Statistics 520, Fall 2025

Assignment 3

1. (10 pt.) Suppose that a random variable Y has a beta distribution with parameters α and β. A standard form for the probability density function of Y is, for $\alpha > 0$ and $\beta > 0$,

$$f(y|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\,\Gamma(\beta)}\, y^{\alpha-1}\,(1-y)^{\beta-1}; \quad 0 < y < 1.$$

   Put this density in canonical exponential family form. Using properties of exponential families, find $E\{\log(y)\}$ and $E\{\log(1-Y)\}$ expressed in terms of the original α and β parameters.

   Note: use $\Gamma'(x)$ to denote the derivative of the gamma function, $\frac{d}{dx}\Gamma(x)$.

2. (5 pt.) Suppose that a random variable Y has a Poisson distribution with parameter λ. A standard form for the probability mass function of Y is, for $\lambda > 0$,

$$f(y|\lambda) = \frac{1}{y!}\,\lambda^{y}\,\exp(-\lambda); \quad y = 0, 1, 2, \ldots$$

   Put this probability mass function in canonical exponential family form. Using properties of exponential families, verify that $E(Y) = \lambda$.

3. Suppose that a random variable Y has a gamma distribution with parameters α and β. A standard form for the probability density function of Y is, for $\alpha > 0$ and $\beta > 0$,

$$f(y|\alpha,\beta) = \frac{\beta^{\alpha}}{\Gamma(\alpha)}\, y^{\alpha-1}\,\exp(-\beta y); \quad y > 0. \tag{1}$$

   Note: You may have seen a gamma density written with a parameter that is equal to $1/\beta$ in the above expression. Use the parameterization given above to *answer this question (I think it will be easier).*

(a) (5 pts.) Write the gamma density in the form of a two parameter exponential family. Using properties of exponential families, derive the expected values of Y and $\log(Y)$.

(b) (5 pts.) Write the gamma density in the form of an exponential dispersion family with parameters θ and ϕ. Derive the expected value of Y.