Main Effects

A *main effect* is the difference between marginal means associated with two levels of a factor.

In our two-factor example, the *main effect* of Diet is $\bar{\mu}_{1\cdot} - \bar{\mu}_{2\cdot}$.

	Drug 1	Drug 2	Drug 3	
Diet 1	μ_{11}	μ_{12}	μ_{13}	$\bar{\mu}_{1\cdot}$
Diet 2	μ_{21}	μ_{22}	μ_{23}	$\bar{\mu}_{2\cdot}$
	$\bar{\mu}_{\cdot 1}$	$\bar{\mu}_{\cdot 2}$	$\bar{\mu}_{\cdot 3}$	$\bar{\mu}_{\cdot\cdot}$

averaging over all levels of drug, and average, obtain difference in outcomes between diet 1 & 2

Main Effects (continued)

In our two-factor example, the *main effects* of Drug involve the differences $\bar{\mu}_{\cdot 1} - \bar{\mu}_{\cdot 2}$, $\bar{\mu}_{\cdot 1} - \bar{\mu}_{\cdot 3}$, and $\bar{\mu}_{\cdot 2} - \bar{\mu}_{\cdot 3}$.

	Drug 1	Drug 2	Drug 3	
Diet 1	μ_{11}	μ_{12}	μ_{13}	$\bar{\mu}_{1\cdot}$
Diet 2	μ_{21}	μ_{22}	μ_{23}	$\bar{\mu}_{2\cdot}$
	$\bar{\mu}_{\cdot 1}$	$\bar{\mu}_{\cdot 2}$	$\bar{\mu}_{\cdot 3}$	$\bar{\mu}_{\cdot\cdot}$

to establish that there is no Drug main effect we need to compare all 3 levels using pairwise comparisons.

of interest at first: is there a difference? less concerned which levels differ (this analysis comes later)

Main Effects (continued)

If $\bar{\mu}_{1\cdot} = \bar{\mu}_{2\cdot}$, it would be customary to say, “There is no Diet main effect.”

If $\bar{\mu}_{\cdot 1} = \bar{\mu}_{\cdot 2} = \bar{\mu}_{\cdot 3}$, it would be customary to say, “There are no Drug main effects.”

	Drug 1	Drug 2	Drug 3	
Diet 1	μ_{11}	μ_{12}	μ_{13}	$\bar{\mu}_{1\cdot}$
Diet 2	μ_{21}	μ_{22}	μ_{23}	$\bar{\mu}_{2\cdot}$
	$\bar{\mu}_{\cdot 1}$	$\bar{\mu}_{\cdot 2}$	$\bar{\mu}_{\cdot 3}$	$\bar{\mu}_{\cdot\cdot}$

Interaction Effects

The linear combination $\boxed{\mu_{ij} - \mu_{ij'} - \mu_{i'j} + \mu_{i'j'}}$ for $i \neq i'$ and $j \neq j'$ is an *interaction effect*.

every interaction can be expressed using this format

For example,

$$\mu_{11}-\mu_{12}-\mu_{21}+\mu_{22} = \boxed{(\mu_{11}-\mu_{12})}-\boxed{(\mu_{21}-\mu_{22})} = (\mu_{11}-\mu_{21})-(\mu_{12}-\mu_{22})$$

is an interaction effect.

equal if no interaction

if there is no interaction, then $(\mu_{11} - \mu_{21}) = (\mu_{12} - \mu_{22})$ $= (\mu_{13} - \mu_{23})$, i.e., drug has no effect on the effect of diet

	Drug 1	Drug 2	Drug 3
Diet 1	μ_{11}	μ_{12}	μ_{13}
Diet 2	μ_{21}	μ_{22}	μ_{23}

Interaction Effects (continued)

so differences between any of the levels of drug are the same under

When all interaction effects are zero, we may say there are "no interactions" between the factors or that the two factors do not "interact."

diet 1 and diet 2!

When there are no interactions between factors, the simple effects of either factor are the same across all levels of the other factor.

For example, when there are no interactions between the factors Diet and Drug, the simple effect of Diet is the same for each level of Drug. Likewise, any simple effect of Drug is the same for both diets.

Testing for Non-Zero Effects

To properly set up C, look at β and how the parameters are arranged in β: $\beta_1 = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \end{pmatrix}$ $\beta_1^* = \begin{pmatrix} \mu_2 \\ \mu_1 \\ \mu_2 \end{pmatrix}$

We can test whether simple effects, main effects, or interaction effects are zero vs. non-zero using tests of the form

$$\mathrm{H}_0 : \boldsymbol{C}\boldsymbol{\beta} = \boldsymbol{0} \text{ vs. } \mathrm{H}_A : \boldsymbol{C}\boldsymbol{\beta} \neq \boldsymbol{0}.$$

The following slides give appropriate $\boldsymbol{C}$ matrices for several examples.

Whenever a factor has only 2 levels, C turns into a row vector

Testing for Non-Zero Effects

	Dr1	Dr2	Dr3
D1	μ_{11}	μ_{12}	μ_{13}
D2	μ_{21}	μ_{22}	μ_{23}

H_0: No simple effect of Diet for Drug 1 $(\mu_{11} = \mu_{21})$

$$\underbrace{\begin{array}{cccccc} \mu_{11} & \mu_{12} & \mu_{13} & \mu_{21} & \mu_{22} & \mu_{23} \\ \hline 1 & 0 & 0 & -1 & 0 & 0 \end{array}}_{C^T} \begin{bmatrix} \mu_{11} \\ \mu_{12} \\ \mu_{13} \\ \mu_{21} \\ \mu_{22} \\ \mu_{23} \end{bmatrix} \begin{array}{l} \text{diet 1} \\ \\ \text{diet 2} \end{array} = \begin{bmatrix} 0 \end{bmatrix}$$

within each diet, drugs follow in order
1 2 3

H_0: No simple effect of Drug 2 vs. Drug 3 for $\boxed{\text{Diet 2}}$ $(\mu_{22} = \mu_{23})$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} \mu_{11} \\ \mu_{12} \\ \mu_{13} \\ \mu_{21} \\ \mu_{22} \\ \mu_{23} \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$$

We will average across all levels of the other factor

difference in marginal diet means

H$_0$: No Diet Main Effect $(\bar{\mu}_{1\cdot} = \bar{\mu}_{2\cdot})$

$$\underbrace{\left[\ \overbrace{\tfrac{1}{3}\quad \tfrac{1}{3}\quad \tfrac{1}{3}}^{\bar{\mu}_{1\cdot}}\quad \overbrace{-\tfrac{1}{3}\quad -\tfrac{1}{3}\quad -\tfrac{1}{3}}^{\bar{\mu}_{2\cdot}}\ \right]}_{\text{these need to add up to zero}} \begin{bmatrix} \mu_{11} \\ \mu_{12} \\ \mu_{13} \\ \mu_{21} \\ \mu_{22} \\ \mu_{23} \end{bmatrix} = \left[\ 0\ \right]$$

$$\bar{\mu}_{1\cdot} = \frac{1}{3}\left(\mu_{11} + \mu_{12} + \mu_{13}\right)$$

Testing for Non-Zero Effects

has 3 levels $\Rightarrow$ we need to make more than $\underline{1}$ comparison

H_0: No Drug Main Effects $(\bar{\mu}_{\cdot 1} = \bar{\mu}_{\cdot 2} = \bar{\mu}_{\cdot 3})$ $\Rightarrow$ # of necessary comparisons $=$ # of rows in C

$\bar{\mu}_{\cdot 1} = \frac{1}{2}(\mu_{11} + \mu_{12})$

$$
\begin{array}{cccccc}
\mu_{11} & \mu_{12} & & \mu_{21} & \mu_{22} & \\
\end{array}
$$

$$
\begin{bmatrix}
\frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\
\frac{1}{2} & 0 & -\frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2}
\end{bmatrix}
\begin{bmatrix}
\mu_{11} \\ \mu_{12} \\ \mu_{13} \\ \mu_{21} \\ \mu_{22} \\ \mu_{23}
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0
\end{bmatrix}
$$

$$
\begin{array}{cccccc}
\mu_{11} & & \mu_{13} & \mu_{21} & & \mu_{23}
\end{array}
$$

μ_{11}	μ_{12}	μ_{13}
μ_{21}	μ_{22}	μ_{23}

$\bar{\mu}_{\cdot 1} = \bar{\mu}_{\cdot 2} = \bar{\mu}_{\cdot 3}$

row 1 of C : $\bar{\mu}_{\cdot 1}$ vs. $\bar{\mu}_{\cdot 2}$ $\quad \bar{\mu}_{\cdot 1} - \bar{\mu}_{\cdot 2} = 0$

row 2 of C : $\bar{\mu}_{\cdot 1}$ vs. $\bar{\mu}_{\cdot 3}$ $\quad \bar{\mu}_{\cdot 1} - \bar{\mu}_{\cdot 3} = 0$

Testing for Non-Zero Effects

H_0: No Drug Main Effects $(\bar{\mu}_{\cdot 1} = \bar{\mu}_{\cdot 2} = \bar{\mu}_{\cdot 3})$

Here is an alternative specification that will yield the same test.

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \mu_{11} \\ \mu_{12} \\ \mu_{13} \\ \mu_{21} \\ \mu_{22} \\ \mu_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

equivalent now we compare

Drug 1 vs. Drug 2 & Drug 2 vs. Drug 3

Testing for Non-Zero Effects

end lecture

end Wednesday February 7th

H_0: No Drug Main Effects $(\bar{\mu}_{\cdot 1} = \bar{\mu}_{\cdot 2} = \bar{\mu}_{\cdot 3})$

Here is yet another alternative specification that will yield the same test.

$$\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & 0 \\ \frac{1}{4} & \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} & \frac{1}{4} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \mu_{11} \\ \mu_{12} \\ \mu_{13} \\ \mu_{21} \\ \mu_{22} \\ \mu_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\frac{1}{2}(\bar{\mu}_{\cdot 1} + \bar{\mu}_{\cdot 2}) = \bar{\mu}_{\cdot 3}$ under $H_0: \bar{\mu}_{\cdot 1} = \bar{\mu}_{\cdot 2} = \bar{\mu}_{\cdot 3}$

Testing for Non-Zero Effects

H_0: No Diet-by-Drug Interactions:
$(\mu_{ij} - \mu_{ij'} - \mu_{i'j} + \mu_{i'j'} = 0$ for all $i \neq i'$ and $j \neq j')$

$$\begin{bmatrix} 1 & -1 & 0 & -1 & 1 & 0 \\ \\ 1 & 0 & -1 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_{11} \\ \mu_{12} \\ \mu_{13} \\ \mu_{21} \\ \mu_{22} \\ \mu_{23} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

R/SAS Code and Output

- The R/SAS code and output for the above sample are given as separate handouts.

- We will discuss alternative parameterizations of the cell-means model as part of the implementation in SAS and R