Lecture 1

Monday, Aug 26

STAT 542: Theory of Probability and Statistics I

Fall 2024 (4 Cr.)

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Office Hours: to be announced or by appointment

Assistants: Zheming (Jamie) Cao
Email: zmcao@iatate.edu
Office hours: Tuesday & Thursday from 10:50–11:50 A.M.

Lecture: MF 8:50 AM - 9:40 AM (Snedecor Hall 3105)
& TR 9:00 AM - 9:50 AM (Snedecor Hall 3105)

Textbook: *Statistical Inference* - (2nd Edition) by G. Casella & R. L. Berger, Duxbury Press.

Course Topics: Basic Probability Theory Concepts (Chapter 1)
Random Variable Expectations and Transformations (Chapter 2)
Common probability models (Chapter 3)
Multiple Random Variables (Chapter 4)
Random Samples, Normal Theory, Large Sample Convergence (Chapter 5)

Course Information

Objectives: As the discipline of statistics focuses heavily on data analysis based on probability, the aim of the course is to develop an understanding of basic results in probability theory. We emphasize random variables, probability distributions, and construction of probability models. The presentation of the material underscores its application to statistical methods and theory that will be further covered in STAT 543 during the spring.

Prerequisites: MATH 414: Students are assumed to have some background in calculus and advanced calculus - including limits, derivatives, Taylor series, some multivariate calculus (e.g., two-dimensional integration) and linear algebra (e.g., matrices, determinants). We will also use set theory and combinatorics at the beginning of the course.

Lecture Materials: Lecture notes and material will be posted on the course page in Canvas as PDF files. There are lecture-note skeletons for each lecture (involving some "blanks" for additional notes that you may fill-in yourself while viewing a lecture) and there are also completed ("filled-in") lecture notes posted after each lecture.

Exams: There will be one midterm exam and a final exam. The tentative date is Monday, October 21. You may use a calculator and formula sheet for the exams (more details later). If you know that you have a conflict with an exam due to circumstances beyond your control, you must notify me **before** the scheduled exam. Make-up exams are permitted only at the instructor's discretion.

Homework: Weekly homework problems will be assigned throughout the semester to provide opportunities for practice and application (typically due Fridays with some schedule changes around exams). Both the homework problems and their solutions will be posted on the course website in Canvas. You are welcome to discuss the homework problems with other students but everyone must **independently** write up and submit a homework assignment (do *not* copy the work of others). The lowest homework score will be dropped at the end of the semester.

Grading: Grades will be based on the following criteria:

Component	**Weight**	**Tentative Date**
Homework	20%	
Midterm Exam *	40%	Monday, October 21
Final Exam	40%	Tuesday, December 17, 7:30-9:30 AM

* Please note that the dates/times for the midterm exams are tentative and may be changed based on the pace of the course. The final exam date/time are set by the university.[1] You will always be given sufficient notice of an exam (e.g., two weeks). Locations of exams are to be announced.

ISU Canvas: Students can keep track of their grade scores and access all other class material through a Canvas page for STAT 542. To enter Canvas, go to "http://www.iastate.edu" and click on "Canvas". Then enter your ISU username (i.e., your ISU email address without "@iastate.edu") and ISU password. If you have no ISU Net-ID, you need to get one (http://www.it.iastate.edu/services/passwords).

Mutual Respect and Professionalism: You are expected to treat your instructor and all other participants in the course with courtesy and respect. Your comments to others should be constructive and free from harassing statements, and your conduct in the classroom should be considerate, avoiding actions that could be potentially disruptive to others. It is your instructor's goal to promote a welcoming atmosphere of appreciation and support for all in the classroom. Please contact Prof. Nordman if you have suggestions for improving the classroom environment.

[1] According to University policy: "Final exams may not be given at a time other than that for which the exam is scheduled by the registrar." You will find other Exam information and policies here: http://www.registrar.iastate.edu/students/exams

Tentative Set-up of Course

Week	Dates	Material	Text
1-2	M Aug 26 - F Sept 6	Probability - introduction	Sections 1.1-1.3
3	M Sept 9 - F Sept 13	Random variables - definitions	Sections 1.4-1.6
4-5	M Sept 16 - F Sept 27	Expected values, moment generating functions, inequalities	Sections 2.1-2.3, 3.6
6	M Sept 30 - F Oct 4	Families of distributions	Sections 3.1-3.5
7	M Oct 7 - F Oct 11	Bivariate distributions - joint & marginal distributions	Section 4.1
8	M Oct 14 - F Oct 18	Conditional distributions, independence, hierarchical models	Sections 4.2, 4.4
9	M Oct 21	Midterm Exam	
9	M Oct 21 - F Oct 25	Covariances, multivariate distributions	Sections 4.5-4.6
10	M Oct 28 - F Nov 1	Transformations, order statistics	Sections 4.3, 5.4
11	M Nov 4 - F Nov 8	Random samples, sums of independent random variables	Sections 5.1-5.2
12	M Nov 11 - F Nov 15	Convergence concepts	Section 5.5
13	M Nov 18 - F Nov 22	Sampling from normal distributions	Section 5.3
	M Nov 25 - F Nov 29	No class - Thanksgiving break	
14-15	M Dec 2 - F Dec 13	Normal theory, multivariate normal, stochastic processes	Section 5.3
16	T Dec 17	Final Exam (7:30-9:30 a.m.)	

Additional References: The course text is a fine book, but no book works perfectly for everyone. Below is a list of texts which cover the same essential material.

1. *Introduction to Probability and Mathematical Statistics* - L.J. Bain & M. Engelhardt, 2nd edition, 1992: Perhaps a bit less theoretical than our text but similar; this text also gives answers to odd-numbered exercises.
2. *Introduction to Mathematical Statistics* - R.V. Hogg & A.T. Craig, 5th edition, 1995: A bit more theoretical than our text with perhaps less emphasis on applications.
3. *Probability* - J. Pitman, 1993: This text only deals with probability (no STAT 543 material).
4. *Mathematical Statistics* - K. Knight, 1999: The text covers the main probability ideas but with less detail than our text and has a slightly different emphasis on statistical inference topics.
5. *Mathematical Statistics and Data Analysis* - J.A. Rice, 1995: This book includes probability and is more applied than our text.

General ISU Policies for this Course

Academic Integrity
The class will follow Iowa State University's policy on academic conduct with integrity. Anyone suspected of academic dishonesty or misconduct will be reported to the Dean of Students Office and receive zero credit on any assignment/exam where misconduct has occurred. More information can be found at
http://www.studentconduct.dso.iastate.edu/academic/misconduct.html

Accessibility Statement
Iowa State University is committed to assuring that all educational activities are free from discrimination and harassment based on disability status. Students requesting accommodations for a documented disability are required to work directly with staff in Student Accessibility Services (SAS) to establish eligibility and learn about related processes before accommodations will be identified. After eligibility is established, SAS staff will create and issue a Notification Letter for each course listing approved reasonable accommodations. This document will be made available to the student and instructor either electronically or in hard-copy every semester. Students and instructors are encouraged to review contents of the Notification Letters as early in the semester as possible to identify a specific, timely plan to deliver/receive the indicated accommodations. Reasonable accommodations are not retroactive in nature and are not intended to be an unfair advantage. Additional information or assistance is available online at www.sas.dso.iastate.edu, by contacting SAS staff by email at accessibility@iastate.edu, or by calling 515-294-7220. Student Accessibility Services is a unit in the Dean of Students Office located at 1076 Student Services Building.

Prep Week (the last week of the semester prior to final exam week)
This class follows the Iowa State University Prep Week policy as noted in section 10.6.4 of the Faculty Handbook.
http://www.provost.iastate.edu/resources/faculty-handbook. (There will be no assignments due in the week before Finals.)

Harassment and Discrimination
Iowa State University does not discriminate on the basis of race, color, age, ethnicity, religion, national origin, pregnancy, sexual orientation, gender identity, genetic information, sex, marital status, disability, or status as a U.S. Veteran. Inquiries regarding non-discrimination policies may be directed to Office of Equal Opportunity, 3410 Beardshear Hall, 515 Morrill Road, Ames, Iowa 50011, Tel. 515-294-7612, Hotline 515-294-1222, email eooffice@iastate.edu

Religious Accommodation
Iowa State University welcomes diversity of religious beliefs and practices, recognizing the contributions differing experiences and viewpoints can bring to the community. There may be times when an academic requirement conflicts with religious observances and practices. If that happens, students may request the reasonable accommodation for religious practices. In all cases, you must put your request in writing. The instructor will review the situation in an effort to provide a reasonable accommodation when possible to do so without fundamentally altering a course. For students, you should first discuss the conflict and your requested accommodation with your professor at the earliest possible time. You or your instructor may also seek assistance

from the Dean of Students Office at 515-294-1020 or the Office of Equal Opportunity at 515-294-7612.

Free Expression
Iowa State University supports and upholds the First Amendment protection of freedom of speech and the principle of academic freedom in order to foster a learning environment where open inquiry and the vigorous debate of a diversity of ideas are encouraged. Students will not be penalized for the content or viewpoints of their speech as long as student expression in a class context is germane to the subject matter of the class and conveyed in an appropriate manner.

Contact Information for any Issues Above
If you are experiencing, or have experienced, a problem with any of the above issues, email academicissues@iastate.edu

STAT 542: Theory of Probability and Statistics

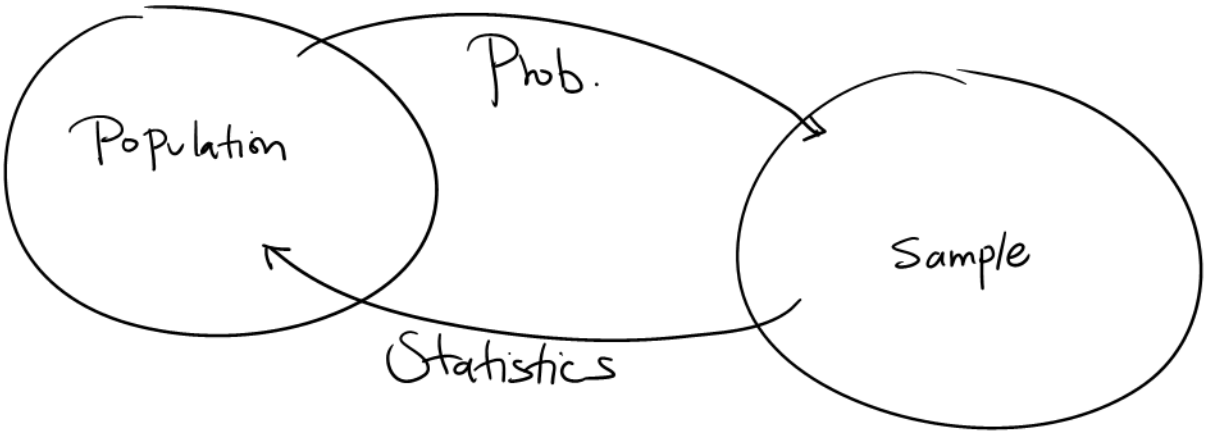
- **Probability** is a branch of mathematics concerned with the study of *random* phenomenon (e.g., experiments, models of populations)

- We are primarily interested in probability as it relates to **statistical inference**, the science of drawing inferences about populations based on only a part of the population (i.e., a sample)

- Applying probability to develop statistical inference is the topic of STAT 543.

  Learning the probability notions useful for statistical inference is topic of STAT 542.

- Little picture here



Introduction to Probability

Terminology

Some Definitions

1. **population**: the entire set of objects that we are interested in studying

   e.g. all ISU students

2. **sample**: the subset of the population available for observation

   e.g. STAT 542 students,

   Note: population and sample are crucial terms in understanding statistics (i.e., STAT 543), but will not occur very often in our discussions of probability theory (i.e., STAT 542)

3. **experiment**: process of obtaining an observed result of a random phenomenon

   Some examples:

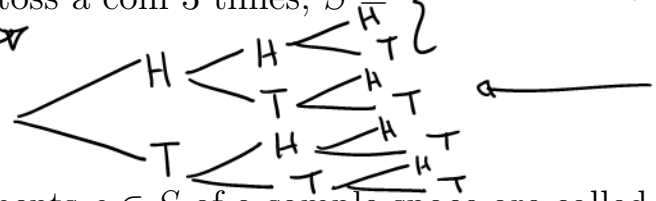
   - recording the outcome of a single coin toss
   - recording the number of children in a randomly chosen U.S. household
   - determining corn yield of a plot of land using fertilizer type A

Introduction to Probability

Terminology (cont'd)

4. **sample space** S: the set of all possible outcomes of the experiment

   - Some examples:
     - for a single coin toss $S =$ $\{H, T\}$
     - number of children $S =$ $\{0, 1, 2, 3, \cdots\}$
     - corn yield $S = [0, \infty)$ ← Continuous
     - toss a coin 3 times, $S =$ $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

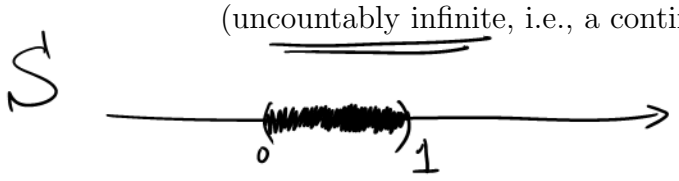


   - elements $s \in S$ of a sample space are called **sample points** (s)

S → Sample space

e.g. "HTH" is a Sample point

   - a sample space may be
     - **discrete**
       (finite or countably infinite, i.e., listable as a finite/infinite sequence)
       $S = \{s_1, s_2, \cdots s_n\}$
       $S = \{s_1, s_2, s_3, \cdots\}$
     - or **continuous**
       (uncountably infinite, i.e., a continuum of sample points like $S = [0, \infty)$)



   - multiple sample spaces may be defined for the same experiment

e.g. flip a coin 3X $S = \{HHH, HT, H, \cdots, TTT\}$

* of heads, $S = \{0, 1, 2, 3\}$

Introduction to Probability

Terminology (cont'd)

5. **event** (e.g., $A, B, \ldots$): subset of the sample space S

   $A \subset S$

   - Some examples:
     - $A = \{H\}$, the event that a coin lands H up

       "H" $\{H\}$ $S = \{H, T\}$, $A \subset S$
     - $B = \{\text{number of children is even}\} = \{0, 2, 4, \ldots\}$

       $B \subset S$

       $S = \{0, 1, 2, 3, \cdots\}$
     - $C = \{\text{corn yield is more than 100 bushels per acre}\} = (100, \infty)$

       $S = [0, \infty)$

   - We say that the event occurs if it contains the observed outcome of the experiment

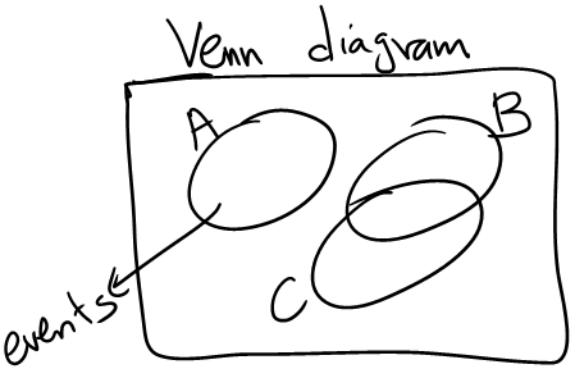
     # of children $S = \{0, 1, 2, \cdots\}$

     "$B \equiv$ even", "$C \equiv \leq 10$"

     Outcome $s = 2$, then event B occurs + C occurs

     Outcome $s = 3$, then the event B doesn't occur.

     But, C occurs



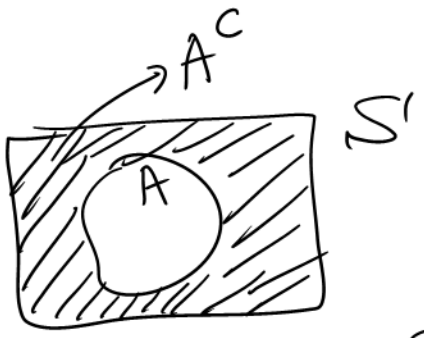
Introduction to Probability

Set Theory: Definitions

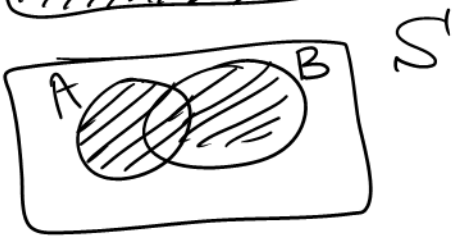
- set: A is a collection of elements
  (in our case A is a collection of outcomes)

- membership: $x \in A$ or $x \notin A$
  (x is in A or x is not in A)

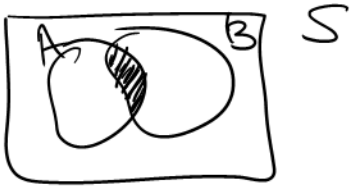
- complement: $A^c = \{x : x \notin A\}$
  (x such that x is not in A)



- union: $A \cup B = \{x : x \in A \text{ or } x \in B\}$
  (x is in A or B or both)



- intersection: $A \cap B = \{x : x \in A \text{ and } x \in B\}$



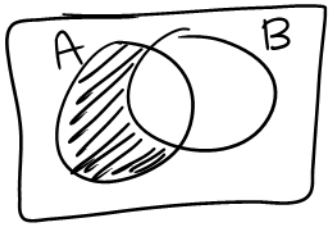
- subset: $A \subset B$ means that A is contained in B
  (formally, $x \in A \Rightarrow x \in B$)

$x \in A \overset{?}{\Longrightarrow} x \in B$

- equality: $A = B$ if $A \subset B$ and $B \subset A$

"$\subseteq$"

$x \in A \iff x \in B$



- empty set: $\emptyset$

- Set subtraction $A \setminus B \equiv A \cap B^c = \{x : x \in A \text{ and } x \notin B\}$

Introduction to Probability

Set Theory: Algebra (without proof)

- Algebraic Laws
  - commutativity:

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

  - associativity:

$$A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C$$
$$A \cap (B \cap C) = (A \cap B) \cap C = A \cap B \cap C$$

  - distributive law:

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

To show equality here: (i) $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$ (ii) $(A \cup B) \cap (A \cup C) \subset A \cup (B \cap C)$

To check (i): $x \in A \cup (B \cap C) \stackrel{?}{\Longrightarrow} x \in (A \cup B) \cap (A \cup C)$

$x \in A \Rightarrow x \in A \cup B,\ x \in A \cup C \Rightarrow x \in (A \cup B) \cap (A \cup C)$ (I)

$x \in B \cap C \Rightarrow x \in B,\ x \in C \Rightarrow x \in A \cup B$ and $x \in A \cup C \Rightarrow x \in (A \cup B) \cap (A \cup C)$ (II)

$\Rightarrow$ (I) & (II) $x \in A \cup (B \cap C) \Rightarrow x \in (A \cup B) \cap (A \cup C)$.

we can show part (ii) similar to that part (i)

  - DeMorgan's laws:

$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$