PS2

Assignment 2

In contrast to inverse gamma distributions, an inverse Gaussian distribution is **not** the distribution of the reciprocal of a random variable having a Gaussian distribution. It is an entirely different distribution.

There are any number of ways that people have parameterized inverse Gaussian probability density functions. One way is, for parameters $\mu > 0$ and $\lambda > 0$,

$$f(y \mid \mu, \lambda) = \left(\frac{\lambda}{2\pi y^3}\right)^{1/2} \exp\left[-\frac{\lambda}{2\mu^2 y}(y-\mu)^2\right], \quad y > 0. \tag{1}$$

Question 1 (5 pts)

Write the density (1) in exponential dispersion family form. Identify the natural parameter θ and dispersion parameter ϕ in terms of the original parameters μ and λ.

Answer

Question 2 (5 pts)

Using the result from question 1, find the expected value and variance of a random variable Y that follows an inverse Gaussian distribution. Write these moments in terms of θ and ϕ and then also in terms of μ and λ.

How is the variance related to the expected value for this distribution?

Answer

Question 3 (5 pts)

To get a feel for this distribution with the same location but different values of the dispersion parameter, produce a graph with three overlaid density functions having $\mu = 1$ and $\lambda = 4, 8$, and 16.

Answer

```
# Placeholder for your code
# Example: curve plotting densities with dIG from a package
```

Question 4 (10 pts)

Now compare the inverse Gaussian distributions from Question 3 with the corresponding gamma distributions, which means gamma distributions having the same mean and variance as the inverse Gaussian distributions.

A gamma density with parameters $\alpha > 0$ and $\beta > 0$ is

$$f(y \mid \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \exp(-\beta y), \quad y > 0.$$

The expected value and variance for a random variable Y having this distribution are

$$E(Y) = \frac{\alpha}{\beta}, \quad \mathrm{Var}(Y) = \frac{\alpha}{\beta^2}.$$

To find the corresponding gamma distribution for an inverse Gaussian distribution with parameter values μ and λ, first determine the expected value and variance that result from your answer to Question 2, equate these with the expected value and variance for a Gamma(α, β) distribution as just given, and solve for α and β.

Finally, produce three graphs with the three inverse Gaussian distributions already computed in Question 3 and overlay the corresponding gamma distribution.

For the three cases of Question 3, do you notice any systematic difference in inverse Gaussian and gamma distributions?

Answer

```r
# Placeholder code
# Example structure (fill in with actual functions later):

# Define parameters
mu <- 1
lambdas <- c(4, 8, 16)

# Example plotting framework
# for (lambda in lambdas) {
#   # Compute IG density (statmod::dinvgauss)
#   # Compute Gamma density (dgamma with matched mean/var)
#   # Overlay plots
# }
```

To find the corresponding gamma distribution for an inverse Gaussian distribution with parameter values μ and λ, first determine the expected value and variance that result from your answer to Question 2, equate these with the expected value and variance for a Gamma(α, β) distribution as just given, and solve for α and β.

Finally, produce three graphs with the three inverse Gaussian distributions already computed in Question 3 and overlay the corresponding gamma distribution.

For the three cases of Question 3, do you notice any systematic difference in inverse Gaussian and gamma distributions?