RSA Cryptosystem Generation of Public and Private Keys in Rust

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***Abstract*—The goal of our project was to develop two programs. The first program would be for determining large prime numbers and the second program for using the first application to create public and private key pairs and using these pairs to encrypt/decrypt a message.**

***Index Terms*—RSA, rust, key generation, large prime numbers**

I. Introduction

This project under the supervision of Dr. Elise de Doncker is to implement the numerous algorithms needed to create a public and private RSA key pair set.

II. Research

At the start of our project we first researched what an RSA key is and how to generate them and discovered it only takes a few steps. The first step is to create two prime numbers. The larger the prime number the better the encryption. Then with these two prime numbers we would be able to create a public and private key [1].

We found two algorithms for finding if a number is prime or not. The first is the AKS Primality Test. This primality test is the best deterministic primality test known in terms of time complexity [2]. This being said it is still very slow when compared to non deterministic primality testing algorithms. So we also looked at the Miller Rabin probabilistic primality tests to also test numbers which runs much faster than the AKS primality test. In the end, and explained later, we chose the Miller-Rabin test, in part because of performance, and in part because the language we were using has not yet implemented any logarithm operations for its BigInt types.

III. Design

First we needed to select a programming language. We both had expressed interest in the new beta programming language Rust and after some research we discovered that it would be a good language to use. Rust is an open-source compiled language that is syntactically similar to C and C++ with an emphasis on control of memory layout and safety.

After choosing what language we were going to use the first thing we did was see if someone had already created a primality testing library that we could use. Upon inspecting the algorithms that were available in the libraries on crates.io used the Sieve of Eratosthenes technique. This algorithm calculates all numbers that are less than n that are prime. This is good for if you need small prime numbers, but our goal was to generate large prime numbers.

With no library available we decided that we would have to create our own large prime generator. Because there was no library available we decided to pull that section of code out and create our own library. Once the project is completed that section of code will be publicly available on GitHub under MIT/Apache-2.0 license.

IV. Implementation

The prime generator used the Miller-Rabin testing for our generation of prime numbers, for reasons discussed below. Before any test was used we made sure the number wasn't even to make sure we aren't using lots of time for easy to find compound numbers.

Here we have a slight discrepancy between the initial design proposal and our finished product. In the design document our group submitted, we said we would be using the AKS deterministic polynomial-time test for primality. Unfortunately, this was not possible using Rust. For encryption keys, we want arbitrarily large numbers, which requires the use of a BigInt type. However, there are no logarithm operations supported on this data type, and logarithms are essential to the calculations involved in the AKS test for primality. Rather than expend effort contributing code to the core of the language we chose to use so that we could implement this primality test, we decided instead that the Miller-Rabin probabilistic test for primality would meet our needs for the purposes of this project. Had we chosen a different, more mature language at the start of our project, we would have been able to implement the AKS test. However, by the time we realized that implementing this test would be impossible in the language that we chose, we had already implemented too much of the rest of the program for us to discard what we had accomplished.

The key generator calls the prime generation method of the first program and that takes care of all the work for prime number generation. After it gets the prime numbers p and q we use those to compute n which is simply p times q. Next we determine (n) which is simply $n - (p + q - 1)$. After these are all computed we create a good e value and compute d. The e value is simply a number
$1 <$ e $< (n)$. Then d = e^{-1}(mod((n)), which is the private key part of the encryption.

V. Testing

Since the two programs were separate we had a rather easy time testing each one out individually. For our prime number generator it was fairly simple to determine that our test is working correctly. All that we had to do was send in some numbers that we know are prime and some that are not prime and make sure that the test responded accordingly.

To test our program we took an example and tested that our input and output matched. For the example we took some small prime numbers just for testing. So for p and q we took the values 61 and 53 respectively. Then we know that it should calculate for n as 3233 and we know that the totient that it creates should be 3120. For e we need to assign it a value so we can calculate the other values correctly. So for this example we chose 17 and when we calculate the d value we get 2753.

Now using those values we can compute what A should encrypt to and determine if our program is encrypting the value correctly. So the ASCII value of A is 65, so our ciphertext would be $65^{17} \mathrm{mod} 3233 = 2790$. So for our test values we know that A should be encrypted to 2790. For testing our decryption part we just have to make sure it calculates 2790 back to A.

A. *Performance Measurements*

As we were implementing this program, it became clear that modulo exponentiation was the bottleneck in the performance of our program. Modulo exponentiation is used in the process of generating prime numbers, as well as in the processes of encryption and decryption. With such a ubiquitous algorithm, it was important that we had an efficient algorithm. We began with a naive implementation of modulo exponentiation, and we also implemented an optimized algorithm documented by Bruce Schneier [3].

It is clear to see that encryption and decryption both follow polynomial curves as the number of bits used in the encryption process increase, with decryption increasing at a much faster rate than decryption. This is exhibited in Figure 1.

There is a substantial difference in the rates of growth in reference to the time these two algorithms take. The naive algorithm grows exponentially, as it is an exponentiation function. While the complexity scales linearly with the power to which a number is being brought, in the context of input size, that is the number of bits in the power, the rate of growth is approximately 2^s

We can see in this code that the time complexity of this algorithm is going to be linear in relation to the

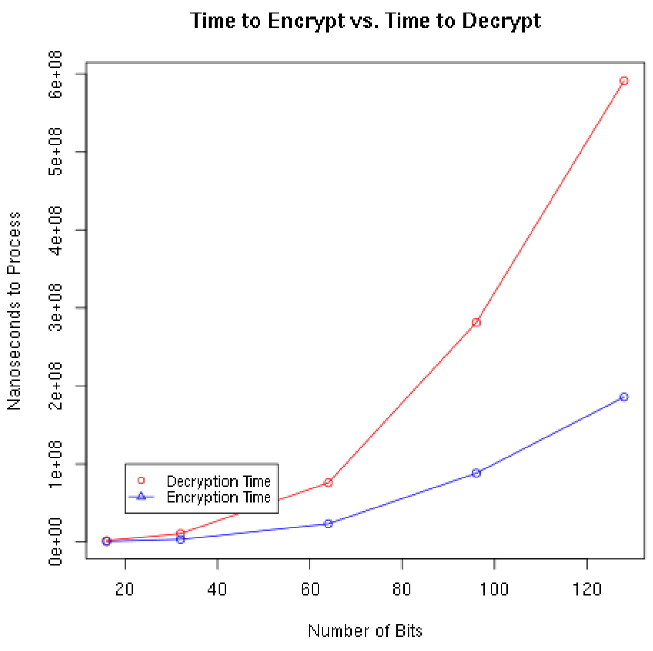


Fig. 1. Increasing time with increasing bit-size of keys

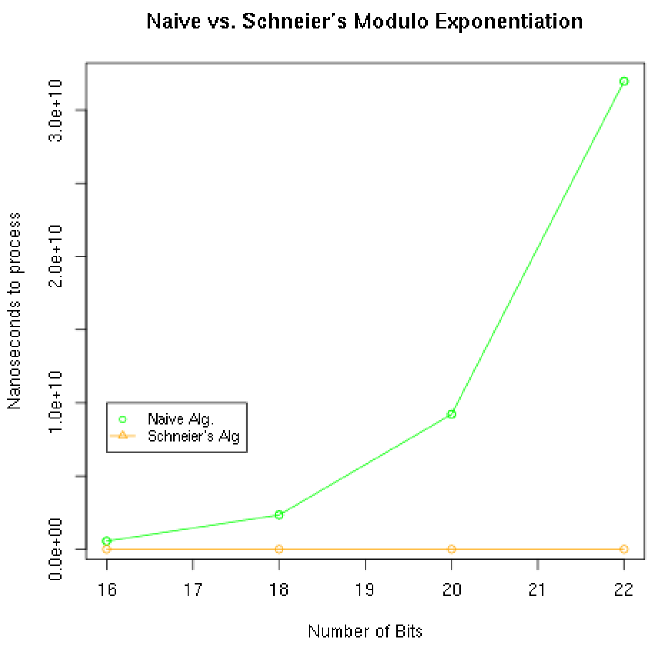


Fig. 2. Time complexity between a naive implementation of modulo exponentiation and Schneier's modulo exponentiation[3]

```
/**
 * Modulo Exponentiation based on Bruce Schneier's
 * algorithm discussed in his book Applied Cryptography
 */
pub fn schneier_mod_exp(base: BigUint, power: BigUint,
                        modulo: BigUint) -> BigUint{
    let mut result = BigUint::one();
    let mut pow = power.clone();
    let mut basic = base.clone();
    let two = BigUint::from_usize(2).unwrap();
    let zero = BigUint::zero();
    while pow.gt(&zero){
        if pow.clone().rem(&two).eq(&BigUint::one()){
            result = (result.mul(&basic)).rem(&modulo);
        }
        pow = pow.clone().shr(1);
        basic = basic.clone().mul(&basic).rem(&modulo);
    }
    return result;
```

Fig. 3. Schneier's Modulo Exponentiation Implemented in Rust[3]

input size s, which is defined as the number of bits required to represent the power to which the base is being brought. Each iteration through the loop, the power is bit-shifted to the right by one bit, until eventually the power reaches zero. This complexity for the algorithm should be immediately apparent when studying the code in Figure 3, and studying the chart in Figure 2 will provide a clue that this algorithm is likely linear in time complexity.

We were able to generate the data used for this analysis using the Rust language's built in testing suite. We made extensive use of Rust's testing capabilities, following a test-drived development lifecycle, and using tests to verify the correctness of the algorithms being implemented. The actual analysis of the data was conducted using the R programming language, which is particularly well-suited for this type of analysis.

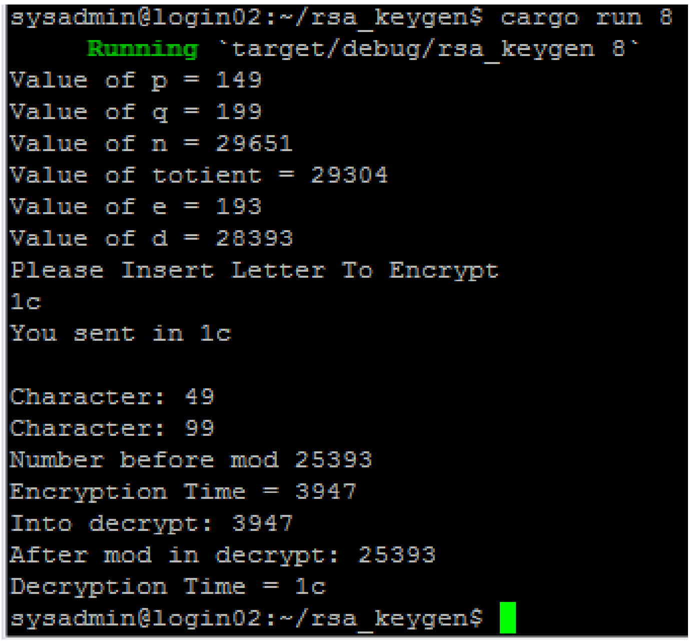
VI. Goals Reached

Our initial goals were to create a primality testing section and to create public and private keys. We were successful in testing for prime numbers with a a non-deterministic method. Another goal of ours was to encrypt and decrypt an input string and we were also successful in this goal. Using the fast modulo exponentiation shown in Figure 3, we were able to use keys of up to 224 bits in a practical amount of time.

One of our stretch goals was not reached. We were hoping to be able to use our program with other programs that use public and private key pairs such as ssh. We determined this to be out of scope because of the strict standards set up by the IEEE.

VII. User Guide

To run this program it is very simple. First you will need to build the program so cd to the directory and run



```
sysadmin@login02:~/rsa_keygen$ cargo run 8
     Running `target/debug/rsa_keygen 8`
Value of p = 149
Value of q = 199
Value of n = 29651
Value of totient = 29304
Value of e = 193
Value of d = 28393
Please Insert Letter To Encrypt
1c
You sent in 1c

Character: 49
Character: 99
Number before mod 25393
Encryption Time = 3947
Into decrypt: 3947
After mod in decrypt: 25393
Decryption Time = 1c
sysadmin@login02:~/rsa_keygen$
```

Fig. 4. Screenshot of the RSA program output

```
cargo build
```

This compiles the program then to run it we simply do

```
cargo run X
```

Where X is the bit size. The larger the bit size the longer the program will run. After you run the application it will ask for a character you would like encrypted then decrypted. Figure 4 exhibits an example run of the program.

VIII. Conclusion

The program does generate large prime numbers as it is advertised to do, however with the larger bit sizes the time to calculate a prime number takes a very long time. Keeping in mind that finding a prime number is polynomial in terms of complexity it should be understood that there is no better way to do this. This program is capable of encrypting using arbitrary key sizes, but the performance of Rust's arbitrarily sized integers is still lacking, so performance degrades quickly the larger the key size gets. Additionally, we focused on code readability and reusability, and so did not implement any particularly sophisticated optimizations in our encryption program. As a result, we are only able to use keys up to 224 bits in a practical amount of time. This is greatly already greatly improved by Schneier's modulo exponentiation algorithm, and is a better result than we had anticipated. However the language is still in its beta phases so as it becomes more developed there will be more support for complex computations. Even with these challenges we were able to create a program that met most of our goals and was able to actually encrypt and decrypt information so we think that the project was successful. [3]

References

[1] S. Mohan and B. Adiga, “Fast algorithms for implementing rsa public key cryptosystem,” *Electronic Letters*, vol. 21, p. 761, Aug. 1985.

[2] R. E. Neapolitan, *Foundations of Algorithms*. Burlington, MA: Jones & Bartlett Learning, 2015.

[3] B. Schneier, *Applied Cryptography (2Nd Ed.): Protocols, Algorithms, and Source Code in C*. New York, NY, USA: John Wiley & Sons, Inc., 1995.