \xi_{N_i t_k}^{(c)(\tau)(\nu+1)} = \hat{v}_{N_i t_k}^{(c)(\tau)(\nu)} + \hat{\theta}_{N_i t_k}^{(c)(\tau)(\nu+1)} \\
& u_{N_i t_k}^{(c)(\tau)(\nu+1)} = u_{N_i t_k}^{(c)(\tau)(\nu)} + (P_{N_i t_k}^{(c)(\tau)(\nu+1)} - z_{N_i t_k}^{(c)(\tau)(\nu+1)}) \\
& v_{N_i t_k}^{(c)(\tau)(\nu+1)} = v_{N_i t_k}^{(c)(\tau)(\nu)} + (\theta_{N_i t_k}^{(c)(\tau)(\nu+1)} - \xi_{N_i t_k}^{(c)(\tau)(\nu+1)})
\end{aligned}$$

Note

~~The way the algorithm works is~~ that, not only do all the devices update their variables in parallel, but also, except the generators, all devices have associated with them the base-case and the contingency scenarios in each dispatch interval, each of which in turn update their respective variables in parallel as well. For the generators, the present dispatch interval and the next one are related through the ramps ~~rate~~ rate constraint ($\Delta(\cdot)$ function). Then all the nets and the base-case/contingency scenarios associated with them update the first two set of variables in parallel and then update the next two in parallel. The proximal function for a function g is given by

$$\operatorname{prox}_{g, \rho}(\mathbf{v}) = \operatorname{argmin}_{\mathbf{x}} (g(\mathbf{x}) + (\rho/2)\|\mathbf{x} - \mathbf{v}\|_2^2).$$

For this case, the prox messages and the Proximal Message Passing Algorithm is as follows:

Separate the labels for the steps in the algorithm from the actual update. It might be better to put this into a figure and refer to the figure.

$$\begin{aligned}
& \xrightarrow{\text{red arrow}} 1. (P_{g_q t_k}^{(0)(\tau)(\nu+1)}, \theta_{g_q t_k}^{(c)(\tau)(\nu+1)}) \\
& = \mathbf{prox}_{C(P)+\Delta(P), \rho} (P_{g_q t_k}^{(0)(\tau)(\nu)} - \hat{P}_{g_q t_k}^{(c)(\tau)(\nu)} - u_{g_q t_k}^{(c)(\tau)(\nu)}, \\
& \quad \hat{v}_{g_q t_k}^{(c)(\tau)(\nu-1)} + \hat{\theta}_{g_q t_k}^{(c)(\tau)(\nu)} - v_{g_q t_k}^{(c)(\tau)(\nu)}), \\
& \quad \forall g_q \in G, \forall (c) \in \mathcal{L}, \forall \tau \in \Omega \\
& 2. (P_{T_r t_k}^{(c)(\tau)(\nu+1)}, \theta_{T_r t_k}^{(c)(\tau)(\nu+1)}, P_{T_r t_{k'}}^{(c)(\tau)(\nu+1)}, \theta_{T_r t_{k'}}^{(c)(\tau)(\nu+1)}) \\
& = \mathbf{prox}_{F(P)+\chi(P, \theta), \rho} (P_{T_r t_k}^{(c)(\tau)(\nu)} - \hat{P}_{T_r t_k}^{(c)(\tau)(\nu)} - u_{T_r t_k}^{(c)(\tau)(\nu)}, \\
& \quad \hat{v}_{T_r t_k}^{(c)(\tau)(\nu-1)} + \hat{\theta}_{T_r t_k}^{(c)(\tau)(\nu)} - v_{T_r t_k}^{(c)(\tau)(\nu)}), \\
& \quad \forall T_r \in T, \forall (c) \in \mathcal{L}, \forall \tau \in \Omega \\
& 3. (P_{D_d t_k}^{(c)(\tau)(\nu+1)}, \theta_{D_d t_k}^{(c)(\tau)(\nu+1)}) \\
& = \mathbf{prox}_{-D, \rho} (\hat{v}_{D_d t_k}^{(c)(\tau)(\nu-1)} + \hat{\theta}_{D_d t_k}^{(c)(\tau)(\nu)} - v_{D_d t_k}^{(c)(\tau)(\nu)}), \\
& \quad \forall D_d \in L, \forall (c) \in \mathcal{L}, \forall \tau \in \Omega \\
& 4. u_{N_i t_k}^{(c)(\tau)(\nu+1)} = u_{N_i t_k}^{(c)(\tau)(\nu)} + \hat{P}_{N_i t_k}^{(c)(\tau)(\nu+1)}, \\
& \quad \forall N_i \in \mathcal{N}, \forall (c) \in \mathcal{L}, \forall \tau \in \Omega \\
& 5. v_{N_i t_k}^{(c)(\tau)(\nu+1)} = \tilde{v}_{N_i t_k}^{(c)(\tau)(\nu)} + \tilde{\theta}_{N_i t_k}^{(c)(\tau)(\nu+1)}, \\
& \quad \forall N_i \in \mathcal{N}, \forall (c) \in \mathcal{L}, \forall \tau \in \Omega
\end{aligned}$$

7. Numerical Results

In this section, we present the results of the APMP algorithm for the LASCOFF problem instance for tracking demand variation for the IEEE Test cases [38] ~~Only for the 5 bus system, which isn't present there, we have presented in tables 1 and 2, respectively, the data pertaining to generators and lines.~~ ^{and for a} ~~with data shown~~

Table 1: Generator Data for 5 Bus System.

Node	A (\$/MWh ²)	B(\$/MWh)	C(\$/h)	\bar{P}	\underline{P}	\bar{R}	\underline{R}	Sched. MW
1	0.0430293	20	0	332.4	0	20	-20	140.765
2	0.25	20	0	140	0	15	-15	24.2275

no load coefficients of the cost curve, respectively. \bar{P} , \underline{P} , \bar{R} , and \underline{R} are respectively, the maximum, minimum generating limits and maximum and minimum ramping limits expressed in MW/dispatch interval. While performing the actual simulations, we have observed that it is very hard to solve the $(N - 1)$ SCOPFs just by using ADMM-PMP, perhaps due to difficulty ⁱⁿ ~~at~~ choosing the appropriate tuning parameter values. So, we have used a version of APMP for solving the SCOPFs as well, as illustrated in figure ~~(3)~~ ¹. Here, we apply the ADMM-PMP to solve OPFs for the base case as well as each contingency scenario separately and use an outer layer of APP over all the scenarios to reach a consensus regarding the pre-contingency dispatch (the message exchanges regarding the beliefs of each scenario for

Table 2: Line Data for 5 Bus System.

From Node	To Node	Resistance (pu)	Reactance (pu)	Flow Limit (MW)
1	2	0.02	0.06	100
1	3	0.08	0.24	100
2	3	0.06	0.18	100
2	4	0.06	0.18	100
2	5	0.04	0.12	100
3	4	0.01	0.03	100
4	5	0.08	0.24	100

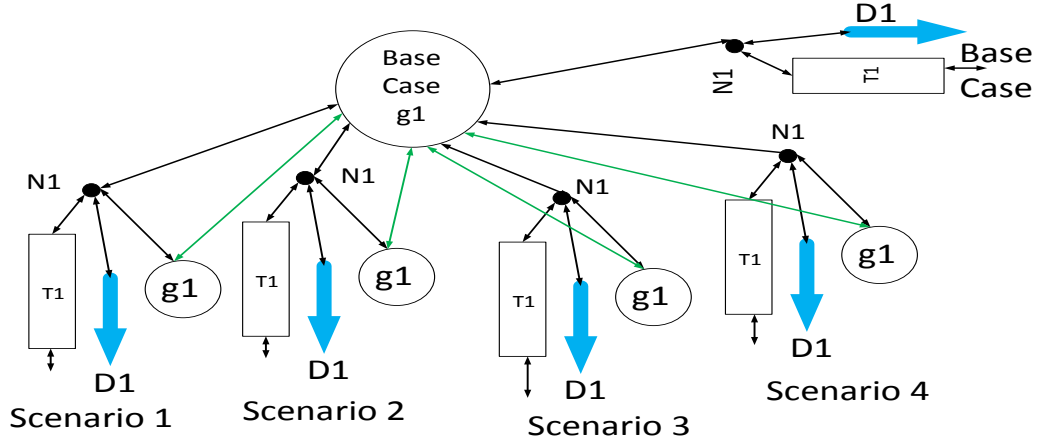


Figure 3: APMP for SCOPF in each dispatch interval

the value of dispatch, to reach consensus, are shown with the green lines). We have been successful at solving the SCOPFs using this approach for the IEEE 5, 14, 30, 48, 118, and 300 bus test cases with different lines marked for contingency, and also were able to identify those problem instances that are infeasible. Thereafter, for solving the LASCOPF, we can now treat the SCOPF solver as one unit, and as figure 4 shows, use the outermost APP layer to exchange messages about the beliefs of the different interval-wise generation values to reach a consensus. All the other LASCOPF problems can be solved in the exact same manner. We have run all these simulations on a Dell Inspiron 17R laptop computer powered by a 4x Intel(R) Core(TM) i7-4500U CPU running at 1.80 GHz, with a RAM of capacity 8054 MB and the OS is Ubuntu 16.04.4 LTS. We have coded all the simulation programs in C++11, with the generator optimization solvers being implemented in two modes:

- CVXGEN custom solvers for generators' optimization for fully distributed APMP (APP+ADMM-PMP)

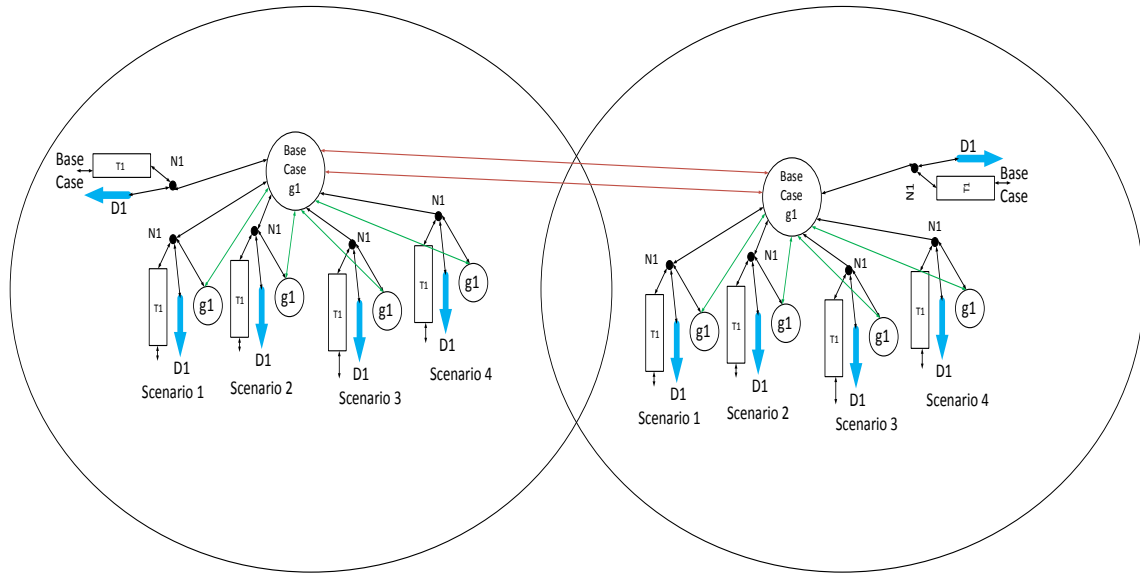


Figure 4: LASCOPF for Demand Variation with APMP Schematic for Outer APP Iterations

- GUROBI solvers for generators' optimization for fully distributed APMP (APP+ADMM-PMP)

The compiler used is GCC version 4.8.4. In the above-mentioned two methods, only the solvers are different. However, the implementation is a combination of two outer layers of coarse grained APP and one innermost layer of fine-grained ADMM-PMP. We have observed no or minimal disagreement between the results obtained by using these two solvers. However, the time taken by GUROBI is much longer than that by CVXGEN custom solvers. Hence, ~~in the rest of this material~~, we will only present the results we obtained using CVXGEN. We have experimented with IEEE test cases with 5, 14, 30, 48, 118, and 300 bus systems for ~~any number of the lines (for none to all) analyzed for contingencies~~, and for several ~~number~~ of dispatch intervals. The simulation software that we have designed can ~~tell us~~ (based on a combination logic of the number of iterations and convergence trend of the residuals of the different stages of the algorithm, as well as the different parts of the system) whether a particular problem instance is solvable or feasible or not. Out of those cases ~~that we found out to be feasible~~, we will present here the results of the following cases:

- 5 bus system with all lines ~~marked~~ ^{each analyzed} for contingency
- 14 bus system with all lines ~~marked~~ for contingency
- 30 bus system with lines (from node-to node) (5-7), (6-28), (10-21), and (12-16) ~~marked~~ for contingency

- 48 bus system with lines (1-5), (2-6), and (3-24) ^{analyzed} marked for contingency
- 118 bus system with lines (1-3) and (2-12) marked for contingency
- 300 bus system with lines (7-131) and (201-204) marked for contingency

Although we weren't able to do effective multithreading due to running the code on a small laptop, however, the table 3 below lists the simulation metrics of the above-mentioned cases and the Virtual Time is approximately the time in seconds that would have taken to solve, if we ~~would have been able to do~~ ^{had implemented} a complete nested parallelization. The other time represents the actual time taken on our laptop. The numbers there represent the residual of the outermost APP layer of the algorithm, which is the norm-2 of the disagreement between the different generation MW beliefs regarding the present, previous, and next interval generation values. The ** sign next to the residuals mean that the outer iteration has already converged and

Table 3: Convergence Metrics for the IEEE Test Cases

^{by the corresponding iteration count}

Iteration Count	5 Bus	14 Bus	30 Bus	48 Bus	118 Bus
1	0.796539	3.39049	0.961436	23.5966	8.86479
2	0.633823	2.83768	0.478086	12.6472	6.65057
3	0.498171	1.71538	**0.478086	8.52821	3.54619
4	**0.498171	0.971522	**0.478086	4.02483	2.29922
5	**0.498171	1.62635	**0.478086	1.77445	1.30795
6	**0.498171	1.26321	**0.478086	0.787015	0.812944
7	**0.498171	0.513289	**0.478086	0.294991	0.510176
Time (s)	0.973268	145.198	2.28523	55.6861	6229.04
Virtual Time (s)	0.464014	141.854	1.08964	22.9299	3537.62

we have fixed the subsequent residual values, just to help us draw the plot, that appears in fig 5. The load variation data for the 5 bus and 14 bus cases over 5 dispatch intervals, that we have simulated for, are explicitly shown in tables 4 and 5 respectively. The ones for the 30, 48, 118, and 300 bus systems are shown as only the variations from the base case value (that appear on the UW IEEE test case archive <https://labs.ece.uw.edu/pstca/>) of the particular loads in particular intervals in tables 6, 7, 8, and 9 respectively. The base case values occur in the first interval. For the OPF simulations, we have used the primal and dual residual tolerances to be 0.06 and 0.6, respectively (the reason, we use a higher value for the dual residual is that, we have observed that we get accurate enough solution for even somewhat higher value of the dual residual) and we have used a discrete version of proportional+derivative controller to adjust the value of ρ for the first 3000 iterations, such that at each iteration the relationship $\rho \times \epsilon_{primal} = \epsilon_{dual}$ is maintained, where ϵ_{primal} and ϵ_{dual} are respectively the primal and dual residuals. After the first 3000 iterations, if the algorithm hasn't still converged, then ρ is held fixed at the last value. The initial value of ρ

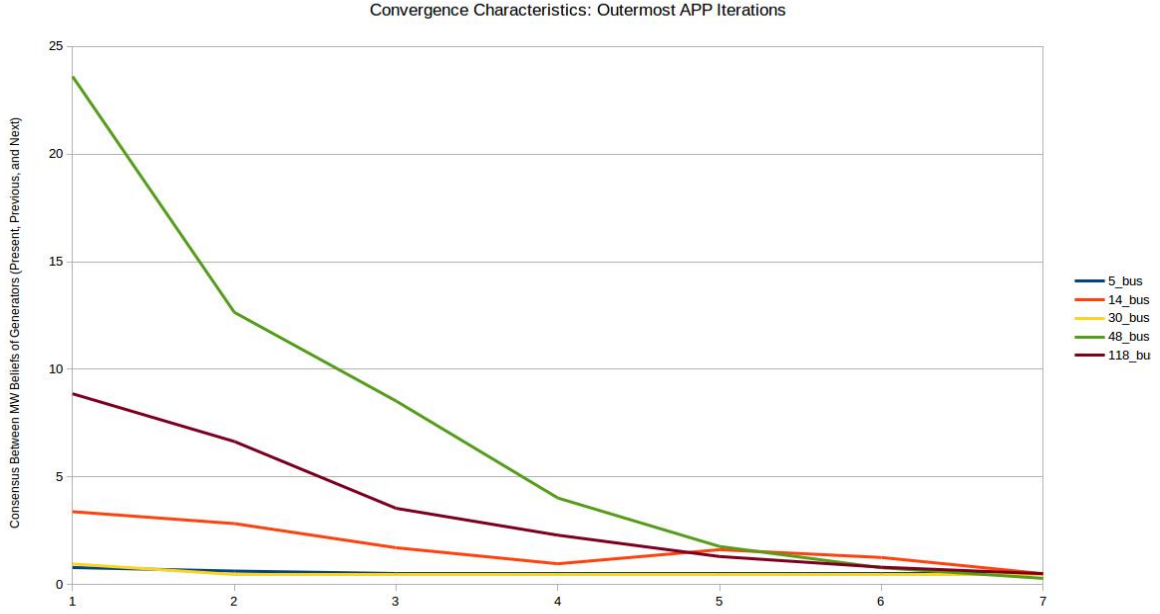


Figure 5: Convergence Characteristics of IEEE Test Systems

at the beginning of the iterations is taken as 1. The following are the APP parameters for outer iterations for attaining consensus among the different base/contingency scenarios for solving SCOPFs.

- $\alpha_{\text{SCOPF}} = 5$ for $\nu \leq 5$, $\alpha_{\text{SCOPF}} = 3$ for $5 < \nu \leq 10$, $\alpha_{\text{SCOPF}} = 2.5$ for $10 < \nu \leq 15$, $\alpha_{\text{SCOPF}} = 1.25$ for $15 < \nu \leq 20$, and $\alpha_{\text{SCOPF}} = 0.5$ for $\nu > 20$, $\beta_{\text{SCOPF}} = 200$, $\gamma_{\text{SCOPF}} = 100$ (ν is the APP iteration count)
- Final Tolerance ϵ_{SCOPF} : 0.7

Following are the APP parameters for outermost iterations for attaining consensus among the different MW outputs in different intervals, limited by ramp-rate constraints.

- $\alpha_{\text{LASCOF}} = 10$ for $\mu_{\text{APP}} \leq 5$, $\alpha_{\text{LASCOF}} = 5$ for $5 < \mu_{\text{APP}} \leq 10$, $\alpha_{\text{LASCOF}} = 2.5$ for $10 < \mu_{\text{APP}} \leq 15$, $\alpha_{\text{LASCOF}} = 1.25$ for $15 < \mu_{\text{APP}} \leq 20$, and $\alpha_{\text{LASCOF}} = 0.5$ for $\mu_{\text{APP}} > 20$, $\beta_{\text{LASCOF}} = 200$, $\gamma_{\text{LASCOF}} = 100$ (μ_{APP} is the outermost APP iteration count)
- Final Tolerance ϵ_{LASCOF} : 0.6

As it can be seen, for both the inner and outer APP iterations, we have used changing step-length. The way we chose those values are ad-hoc and gives a reasonable balance between accuracy and solve-time. The figure 5 shows the outer APP convergence for this example problem. We have also observed in our simulations that if we take more previous dummy intervals than the actual number of intervals, we get a higher accuracy. For instance,

Comment: This needs more explanation. This is the first time you mention "dummy intervals," I think

Table 4: Variation of Load for 5 Bus System.

Conn. Node	MW, Int.-1	MW, In.-2	MW, Int.-3	MW, Int.-4	MW, Int.-5
2	20	30	20	20	20
3	45	40	43	43	43
4	40	40	45	45	45
5	60	65	65	65	65

Table 5: Variation of Load for 14 Bus System.

Conn. Node	MW, Int.-1	MW, In.-2	MW, Int.-3	MW, Int.-4	MW, Int.-5
2	21.7	16.7	26.7	24.2	14.2
3	94.2	89.2	99.2	96.7	86.7
4	47.8	42.8	52.8	50.3	40.3
5	7.6	2.6	12.6	10.1	0.1
6	11.2	6.2	16.2	13.7	3.7
9	29.5	24.5	34.5	32	22
10	9	4	14	11.5	1.5
11	3.5	1.5	8.5	6	4
12	6.1	1.1	11.1	8.6	1.4
13	13.5	8.5	18.5	16	6
14	14.9	9.9	19.9	17.4	7.4

if we want to simulate for 5 dispatch intervals, then we can get convergence and better accuracy, if we simulate for at least 6 intervals, with the first interval being a dummy interval with the same load values as that for the actual first interval. As future research, we would like to explore this issue thoroughly and also devise proven means of tuning the path length for minimum number of iterations to converge. All the results presented above have been calculated using a single thread, although, the code is written in such a way, that, if desired, it can easily be converted to a multithreaded one. Implementing fully multithreaded code is one of the proposed future works.

8. Conclusion

Comment: I think that a reviewer is going to demand that we do a full multithreaded implementation.

In this paper, we have presented the look-ahead SCOPF considering variation of load demand over several dispatch time intervals and ability of the LASCOPF model to successfully track the temporal demand variation. We have also presented a completely decentralized or distributed computational algorithm to implement the calculations for such a massive scale problem. Through the simulation examples, we have demonstrated the scalability of the algorithm and the effectiveness of the methodology for dispatching systems where load varies over time. The next step in the research in this direction will be to represent the post-contingency states corresponding to each contingency scenario and the system

Table 6: Variation of Load for 30 Bus System.

Conn. Node	MW, Int.-2	MW, In.-3	MW, Int.-4	MW, Int.-5
10	0	0	10	0
12	5	0	0	0
18	0	3	0	0
24	0	0	0	12

Table 7: Variation of Load for 48 Bus System.

Conn. Node	MW, Int.-2	MW, In.-3	MW, Int.-4	MW, Int.-5
5	0	100	0	0
7	0	0	0	100
19	0	22	0	0
20	40	0	0	0
27	0	0	0	-20
29	0	0	-47	0
32	20	0	-14	0
38	0	-100	0	0
44	0	0	100	0

restoration to security over multiple dispatch intervals. We will be presenting that topic in our next paper.

Appendix A. Acknowledgment

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Table 8: Variation of Load for 118 Bus System.

Conn. Node	MW, Int.-2	MW, In.-3	MW, Int.-4	MW, Int.-5
36	0	0	20	0
51	-20	0	0	0
70	0	-10	0	0
77	0	0	0	40

Table 9: Variation of Load for 300 Bus System.

Conn. Node	MW, Int.-2	MW, In.-3	MW, Int.-4	MW, Int.-5
1	1	0	0	0
15	0	4	0	0
145	1	0	0	0
209	0	0	12	0
217	0	0	0	20
9533	1	0	0	0

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