Statistics 520

Exam 1 – Example Question Solutions

Fall 2025

Short Answer Questions

(5 pts. each)

1. Probability of event E under Laplacian probability,

$$Pr(E) = \frac{|E|}{|\mathcal{S}|}.$$

2. (a) Convergence of estimates

   (b) Convergence of likelihood or log likelihood

   (c) Convergence of gradient of likelihood or log likelihood

3.

$$p(\theta|\boldsymbol{y}) = \pi_\theta[\theta|h(\boldsymbol{\lambda}, \boldsymbol{y})]$$

for some function h.

4.

$$\frac{\partial}{\partial \theta_j} B(\boldsymbol{\theta}) = E[T_j(y)].$$

5.

$$I_{u,v}(\boldsymbol{\theta}) = E\left(\frac{\partial}{\partial \theta_u} \log\{f(y|\boldsymbol{\theta})\} \frac{\partial}{\partial v} \log\{f(y|\boldsymbol{\theta})\} \right).$$

6. Because all legitimate concepts of probability must obey the same mathematical rules. So the manipulation of prior and posterior distributions in terms of epistemic probability "on paper" looks the same as the manipulation of distributions for random variables relative to relative frequency probability.

Multiple Choice

(15 pts.)

Best answer is (g). Answers (f) and (j) get 10 points, answers (i) and (k) get 5 points, answer (h) gets 0 points.

Additional Questions

1. (25 pts.)

$$
\begin{aligned}
p(\mu|\boldsymbol{y}) &\propto f(\boldsymbol{y}|\mu)\pi(\mu) \\
&\propto \exp\left[-\frac{1}{2\sigma^2}(y-\mu)^2 - \frac{1}{2\tau^2}(\mu-\lambda)^2\right] \\
&\propto \exp\left[-\frac{1}{2}\left\{\mu^2\left(\frac{1}{\sigma^2}+\frac{1}{\tau^2}\right) - 2\mu\left(\frac{y}{\sigma^2}+\frac{\lambda}{\tau^2}\right)\right\}\right].
\end{aligned}
$$

By the result on completing the square, this is a normal distribution with mean M and variance V, where,

$$
\begin{aligned}
M &= \frac{\tau^2 y + \sigma^2\lambda}{\sigma^2+\tau^2}, \\
V &= \frac{\sigma^2\tau^2}{\sigma^2+\tau^2}.
\end{aligned}
$$

2. (30 pts.)

Beginning with the hint,

$$
\begin{aligned}
&\frac{d}{d\theta}\int \exp[\phi\{\theta y - b(\theta)\} + c(y,\phi)]\,dy = 0 \\
\Rightarrow &\int \frac{d}{d\theta}\exp[\phi\{\theta y - b(\theta)\} + c(y,\phi)]\,dy = 0 \\
\Rightarrow &\int \exp[\phi\{\theta y - b(\theta)\} + c(y,\phi)][\phi y - \phi b'(\theta)]\,dy = 0 \\
\Rightarrow &\phi E(Y) - \phi b'(\theta) = 0 \\
\Rightarrow &E(Y) = b'(\theta).
\end{aligned}
$$

Similarly,

$$\frac{d^2}{d\theta^2}\int \exp[\phi\{\theta y - b(\theta)\} + c(y,\phi)]\,dy = 0$$

$$\Rightarrow \int \frac{d^2}{d\theta^2}\exp[\phi\{\theta y - b(\theta)\} + c(y,\phi)]\,dy = 0$$

$$\Rightarrow \int \frac{d}{d\theta}\exp[\phi\{\theta y - b(\theta)\} + c(y,\phi)][\phi y - \phi b'(\theta)]\,dy = 0$$

$$\Rightarrow \int \exp[\phi\{\theta y - b(\theta)\} + c(y,\phi)][\phi y - \phi b'(\theta)]^2 + \exp[\phi\{\theta y - b(\theta)\} + c(y,\phi)][-\phi b''(\theta)]dy = 0$$

$$\Rightarrow \int \exp[\phi\{\theta y - b(\theta)\} + c(y,\phi)][\phi y - \phi E(Y)]^2 + \exp[\phi\{\theta y - b(\theta)\} + c(y,\phi)][-\phi b''(\theta)]dy = 0$$

$$\Rightarrow \phi^2 var(Y) = \phi b''(\theta)$$

$$\Rightarrow var(Y) = \frac{1}{\phi} b''(\theta).$$