# HW8

Sam Olson

## Q1

Suppose there is one observation $X$ with pdf

$$f(x) = 2\theta(1 - 2x) + 2x, \quad \text{for } x \in [0, 1],\ \theta \in [0, 1].$$

Find the **Bayes test** for

$$H_0 : \theta \leq 0.4 \quad \text{vs.} \quad H_1 : \theta > 0.4$$

with respect to the **uniform prior** on $[0, 1]$.

### Answer

# Q2

Problem 9.13, *Casella and Berger (2nd Edition)*

Let $X$ be a single observation from the $\mathrm{Beta}(\theta, 1)$ pdf.

## a)

Let $Y = -(\log X)^{-1}$. Evaluate the confidence coefficient of the set $[y/2, y]$.

**Answer**

## b)

Find a pivotal quantity and use it to set up a confidence interval having the same confidence coefficient as the interval in part (a).

**Answer**

## c)

Compare the two confidence intervals.

**Answer**

# Q3

Problem 9.16, *Casella and Berger (2nd Edition)*

Let $X_1, \ldots, X_n$ be i.i.d. $\mathrm{N}(\theta, \sigma^2)$, where $\sigma^2$ is known. For each of the following hypotheses, write out the acceptance region of a level $\alpha$ test and the $1 - \alpha$ confidence interval that results from inverting the test.

## a)

$H_0 : \theta = \theta_0$ versus $H_1 : \theta \neq \theta_0$

**Answer**

## b)

$H_0 : \theta \geq \theta_0$ versus $H_1 : \theta < \theta_0$

**Answer**

## c)

$H_0 : \theta \leq \theta_0$ versus $H_1 : \theta > \theta_0$

**Answer**

# Q4

Problem 9.11, *Casella and Berger (2nd Edition)*

If $T$ is a continuous random variable with cdf $F_T(t \mid \theta)$ and $\alpha_1 + \alpha_2 = \alpha$, show that an $\alpha$-level acceptance region of the hypothesis $H_0 : \theta = \theta_0$ is

$$\{t : \alpha_1 \leq F_T(t \mid \theta_0) \leq 1 - \alpha_2\},$$

with associated confidence $1 - \alpha$ set

$$\{\theta : \alpha_1 \leq F_T(t \mid \theta) \leq 1 - \alpha_2\}.$$

## Answer