Homework 7 – STAT 5430

Due Monday, April 7 by midnight in gradescope;

1. Problem 8.6(a)-(b), Casella and Berger (2nd Edition)

2. Problem 8.28, Casella and Berger (2nd Edition)

3. Problem 8.29(a)-(b), Casella and Berger (2nd Edition)

   The cauchy(θ) pdf is given by

   $$f(x|\theta) = \frac{1}{\pi}\frac{1}{1+(x-\theta)^2} \qquad x \in \mathbb{R}, \quad -\infty < \theta < \infty.$$

   In part(a), you are asked to argue that the cauchy(θ) family $\{f(x|\theta) : \theta \in \mathbb{R} = \Theta\}$ (based on one observation X) does *not* have MLR in $t(X) = X$ or $t(X) = -X$; that is, the ratio $f(x|\theta_2)/f(x|\theta_1)$ might not be monotone (either increasing or decreasing) in x. For part(b), show that the test given is equivalent to rejecting H_0 if $f(x|\theta = 1) > 2f(x|\theta = 0)$ and not rejecting otherwise; this implies the test must be MP for its size (why?).

4. Consider *one observation* X from the probability density function

   $$f(x|\theta) = 1 - \theta^2\left(x - \frac{1}{2}\right), \quad 0 \le x \le 1, \qquad 0 \le \theta \le 1.$$

   Suppose we wish to test $H_0 : \theta = 0$ vs. $H_1 : \theta > 0$.

   (a) For the above hypotheses, find the UMP test of size $\alpha = 0.05$ based on X. Carefully justify your answer.

   (b) For the above hypotheses, find the likelihood ratio test statistic $\lambda(X)$ based on X (i.e., as a function of X)

   (c) Find the size $\alpha = 0.05$ likelihood ratio test (LRT) of the above hypotheses.