Samuel TAP

10.10.2023

# Fully Homomorphic Encryption I

## Zama’s FHE compiler

ZAMA

# Agenda

# FHE

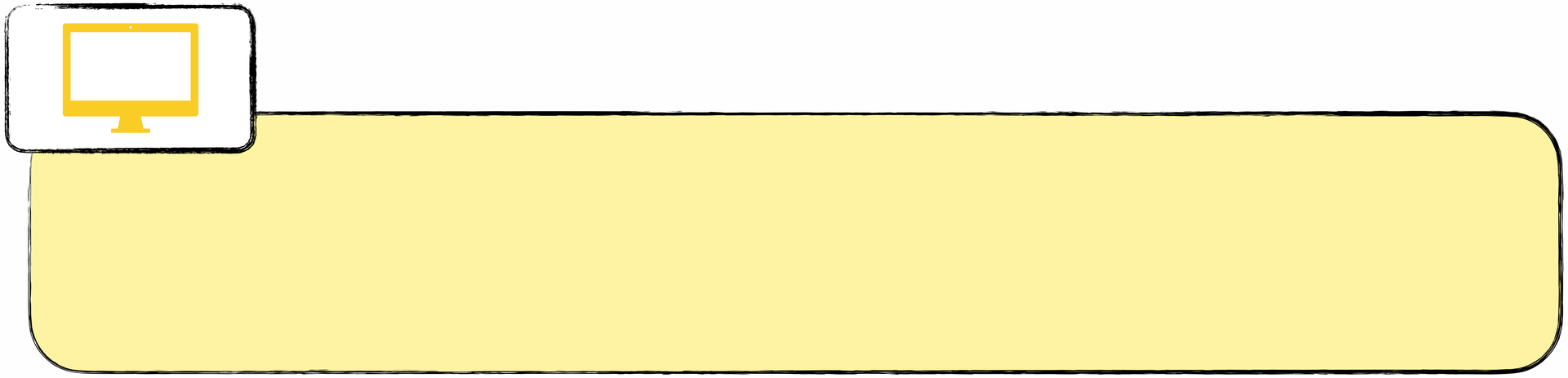
What is FHE ?

Past

# Cloud Computing

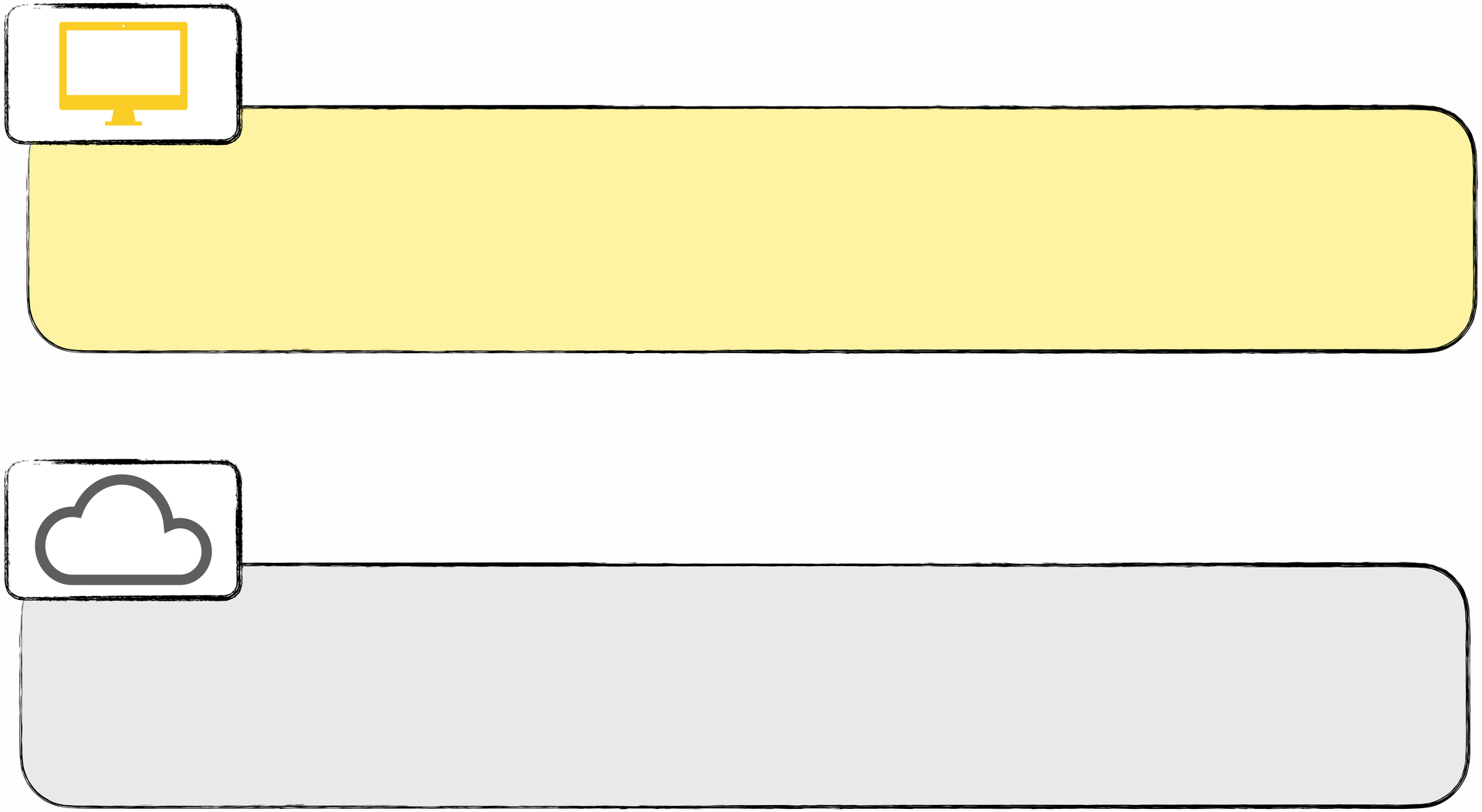
Past

# Cloud Computing



Past

# Cloud Computing



Past

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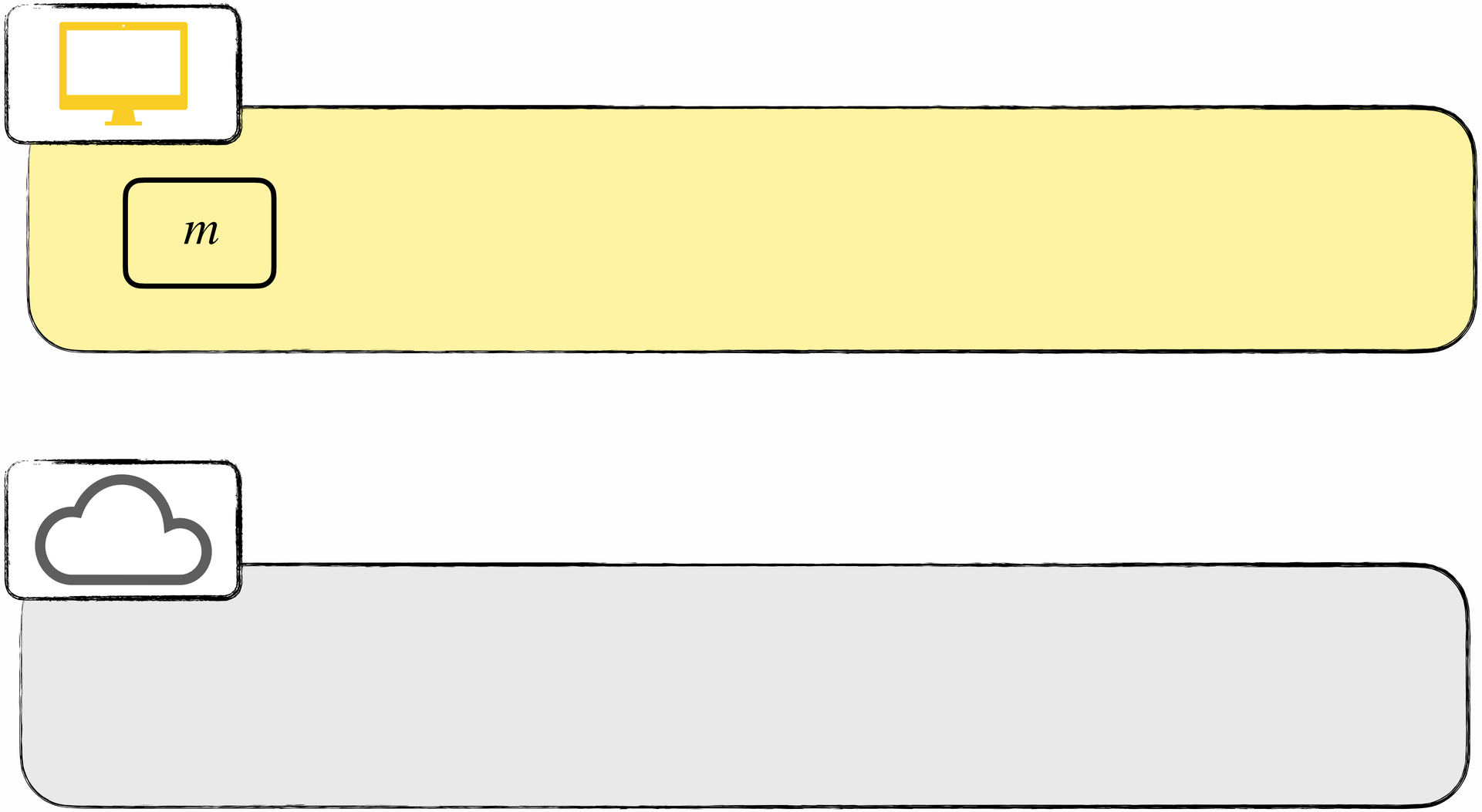

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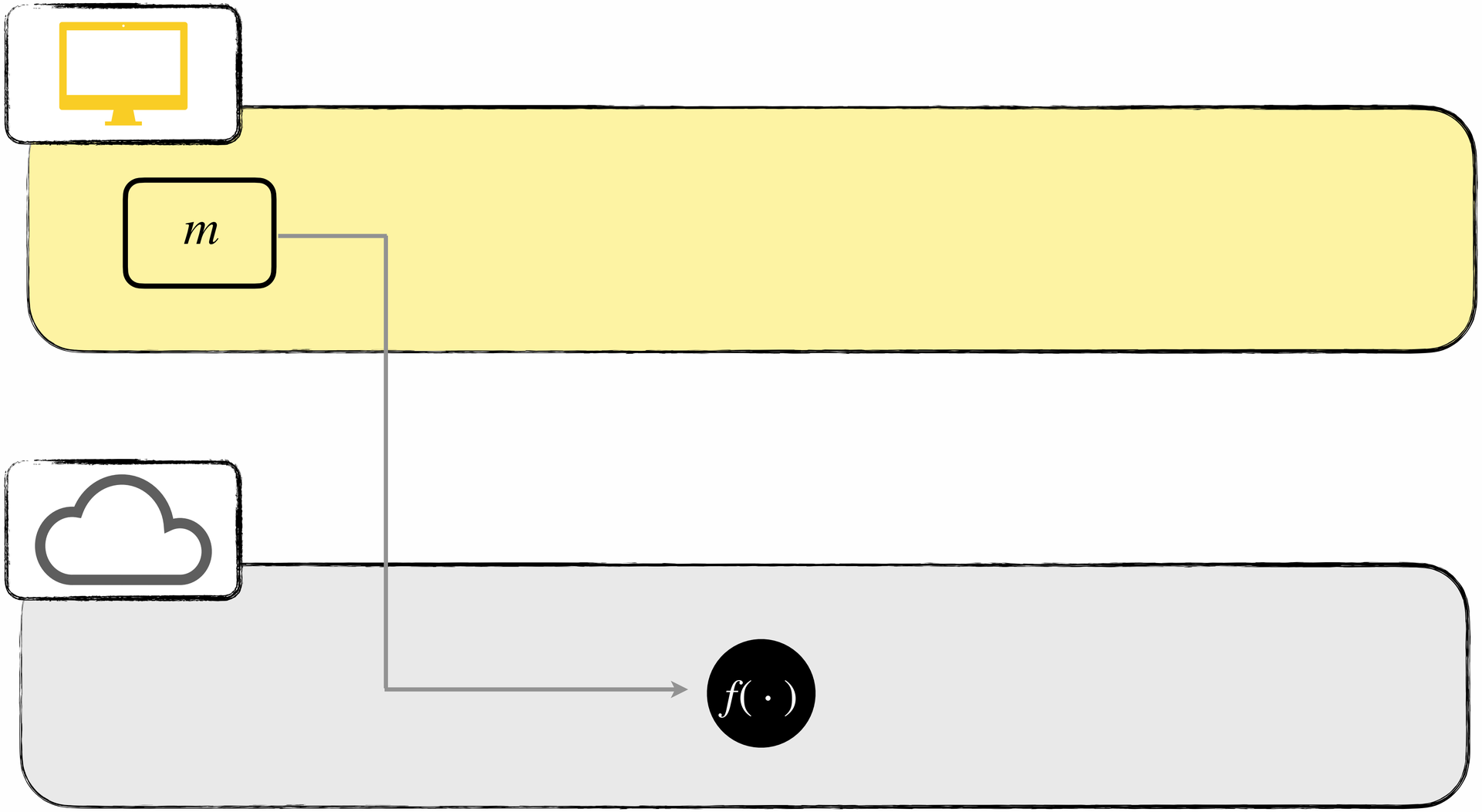

Past

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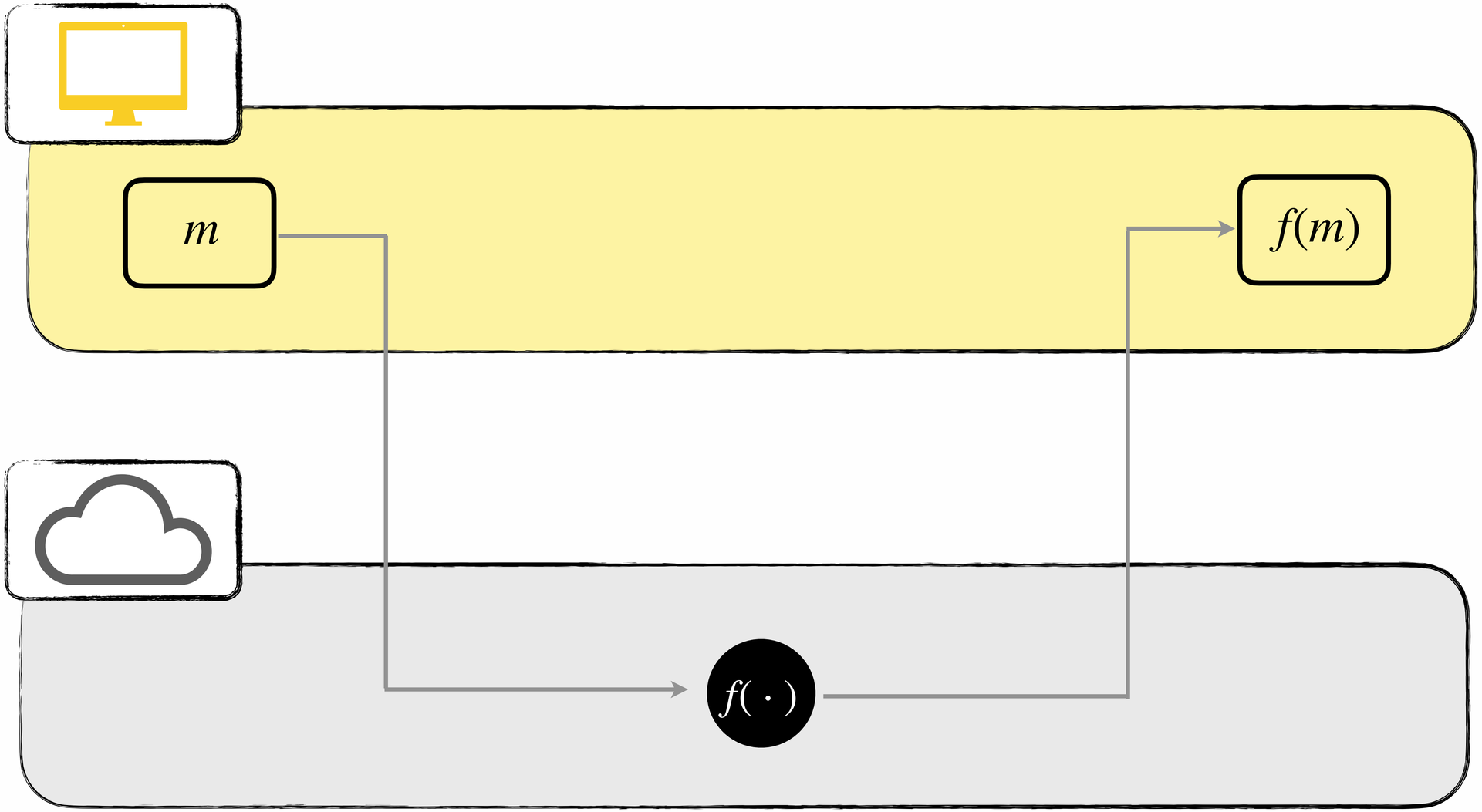

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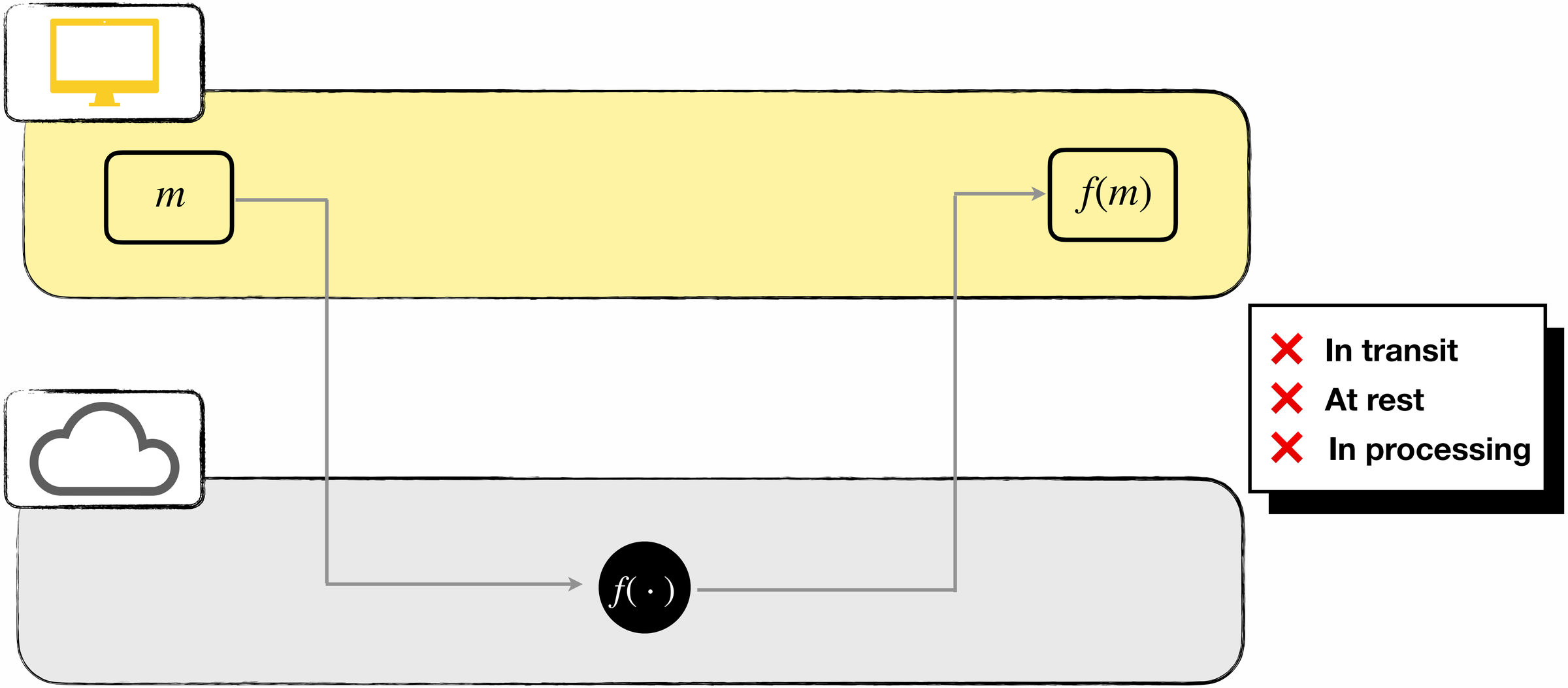

Past Present

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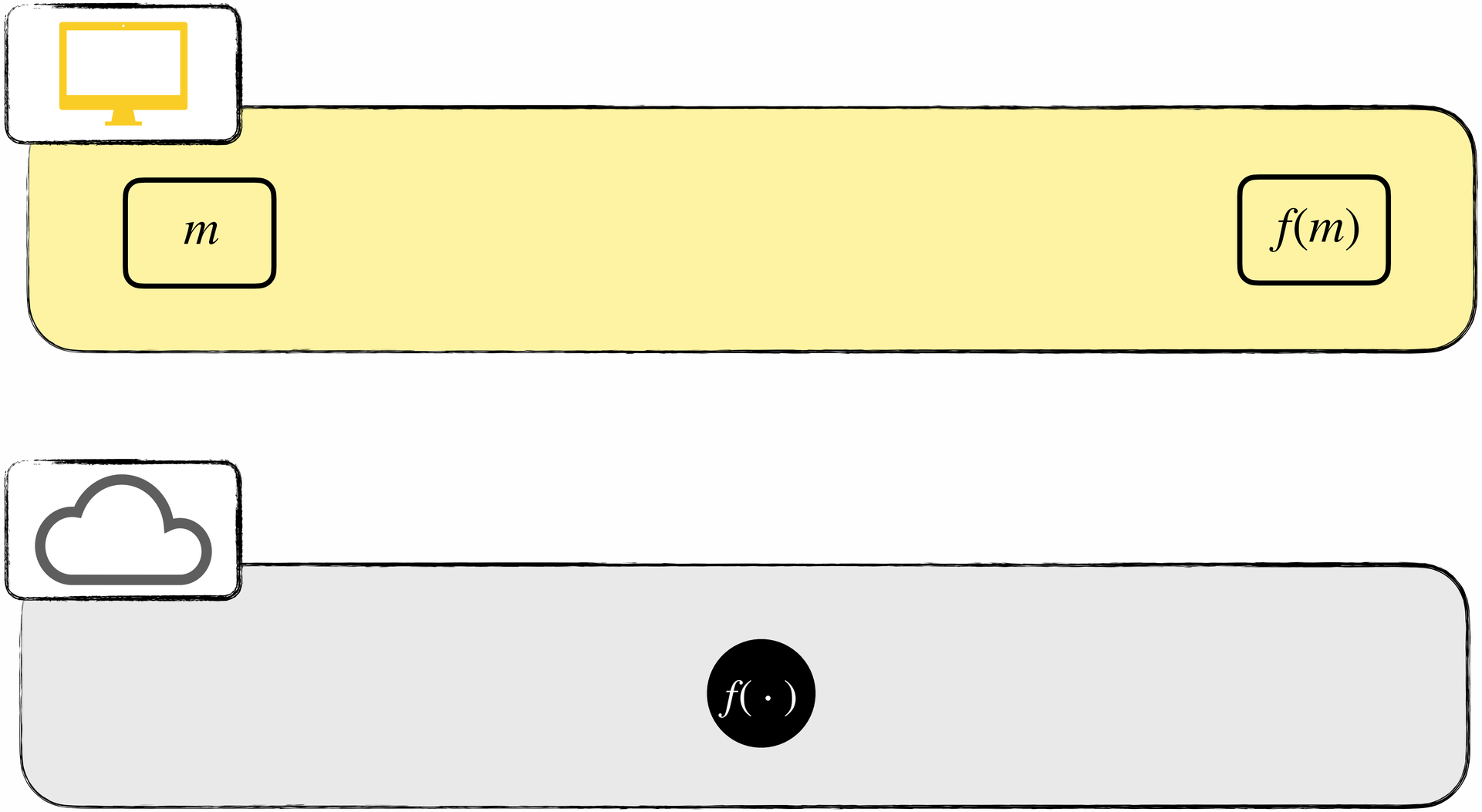

Past Present

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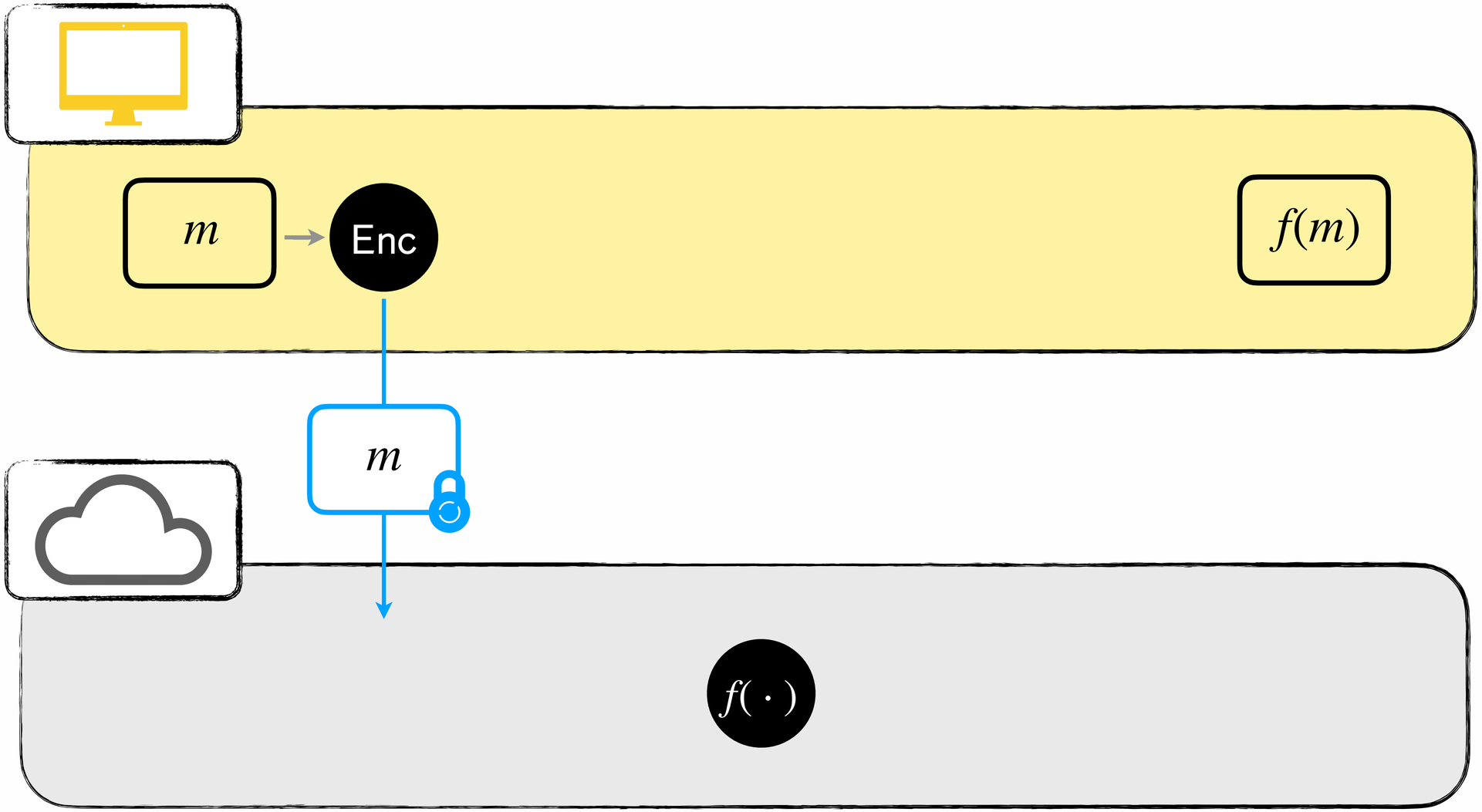

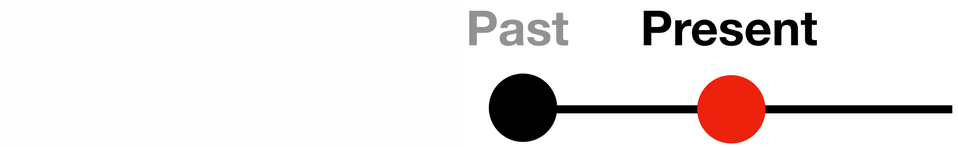


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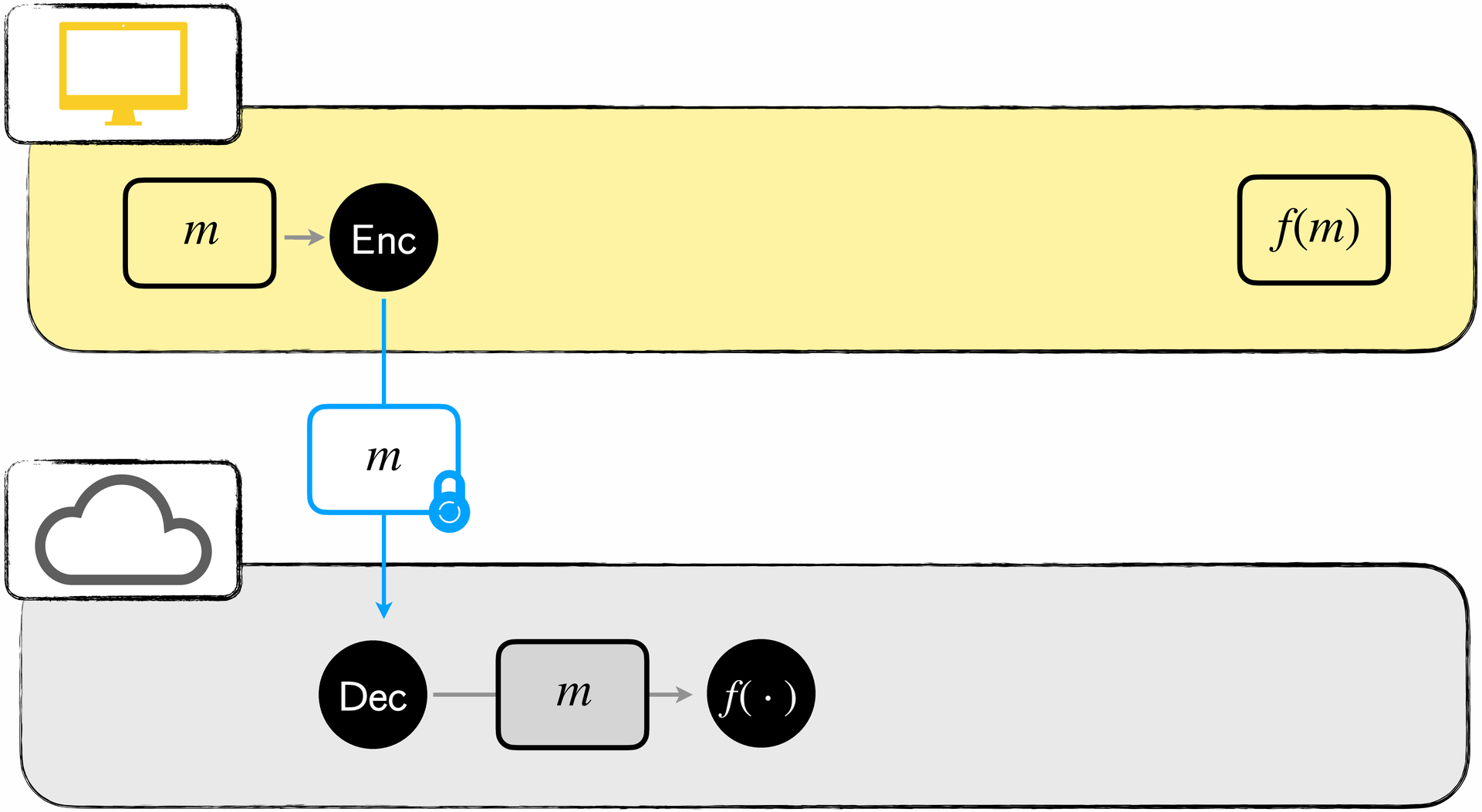

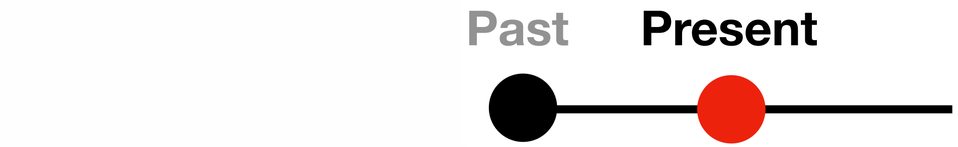


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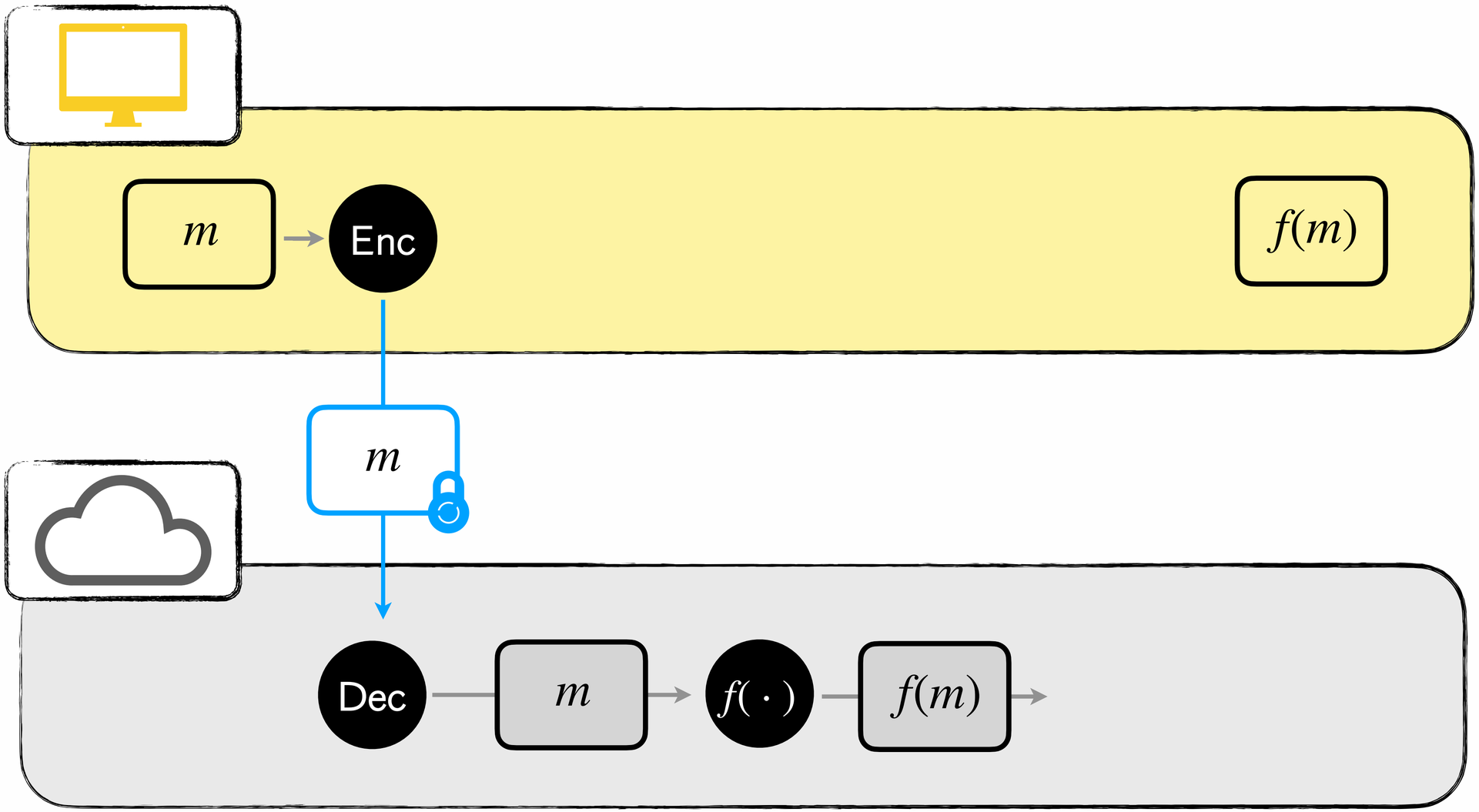

Past Present

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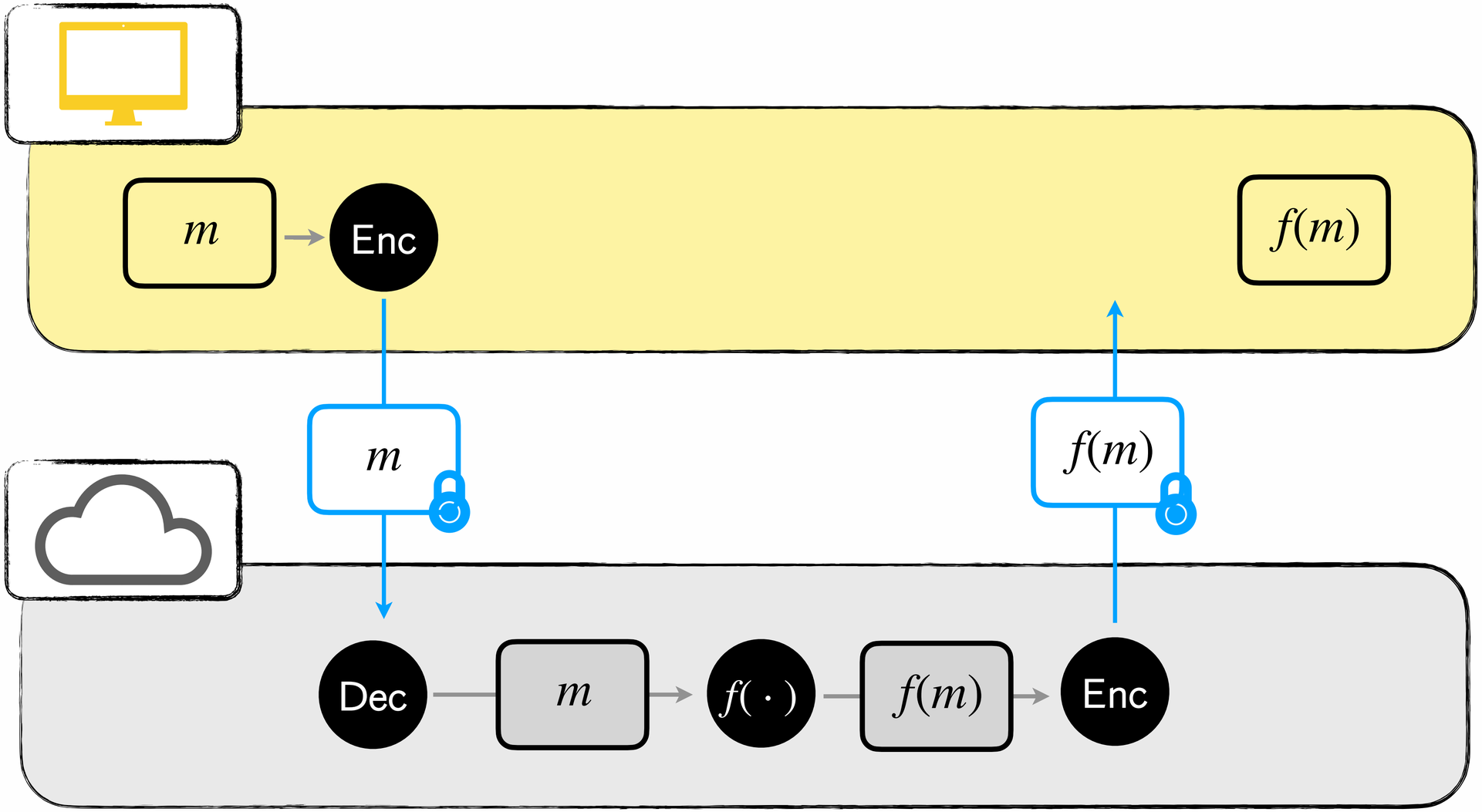

Past Present

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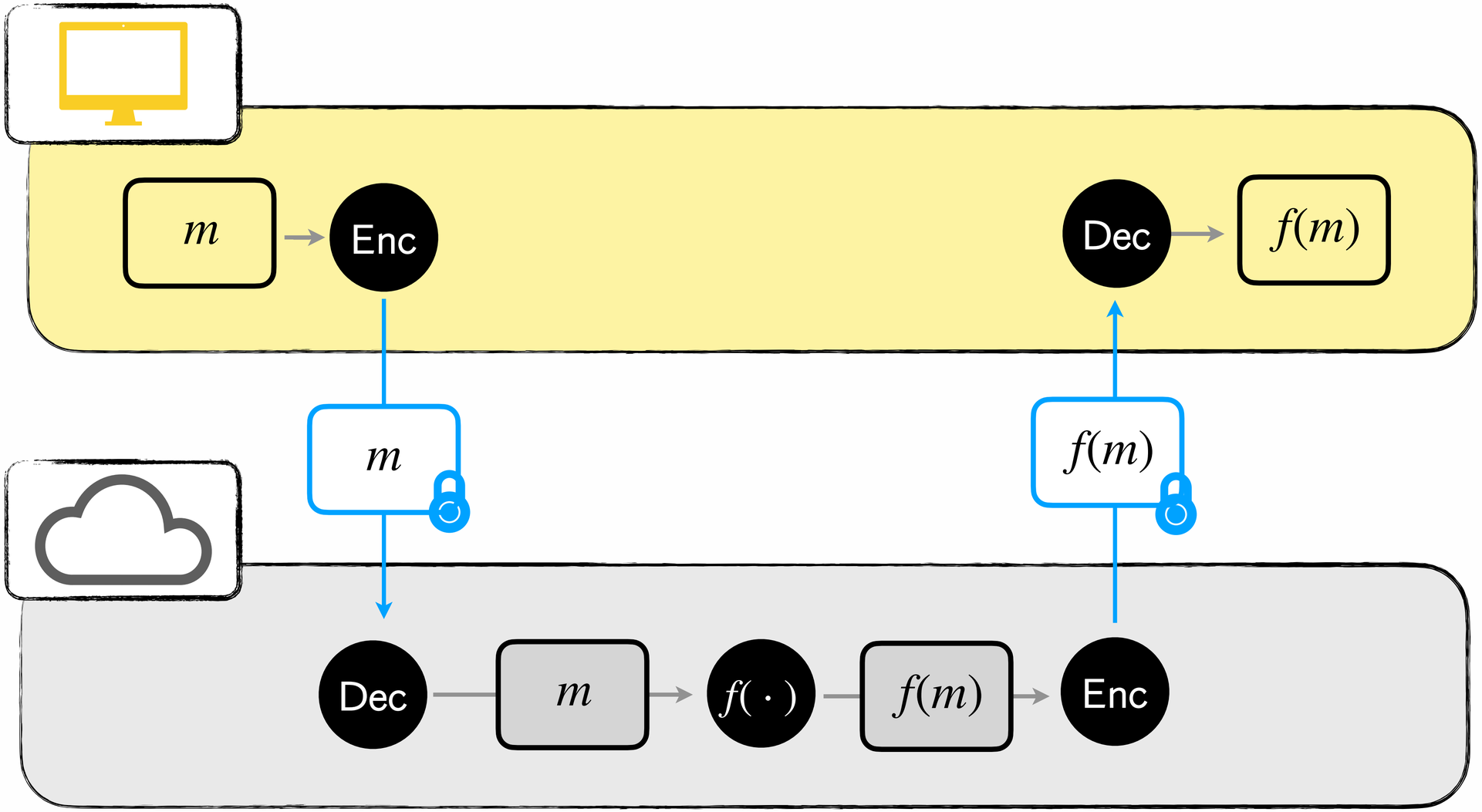
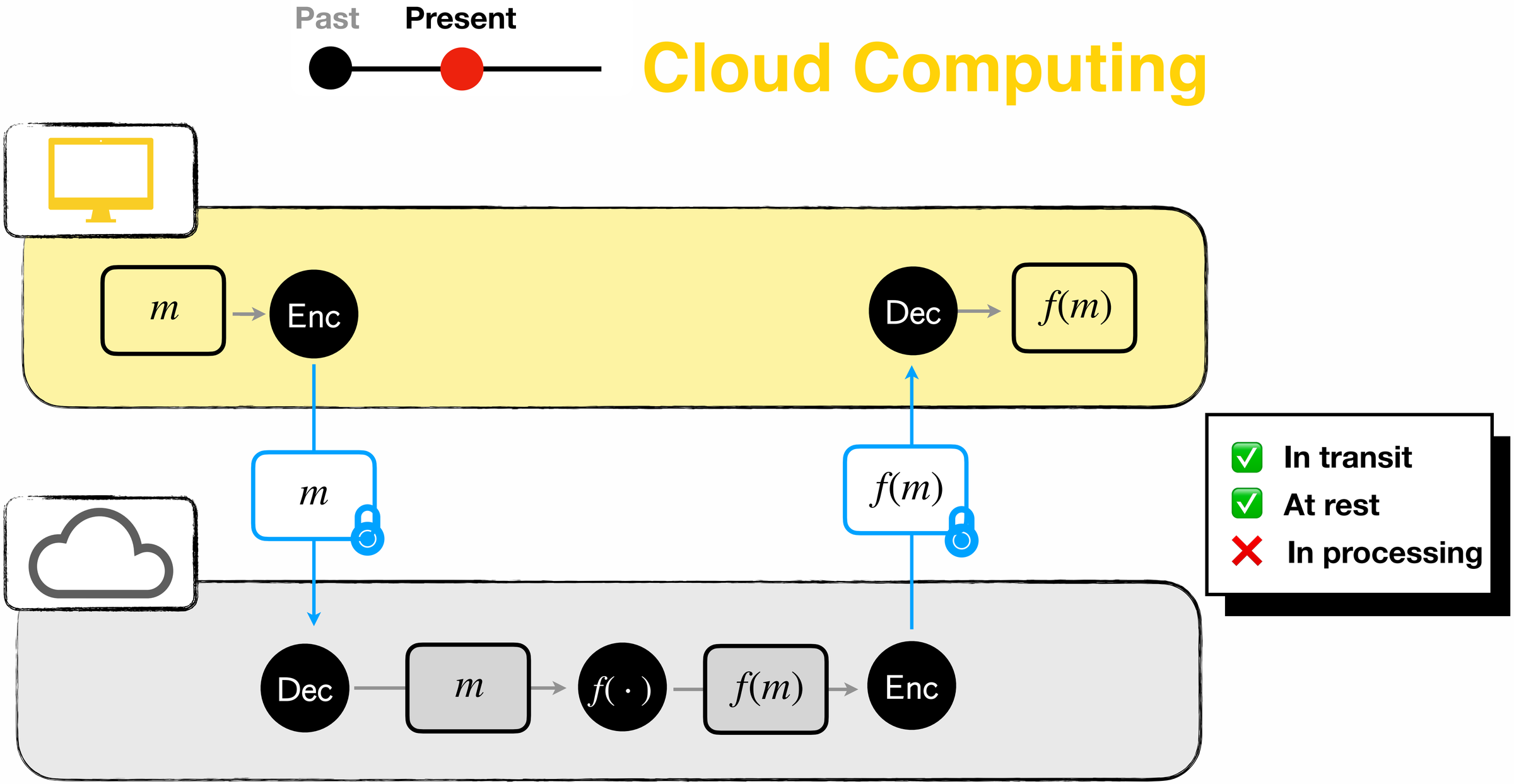
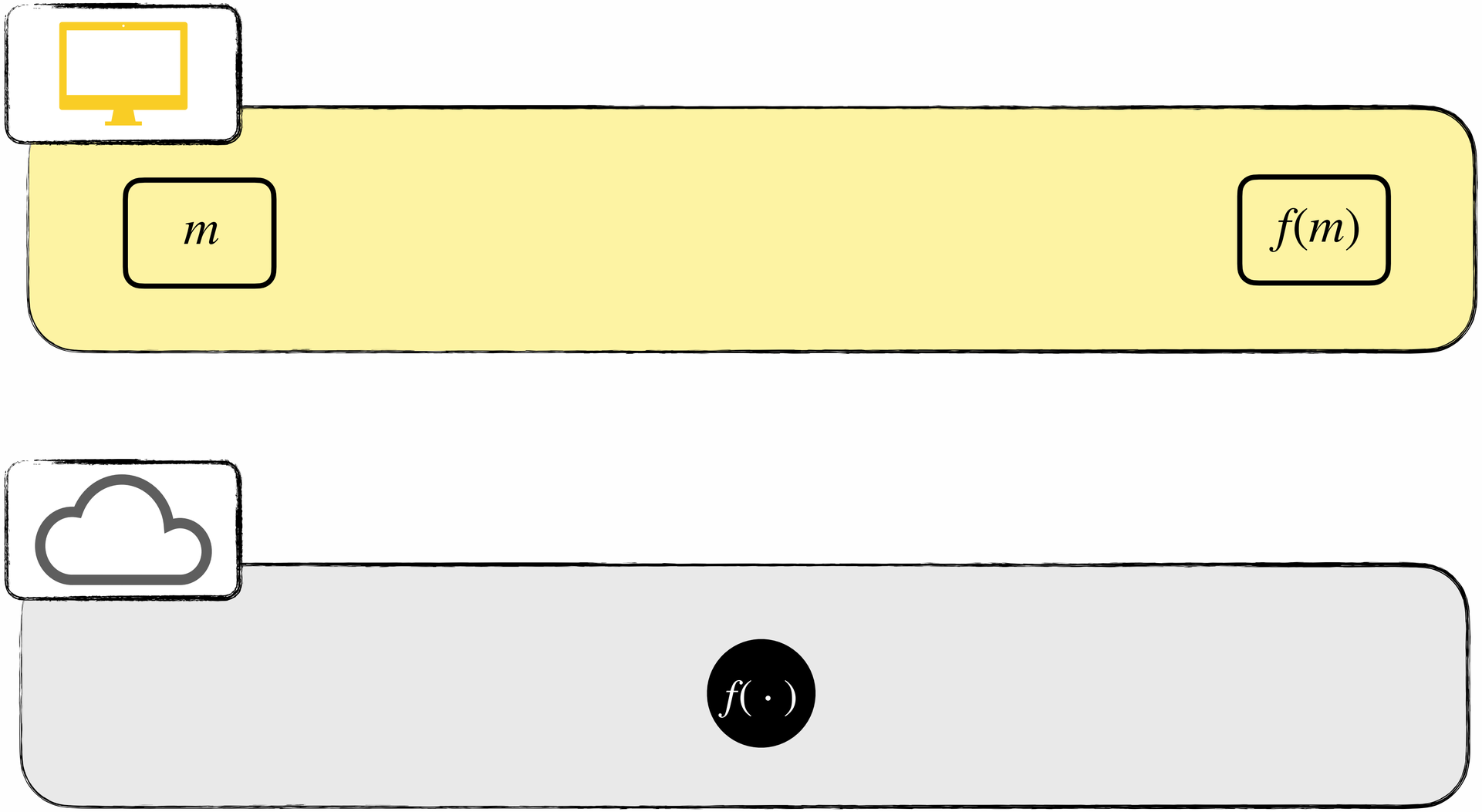



Past
Present
Cloud Computing
m
Enc
Dec
f(m)
m
f(m)
Dec
m
f( · )
f(m)
Enc
In transit
At rest
In processing

Past Present FHE

# Cloud Computing



Past Present FHE

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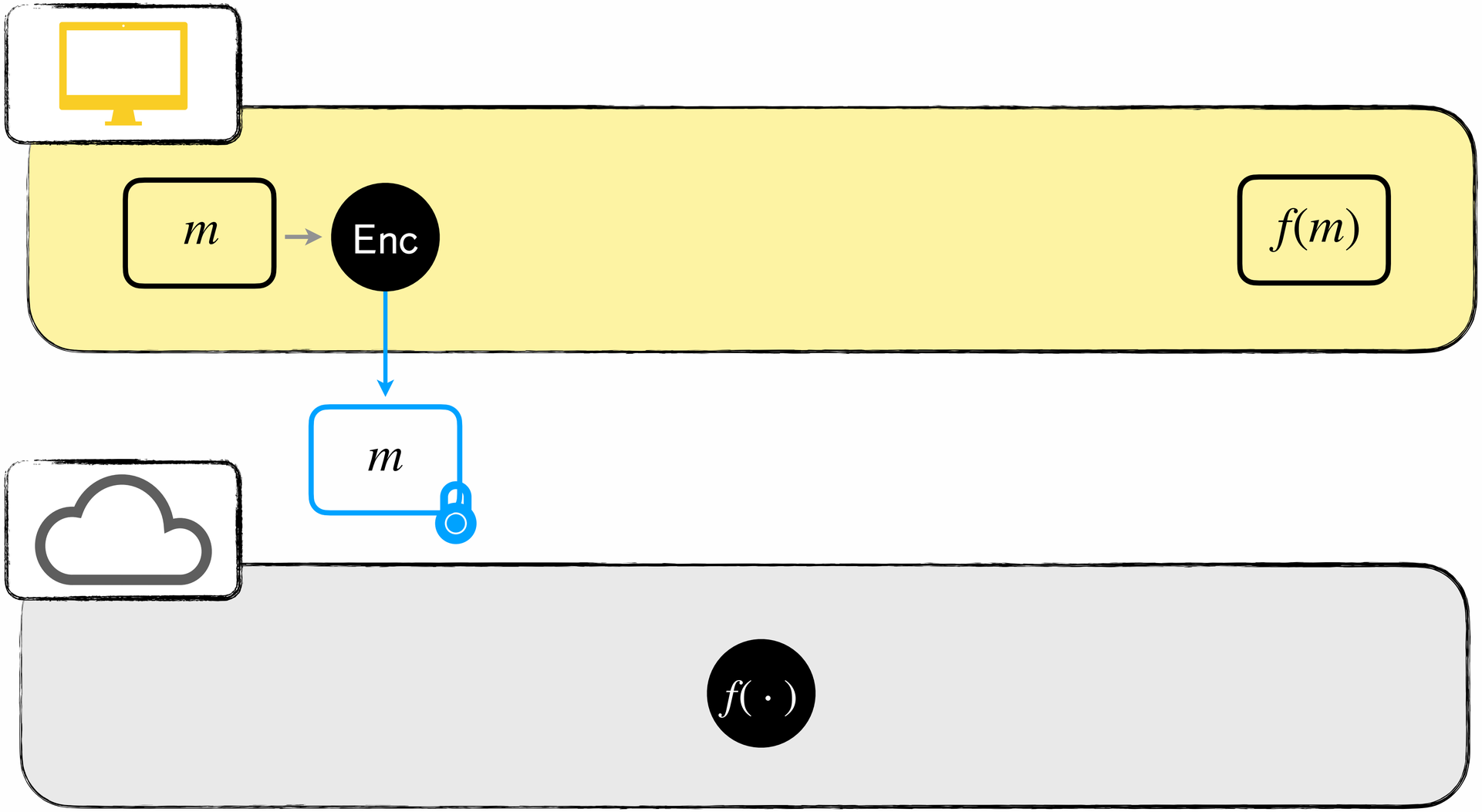

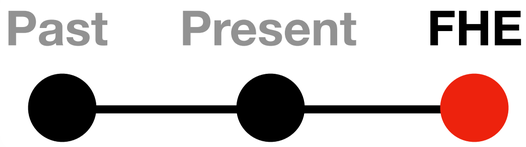


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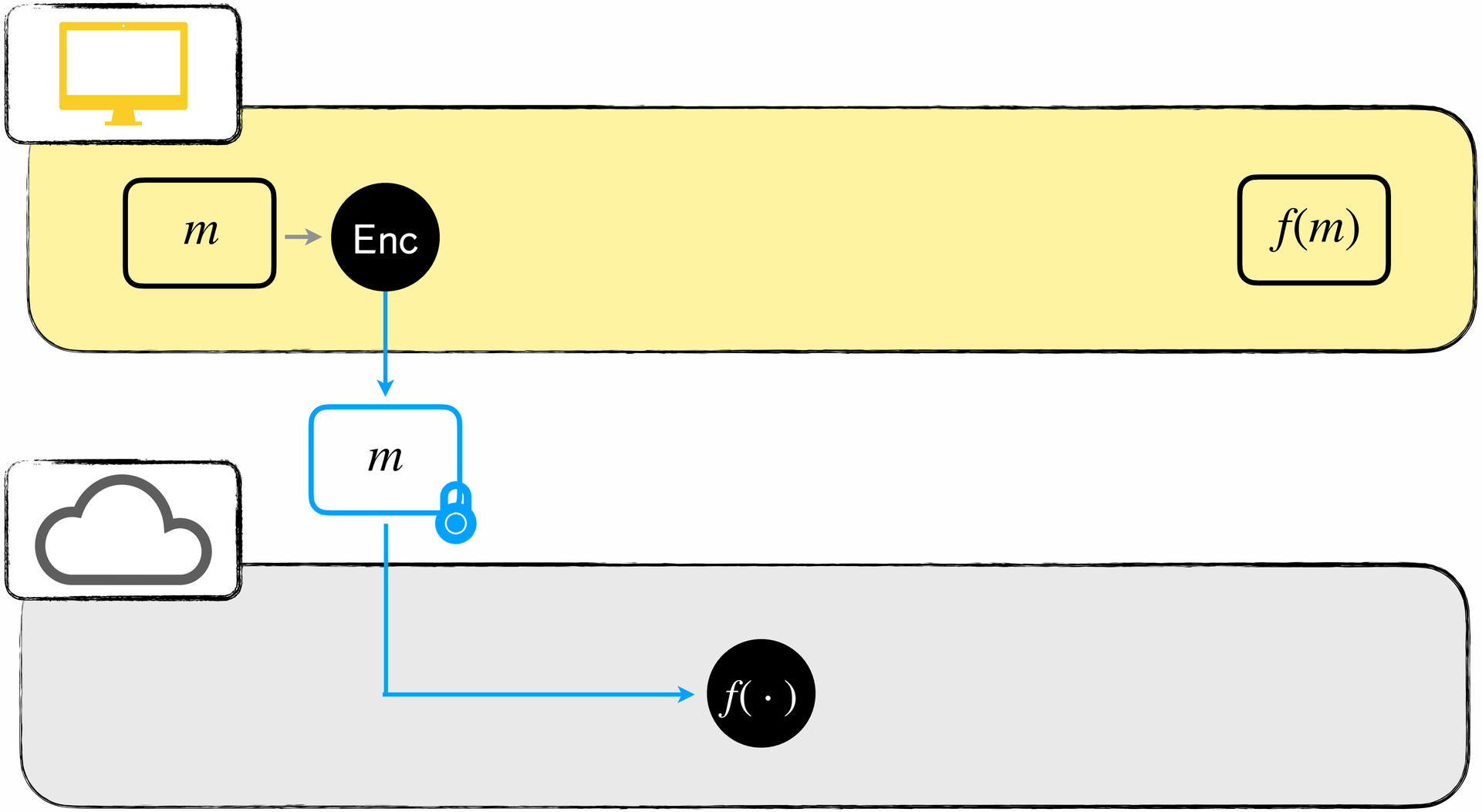

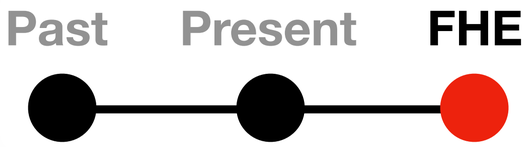


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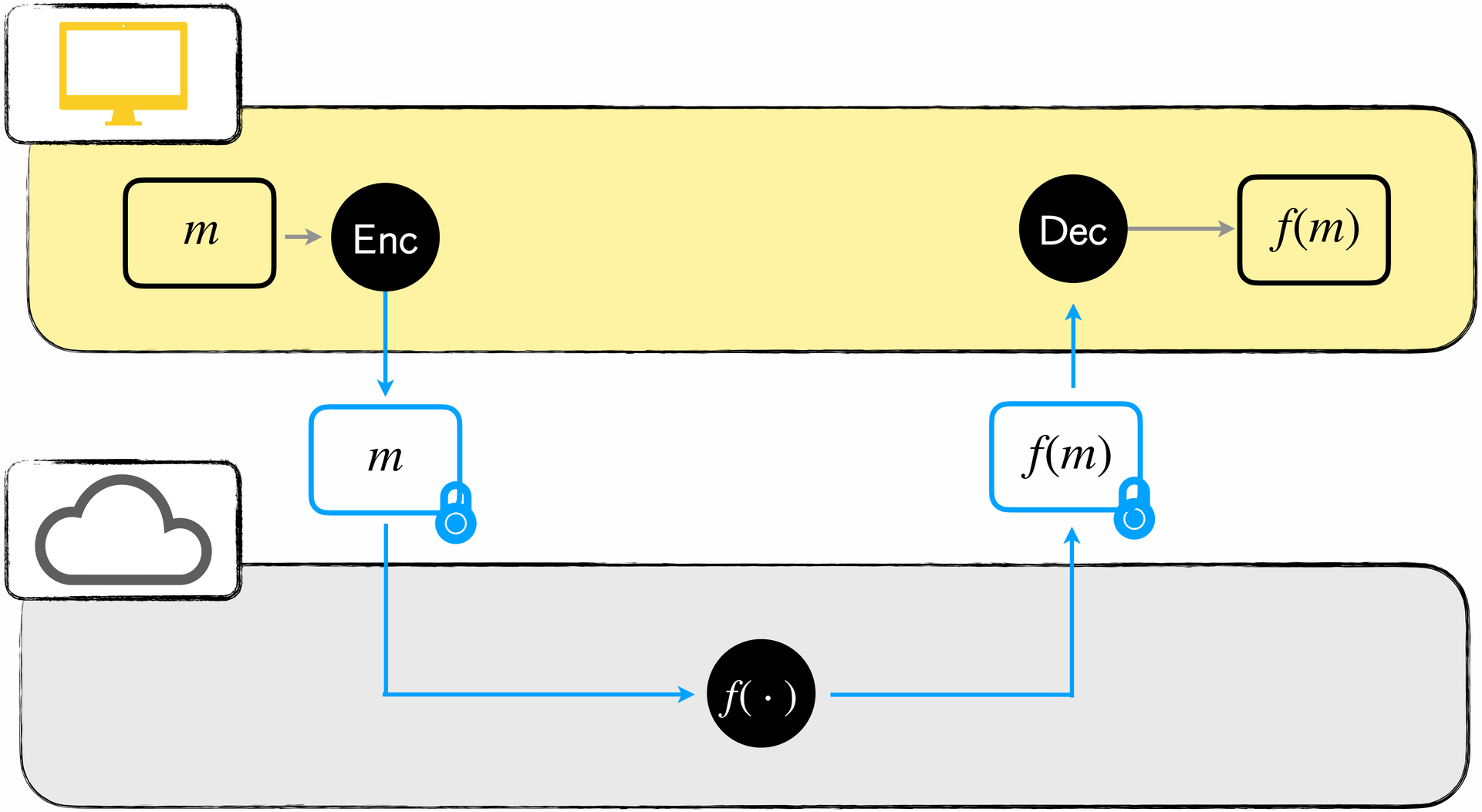

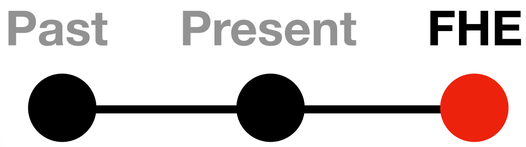


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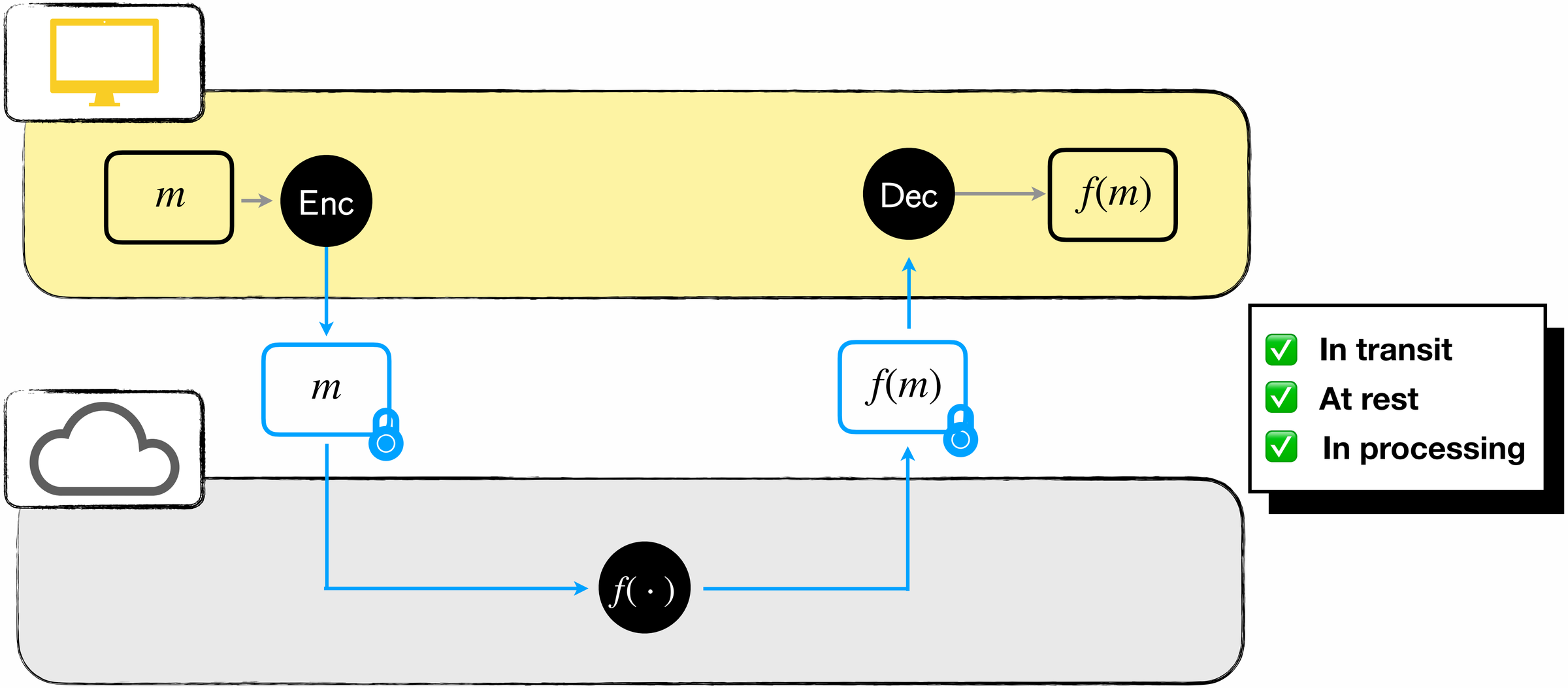
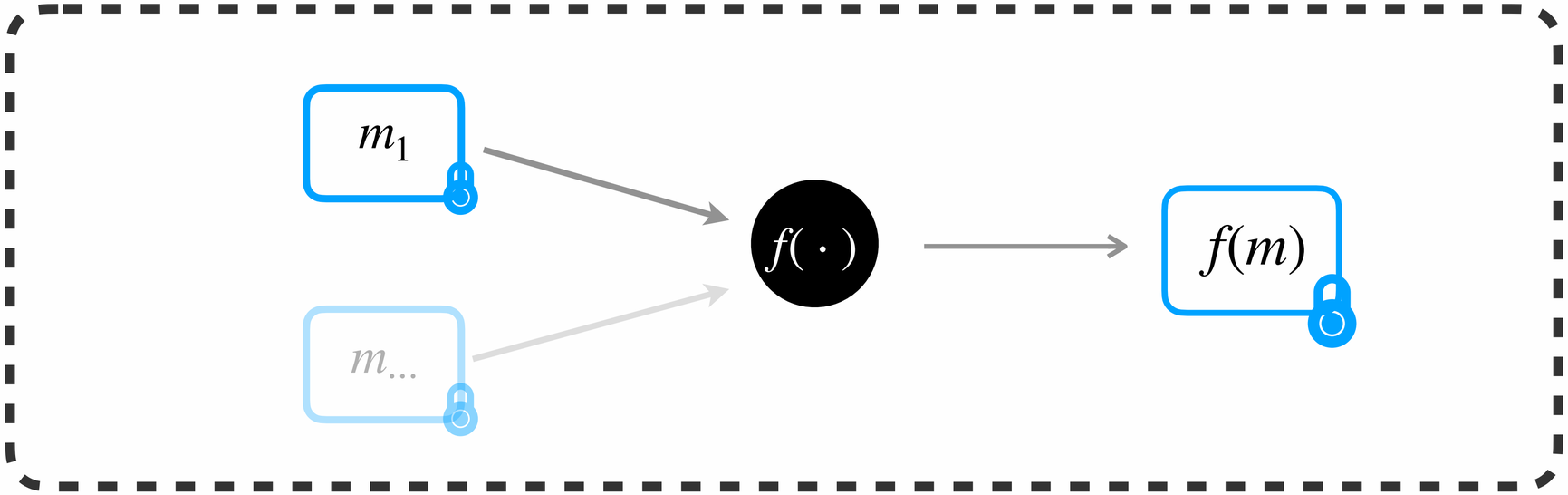



- ✅ In transit
- ✅ At rest
- ✅ In processing

# FHE



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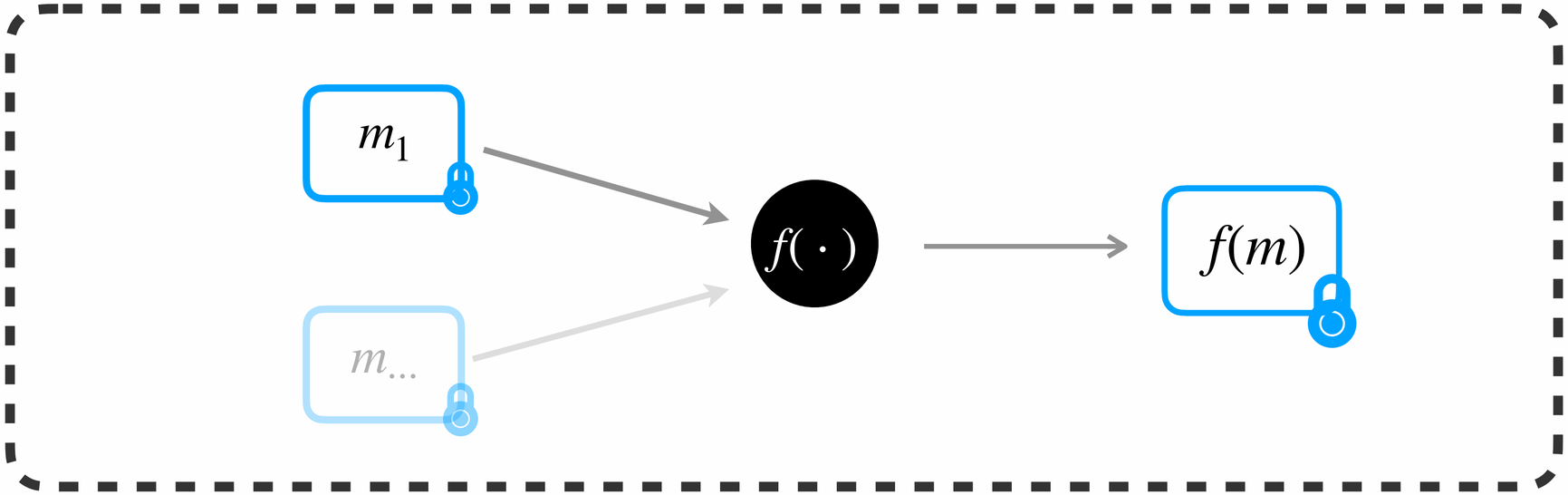

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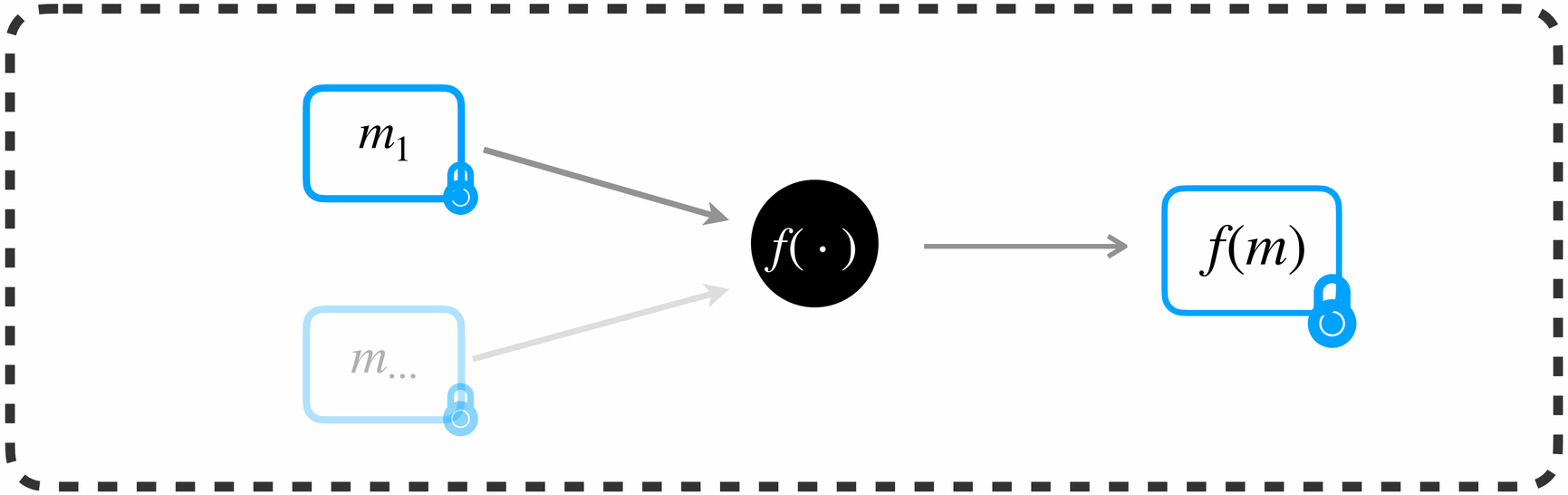
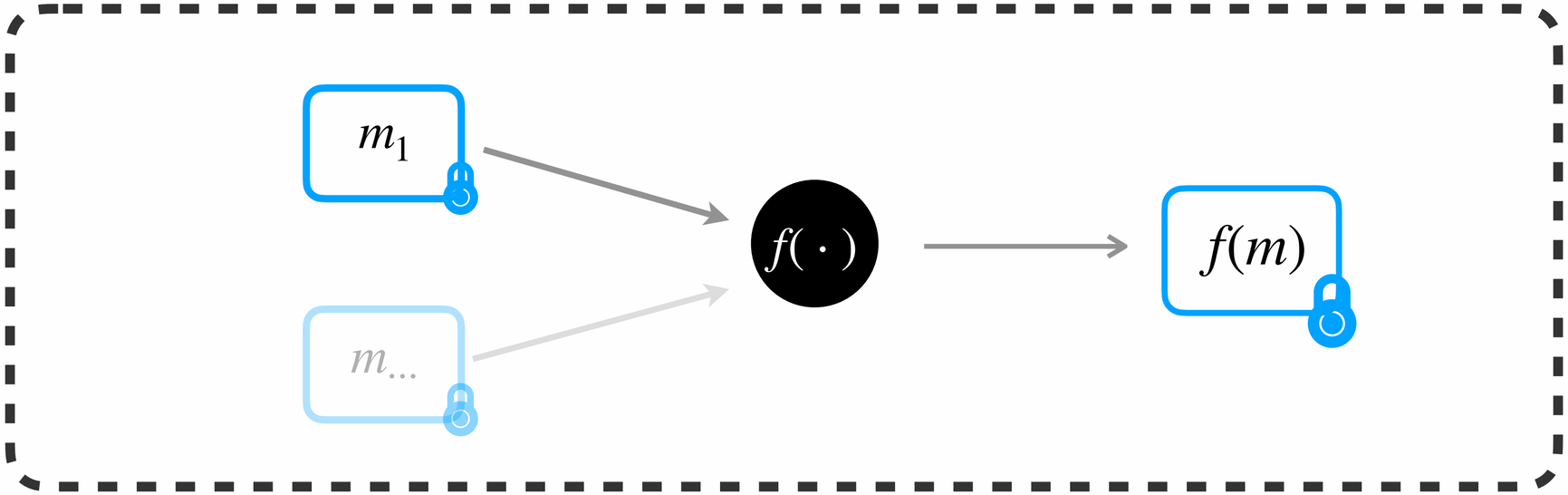



$f$, a boolean circuit

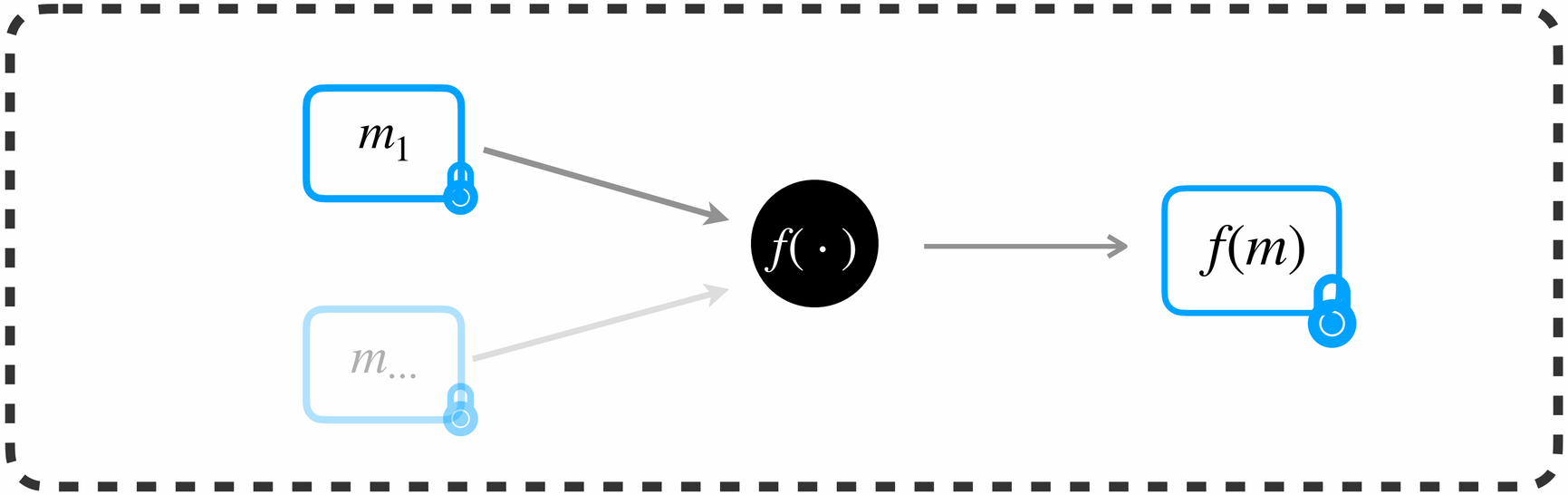
# FHE




$f$, a boolean circuit

$f$, a neural network

# FHE

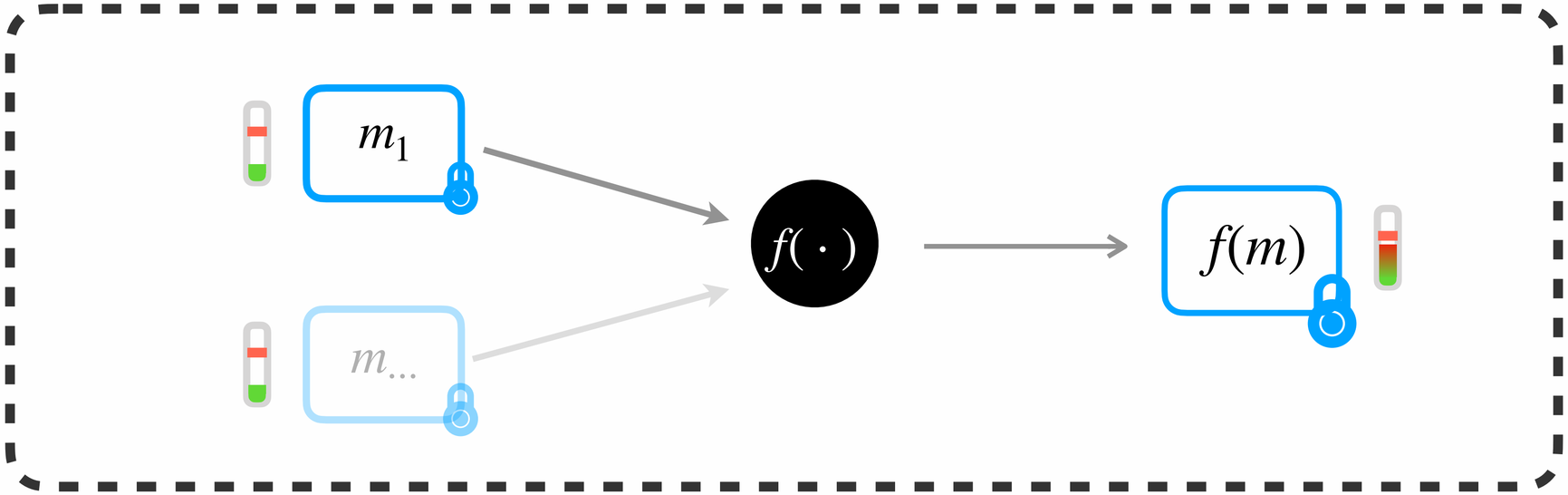



$f$, a boolean circuit

$f$, a neural network

$f$, any function

# FHE



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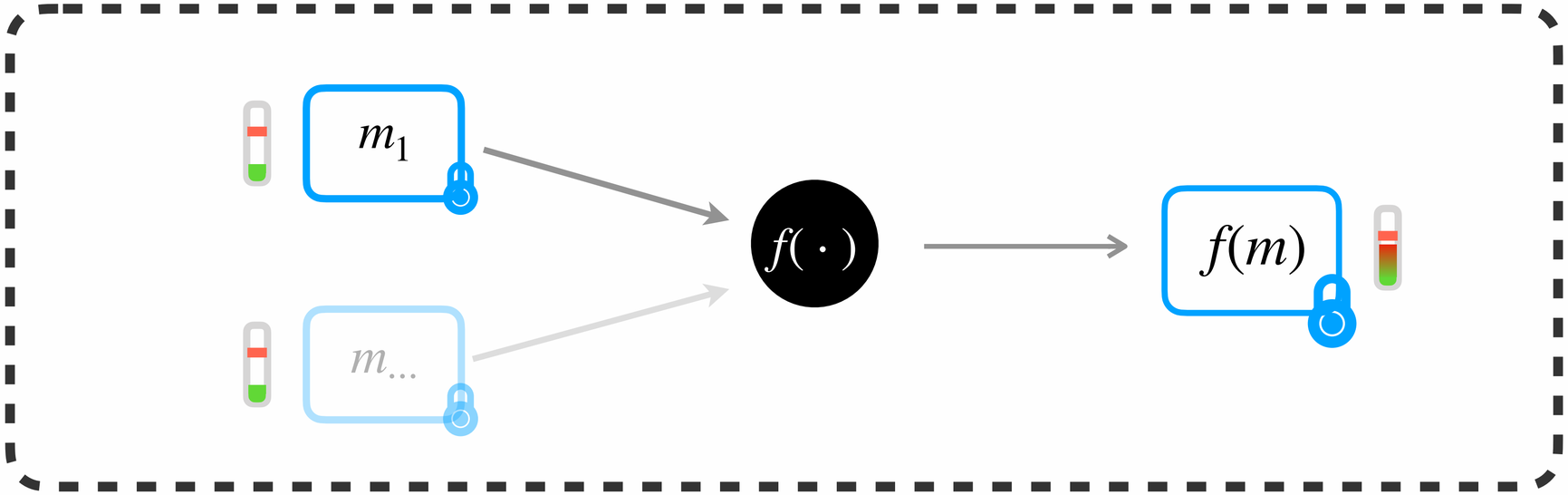

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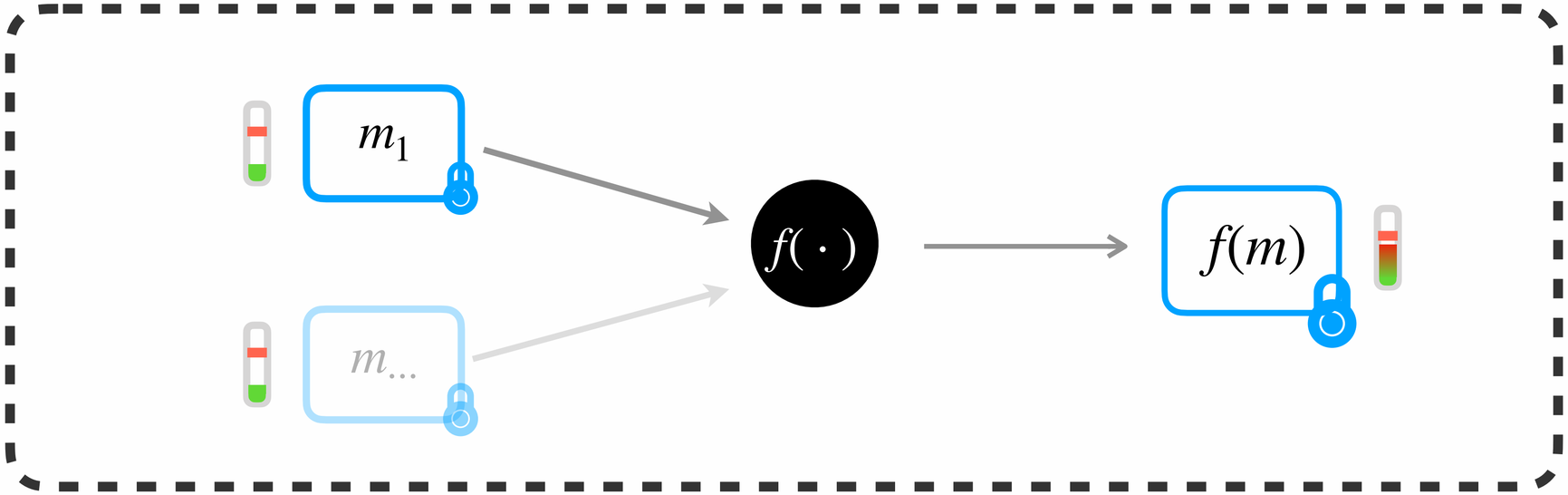
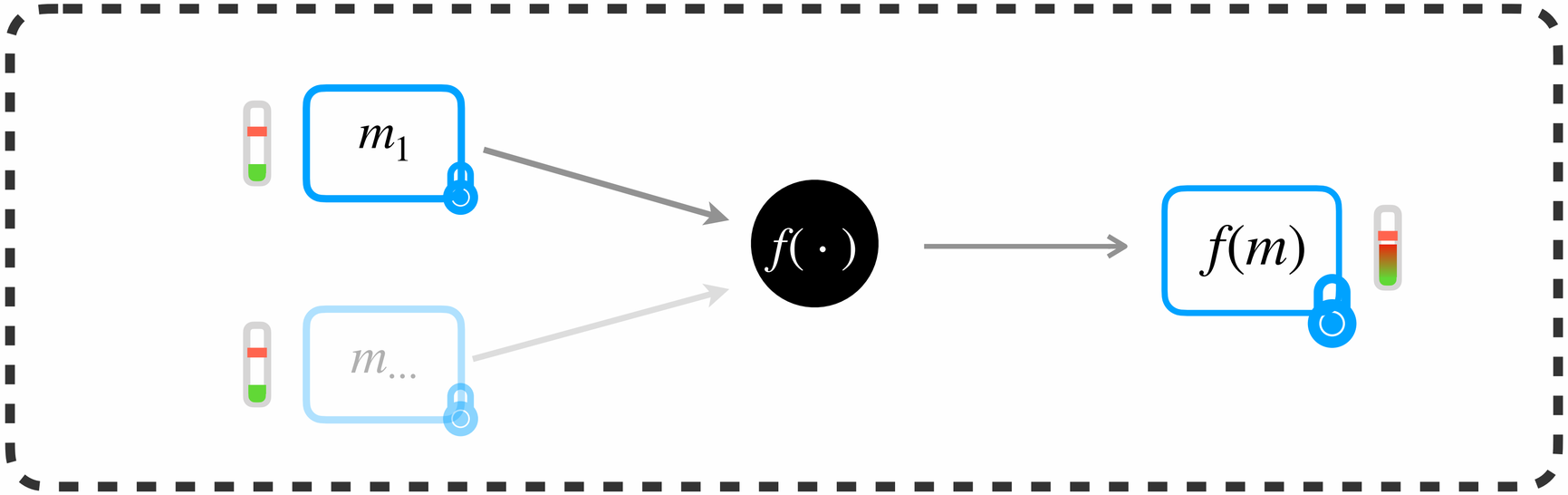



**Most operations increase the noise**

# FHE




Most operations increase the noise

Too much noise leads to incorrect decryption

# Leveled Homomorphic Encryption

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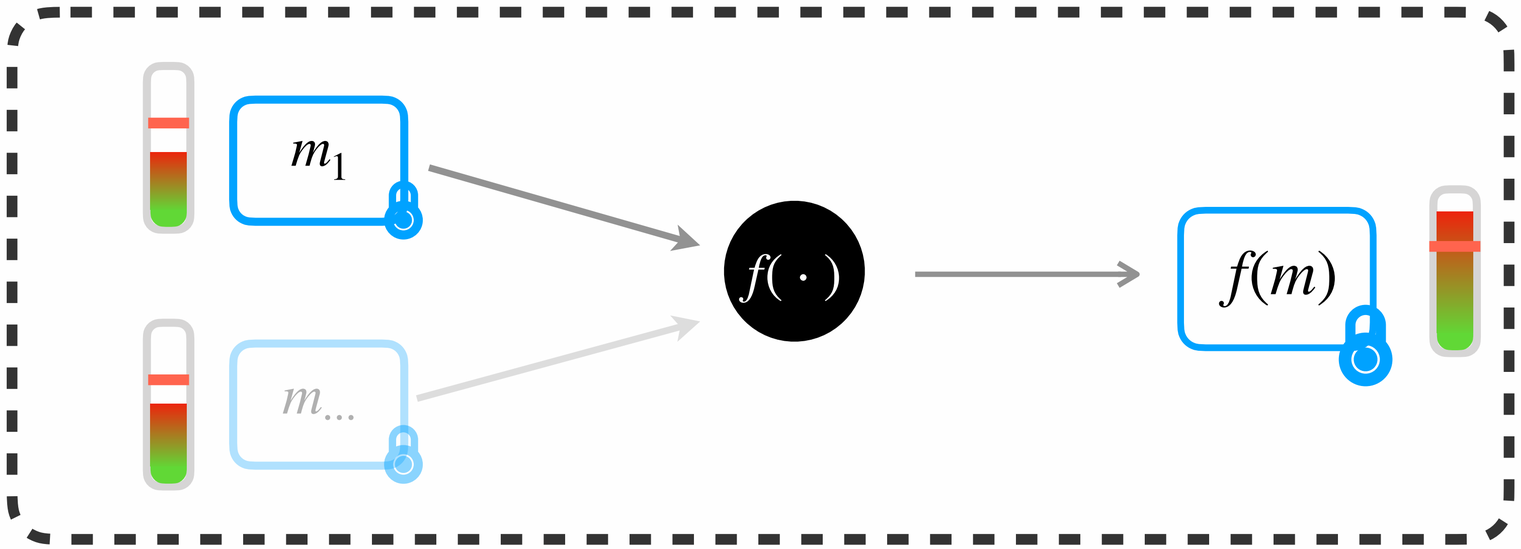

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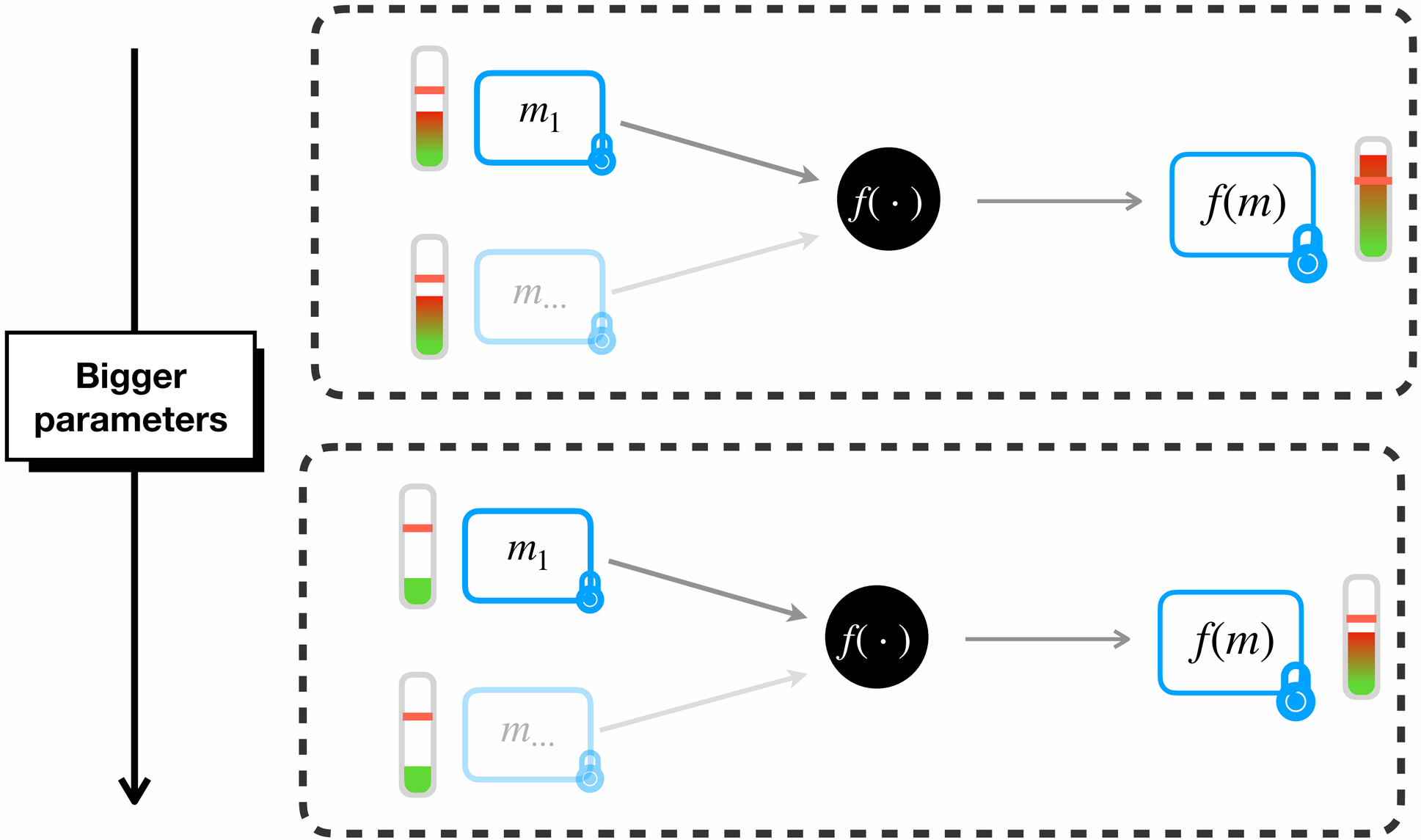

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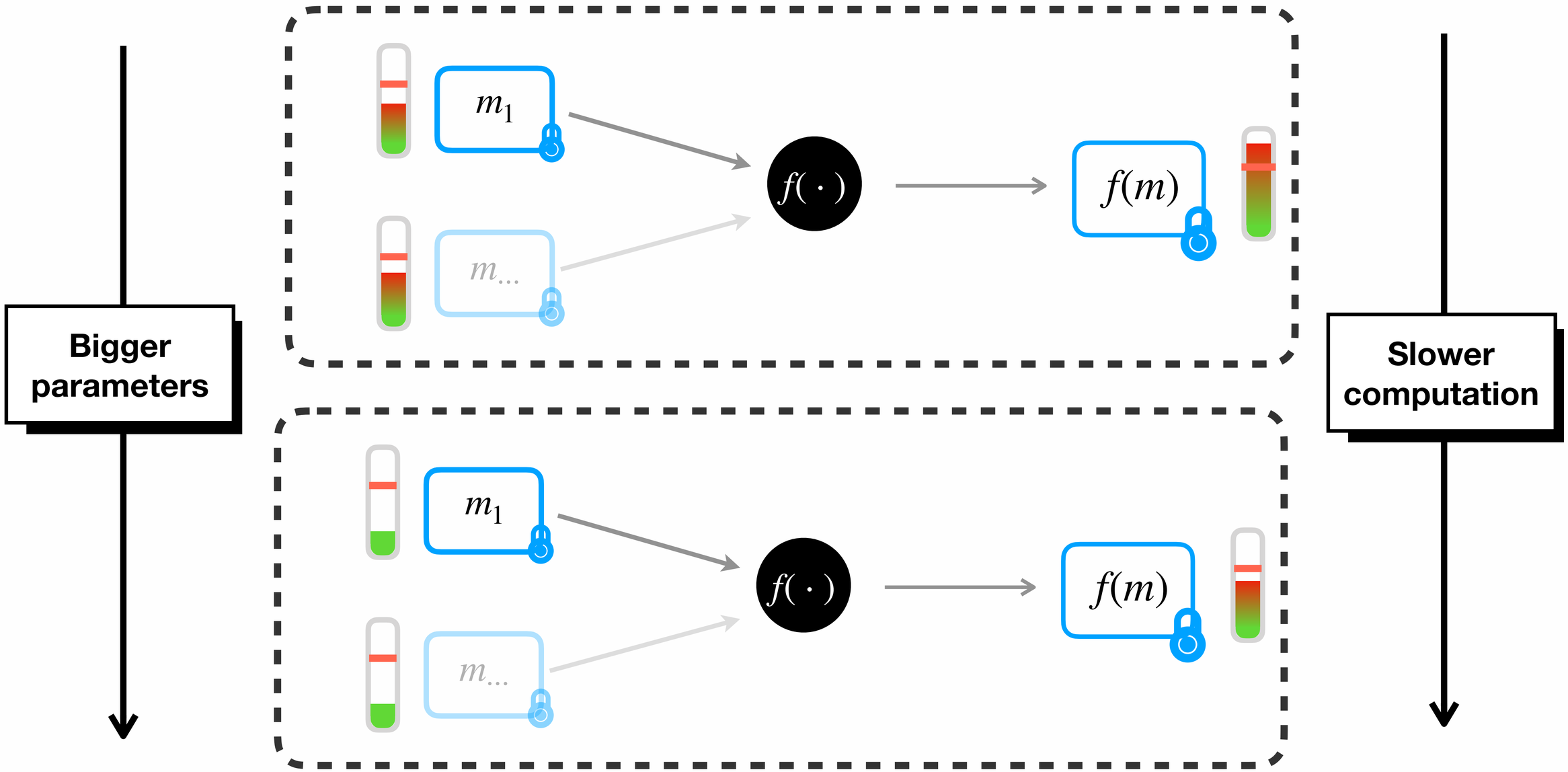

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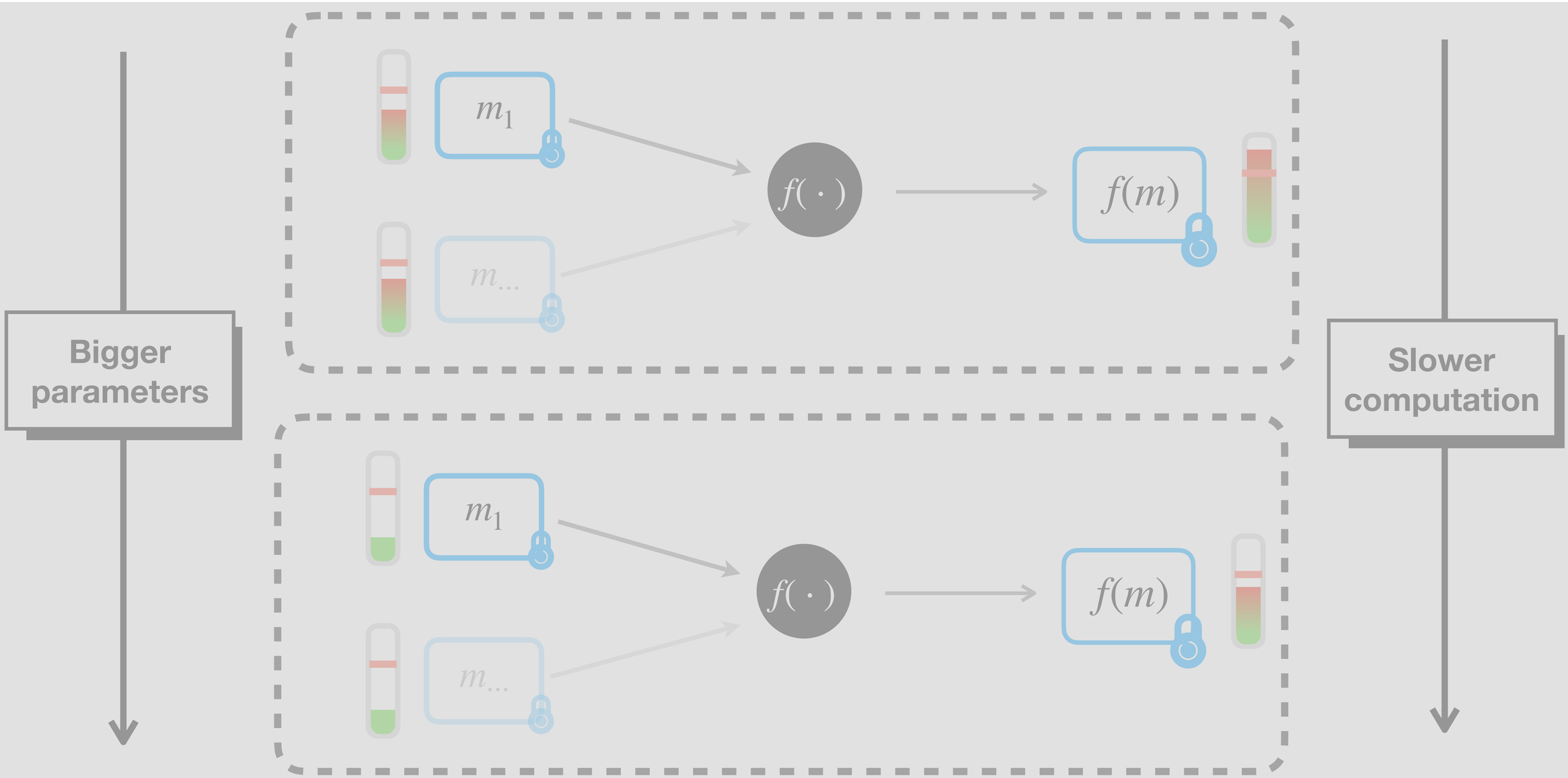

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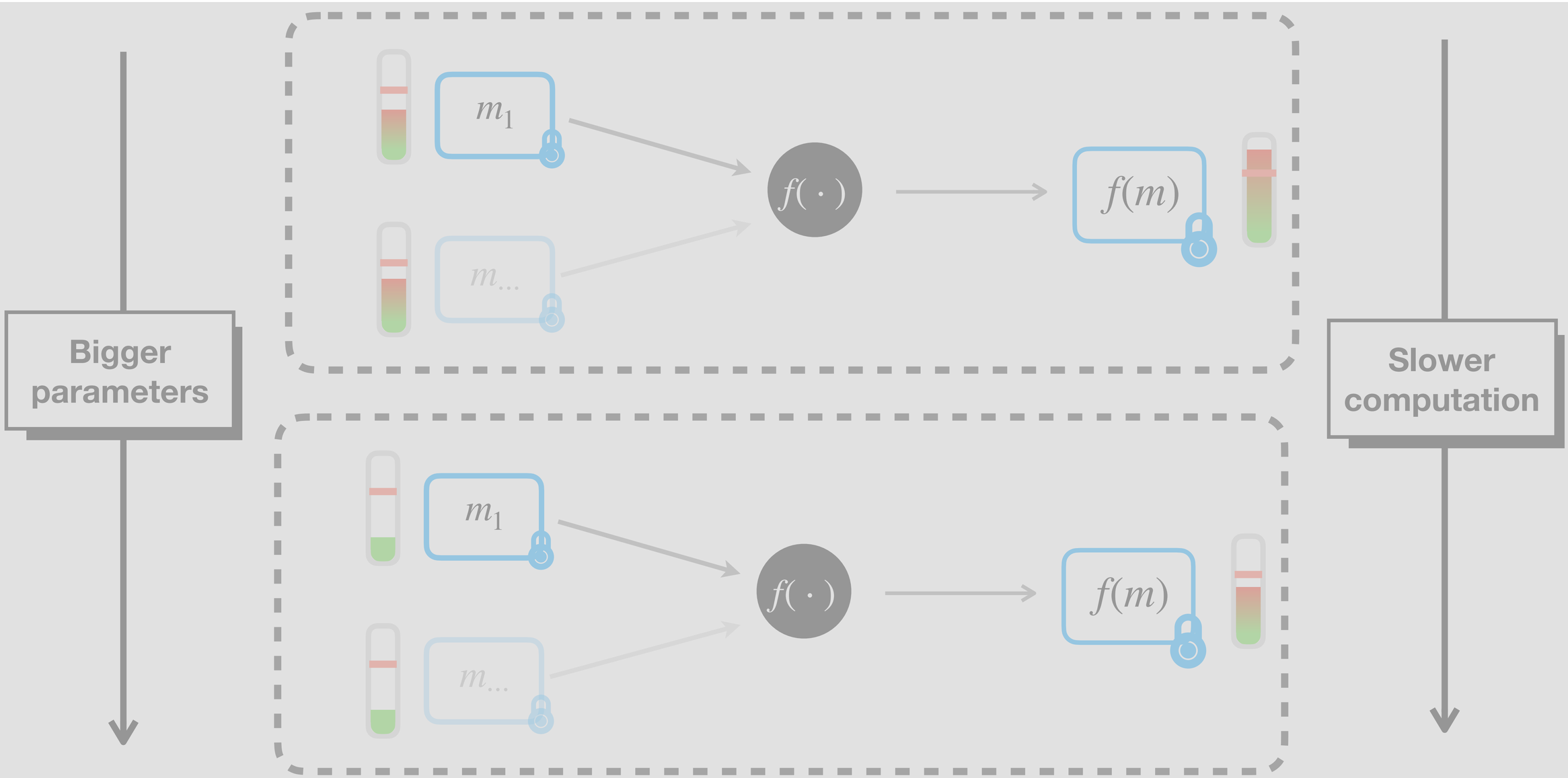

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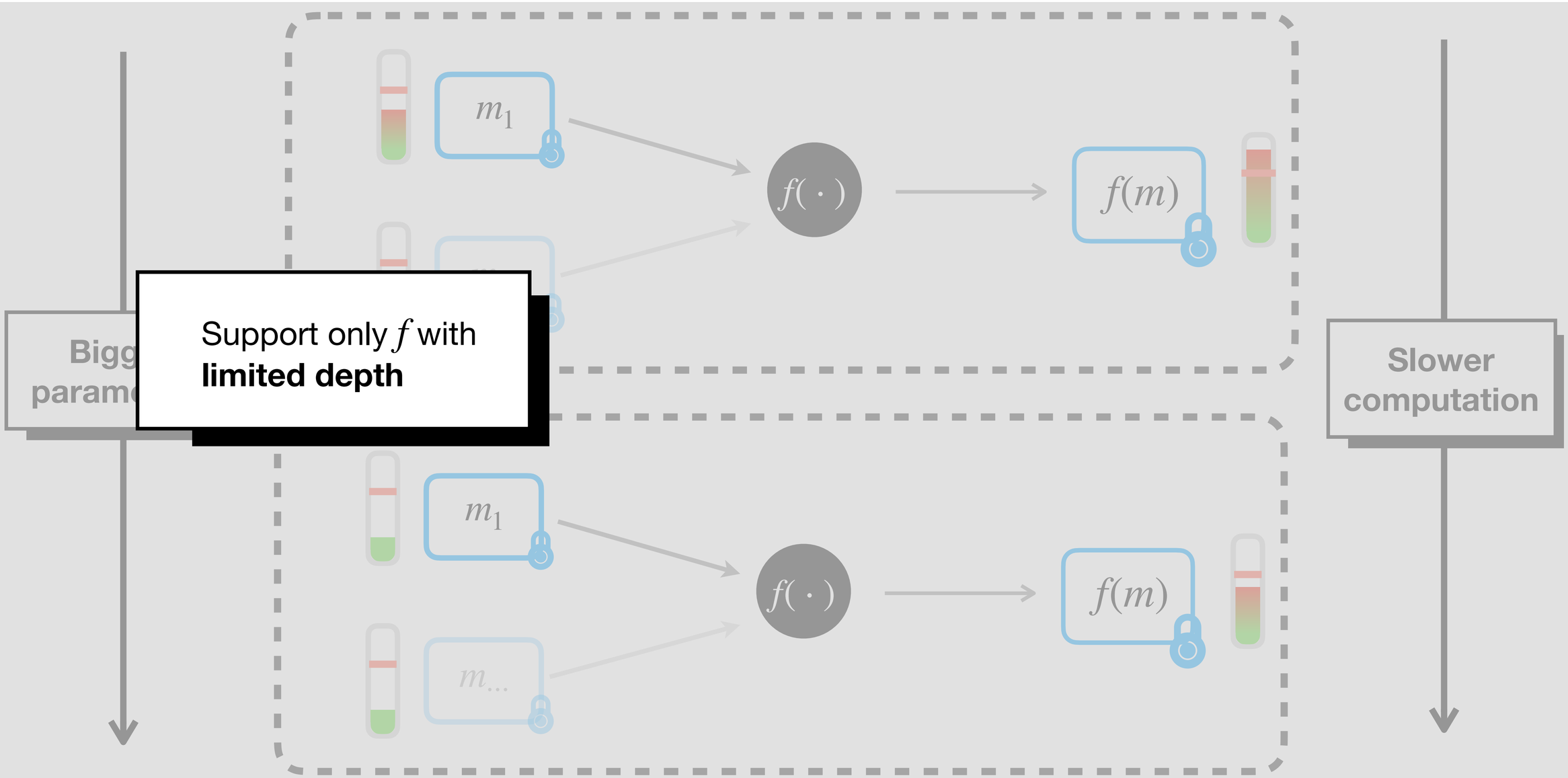

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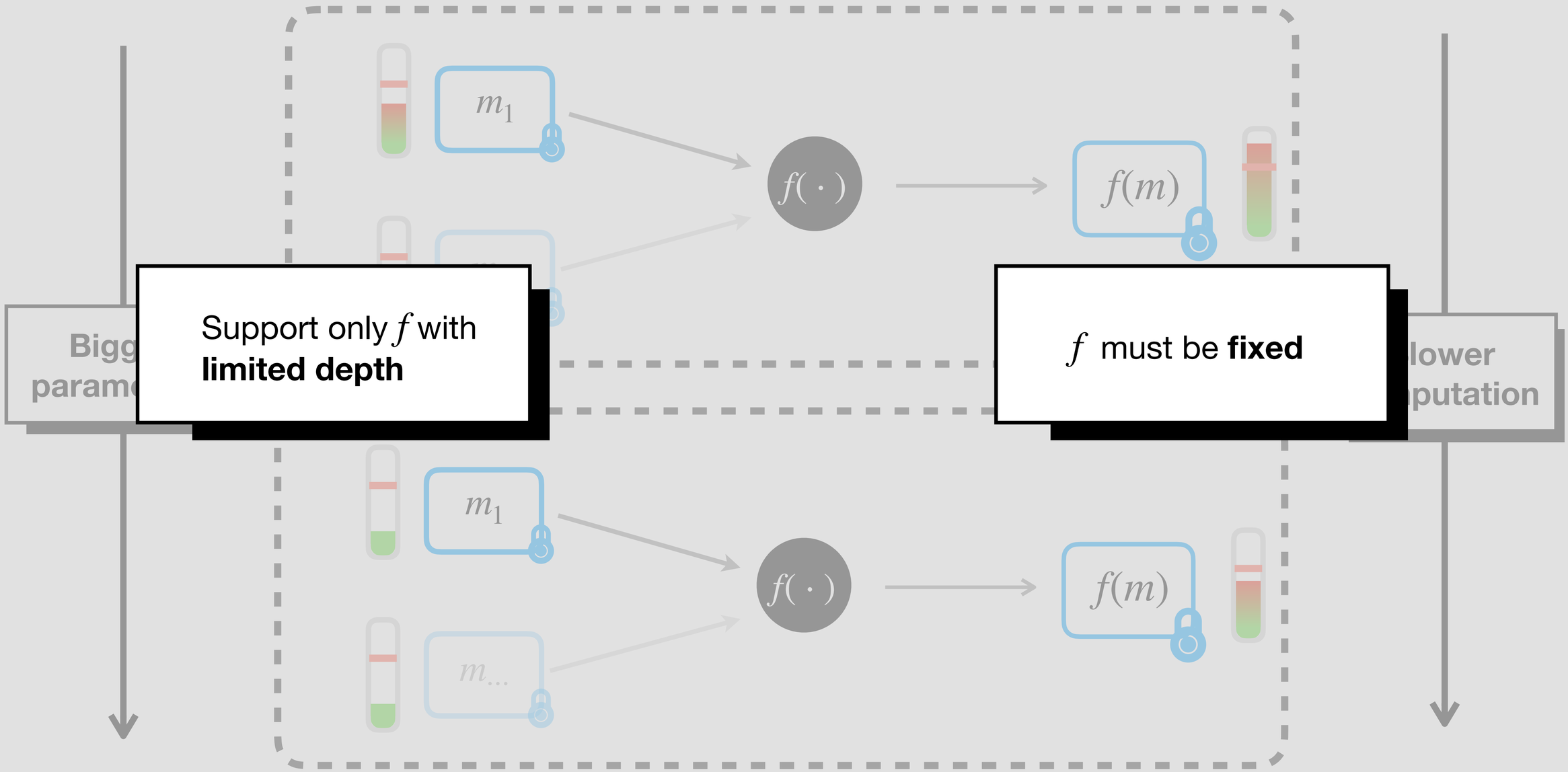
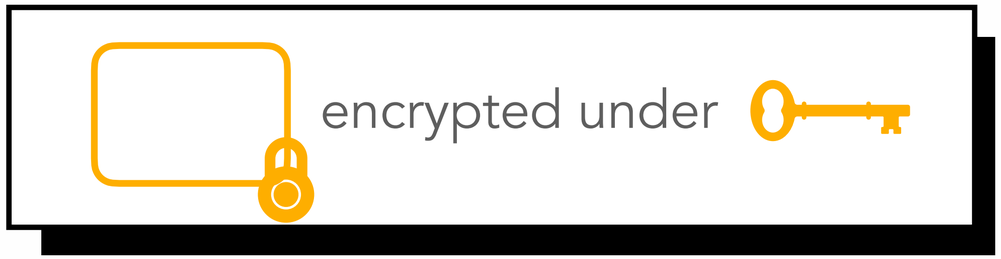
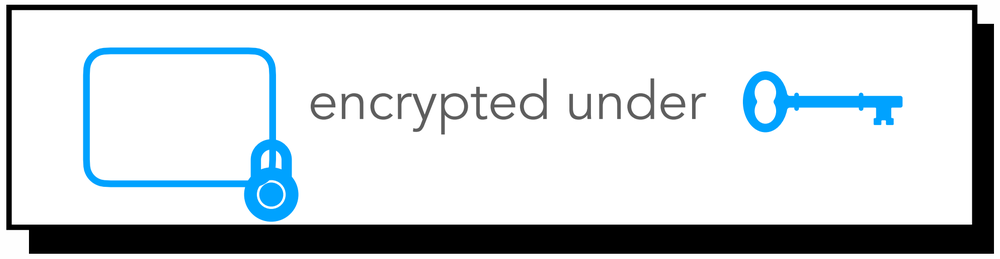


# Bootstrapping [Gen09]




**[Gen09]** C. Gentry. Fully homomorphic encryption using ideal lattices. STOC 2009.

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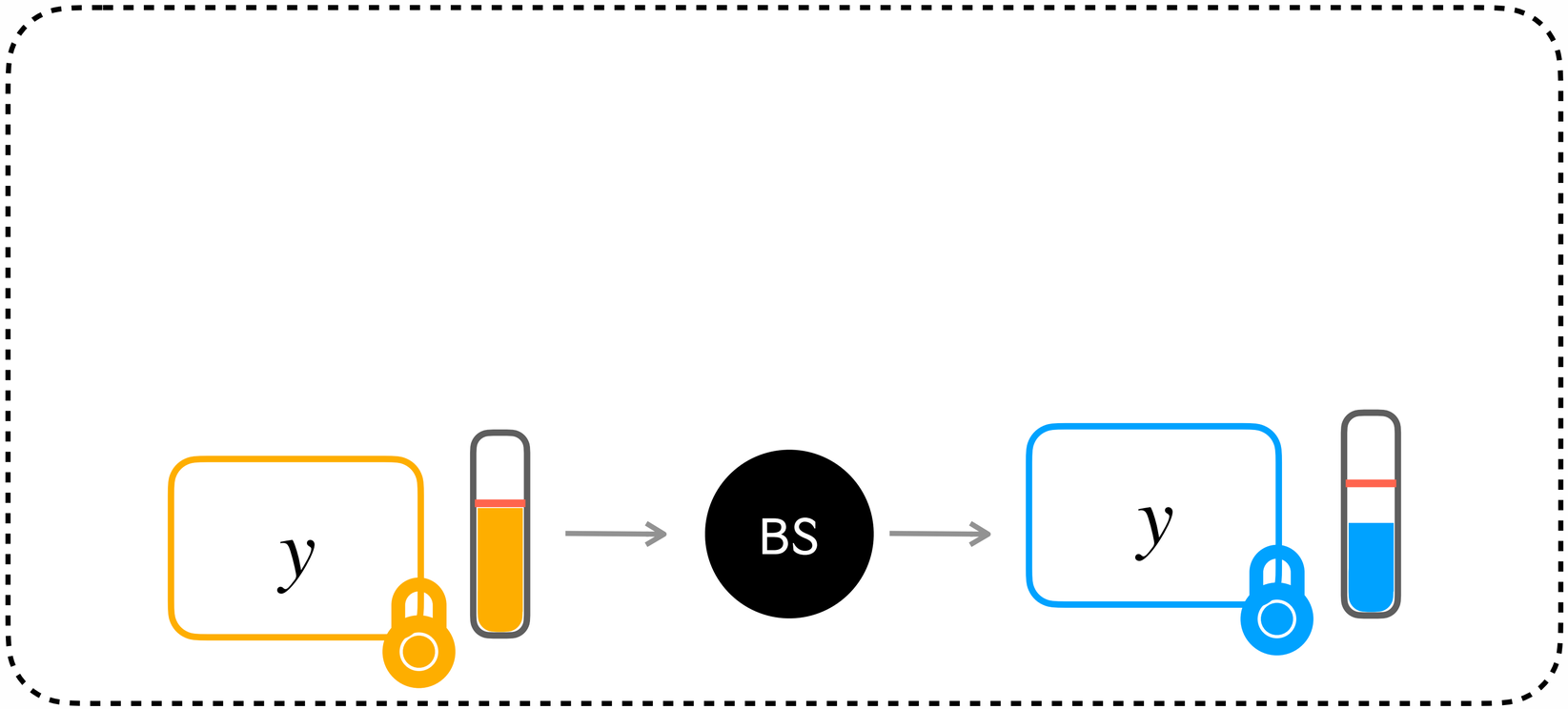


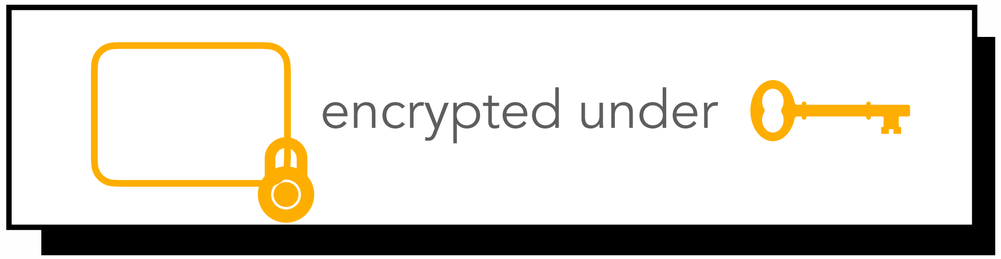


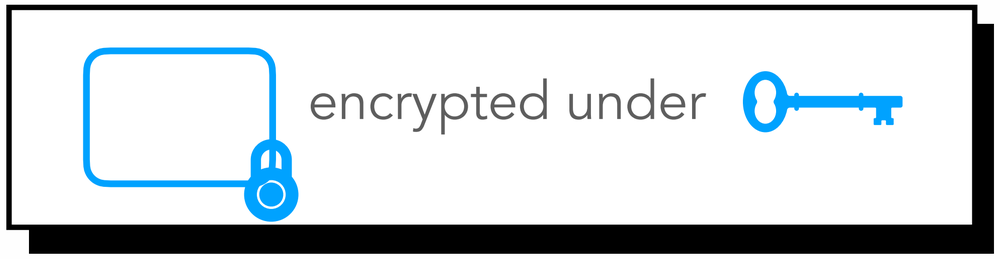


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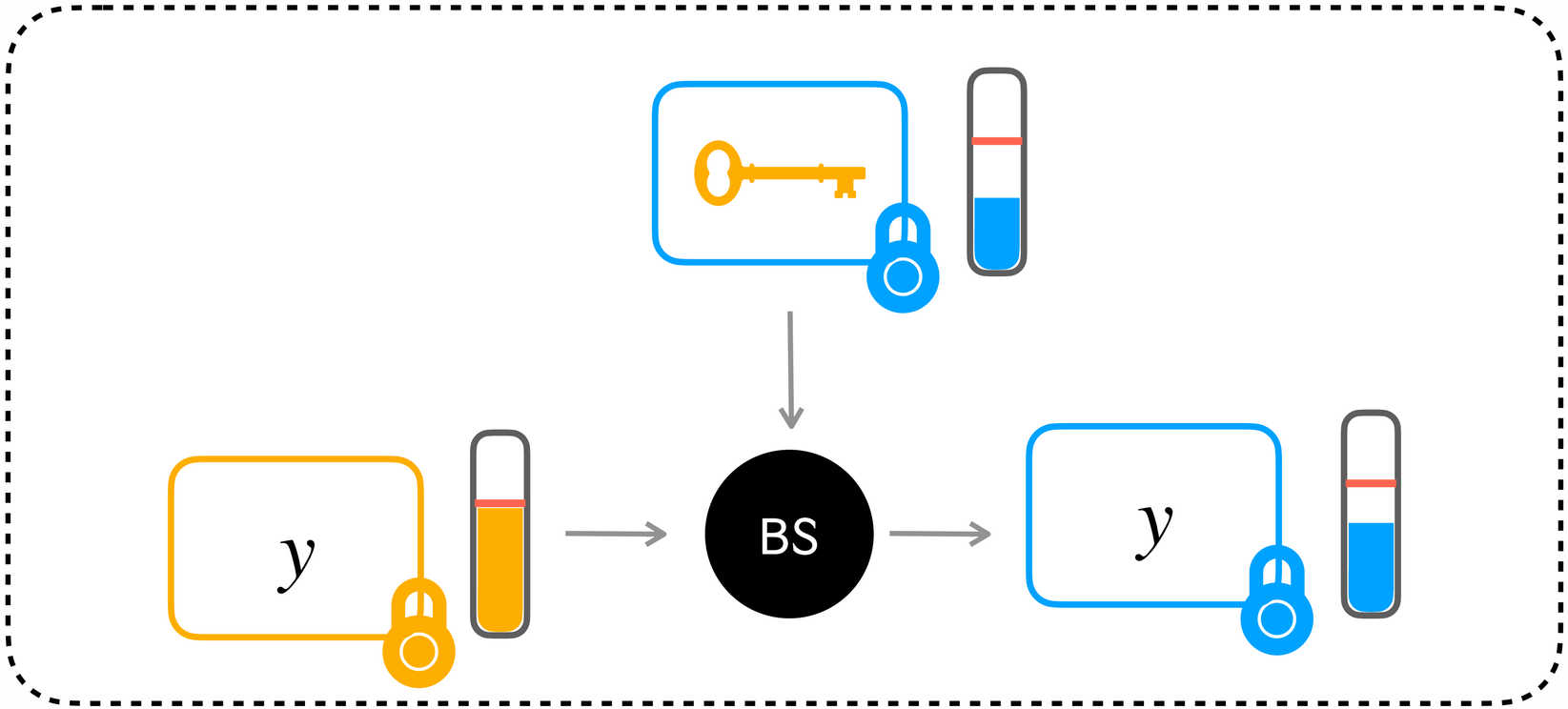


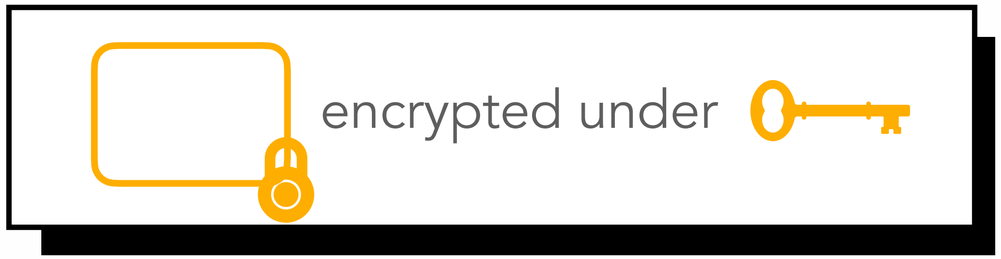


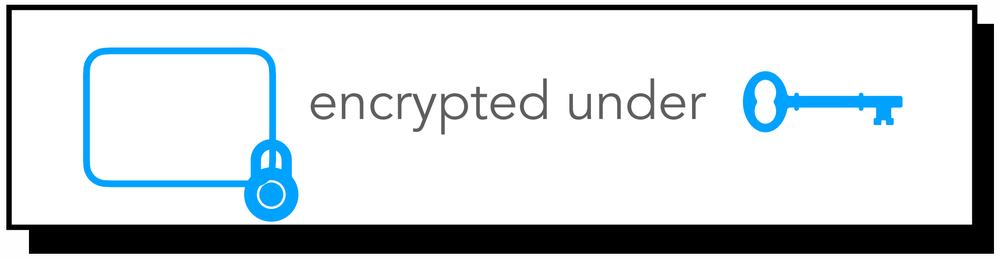


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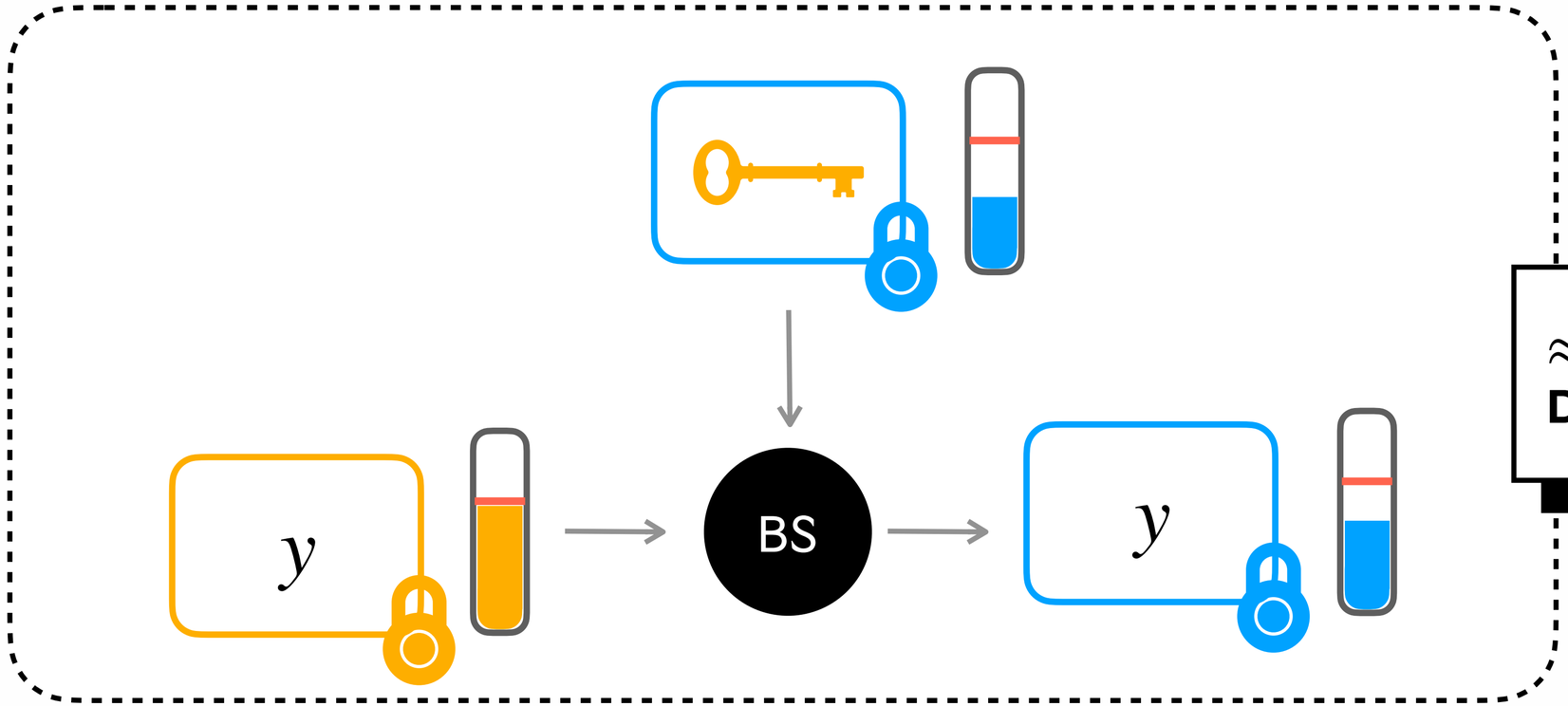
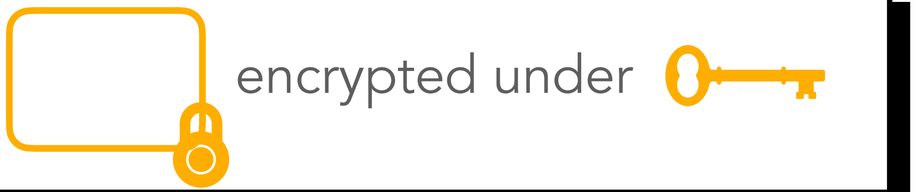
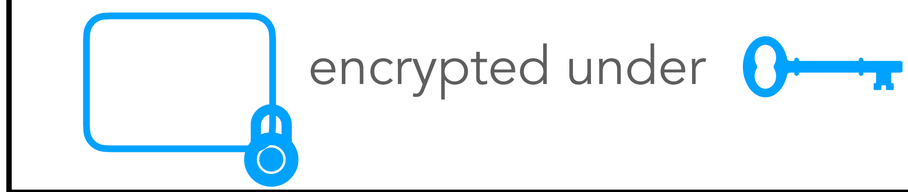



≈ Homomorphic Decryption

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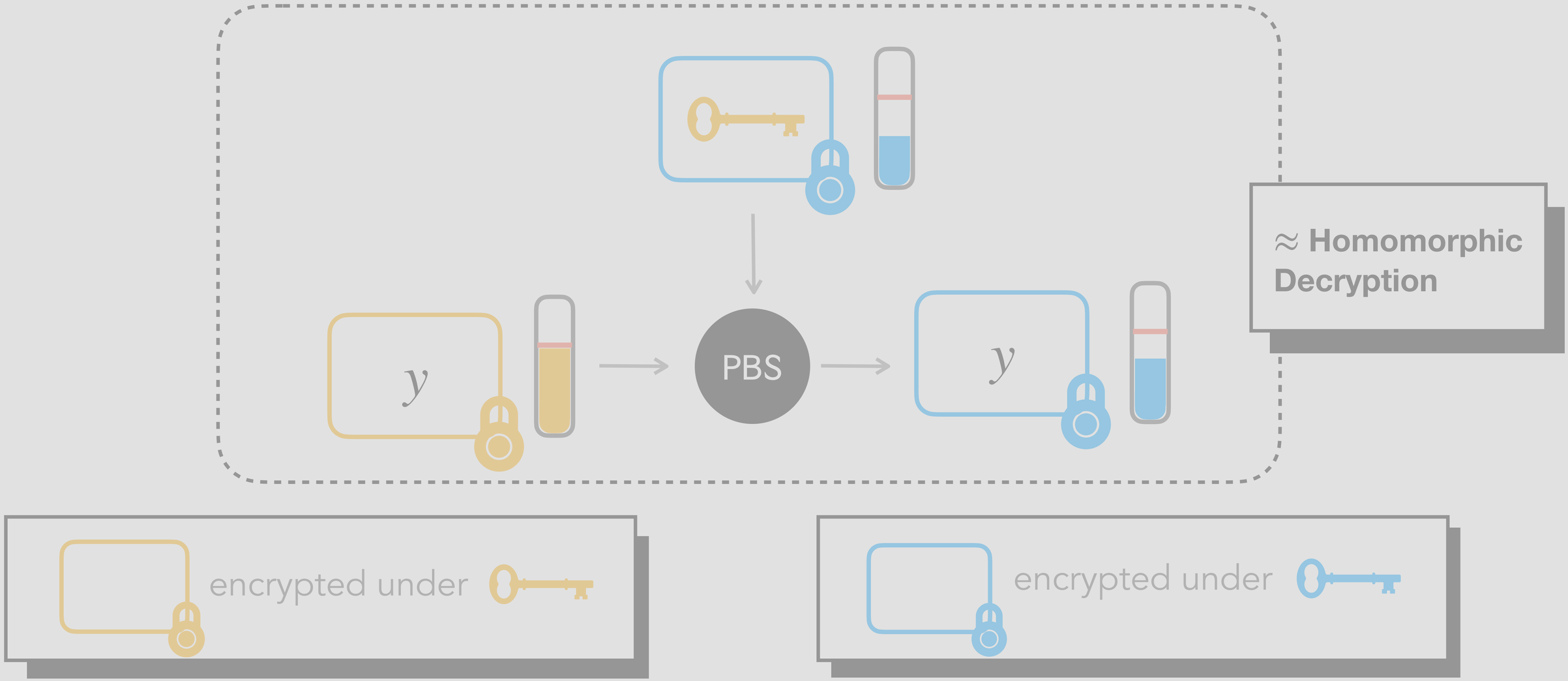


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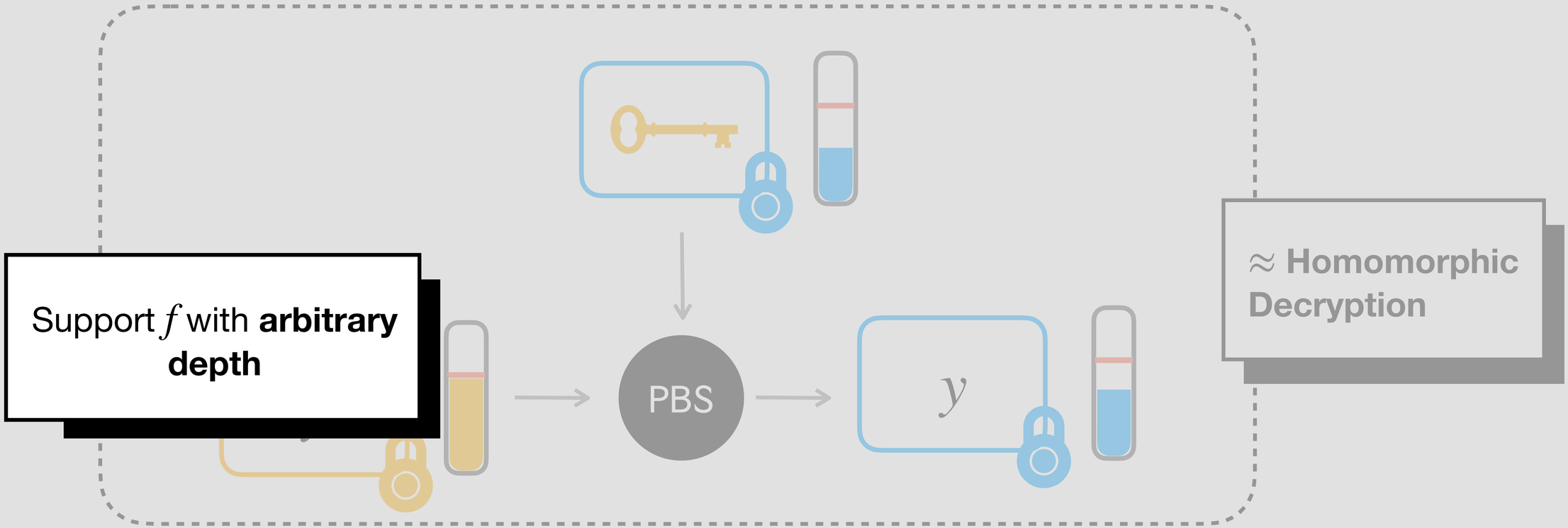


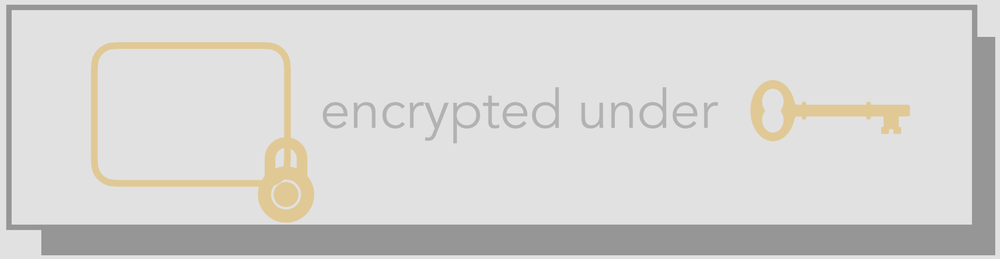


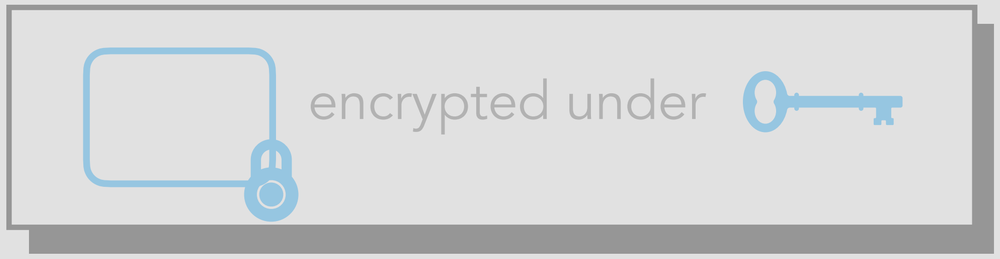


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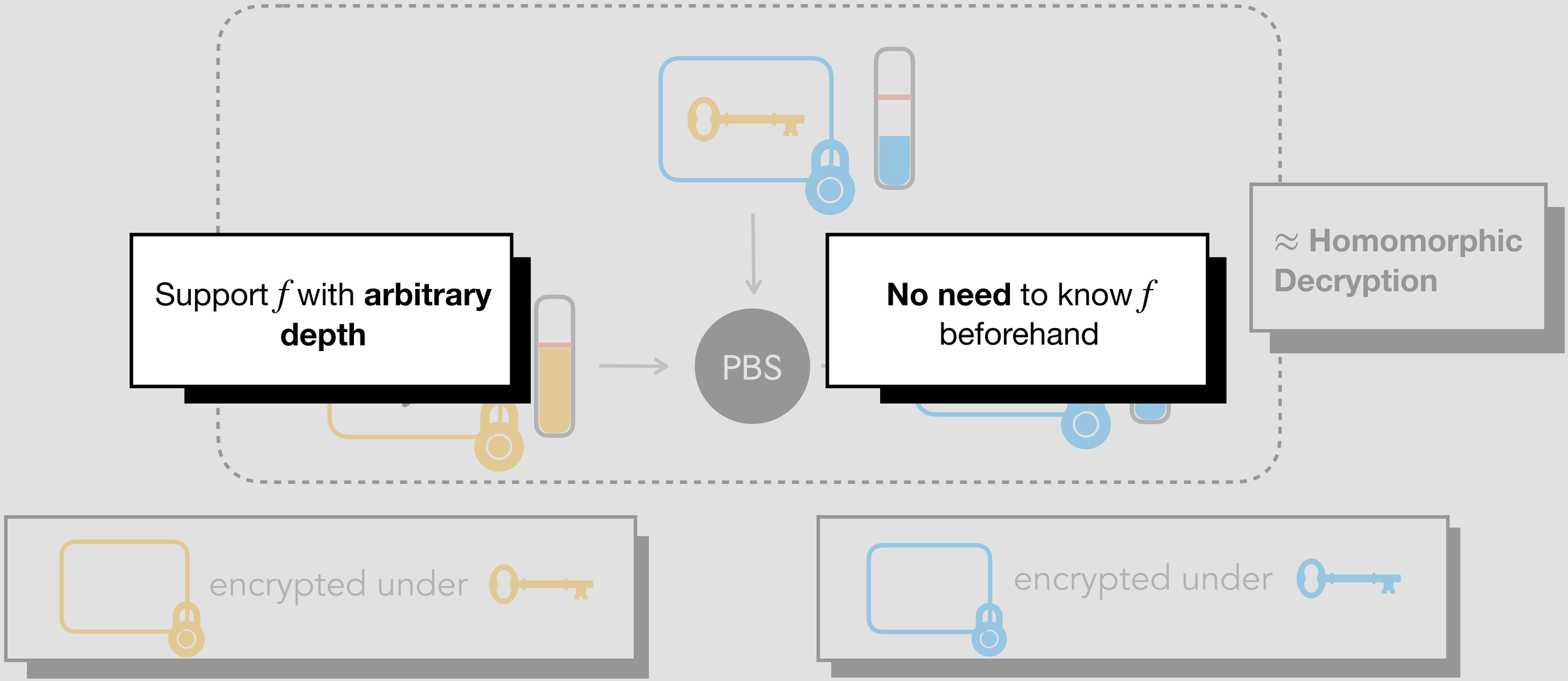


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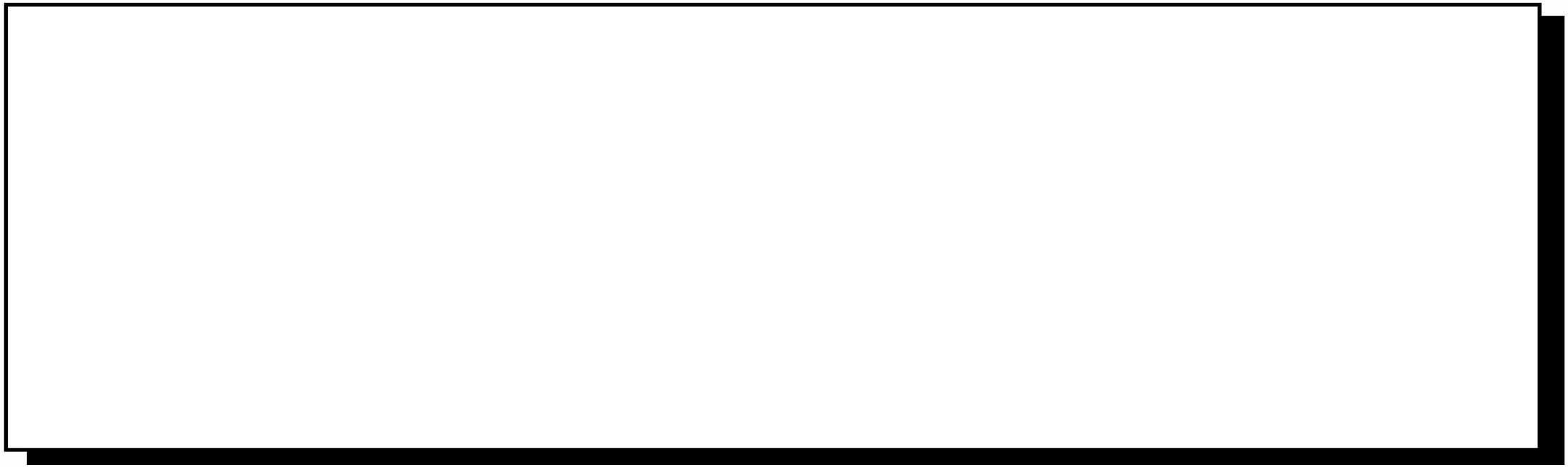
# TFHE

Intuition and main algorithms

# Encryption



Δ

MSB LSB



**[CGGI20]** I. Chillotti, N. Gama, M. Georgieva, M. Izabachène. TFHE: Fast Fully Homomorphic Encryption over the Torus. Journal of Cryptology 2020.

# Encryption

1. Draw a mask $\vec{a} = (a_0, \cdots, a_{n-1})$ from $\mathbb{U}([\![0; q-1]\!])$ and
   a secret key $\vec{s} = (s_0, \cdots, s_{n-1})$ from $\mathbb{U}(\{0,1\})$
   a noise $e$ from $\mathcal{N}(0, \sigma^2)$

$\Delta$

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2. $b = \langle \vec{a}, \vec{s} \rangle + \Delta m + e \pmod{q}$

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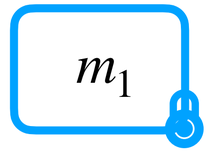


$= (\vec{a}, b)$

Δ

MSB LSB



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LWE Dimension

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$m_1$ $= (\vec{a}, b)$

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MSB LSB



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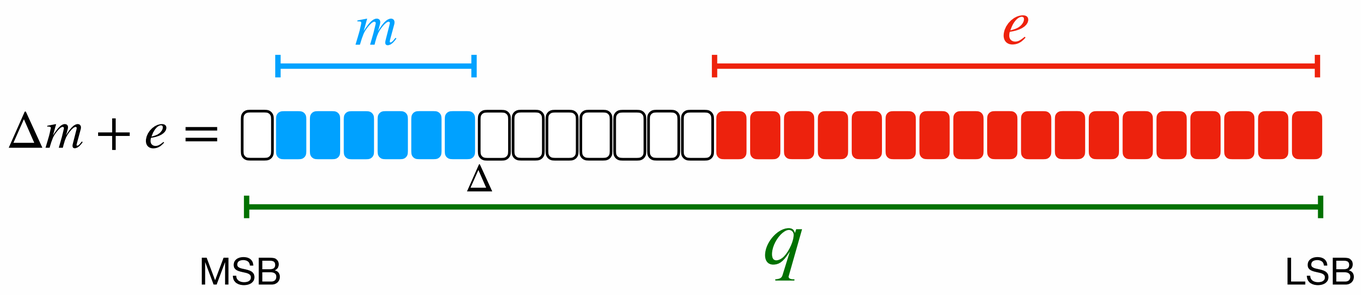




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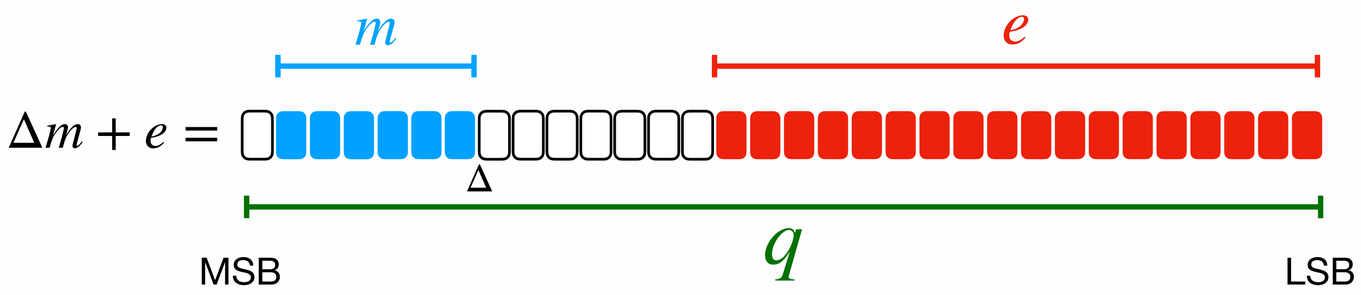


For a security level $\lambda$,

$n \nearrow \implies \sigma \searrow$



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# Decryption



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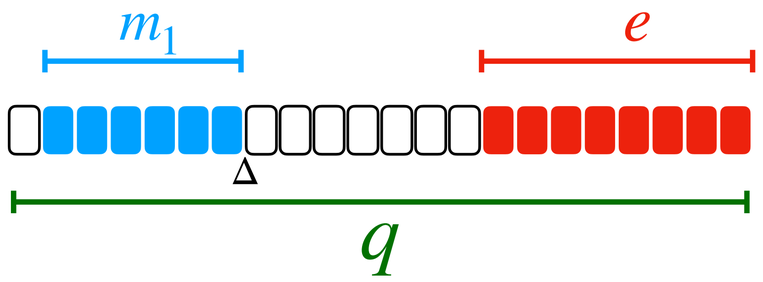




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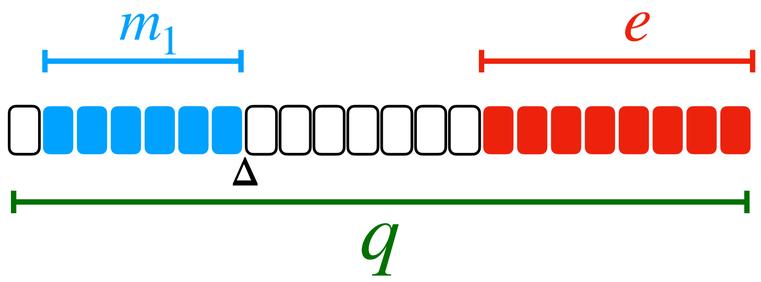
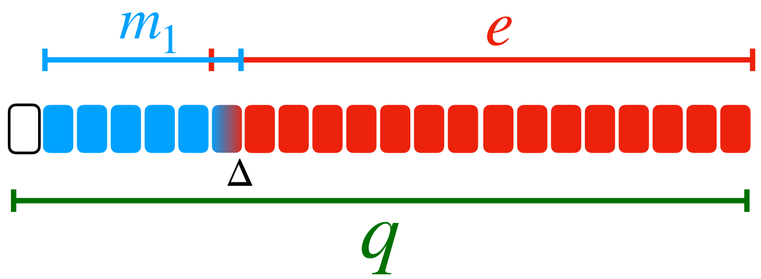



2b $$|e| \geq \frac{\Delta}{2} \Longrightarrow \left\lceil \frac{\Delta m + e}{\Delta} \right\rfloor \neq m$$



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# Decryption

**Condition to guarantee the correctness of a computation**

1 $$b - \langle \vec{a}, \vec{s} \rangle = \cancel{\langle \vec{a}, \vec{s} \rangle} + \Delta m + e - \cancel{\langle \vec{a}, \vec{s} \rangle} \pmod{q}$$

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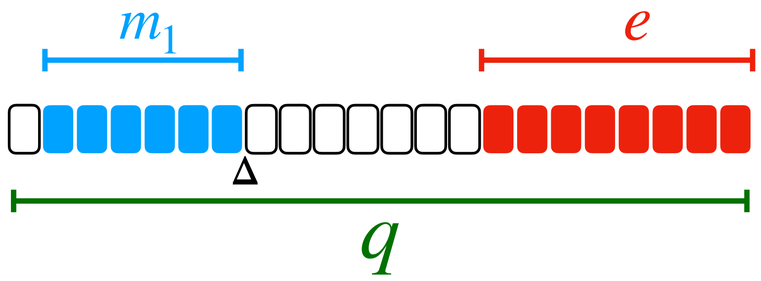


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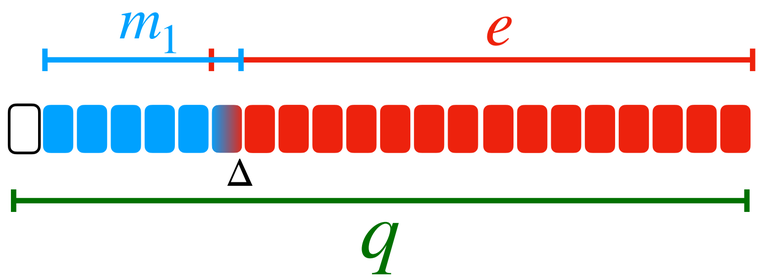




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# Addition



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# Addition

Consider $\mathsf{ct}_1 = (\overrightarrow{a_1}, b_1)$ and $\mathsf{ct}_2 = (\overrightarrow{a_2}, b_2)$



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Consider $\mathsf{ct}_1 = (\overrightarrow{a_1}, b_1)$ and $\mathsf{ct}_2 = (\overrightarrow{a_2}, b_2)$

Define $\mathsf{ct}_3$ such that

$$\mathsf{ct}_3 \leftarrow \mathsf{ct}_1 \oplus \mathsf{ct}_2 = \left(\overrightarrow{a_1} + \overrightarrow{a_2} \pmod q, b_1 + b_2 \pmod q\right)$$



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$$= \cancel{\left\langle \vec{a_1} + \vec{a_2}, \vec{s} \right\rangle} + \underbrace{m_1 + m_2}_{\text{message}} + \underbrace{e_1 + e_2}_{\text{noise}} - \cancel{\left\langle \vec{a_1} + \vec{a_2}, \vec{s} \right\rangle} \pmod q$$



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# Dot Product

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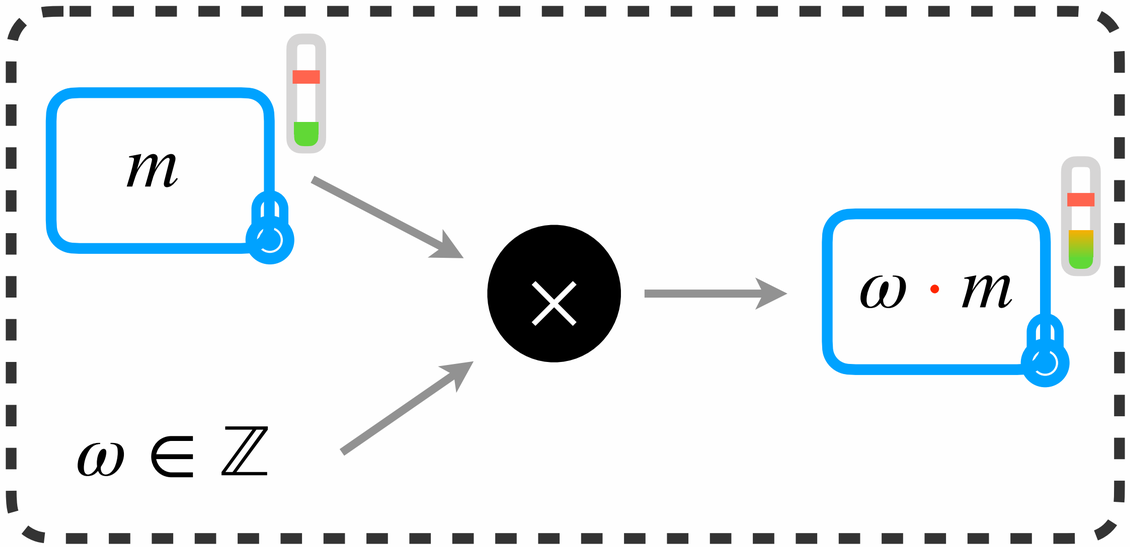

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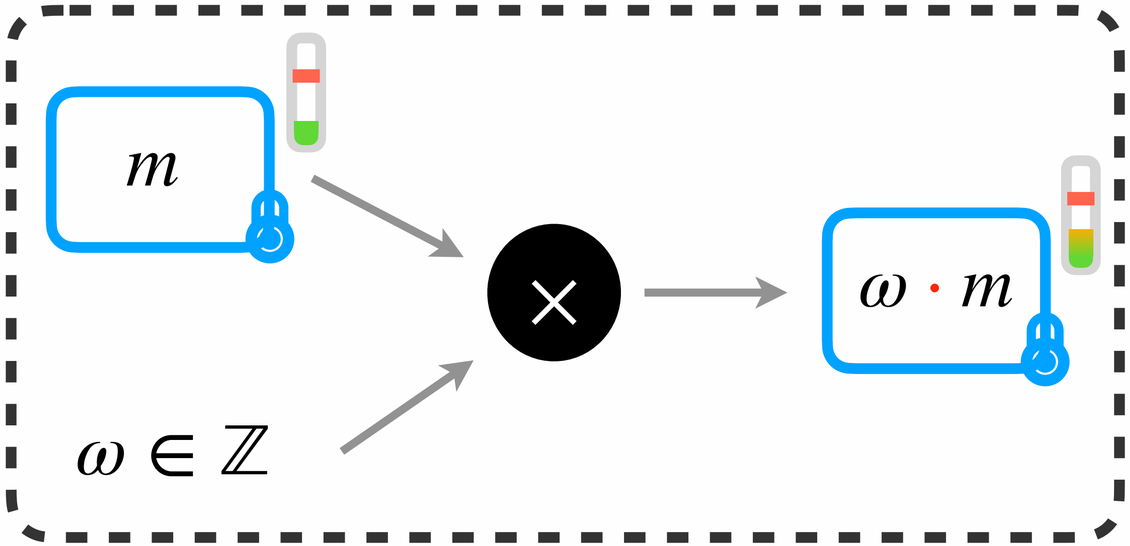
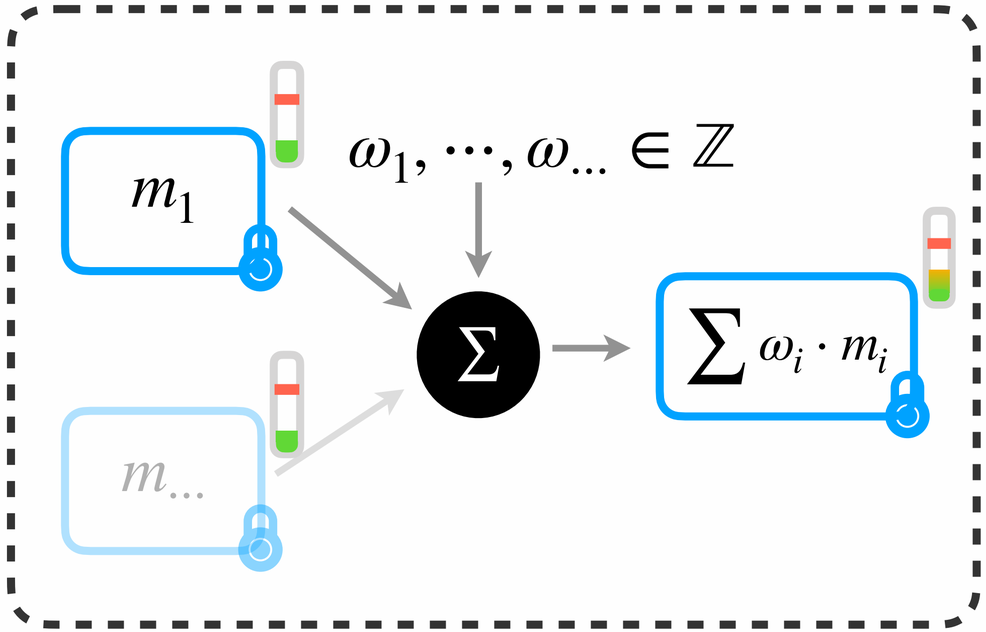

$m$
$\omega \in \mathbb{Z}$
$\times$
$\omega \cdot m$


$m_1$
$m_{...}$
$\omega_1, \cdots, \omega_{...} \in \mathbb{Z}$
$\Sigma$
$\sum \omega_i \cdot m_i$

# Keyswitch

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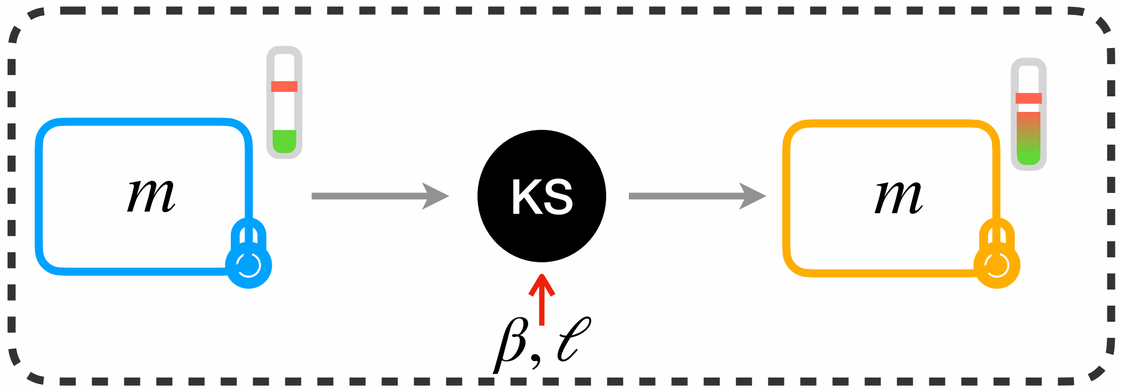

# Keyswitch

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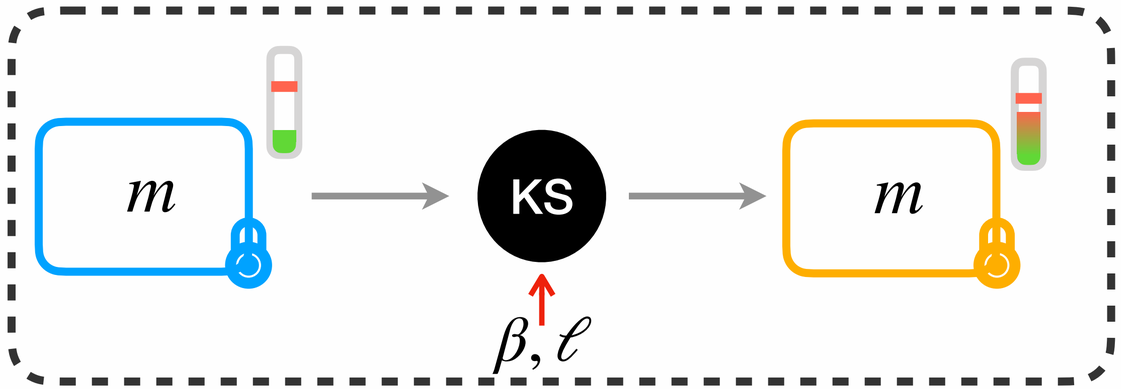

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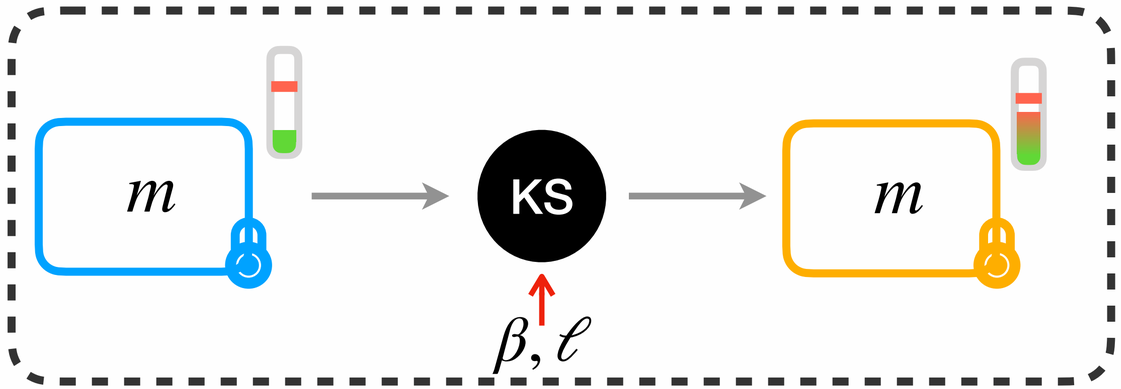
← Only require homomorphic dot product



# Keyswitch

Non-linear operation →

1 $b - \langle \vec{a}, \vec{s} \rangle = \Delta m + e \pmod{q}$

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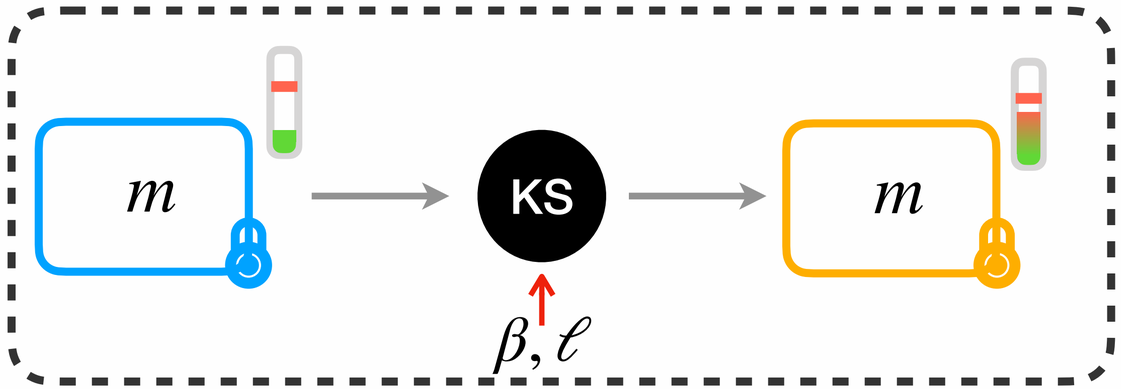

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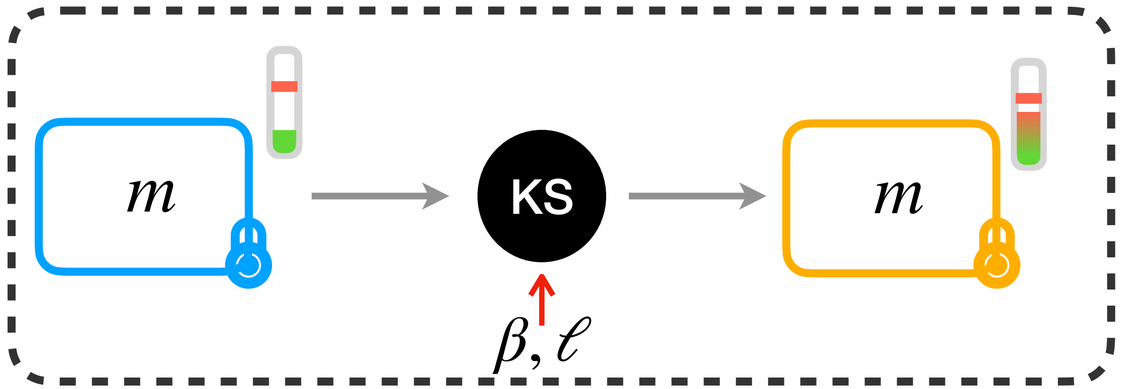

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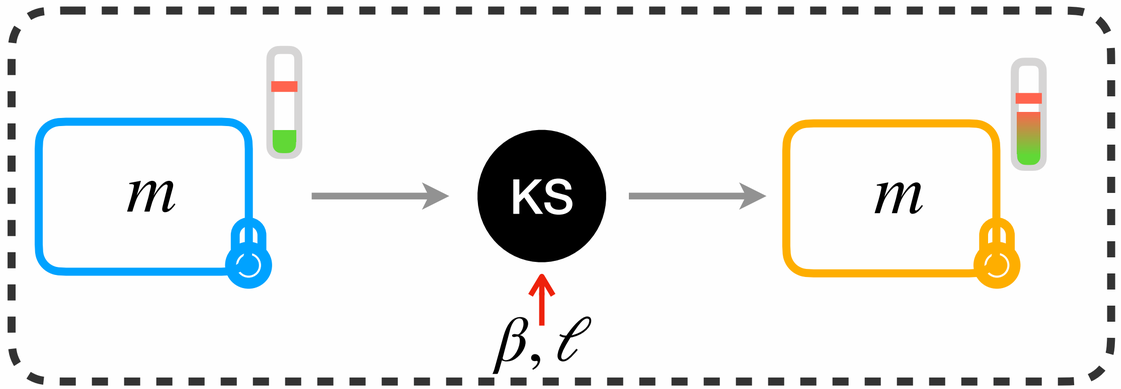


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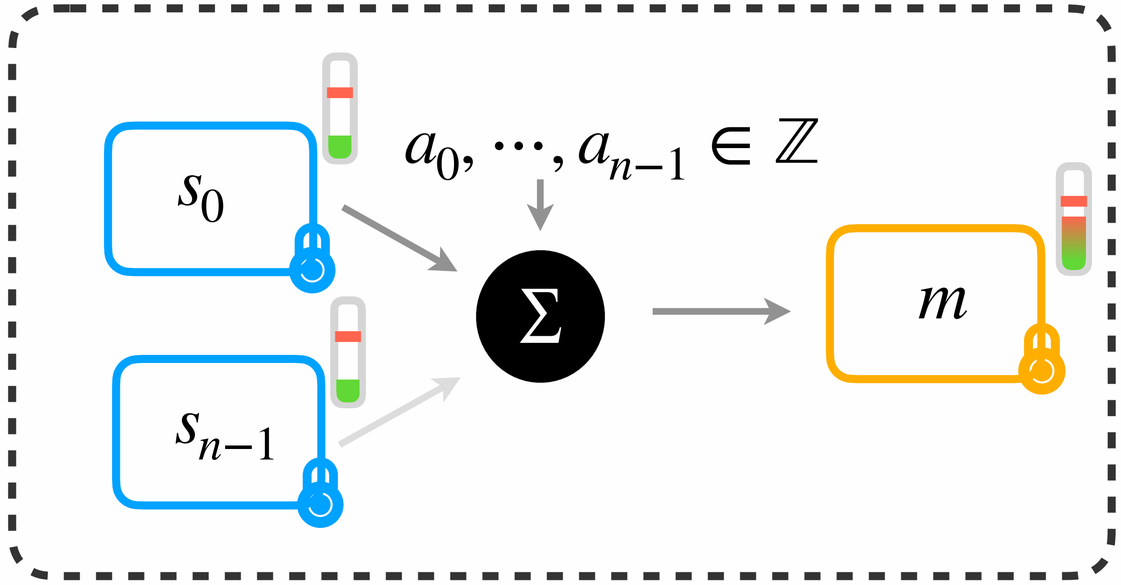
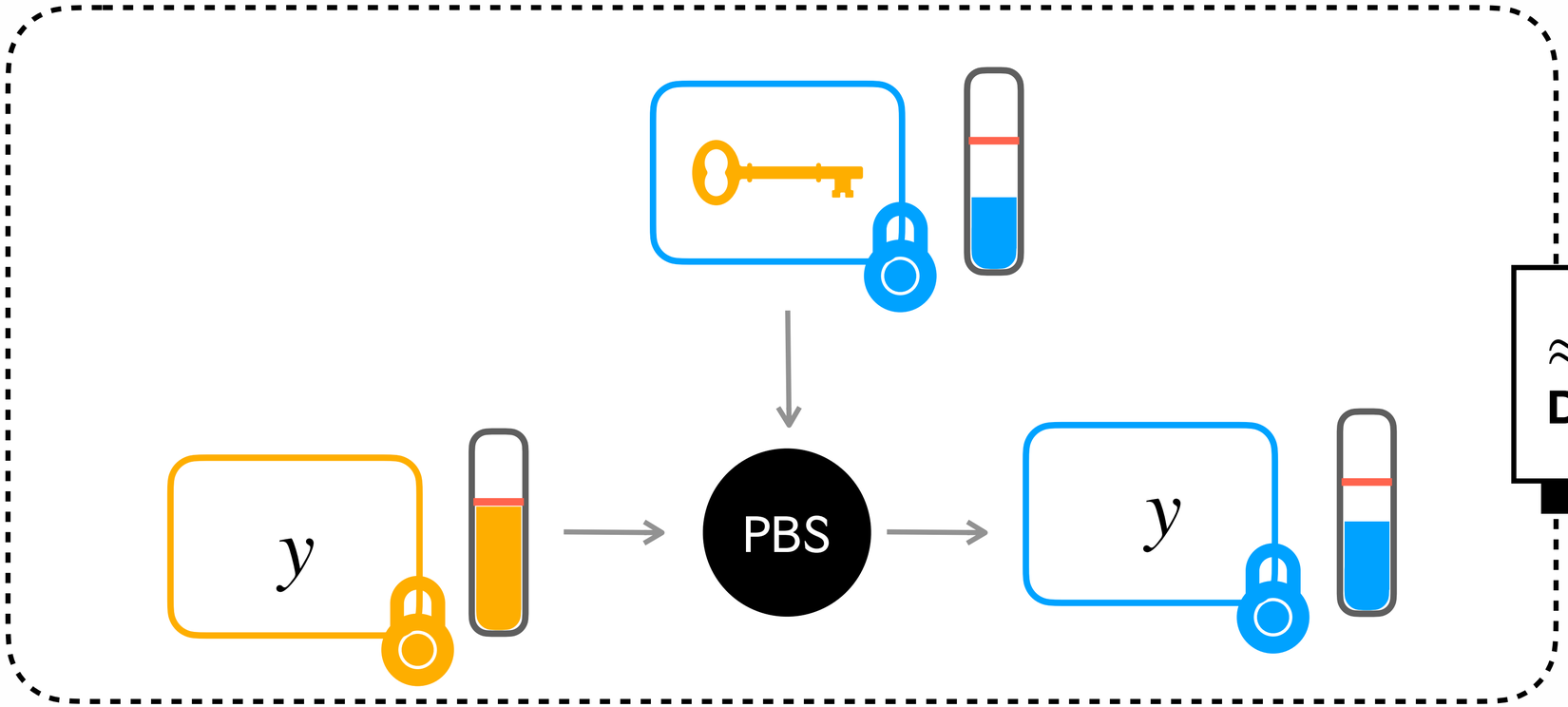
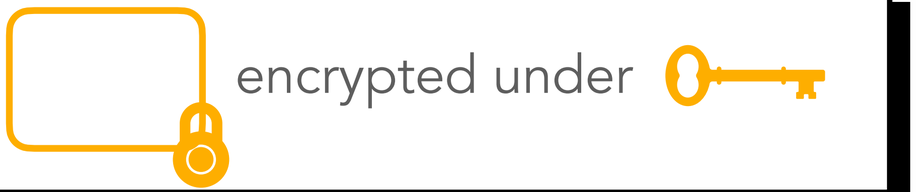
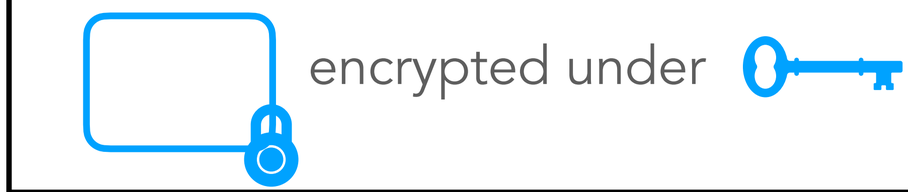


# Programmable Bootstrapping (PBS)




≈ **Homomorphic Decryption**



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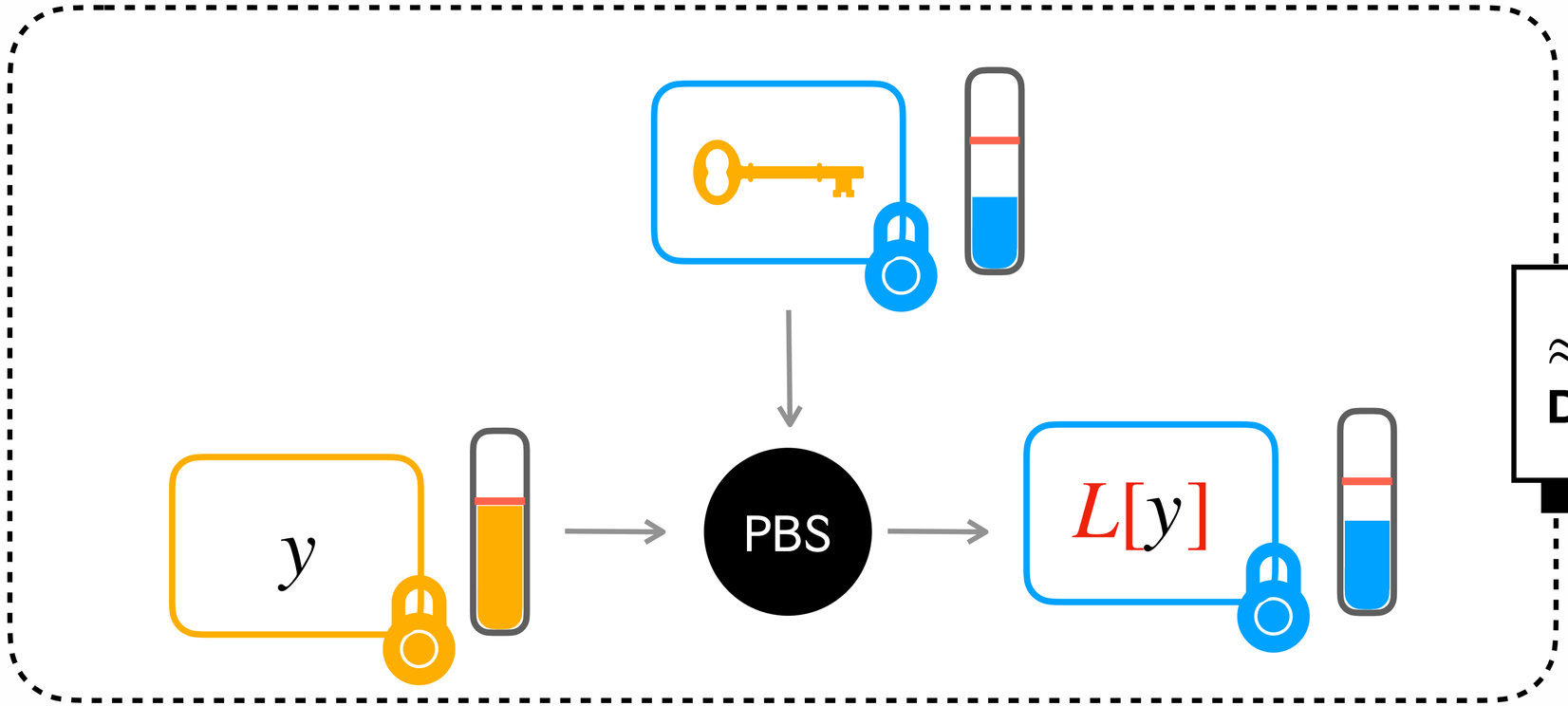


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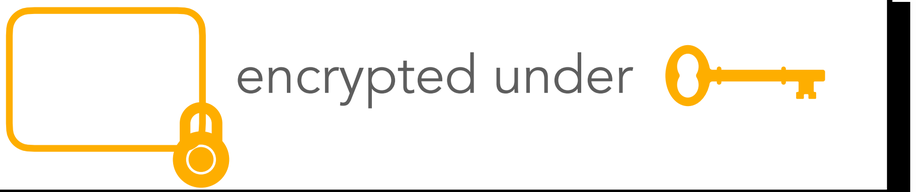


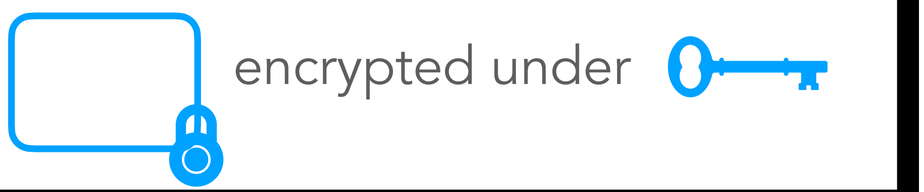




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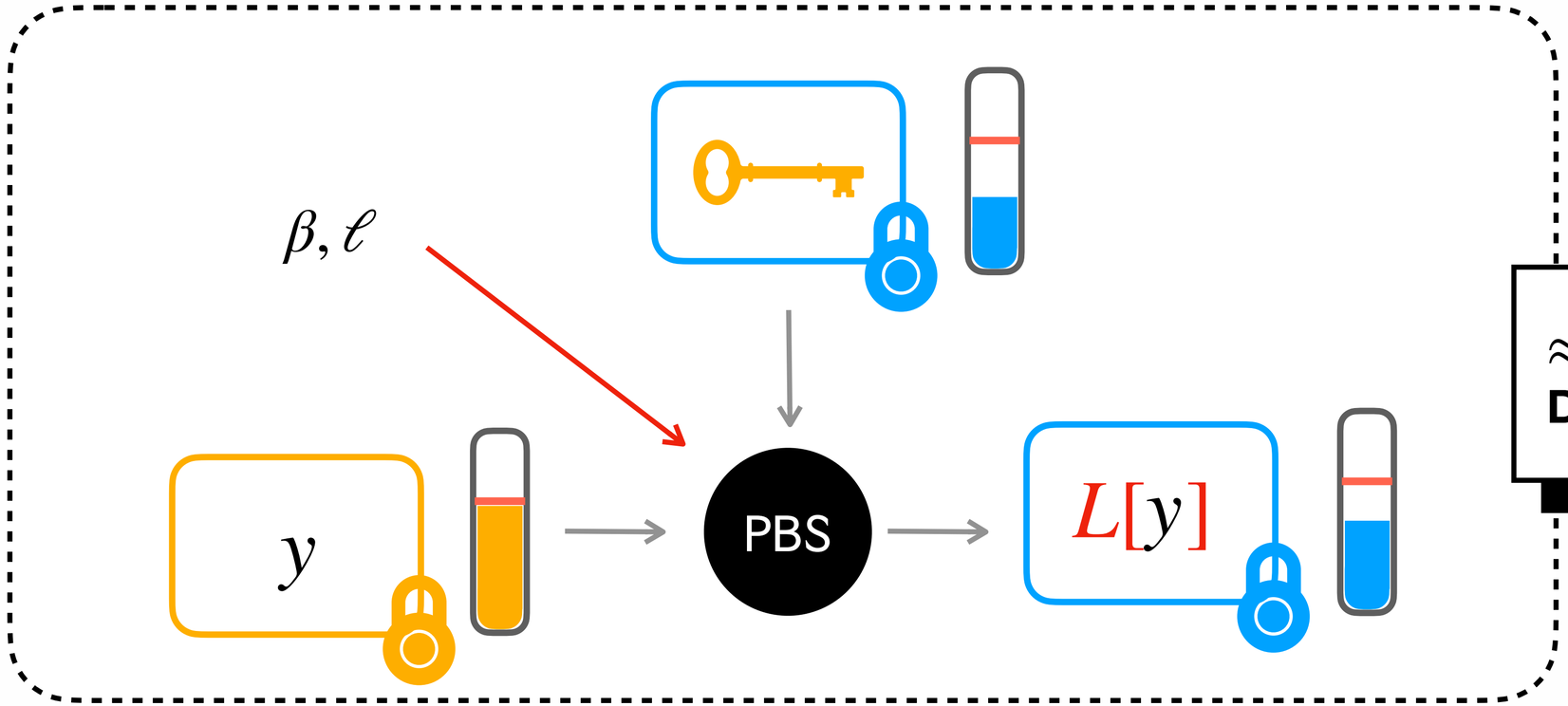


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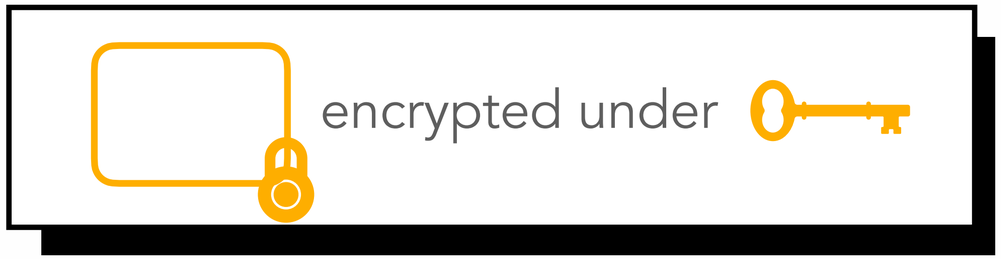


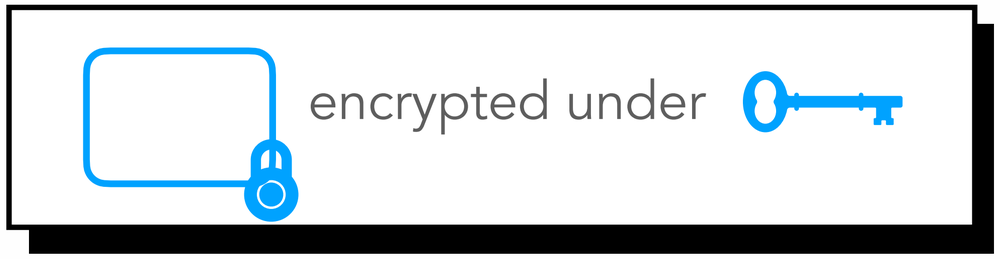




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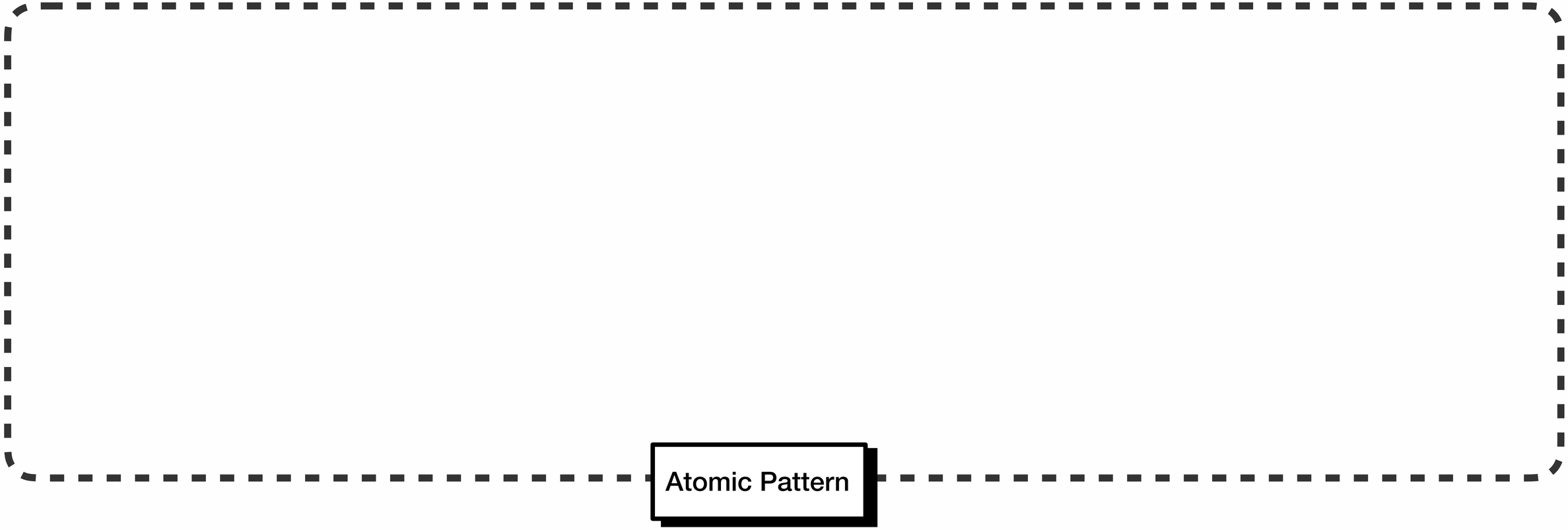
# Optimal arrangement of TFHE building blocks

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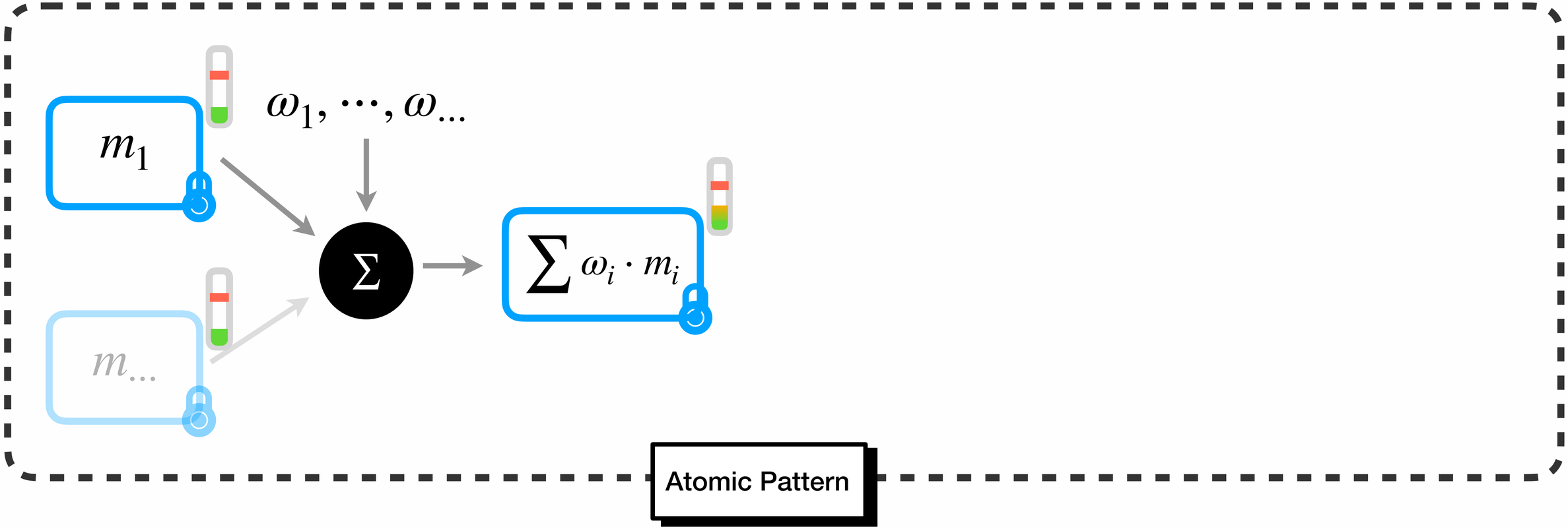


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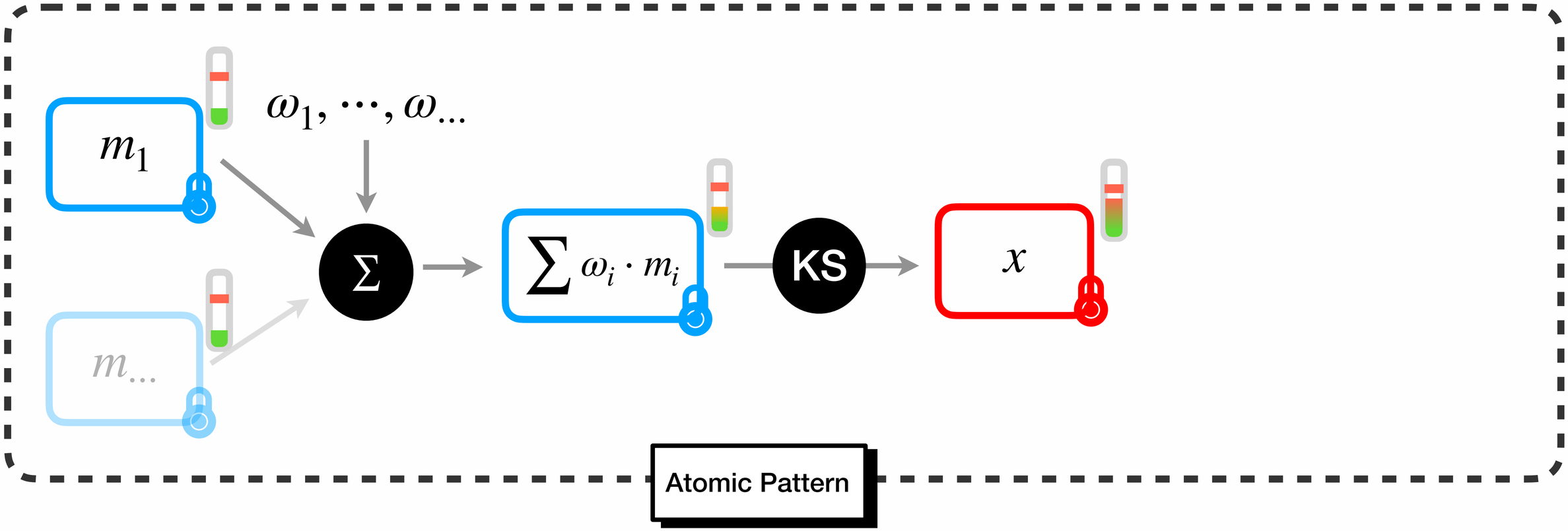


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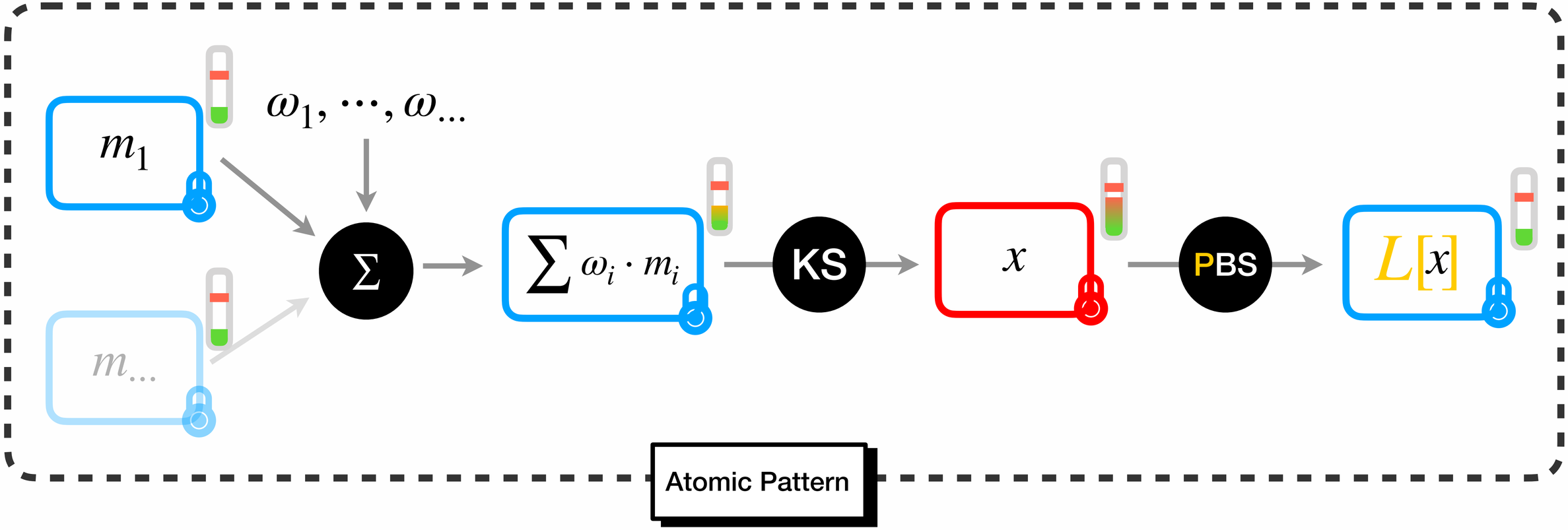


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# Concrete Optimizer

An optimizer for TFHE

# Goals

Translate a function $f$ into a graph of TFHE operators

# Translation

**FHE world**

**TFHE world**

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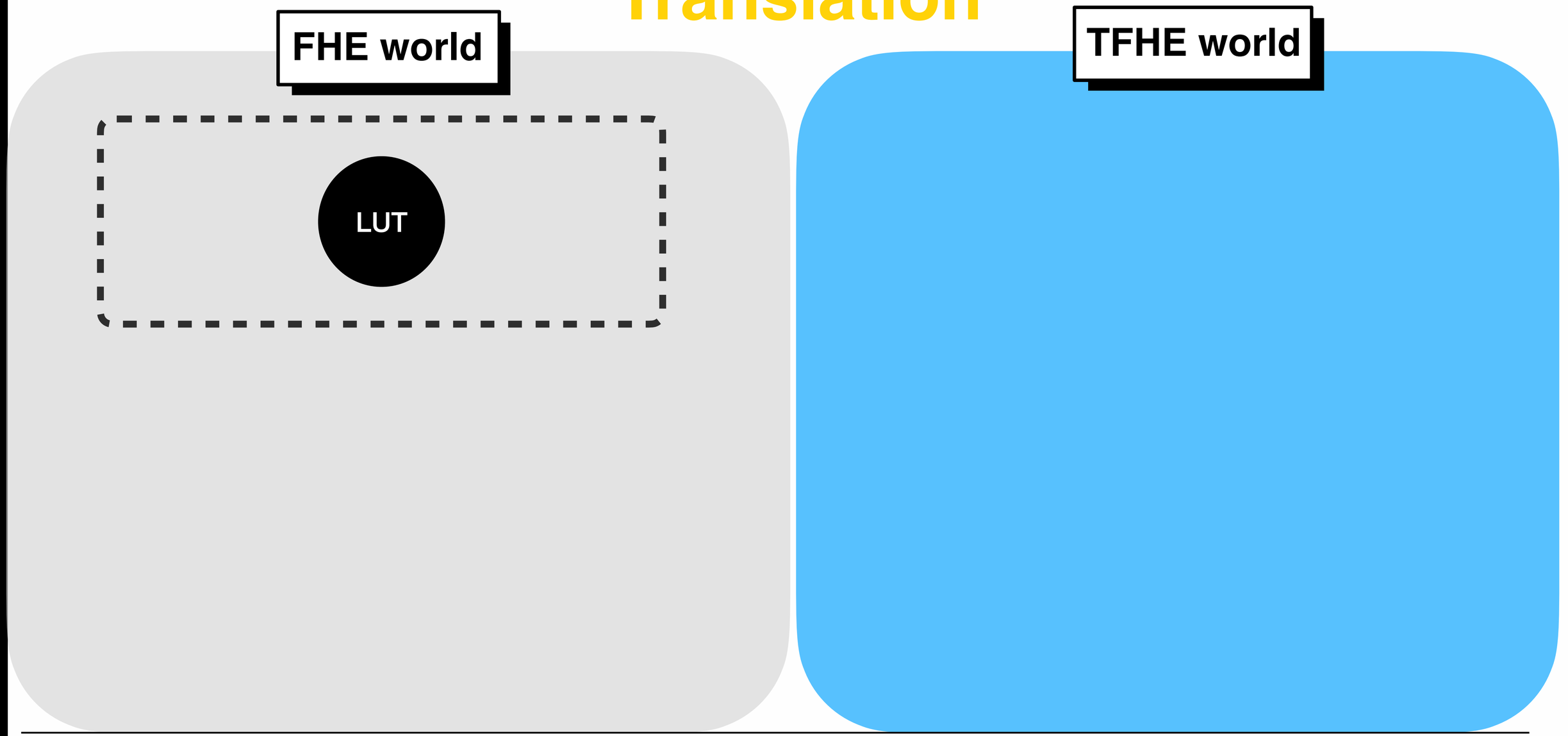


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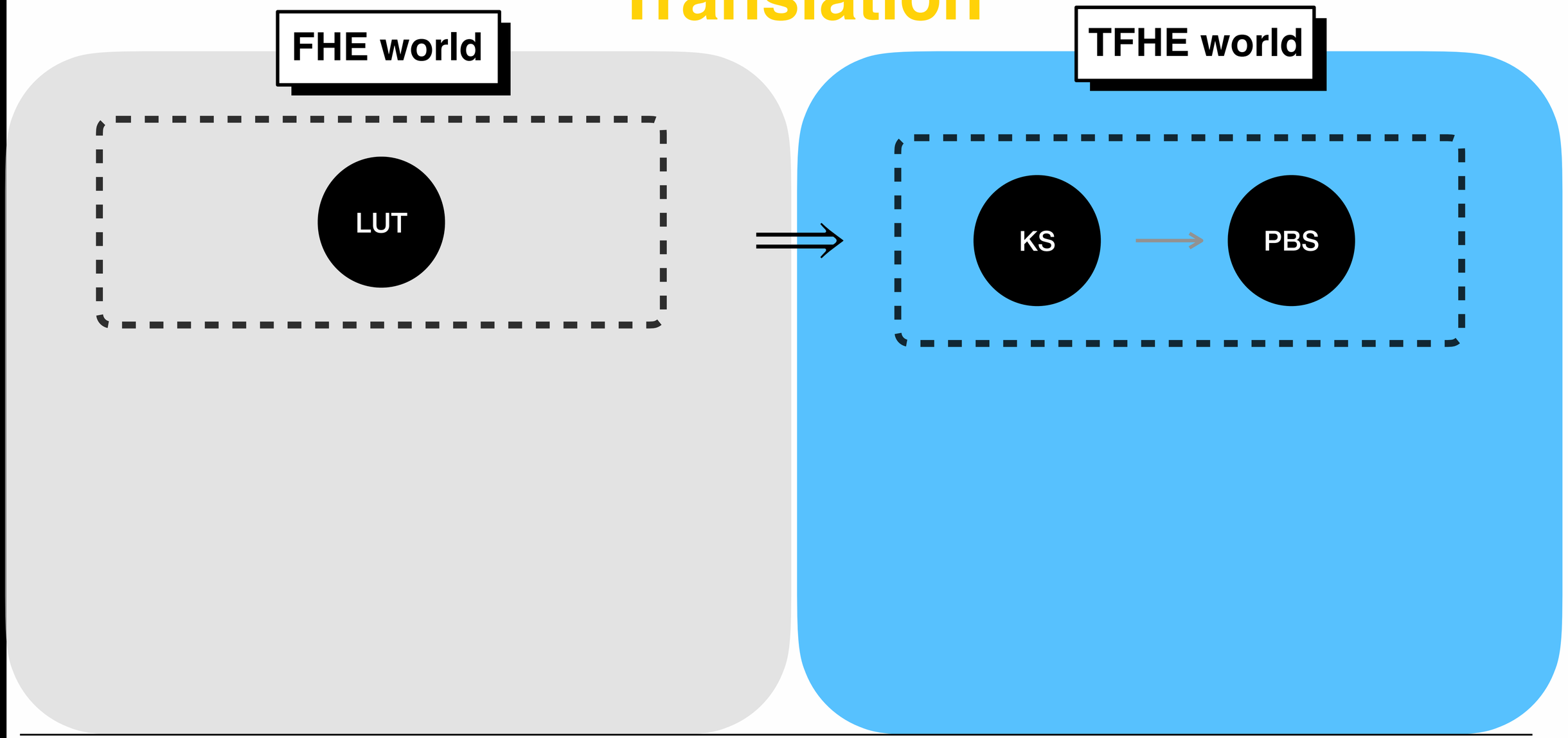


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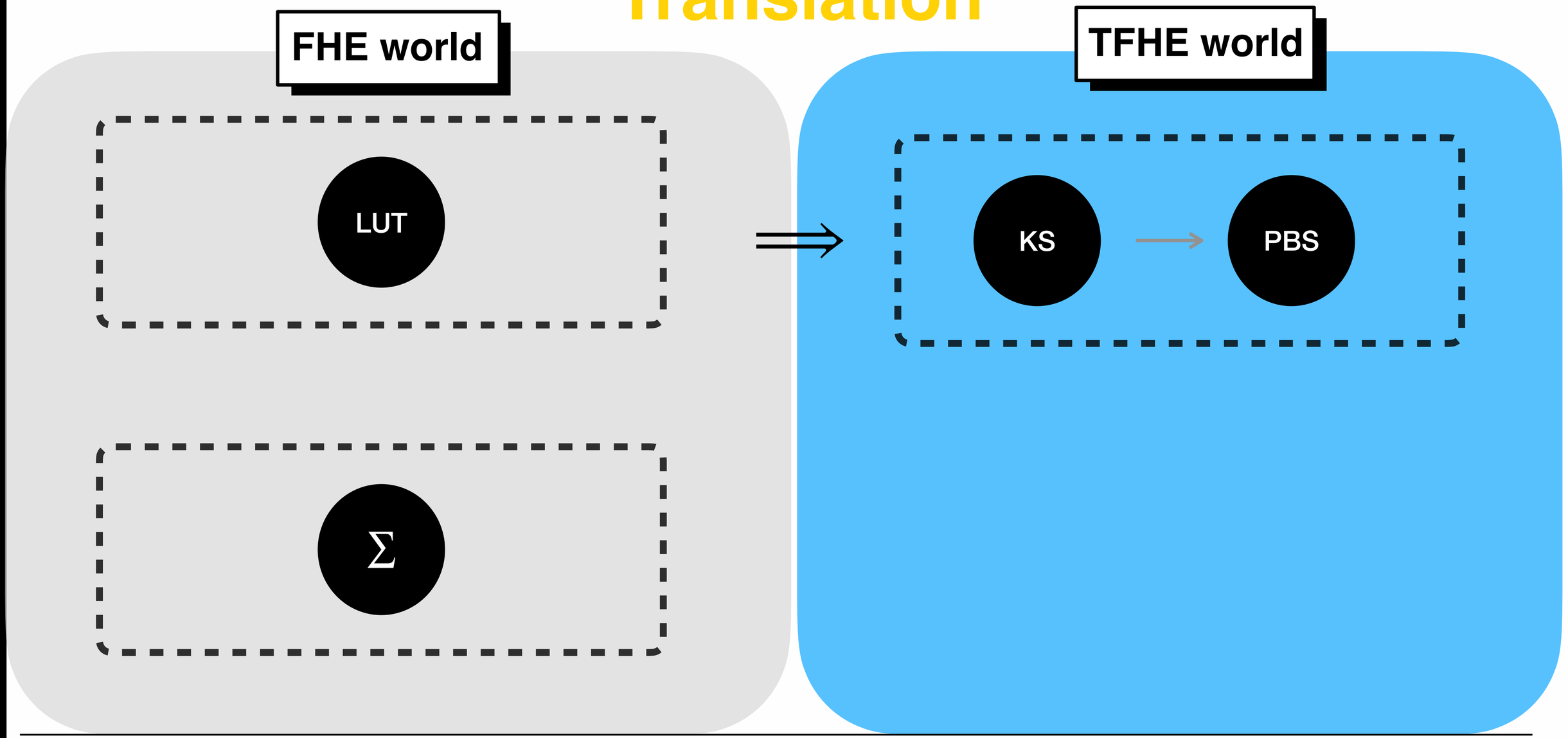


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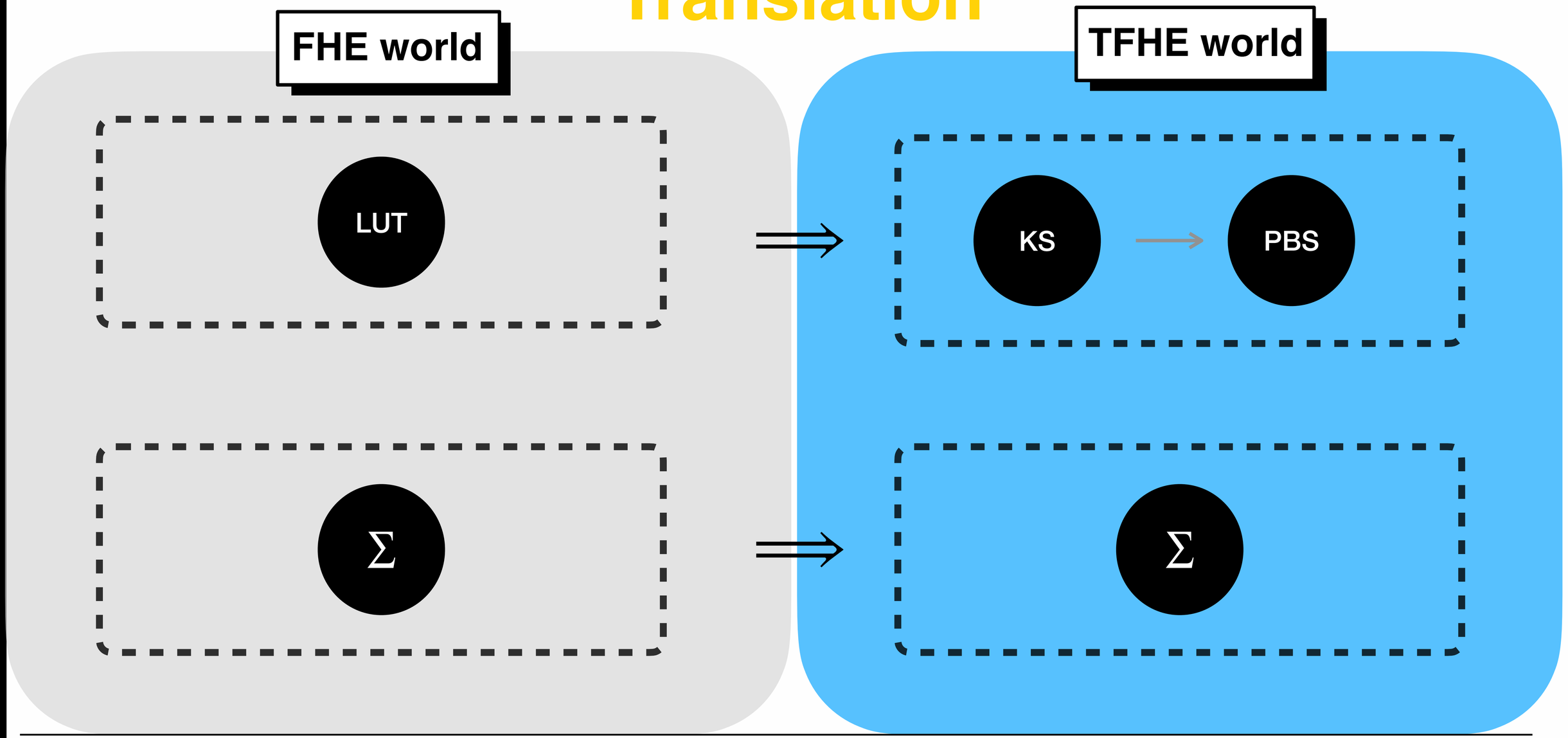


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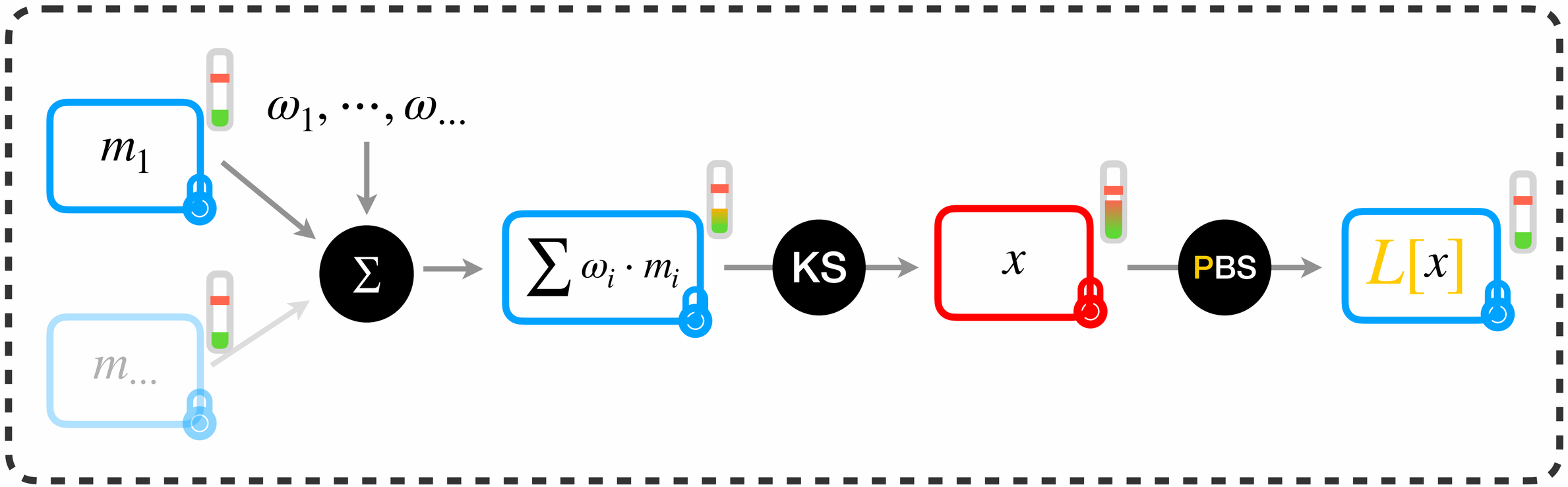


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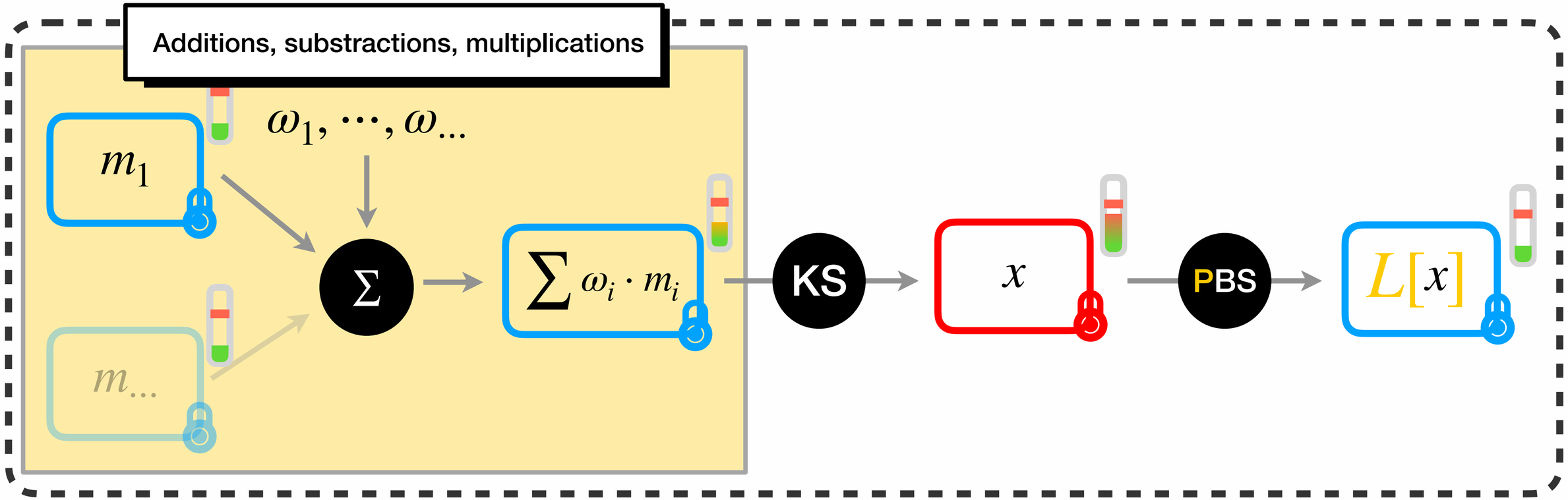


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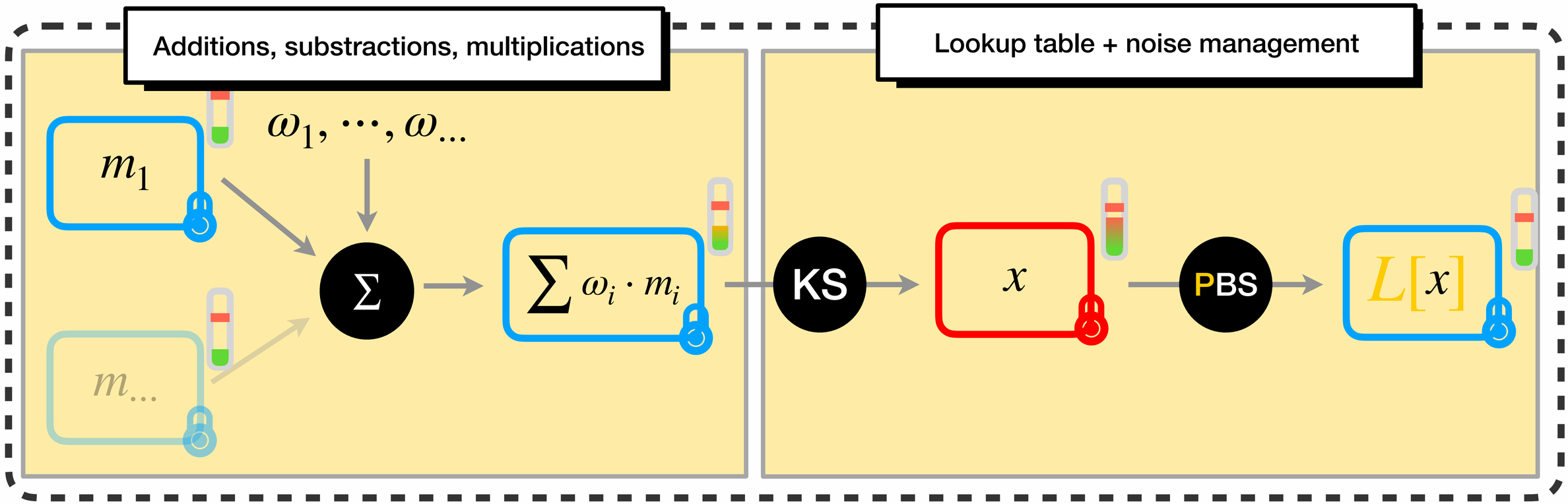


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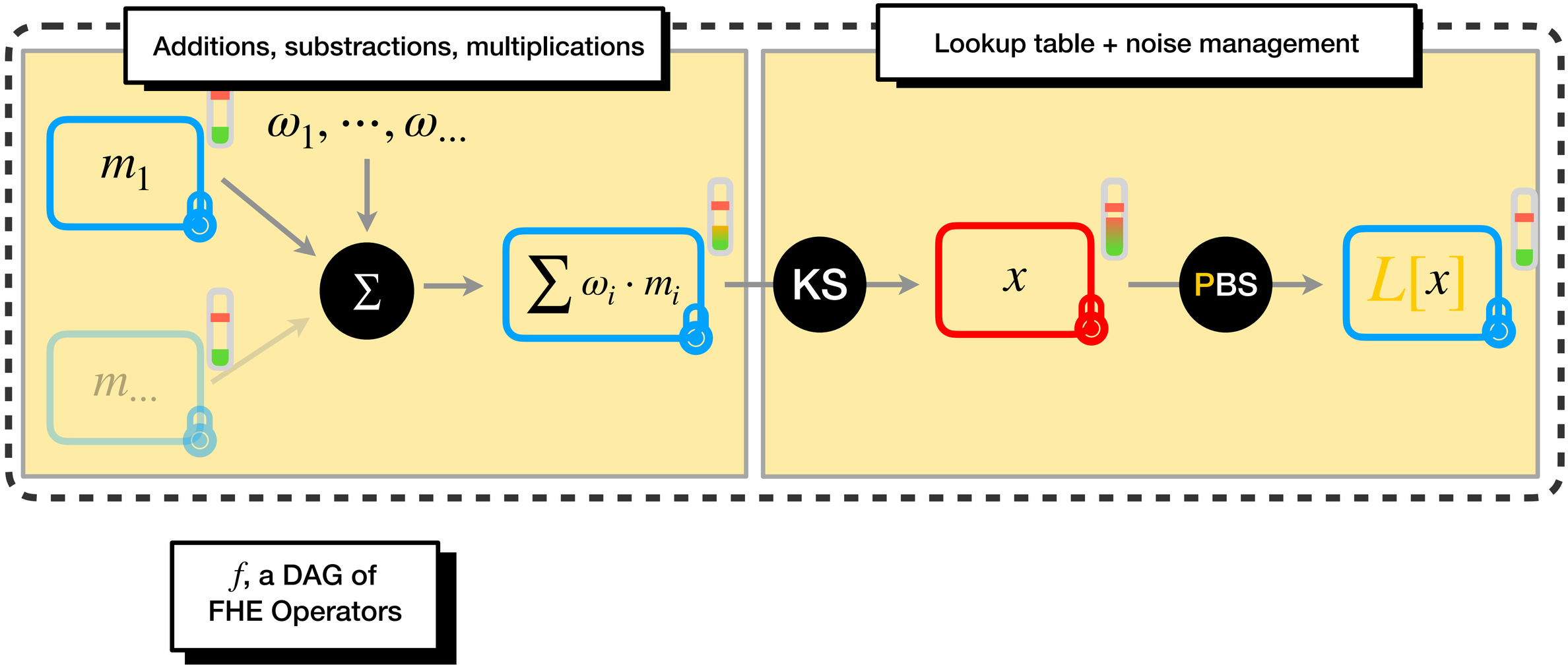


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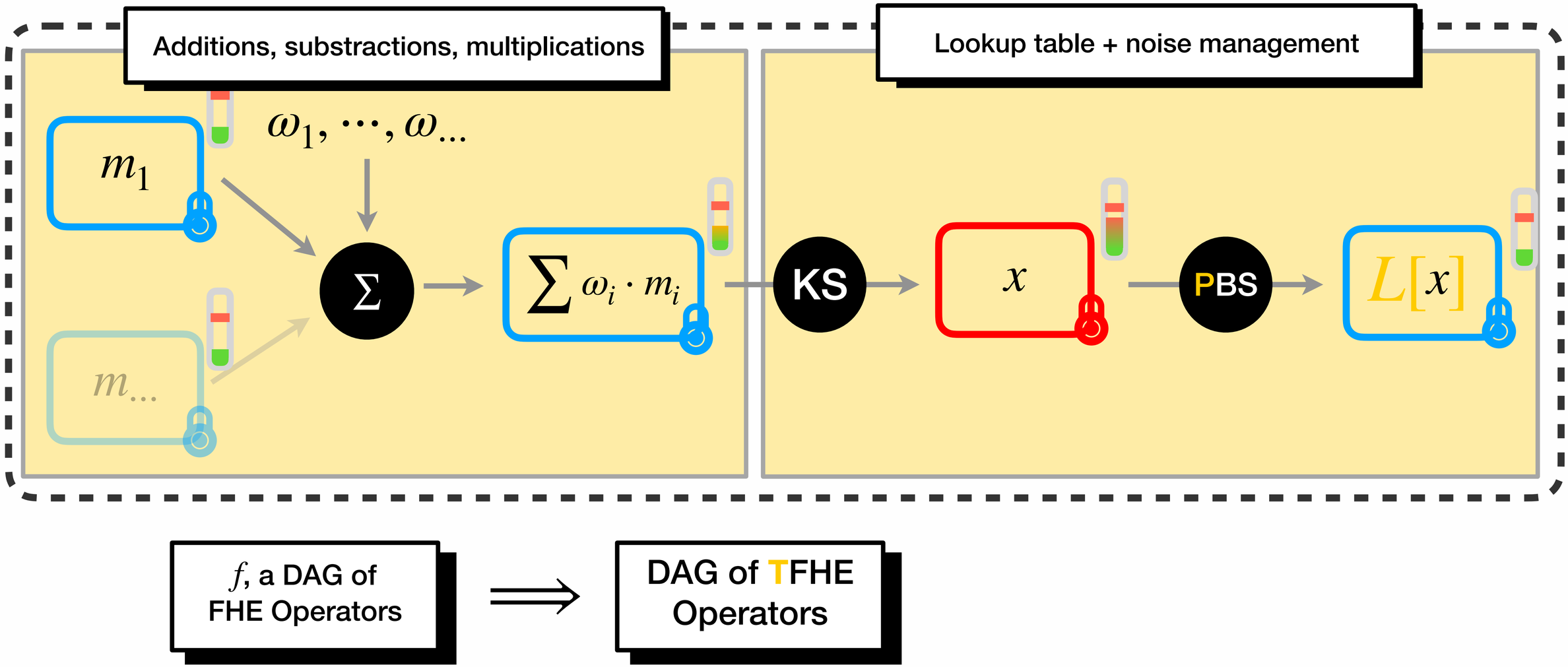


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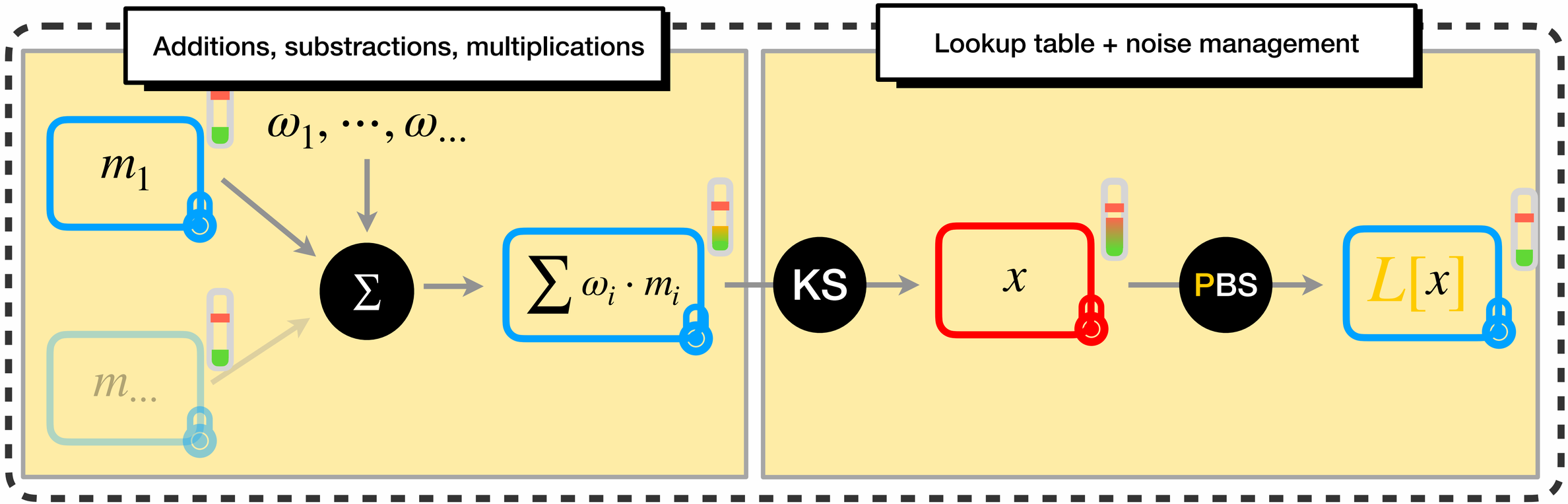


$f$, a DAG of FHE Operators $\Longrightarrow$ DAG of TFHE Operators $\Longrightarrow$ DAG of atomic patterns

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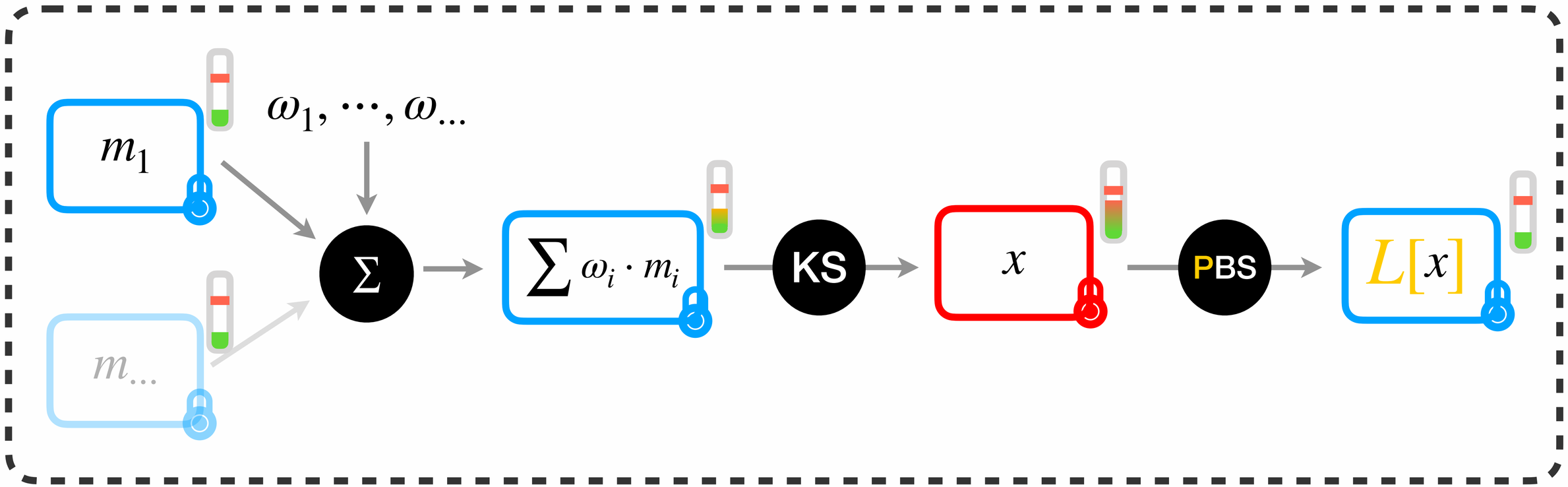


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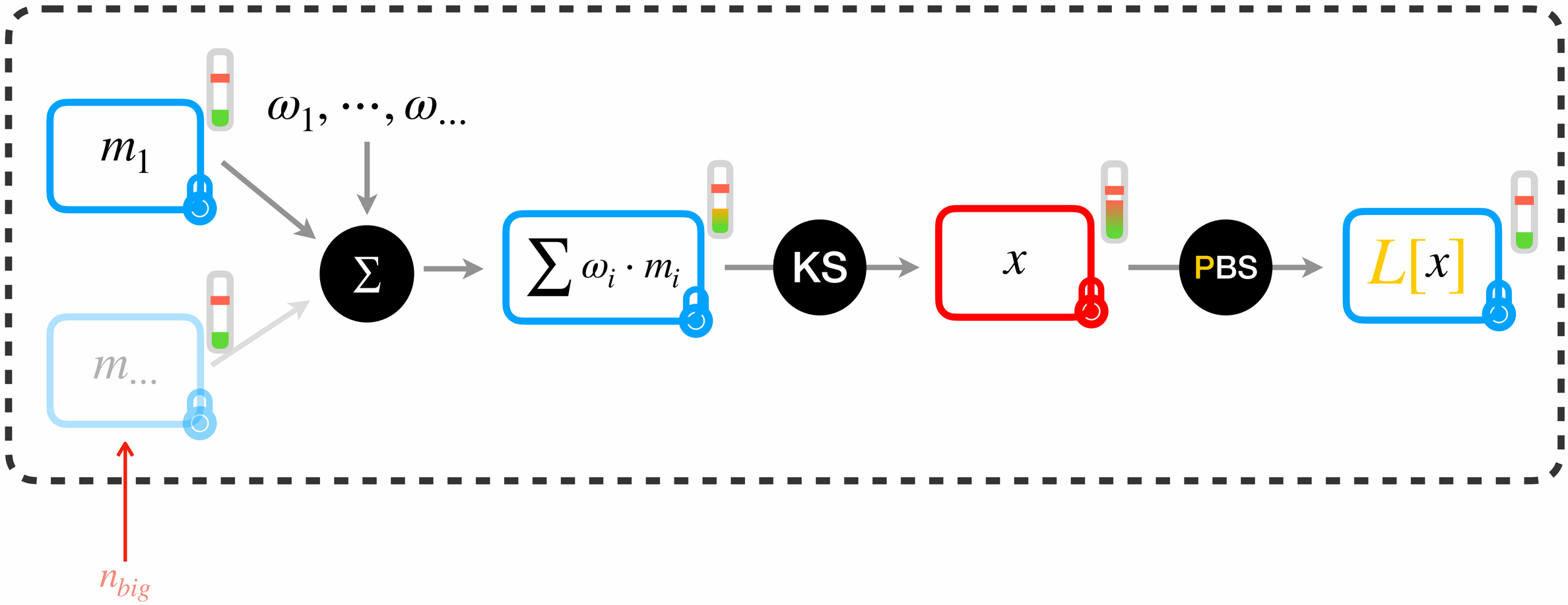


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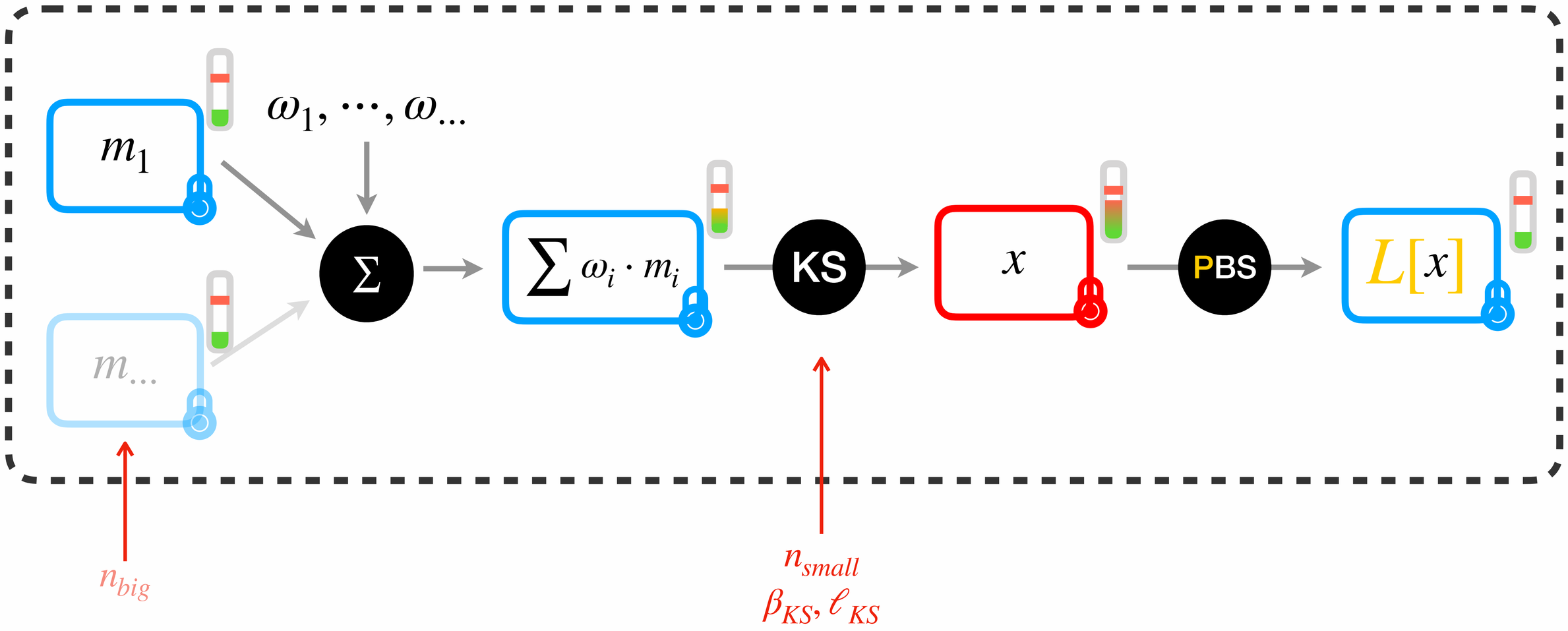


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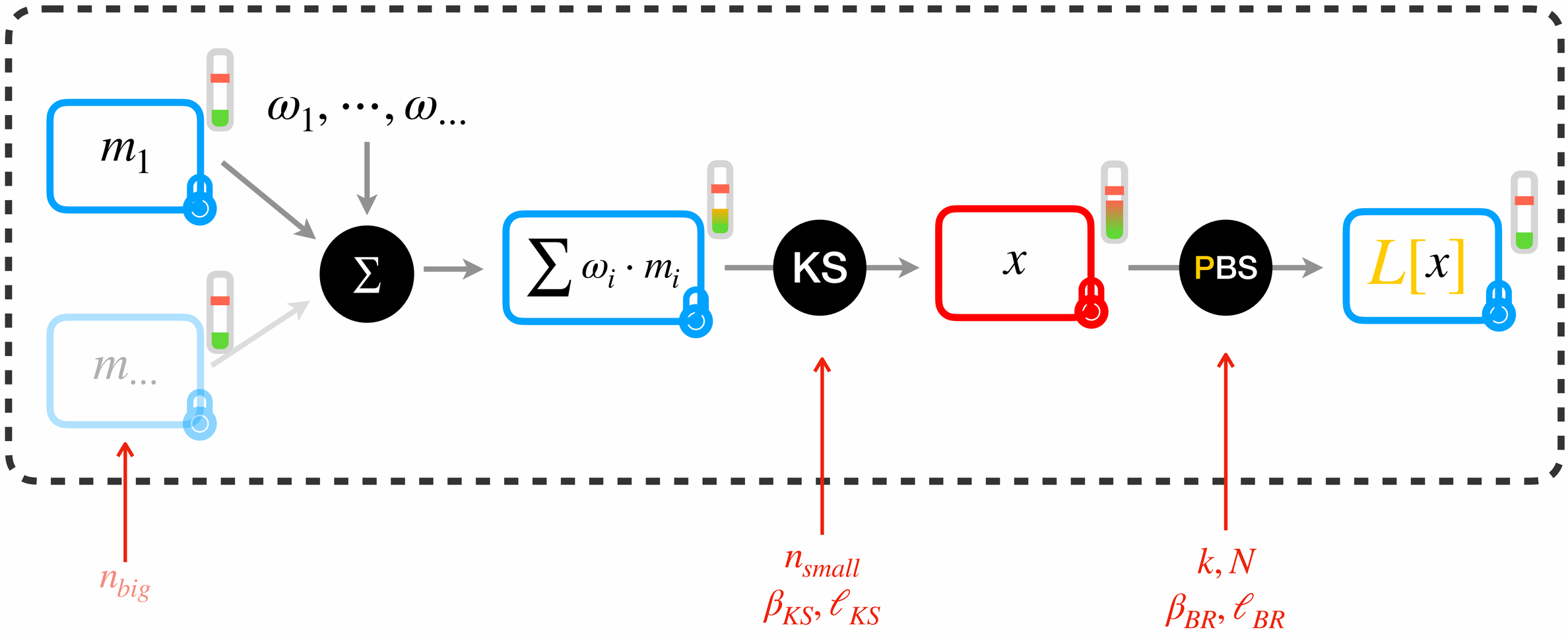


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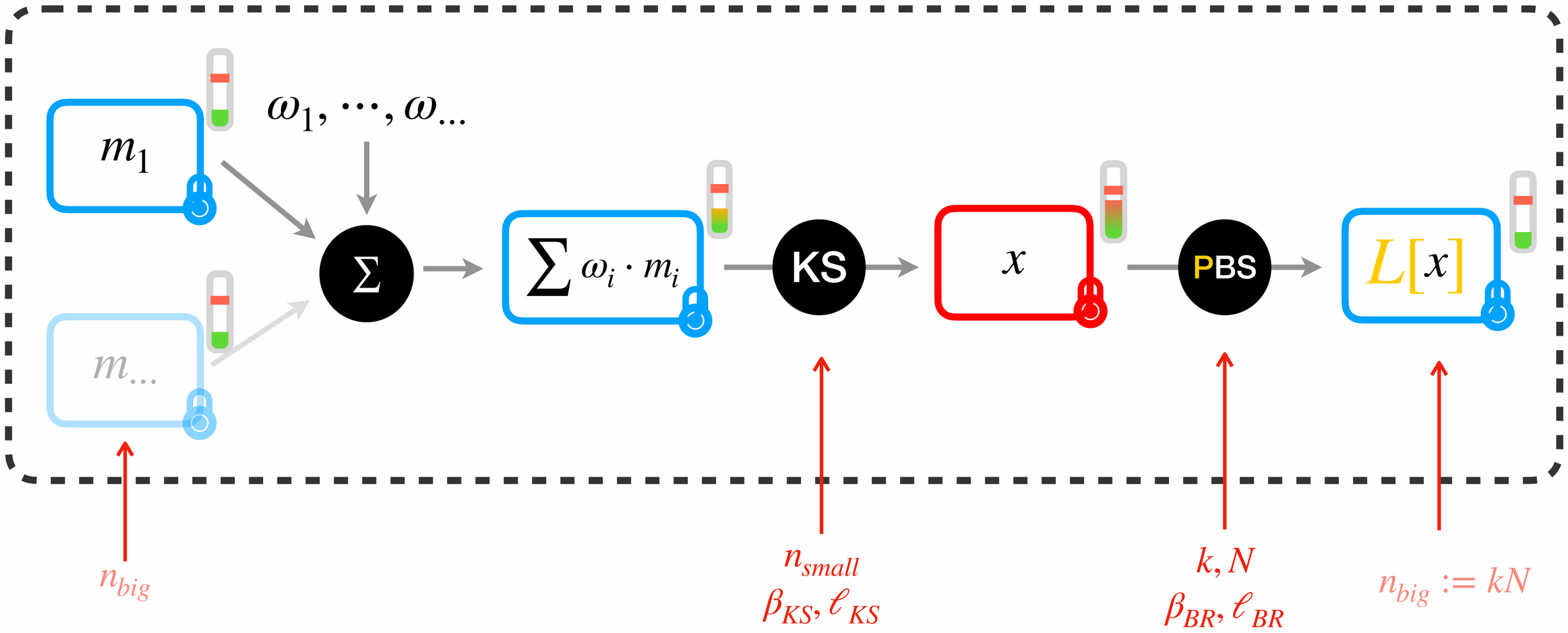


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# Goals

Translate a function $f$ into a graph of TFHE operators

with



**[APS]** M. R. Albrecht, R. Player and S. Scott. On the concrete hardness of Learning with Errors. Journal of Mathematical Cryptology 2015

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Security



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# Goals

Translate a function $f$ into a graph of TFHE operators

with

Security

Correctness

Efficiency



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# Security 



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# Security 

## Lattice Estimator [APS]

Reference tool to estimate the security of lattice-based encryption scheme **[APS]**



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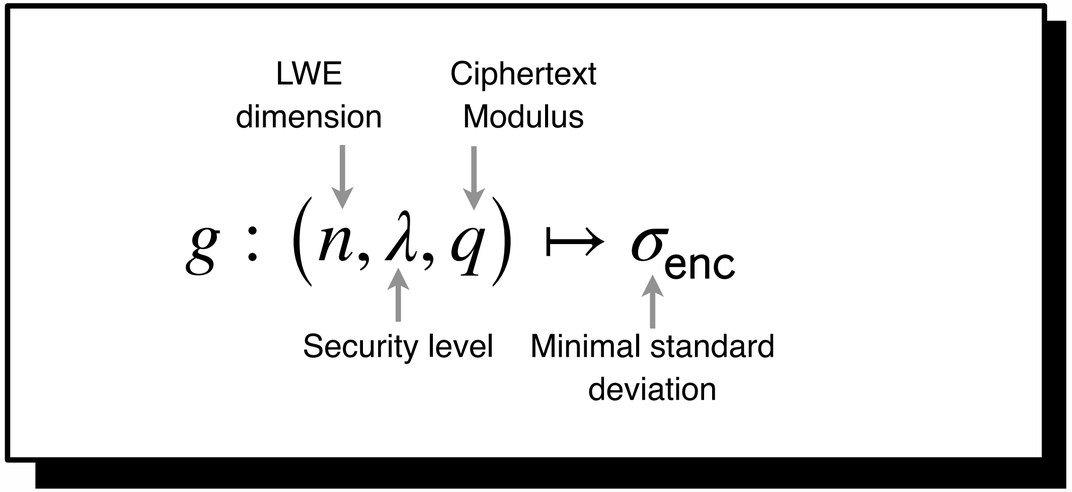




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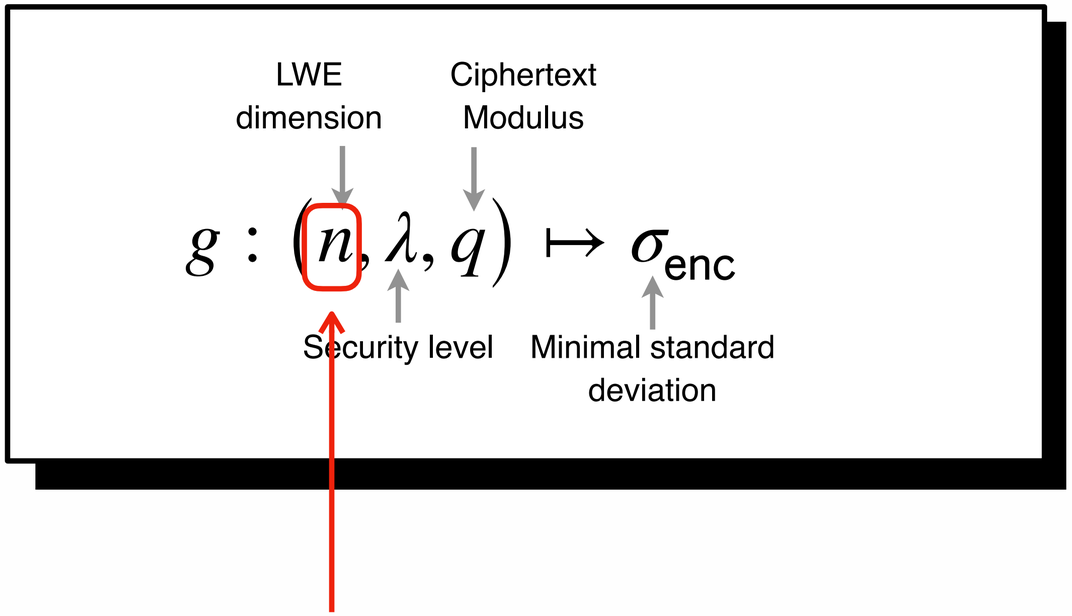




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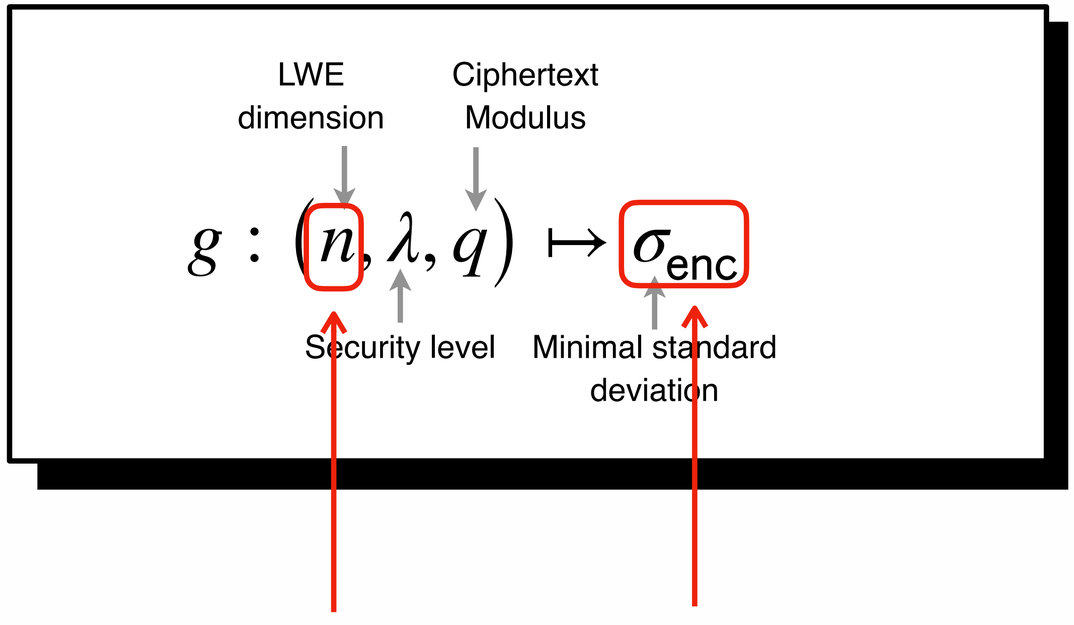




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# Efficiency 🚀



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# Efficiency 🚀

**Rule of thumb**

The smaller the parameters, the faster the computation



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# Efficiency 🚀

**Rule of thumb**

The smaller the parameters, the faster the computation

🤔 **How can we rank the feasible parameters ?**



**[APS]** M. R. Albrecht, R. Player and S. Scott. On the concrete hardness of Learning with Errors. Journal of Mathematical Cryptology 2015

# Efficiency 🚀

**Rule of thumb**

The smaller the parameters, the faster the computation

🤔 **How can we rank the feasible parameters ?**

$\Longrightarrow$

**Cost Model** as a surrogate of the execution time

*Ex: algorithmic complexities*



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# Correctness ✅

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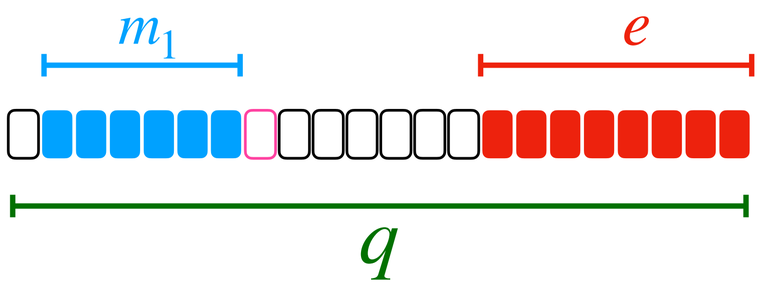
**Correctness Condition**

$$|e| < \frac{\Delta}{2}$$



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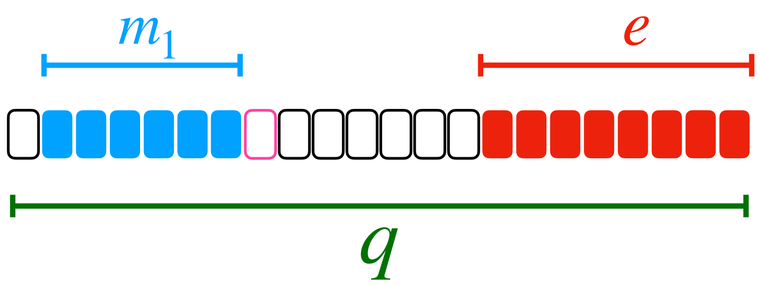


⚠️ $e$ can not be publicly known without compromising the security

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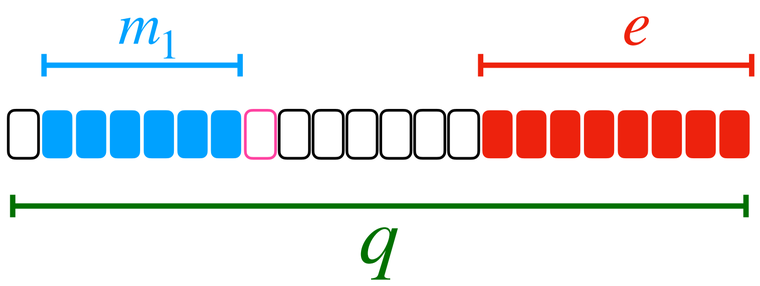


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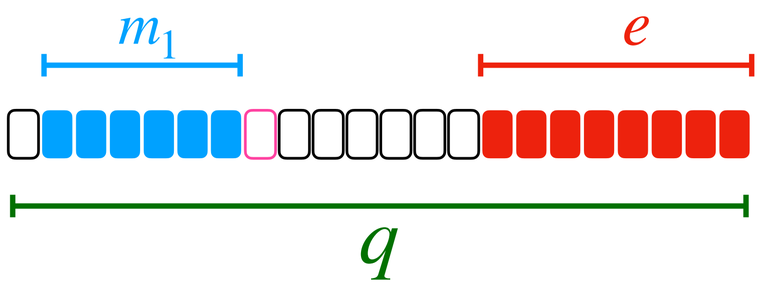


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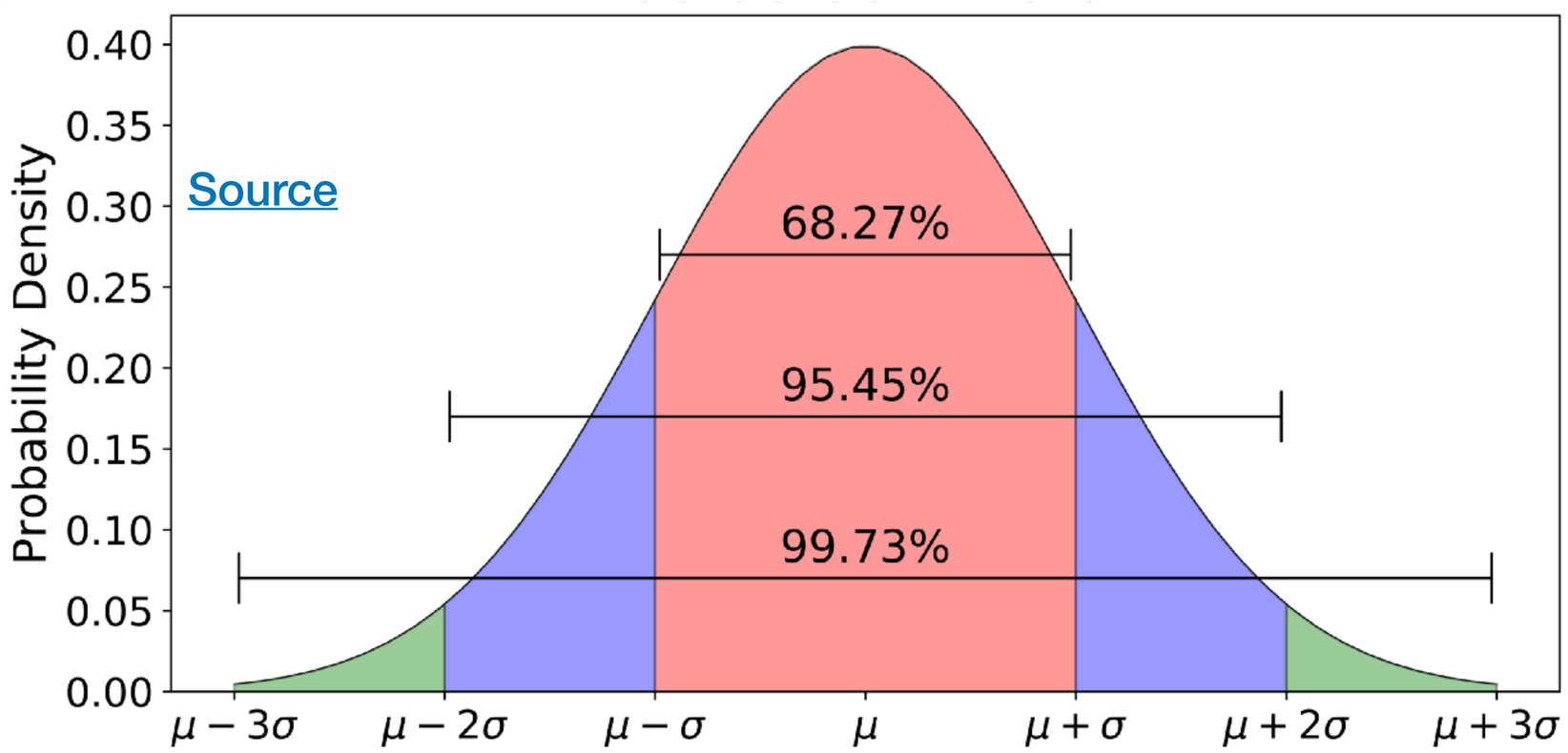
$\Longrightarrow$ **Noise Model** to track the noise distribution throughout the computation

# Correctness ✅

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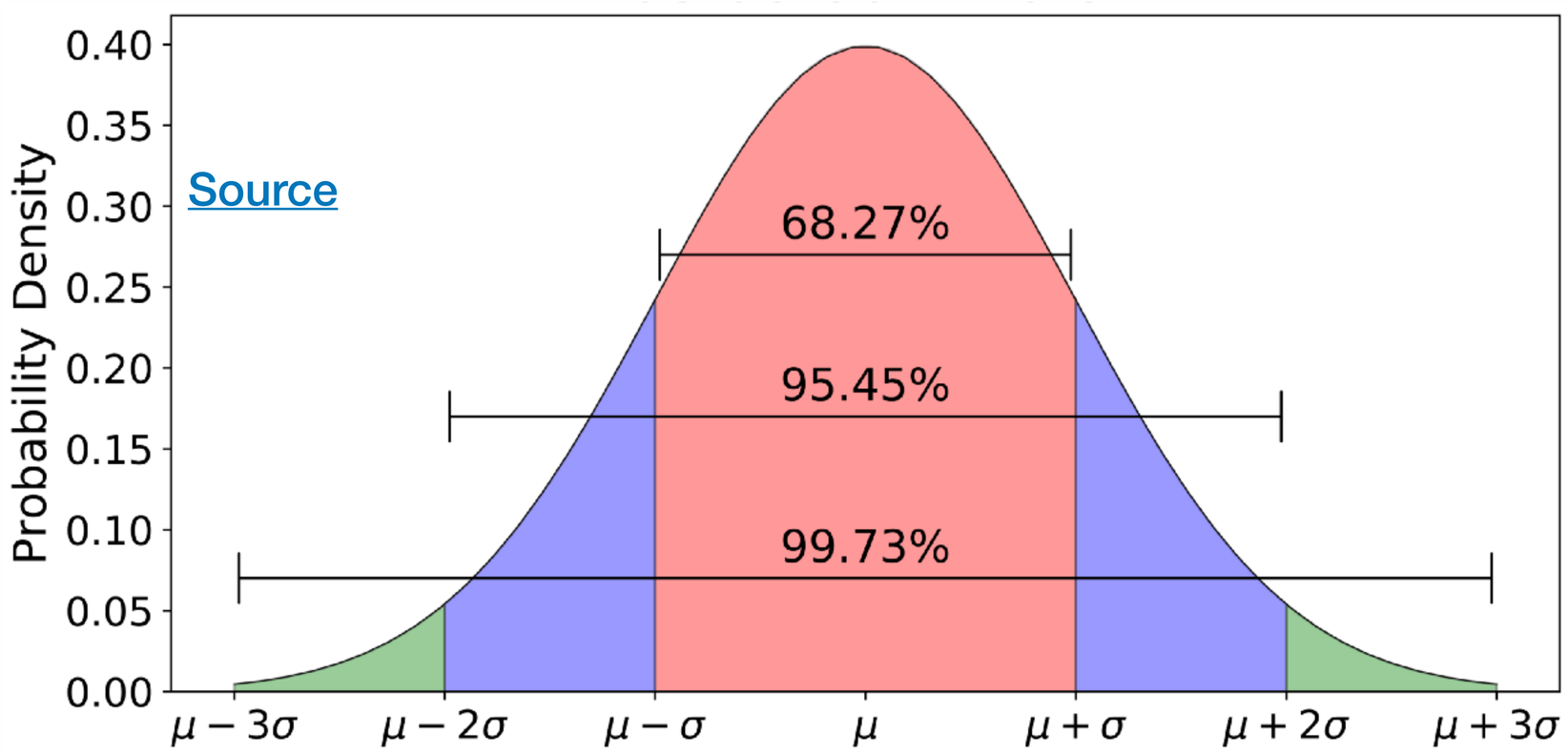


e is drawn from $\mathcal{N}\left(\mu = 0, \sigma^2\right)$ $\Longrightarrow$ $\Pr\left[e \notin [-\kappa \cdot \sigma, \kappa \cdot \sigma]\right] = p_{\text{fail}}$

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Noise Distribution (public)

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$\Leftrightarrow$

Noise bound

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# Optimization Problem

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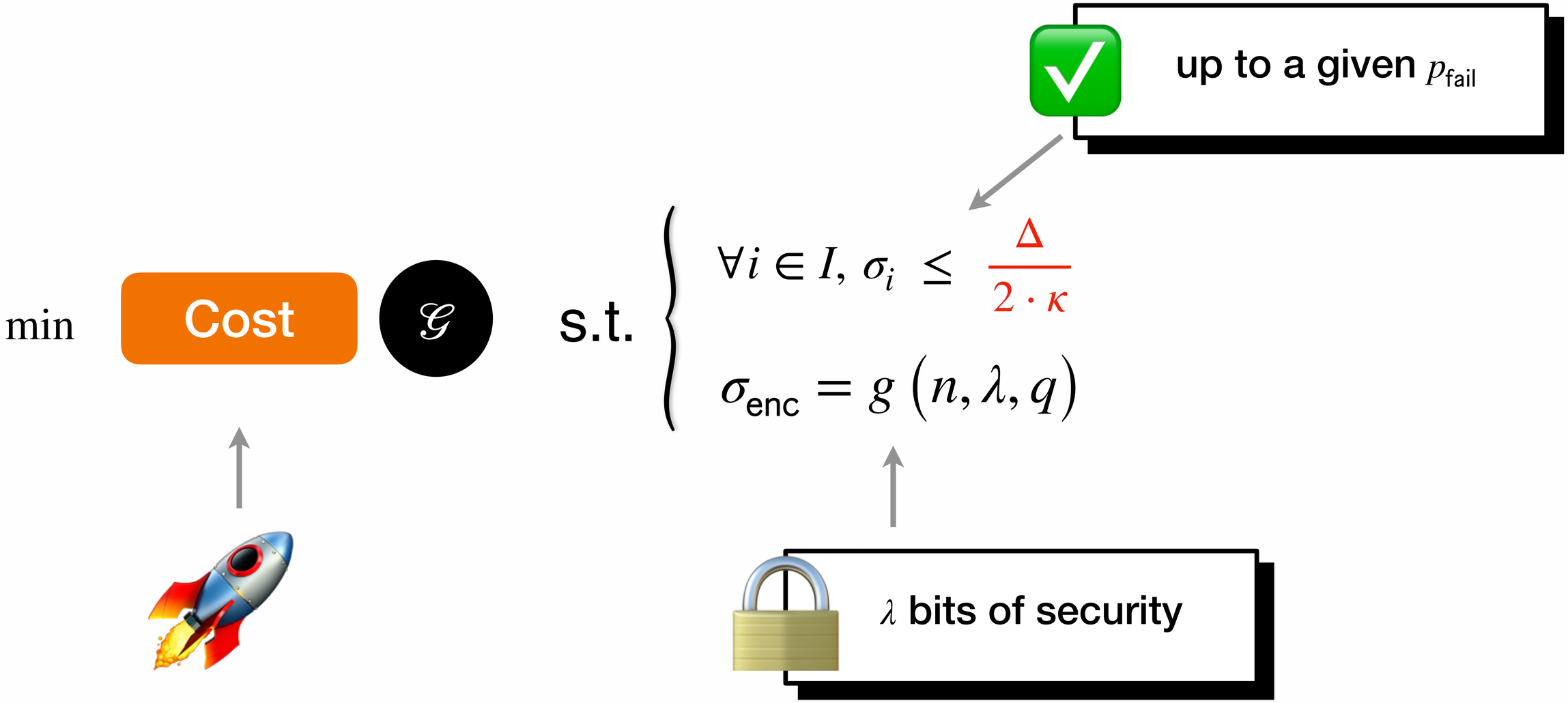

# Conclusion

# What's next ?

## Translation

More than one possible translation: how to find the best one ?

## Multi Parameter Set

One parameter set vs several parameter set

## Failure Probability

At operator level vs at graph level

## Optimization Problem

Simplification with domination reasoning

# Part II

How to translate an arbitrary DAG into an FHE DAG i.e., a DAG only composed of

linear operations and 

?

How to convert a TFHE DAG into a binary executable ?

# Contact and Links

samuel.tap@zama.ai

zama.ai

github.com/zama-ai/concrete

community.zama.ai

ZAMA