

Q 2:

```
def example 1 (lst):
    n = len(lst)
    total = 0
    for j in range(n) #  $\theta(n)$ 
        for k in range (1+j) #  $\theta(n)$ 
            total += lst[k]
    return total
```

$\theta(n^2)$

```
def example 2 (lst):
    n = len(lst)  $\theta(1)$ 
    prefix = 0  $\theta(1)$ 
    total = 0  $\theta(1)$ 
    for j in range(n):  $\theta(n)$ 
        prefix += lst[j]
        total += prefix
    return total
```

$1+1+1+1+1$

$\theta(n)$

```
def example 3 (n):
```

$i = 1$

Sum = 0

while ($i < n * n$):

$i *= 2$

Sum += i

return Sum.

$i < n^2$ $k = \log_2 n$

$1+2+3+...+\log_2 n$

$\frac{k(k+1)}{2} = \theta(k^2)$

$\boxed{\theta(\log_2(n))}$ $\theta((\log_2 n)^2)$

a. $5n^3 + 2n^2 + 3n = O(n^3)$

$$5n^3 + 2n^2 + 3n < 5n^3 + 2n^3 + 3n^3 < C \cdot n^3$$

$$5n^3 + 2n^2 + 3n < 5n^3 + 2n^3 + 2n^3$$

$$5n^3 + 2n^2 + 3n < 10n^3 \quad n > 1$$

$$C = 5 + 2 + 3$$

$$= 10$$

$$n_0 = 1$$

b). $\sqrt{7n^2 + 2n - 8} = O(n)$

~~$$cn < \sqrt{7n^2 + 2n - 8} < dn$$~~

~~$$c^2 n^2 < 7n^2 + 2n - 8 < d^2 n^2$$~~

~~$$\sqrt{7n^2 + 2n - 8} \leq \sqrt{7n^2 + 2n} \text{ for big } n$$~~

~~$$\sqrt{7n^2 + 2n} \leq \sqrt{7n^2 + 2n^2} \quad n \geq 1$$~~

~~$$\sqrt{9n^2} = 3n \text{ for all } n \geq 1$$~~

for big n :

$$3n \leq Cn \text{ (upper bound)}, C \geq 3, n \geq 1$$

$$\sqrt{7n^2 + 2n - 8} \geq \sqrt{7n^2}$$

$$\sqrt{7n^2} \geq Cn$$

$$C \leq \sqrt{7}$$

$$C_2 = \sqrt{7}$$

$$\begin{aligned} 2n - 8 &\geq 0 \\ n &\geq 4 \end{aligned}$$

c). $d(n) = O(f(n))$

$$e(n) = O(g(n))$$

$$dn \leq C \cdot f(n) \text{ for all } n \geq n_0$$

$$en \leq C \cdot g(n) \text{ for all } n \geq n_0$$

$$e^n d^n = C \cdot f(n) \cdot C \cdot g(n)$$

$$= O(f(n) \cdot g(n))$$

$$n_0 = 4$$