Homework 5 – STAT 542

No Due Date)

1. For a family of cdfs $F(x|\theta)$, $\theta \in \Theta$, we can that the cdfs are stochastically ordered if $\theta_1 > \theta_2$ implies $F(x|\theta_1) \leq F(x|\theta_2)$ for any $x \in \mathbb{R}$, i.e., $1 - F(x|\theta_1) \geq 1 - F(x|\theta_2)$.

   (a) Show that a location family is stochastically ordered in terms of the location parameter.

   (b) If the sample space (or range of the random variable) is $[0, \infty)$, show that a scale family is stochastically ordered in terms of the scale parameter.

2. 4.1, Casella & Berger. (The joint pdf is $f(x, y) = \frac{1}{4}$ (constant) for $-1 < x < 1$, $-1 < y < 1$ here; probabilities (integrals under joint pdf) will be determined as "an (x, y) subregion of $(-1, 1) \times (-1, 1)$" multiplied by $1/4$.)

3. 4.4, Casella & Berger

4. 4.5, Casella & Berger

5. Suppose that an urn contains 4 balls 1,3,5,8. We choose two balls at random from the urn without replacement. Let X be the number on the first ball chosen and let Y represent the larger of the numbers appearing on the two balls.

   (a) State the joint pmf for the random vector (X, Y).
   Hint: It might be easier to start with the joint distribution of (X, Z), where X is the number on the first ball chosen and Z is the number on the second ball chosen. There are 12 outcomes possible for (X, Z), each with probability $1/12$; these outcomes and probabilities determine the distribution of (X, Y).

   (b) Give the marginal distribution of X and the marginal distribution of Y.

   (c) Find the expected value of X and the expected value of $Y - X$.

   (d) Give the covariance of X and Y

6. Consider (X, Y) having the distribution in problem 2 above. Find mean μ_X and variance σ_X^2 of X and the mean μ_Y and variance σ_Y^2 of Y, and determine $\mathrm{E}[(X - \mu_X)^2(Y - \mu_Y)^2]$.

7. Prove or disprove the following.

   (a) If $\mathrm{E}X > \mathrm{E}Y$ then $P(X > Y) > 0$.
   Hint: Consider that $P(X > Y) = 0$ means that $Y \geq X$ with probability 1.

   (b) Suppose that $F_X(x)$ and $F_Y(y)$ are univariate cdfs. Is $F(x, y) \equiv \max\{F_X(x), F_Y(y)\}$ a legitimate bivariate cdf?

   (c) Suppose that $F_X(x)$ and $F_Y(y)$ are univariate cdfs. Is $F(x, y) \equiv \min\{F_X(x), F_Y(y)\} = [F_X(x) + F_Y(y) - |F_X(x) - F_Y(y)|]/2$ a legitimate bivariate cdf?