

Manuscript Details

Manuscript number	IJEPES_2019_930
Title	Look-Ahead SCOPF (LASCOPF) for Tracking Demand Variation via Auxiliary Proximal Message Passing (APMP) Algorithm
Article type	Research paper

Abstract

In this paper, we will consider the Look-Ahead Security Constrained Optimal Power Flow (LASCOPF) problem looking forward multiple dispatch intervals, in which the load demand varies over dispatch intervals according to some forecast. We will consider the base-case and several contingency scenarios in the upcoming as well as in the subsequent dispatch intervals. We will formulate and solve the problem in a Model Predictive Control (MPC) paradigm. We will present the Auxiliary Proximal Message Passing (APMP) algorithm to solve this problem, which is a bi-layered decomposition-coordination type distributed algorithm, consisting of an outer Auxiliary Problem Principle (APP) layer and an inner Proximal Message Passing (PMP) layer. The APP part of the algorithm distributes the computation across several dispatch intervals and the PMP part performs the distributed computation within each of the dispatch interval across different devices (i.e. generators, transmission lines, loads) and nodes or nets. We will demonstrate the effectiveness of our method with a series of numerical simulations.

Keywords Alternating Direction Method of Multipliers (ADMM); Proximal Message Passing (PMP); Auxiliary Problem Principle (APP); contingency scenarios; Model Predictive Control (MPC)

Corresponding Author Sambuddha Chakrabarti

Corresponding Author's Institution KTH Royal Institute of Technology

Order of Authors Sambuddha Chakrabarti, Ross Baldick

Suggested reviewers Amin Kargarian, Ramtin Madani

Submission Files Included in this PDF

File Name [File Type]

Cover_Elsevier_LASCOPFDV (1).docx [Cover Letter]

Highlights_Elsevier_LASCOPFDV.docx [Highlights]

Look_Ahead_Demand_ADMM_journal (1).pdf [Manuscript File]

Submission Files Not Included in this PDF

File Name [File Type]

Look_Ahead_Demand_ADMM_journal.zip [LaTeX Source File]

To view all the submission files, including those not included in the PDF, click on the manuscript title on your EVISE Homepage, then click 'Download zip file'.

Research Data Related to this Submission

There are no linked research data sets for this submission. The following reason is given:
Data will be made available on request

Cover Letter for the manuscript, “Look-Ahead SCOPF (LASCOPF) for Tracking Demand Variation via Auxiliary Proximal Message Passing (APMP) Algorithm”

Sambuddha Chakrabarti & Ross Baldick

Title: Look-Ahead SCOPF (LASCOPF) for Tracking Demand Variation via Auxiliary Proximal Message Passing (APMP) Algorithm

Abstract: In this paper, we will consider the Look-Ahead Security Constrained Optimal Power Flow (LASCOPF) problem looking forward multiple dispatch intervals, in which the load demand varies over dispatch intervals according to some forecast. We will consider the base-case and several contingency scenarios in the upcoming as well as in the subsequent dispatch intervals. We will formulate and solve the problem in a Model Predictive Control (MPC) paradigm. We will present the Auxiliary Proximal Message Passing (APMP) algorithm to solve this problem, which is a bi-layered decomposition-coordination type distributed algorithm, consisting of an outer Auxiliary Problem Principle (APP) layer and an inner Proximal Message Passing (PMP) layer. The APP part of the algorithm distributes the computation across several dispatch intervals and the PMP part performs the distributed computation within each of the dispatch interval across different devices (i.e. generators, transmission lines, loads) and nodes or nets. We will demonstrate the effectiveness of our method with a series of numerical simulations.

Significance of the Present Work: Other than its own academic, intellectual, and research merit, the present work will act as the foundation, based on which, we will be developing more advanced versions of SCOPF, which will enable us to actively implement post-contingency restorative and corrective actions within the dispatch optimization. The algorithm, that we have presented here, will also be useful in several other large-scale optimization problems, which inherently have a nested-parallelization capability, to speed up computation time.

Contribution of Sambuddha Chakrabarti in the present work: Sambuddha Chakrabarti (the first and corresponding author) has been primarily responsible for carrying out the software development, fine-tuning the algorithm, and also developing the details of the mathematical model.

Contribution of Ross Baldick: Ross Baldick has been mainly responsible for generating the idea, master-planning of the project, part of which is this submission, inviting the funding, and coordinating the entire project. He has a profound contribution at refining the mathematical model and suggesting the particular algorithms, which has been made use of, in this paper.

Funding Information: The authors were supported, in part, by the National Science Foundation (NSF) under grant ECCS-1406894.

Corresponding Author Affiliation and Contact Details: Sambuddha Chakrabarti works as a postdoctoral researcher with the Electricity Market Research group (EMReG), School of Electrical Engineering & Computer Science (EECS), KTH Royal Institute of Technology, Stockholm, Sweden
email id: schakrabarti@utexas.edu

Address: Kungshamra 2, Room-1004, Solna-17070, Stockholm, Sweden (C/O: L Varawala)
Phone: +46-723947870

Co-Author Affiliation and Contact Details: Ross Baldick is a professor with the Department of Electrical and Computer Engineering, The University of Texas at Austin, Austin, Texas, USA

email id: baldick@ece.utexas.edu

Address: Ross Baldick, Engineering Education and Research Center EER 7.872, UT Austin, Austin, TX 78712-1084

Phone: 001-5124715879

Highlights for the manuscript, “Look-Ahead SCOPF (LASCOPF) for Tracking Demand Variation via Auxiliary Proximal Message Passing (APMP) Algorithm”

Sambuddha Chakrabarti & Ross Baldick

- The main objective of this paper is to present the $(N-1)$ *Security Constrained Optimal Power Flow (SCOPF)* for multiple dispatch time intervals-problem and a solution scheme, based on the *Model Predictive Control (MPC)* paradigm to solve it. At the onset of every time we solve it, we do so for the upcoming and several subsequent dispatch intervals, for which we have the load forecast, such that the power generators track the demand variation at the minimum possible cost, while abiding by the different network constraints, as well as generation limits.
- While solving the above-mentioned problem, we also consider the possibility of vulnerable lines, that may go out of service, during the actual operation. We dispatch the generators in such a way, that even if one of these outages, or “*contingencies*,” as we will henceforth call them, happens (one at a time), the post-contingency power flows will be well below the maximum limits allowed on each of the transmission lines.
- In order to solve such a huge size of the problem (even for small networks) within a reasonable time, we have proposed the *Auxiliary Proximal Message Passing (APMP)* algorithm. It is a bi-layered distributed decomposition-coordination type algorithm, which consists of an outer coarse-grained *Auxiliary Problem Principle (APP)* layer, which splits the computation across different dispatch intervals, and an inner fine-grained *Alternating Direction Method of Multipliers-Proximal Message Passing (ADMM-PMP)* layer, which splits the computation across the different generators, transmission lines, loads, and nodes.
- We have provided a detailed mathematical analysis and/or cited appropriate published references for the derivation of important theoretical results.
- We have validated our models and algorithms and demonstrated their capabilities with some numerical simulations, performed on the IEEE test cases.

Look-Ahead SCOPF (LASCOF) for Tracking Demand Variation via Auxiliary Proximal Message Passing (APMP) Algorithm [☆]

Sambuddha Chakrabarti^{a,*}, Ross Baldick^b

^a*Electricity Market Research Group (EMReG), School of Electrical Engineering and Computer Science (EECS), KTH Royal Institute of Technology, Stockholm 10044, Sweden*

^b*Department of Electrical & Computer Engineering (ECE), The University of Texas at Austin, Austin, TX 78705, USA*

Abstract

In this paper, we will consider the Look-Ahead Security Constrained Optimal Power Flow (LASCOF) problem looking forward multiple dispatch intervals, in which the load demand varies over dispatch intervals according to some forecast. We will consider the base-case and several contingency scenarios in the *upcoming* as well as in the *subsequent* dispatch intervals. We will formulate and solve the problem in a Model Predictive Control (MPC) paradigm. We will present the *Auxiliary Proximal Message Passing (APMP)* algorithm to solve this problem, which is a bi-layered decomposition-coordination type distributed algorithm, consisting of an outer *Auxiliary Problem Principle (APP)* layer and an inner *Proximal Message Passing (PMP)* layer. The *APP* part of the algorithm distributes the computation across several dispatch intervals and the *PMP* part performs the distributed computation within each of the dispatch interval across different devices (i.e. generators, transmission lines, loads) and nodes or nets. We will demonstrate the effectiveness of our method with a series of numerical simulations.

Keywords: Alternating Direction Method of Multipliers (ADMM), Proximal Message Passing (PMP), Auxiliary Problem Principle (APP), contingency scenarios, Model Predictive Control (MPC)

1. Introduction

In this paper, we will consider the $(N-1)$ *Security-Constrained Optimal Power Flow (SCOPF)* problem, in which devices (i.e. generators, transmission lines, loads etc.) are connected on

[☆]The objective of this paper is to present novel algorithmic and computational tools for solving the Look-Ahead Security Constrained Optimal Power Flow (LASCOF) accurately, provably, and reasonably fast. The authors were supported, in part, by the National Science Foundation (NSF) under grant ECCS-1406894.

*I am the corresponding author

Email addresses: schakrabarti@utexas.edu (Sambuddha Chakrabarti), baldick@ece.utexas.edu (Ross Baldick)

the power network and there exists a set of *scenarios* — each corresponding to the failure of a particular transmission line — over which we must ensure optimal feasible operation of the network. But, instead of solving the SCOPF for the upcoming dispatch interval only, at the start of each such interval, we will also look forward several subsequent dispatch intervals, in which the load demands change according to a forecast. We will take into account the ramp rates of generators and operate the system such that the generation cost is minimized, subject to the constraints of satisfying the changing load demand, line power flow limit constraints, minimum and maximum generation constraint of each generator, enforced in each of the upcoming and subsequent dispatch intervals considered.

The goal is to minimize a composite cost function that includes the cost (and constraints) of nominal operation, as well as those associated with constraints on operation in any of the (adverse) scenarios, for multiple dispatch intervals. This results in a large optimization problem, since variables in the network, namely, real power injection and bus voltage phase angle, are repeated $|\mathcal{L}|$ times for each of the $|\Omega|$ dispatch intervals, where $|\mathcal{L}|$ is the number of contingencies. Therefore we present a distributed algorithm to solve this problem, whereby we break the problem into several smaller independent sub-problems and solve them in parallel, such that the only co-ordination required is local, through the Locational Marginal Prices (LMP) updates. Specifically, we use a suitably modified version of the message passing algorithm from [1] and [2] to solve this problem efficiently.

For simplicity, we consider only DC power flow in this paper. The extension to AC power flow, involves simply applying the AC-OPF model from [3], [4], [5], [6], [7], and [8] to each scenario and requiring that the phase angles of a given device are equal across all scenarios in the respective time periods.

In the previous works in [1], [9], and [2], Kraning *et al.*, Liu *et al.* and Chakrabarti *et al.* respectively applied the Proximal Message Passing Algorithm to solving the standard Static Optimal Power Flow (OPF) Problem to solving the $(N - 1)$ Security Constrained OPF (SCOPF). In this paper, we extend this approach to solving a look-ahead dispatch problem, which considers the variation of load over a time horizon. There can be interesting and direct underpinnings of solving such problems, when it comes to design and allocation of Financial Transmission Rights (FTRs) under random outages, as well [10]. Our formulation for the LASCOPF is based on the Model Predictive Control (MPC) or Receding Horizon Control (RHC) methodology [11, 12, 13, 14, 15]. The rest of the paper is organized as follows: In section 2 we present a brief literature survey, after which, in section 3 we introduce the system of notations and present the conventional formulation of look-ahead SCOPF in the “Angles Represented” version, where the different real power flows on the Transmission Lines are represented in terms of the real powers injected at the buses and the voltage phase angles at the buses. In section 4, we apply the coarse-grained APP decomposition to split the problem into different dispatch intervals. In section 5, we reformulate all the preceding SCOPF problems (belonging to different dispatch intervals) into a different framework, the \mathcal{DTN} (Devices-Terminals-Nets) formulation, which is particularly suitable for applying the ADMM (Alternating Direction Method of Multipliers) [16] based Proximal Message Passing algorithm to the problems. In section 6, we present the Proximal Message Passing fine-grained decomposition for the present problem. In section 7, we present the results of

some simulation studies conducted on the IEEE test systems and we draw the concluding remarks and point to future research in section 8.

2. Literature Survey and Related work

The Optimal Power Flow Problem is at the heart of Power Systems planning and operations. Initially formulated by Carpentier in [17], it has been studied for more than half a century, and has extensively been applied in the industry as well, including solving the electricity market scheduling and dispatch calculations by the ISOs/TSOs (which involves solving the DC-OPF) and applications by the market participants for planning using production-cost modeling systems, like Uplan [18]. Happ in [19] and Chowdhury & Rahman in [20] trace the historical development of OPF upto the 90s. A recent reference that provides a good summary of the historical development of the problem is Cain, O'Neill and Castillo [21]. The references cited there also provide good insights into formulation and modeling particularly, of the ACOPF. There has been recently a surge in the interest in ACOPF following the works of Bai *et al.* in [3] and Lavaei *et al.* in [4], [5], [6], [7], where the authors showed that the ACOPF problem can be solved accurately by a convex relaxation of the original problem into a Semidefinite Programming problem. In the recent reference [22], the authors presented a modified version of the ADMM, showing that a global optimum for the highly non-convex AC-OPF can be obtained by applying this algorithm. The recent trend of applying distributed optimization algorithms to power flow problem, in order to speed up the computation time has gained ample momentum already. Two of the recent papers, that provide very good and comprehensive comparisons between the performance of several different algorithms from solving the OPF and SCOPF problems are [23] and [24], where the authors mentioned of ADMM, Proximal Message passing (PMP), APP, Consensus+Innovation (C+I), Optimality Condition Decomposition (OCD) etc. algorithms.

The pioneering work on the Security Constrained OPF (SCOPF) was done by Stott *et al* in [25]. Some of the recent references include works by Chiang *et al.* [26], Phan *et al.* [27], Chakrabarti *et al.* [2] etc. The last two deal with application of distributed algorithms to the SCOPF problem. In [8] and in [9], the authors have applied ADMM to solve the ACOPF and SCOPF, but not to a look-ahead problem. Significant early works on ADMM method during the 70s and 80s were [28], [29], [30] etc. followed by work during the 90's which include [31], [32]. Combining these two fields gives rise to the Distributed Computational methods for OPF [33], [34]. The last reference provides a good comparison of the distributed methods till the end of 90s. Good references on Model Predictive Control (MPC) or Receding Horizon Control (RHC) are [11, 12, 13, 14, 15]. In our present work, we will be solving the Look-Ahead Security Constrained Optimal Power Flow (LASCOPF) problem, formulated in a MPC/RHC format, using a combination of APP and ADMM-PMP algorithms, which we will call the APMP algorithm. As in all MPC/RHC formulation, the implicit assumption will be that, this problem for several look-ahead dispatch intervals is solved at the beginning of each dispatch interval, updating the forecasts, thereby reaching the best accuracy and ability to track the time-varying load demand.

3. Conventional Look-Ahead SCOPF Formulation

In this section, we will formulate the models for the look-ahead SCOPF in the conventional framework. First we will introduce the notations and symbols to be used for the rest of the paper, which are same as those introduced in [2] with a few additions.

3.1. Notations and Conventions

We have categorized the entities used in the subsequent formulations into five different groups: Sets, Elements, Index, Parameters, and Variables.

- *Sets*

\mathcal{D} : Set of Devices The next three sets form partitions of the set of devices:

$G \subseteq \mathcal{D}$: Set of Generators

$T \subseteq \mathcal{D}$: Set of Transmission Lines

$L \subseteq \mathcal{D}$: Set of Loads

\mathcal{T} : Set of Terminals

\mathcal{N} : Set of Nets or Buses or Nodes

$J(N_i)$: Set of buses/nodes/nets directly connected to node $N_i \in \mathcal{N}$

$\mathcal{L} = \{0, 1, 2, \dots, |\mathcal{L}|\}$: Set of possible $(N - 1)$ Contingencies. The element 0 indicates the base case

$\Omega = \{0, 1, 2, \dots, |\Omega|\}$: Set of Dispatch intervals or, the net Dispatch Horizon under consideration. 1 indicates the upcoming dispatch interval under consideration, for which a dispatch decision will definitely be implemented. The rest, $\{2, 3, \dots, |\Omega|\}$, are subsequent intervals in future. Hence, dispatch interval 0 is the current running one.

\dagger will be used to denote the transpose of a vector or matrix.

- *Elements*

t : Elements of \mathcal{T}

g : Elements of G

D : Elements of L

T : Elements of T

N : Elements of \mathcal{N}

- *Indices*

i, j : Nets

k : Terminals

q : Generators
 r : Transmission Lines
 d : Loads
 c : Contingencies
 τ : Dispatch Intervals
 ν : Iteration count for ADMM/PMP algorithm
 μ_{APP} : Iteration count for APP algorithm

- *Parameters*

$R_{T_r}, X_{T_r}, Z_{T_r} = R_{T_r} + (\sqrt{-1})X_{T_r}, B_{T_r}$: Resistance, Reactance, Impedance, and Susceptance of the r^{th} Transmission Line, respectively

α, β, γ : APP tuning parameters, ρ : ADMM-PMP tuning parameter

$\alpha_{g_q}, \beta_{g_q}, \gamma_{g_q}$: Quadratic, Linear, and Constant Cost Co-efficients of the q^{th} Generator

$C_{g_q}(\cdot), f_{dev}(\cdot)$ will be used to denote the cost function of the g_q^{th} generator and that of a generic device, respectively, throughout. We will introduce the other cost functions in the appropriate sections

$I_{\leq}(\cdot), I_{\geq}(\cdot), I_{=}(.)$: Indicator functions such that $I_{\leq}(x) = 0$, if $0 \leq x$, and $= \infty$ otherwise, and similarly for $I_{\geq}(\cdot)$ and $I_{=}(.)$

$\overline{P}_{g_q}, \underline{P}_{g_q}, \overline{R}_{g_q}, \underline{R}_{g_q} (= -\overline{R}_{g_q}, \text{usually}), \overline{L}_{T_r}$ denote the maximum and minimum generating limits of generators, maximum ramp-up and ramp-down limits of generators, and power carrying capacity of transmission lines respectively

- *Variables*

P : The real power or, MW associated with different devices

θ : The bus voltage phase angles associated with different nodes, as well as the devices

The following is the convention we follow in order to identify the associations of any particular variable to the sets: $(P_{N_t(c_1)(\tau_1)}^{(c_2)(\tau_2)})^{(\nu)}$. The foregoing notation refers to a variable (P) associated with a particular terminal indexed by the t of either a net or a device for the contingency scenario indexed by c_2 during τ_2 , as estimated by a computing unit linked to τ_1 and contingency scenario, c_1 . Sometimes we will use the net number instead of the terminal number in the above convention, when we want to indicate several devices connected to a particular net. If it is part of an iterative algorithm, then the outermost superscript ν indicates the iteration count. Whenever a variable is boldface, one or more of the indices will be missing and that means the boldface variable is a vector each of whose components will have all or some of the missing indices (the components themselves can be vectors or scalars). When the variable

is not bold-face and still some of the indices are missing, that means it is a scalar and the missing indices are either irrelevant or their values are implied from the context. Also it is to be observed that since generators and loads are single terminal devices, it is not necessary to specify the terminals for these, unless absolutely required.

3.2. Look-Ahead Dispatch: Generalized Case of Demand Variation for Multi-Bus Case

In this paper we build on the static SCOPF problem formulation in [2]. We will develop the mathematical model for a look-ahead dispatch calculation that considers an upcoming and several subsequent dispatch intervals at the onset of each current dispatch interval and takes into account the possible variations of operating parameters across different scenarios represented in different subsequent dispatch intervals, so that at each interval the entire system is secure. At the end of each current dispatch interval, the calculation “rolls forward” and the whole look-ahead calculation is repeated with updated demand forecast, just as in a “Model Predictive Control (MPC)” or a “Receding Horizon Control (RHC).” The generalized version of this problem for a multi-time horizon, arbitrary network is as follows:

$$\min_{P_{gq}^{(\tau)}, \theta} \sum_{\tau \in \Omega} \sum_{gq \in G} C_{gq}(P_{gq}^{(\tau)}) \quad (1a)$$

$$\text{Subject to : } \forall (c) \in \mathcal{L}, \forall \tau \in \Omega, \forall T_r \in T$$

$$P_{gqN_i}^{(\tau)} - P_{DdN_i}^{(\tau)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(0)}(\theta_{N_i}^{(0)(\tau)} - \theta_{N_{\bar{i}}}^{(0)(\tau)}); \forall N_i \in \mathcal{N} \quad (1b)$$

$$P_{gqN_i}^{(\tau)} - P_{DdN_i}^{(\tau)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(c)}(\theta_{N_i}^{(c)(\tau)} - \theta_{N_{\bar{i}}}^{(c)(\tau)}); \forall N_i \in \mathcal{N} \quad (1c)$$

$$|B_{T_r}^{(0)}(\theta_{T_{rt_1}}^{(0)(\tau)} - \theta_{T_{rt_2}}^{(0)(\tau)})| \leq \bar{L}_{T_r}^{(0)}, \forall T_r \in T \quad (1d)$$

$$|B_{T_r}^{(c)}(\theta_{T_{rt_1}}^{(c)(\tau)} - \theta_{T_{rt_2}}^{(c)(\tau)})| \leq \bar{L}_{T_r}^{(c)}, \forall T_r \in T \quad (1e)$$

$$\underline{R}_{gq} \leq (P_{gq}^{(\tau+1)} - P_{gq}^{(\tau)}) \leq \bar{R}_{gq} \quad \forall gq \in G \quad (1f)$$

4. APMP Algorithm: The Coarse Grained Decomposition Component

In this section, we will provide the mathematical formulation of the Auxiliary Proximal Message Passing (APMP) algorithm, as applied to the LASCOPF problem. We will start by describing the APP based coarse-grained component of the algorithm, which will be applied to the LASCOPF problems, followed by the PMP fine-grained component, which decomposes the SCOPF problems (and the OPF problems) of each dispatch interval into the device level computations. The basic idea behind the APP based coarse-grained distributed component of the algorithm is depicted in figure 1, where the two circles correspond to different dispatch intervals and/or scenarios, across which the computation is split, and the arrows indicate the exchange of messages between those. The messages correspond to the beliefs of each circle or coarse grain about the values of power generations and/or injections/line flows within itself and also of those belonging to the neighboring circles.

4.1. Generalized Case of Demand Variation for Multi-Bus Case

Referring to the figure 2, the different overlapping circles here represent the different dispatch intervals, and with $\tau = 0$ and $\tau = 4$ representing respectively, the latest dispatch interval for which the LASCOPF problem is already solved & the results are known, and the dispatch interval following the concerned time horizon. In the coarse-grained decomposition, when $\tau = 1$, $P_{g_2(\tau)}^{(\tau-1)}$ and $P_{g_1(\tau)}^{(\tau-1)}$ are replaced by $P_{g_2}^{(0)}$ and $P_{g_1}^{(0)}$ respectively, which are the (known) MW outputs from the current dispatch interval. When $\tau = 3$, $P_{g_2(\tau)}^{(\tau+1)}$ and $P_{g_1(\tau)}^{(\tau+1)}$ are replaced by $P_{g_2}^{(4)}$ and $P_{g_1}^{(4)}$ respectively, which are the last but one iterate values of the MW outputs from the last dispatch interval. We will now apply the Auxiliary Problem Principle (APP)[35], [36] to the above optimization problem to derive a set of expressions for the iterate updates. This reformulation is very similar in flavor to the ones presented previously in [33], [37], [38], [39] etc. We will state the coarse-grained APP based decomposition of the problem (the one for which the conventional formulation has been presented in equation (1)). We will distribute the SCOPF across each dispatch interval and then exchange messages regarding what a particular interval thinks about the optimal values of the decision variables for the intervals immediately preceding and succeeding it, eventually attempting to achieve a consensus between those. In the figure, the values shown at the two sides of the arrows are those “beliefs” among which we are trying to achieve consensus. We follow the same convention as introduced in section 3.1. As before, notice that each of the subproblems is very similar to the classical $(N - 1)$ SCOPF problems, but with two important differences. First of all, each of these has ramp rate constraints, to account for the change in generator outputs over multiple intervals. Secondly, there are regularization terms added to the cost function for attaining consensus among different coarse grains about the values of the decision variables, which, in this case, are generator outputs. The second terms in the objective functions are the ones representing the proximity from previous iterates. The third and fourth terms in the objective functions of equations (2) and (4) and the third, fourth, fifth, and sixth terms in the objective function of equation (3) are the ones for attaining consensus among the coarse grains. The last terms in the objective functions of equations (2) and (4) and the last two terms in the objective function of equation (3) correspond to complementary slackness. Note that, in this case also, the real power generation variables are the only ones at which we wish to attain consensus through the application of APP. The bus voltage angles can be thought of as localized to the particular dispatch interval sub-problems, whose values at any iteration are determined by the power generation or injection profiles. We have split the LASCOPF problem into three types of sub-problems, each corresponding to a particular dispatch time interval.

First Dispatch Interval SCOPF:

$$\begin{aligned} \mathbf{P}_{(1)}^{(\mu_{APP}+1)} = \underset{\mathbf{P}_{(1)}, \theta}{\operatorname{argmin}} \sum_{g_q \in G} C_{g_q}(P_{g_q(1)}^{(1)}) + \\ \frac{\beta}{2} \|\mathbf{P}_{(1)} - \mathbf{P}_{(1)}^{(\mu_{APP})}\|_2^2 \\ + \gamma [\mathbf{P}_{(1)}^{(1)\dagger} (\mathbf{P}_{(1)}^{(\mu_{APP})} - \mathbf{P}_{(2)}^{(\mu_{APP})}) + \end{aligned}$$

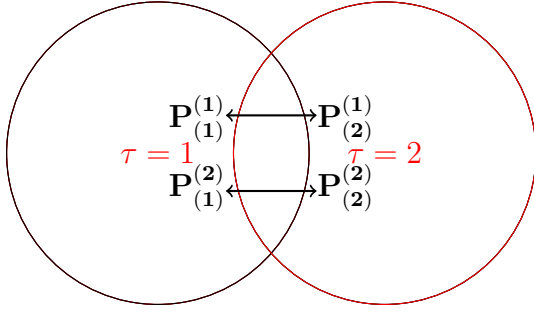


Figure 1: Schematic for APP Message Exchange.

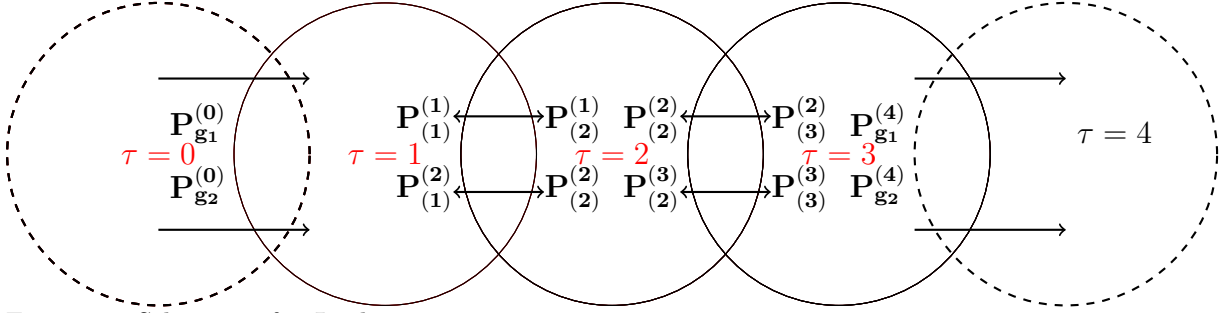


Figure 2: Schematic for Look-Ahead SCOPF for Demand Variation

$$\begin{aligned} & \mathbf{P}_{(1)}^{(2)\dagger} (\mathbf{P}_{(1)}^{(2)(\mu_{APP})} - \mathbf{P}_{(2)}^{(2)(\mu_{APP})}) \\ & + \lambda_1^{(\mu_{APP})\dagger} \mathbf{P}_{(1)}^{(1)} + \lambda_2^{(\mu_{APP})\dagger} \mathbf{P}_{(1)}^{(2)} \end{aligned} \quad (2a)$$

Subject to : $\forall(c) \in \mathcal{L}, \forall T_r \in T$

Power-Balance Constraints (Base-Case & Contingency):

$$P_{gq_{N_i}(1)}^{(1)} - P_{D_{dN_i}}^{(1)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(0)} (\theta_{N_i}^{(0)(1)} - \theta_{N_{\bar{i}}}^{(0)(1)}); \forall N_i \in \mathcal{N} \quad (2b)$$

$$P_{gq_{N_i}(1)}^{(1)} - P_{D_{dN_i}}^{(1)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(c)} (\theta_{N_i}^{(c)(1)} - \theta_{N_{\bar{i}}}^{(c)(1)}); \forall N_i \in \mathcal{N} \quad (2c)$$

Flow Limit Constraints (Base-Case & Contingency):

$$|B_{T_r}^{(0)} (\theta_{T_{rt_1}}^{(0)(1)} - \theta_{T_{rt_2}}^{(0)(1)})| \leq \bar{L}_{T_r}^{(0)}, \forall T_r \in T \quad (2d)$$

$$|B_{T_r}^{(c)} (\theta_{T_{rt_1}}^{(c)(1)} - \theta_{T_{rt_2}}^{(c)(1)})| \leq \bar{L}_{T_r}^{(c)}, \forall T_r \in T \quad (2e)$$

Ramp-Rate Constraints:

$$\underline{R}_{g_q} \leq P_{g_q(1)}^{(2)} - P_{g_q(1)}^{(1)} \leq \bar{R}_{g_q}, \forall g_q \in G \quad (2f)$$

$$\underline{R}_{g_q} \leq P_{g_q(1)}^{(1)} - P_{g_q}^{(0)} \leq \bar{R}_{g_q}, \forall g_q \in G \quad (2g)$$

Intermediate Dispatch Interval $\tau \in \{2, 3, \dots, |\Omega| - 1\}$ SCOPF:

$$\begin{aligned}
\mathbf{P}_{(\tau)}^{(\mu_{APP}+1)} = & \underset{\mathbf{P}_{(\tau)}, \theta}{\operatorname{argmin}} \sum_{g_q \in G} C_{g_q}(P_{g_q(\tau)}^{(\tau)}) + \\
& \frac{\beta}{2} \|\mathbf{P}_{(\tau)} - \mathbf{P}_{(\tau)}^{(\mu_{APP})}\|_2^2 \\
& + \gamma [\mathbf{P}_{(\tau)}^{(\tau-1)\dagger} (\mathbf{P}_{(\tau)}^{(\tau-1)(\mu_{APP})} - \mathbf{P}_{(\tau-1)}^{(\tau-1)(\mu_{APP})}) + \\
& \quad \mathbf{P}_{(\tau)}^{(\tau)\dagger} (\mathbf{P}_{(\tau)}^{(\mu_{APP})} - \mathbf{P}_{(\tau-1)}^{(\mu_{APP})}) \\
& \quad + \mathbf{P}_{(\tau)}^{(\tau)\dagger} (\mathbf{P}_{(\tau)}^{(\mu_{APP})} - \mathbf{P}_{(\tau+1)}^{(\mu_{APP})}) + \\
& \quad \mathbf{P}_{(\tau)}^{(\tau+1)\dagger} (\mathbf{P}_{(\tau+1)}^{(\mu_{APP})} - \mathbf{P}_{(\tau+1)}^{(\mu_{APP})})] \\
& + \lambda_{2\tau-1}^{(\mu_{APP})\dagger} \mathbf{P}_{(\tau)}^{(\tau)} + \lambda_{2\tau}^{(\mu_{APP})\dagger} \mathbf{P}_{(\tau)}^{(\tau+1)} \\
& - \lambda_{2\tau-3}^{(\mu_{APP})\dagger} \mathbf{P}_{(\tau)}^{(\tau-1)} - \lambda_{2\tau-2}^{(\mu_{APP})\dagger} \mathbf{P}_{(\tau)}^{(\tau)}
\end{aligned} \tag{3a}$$

Subject to : $\forall(c) \in \mathcal{L}, \forall T_r \in T$

Power-Balance Constraints (Base-Case & Contingency):

$$P_{g_{qN_i}(\tau)}^{(\tau)} - P_{D_{dN_i}(\tau)}^{(\tau)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(0)}(\theta_{N_i}^{(0)(\tau)} - \theta_{N_{\bar{i}}}^{(0)(\tau)}); \forall N_i \in \mathcal{N} \tag{3b}$$

$$P_{g_{qN_i}(\tau)}^{(\tau)} - P_{D_{dN_i}(\tau)}^{(\tau)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(c)}(\theta_{N_i}^{(c)(\tau)} - \theta_{N_{\bar{i}}}^{(c)(\tau)}); \forall N_i \in \mathcal{N} \tag{3c}$$

Flow Limit Constraints (Base-Case & Contingency):

$$|B_{T_r}^{(0)}(\theta_{T_{rt_1}}^{(0)(\tau)} - \theta_{T_{rt_2}}^{(0)(\tau)})| \leq \bar{L}_{T_r}^{(0)}, \forall T_r \in T \tag{3d}$$

$$|B_{T_r}^{(c)}(\theta_{T_{rt_1}}^{(c)(\tau)} - \theta_{T_{rt_2}}^{(c)(\tau)})| \leq \bar{L}_{T_r}^{(c)}, \forall T_r \in T \tag{3e}$$

Ramp-Rate Constraints:

$$\underline{R}_{g_q} \leq P_{g_q(\tau)}^{(\tau+1)} - P_{g_q(\tau)}^{(\tau)} \leq \bar{R}_{g_q}, \forall g_q \in G \tag{3f}$$

$$\underline{R}_{g_q} \leq P_{g_q(\tau)}^{(\tau)} - P_{g_q(\tau)}^{(\tau-1)} \leq \bar{R}_{g_q}, \forall g_q \in G \tag{3g}$$

Last Dispatch Interval, or Dispatch Interval $|\Omega|$ SCOPF:

$$\begin{aligned}
\mathbf{P}_{(|\Omega|)}^{(\mu_{APP}+1)} = & \underset{\mathbf{P}_{(|\Omega|)}, \theta}{\operatorname{argmin}} \sum_{g_q \in G} C_{g_q}(P_{g_q(|\Omega|)}^{(|\Omega|)}) + \\
& \frac{\beta}{2} \|\mathbf{P}_{(|\Omega|)} - \mathbf{P}_{(|\Omega|)}^{(\mu_{APP})}\|_2^2 \\
& + \gamma [\mathbf{P}_{(|\Omega|)}^{(|\Omega|-1)\dagger} (\mathbf{P}_{(|\Omega|)}^{(|\Omega|-1)(\mu_{APP})} - \mathbf{P}_{(|\Omega|-1)}^{(|\Omega|-1)(\mu_{APP})}) + \\
& \quad \mathbf{P}_{(|\Omega|)}^{(|\Omega|)\dagger} (\mathbf{P}_{(|\Omega|)}^{(\mu_{APP})} - \mathbf{P}_{(|\Omega|-1)}^{(\mu_{APP})})] \\
& - \lambda_{2|\Omega|-3}^{(\mu_{APP})\dagger} \mathbf{P}_{(|\Omega|)}^{(|\Omega|-1)} - \lambda_{2|\Omega|-2}^{(\mu_{APP})\dagger} \mathbf{P}_{(|\Omega|)}^{(|\Omega|)}
\end{aligned} \tag{4a}$$

Subject to : $\forall(c) \in \mathcal{L}, \forall T_r \in T$

Power-Balance Constraints (Base-Case & Contingency):

$$P_{gqN_i(|\Omega|)}^{(|\Omega|)} - P_{DdN_i}^{(|\Omega|)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(0)}(\theta_{N_i}^{(0)(|\Omega|)} - \theta_{N_{\bar{i}}}^{(0)(|\Omega|)});$$

$$\forall N_i \in \mathcal{N} \quad (4b)$$

$$P_{gqN_i(|\Omega|)}^{(|\Omega|)} - P_{DdN_i}^{(|\Omega|)} = \sum_{N_{\bar{i}} \in J(N_i)} B_{T_r}^{(c)}(\theta_{N_i}^{(c)(|\Omega|)} - \theta_{N_{\bar{i}}}^{(c)(|\Omega|)});$$

$$\forall N_i \in \mathcal{N} \quad (4c)$$

Flow Limit Constraints (Base-Case & Contingency):

$$|B_{T_r}^{(0)}(\theta_{T_{rt_1}}^{(0)(|\Omega|)} - \theta_{T_{rt_2}}^{(0)(|\Omega|)})| \leq \bar{L}_{T_r}^{(0)}, \forall T_r \in T \quad (4d)$$

$$|B_{T_r}^{(c)}(\theta_{T_{rt_1}}^{(c)(|\Omega|)} - \theta_{T_{rt_2}}^{(c)(|\Omega|)})| \leq \bar{L}_{T_r}^{(c)}, \forall T_r \in T \quad (4e)$$

Ramp-Rate Constraints:

$$\underline{R}_{gq} \leq P_{gq(|\Omega|)}^{(|\Omega|)(\mu_{APP})} - P_{gq(|\Omega|)}^{(|\Omega|)} \leq \bar{R}_{gq}, \forall gq \in G \quad (4f)$$

$$\underline{R}_{gq} \leq P_{gq(|\Omega|)}^{(|\Omega|)} - P_{gq(|\Omega|)}^{(|\Omega|-1)} \leq \bar{R}_{gq}, \forall gq \in G \quad (4g)$$

Dual Variable Updates :

$$\lambda_{\tau}^{(\mu_{APP+1})} = \lambda_{\tau}^{(\mu_{APP})} + \alpha(\mathbf{P}_{(\tau)}^{(\tau)(\mu_{APP+1})} - \mathbf{P}_{(\tau+1)}^{(\tau)(\mu_{APP+1})}) \quad (4h)$$

$$\lambda_{\tau+1}^{(\mu_{APP+1})} = \lambda_{\tau+1}^{(\mu_{APP})} + \alpha(\mathbf{P}_{(\tau)}^{(\tau+1)(\mu_{APP+1})} - \mathbf{P}_{(\tau+1)}^{(\tau+1)(\mu_{APP+1})}) \quad (4i)$$

We have presented a coarse grained parallelization of the look-ahead SCOPF to cope with demand variation, where the entire problem is decomposed across the different dispatch time intervals. Subsequently, we will present the ADMM-Proximal Message Passing based fine grained decomposition and parallelization of the problem.

5. \mathcal{DTN} Reformulations of the SCOPF Problems

In this section we carry out the reformulation of the SCOPF models that we presented in the last section within each APP coarse grain, in order for us to be able to solve those problems by the Proximal Message Passing method. In order to simplify the presentation and also better clarify the meaning of each term, we will group the terms of the objective into four different categories. We will define them and write down the expressions pertaining to the SCOPF in the coarse grain corresponding to equation (3). These are:

1) Cost of Generation ($C(P)$): This term consists of the actual total cost of generating real power by the different generators as well as the indicator functions corresponding to the lower and upper generating limits of the different generators. For this term, the real power generated is always considered at the base case. This term is given as:

$$C(\mathbf{P}^{(0)(\tau)}) = \sum_{t_k \in gq \cap \mathcal{T}, q=1}^{|G|} \left(C_{gq}(P_{gqt_k}^{(0)(\tau)}) + I_{\leq}(\bar{P}_{gq} - P_{gqt_k}^{(0)(\tau)}) + I_{\leq}(P_{gqt_k}^{(0)(\tau)} - \underline{P}_{gq}) + \frac{\beta}{2} \left[(P_{gqt_k}^{(0)(\tau)} - \right. \right.$$

$$P_{g_{t_k}}^{(0)(\tau)(\mu_{APP})})^2 + \sum_{s=-1,1} (P_{g_{t_k}}^{(0)(\tau+s)} - P_{g_{t_k}}^{(0)(\tau+s)(\mu_{APP})})^2 \Big] + \gamma \Big[\sum_{s=-1,1} (P_{g_{t_k}}^{(0)(\tau)} (P_{g_{t_k}}^{(0)(\tau)(\mu_{APP})} - P_{g_{t_k}}^{(0)(\tau)(\mu_{APP})}) + P_{g_{t_k}}^{(0)(\tau+s)} (P_{g_{t_k}}^{(0)(\tau+s)(\mu_{APP})} - P_{g_{t_k}}^{(0)(\tau+s)(\mu_{APP})})) \Big] + (\lambda_{g_{2\tau-1}}^{(\mu_{APP})} - \lambda_{g_{2\tau-2}}^{(\mu_{APP})}) P_{g_{t_k}}^{(0)(\tau)} + \lambda_{g_{2\tau}}^{(\mu_{APP})} P_{g_{t_k}}^{(0)(\tau+1)} - \lambda_{g_{2\tau-3}}^{(\mu_{APP})} P_{g_{t_k}}^{(0)(\tau-1)} \Big)$$

For the sake of brevity, we are presenting here the terms corresponding to line flow limit constraint ($F(P^{(c)(\tau)})$), power-angle relation ($\chi(P^{(c)(\tau)}, \theta^{(c)(\tau)})$), and ramp constraint ($\Delta(P^{(0)(\tau)})$) in the most general and condensed forms. The details of these can be found in [2] and [40].

2) Line Flow Limit Constraint ($F(P)$): This term consists of the sum of the indicator functions corresponding to enforcing that real power flows on the lines are less than the maximum allowed, both at the base-case as well as under different contingencies. This term can be written as:

$$F(P^{(c)(\tau)}) = \sum_{(c) \in \mathcal{L}} \sum_{T_r \in \mathcal{T}} \sum_{t_k \in T_r \cap \mathcal{T}} I_{\leq}(\bar{L}_{T_r}^{(c)} - |P_{T_r t_k}^{(c)(\tau)}|)$$

3) Power-Angle Relation ($\chi(P, \theta)$): This term consists of the sum of the indicator functions corresponding to the relation of the power flow at each end of the lines and the voltage phase angles at the two ends, both at the base-case and the contingencies. This term can be written as:

$$\chi(P^{(c)(\tau)}, \theta^{(c)(\tau)}) = \sum_{(c) \in \mathcal{L}} \sum_{T_r \in \mathcal{T}} \sum_{t_k, t_{k'} \in T_r \cap \mathcal{T}} I_{=}\left(P_{T_r t_k}^{(c)(\tau)} + \frac{\theta_{T_r t_k}^{(c)(\tau)} - \theta_{T_r t_{k'}}^{(c)(\tau)}}{X_{T_r}^{(c)}}\right)$$

4) Ramp Constraint ($\Delta(P)$): This term corresponds to the change of power output of generator from one time period to another and the maximum rate at which it can go up or down. This term can be written as:

$$\Delta(P^{(0)(\tau)}) = \sum_{s=-1,1} \sum_{t_k \in g_q \cap \mathcal{T}, q=1}^{|G|} (I_{\leq}(\bar{R}_{g_q} - P_{g_{t_k}}^{(0)(\tau+s)} + P_{g_{t_k}}^{(0)(\tau)}) + I_{\leq}(P_{g_{t_k}}^{(0)(\tau+s)} - P_{g_{t_k}}^{(0)(\tau)} - \underline{R}_{g_q}))$$

The reformulated equations for the problem stated in equations (2)-(4) are as follows:

$$\min_{P_{t_k}^{(c)(\tau)}, \theta_{t_k}^{(c)(\tau)}} f(P) = C(P^{(0)(\tau)}) + F(P^{(c)(\tau)}) + \chi(P^{(c)(\tau)}, \theta^{(c)(\tau)}) + \Delta(P^{(0)(\tau)}) \quad (5a)$$

$$\text{Subject to: } \hat{P}_{N_{it_k}}^{(c)(\tau)} = 0, \tilde{\theta}_{N_{it_k}}^{(c)(\tau)} = 0, \\ \forall N_i \in \mathcal{N}, \forall t_k \in \mathcal{T}, \forall (c) \in \mathcal{L} \quad (5b)$$

In the above set of equations, $\hat{P}_{N_{it_k}}^{(c)(\tau)}$ is the average node power injection and $\tilde{\theta}_{N_{it_k}}^{(c)(\tau)}$ is the deviation of the voltage phase angle of each device connected to a particular node from the average node voltage phase angle. Readers are referred to [2], [1] for more information on these. Hence, constraints stated in equation (5b) represent the Kirchhoff's law for node power balance and angle consistency.

6. APMP Algorithm: The Fine Grained Decomposition Component

In this section we state the equations for the Proximal Message Passing algorithm based fine-grained decomposition of the above problem.

6.1. Proximal Message Passing for the Look-Ahead Dispatch: Generalized Case of Demand Variation for the Multi Bus-Case

From equation (5), the slightly reformulated \mathcal{DTN} equations are as follows:

$$\begin{aligned} & \min_{P_{t_k}^{(c)}, \theta_{t_k}^{(c)}} C(P^{(0)(\tau)}) + F(P^{(c)(\tau)}) + \chi(P^{(c)(\tau)}, \theta^{(c)(\tau)}) + \\ & \Delta(P^{(0)(\tau)}) + \sum_{(c) \in \mathcal{L}} \sum_{N_i \in \mathcal{N}} (\hat{I}(z_{N_i t_k}^{(c)(\tau)}) + \tilde{I}(\xi_{N_i t_k}^{(c)(\tau)})) \\ & \text{Subject to: } P_{t_k}^{(c)(\tau)} = z_{t_k}^{(c)(\tau)}, \theta_{t_k}^{(c)(\tau)} = \xi_{t_k}^{(c)(\tau)}, \\ & \forall N_i \in \mathcal{N}, \forall t_k \in \mathcal{T}, \forall (c) \in \mathcal{L} \end{aligned} \quad (6)$$

where $\hat{I}(z_{N_i t_k})$ and $\tilde{I}(\xi_{N_i t_k})$ are indicator functions of the sets $\{z_{t_k} | \hat{z}_{N_i t_k} = 0\}$ and $\{\xi_{t_k} | \tilde{\xi}_{N_i t_k} = 0\}$ respectively. Again, for the sake of brevity, we will skip writing down the iterates for the transmission lines and loads (which are simple projections), and make a mention that they are the same as the ones presented in references [2] and [40].

6.1.1. Iterates for Generators

The iterates for generators consist of the update equations for the real power output and voltage-phase angles of the generator terminals for both the base case and the different $(N - 1)$ contingency scenarios and are as follows:

$$\begin{aligned} & (P_{g_q t_k}^{(0)(\tau)(\nu+1)}, \theta_{g_q t_k}^{(c)(\tau)(\nu+1)}) \\ & = \operatorname{argmin}_{P_{g_q t_k}^{(0)(\tau)}, \theta_{g_q t_k}^{(c)(\tau)}} [C_{g_q}(P_{g_q t_k}^{(0)(\tau)}) + I_{\leq}(\bar{P}_{g_q} - P_{g_q t_k}^{(0)(\tau)}) \\ & + I_{\leq}(P_{g_q t_k}^{(0)(\tau)} - \underline{P}_{g_q}) + \frac{\beta}{2}[(P_{g_q t_k}^{(0)(\tau)} - P_{g_q t_k}^{(0)(\tau)(\mu_{APP})})^2 \\ & + \sum_{s=-1,1} (P_{g_q t_k}^{(0)(\tau+s)} - P_{g_q t_k}^{(0)(\tau+s)(\mu_{APP})})^2] + \\ & \gamma[\sum_{s=-1,1} (P_{g_q t_k}^{(0)(\tau)}(P_{g_q t_k}^{(0)(\tau)(\mu_{APP})} - P_{g_q t_k}^{(0)(\tau)(\mu_{APP})}) \\ & + P_{g_q t_k}^{(0)(\tau+s)}(P_{g_q t_k}^{(0)(\tau+s)(\mu_{APP})} - P_{g_q t_k}^{(0)(\tau+s)(\mu_{APP})})) + \\ & (\lambda_{g_q 2(\tau-1)}^{(\mu_{APP})} - \lambda_{g_q 2(\tau-2)}^{(\mu_{APP})})P_{g_q t_k}^{(0)(\tau)} + \lambda_{g_q 2\tau}^{(\mu_{APP})}P_{g_q t_k}^{(0)(\tau+1)} - \\ & \lambda_{g_q 2(\tau-3)}^{(\mu_{APP})}P_{g_q t_k}^{(0)(\tau-1)} + \Delta(P^{(0)(\tau)}) + \\ & \sum_{(c) \in \mathcal{L}} (\frac{\rho}{2})(\|P_{g_q t_k}^{(0)(\tau)} - z_{g_q t_k}^{(c)(\tau)(\nu)} + u_{g_q t_k}^{(c)(\tau)(\nu)}\|_2^2 \\ & + \|\theta_{g_q t_k}^{(c)(\tau)} - \xi_{g_q t_k}^{(c)(\tau)(\nu)} + v_{g_q t_k}^{(c)(\tau)(\nu)}\|_2^2)], \\ & \forall g_q \in G, \tau \in \Omega, t_k \in \mathcal{T} \cap G \end{aligned}$$

6.1.2. Iterates for Nets

We are writing here just the analytical forms already derived in [1].

$$\begin{aligned} & \forall N_i \in \mathcal{N}, \forall t_k \in \mathcal{T} \cap N_i, \forall (c) \in \mathcal{L}, \tau \in \Omega \\ & z_{N_i t_k}^{(c)(\tau)(\nu+1)} \\ & = u_{N_i t_k}^{(c)(\tau)(\nu)} + P_{N_i t_k}^{(c)(\tau)(\nu+1)} - \hat{u}_{N_i t_k}^{(c)(\tau)(\nu)} - \hat{P}_{N_i t_k}^{(c)(\tau)(\nu+1)} \\ & \xi_{N_i t_k}^{(c)(\tau)(\nu+1)} = \hat{v}_{N_i t_k}^{(c)(\tau)(\nu)} + \hat{\theta}_{N_i t_k}^{(c)(\tau)(\nu+1)} \\ & u_{N_i t_k}^{(c)(\tau)(\nu+1)} = u_{N_i t_k}^{(c)(\tau)(\nu)} + (P_{N_i t_k}^{(c)(\tau)(\nu+1)} - z_{N_i t_k}^{(c)(\tau)(\nu+1)}) \\ & v_{N_i t_k}^{(c)(\tau)(\nu+1)} = v_{N_i t_k}^{(c)(\tau)(\nu)} + (\theta_{N_i t_k}^{(c)(\tau)(\nu+1)} - \xi_{N_i t_k}^{(c)(\tau)(\nu+1)}) \end{aligned}$$

Note that, not only do all the devices update their variables in parallel, but also, except the generators, all devices have associated with them the base-case and the contingency scenarios in each dispatch interval, each of which in turn update their respective variables in parallel as well. For the generators, the present dispatch interval and the next one are related through the ramp rate constraint ($\Delta(\cdot)$ function). Then all the nets and the base-case/contingency scenarios associated with them update the first two set of variables in parallel and then update the next two in parallel. The proximal function for a function g is given by

$$\mathbf{prox}_{g,\rho}(\mathbf{v}) = \underset{\mathbf{x}}{\operatorname{argmin}}(g(\mathbf{x}) + (\rho/2)\|\mathbf{x} - \mathbf{v}\|_2^2).$$

For this case, the prox messages and the Proximal Message Passing Algorithm is as follows:

$$\begin{aligned} & 1. (P_{g_q t_k}^{(0)(\tau)(\nu+1)}, \theta_{g_q t_k}^{(c)(\tau)(\nu+1)}) \\ &= \mathbf{prox}_{C(P)+\Delta(P),\rho}(P_{g_q t_k}^{(0)(\tau)(\nu)} - \hat{P}_{g_q t_k}^{(c)(\tau)(\nu)} - u_{g_q t_k}^{(c)(\tau)(\nu)}, \\ & \hat{v}_{g_q t_k}^{(c)(\tau)(\nu-1)} + \hat{\theta}_{g_q t_k}^{(c)(\tau)(\nu)} - v_{g_q t_k}^{(c)(\tau)(\nu)}), \\ & \forall g_q \in G, \forall (c) \in \mathcal{L}, \forall \tau \in \Omega \\ \\ & 2. (P_{T_r t_k}^{(c)(\tau)(\nu+1)}, \theta_{T_r t_k}^{(c)(\tau)(\nu+1)}, P_{T_r t_k'}^{(c)(\tau)(\nu+1)}, \theta_{T_r t_k'}^{(c)(\tau)(\nu+1)}) \\ &= \mathbf{prox}_{F(P)+\chi(P,\theta),\rho}(P_{T_r t_k}^{(c)(\tau)(\nu)} - \hat{P}_{T_r t_k}^{(c)(\tau)(\nu)} - u_{T_r t_k}^{(c)(\tau)(\nu)}, \\ & \hat{v}_{T_r t_k}^{(c)(\tau)(\nu-1)} + \hat{\theta}_{T_r t_k}^{(c)(\tau)(\nu)} - v_{T_r t_k}^{(c)(\tau)(\nu)}), \\ & \forall T_r \in T, \forall (c) \in \mathcal{L}, \forall \tau \in \Omega \\ \\ & 3. (P_{D_d t_k}^{(c)(\tau)(\nu+1)}, \theta_{D_d t_k}^{(c)(\tau)(\nu+1)}) \\ &= \mathbf{prox}_{-D,\rho}(\hat{v}_{D_d t_k}^{(c)(\tau)(\nu-1)} + \hat{\theta}_{D_d t_k}^{(c)(\tau)(\nu)} - v_{D_d t_k}^{(c)(\tau)(\nu)}), \\ & \forall D_d \in L, \forall (c) \in \mathcal{L}, \forall \tau \in \Omega \\ \\ & 4. u_{N_i t_k}^{(c)(\tau)(\nu+1)} = u_{N_i t_k}^{(c)(\tau)(\nu)} + \hat{P}_{N_i t_k}^{(c)(\tau)(\nu+1)}, \\ & \forall N_i \in \mathcal{N}, \forall (c) \in \mathcal{L}, \forall \tau \in \Omega \\ \\ & 5. v_{N_i t_k}^{(c)(\tau)(\nu+1)} = \tilde{v}_{N_i t_k}^{(c)(\tau)(\nu)} + \tilde{\theta}_{N_i t_k}^{(c)(\tau)(\nu+1)}, \\ & \forall N_i \in \mathcal{N}, \forall (c) \in \mathcal{L}, \forall \tau \in \Omega \end{aligned}$$

In the above, steps 1, 2, and 3 consist of the concurrent calculation of prox functions by the different devices, independently, which happens after the most recently updated values of the dual variables are “broadcast” to the devices from the nodes, as shown in figure 3, in which, we have shown two generators (g_1, g_2), two loads (D_1, D_2), one transmission line (T_1), and two nodes (N_1, N_2). The direction of the arrows show the direction of conveying the most recently updated values of the dual variables. The calculations of steps 4 and 5, which refer to updating of the dual variables corresponding to nodal power balance and angle consistency, respectively follow the above-mentioned prox-function calculation by the devices. Once the devices finish calculating their respective prox-functions, the most recently updated decision variable values are sent back to the respective nodes, through a process called “gather,” as

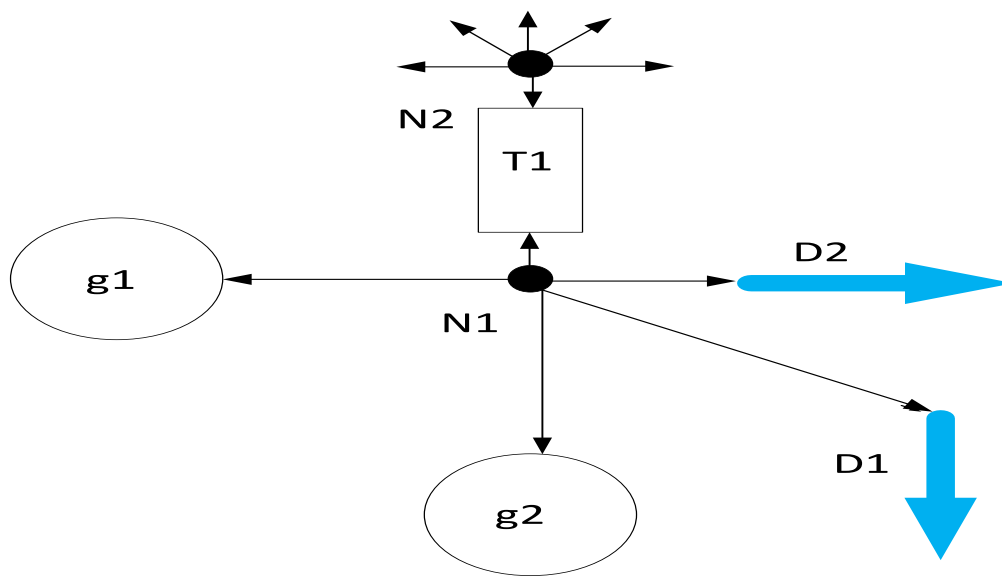


Figure 3: Broadcast: Calculation of Prox Function for Devices

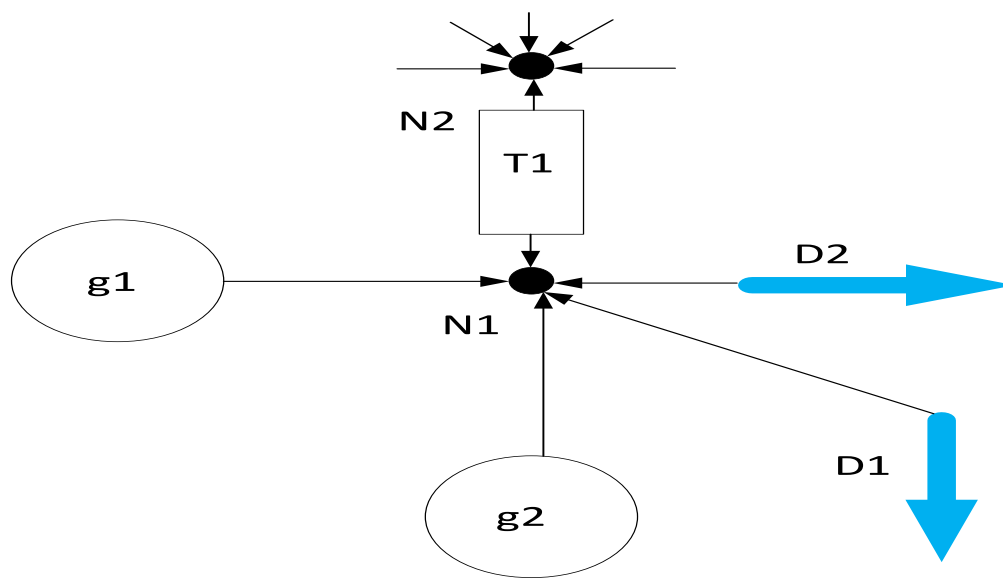


Figure 4: Gather: Updating of Dual variables by Nodes

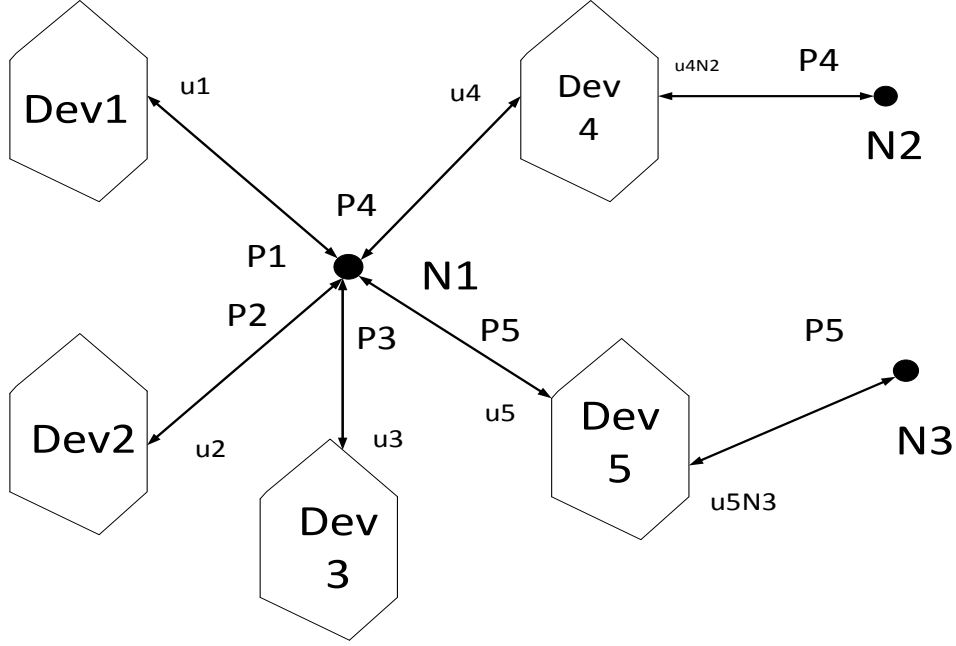


Figure 5: Generic ADMM: Broadcast and Gather

shown in figure 4 The ADMM based PMP can be thought of as a recurring sequence of such broadcast-gather operation until the convergence criteria is satisfied, as represented in figure 5 for a generic set of several devices and nodes. In the next section, we will present the numerical results pertaining to the LASCOPF simulations.

7. Numerical Results

In this section, we present the results of the APMP algorithm for the LASCOPF problem instance for tracking demand variation for the IEEE Test cases [41] and for a 5 bus system, with data shown in tables 1 and 2. In table 1, A , B , and C are the quadratic, linear, and no load coefficients of the cost curve, respectively. \bar{P} , \underline{P} , \bar{R} , and \underline{R} are respectively, the maximum, minimum generating limits and maximum and minimum ramping limits expressed in MW/dispatch interval. While performing the actual simulations, we have observed that it is very hard to solve the $(N - 1)$ SCOPFs just by using ADMM-PMP, perhaps due to

Table 1: Generator Data for 5 Bus System.

Node	A (\$/MWh ²)	B(\$/MWh)	C(\$/h)	\bar{P}	\underline{P}	\bar{R}	\underline{R}	Sched. MW
1	0.0430293	20	0	332.4	0	20	-20	140.765
2	0.25	20	0	140	0	15	-15	24.2275

Table 2: Line Data for 5 Bus System.

From Node	To Node	Resistance (pu)	Reactance (pu)	Flow Limit (MW)
1	2	0.02	0.06	100
1	3	0.08	0.24	100
2	3	0.06	0.18	100
2	4	0.06	0.18	100
2	5	0.04	0.12	100
3	4	0.01	0.03	100
4	5	0.08	0.24	100

difficulty in choosing the appropriate tuning parameter values. So, we have used a version of APMP for solving the SCOPFs as well, as illustrated in figure 6. Here, we apply the ADMM-PMP to solve OPFs for the base case as well as each contingency scenario separately and use an outer layer of APP over all the scenarios to reach a consensus regarding the pre-contingency dispatch (the message exchanges regarding the beliefs of each scenario for the value of dispatch, to reach consensus, are shown with the green lines). We have been successful at solving the SCOPFs using this approach for the IEEE 5, 14, 30, 48, 57, 118, and 300 bus test cases with different lines marked for contingency, and also were able to identify those problem instances that are infeasible. Thereafter, for solving the LASCOPF, we can now treat the SCOPF solver as one unit, and as figure 7 shows, use the outermost APP layer to exchange messages about the beliefs of the different interval-wise generation values to reach a consensus. All the other LASCOPF problems can be solved in the exact same manner. We have run all these simulations on a Dell Inspiron 17R laptop computer powered by a 4x Intel(R) Core(TM) i7-4500U CPU running at 1.80 GHz, with a RAM of capacity 8054 MB and the OS is Ubuntu 16.04.4 LTS. We have coded all the simulation programs in C++11, with the generator optimization solvers being implemented in two modes:

- CVXGEN custom solvers [42], [43], [44], [45] for generators' optimization for fully distributed APMP (APP+ADMM-PMP)
- GUROBI solvers for generators' optimization for fully distributed APMP (APP+ADMM-PMP)

The compiler used is GCC version 4.8.4. In the above-mentioned two methods, only the solvers are different. However, the implementation is a combination of two outer layers of

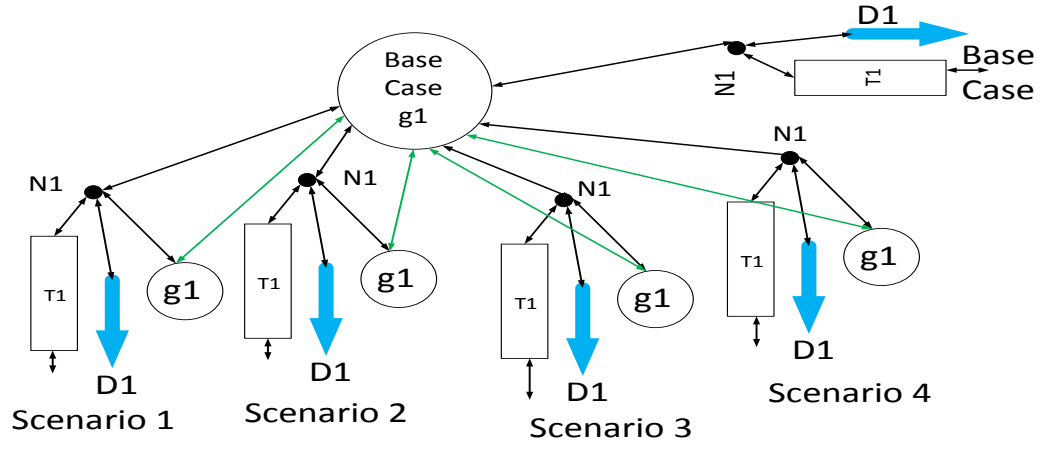


Figure 6: APMP for SCOPF in each dispatch interval

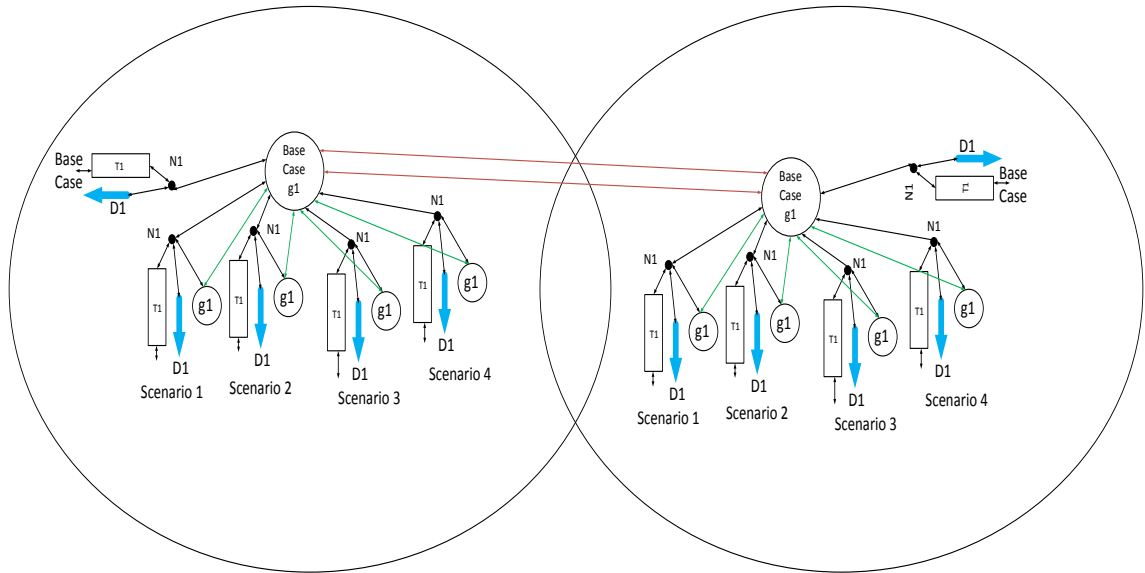


Figure 7: LASCOF for Demand Variation with APMP Schematic for Outer APP Iterations

coarse grained APP and one innermost layer of fine-grained ADMM-PMP. We have observed no or minimal disagreement between the results obtained by using these two solvers. However, the time taken by GUROBI is much longer than that by CVXGEN custom solvers [42], [43], [44], [45]. Hence, we will only present the results we obtained using CVXGEN. We have experimented with IEEE test cases with 5, 14, 30, 48, 57, 118, and 300 bus systems, with contingencies of some or all lines analyzed for contingencies, and for several of dispatch intervals. The simulation software that we have designed can determine (based on a combination logic of the number of iterations and convergence trend of the residuals of the different stages of the algorithm, as well as the different parts of the system) whether a particular problem instance is solvable or feasible or not. Out of those cases that we found out to be feasible, we will present here the results of the following cases:

- 5 bus system with each line analyzed for contingency
- 14 bus system with each line analyzed for contingency
- 30 bus system with lines (from node-to node) (5-7), (6-28), (10-21), and (12-16) analyzed for contingency
- 48 bus system with lines (1-5), (2-6), and (3-24) analyzed for contingency
- 57 bus system with lines (9-12), (12-17), (38-48), and (24-26) analyzed for contingency
- 118 bus system with lines (1-3) and (2-12) analyzed for contingency
- 300 bus system with lines (7-131) and (201-204) analyzed for contingency

Table 3 below lists the simulation metrics of the above-mentioned cases and the ST is the simulation time in seconds that it takes to solve the respective problems, if we implement a fully and complete nested parallelization. The numbers there represent the residual of the outermost APP layer of the algorithm, which is the norm-2 of the disagreement between the different generation MW beliefs regarding the present, previous, and next interval generation values. The ** sign next to the residuals mean that the outer iteration has already converged

Table 3: Convergence Metrics for the IEEE Test Cases

Iter	5 Bus	14 Bus	30 Bus	48 Bus	57 Bus	118 Bus	300 Bus
1	0.2673	3.05719	0.537591	13.493	1.47779	8.17101	0.8805
2	**0.2673	1.32581	**0.537591	6.0792	1.17792	4.18356	0.2317
3	**0.2673	1.50355	**0.537591	2.06586	0.526238	1.6507	**0.2317
4	**0.2673	0.993954	**0.537591	0.111532	**0.526238	0.431	**0.2317
5	**0.2673	0.534623	**0.537591	**0.111532	**0.526238	**0.431	**0.2317
ST (s)	0.676335	205.667	1.28923	79.163	162.108	848.727	918.707

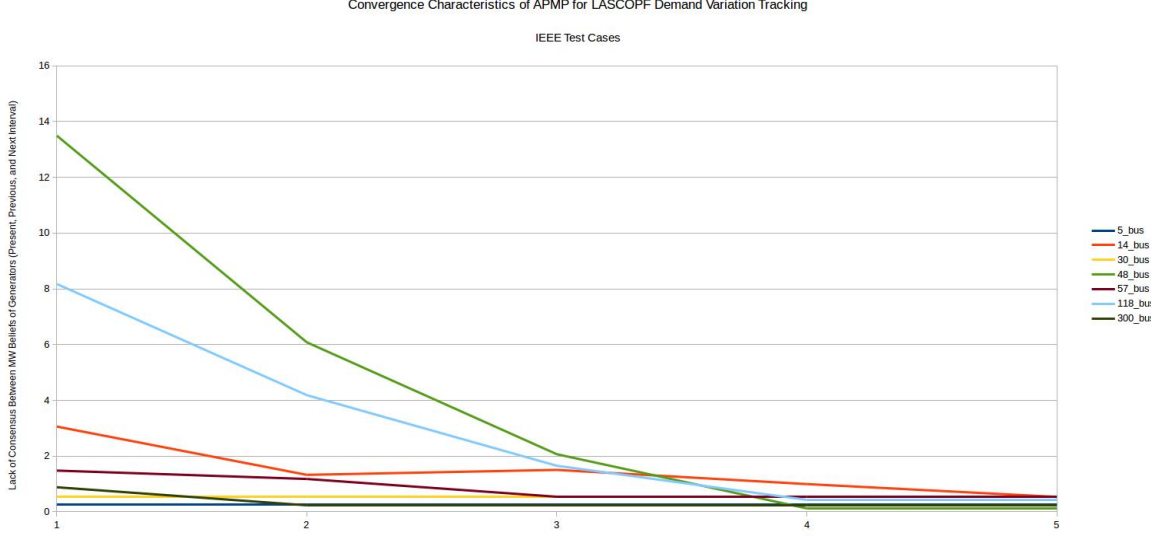


Figure 8: Convergence Characteristics of IEEE Test Systems

by the corresponding iteration count and we have fixed the subsequent residual values, just to help us draw the plot, that appears in fig 8. The load variation data for the 5 bus and 14 bus cases over 5 dispatch intervals, that we have simulated for, are explicitly shown in tables 4 and 5 respectively. The ones for the 30, 48, 57, 118, and 300 bus systems are shown as only the variations from the base case value (that appear on the UW IEEE test case archive <https://labs.ece.uw.edu/pstca/>) of the particular loads in particular intervals in tables 6, 7, 8, 9, and 10 respectively. The base case values occur in the first interval. For the OPF simulations, we have used the primal and dual residual tolerances to be 0.06 and 0.6, respectively (the reason, we use a higher value for the dual residual is that, we have observed that we get accurate enough solution for even somewhat higher value of the dual residual) and we have used a discrete version of proportional+derivative controller to adjust the value of ρ for the first 3000 iterations, such that at each iteration the relationship $\rho \times \epsilon_{primal} = \epsilon_{dual}$ is maintained, where ϵ_{primal} and ϵ_{dual} are respectively the primal and dual residuals. After the first 3000 iterations, if the algorithm hasn't still converged, then ρ is held fixed at the last value. The initial value of ρ at the beginning of the iterations is taken as 1. The following are the APP parameters for outer iterations for attaining consensus among the different base/contingency scenarios for solving SCOPFs.

- $\alpha_{SCOPF} = 5$ for $\nu \leq 5$, $\alpha_{SCOPF} = 3$ for $5 < \nu \leq 10$, $\alpha_{SCOPF} = 2.5$ for $10 < \nu \leq 15$, $\alpha_{SCOPF} = 1.25$ for $15 < \nu \leq 20$, and $\alpha_{SCOPF} = 0.5$ for $\nu > 20$, $\beta_{SCOPF} = 200$, $\gamma_{SCOPF} = 100$ (ν is the APP iteration count)
- Final Tolerance ϵ_{SCOPF} : 0.7

Following are the APP parameters for outermost iterations for attaining consensus among the different MW outputs in different intervals, limited by ramp-rate constraints.

- $\alpha_{\text{LASCOF}} = 10$ for $\mu_{APP} \leq 5$, $\alpha_{\text{LASCOF}} = 5$ for $5 < \mu_{APP} \leq 10$, $\alpha_{\text{LASCOF}} = 2.5$ for $10 < \mu_{APP} \leq 15$, $\alpha_{\text{LASCOF}} = 1.25$ for $15 < \mu_{APP} \leq 20$, and $\alpha_{\text{LASCOF}} = 0.5$ for $\mu_{APP} > 20$, $\beta_{\text{LASCOF}} = 200$, $\gamma_{\text{LASCOF}} = 100$ (μ_{APP} is the outermost APP iteration count)
- Final Tolerance ϵ_{LASCOF} : 0.6

As can be seen, for both the inner and outer APP iterations, we have used changing step-length. The way we chose those values are ad-hoc and gives a reasonable balance between accuracy and solve-time. Figure 8 shows the outer APP convergence for this example

Table 4: Variation of Load for 5 Bus System.

Conn. Node	MW, Int.-1	MW, In.-2	MW, Int.-3	MW, Int.-4	MW, Int.-5
2	20	30	20	20	20
3	45	40	43	43	43
4	40	40	45	45	45
5	60	65	65	65	65

Table 5: Variation of Load for 14 Bus System.

Conn. Node	MW, Int.-1	MW, In.-2	MW, Int.-3	MW, Int.-4	MW, Int.-5
2	21.7	16.7	26.7	24.2	14.2
3	94.2	89.2	99.2	96.7	86.7
4	47.8	42.8	52.8	50.3	40.3
5	7.6	2.6	12.6	10.1	0.1
6	11.2	6.2	16.2	13.7	3.7
9	29.5	24.5	34.5	32	22
10	9	4	14	11.5	1.5
11	3.5	1.5	8.5	6	4
12	6.1	1.1	11.1	8.6	1.4
13	13.5	8.5	18.5	16	6
14	14.9	9.9	19.9	17.4	7.4

Table 6: Variation of Load for 30 Bus System.

Conn. Node	MW, Int.-2	MW, In.-3	MW, Int.-4	MW, Int.-5
10	0	0	10	0
12	5	0	0	0
18	0	3	0	0
24	0	0	0	12

Table 7: Variation of Load for 48 Bus System.

Conn. Node	MW, Int.-2	MW, In.-3	MW, Int.-4	MW, Int.-5
5	0	100	0	0
7	0	0	0	100
19	0	22	0	0
20	40	0	0	0
27	0	0	0	-20
29	0	0	-47	0
32	20	0	-14	0
38	0	-100	0	0
44	0	0	100	0

Table 8: Variation of Load for 57 Bus System.

Conn. Node	MW, Int.-2	MW, In.-3	MW, Int.-4	MW, Int.-5
8	5	0	0	0
10	0	0	0	5
14	0	3	0	0
16	0	0	-20	0
25	0.4	0	0	0
29	0	0	30	0
32	0	1	0	0
41	0	3.7	0	0
47	0	6	0	0

problem. As future research, we would like to explore proven means of tuning the path length for minimum number of iterations to converge.

8. Conclusion

In this paper, we have presented the Look-Ahead SCOPF (LASCOPF) considering variation of demand over several dispatch time intervals and ability of the LASCOPF model to successfully track the temporal demand variation. We have also presented a completely decentralized (i.e. without any central coordinator) computational algorithm to implement the calculations for such a massive scale problem. Through the simulation examples, we have demonstrated the scalability of the algorithm and the effectiveness of the methodology for dispatching systems where load varies over time. The next step in the research in this direction will be to represent the post-contingency states corresponding to each contingency scenario and the system restoration to security over multiple dispatch intervals. We will be presenting that topic in our next paper.

Table 9: Variation of Load for 118 Bus System.

Conn. Node	MW, Int.-2	MW, In.-3	MW, Int.-4	MW, Int.-5
36	0	0	20	0
51	-20	0	0	0
70	0	-10	0	0
77	0	0	0	40

Table 10: Variation of Load for 300 Bus System.

Conn. Node	MW, Int.-2	MW, In.-3	MW, Int.-4	MW, Int.-5
1	1	0	0	0
15	0	4	0	0
145	1	0	0	0
209	0	0	12	0
217	0	0	0	20
9533	1	0	0	0

Appendix A. Acknowledgment

The authors were supported, in part, by the National Science Foundation under grant ECCS-1406894. The authors would like to thank Mahdi Kefayati, PhD graduate (2014), Department of Electrical and Computer Engineering, The University of Texas at Austin, Patryk Radyjowski, PhD student, Department of Mechanical Engineering, The University of Texas at Austin, Matt Kraning, Eric Chu, PhD graduates (2013) from Stanford University, and Prof. Stephen Boyd from Stanford University for help with software and mathematical modeling related issues.

References

- [1] M. Kraning, E. Chu, J. Lavaei, S. Boyd, Dynamic network energy management via proximal message passing, *Foundations and Trends in Optimization* 1 (2).
- [2] S. Chakrabarti, M. Kraning, E. Chu, R. Baldick, S. Boyd, Security constrained optimal power flow via proximal message passing, *Proceedings of the 2014 IEEE PES Clemson Power Systems Conference*.
- [3] K. Bai, H. Wei, Y. Wang, Semidefinite programming for optimal power flow problems, *International Journal of Electric Power & Energy Systems* 30 (6-7) (2008) 383–392.
- [4] J. Lavaei, S. Low, Zero duality gap in optimal power flow problem, *IEEE Transactions on Power Systems* 27 (1) (2012) 92–107.
- [5] S. Sojoudi, J. Lavaei, Physics of power networks makes hard problems easy to solve, To appear, *IEEE Power & Energy Society General Meeting*.
- [6] S. Sojoudi, J. Lavaei, Network topologies guaranteeing zero duality gap for optimal power flow problem, To appear, *IEEE Power & Energy Society General Meeting*.
- [7] J. Lavaei, D. Tse, B. Zhang, Geometry of power flows in tree networks, To appear, *IEEE Power & Energy Society General Meeting*.
- [8] T. Erseghe, Distributed optimal power flow using admm, *Power Systems*, *IEEE Transactions on PP* (99) (2014) 1–11. doi:10.1109/TPWRS.2014.2306495.

- [9] L. Liu, A. Khodaei, W. Yin, Z. Han, A distribute parallel approach for big data scale optimal power flow with security constraints, in: Smart Grid Communications (SmartGridComm), 2013 IEEE International Conference on, 2013, pp. 774–778. doi:10.1109/SmartGridComm.2013.6688053.
- [10] S. Chakrabarti, M. R. Hesamzadeh, Financial transmission rights: A new proposal, in: 2018 15th International Conference on the European Energy Market (EEM), 2018, pp. 1–5. doi:10.1109/EEM.2018.8469941.
- [11] S. J. Qin, T. A. Badgwell, A survey of industrial model predictive control technology, *Control Engineering Practice* 11 (7) (2003) 733–764.
- [12] E. G. Cho, K. A. Thoney, T. J. Hodgson, R. E. King, Supply chain planning: Rolling horizon scheduling of multi-factory supply chains, in: Proceedings of the 35th conference on Winter simulation: driving innovation, 2003, pp. 1409–1416.
- [13] F. Herzog, Strategic portfolio management for long-term investments: An optimal control approach, Ph.D. thesis, ETH, Zurich (2005).
- [14] K. T. Talluri, G. J. V. Ryzin, *The Theory and Practice of Revenue Management*, Springer, 2004.
- [15] M. Kraning, Y. Wang, E. Akuiyibo, S. Boyd, Operation and configuration of a storage portfolio via convex optimization, in: Proceedings of the 18th IFAC World Congress, 2011, pp. 10487–10492.
- [16] S. Boyd, N. Parikh, E. Chu, B. Peleato, J. Eckstein, Distributed optimization and statistical learning via the alternating direction method of multipliers, *Foundations and Trends in Machine Learning* 3 (2011) 1–122.
- [17] J. Carpentier, Contribution to the economic dispatch problem, *Bull. Soc. Francaise Elect.* 3 (8) (1962) 431–447.
- [18] S. Chakrabarti, Study of uplan based resources planning & analysis by power generation utilities in the deregulated electricity market, Master’s thesis, The University of Texas at Austin (2010).
- [19] H. H. Happ, Optimal power dispatch - a comprehensive survey, *IEEE Transactions on Power Apparatus and Systems* 96 (3) (1977) 841–854.
- [20] B. H. Chowdhury, S. Rahman, A review of recent advances in economic dispatch, *IEEE Transactions on Power Systems* 5 (4) (1990) 1248–1259.
- [21] M. B. Cain, R. P. O'Neill, A. Castillo, History of optimal power flow and formulations.
- [22] S. Mhanna, A. C. Chapman, G. Verbi, Component-based dual decomposition methods for the opf problem, *Sustainable Energy, Grids and Networks* 16 (2018) 91 – 110. doi:https://doi.org/10.1016/j.segan.2018.04.003.
URL <http://www.sciencedirect.com/science/article/pii/S2352467718300389>
- [23] D. K. Molzahn, F. Drfler, H. Sandberg, S. H. Low, S. Chakrabarti, R. Baldick, J. Lavaei, A survey of distributed optimization and control algorithms for electric power systems, *IEEE Transactions on Smart Grid* 8 (6) (2017) 2941–2962. doi:10.1109/TSG.2017.2720471.
- [24] A. Kargarian, J. Mohammadi, J. Guo, S. Chakrabarti, M. Barati, G. Hug, S. Kar, R. Baldick, Toward distributed/decentralized dc optimal power flow implementation in future electric power systems, *IEEE Transactions on Smart Grid* 9 (4) (2018) 2574–2594. doi:10.1109/TSG.2016.2614904.
- [25] B. Stott, O. Alsac, A. Monticelli, Security analysis and optimization, *Proceedings of the IEEE* 75 (12) (1987) 1623–1644. doi:10.1109/PROC.1987.13931.
- [26] N.-Y. Chiang, A. Grothey, Solving security constrained optimal power flow problems by a structure exploiting interior point method, Tech. Rep. Technical Report ERGO-11-014, School of Mathematics, The University of Edinburgh (jun 2011).
- [27] D. Phan, J. Kalagnanam, Distributed methods for solving the security-constrained optimal power flow problem, *Proceedings of the 2012 IEEE PES Innovative Smart Grid Technologies Conference*.
- [28] R. Glowinski, A. Marrocco, Sur l’approximation, par elements finis d’ordre un, et la resolution, par penalisation-dualité, d’une classe de problems de Dirichlet non lineares, *Revue Française d’Automatique, Informatique, et Recherche Opérationnelle* 9 (1975) 41–76.
- [29] D. Gabay, Applications of the method of multipliers to variational inequalities, in: M. Fortin, R. Glowinski (Eds.), *Augmented Lagrangian Methods: Applications to the Solution of Boundary-Value Problems*, North-Holland: Amsterdam, 1983.
- [30] M. Fortin, R. Glowinski, *Augmented Lagrangian Methods: Applications to the Numerical Solution of*

- Boundary-Value Problems, North-Holland: Amsterdam, 1983.
- [31] J. Eckstein, M. Fukushima, Some reformulations and applications of the alternating direction method of multipliers, *Large Scale Optimization: State of the Art* (1993) 119–138.
 - [32] J. Eckstein, Parallel alternating direction multiplier decomposition of convex programs, *Journal of Optimization Theory and Applications* 80 (1) (1994) 39–62.
 - [33] B. H. Kim, R. Baldick, Coarse-grained distributed optimal power flow, *IEEE Transactions on Power Systems* 12 (2) (1997) 932–939.
 - [34] B. H. Kim, R. Baldick, A comparison of distributed optimal power flow algorithms, *IEEE Transactions on Power Systems* 15 (2) (2000) 599–604.
 - [35] G. Cohen, Optimization by decomposition and coordination: A unified approach, *Automatic Control, IEEE Transactions on* 23 (2) (1978) 222–232. doi:10.1109/TAC.1978.1101718.
 - [36] G. Cohen, Auxiliary problem principle and decomposition of optimization problems, *Journal of Optimization Theory and Applications* 32 (3) (1980) 277–305. doi:10.1007/BF00934554. URL <http://dx.doi.org/10.1007/BF00934554>
 - [37] R. Baldick, B. Kim, C. Chase, Y. Lao, A fast distributed implementation of optimal power flow, *IEEE Transactions on Power Systems* 31 (3).
 - [38] R. Ebrahimian, R. Baldick, State estimation distributed processing [for power systems], *Power Systems, IEEE Transactions on* 15 (4) (2000) 1240–1246. doi:10.1109/59.898096.
 - [39] J. Batut, A. Renaud, Daily generation scheduling optimization with transmission constraints: a new class of algorithms, *Power Systems, IEEE Transactions on* 7 (3) (1992) 982–989. doi:10.1109/59.207311.
 - [40] S. Chakrabarti, Post-contingency states representation & redispatch for restoration in power systems operation, Ph.D. thesis, The University of Texas at Austin (2017).
 - [41] U. of Washington, Power systems test case archive. URL <https://labs.ece.uw.edu/pstca/>
 - [42] J. Mattingley, S. Boyd, Cvxgen: a code generator for embedded convex optimization, *Springer Optim. Eng.*
 - [43] Y. W. Jacob Mattingley, S. Boyd, Code generation for receding horizon control, Stanford University.
 - [44] J. Mattingley, S. Boyd, Real time convex optimization in signal processing, *IEEE Signal Processing Magazine*.
 - [45] J. Mattingley, S. Boyd, Automatic code generation for real-time convex optimization, *Convex Optimization in Signal Processing and Communications*.