Section A	Concepts and Skills	150 marks

Answer **any five** questions from this section.

Question 1 **(30 marks)**

(a) An Irish rock band played a concert which was attended by 18 000 people.
The average ticket price was €43.

(i) Calculate the total income from the ticket sales.

€774,000

(ii) The total costs for organising the concert were €360 000.
Calculate the profit, as a percentage of the total ticket sales, made by the rock band on this concert. Give your answer to one decimal place.

$\frac{360,000}{774,000} = 46.5\%$

(b) **(i)** The rock band played a concert in America. They made a profit of $270 000.

There are four members in the band: the singer, lead guitarist, bass guitarist and drummer. The singer writes all of the songs, so he received half of all profits and the remainder was divided evenly between the two guitarists and the drummer.

Calculate how much of the $270 000 the drummer received.

270,000 ÷ 2 = 135,000 singer.
− 135,000
135000 ÷ 3 = €45000 for the drummer

(ii) The drummer took $30 000 of his share and converted it to euro at an exchange rate of €1 = $1·12. Calculate how much the drummer received, to the nearest euro.

~~€30,000 = $33600~~ $30,000 ÷ 1.12
€26785.71

Question 2 **(30 marks)**

(a) Simplify $4(3+2i)+i(5-i)$

And express your answer in the form $a+ai$, where $a \in \mathbb{R}$ and $i^2=-1$.

16 + 8i + 5i + 1

~~Fb~~ 15 + 13i

(b) **(i)** If $z=3+3i$, what is $\bar{z}$?

$\bar{z} =$ 3 - 3i

(ii) Express $\frac{z}{\bar{z}}$ in the form $a+bi$, where $a,b \in \mathbb{R}$.

$\frac{(3+3i)(3-3i)}{(3-3i)(3-3i)} = \frac{3(3-3i)+3i(3-3i)}{3(3-3i)-3i(3-3i)}$

$\frac{9-9i-9i+9i^2}{3-9i-9i+9i^2} = \frac{9-18i+8}{9-18i+8}$

1 - 8i

X

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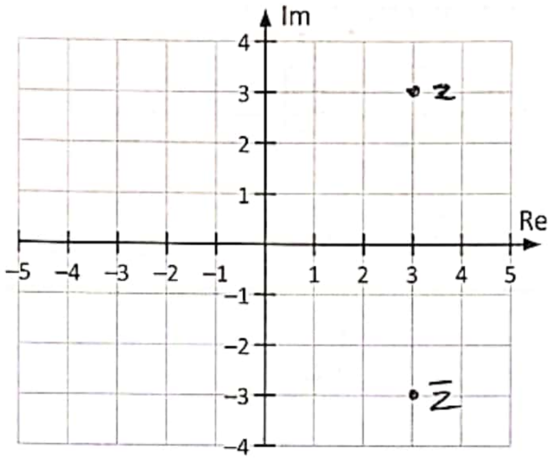
(iii) Hence, plot z, $\bar{z}$ and $\frac{z}{\bar{z}}$ on the Argand diagram. Label each point clearly.



(iv) Calculate $|z|$. Give your answer in simplest surd form.

Question 3 **(30 marks)**

(a) The first four terms of an arithmetic sequence are given as:

$\boxed{-8}$, -3 , $\overset{+10}{\boxed{2}}$, 7 , ...

Find the common difference of this arithmetic sequence and hence find the two missing values in this sequence and enter them in the boxes above.

(b) A sequence of values follows the rule: **Any term = 3(previous term) + 5**

For example, Second term = 3(first term) + 5

(i) Use this rule to complete the table below.

Term number	1	2	3	4	5
Value	-2	-1	2	11	38

(ii) Is the sequence of values linear, quadratic or neither?

Tick (✓) the appropriate box and give a reason for your answer.

Linear ☐ Quadratic ☐ Neither ☑

Reason: The first or second difference isn't the same

Question 4 (30 marks)

(a) Express $16^{\frac{1}{2}}$ and 3^{-5} without indices.

$16^{\frac{1}{2}} =$ 4	$3^{-5} =$ 0·004

(b) Solve the equation $3^{5x-2} = 27^{2x}$ and verify your answer.

$3^{3x} = 27^{2x}$

a

(c) Find the solutions of the equation $3x^2 - 2x - 3 = 0$, where $x \in \mathbb{R}$.

Give each answer in its simplest surd form.

$\underset{a}{3x^2} - \underset{b}{2x} - \underset{c}{3}$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-3)}}{2(3)}$

$\frac{2 \pm \sqrt{4 + 36}}{6}$

$+$ / \ $-$

1.39 -0.72

or or

$\frac{1 + \sqrt{10}}{3}$ $\frac{1 - \sqrt{10}}{3}$

30

Question 5 **(30 marks)**

(a) Let $f(x) = x^2 - 5x, \quad \text{for } x \in \mathbb{R}.$

(i) Find the value of $f(-2)$.

$f(-2) = (-2)^2 - 5(-2). \quad = \underline{14.}$

(ii) Find $f'(x)$, the derivative of $f(x)$.

(iii) For what value of x is $f'(x) = 0$?

(b) A curve has equation $y = x^3 - 4x + 7$.

(i) Find $\frac{dy}{dx}$, the slope of the tangent to the curve at any point along the curve.

$\frac{dy}{dx} = 3x - 4$

(ii) Hence, find the equation of the tangent to the curve $y = x^3 - 4x + 7$ at the point $(2, -3)$.

~~$(x-h)(x_1-h)+(y+k)(y$~~

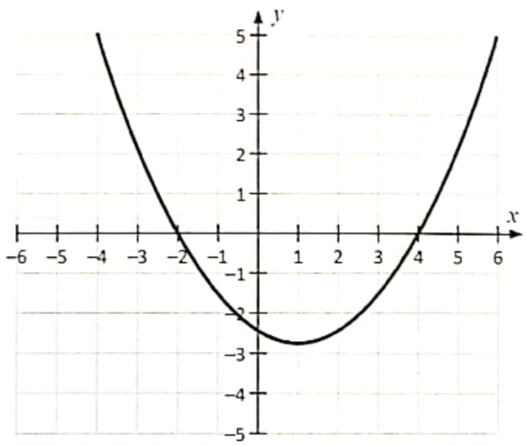
$(x-h)(x_1-h)+(y-k)(y_1-k)=r^2$

~~$4x+2y$~~

Question 6 **(30 marks)**

(a) **(i)** The function $f(x)$ is graphed on the coordinate plane.

What type of function is $f(x)$? Tick (✓) the appropriate box.



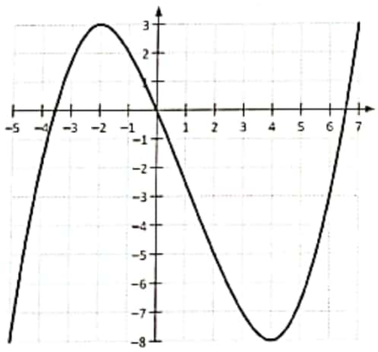
Linear ☐

Quadratic ☐

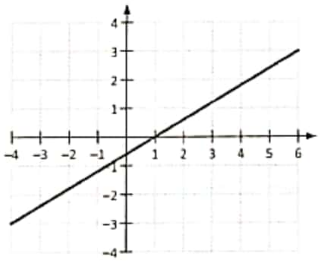
Cubic ☑

(ii) What are the roots of $f(x)$? $\boxed{-2}$ and $\boxed{4}$

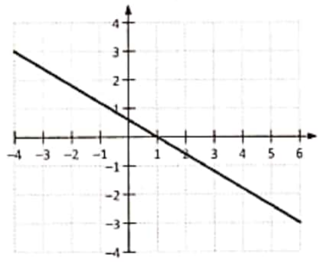
(iii) Which of the graphs A, B or C shown below represents $f'(x)$, the first derivative of $f(x)$? Give a reason for your answer.



Graph A



Graph B



Graph C

Graph = $\boxed{A}$

Reason: Cubic

(b) The equation of a line, l, is given by $y = 3x - 2$

The equation of a curve, c, is given by $6x^2 + x - y^2 = 0$

Without graphing these equations, use simultaneous equations to find the points where the line l intersects the curve c.

$y = 3x - 2$

$6x^2 + x - y^2 = 0$

$6x^2 + x (-3x - 2)^2$

$7x^2 (-3x - 2)^2$

$-21x^2 + 14x^2$

$6x^2 + x - y^2 = 0$

$\cancel{6(3x-2)^2 +}$

$6x^2 + x(-3x - 2)^2 = 0$

$6x^2 + (3x^2 - 2x)$

Section B	Contexts and Applications	150 marks

Answer **any three questions** from this section.

Question 7 **(50 marks)**

Cillian is spending the summer on his uncle Donal's farm.
Donal is going to pay Cillian for helping out on the farm and he gives Cillian two options:

Option A: €10 per day

Option B: €1 on day 1, €2 on day 2, €3 on day 3, etc.

(a) If Cillian will be on the farm for 7 days, which option will give him more money?
Clearly show all calculations used.

Option A = €70 (€10 × 7)

Option B = 1
2
3
4
5
6
+ 7

€28.

∴ Option A will give him more money.

(b) Write an expression for the total amount earned by Cillian after n days, using option A.

10n

~~Tn = a + (n−1) d~~
~~Tn = 10 (n−1) 10~~

(c) The monies paid in option B form an arithmetic sequence: €1, €2, €3, €4, ...

(i) Identify the first term, a, and the common difference, d.

$a = \boxed{1}$ $d = \boxed{1}$

(ii) The total amount Cillian would earn, after n days, using option B can be calculated using the following formula:

$$\text{Total earned using option B} = \frac{n^2 + n}{2}$$

If Cillian chooses option B and works on the farm for 30 days, how much would he earn?

$$\frac{30^2 + 30}{2} = \frac{930}{2} = €465.$$

(iii) For how many days would Cillian have to work such that the pay he would receive by either option is the same? (Hint: Let the total earned by option A equal the total earned by option B.)

$$\frac{n^2 + n}{2} = 0$$

$$10n = 0$$

(d) Cillian's older cousin, Roisin, works full-time on the farm.

She earns an annual salary of €43 000.

The standard cut-off point is €36 000, the standard rate of tax is 20% and the higher rate is 40%. Roisin has annual tax credits of €2 600.

(i) Calculate Roisin's net income for the year (i.e. how much money she has remaining after she pays the income tax due).

43,000
36,000
7000

36,000 x 20% = 7200
7000 x 40% = 2800
+ 10,000
- 2,600
7,400

43,000
- 7,400
€35,600 remaining

(ii) The farm has a particularly profitable year and Donal pays Roisin a bonus.
Her net income increased by €3 840 in the year during which she received the bonus.

Calculate the bonus she received, before tax was deducted.

35,600
3 840
€ 39,440

Net income = after tax

Question 8 **(50 marks)**

(a) A triathlon is a race where an athlete needs to complete three different segments: swimming, cycling and running.

(i) Larry took part in a triathlon and his performance for each segment is recorded below. Calculate the two missing values and enter them into the table.

	Time taken (mins)	**Distance (km)**	**Speed (km/h)**
Swimming	50 mins	1·5 km	1·8 km/h
Cycling	30 mins		25 km/h
Running		8 km	10 km/h

D
S x T

Cycling = 750
Running = 0·8

(ii) Using Larry's total time and total distance for all three segments, determine his average speed for the whole triathlon.
Give your answer in km/h, to one decimal place.

Speed = Distance ÷ time.

(b) Larry is a keen golfer. He wants to improve his game, so he goes for some specialist golf lessons where a computer simulator analyses his swing and makes suggestions for improvement.

When Larry hits the ball with the driver club, in the simulator, the ball follows a path as described by the function

$$h(x) = 0 \cdot 525\, x \; - \; 0 \cdot 0025\, x^2$$

where h is the height of the ball and x is the distance the ball has travelled horizontally from the point where it was struck by the golf club.

(i) When the ball lands, its height above the ground will be 0 metres.
Use this information to find how far away Larry's ball would expect to land.

$h(0) = 0 \cdot 525x - 0 \cdot 0025x^2$.

(ii) Find $h'(x)$ and hence find the maximum height Larry's ball will reach.

(c) After some time using the simulator, the function describing the motion of Larry's golf ball is

$$g(x) = p\,x - 0 \cdot 002x^2, \qquad \text{where } p \in \mathbb{R}$$

Based on this function, the ball would now land at the point (225, 0).
Use this information to find the value of p.

(d) Larry's golf club have the following offers on golf lessons.

- Offer 1: Buy two lessons for €80 each and get a third lesson for free
- Offer 2: Buy one lesson for €70 and get a second half price.
- Offer 3: Buy a set of five lessons for €240

(i) Which offer results in the lowest cost per lesson? Tick (✓) the appropriate box and justify your answer.

Offer 1 ☐ Offer 2 ☐ Offer 3 ☑

3 lessons for ~~€~~ €160 = 6 for €320

2 lessons for €105. = 6 for €315.

5 lessons for €240 = 6 for €288

?

(ii) Larry decides to not go with the lowest-cost option.
Give a reason why you think this may be the case.

Because ~~Bu~~ when he gets to a certain amount of lessons after the set 5 lessons, it will end up still costing more in the long run.

Question 9 **(50 marks)**

(a) The area of algae growing on a lake can be calculated using the formula

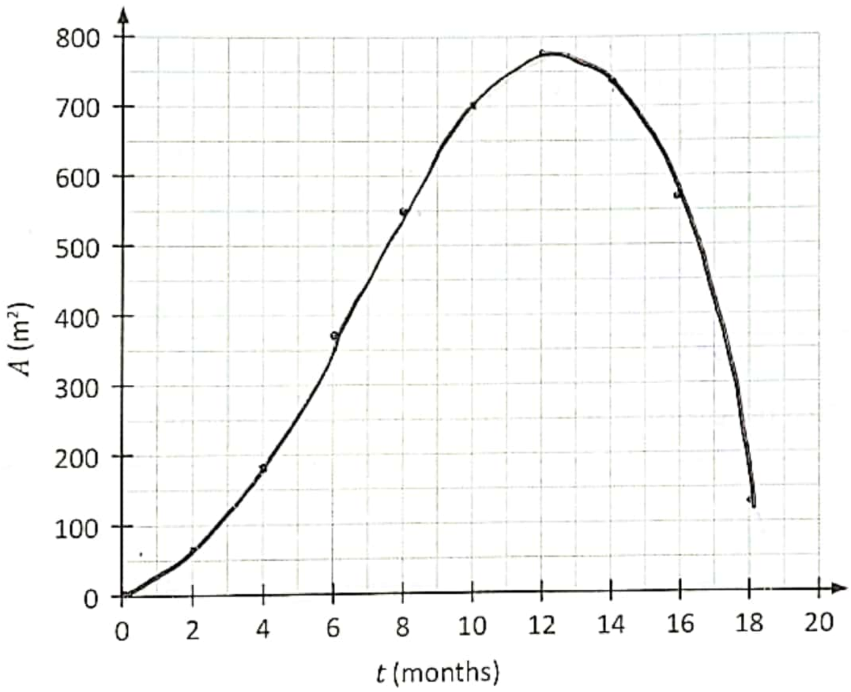
$$A = 15\,t^2 - 0\cdot8\,t^3$$

where A is the area in square metres of the algae and t is the time in months after the algae starts growing.

(i) Use this formula to complete the table below. Give each value to the nearest whole number. One entry has been done for you.

t (months)	0	2	4	6	8	10	12	14	16	18
A (m^2)	0	54	189	367	550	700	778	745	563	194

(ii) Graph $A = 15\,t^2 - 0\cdot8\,t^3$ in the domain $0 \le t \le 18$.



(b) **(i)** Find $\frac{dA}{dt}$.

$\cancel{\frac{dy}{dx}}$

$13t^2 - 0.8t^3$

$\frac{dy}{dx} = 30t - 2.4$

(ii) Hence, find the rate of change in the area of the algae after 2 months.

What can you conclude about the algae at this time?

$30(2) - 2.4(2)$

$58\ m^2$

Conclusion: It is ~~decreasing~~. increasing

(iii) Find the value of t when the maximum area is covered by the algae.

(c) At 6 months, when the area covered is 367 m^2, the local county council pour an algaecide into the lake to kill the algae.

Following this, the area of the algae can be calculated using the formula

$A = 367 - 3 \cdot 2\, t^2$, where t is the number of months after the algaecide is added to the lake.

16 12 14 16 .

(i) Show that the area of the algae is decreasing for all values of $t \geq 0$.

$367 - 3 \cdot 2 (10)^2 = 700$ m^2
$367 - 3 \cdot 2 (12)^2 = 778$ m^2
$367 - 3 \cdot 2 (14)^2 = 745$ m^2
$367 - 3 \cdot 2 (16)^2 = 563$ m^2

decreasing

(ii) After how many months, to the nearest month, will the area of algae be zero?

$30t - 2 \cdot 4 \cdot = 0$
$30t = 2 \cdot 4$
$t = 13$ months

Question 10 **(50 marks)**

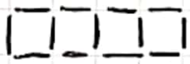
(a) Maya has some sticks that are all of the same length. She arranges them in squares and has made the following three rows of patterns:

Row 1

Row 2

Row 3

(i) Draw the fourth pattern in the sequence into the box below.



(ii) Maya sees that 4 sticks are required to make the single square in the first row, 7 sticks to make 2 squares in the second row and in the third row she needs 10 sticks to make 3 squares. Use this information to complete the following table.

Row number	1	2	3	4	5
Number of sticks	4	7	10	13	16

(iii) Find a formula for T_n, the number of sticks required to make a similar arrangement of n squares in the n^{th} row.

$T_n = a + (n-1)d.$

$T_n = 4 + (n-1)3.$

(iv) Pattern k uses 67 sticks, where $k \in \mathbb{N}$. Find the value of k.

$4+(n-1)3.$

$4+(k-1)3 = 67.$

(b) **(i)** Maya continues to make squares following the same pattern. She makes 4 squares in the 4th row and so on until she has completed 10 rows.

Find the **total** number of sticks Maya uses in making these 10 rows.

~~S_T~~ $\cancel{T_{10} = 4+(10-1)3}$

$\cancel{4+9(3)}$

4, 7, 10, 13, 16, 19, 22, 25, 28, 31

Total number = 31 sticks.

(ii) Maya has 1750 sticks. Find the greatest number of rows Maya can make using these sticks.

$S_n = \frac{a(1-d^2)}{1-d}$ $\frac{4(1-3^2)}{1-3.}$ $= 16.$

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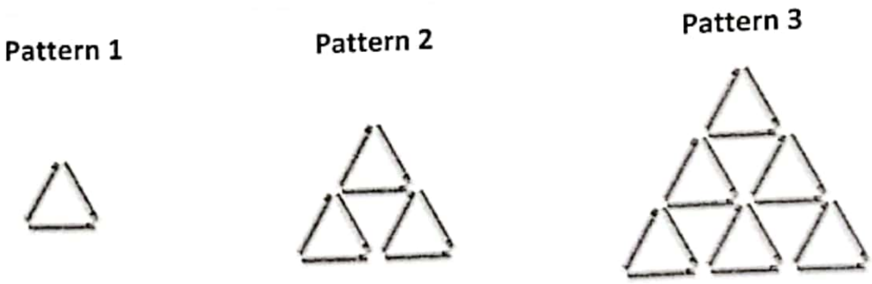
(c) Maya uses the sticks to create a pattern of triangles, as shown.



The number of sticks in pattern n is T_n.

The general term describing T_n can be written in the form:

$$T_n = \frac{3}{2}n^2 + bn + c, \text{where } b, c \in \mathbb{R}$$

(i) Use substitution for n to write T_1 and T_2 in terms of b, c and a number.

$T_1 = \frac{3}{2}(1)^2 + b(1) + c = 1 \cdot 5 + b + c$

$T_2 = \frac{3}{2}(2)^2 + b(2) + c. = 6 + b + c.$

(ii) Hence, or otherwise, find the value of b and the value of c.

$1 \cdot 5 + b + c = 3.$

$6 + b + c = 9.$

$b + c = 1 \cdot 5.$

$b + c = 3$

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You can use this page for extra work