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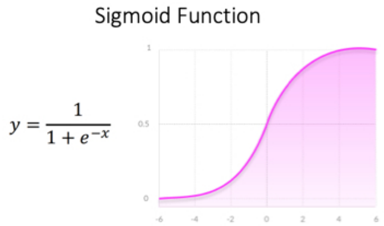



```
* If X is high, the value is approximately 1
* if X is small, the value is approximately 0
```

Import Libraries

In [2]:
```python
import numpy as np
```

Define the Sigmoid Function

In [3]:
```python
def sigmoid(sum_func):
  return 1 / (1 + np.exp(-sum_func))
```

In [4]:
```python
sigmoid(0)
```

Out[4]:
```
0.5
```

In [5]:
```python
np.exp(2)
```

Out[5]:
```
7.38905609893065
```

In [6]:
```python
np.exp(1)
```

Out[6]:
```
2.718281828459045
```

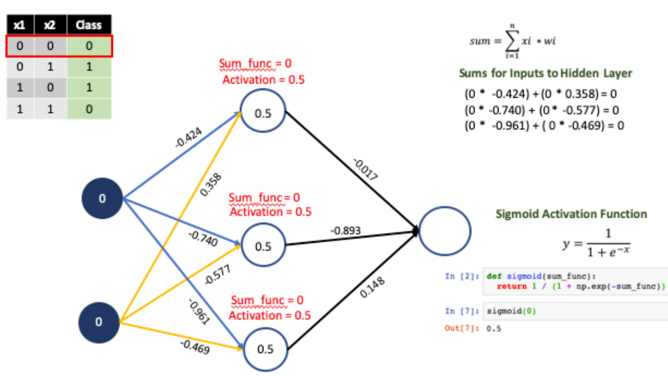
In [7]:
```python
sigmoid(40)
```

Out[7]:
```
1.0
```

In [8]:
```python
sigmoid(-20.5)
```

Out[8]:
```
1.2501528648238605e-09
```

Input Layer to Hidden Layer



Define "Inputs, outputs and weights" as Numpy arrays

Inputs

In [9]:
```python
inputs = np.array([[0,0],
                   [0,1],
                   [1,0],
                   [1,1]])
```

In [10]:
```python
inputs
```

Out[10]:
```
array([[0, 0],
       [0, 1],
       [1, 0],
       [1, 1]])
```

In [11]:
```python
inputs.shape
```

Out[11]:
```
(4, 2)
```

Outputs

In [12]:
```python
outputs = np.array([[0],
```

```
                  [1],
                  [1],
                  [0]])
```

In [13]:
```
outputs.shape
```

Out[13]:
```
(4, 1)
```

Weights

These weights are for the connection between the inputs and the hidden layer

In [14]:
```
# First row holds the weights for x1, 2nd row contains the weights for x2

weights_0 = np.array([[-0.424, -0.740, -0.961],
                      [0.358, -0.577, -0.469]])
weights_0.shape
```

Out[14]:
```
(2, 3)
```

These weights are for the connection between the hidden layer and the output

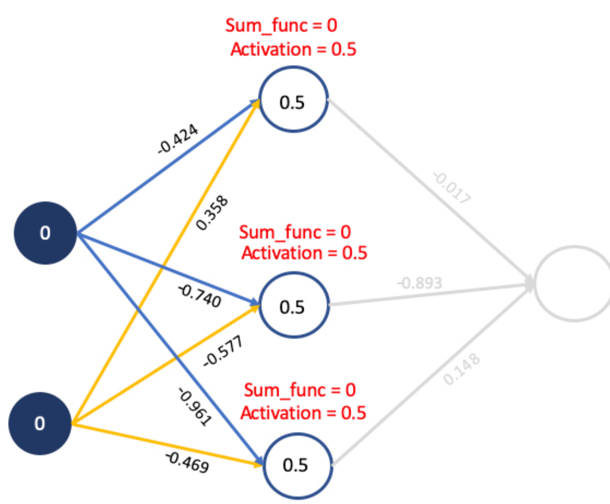
In [15]:
```
weights_1 = np.array([[-0.017],
                      [-0.893],
                      [0.148]])
weights_1.shape
```

Out[15]:
```
(3, 1)
```

Epochs & Learning Rate

In [16]:
```
epochs = 100
learning_rate = 0.3
```

In [17]:
```
#for epoch in epochs:
```



Dealing with the first side

In [18]:
```
input_layer = inputs
input_layer
```

Out[18]:
```
array([[0, 0],
       [0, 1],
       [1, 0],
       [1, 1]])
```

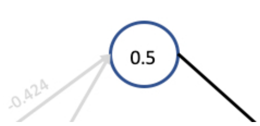
In [19]:
```
# "sum_synapse_0" This holds the sum function total of weights for the hidden layer
# For the Output: Each row holds the sum_func for each input data  [0,0,0 -> data 0,0],[0.358, -0.577, -0.469 --> 0,1]
# The dot product does the matrix multiplication and also the sum

sum_synapse_0 = np.dot(input_layer, weights_0)
sum_synapse_0
```

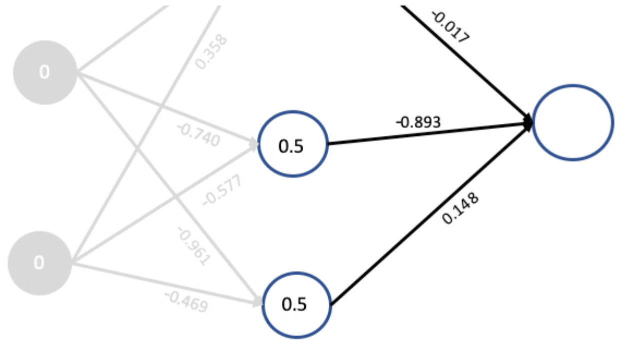
Out[19]:
```
array([[ 0.   ,  0.   ,  0.   ],
       [ 0.358, -0.577, -0.469],
       [-0.424, -0.74 , -0.961],
       [-0.066, -1.317, -1.43 ]])
```

In [20]:
```
# Computing the Sigmoid function for the Hidden layer

hidden_layer = sigmoid(sum_synapse_0)
hidden_layer
```

Out[20]:
```
array([[0.5       , 0.5       , 0.5       ],
       [0.5885562 , 0.35962319, 0.38485296],
       [0.39555998, 0.32300414, 0.27667802],
       [0.48350599, 0.21131785, 0.19309868]])
```

Dealing with the 2nd side





In [21]:
```
weights_1
```

Out[21]:
```
array([[-0.017],
       [-0.893],
       [ 0.148]])
```

In [22]:
```
# "sum_synapse_1" This holds the sum function total of weights for the output layer
# For the Output: Each row holds the sum_func for each input data

sum_synapse_1 = np.dot(hidden_layer, weights_1)
sum_synapse_1
```

Out[22]:
```
array([[-0.381     ],
       [-0.27419072],
       [-0.25421887],
       [-0.16834784]])
```

In [23]:
```
output_layer = sigmoid(sum_synapse_1)
output_layer
```

Out[23]:
```
array([[0.40588573],
       [0.43187857],
       [0.43678536],
       [0.45801216]])
```

XOR Operator – Error (Loss Function)

Error = correct (class) - prediction

x1	x2	Class	Prediction	Error
0	0	0	0.405	-0.405
0	1	1	0.431	0.569
1	0	1	0.436	0.564
1	1	0	0.458	-0.458
		Average Error = abs(error) = 0.499		

In [24]:
```
outputs
```

Out[24]:
```
array([[0],
       [1],
       [1],
       [0]])
```

In [25]:
```
output_layer
```

Out[25]:
```
array([[0.40588573],
       [0.43187857],
       [0.43678536],
       [0.45801216]])
```

In [26]:
```
error_output_layer = outputs - output_layer
error_output_layer
```

Out[26]:
```
array([[-0.40588573],
       [ 0.56812143],
       [ 0.56321464],
       [-0.45801216]])
```

In [27]:
```
average_error = np.mean(abs(error_output_layer))
average_error
```

Out[27]:
```
0.49880848923713045
```

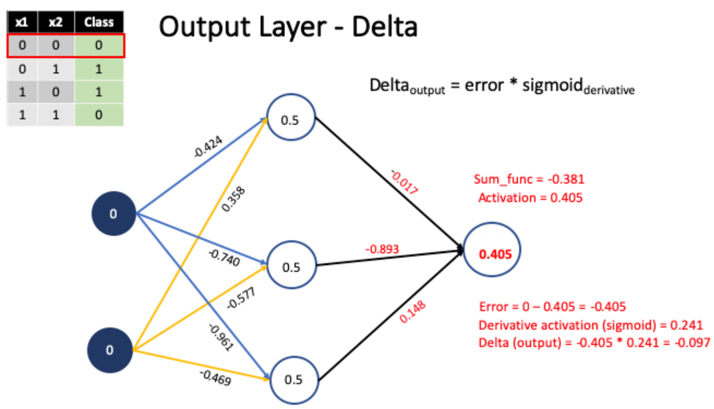
Sigmoid Derivative

$$y = \frac{1}{1 + e^{-x}}$$

$$d = y * (1 - y)$$

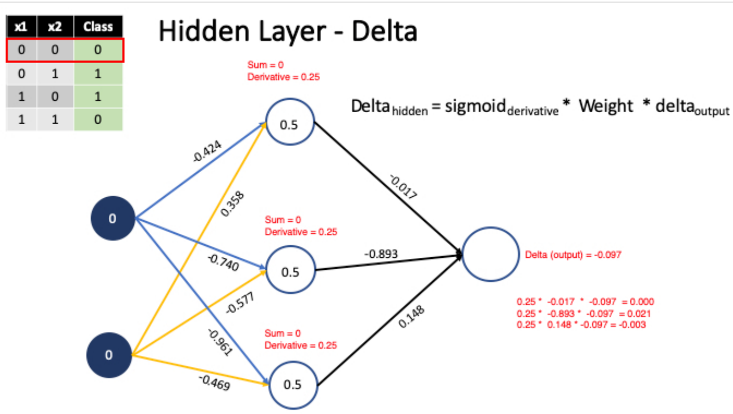
```
In [28]: def sigmoid_derivative(sigmoid):
             return sigmoid * (1 - sigmoid)
```

Delta output Calculation




```
In [29]: # output_layer holds the results of our application of the sigmoid, computed above

         output_layer

Out[29]: array([[0.40588573],
                [0.43187857],
                [0.43678536],
                [0.45801216]])

In [30]: # derivative_output is our Derivative of the activation function (sigmoid) which we have on the slide
         # each row is for each instance of our input dataset

         derivative_output = sigmoid_derivative(output_layer)
         derivative_output

Out[30]: array([[0.2411425 ],
                [0.24535947],
                [0.24600391],
                [0.24823702]])

In [31]: error_output_layer

Out[31]: array([[-0.40588573],
                [ 0.56812143],
                [ 0.56321464],
                [-0.45801216]])

In [32]: # Delta output
         # each row is for each instance of our input dataset

         delta_output = error_output_layer * derivative_output
         delta_output

Out[32]: array([[-0.0978763 ],
                [ 0.13939397],
                [ 0.138553  ],
                [-0.11369557]])
```

Delta calculations for the Hidden Layer




```
In [33]: delta_output

Out[33]: array([[-0.0978763 ],
                [ 0.13939397],
                [ 0.138553  ],
```

```
       [-0.11369557]])
```

In [34]:
```
weights_1
```

Out[34]:
```
array([[-0.017],
       [-0.893],
       [ 0.148]])
```

NOTE THAT:

```
* Lets deal with this part first (Weight  * delta_output)
* Notice that we will get an error below becuase of the shape of the weights_1  (Transpose)
```

In [35]:
```
delta_output_x_weight = delta_output.dot(weights_1)
```

```
---------------------------------------------------------------------------
ValueError                                Traceback (most recent call last)
<ipython-input-35-50b740e5a31c> in <module>
----> 1 delta_output_x_weight = delta_output.dot(weights_1)

ValueError: shapes (4,1) and (3,1) not aligned: 1 (dim 1) != 3 (dim 0)
```

In [36]:
```
weights_1.shape
```

Out[36]:
```
(3, 1)
```

In [37]:
```
weights_1T = weights_1.T
weights_1T
```

Out[37]:
```
array([[-0.017, -0.893,  0.148]])
```

In [38]:
```
weights_1T.shape
```

Out[38]:
```
(1, 3)
```

Each one of the weights will have to be multiplied by each delta_output for each data instance

```
array([[-0.017],
       [-0.893],
       [ 0.148]])
```

In [39]:
```
delta_output_x_weight = delta_output.dot(weights_1T)
delta_output_x_weight
```

Out[39]:
```
array([[ 0.0016639 ,  0.08740354, -0.01448569],
       [-0.0023697 , -0.12447882,  0.02063031],
       [-0.0023554 , -0.12372783,  0.02050584],
       [ 0.00193282,  0.10153015, -0.01682694]])
```

NOTE THAT:

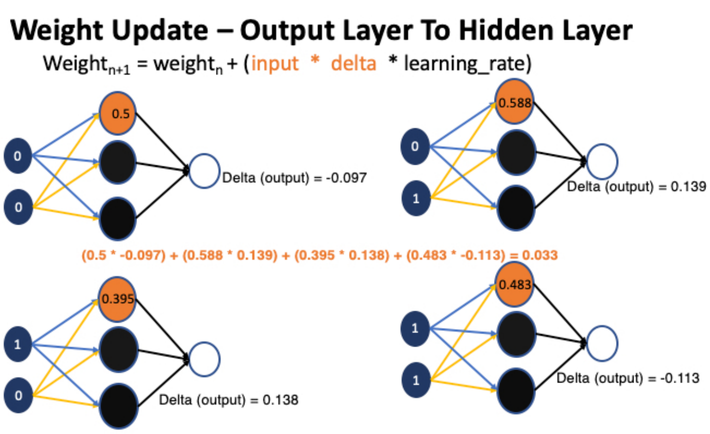
```
* Now we need to deal with the last part of the equation
* sigmoid_derivative * delta_output_x_weight
```

In [40]:
```
hidden_layer
```

Out[40]:
```
array([[0.5       , 0.5       , 0.5       ],
       [0.5885562 , 0.35962319, 0.38485296],
       [0.39555998, 0.32300414, 0.27667802],
       [0.48350599, 0.21131785, 0.19309868]])
```

In [41]:
```
#  Each row in the output of delta_hidden_layer is for the data input values

delta_hidden_layer = delta_output_x_weight * sigmoid_derivative(hidden_layer)
delta_hidden_layer
```

Out[41]:
```
array([[ 0.00041597,  0.02185088, -0.00362142],
       [-0.00057384, -0.02866677,  0.00488404],
       [-0.00056316, -0.02705587,  0.00410378],
       [ 0.00048268,  0.01692128, -0.00262183]])
```



We will deal with the (input * delta) first

- The first column in "hidden_layer" holds the activation value for the first neuron

In [42]:
```
hidden_layer
```

Out[42]:
```
array([[0.5       , 0.5       , 0.5       ],
       [0.5885562 , 0.35962319, 0.38485296],
```

```
       [0.39555998, 0.32300414, 0.27667802],
       [0.48350599, 0.21131785, 0.19309868]])
```

In [43]:
```python
delta_output
```

Out[43]:
```
array([[-0.0978763 ],
       [ 0.13939397],
       [ 0.138553  ],
       [-0.11369557]])
```

- We need to multiply the "inputs" by "delta" however, for the matrix multiplication we need to transpose the values in the hidden_layer, so we have all of them on one row for each neuron

In [44]:
```python
hidden_layerT = hidden_layer.T
hidden_layerT
```

Out[44]:
```
array([[0.5       , 0.5885562 , 0.39555998, 0.48350599],
       [0.5       , 0.35962319, 0.32300414, 0.21131785],
       [0.5       , 0.38485296, 0.27667802, 0.19309868]])
```

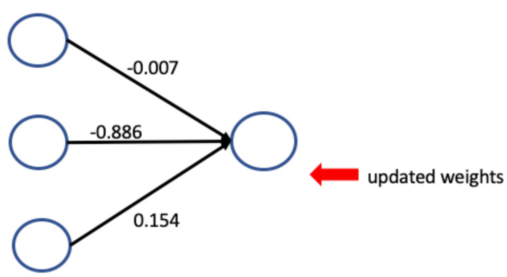
In [45]:
```python
input_x_delta1 = hidden_layerT.dot(delta_output)
input_x_delta1
```

Out[45]:
```
array([[0.03293657],
       [0.02191844],
       [0.02108814]])
```

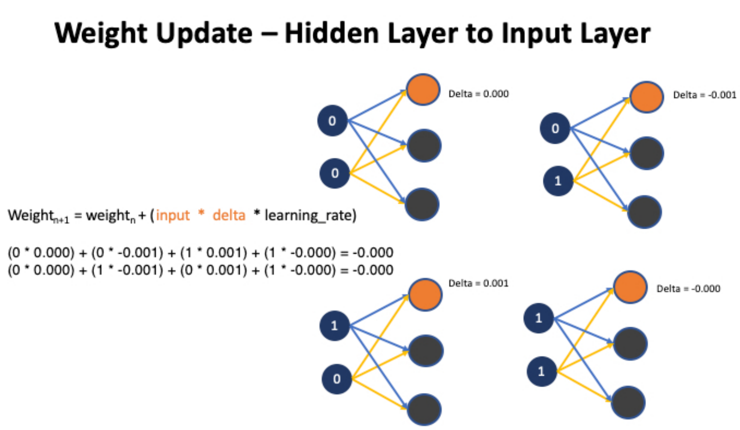
Let us now update the "weights_1"

In [48]:
```python
weights_1 = weights_1 + (input_x_delta1 * learning_rate)
weights_1
```

Out[48]:
```
array([[-0.00711903],
       [-0.88642447],
       [ 0.15432644]])
```



Dealing with the Hidden Layer to Input Layer



In [54]:
```python
# First column is X1, and 2nd column is X2 (our input values )

input_layer
```

Out[54]:
```
array([[0, 0],
       [0, 1],
       [1, 0],
       [1, 1]])
```

In [50]:
```python
delta_hidden_layer
```

Out[50]:
```
array([[ 0.00041597,  0.02185088, -0.00362142],
       [-0.00057384, -0.02866677,  0.00488404],
       [-0.00056316, -0.02705587,  0.00410378],
       [ 0.00048268,  0.01692128, -0.00262183]])
```

In [51]:
```python
# we need to transpose the values just as we did before

input_layerT = input_layer.T
input_layerT
```

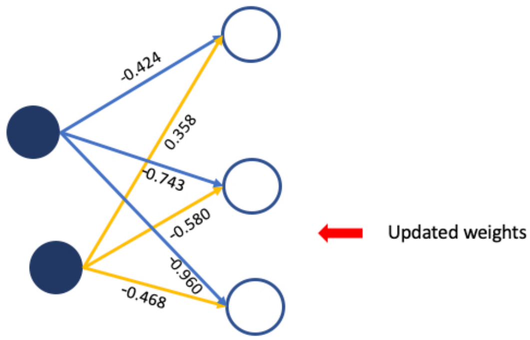
Out[51]:
```
array([[0, 0, 1, 1],
       [0, 1, 0, 1]])
```

In [52]:
```python
input_x_delta0 = input_layerT.dot(delta_hidden_layer)
input_x_delta0
```

Out[52]: array([[-8.04778516e-05, -1.01345901e-02, 1.48194623e-03],
       [-9.11603819e-05, -1.17454886e-02, 2.26221011e-03]])

In [53]:
```
weights_0 = weights_0 + (input_x_delta0 * learning_rate)
weights_0
```

Out[53]:
```
array([[-0.42402414, -0.74304038, -0.96055542],
       [ 0.35797265, -0.58052365, -0.46832134]])
```



So all the lines of code above, has allowed us to complete our first epoch. we will need to put all the code together so we can run multiple epochs

Complete Artificial Neural Network

In [55]:
```
#Importing Numpy
import numpy as np

# This is the sigmoid Function
def sigmoid(sum):
  return 1 / (1 + np.exp(-sum))

#This is the sigmoid derivative as used before
def sigmoid_derivative(sigmoid):
  return sigmoid * (1 - sigmoid)

# Our input values
inputs = np.array([[0,0],
                   [0,1],
                   [1,0],
                   [1,1]])
#Our output values
outputs = np.array([[0],
                    [1],
                    [1],
                    [0]])
```

In [89]:
```
# weights_0 = np.array([[-0.424, -0.740, -0.961],
#                       [0.358, -0.577, -0.469]])

# weights_1 = np.array([[-0.017],
#                       [-0.893],
#                       [0.148]])
```

Initializing our weights with random values

- **Note:** Multiplying the random number by 2 and subtracting by 1, allows us to have a mix of both positive and negative random numbers for the weights

In [90]:
```
weights_0 = 2 * np.random.random((2, 3)) - 1
weights_1 = 2 * np.random.random((3, 1)) - 1
```

In [91]:
```
epochs = 400000
learning_rate = 0.6
error = []

for epoch in range(epochs):
  input_layer = inputs
  sum_synapse0 = np.dot(input_layer, weights_0)
  hidden_layer = sigmoid(sum_synapse0)

  sum_synapse1 = np.dot(hidden_layer, weights_1)
  output_layer = sigmoid(sum_synapse1)

  error_output_layer = outputs - output_layer
  average = np.mean(abs(error_output_layer))
    #print after every specified range of the value
  if epoch % 100000 == 0:
    print('Epoch: ' + str(epoch + 1) + ' Error: ' + str(average))
    error.append(average)

  derivative_output = sigmoid_derivative(output_layer)
  delta_output = error_output_layer * derivative_output

  weights1T = weights1.T
  delta_output_weight = delta_output.dot(weights1T)
  delta_hidden_layer = delta_output_weight * sigmoid_derivative(hidden_layer)

  hidden_layerT = hidden_layer.T
  input_x_delta1 = hidden_layerT.dot(delta_output)
  weights_1 = weights_1 + (input_x_delta1 * learning_rate)

  input_layerT = input_layer.T
  input_x_delta0 = input_layerT.dot(delta_hidden_layer)
  weights_0 = weights_0 + (input_x_delta0 * learning_rate)
```

```
Epoch: 1 Error: 0.4999269690201742
Epoch: 100001 Error: 0.015563199459768319
Epoch: 200001 Error: 0.010326071808337757
Epoch: 300001 Error: 0.008192022809586367
```

At this point after runing for 1million epochs you can see the value is very low.

In [67]:
```
#1 million epochs with a learning rate of 0.3
1 - 0.009670967930930745
```

Out[67]:
```
0.9903290320690693
```

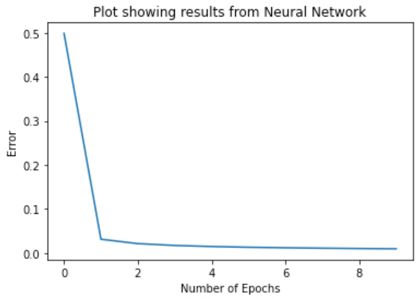
In [92]:
```
#after 400,000 epochs, with a learning rate of 0.6
1- 0.008192022809586367
```

Out[92]:
```
0.9918079771904136
```

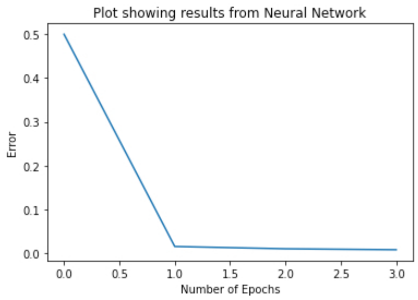
Let's visualize this result

In [68]:
```
import matplotlib.pyplot as plt
```

In [70]:
```
plt.xlabel('Number of Epochs')
plt.ylabel('Error')
plt.title('Plot showing results from Neural Network')
plt.plot(error)
plt.show()
```



In [93]:
```
plt.xlabel('Number of Epochs')
plt.ylabel('Error')
plt.title('Plot showing results from Neural Network')
plt.plot(error)
plt.show()
```



Compearing the outputs and the predictions

In [74]:
```
outputs
```

Out[74]:
```
array([[0],
       [1],
       [1],
       [0]])
```

In [75]:
```
output_layer
```

Out[75]:
```
array([[0.0105802 ],
       [0.9826343 ],
       [0.98124727],
       [0.02222762]])
```

* We see that our neural network was able to get values close to the actual values from the results.

* This shows that our neural network can handle the complexity of the XOR operator dataset.

- Let us see the updated weights. These are the weights we will require if we want to make future predictions

In [77]:
```
weights_0
```

Out[77]:
```
array([[-0.04578885, -5.45125419, -1.01868082],
       [ 1.04772434, -5.25578328, -0.5958262 ]])
```

In [78]:
```
weights_1
```

Out[78]:
```
array([[-14.93268248],
       [-37.16040415],
       [ 43.01681387]])
```

In [94]:
```
# This function accepts an instance of a dataset
```

```
def calculate_output(instance):
    #input to hidden layer
    hidden_layer = sigmoid(np.dot(instance, weights_0))
    #hidden to output layer
    output_layer = sigmoid(np.dot(hidden_layer, weights_1))
    return output_layer[0]
```

In [95]:
```
round(calculate_output(np.array([0, 0])))
```

Out[95]: 0

In [96]:
```
round(calculate_output(np.array([0, 1])))
```

Out[96]: 1

In [97]:
```
round(calculate_output(np.array([1, 0])))
```

Out[97]: 1

In [98]:
```
round(calculate_output(np.array([1, 1])))
```

Out[98]: 0

In [ ]: