Statistics 520 - Assignment 3

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Assignment 3

1. **(10 pt.)** Suppose that a random variable Y has a beta distribution with parameters α and β. A standard form for the probability density function of Y is, for $\alpha > 0$ and $\beta > 0$,

$$f(y \mid \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}, \quad 0 < y < 1.$$

Put this density in **canonical exponential family form**.

Using properties of exponential families, find $E\{\log(Y)\}$ and $E\{\log(1-Y)\}$ expressed in terms of the original α and β parameters.

Note: Use $\Gamma'(x)$ to denote the derivative of the gamma function,

$$\frac{d}{dx}\Gamma(x).$$

Answer

2. **(5 pt.)** Suppose that a random variable Y has a Poisson distribution with parameter λ. A standard form for the probability mass function of Y is, for $\lambda > 0$,

$$f(y \mid \lambda) = \frac{1}{y!}\lambda^y \exp(-\lambda), \quad y = 0, 1, 2, \ldots$$

Put this probability mass function in **canonical exponential family form**.

Using properties of exponential families, verify that $E(Y) = \lambda$.

Answer

3. Suppose that a random variable Y has a gamma distribution with parameters α and β. A standard form for the probability density function of Y is, for $\alpha > 0$ and $\beta > 0$,

$$f(y \mid \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \exp(-\beta y), \quad y > 0.$$

Note: You may have seen a gamma density written with a parameter that is equal to $1/\beta$ in the above expression. Use the parameterization given above to answer this question (I think it will be easier).

(a) (5 pts.) Write the gamma density in the form of a two-parameter exponential family.
Using properties of exponential families, derive the expected values of Y and $\log(Y)$.

(b) (5 pts.) Write the gamma density in the form of an exponential dispersion family with parameters θ and ϕ.
Derive the expected value of Y.

Answer

(a)
(b)