Stat 5100 Assignment 1

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Due: Wednesday, January 29th 11:59PM in Gradescope

Instructions

- Please note that a template for the assignment will be provided, but its use is not required.
- When submitting your assignment in Gradescope, ensure that every problem set (3–8) starts on a new page.

Purpose

The main purpose of this assignment is to familiarize yourself with the Preliminary Knowledge on Linear Algebra and Statistics posted for Lecture 1. Aside from Question 2, all questions are related to Linear Algebra. Questions on statistical concepts will follow on Homework 2.

Problem 1

Search the online catalog of Parks Library for a Linear Algebra book specifically for Statistics. I found at least one that is available online through your ISU account. Feel free to search elsewhere. I am not asking you to purchase any books, but I want you to have access to at least one as a resource.

Problem 2

Read through the notes posted for Lecture 1 (15-page document). Post any questions you have on the discussion board in the designated space. Grant and I, or your peers, will answer your questions.

Problem 3

Let $\mathbf{A}$ be an $m \times m$ idempotent matrix. Show that:

a) $\underset{m\times m}{\mathbf{I}} - \mathbf{A}$ is idempotent.

b) $\mathbf{B}\mathbf{A}\mathbf{B}^{-1}$ is idempotent, where $\mathbf{B}$ is any $m \times m$ nonsingular matrix.

Problem 4

A matrix $\mathbf{A}$ is symmetric if $\mathbf{A} = \mathbf{A}^\top$. Determine the truth of the following statements:

a) If $\mathbf{A}$ and $\mathbf{B}$ are symmetric, then their product $\mathbf{AB}$ is symmetric.

b) If $\mathbf{A}$ is not symmetric, then $\mathbf{A}^{-1}$ is not symmetric.

c) When $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are symmetric, the transpose of $\mathbf{ABC}$ is $\mathbf{CBA}$.

If $\mathbf{A} = \mathbf{A}^\top$ and $\mathbf{B} = \mathbf{B}^\top$, which of these matrices are certainly symmetric?

- $\mathbf{A}^2 - \mathbf{B}^2$
- $\mathbf{ABA}$
- $\mathbf{ABAB}$
- $(\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B})$

Problem 5

Consider the matrix

$$\mathbf{X} = \begin{bmatrix} 1 & -3 & 0 & -3 \\ 1 & -2 & -1 & 2 \\ 2 & -5 & -1 & -1 \end{bmatrix}$$

a) Show that the columns of $\mathbf{X}$ are linearly dependent.

b) Find the rank of $\mathbf{X}$.

c) Use the generalized inverse algorithm in Slide Set 1 to find a generalized inverse of $\mathbf{X}$.

d) Use the R function `ginv` in the `MASS` package to find a generalized inverse of $\mathbf{X}$.

- To load the `MASS` package into your R workspace, use the command `library(MASS)`.
- If the `MASS` package is not already installed, use `install.packages("MASS")` to install it.

e) Provide one matrix $\mathbf{X}^*$ that satisfies both of the following characteristics:

- $\mathbf{X}^*$ has full-column rank.
- $\mathbf{X}^*$ has column space equal to the column space of $\mathbf{X}$.

Problem 6

Prove the following result:

Suppose the set of $m \times 1$ vectors $\mathbf{x}_1, \ldots, \mathbf{x}_n$ is a basis for the vector space $\mathcal{S}$. Then any vector $\mathbf{x} \in \mathcal{S}$ has a unique representation as a linear combination of the vectors $\mathbf{x}_1, \ldots, \mathbf{x}_n$.

Problem 7

Am I a vector space? (The basic question here is whether every linear combination is in the space. If there is no zero, then I'm for sure not a vector space.)

a) All vectors in $\mathbb{R}^n$ whose entries sum to 0.

b) All matrices in $\mathbb{R}^{m \times n}$ whose entries, when squared, sum to 1.

Problem 8

Let $\mathbf{A}$ represent any $m \times n$ matrix, and let $\mathbf{B}$ represent any $n \times q$ matrix. Prove that for any choices of generalized inverses $\mathbf{A}^-$ and $\mathbf{B}^-$, $\mathbf{B}^-\mathbf{A}^-$ is a generalized inverse of $\mathbf{AB}$ if and only if $\mathbf{A}^-\mathbf{ABB}^-$ is idempotent.