STAT 5460: Homework IV FALL 2025

Write your name on each page. This homework has 4 questions for a total of 30 points, due date is **Monday November 17, 2025**. Please turn in your solutions (CLEAN version) on Monday November 17, 2025 during class.

1. [10 points] Write the conditional bias of the local polynomial regression estimator for $p - \nu$ odd

$$\text{bias}[\widehat{m}_\nu(x_0)|\mathbb{X}] = \varepsilon_{\nu+1}^T S^{-1} c_p \frac{\nu!}{(p+1)!} m^{(p+1)}(x_0) h^{p+1-\nu} + o_p(h^{p+1-\nu})$$

   in terms of the equivalent kernel $K^\star_{\nu,p}$ (see p. 60 Eq. (4.29) in the notes).

2. [10 points] Write the conditional variance of the local polynomial regression estimator

$$\mathbf{Var}[\widehat{m}_\nu(x_0)|\mathbb{X}] = \varepsilon_{\nu+1}^T S^{-1} S^\star S^{-1} \varepsilon_{\nu+1} \frac{\nu!^2 \sigma^2(x_0)}{f_X(x_0) n h^{1+2\nu}} + o_p\left(\frac{1}{nh^{1+2\nu}}\right)$$

   in terms of the equivalent kernel $K^\star_{\nu,p}$ (see p. 60 Eq. (4.30) in the notes).

3. [5 points] Show that the equivalent kernel satisfies the following moment condition

$$\int u^q K^\star_{\nu,p}(u)\, du = \delta_{\nu,q} \quad 0 \le \nu, q \le p,$$

   where $\delta_{\nu,q} = 1$ if $\nu = q$ and 0 else.

4. [5 points] Show that the weights W_ν^n satisfy the following discrete moment condition

$$\sum_{i=1}^n (X_i - x_0)^q W_\nu^n\left(\frac{X_i - x_0}{h}\right) = \delta_{\nu,q}, \quad 0 \le \nu, q \le p$$