Spacecraft Mission Design Project - Musical Masterminds

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Abstract

The purpose of this project was to launch a rocket containing a 1000lb spacecraft into an elliptical orbit and then executing a Hohmann transfer to get that spacecraft to the Moon. An additional portion of the project was to execute a double Hohmann transfer, similar to that of the Israeli spacecraft Beresheet, and compare which is better. Better being indicated by which method uses less fuel. We were able to find a Thrust and Flight Path Angle of 3,033,488 N and 83.0515° respectively. A single Hohmann transfer with a ΔV_t of 1757.9 m/s to be the ideal transfer going from the perigee of the elliptical orbit to the 6 o'clock location of the Moon's orbit. We were also able to find the force for the first and second impulses to be 15948.7 N and 37584.7 N. It was also found that the changes in velocities for a single and double Hohmann transfer were similar values. Thus, using either a single or double transfer would yield the same fuel consumption.

Introduction

The Israeli Spacecraft, Beresheet, is the first spacecraft launched from a private corporation that applied the concepts of the Hohmann transfer maneuver to travel from the Earth to the Moon. Generally speaking, the spacecraft was to circulate around the Earth's orbit until the spacecraft increased in speed and transformed its trajectory from a circular to an elliptical orbit. As the spacecraft traveled in an elliptical orbit, the elliptical path continued to increase[6] until the spacecraft entered into the Moon's sphere of influence. Once the spacecraft was captured by the Moon's gravitational force, the spacecraft would descend to the Moon's surface[5]. Once the spacecraft successfully landed, the spacecraft would take pictures of the surface of the Moon and measure the magnetic field or the Mare Serenitatis[6] of its location. Unfortunately, the Beresheet was unable to accomplish the mission due to a technical failure in the controlled landing procedure. The engine utilized in the spacecraft provided enough power to propel the spacecraft to the moon, but the efficiency of the engine provided a challenge for the spacecraft to decrease its velocity. According to Rob Westcott, a senior propulsion engineer at Nammo[6], the engine had to continually turn on and off in short time intervals and in an accurate and precise manner to slow the spacecraft and allow it to come to a gentle stop. However, consistently turning the engine on and off built up heat inside the engine. Ultimately this led to malfunctions within the engine and communication system of the spacecraft.

The assigned problem was solved by applying the following assumptions:
Consider a 1000lb spacecraft. Find the thrust needed and flight path angle to place this spacecraft to an orbit with perigee 600 miles and eccentricity 0.8. Then, execute a Hohmann transfer to place the spacecraft into an orbit that is 200 miles over the Moon. Comment on the force needed and discuss whether you want to do this transfer in a single Hohmann transfer or whether two transfers (as in the Israeli spacecraft) will be better. Criterion for better is using less fuel
The thrust and flight path angle values were computed using the parameters and characteristics of the Beresheet spacecraft that was launched by the SpaceX Falcon 9. Meanwhile, for the orbital portion the moon is positioned at its closest location so the spacecraft travels a minimum distance to save fuel. Additionally, the impulsive maneuver was executed at the same location for each transfer. To further elaborate, from Figure 1, the perigee location is the same for each transfer even though the apogee location increases. The same technique was applied when solving for a single Hohmann transfer and a two transfer.

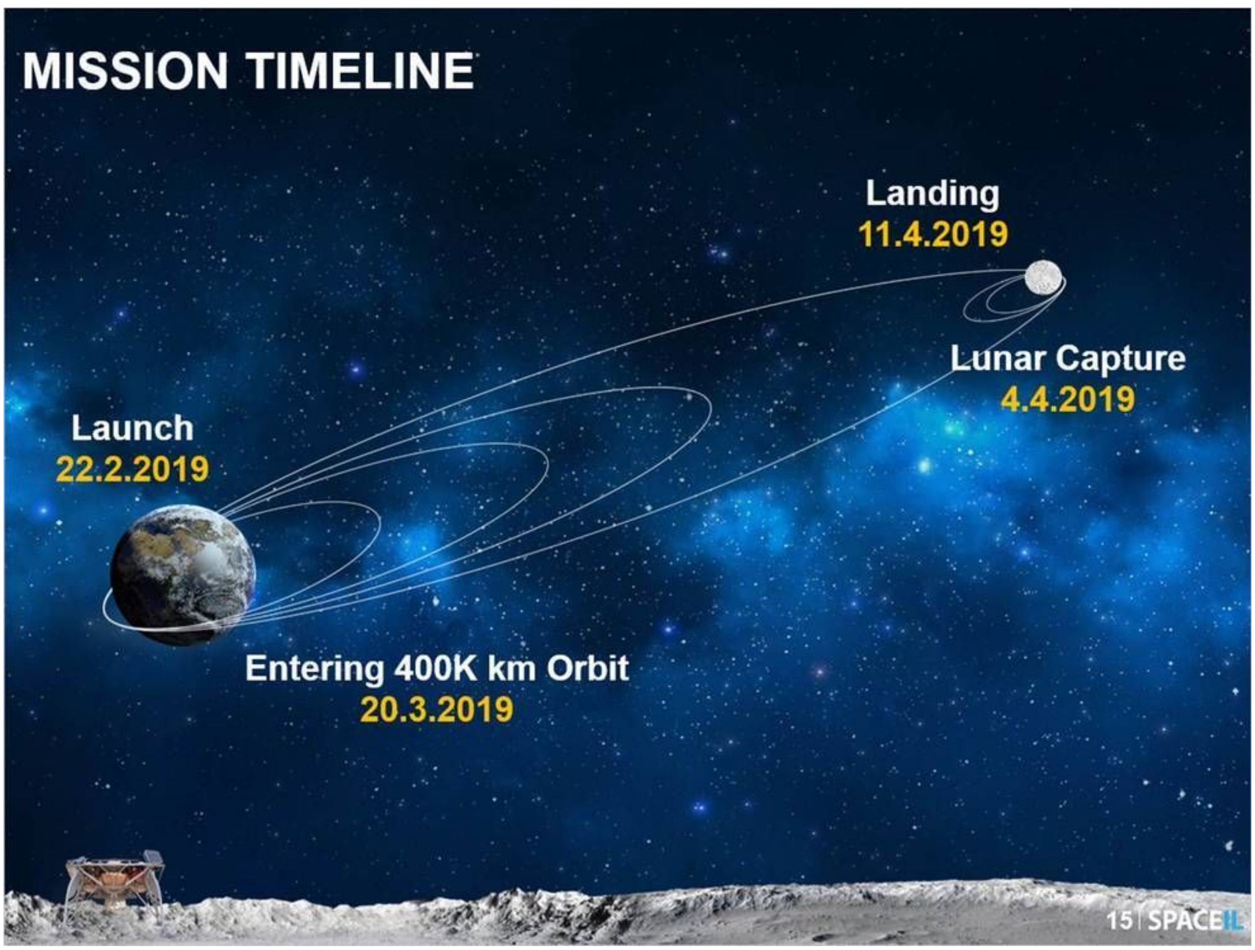

Figure 1: Beresheet Flight Path

Theory

A transfer orbit is required to move a spacecraft between non-intersecting orbits. The Hohmann transfer orbit is the basic elliptical orbit that transfers a spacecraft between two circular orbits at different altitudes. The perigee of the orbit is located at the inner orbit while the apogee of the orbit is located at the outer orbit. Depending on type of transfer, e.g. inner to outer orbit or outer to inner orbit, an increase or decrease in energy and a change in velocity is required.

For a Hohmann transfer from an inner circular orbit to an outer circular orbit, there must be two spontaneous changes in velocities (ΔV_1 and ΔV_2). The first change in velocity and the second change in velocity are projected collinearly from the perigee and apogee. Additionally, both changes in velocities at their respective locations increase the energy of the spacecraft. The first change in velocity increases the energy of the spacecraft to counterpart the energy of the transfer ellipse. While the second change in velocity increases the energy of the spacecraft to match the energy of the outer orbit. Without the second change in velocity, the objective will continue to orbit the transfer ellipse and the earth. For orbits transferring from the outer to inner circular orbit, the values for (changes in velocities) is less compared to the inner to outer orbit transfer. The energy value needs to decrease in order for the spacecraft to travel from the outer orbit to the transfer ellipse, and then the transfer ellipse to the inner orbit.

Additionally, the Hohmann transfer orbit is applicable for the transfer between two elliptical orbits if two elliptical orbits share a common major axis. An example of a shared common major axis is the apsides if the apsides of the two orbits are on the same line. This is essential because both the changes in velocities must be collinear when fired from the perigee and apogee. Therefore, there are two methods that could be used to transfer an object between two elliptical orbits. The first way is to circularize the departure orbit prior to the Hohmann transfer. This method conserves more energy. The second approach is circularizing the Hohmann transfer orbit so that the orbit intersects the destination orbit. After the two orbits intersect, the circular orbit will convert back to an ellipse. At the end, no matter what method is implemented both require a collinear third change in velocity (ΔV_3).

Relevant Equations:

$r_1 = r_E + h_1$	(1)

$a = \frac{r_p}{(1-\varepsilon)}$	(2)

$$E = -\frac{\mu}{2a} \tag{3}$$

$$V = \sqrt{2(E + \frac{\mu}{r_p})} \tag{4}$$

$$H = \sqrt{\mu * a * (1 - \varepsilon^2)} \tag{5}$$

$$\gamma = cos^{-1}(\frac{H}{r_p V}) \tag{6}$$

$$2a = r_p + r_a \tag{7}$$

$$T = \frac{dm}{dt} * I_{sp} * g_0 \tag{8}$$

$$V_{b1} = \frac{(ms_1 + m_{p1} + m_{s2} + m_{p2} + m_{payload})}{(m_{s1} + m_{p2} + m_{s2} + m_{payload})} \tag{9}$$

$$V_{b2} = \frac{(m_{s2} + m_{p2} + m_{payload})}{(m_{s2} + m_{payload})} \tag{10}$$

$$V_t = V_{b1} + V_{b2} \tag{11}$$

$$V_{CS} = \sqrt{\frac{\mu}{r_E}} \tag{12}$$

$$\Delta V_1 = \left| V_{tp} - V_{ics} \right| \tag{13}$$

Where:
tp = transfer orbit perigee
ics = inner circular orbit

$$\Delta V_2 = \left| V_{osc} - V_{ta} \right| \tag{14}$$

Where:
ta = transfer orbit apogee

$ocs = outer\ circular\ orbit$	

$\Delta V = \Delta V_1 + \Delta V_2$	(15)

$r_{SOI} = r * (\frac{m_2}{m_1})^{0.4}$	(16)

$\Delta v = v_e ln\frac{m_0}{m_f} = I_{sp} g_o ln\frac{m_0}{m_f}$	(17)

$m_{original} = \frac{m_{final\ payload}}{(1-(1-e^{-\frac{\Delta V_1}{g_0 * I_{sp}}}))*...*(1-(1-e^{-\frac{\Delta V_{n-1}}{g_0 * I_{sp}}}))*(1-(1-e^{-\frac{\Delta V_n}{g_0 * I_{sp}}}))}$	(18)

$m_{propellant\ n} = m_{n-1}(1 - e^{-\frac{\Delta V_n}{g_0 * I_{sp}}})$	(19)

$m_n = m_{n-1} - m_{propellant\ n}$	(20)

$\frac{dm}{dt} = \frac{m_{propellant}}{t_{burn}}$	(21)

Results

For simplification purposes, the overall problem was divided into parts:
Consider a 1000lb spacecraft. Find the **(a)** thrust needed and **(b)** flight path angle to place this spacecraft to an orbit with perigee 600 miles and eccentricity 0.8. Then, **(c)** execute a Hohmann transfer to place the spacecraft into an orbit that is 200 miles over the Moon. **(d)** Comment on the force needed and **(e)** discuss whether you want to do this transfer in a single Hohmann transfer or whether two transfers (as in the Israeli spacecraft) will be better. Criterion for better is using less fuel

Part A: Thrust Calculations

- Given variables
  - Weight of spacecraft = 1000 lb
  - Gravity: $g_0 = 9.81\ \frac{m}{s^2}$
- Mass of spacecraft

- $W = \frac{Weight\ of\ Spacecraft}{Conversion\ to\ kg} = \frac{1000\ lb}{2.205} = 453.5\ kg$
- Estimated fuel weight aboard:796.7 kg
- Total mass: $m_{pay} = 453.5 + 796.7 = 1250.2\ kg$

- Thrust
  - $T = \frac{dm}{dt} * I_{sp} * g_0$
- Following Criteria is from Falcon 9 Rocket[7] with Merlin 1D and Merlin 1D Vacuum engines used in Beresheet launch
  - First Stage - Merlin 1D
    - Burn Time: $dt_1 = 180\ s$
    - Mass: $m_{s1} = 25,401.18\ kg$
    - Specific Impulse: $I_{sp,1} = 282\ s$
    - Max Propellant Weight: $m_{p1,max} = 372,853\ kg$
    - Minimum Engine Throttle: 40%
  - Second Stage Burn Time - Merlin 1D Vacuum
    - Burn Time: $dt_2 = 346\ s$
    - Mass: $m_{s2} = 4263.769\ kg$
    - Specific Impulse: $I_{sp,2} = 340\ s$
    - Max Propellant Weight: $m_{p2,max} = 66,587\ kg$
    - Minimum Engine Throttle: 39%
- Propellant Mass
  - Necessary Velocity (Achieved by Falcon 9 on Beresheet Launch[4])= 10 km/s
  - Velocity Calculations:
    - Vary Stage 1 Propellant Mass as follows:
      - $0.4 * mp_{1,max} < m_{p1} < m_{p1,max}$
      - $149,140 < m_{p1} < 372.853$
    - Vary Stage 2 Propellant Mass as follows:
      - $0.39 * mp_{2,max} < m_{p2} < m_{p2,max}$
      - $25,969 < m_{p1} < 66,587$
    - $V_{b1} = \frac{(ms_1 + m_{p1} + m_{s2} + m_{p2} + m_{payload})}{(m_{s1} + m_{p2} + m_{s2} + m_{payload})}$
    - $V_{b2} = \frac{(m_{s2} + m_{p2} + m_{payload})}{(m_{s2} + m_{payload})}$
    - $V_t = V_{b1} + V_{b2}$
  - Parametric study performed on various masses. Result shown in Figure 1
  - When $V_t = 10000(+-50)\ m/s$
    - $mp_1 = 172,090\ kg$
    - $mp_2 = 30,135\ kg$
- Thrust Calculations
  - First Stage

- $\frac{dm_1}{dt_1} = \frac{172,090}{180} = 956.03\ kg/s$
- $T_1 = 956.03 * 9.81 * 282 = 2,644,780\ N$

- Second Stage
  - $\frac{dm_2}{dt_2} = \frac{30,135}{346} = 87.1\ kg/s$
  - $T_2 = 87.1 * 9.81 * 340 = 290,513\ N$
- Total Required Thrust
  - $T = T_1 + T_2 = 2,935,293\ \ N$
- Supporting Figure:

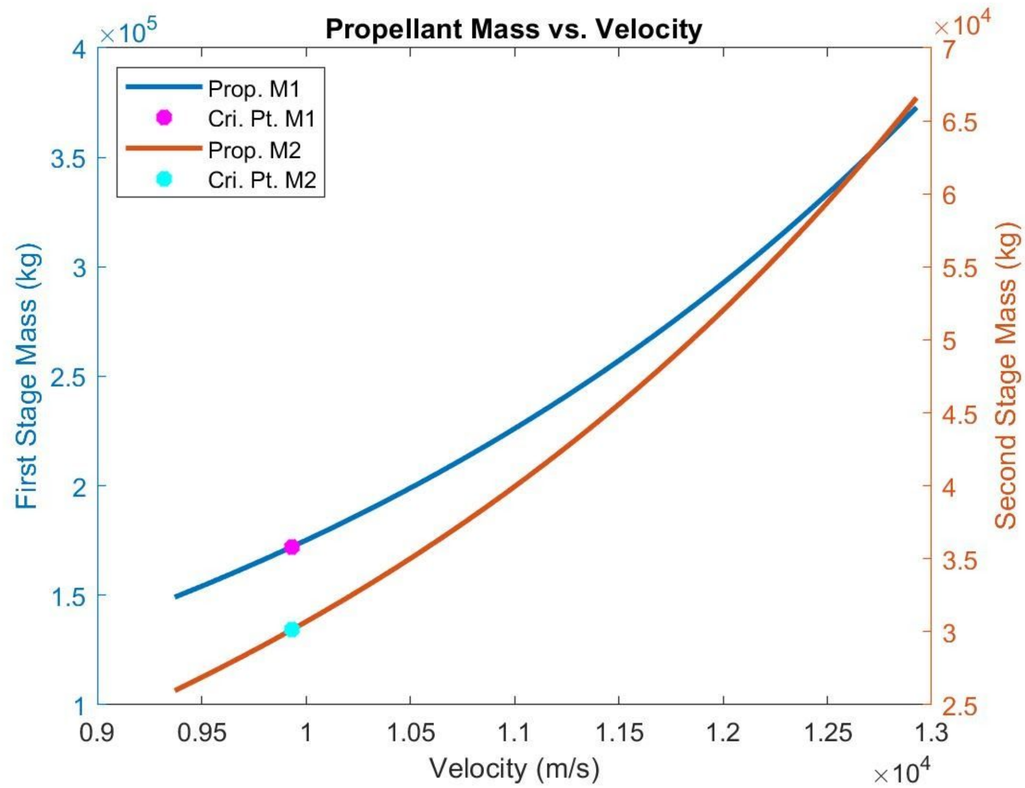

Figure 2: Propellant Mass Calculations

NOTE: Fuel mass for the 1,000lb spacecraft is calculated in Part D, where reverse analysis is performed to understand the total mass of the payload necessary to transport a 1,000lb mass to the designated moon orbit. This value ultimately affects the mass of the payload aboard the Falcon 9, and is hence incorporated into the calculations of the thrust.

NOTE: Merlin 1D engines were utilized for the Falcon 9 rocket version 1.1. These properties match very closely with the Merlin 1D FT engines used on the Falcon 9 version 1.2, which were utilized from 2016 onward. However, to ensure full reliability of the propulsion system, a dependable older variant of the Merlin engine series (1D) was selected.

Part B: Flight Path Angle Calculations

- Given variables
  - Radius spacecraft deployed from SpaceX Falcon 9 = 60,000 km from Earth surface[4]
  - Radius of Earth (r_E) = 6.378E06 meters
  - Standard gravitational parameter (μ_E) = 3.986E14 m^3/s^2
  - Minimum velocity of the Beresheet = 10 km/s[4]
  - Mass of moon (m_M)= 7.3464E22 kg
  - Mass of Earth (m_E) = 5.9724E24 kg
- Semi-major axis
  - $a = \frac{r_p}{(1-\varepsilon)} = \frac{7343606\ m}{(1-0.80)} = 36718030\ meters$
  - **a = 36718030 m**
- Specific Angular Momentum
  - $H = \sqrt{\mu * a * (1-\varepsilon^2)} = \sqrt{(3.986E14 \frac{m^3}{s^2}) * (36718030\ m) * (1-0.8^2)} = 7.259E10\ \frac{m^2}{s}$
  - **H = 7.259E10 m^2/s**
- Flight path angle
  - $\gamma = cos^{-1}(\frac{H}{r_p V}) = cos^{-1}(\frac{7.259E10\ \frac{m^2}{s}}{(60000E3\ m)(10000\ m/s)})\ = cos^{-1}(0.121)\ = 83.0515°$
  - **γ = 83.0515°**

Part C: Hohmann Transfer

- Given variables
  - Radius of the Moon (r_m) = 1738E3 m
  - Distance from Earth to Moon (r_{EM}) = 356794E3 m
  - Height from the Moon (h_M) = 200 miles = 321800 m
  - Gravitational parameter of the Moon (μ_M) = 4.90339E12 $\frac{m^3}{s^2}$
- Radius at Apogee
  - $r_a = a * (1+\varepsilon)\ = 3.67E7$ m * (1+0.8) = 6.606E8 m
  - **r_a = 6.606E8 m**
- Velocity at Apogee
  - $V_a = \frac{H}{r_a} = \frac{7.259E10\ \frac{m^2}{s}}{6.606E8\ m} = 1098\ m/s$
  - **V_a = 1098 m/s**
- Earth's Center to the Moon's surface
  - $R_{E_C, M} = r_{EM} - r_m = 356794E3\ m\ - 1738E3\ m\ = 3.55056E8\ m$
  - **$R_{Ec', M}$ = 3.55056E8 m**
- Distance to the orbit around the Moon
  - $R_{mo} = R_{E_C M} - h_M = 3.55056E8\ m\ - 321800\ m\ =\ 3.547E8\ m$
  - **R_{mo} = 3.547E8 m**
- Semi-major axis of the Hohmann transfer orbit

- $a_t = \frac{r_a + R_{mo}}{2} = \frac{6.6091E7\ m + 3.588E8\ m}{2} = 2.104E8\ m$
  - **a_t = 2.104E8 m**
- Energy of the transfer orbit
  - $E_t = \frac{-\mu}{2*a_t} = \frac{-3.986E14 \frac{m^3}{s^2}}{2(2.125E8\ m)} = -9.4719E5\ \frac{m^2}{s^2}$
  - **E_t = -9.4719E5 m^2/s^2**
- Velocity of leaving the orbit
  - $V_{Dep} = \sqrt{2(E_t + \frac{\mu}{r_a})} = \sqrt{2((-9.3801E5\ \frac{m^2}{s^2}) + \frac{3.986E14\ \frac{m^3}{s^2}}{6.6091E7\ m}} = 3.1887E3\ \frac{m}{s}$
  - **V_{Dep} = 3.1887E3 m/s**
- Specific angular momentum of the transfer orbit
  - $H_t = r_a * V_{Dep} = (6.6091E7\ m) * (3.1916E3\ \frac{m}{s}) = 2.1074E11\ \frac{m^2}{s}$
  - **H_t = 2.1074E11 m^2/s**
- Arrival velocity
  - $V_{Arr} = \frac{H_t}{R_{mo}} = \frac{2.1093E11\ \frac{m^2}{s}}{3.588E8\ m} = 594.1\ \frac{m}{s}$
  - **V_{Arr} = 594.1 m/s**
- Epsilon of the transfer orbit
  - $\varepsilon_t = \frac{R_{mo} - r_a}{R_{mo} + r_a} = \frac{3.588E8\ m - 6.6091E7\ m}{3.588E8\ m + 6.6091E7\ m} = 0.6859$
  - **ε_t = 0.6859 (elliptical orbit)**
- Velocity of the circular orbit
  - $V_f = \sqrt{\frac{\mu_M}{(r_m + h_m)}} = \sqrt{\frac{4.90339E12\ \frac{m^3}{s^2}}{(1.738E6\ m + 3.218E5\ m)}} = 1.5429E3\ \frac{m}{s}$
  - **V_f = 1.5429E3 m/s**
- Velocity change between circular and elliptical transfer
  - $\Delta V_1 = V_{Dep} - V_a = 3.1887E3\ \frac{m}{s} - 1098\ \frac{m}{s} = 2.0904E3\ \frac{m}{s}$
  - **ΔV_1 = 2.0904E3 m/s**
- Velocity change between elliptical and final circular orbit
  - $\Delta V_2 = V_f - V_{Arr} = 1.5429E3\ \frac{m}{s} - 594.1\ \frac{m}{s} = 948.8\ \frac{m}{s}$
  - **ΔV_2 = 948.8 m/s**
- Total velocity change
  - $\Delta V_t = |\Delta V_1| + |\Delta V_2| = |2.0904E3\ \frac{m}{s}| + |948.8\ \frac{m}{s}| = 3.0392E3\ \frac{m}{s}$
  - **ΔV_t = 3.0392E3 m/s**

NOTE: Justification for utilizing the Moon's Parameter
From F.Hale[1], the sphere of influence is defined as the region where the gravitational influence of a planet is sufficiently large enough to be considered the central force field. Even though in the Solar System the Sun has the largest mass, its gravitational field is inferior in regions between two planets, as shown in Figure 3. Applying this simplification portrays that the force presence exerted from m to m_2 is greater than the force exerted from m_1. Essentially, this

particular spacecraft (m) enters into the region where the Moon's gravitational field (m_2) is considered the primary gravitational influence. Therefore, when determining the circular orbit around the moon the parameters and characteristics of the moon, such as the standard gravitational parameter (μ_M), were implemented into the calculations.

NOTE: Additional Calculations to validate the Moon Sphere of Influence (M is moon, E is Earth, m is mass)

- Radius of Sphere of Influence of the Moon
  - $r_{SOI} = r_{M,E} * (\frac{m_M}{m_E})^{0.4} = 6.143E7\ m$
- Radius of Sphere of Influence of the Earth
  - $r_{SOI} = r_{E,M} * (\frac{m_E}{m_M})^{0.4} = 2.072E9\ m$

NOTE: The Moon's Sphere of Influence was simplified for the calculations in this project. While the Moon's Sphere of Influence should affect the spacecraft at an earlier moment of the transfer orbit, for this project it was only taken into consideration when the spacecraft was switching from the transfer orbit to the Moon orbit.

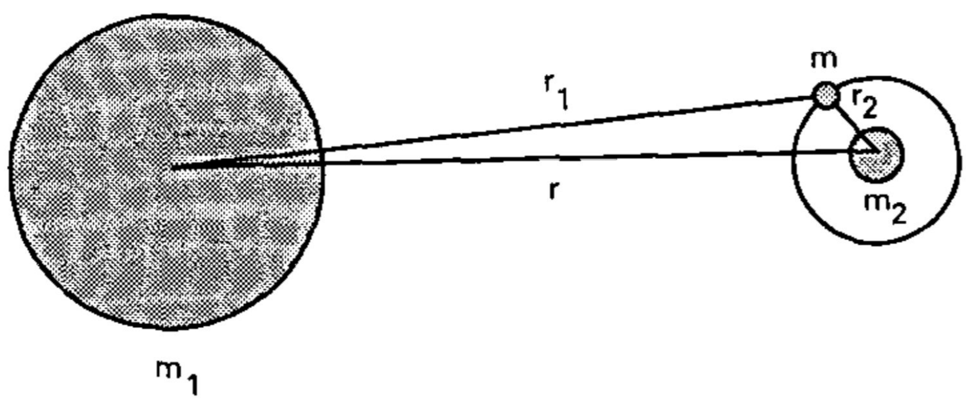

Figure 3: Sphere of influence[1]

Apogee to 6 o'clock location

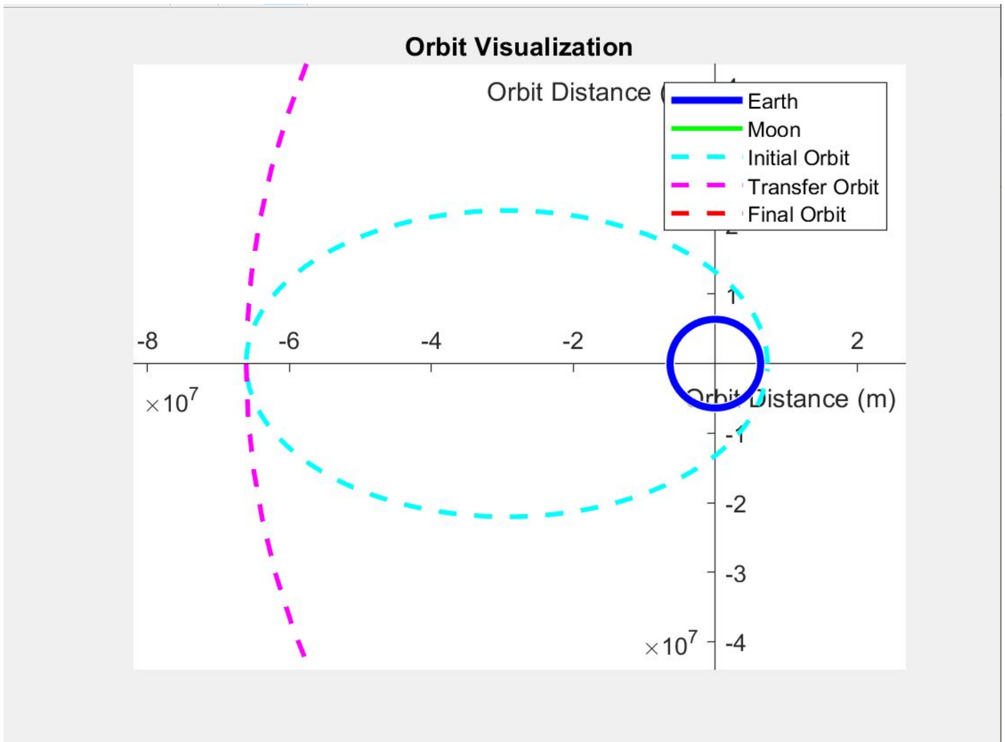

Figure 4: The spacecraft 's initial orbit starting from the Earth entering into the transfer orbit

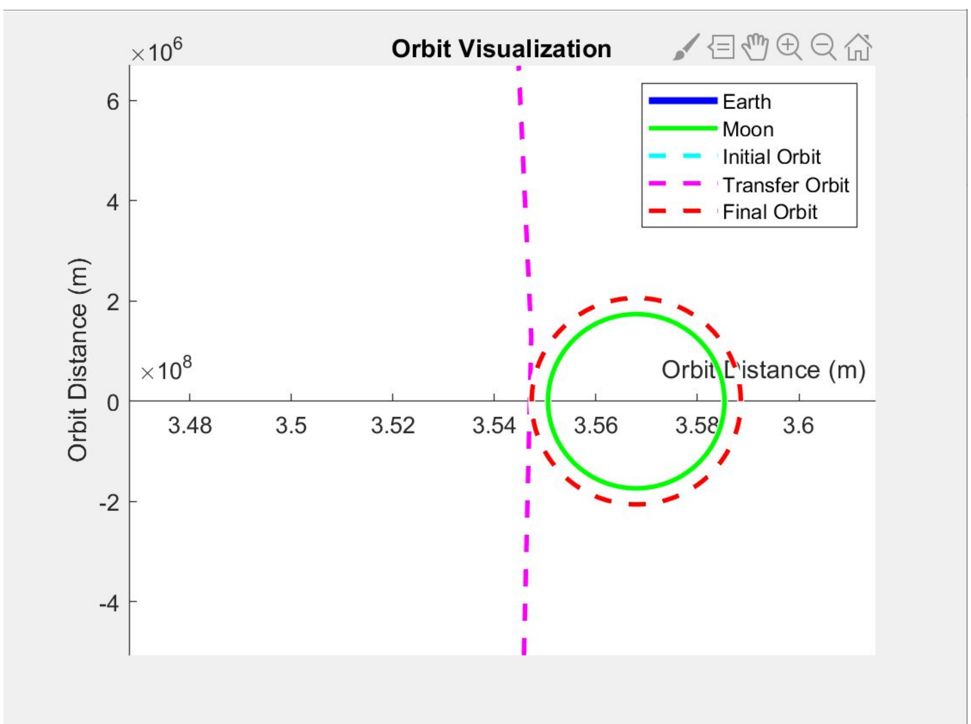

Figure 5: The spacecraft exiting the transfer orbit and entering into the final orbit at a 6 o'clock location (arrival to the moon)

Apogee to 6 o'clock		
Property	**Value**	**Units**
Eccentricity (ε)	0.6859	
Initial Orbit Apogee Velocity (V_a)	1098.3	m/s
Transfer Orbit Entrance Velocity(V_{Dep})	3188.7	m/s
First Velocity Change(dV_1)	2090.4	m/s
Transfer Orbit Arrival Velocity(V_{Arr})	594.1	m/s
Final Orbit Velocity(V_f)	1542.9	m/s
Second Velocity Change(dV_2)	948.8	m/s
Total Velocity Change(dV_t)	3039.2	m/s

Table 1: Orbital parameters for arrival at the moon's apogee at 6 o'clock

Apogee to 12 o'clock

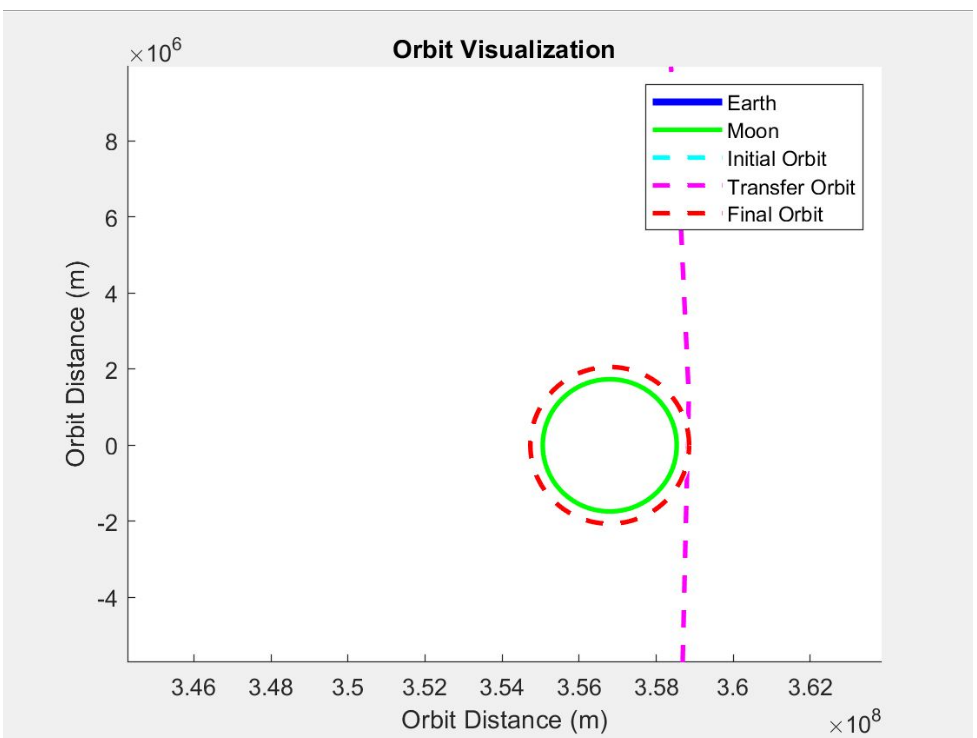

Figure 6: The spacecraft exiting the transfer orbit and entering into the final orbit at a 12 o'clock location (arrival to the moon)

Apogee to 12 o'clock		
Property	**Value**	**Units**
Eccentricity (ε)	0.6889	
Initial Orbit Perigee Velocity (V_a)	1098.3	m/s
Transfer Orbit Entrance Velocity(V_{Dep})	3191.6	m/s
First Velocity Change(dV_1)	2093.3	m/s
Transfer Orbit Arrival Velocity(V_{Arr})	587.8	m/s
Final Orbit Velocity(V_f)	1542.9	m/s
Second Velocity Change(dV_2)	995.1	m/s
Total Velocity Change(dV_t)	3048.4	m/s

Table 2: Orbital parameters for arrival at the moon's apogee at 12 o'clock

Perigee to 12 o'clock

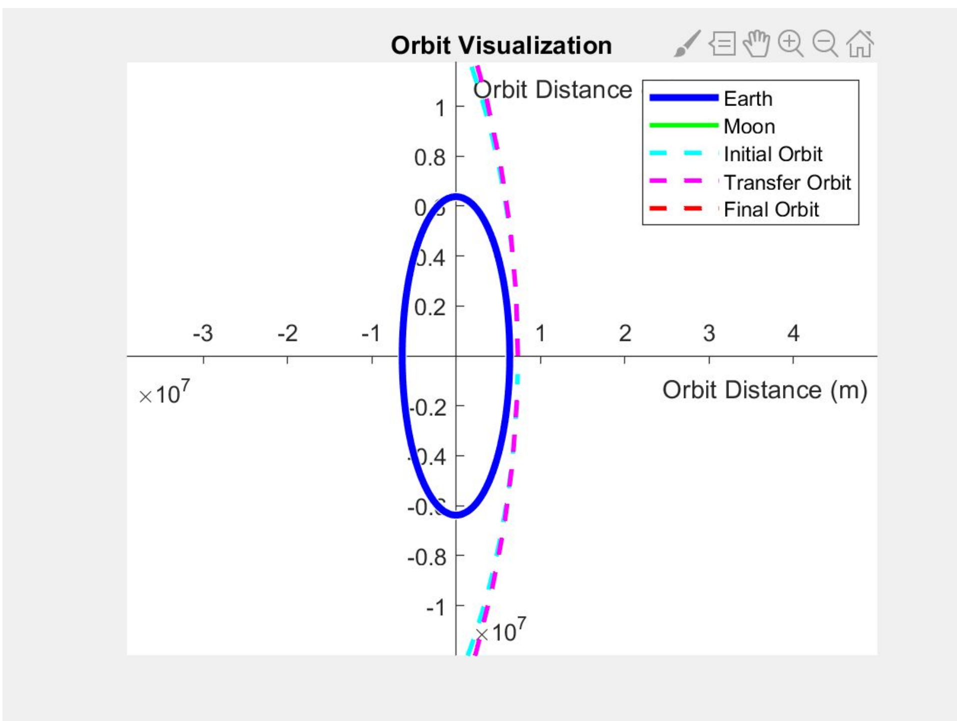

Figure 7: The spacecraft 's initial orbit starting from the Earth entering into the transfer orbit

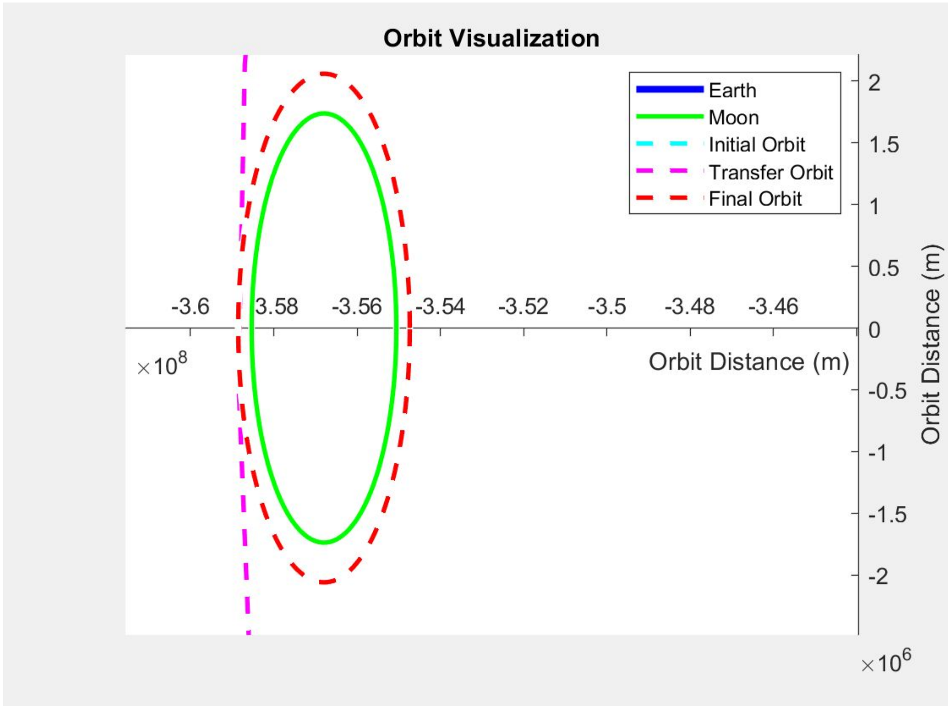

Figure 8: The spacecraft exiting the transfer orbit and entering into the final orbit at a 6 o'clock location (arrival to the moon)

Perigee to 12 o'clock		
Property	**Value**	**Units**
Eccentricity (ε)	0.9599	
Initial Orbit Perigee Velocity (V_p)	9884.5	m/s
Transfer Orbit Entrance Velocity(V_{Dep})	10314	m/s
First Velocity Change(dV_1)	429.6818	m/s
Transfer Orbit Arrival Velocity(V_{Arr})	211.06	m/s
Final Orbit Velocity(V_f)	1542.9	m/s
Second Velocity Change(dV_2)	1331.8	m/s
Total Velocity Change(dV_t)	1761.5	m/s

Table 3: Orbital parameters for arrival at the moon's perigee at 12 o'clock

Perigee to 6 o'clock

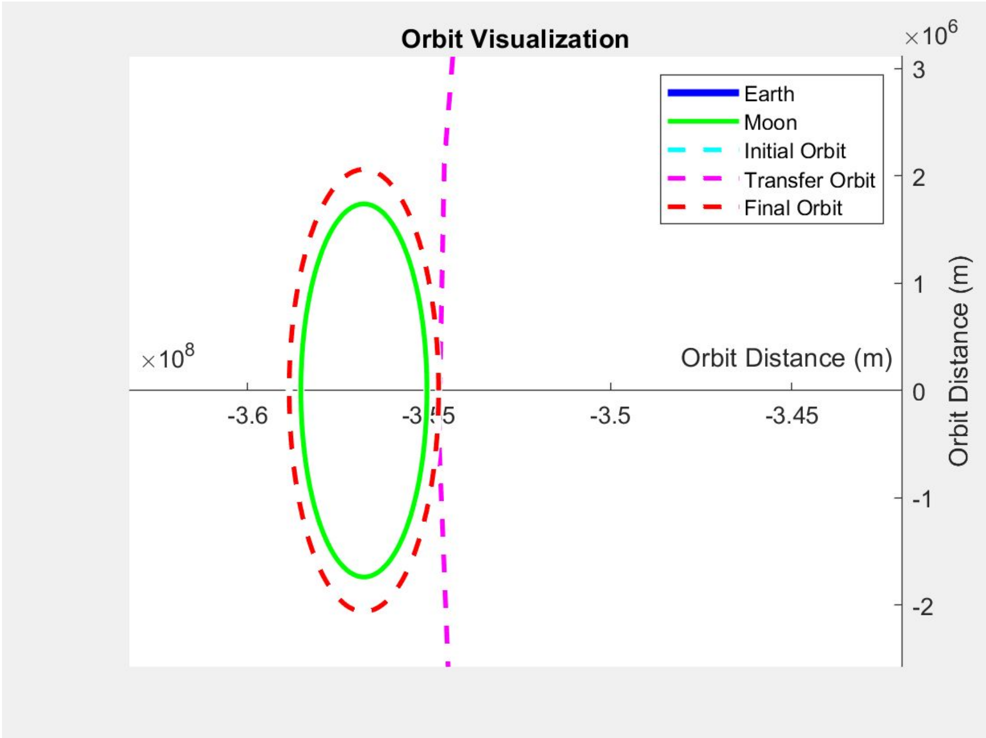

Figure 9: The spacecraft exiting the transfer orbit and entering into the final orbit at a 6 o'clock location (arrival to the moon)

Perigee to 6 o'clock		
Property	**Value**	**Units**
Eccentricity (ε)	0.9594	
Initial Orbit Perigee Velocity (V_p)	9884.5	m/s
Transfer Orbit Entrance Velocity(V_{Dep})	10313	m/s
First Velocity Change(dV_1)	428.48	m/s
Transfer Orbit Arrival Velocity(V_{Arr})	213.4912	m/s
Final Orbit Velocity(V_f)	1542.9	m/s
Second Velocity Change(dV_2)	1329.4	m/s
Total Velocity Change(dV_t)	1757.9	m/s

Table 4: Orbital parameters for arrival at the moon's perigee at 6 o'clock

A Hohmann transfer was executed between the apogee of the given orbit and the intended final circular orbit around the moon.

Part D: Required Forces

Figures:

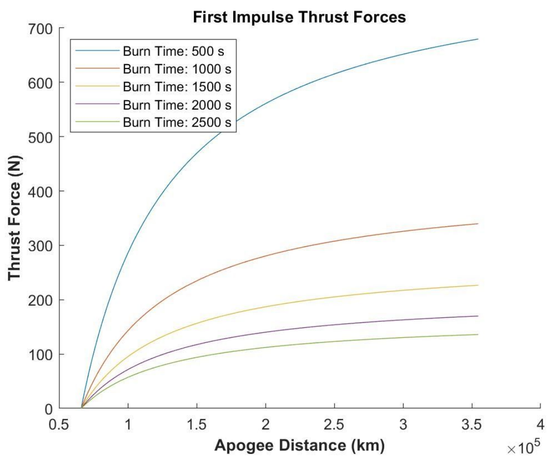

Figure 10: Thrust required 500-2500s

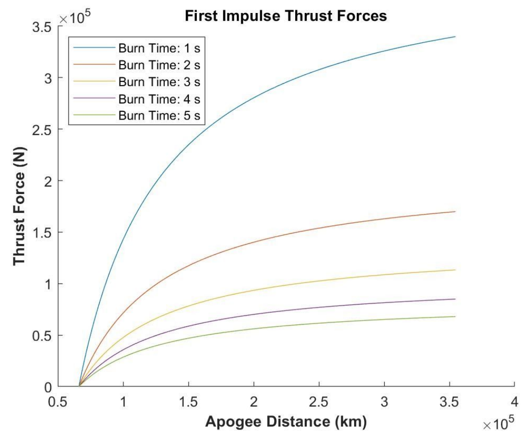

Figure 11: Thrust required 1-5s

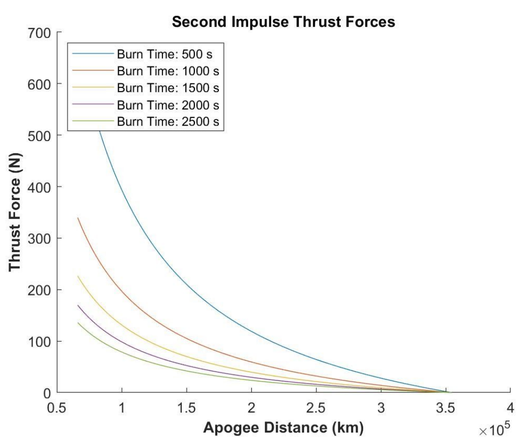

Figure 12: Thrust required at second impulse (500-2500s)

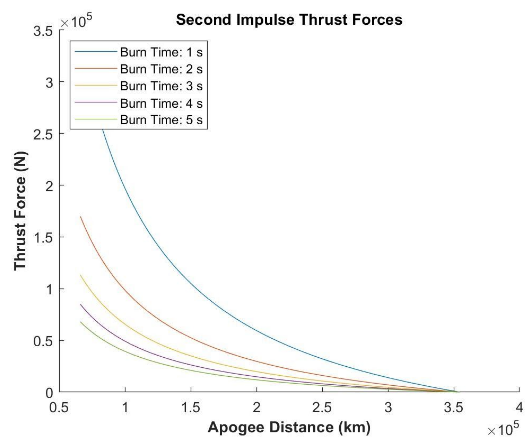

Figure 13: Thrust required at second impulse (1-5s)

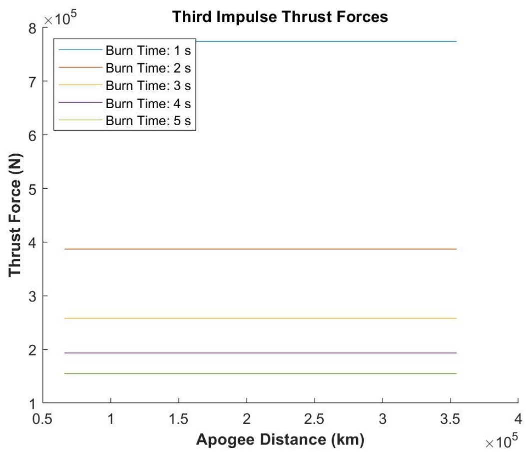

Figure 14: Third Impulse Thrust Forces (1-5s)

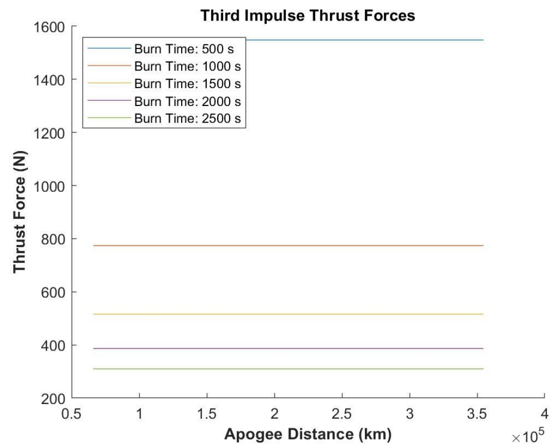

Figure 15: Third Impulse Thrust Forces(500-2500s)

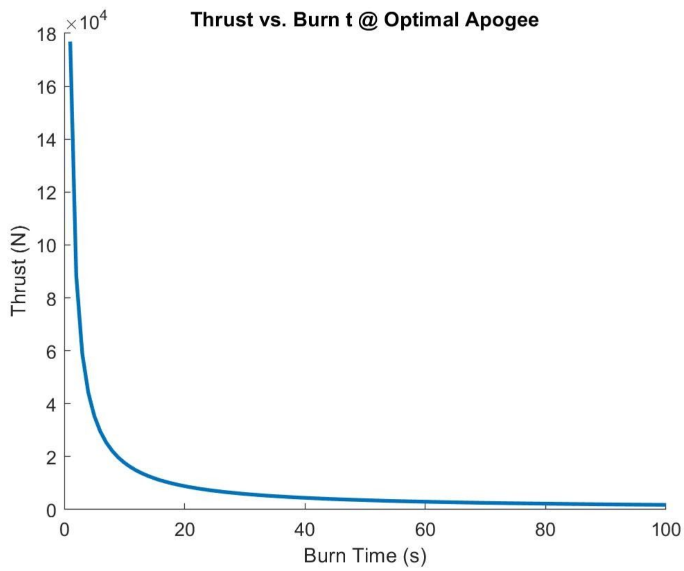

Figure 16: Optimal maneuver thrust vs. burn time calculation

After calculating the changes in velocity required for each impulse maneuver, the forces required at each of these transfers could be determined. The equation [18] was used to first calculate the total mass required post launch, $m_{original}$. This mass includes the $m_{final} = 1000lb$ (453.5kg) payload, as well as the propellant required to change orbits, but excludes any additional structural mass for the propellant housing. The initial mass of this spacecraft before performing any orbital adjustments is therefore $m_{original} = 796.7kg$. With this in mind, the masses of the propellant required for each impulse can be calculated using equation [19], and subsequently the masses of the spacecraft after each impulse can be calculated with equation [20], where 'n' is the current impulse stage the spacecraft is on. These mass propellant values for the single transfer case can be found in Table 5.

From the propellant mass values, a mass flow rate can be calculated, if a burn time is assumed. In a brief exchange with the professor on 12/21, it was mentioned that these impulses should be conducted almost instantaneously, with a burn time of around two seconds being brought up. As such, for the single calculations of thrust, a burn time of two seconds was assumed. This was used in equation [21] to obtain the mass flow rate of the spacecraft. From here, all of the previously gathered factors could be used in equation [8] to determine the thrust. The thrust values for the single transfer orbit can also be found in Table 5.

Single Hohmann Transfer from Perigee to 6 o'clock		
Property	**Value**	**Units**
Mass of Propellant for First Impulse (m_{p1})	122.8	kg
Mass of Propellant for Second Impulse (m_{p2})	279.8	kg
Burn Time of each Impulse (t_{burn})	2	s
Thrust Required at First Impulse (T_1)	15948.7	N
Thrust Required at Second Impulse (T_2)	37584.7	N

Table 5: Propellant and thrust values for arrival at the moon's perigee at 6 o'clock

Similarly, this process to find the required thrust values can also be done for the double transfer orbit, using a similar process. The only major difference being that there is now an additional parameter to account for, the radius of the first transfer orbit's apogee, r_{A2} . This apogee can be positioned anywhere between the r_{A1} (original orbit's apogee) and r_{A3} (second transfer orbit's apogee, 200mi above the moon), because its location was not specified in the problem. Thus, this radius r_{A2} was made into a variable and plotted. Additionally, different burn times were also tested to evaluate how much thrust was potentially needed to adjust the orbits. These plots produced Figures 10-15, which show the different thrust values needed at each impulse, for different apogee and burn time values. The values used in these plots were calculated in the same process as the values in Table 5. From these plots it can be seen that while the thrust for the final impulse T_3 is constant, the thrust at the two transfer orbit impulses, T_1 and T_2 , directly correlate to one another. From analysis of the data, it can be found that

Part E: Two Transfers

According to Griffin and French[8], the efficiency between implementing a single Hohmann transfer or a two transfer is significant if the eccentricity is large. According to Figure 17 and the specified example[8] that uses an eccentricity value of 0.7 and a central angle (ω) of 90°, if the spacecraft travels in the optimal maneuver the spacecraft will save 29% of its fuel power. For a single Hohmann transfer, a transfer from r_1 to r_3, as shown in Figure 18, the eccentricity increases from 0.8 to 0.9594. While a two transfer trajectory, a transfer from r_1 to r_2 and then from r_2 to r_3, would have an eccentricity value between 0.8 and 0.9594. Therefore, the decision of whether a single Hohmann transfer or a two transfer would conserve less fuel is still inconclusive for this given eccentricity value. Additional calculations, such as a bi-elliptical orbit transfer or more than two transfers at the moon region, and comparing those values may provide information that will aid in producing conclusive results. Inserting more transfers when the spacecraft approaches the moon will reduce the number of required engines by decreasing the change in velocities needed to enter into the moon sphere of influence. From ‘Appendix C’, the changes in velocities for a single and a two transfer were similar values. These calculated values support the previously stated statement that the significance between the transfers is relevant if the eccentricity value is high. Both the calculated eccentricities are fairly high. Using a single or two transfer would most likely provide the same amount of fuel conservation.

Optimal Maneuver Orbit Parameters		
Property	**Value**	**Units**
Eccentricity (ε)	0.8788	
Initial Orbit Perigee Velocity (V_p)	9,884.5	m/s
First Transfer Orbit Entrance Velocity(V_{Dep})	10,098.46	m/s
First Velocity Change(dV_1)	213.92	m/s
Second Orbit Velocity(V_{sec})	10,313.02	m/s
Second Velocity Change(dV_2)	214.56	m/s
Arrival Velocity (V_{Arr})	213.49	m/s
Final Velocity (V_f)	1,542.8	m/s
Third Velocity Change (dV_3)	1329.3	m/s
Total Velocity Change (dV_t)	1,757.8	m/s

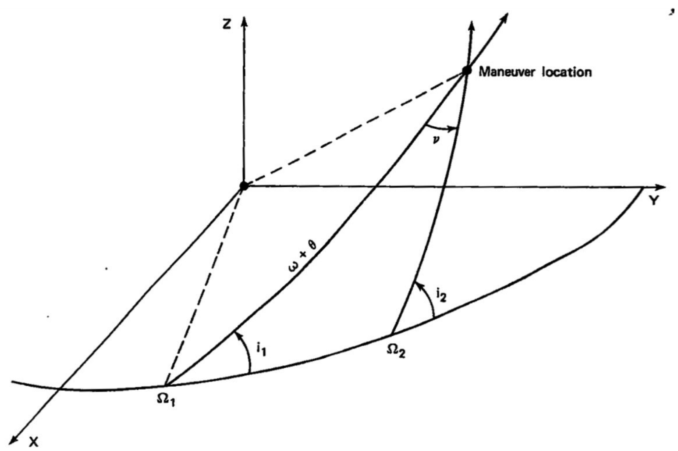

Figure 17: Diagram for the location of the central angle

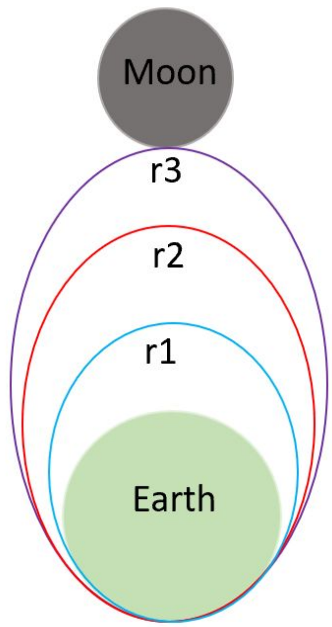

Figure 18: Diagram of the different orbit transfers

Conclusion

In conclusion, we were able to calculate the thrust and flight path angle for a spacecraft to reach an orbit with a perigee of 600 miles and eccentricity of 0.8. Using the data found for the Falcon 9 rocket, a MATLAB code was created that would find the final burnout velocity from the varying propellant mass in both stages of the rocket. When the propellant mass combination allowed for the required velocity was found, we were then able to find the thrust produced by each stage. With the total thrust equal to 3,033,488 N. We were also able to find the flight angle by finding the burnout velocity and radius at which burnout occurs for the SpaceX Falcon 9. This rocket was used for the Israeli spacecraft Beresheet. With those values and by solving for the specific angular momentum with the given perigee and eccentricity we were able to find a flight angle of

83.0515 degrees. After multiple calculations to determine the best flight trajectory. It was determined that the best flight path was to start at the perigee of the elliptical orbit and end at the perigee of the Moon's orbit. More specifically at the area which we refer to as 6 o'clock. Further analysis of the ideal transfer orbit yielded the thrust requirements for the first and second impulses which were 15948.7 N and 37584.7 N respectively. Final results for a single and double Hohmann transfer showed that they will require the same amount of fuel. This is likely due to the fact that the eccentricity changes between the transfer orbits is not very large. Potentially if there were more than two transfer orbits, then there would be a benefit that would lead to less fuel being used.

References

[1] F. Hale, *Introduction to Space Flight*

[2] Perigee of the Moon's orbit. Retrieved December 22, 2020, from https://earthsky.org/?p=319950#:~:text=The%20moon%27s%20distance%20from%20Earth,Earth%20in%20its%20monthly%20orbit

[3] "IADC Space Debris Mitigation Guidelines" (PDF). INTER-AGENCY SPACE DEBRIS COORDINATION COMMITTEE: Issued by Steering Group and Working Group 4. September 2007.

[4] Beresheet Lunar Lander. (n.d.). Retrieved December 22, 2020, from https://directory.eoportal.org/web/eoportal/satellite-missions/b/beresheet

[5] In Depth. (2019, May 15). Retrieved December 22, 2020, from https://solarsystem.nasa.gov/missions/beresheet/in-depth/

[6] Morelle, R. (2019, April 11). Israel's Beresheet spacecraft crashes on Moon. Retrieved December 22, 2020, from https://www.bbc.com/news/science-environment-47879538

[7] Kyle, E. (2017, May 01). Space Launch Report: SpaceX Falcon 9 Data Sheet. Retrieved December 22, 2020, from https://www.spacelaunchreport.com/falcon9.html

[8] Griffin, M.D., & French, J.R. (2004). Space Vehicle Design, Second Edition. Reston, Virginia: American Institute of Aeronautics and Astronautics

APPENDIX A: MAIN MATLAB CODE PROJECT

```
%% Introduction
% Spacecraft Mission Design
% Final Project
% Donald Barnickel, Steven Calalpa, Samuel Chernov, Daniella Chung

%% Part 0: Constant Initialization
mu=3.986e14; % Earth's gravitational parameter
rE=6.378e6; % m, Earth's Radius
Wlb=1000; % lb, mass of spacecraft
W=Wlb/2.205; %kg, mass of spacecraft
hp=600*1.609*1000; %m, height at perigee
rp=(600*1.609)*1000+rE; %m, radius at perigee
eps=0.8; % eccentricity
hm=(200*1.609)*1000; %m, Hohmann Height to moon
rmoon=1738e3; % m, radius of moon
rEM=356794e3; %m, distance from earth to moon
vEsc=sqrt((2*mu)/rE); % m/s

%% Part 1: Thrust and Flight Path Angle
% Perigee Parameters
a=rp/(1-eps); %m, radius of apogee
E=-mu/(2*a); %m^2/s^2, energy of the orbit
H=sqrt(mu*a*(1-eps^2)); % kg*m^2/sec, specific angular momentum
V=sqrt(2*(E+mu/rp)); %m/s, velocity at perigee

% Flight path angle
Rreq=60000*1000; % m, radius @ entrance
Vreq=10000; % m/s, velocity @ entrance
phi=acos(H/(Rreq*Vreq));

% Thrust & Propellant Mass
mPay=W+796.7; % kg, mass of payload- payload & booster fuel
Tmp=thrust(mPay); % separate function, APPENDIX B
T=Tmp(1); % N, Falcon 9 Thrust
mp1=Tmp(2); % kg, Falcon 9 Stage 1 Prop. Mass
mp2=Tmp(3); % kg, Falcon 9 Stage 2 Prop. Mass
```

```
%% Part 2: Hohmann Transfer
% Transfer from 600 mile perigee to
% 200 miles away from the moon

% Properties of leaving orbit
ra=a*(1+eps); %m, transfer pt. from current orbit
va=H/ra; % m/s, circularized orbit velocity

% Radii from Earth
% ra- already achieved prior -> dist. of first orbit to earth
rmp=rEM-rmoon; %m, Earth's center to moon surface
rmo=rmp+hm+2*rmoon;%(ADD for 12 o'clock) %m, 200 mi. away from the moons orbit, closer
to earth

% Transfer Orbit Properties
at=(ra+rmo)/2; %m, semi-major axis of the Hohmann transfer orbit
Et=-mu/(2*at); % energy of the transfer orbit
vDep=sqrt(2*(Et+(mu/ra))); % m/s, velocity of leaving orbit
Ht=ra*vDep; % specific angular momentum of the transfer orbit
vArr=Ht/rmo; % m/s, arrival velocity
epsT=(rmo-ra)/(rmo+ra); % epsilon of the transfer orbit

% Final Orbit properties
muM=4.90339e12; % gravitational constant of the moon
vf=sqrt(muM/(rmoon+hm)); % velocity of the circular orbit

% Velocity Change
dv1=vDep-va; % Velocity change between circular and elliptical transfer
dv2=vf-vArr; % Vel. chng btwn elliptical and final circular
dvt=abs(dv1)+abs(dv2); % total velocity change

%% Orbit Plot
% Sketch this orbit
figure
hold on

% First, plot the Earth
center=[0 0]; % center point of the graph
colorRGB=[0 0 1]; % color to fill in the Earth
p1e=viscircles(center,rE,'Color',colorRGB,'LineWidth',3); % Create circles with Earth's radius
```

```
% Plot moon
center=[rEM 0];
colorRGB=[0 1 0];
p1m=viscircles(center,rmoon,'Color',colorRGB,'LineWidth',3); % create circles with moon's
radius

% Global theta
theta=0:0.05:2*pi; % define theta range in small increments

% Plot initial orbit
orbXi=a*(cos(theta)-eps); % take the x component of orbit, Kepler's Laws
orbYi=a*sqrt(1-eps^2)*sin(theta); % y component of orbit, Kepler's Laws
p1i=plot(orbXi,orbYi,'c--','LineWidth',2); % plot the initial orbit

% Plot transfer orbit
orbXt=-at*(cos(theta)-epsT); % take the x component of orbit, Kepler's Laws
orbYt=-at*sqrt(1-epsT^2)*sin(theta); % y component of orbit, Kepler's Laws
p1t=plot(orbXt,orbYt,'m--','LineWidth',2); % plot the initial orbit

% Plot final circular orbit
colorRGB=[1 0 0];
cirR=rmoon+hm;
p1f=viscircles(center,cirR,'Color',colorRGB,'LineWidth',2,'LineStyle','--');

% Set the Earth to be the origin
ax = gca;
ax.XAxisLocation = 'origin';
ax.YAxisLocation = 'origin';

% Plot labels
title('Orbit Visualization');
ylabel('Orbit Distance (m)');
xlabel('Orbit Distance (m)');
legend([p1e p1m p1i p1t p1f],'Earth','Moon','Initial Orbit','Transfer Orbit','Final Orbit');
hold off
```

APPENDIX B: THRUST CALCULATION SUPPLEMENT

```
function [T] = thrust(mPay)
%% Introduction Information
%{
* Calculate the thrust produced by the rocket
* Taking parameters of the SpaceX Falcon 9 Launcher
    - Comparable payload weights
    - Comparable orbit lift off
* Thurst Eqn: F = q × Ve + (Pe - Pa) × Ae
    - F= Thrust
    - q= Propellant Mass Flow Rate
    - Ve= Velocity of exhaust gas
    - Pe= Pressure @ exit Nozzle
    - Pa= Ambient Pressure
    - Ae= Area of nozzle exit
* Alternatively
    - Isp=F/mdot*g0
    - F=Isp*mdot*g0
    - Only need the Isp's of the rocket

%}

% Alternate Way
%% Constants
g0=9.81; %m/sec^2, gravitational constant

tb1=180; %s, first burn time
tb2=346; %s, second burn time

%% Rocket Properties
tkg=907.185; % ton to kg
ms1=28*tkg; % stage 1 F9 mass
ms2=4.7*tkg; % stage 2 F9 mass
% Payload mass inputted- mp
IspF91=282; % sec, sea level
IspF92=340; % sec, vacuum

%% Propellant Properties
% Stage 1
mp1max=411*tkg; %kg, max mass of Merlin 1D
```

```
mp1min=0.4*mp1max; % 40% of maximum mass
mp1=linspace(mp1min,mp1max,40);

% Stage 2
mp2max=73.4*tkg; %kg, max mass of Merlin 1D Vac.
mp2min=0.39*mp2max; % 39% of the max as specified by props.
mp2=linspace(mp2min,mp2max,40);

% Totals
Vt=zeros(1,40);
Vtcrit=10000; % Velocity req'd to obtain
VtcritE=Vtcrit*0.005; % allowable error
mp1f=0;
mp2f=0;

%% Velocity
for i=1:length(mp1)
   tempMP1=mp1(i);
   tempMP2=mp2(i);


% Stage 1: Merlin 1D
   lnFrac1=(tempMP1+ms1+tempMP2+ms2+mPay)/(ms1+tempMP2+ms2+mPay);
   Vb1=g0*IspF91*log(lnFrac1); % Obtain the velocity of first stg.

% Stage 2: Merlin 1D- Vacuum
   lnFrac2=(tempMP2+ms2+mPay)/(ms2+mPay);
   Vb2=g0*IspF92*log(lnFrac2); % velocity of scnd stg

   Vt(i)=Vb1+Vb2; % total rocket velocity

% Check Critical cond, select those values for mass

   if (Vt(i)-Vtcrit)<VtcritE
      mp1f=tempMP1;
      mp2f=tempMP2;
      ic=i;
   end
end
% Plot of possible Propellant mass configs
```

```
figure
yyaxis right
plot(Vt,mp2,'LineWidth',2);
hold on
plot(Vt(ic),mp2f, 'c*', 'LineWidth',3);
hold on
ylabel('Second Stage Mass (kg)');
yyaxis left
plot(Vt,mp1,'LineWidth',2);
hold on
plot(Vt(ic),mp1f,'m*','LineWidth',3);
hold on
ylabel('First Stage Mass (kg)');
xlabel('Velocity (m/s)');
title('Propellant Mass vs. Velocity');
legend('Prop. M1','Cri. Pt. M1','Prop. M2','Cri. Pt. M2',...
    'Location','northwest');
hold off

%% Total Thurst

% Mass Flow Rate
mdot1=mp1f/tb1;
mdot2=mp2f/tb2;

% Thrust
T1=mdot1*IspF91*g0; % First Thrust
T2=mdot2*IspF92*g0; % Second Thrust

T=[T1+T2 mp1f mp2f]; % total req'd thrust

return



end
```

APPENDIX C: HOHMANN TRANSFER CALCULATION SUPPLEMENT

```
mu1 = 3.986*10^14;
rE = 6.378*10^6;
Vesc = sqrt(2*mu1/rE);
EEarth = -mu1/(2*rE);

mass = 1000/2.205; %Mass in pounds to kg
altitude = 600*1.609*1000;
rP1 = rE+altitude;

epsilon1 = .8;
rA1 = (rP1*(-1-epsilon1))/(epsilon1-1);
check = rA1/9;
a1 = (rP1+rA1)/2;
H1 = sqrt(a1*mu1*(1-(epsilon1^2)));
E1 = -mu1/(2*a1);
VP1 = H1/rP1;
VA1 = H1/rA1;
Isp = 318; %Found from Beresheet rocket parameters

MoonToEarth = 356794*1000; %Meters from Earth
MoonAltitude = 200*1.609*1000; %Meters above Moon
r4 = (MoonAltitude+(1.738*(10^6)));

% deltaE = abs(EEarth) - abs(E1);
% LaunchThrustSpecific = deltaE/altitude;
% LaunchThrust = LaunchThrustSpecific*mass;
rA3 = MoonToEarth-r4;

%Create an array to vary Apogee positions
ArbitraryApogee = (rA1):(100*1000):(rA3);

BurnTime1 = 1:1:3600; %Seperated into Burn 1 and 2, in case you want to
BurnTime2 = 1:1:3600; %burn the 3rd impulse for longer, seeing as it's a much larger deltaV

VDiff12 = [];
VDiff23 = [];
VDiff34 = [];
VTotal = [];
Mass0 = [];
```

```
MassP1 = [];
MassP2 = [];
MassP3 = [];
Compare = [];
T1 = [];
T2 = [];
T3 = [];



for i=1:length(ArbitraryApogee)

    %Depending on where you are leaving from, Perigee or Apogee, change rP2 =
    %                                   rP1 or rA1, respectively.
    rP2 = rP1;
    rA2 = ArbitraryApogee(i);
    a2 = (rP2+rA2)/2;
    epsilon2 = (rA2-rP2)/(rA2+rP2);
    E2 = -mu1/(2*a2);
    H2 = sqrt(a2*mu1*(1-(epsilon2^2)));
    VP2 = H2/rP2;
    VA2 = H2/rA2;

    %Depending on where you are leaving from, Perigee or Apogee, change rP2 =
    %                                   VP1 or VA1, respectively.
    VDiff12(i) = VP2 - VP1;

    rP3 = rP2;

    %If we assume we're entering at 6 'oclock, the do rA2 = MoonToEarth-r3
    %If we assume we're entering at 12 'oclock, the do rA2 = MoonToEarth+r3
    rA3 = MoonToEarth-r4;
    a3 = (rP3+rA3)/2;
    epsilon3 = (rA3-rP3)/(rA3+rP3);
    E3 = -mu1/(2*a3);
    H3 = sqrt(a3*mu1*(1-(epsilon3^2)));
    VP3 = H3/rP3;
    VA3 = H3/rA3;
```

```
%Depending on where you are leaving from, Perigee or Apogee, change rP2 =
%                                       VP2 or VA2, respectively.
VDiff23(i) = VP3 - VP2;

a4 = (r4+r4)/2;
epsilon4 = (r4-r4)/(r4+r4);
mu2 = 4.903*10^12;
E4 = -mu2/(2*a4);
H4 = sqrt(a4*mu2*(1-(epsilon4^2)));
V4 = H4/r4;

VDiff34(i) = V4 - VA3;
VTotal(i) = VDiff12(i) + VDiff23(i) + VDiff34(i);

Mass3 = mass;
Mass0(i) = Mass3/((1-(1 - (exp((-VDiff12(i))/(9.81*Isp)))))*...
   (1-(1 - (exp((-VDiff23(i))/(9.81*Isp)))))*...
   (1-(1 - (exp((-VDiff34(i))/(9.81*Isp))))));


% Mass0 = 2500/2.205;
MassP1(i) = Mass0(i)*(1 - (exp((-VDiff12(i))/(9.81*Isp))));
Mass1 = Mass0(i)-MassP1(i);
MassP2(i) = Mass1*(1 - (exp((-VDiff23(i))/(9.81*Isp))));
Mass2 = Mass1-MassP2(i);
MassP3(i) = Mass2*(1 - (exp((-VDiff34(i))/(9.81*Isp))));
Mass3 = Mass2-MassP3(i);

Compare(i) = VDiff12(i)-VDiff23(i);

if Compare(i)<=1 && Compare(i)>=-1
   MiddlePoint = ArbitraryApogee(i)/1000;
   MiddlePointIndex = i;
end

for j=1:length(BurnTime1)
   Mdot1 = MassP1(i)/BurnTime1(j);
   Mdot2 = MassP2(i)/BurnTime1(j);
   T1(j,i) = Mdot1*9.81*Isp;
   T2(j,i) = Mdot2*9.81*Isp;
```

```
    end
    for k=1:length(BurnTime2)
        Mdot3 = MassP3(i)/BurnTime2(k);
        T3(k,i) = Mdot3*9.81*Isp;
    end

end

% Plot
    % Row- constant apogee, different time
    % Cols.- constant time, different apogee
figure
hold on
% Constant Burn Time, Different Apogee L
for i=1:5
    loc=i*500;
    y=T1(loc,:)';
    txt=['Burn Time: ',num2str(BurnTime1(loc)),' s'];
    plot(ArbitraryApogee./1000,y,'DisplayName',txt);
end
hold off
legend show
legend('Location','northwest');
ylabel('Thrust Force (N)','FontWeight','bold');
xlabel('Apogee Distance (km)','FontWeight','bold');
title('First Impulse Thrust Forces',...
    'FontWeight','bold');
```