STAT 542: Summary to date
Where we have been
& where we are headed

- Completed
  - Intro to probability
    - axioms and properties using set theory
    - conditional probability and independence
  - Random variables
    - definition
    - discrete/continuous
    - cdf, pdf/pmf

- Next
  - Transformations (an intro)
    "new" r.v. $Y = g(X)$ of "old" r.v. X
  - Expected values (mean, variance, moment generating function)
  - Probability-moment inequalities (Markov, Chebychev, Jensen)

Functions of a random variable

Introduction

- Consider a random variable $X \sim F_X(\cdot)$ and a function $g : \mathbb{R} \to \mathbb{R}$.

is a R.V. any function

- Then, $Y = g(X)$ is also a r.v., having its own cdf $F_Y(\cdot)$

Y is a function of X $\Rightarrow$ we can describe the probabilistic behavior of Y in terms of that X.

- Formally, there is also an inverse mapping g^{-1} defined by

$$g^{-1}(A) = \{x \in \mathbb{R} : g(x) \in A\} \quad \text{for any } A \subset \mathbb{R}.$$

$y = g(x) \Rightarrow g(x) : \mathcal{X} \longrightarrow \mathcal{Y}$ the sample space of Y

the sample space of X $\quad g^{-1}(A) = \{x \in \mathcal{X} : g(x) \in A\}$

- Distribution of Y is determined by the distribution of X and the function g

$$P_Y(Y \in A) = P_X(g(X) \in A) = P_X(X \in g^{-1}(A)) \quad \text{for } A \subset \mathbb{R}$$

This means the distribution of Y dependens on the functions F_X and g.

- If X has pdf/pmf $f_X(x)$, then the **range** or **support** of X is

$$\mathcal{X} = \{x \in \mathbb{R} : f_X(x) > 0\}.$$

If Y has pdf/pmf $f_Y(y)$, then the range or support of Y will be

$$\mathcal{Y} = \{y \in \mathbb{R} : f_Y(y) > 0\} = \{g(x) : x \in \mathcal{X}\}.$$

Note: The mapping g^{-1} takes setes into setes; $g^{-1}(A)$ is the set of points in $\mathcal{X}$ that $g(x)$ takes into the set A.

Note: $A = \{y\}$ (A is a point set) $\quad g^{-1}(\{y\}) = \{x \in \mathcal{X} : g(x) = y\}$