Homework 6 – STAT 5420

Due Monday, Nov 3 by 11:59 PM (to be scanned and uploaded in Canvas under "Assignments")

1. 4.17, Casella & Berger
2. 4.32(a), Casella & Berger
3. Expectation
   (a) Show that any random variable X (with finite mean) has zero covariance with any real constant c, i.e., $\mathrm{Cov}(X, c) = 0$.
   (b) Using the definition of conditional expectation, show that $\mathrm{E}[g(X)h(Y)|X = x] = g(x)\mathrm{E}[h(Y)|X = x]$ for an x with $f_X(x) > 0$. (You may assume (X, Y) are jointly discrete.)
4. Suppose that X_i has mean μ_i and variance σ_i^2 for $i = 1, 2$, and that the covariance of X_1 and X_2 is σ_{12}. Compute the covariance between $X_1 - 2X_2 + 8$ and $3X_1 + X_2$.
5. The joint distribution of X, Y is given by the joint pdf

$$f(x, y) = 3(x + y) \quad 0 < x < 1, 0 < y < 1, 0 < x + y < 1$$

   (a) Find the marginal distribution $f_X(x)$.
   (b) Find the conditional pdf of $Y|X = x$, given some $0 < x < 1$.
   (c) Find $\mathrm{E}[Y|X = x]$.
   (d) Given the results in (a),(b),(c), explain how you know $\mathrm{E}[X|Y = y]$ without any further calculation.
   (e) Find $\mathrm{E}[\mathrm{E}[2XY - Y|X]]$.
6. Suppose that $f(x, y) = e^{-y}$ for $0 < x < y < \infty$ (this was one example mentioned in class).
   (a) Find the joint moment generating function for (X, Y).
   (b) Use the joint moment generating function to find the variance of X, the variance of Y, and the covariance of X and Y.
   (c) Based on the joint moment generating function, identify the marginal distribution of X and the marginal distribution of Y.
7. Beta-binomial model: Suppose that the conditional distribution $X|\mathrm{P} = p$ is Binomial(n, p) and suppose P has a Beta(α, β) distribution.
   (a) Using the EVVE formula, find $\mathrm{Var}(X)$.
   (b) Suppose that W has a Binomial$(n, \tilde{p})$ distribution having the same mean as X above. For $n > 1$, show that X has a larger variance than W by a multiplicative factor of

$$\frac{\alpha + \beta + n}{\alpha + \beta + 1} > 1$$

   (Sometimes in modeling counts of successes in n trials, data may exhibit more variability than one would expect for a Binomial model; in which case, it may be helpful to "allow more variability" in an extension of a basic Binomial model. The Beta-Binomial model has more variability than a Binomial model with the same mean.)