Homework 8 – STAT 542

Due Friday, Nov 18 by 11:59 PM

1. X_1, X_2 are independent exponential random variables with mean θ

   (a) Find the joint moment generating function of X_1, X_2.

   (b) Give the definition of the moment generating function of $X_1 - X_2$ and show this can be obtained from (a).

   (c) Find the distribution of $Y = X_1 - X_2$ (using the mgf of Y, one can find that this is a so-called Laplace or double-exponential distribution.)

2. 4.30, Casella & Berger

3. 4.54, Casella & Berger (Hint: Show first if $X \sim$ uniform$(0, 1)$, then $-\log X \sim$ exponential(1).)

4. 4.47, Casella & Berger

5. 4.52, Casella & Berger

6. 4.55, Casella & Berger

7. 4.28, Casella & Berger

8. 4.50, Casella & Berger (Hint: Don't use the joint pdf directly. Use the fact that (X, Y) are MVN with means, variances and covariances implied by the joint pdf. Further, X^2 and Y^2 have χ^2_1 distributions here for finding the variances of X^2 and Y^2. Find $\mathrm{E}[X^2Y^2] = \mathrm{E}[\mathrm{E}(X^2Y^2|Y)]$ using that $X|Y$ has a (conditional) normal distribution.)