

Homework 7 – STAT 542

Due Monday, November 10 by 11:59 PM

Note: Problem 1 involves hierarchical (conditional) specification of distributions (Lecture 32). The remaining problems deal with aspects of independence (Lecture 33 - Lecture 34), but some involve concepts of marginal distributions, conditional distributions and finding probabilities for events involving two or more random variables (Lecture 24-Lecture 32). Problems 4 involve transformations of more than 1 random variable and finding the distribution of these (Lecture 34 - Lecture 35).

1. Consider continuous random vector (X, Y) where $X \sim N(0, 1)$ (standard normal) and $Y|X = x \sim N(x, 1)$ (the conditional distribution of Y given $X = x$ is normal with mean x and variance 1).
 - (a) Find the joint pdf of (X, Y) .
 - (b) Find the marginal distribution of Y .
 - (c) Consider random vector (V, W) where $W \sim N(0, 2)$ (normal with mean 0 and variance 2). How should the conditional distribution $V|W = w$ be chosen so that (V, W) has the same distribution as (X, Y) ?
2. 4.7, Casella & Berger
3. 4.10, Casella & Berger
4. Suppose X and Y are independent random variables, where both have the same (marginal) geometric(p) distribution for $0 < p < 1$. Find the conditional distribution of X given $X + Y = k$ (integer $k \geq 2$).
5. 4.27, Casella & Berger
6. 4.42, Casella & Berger
7. Suppose that random variables X_1, X_2, X_3 have joint pdf $f(x_1, x_2, x_3) = 6$ for $0 < x_1 < x_2 < x_3 < 1$.
 - (a) Are X_1, X_2, X_3 independent? Give an intuitive explanation.
 - (b) Find the marginal pdf of X_2 and identify the distribution of X_2 as a member of an important family of distributions.
 - (c) Find the conditional pdf $f(x_1, x_3|x_2)$ of X_1, X_3 given $X_2 = x_2 \in (0, 1)$.
 - (d) Show that X_1, X_3 are independent given $X_2 = x_2 \in (0, 1)$ (i.e., think of what must be true of the conditional pdf $f(x_1, x_3|x_2)$ in this case).
 - (e) Find the covariance of X_1 and X_3 given $X_2 = x_2$.