Homework 4 – STAT 5430

Due Monday, March 10 by midnight in gradescope;

1. Problem 6.2, Casella and Berger (2nd Edition)
2. *Example of Rao-Blackwell theorem, which is largely a STAT 5420 problem in computation.*

   Let X_1 and X_2 be iid Bernoulli(p), $0 < p < 1$.

   (a) Show $S = X_1 + X_2$ is sufficient for p.

   (b) Identify the conditional probability $P(X_1 = x|S = s)$; you should know which values of x, s to consider.

   (c) Find the conditional expectation $T \equiv E(X_1|S)$, i.e., as a function of the possibilities of S. Note that T is a statistic.

   (d) Show X_1 and T are both unbiased for p.

   (e) Show $\mathrm{Var}_p(T) \leq \mathrm{Var}_p(X_1)$, for any p.
3. Problem 6.21(a)-(b), Casella and Berger (2nd Edition)
4. Problem 6.24, Casella and Berger (2nd Edition)
5. Problem 7.57, Casella and Berger (2nd Edition) You may assume $n \geq 3$.
   One has to Rao-Blackwellize on the complete/sufficient statistic here $\sum_{i=1}^{n+1} X_i$.