HW5

2024-09-29

Homework 5

NOT DUE

Q1: (a), (b)

For a family of cdfs $F(x|\theta), \theta \in \Theta$, we can say that the cdfsare stochastically ordered if $\theta_1 > \theta_2$ imples that

$$F(x|\theta_1) \leq F(x|\theta_2)$$

for any $x \in \mathbb{R}$, i.e.

$$1 - F(x|\theta_1) \geq 1 - F(x|\theta_2)$$

Q1: (a)

Show that a location family is stochastically ordered in terms of the location parameter.

A1: (a)

Q1: (b)

If the sample space (or range of the random variable) is $[0, \infty)$, show that a scae family is stochastically ordered in terms of the scale parameter.

A1: (b)

Q2: 4.1, Casella & Berger, (a), (b), (c)

A random point (X, Y) is distributed uniformly on the square with vertices (1,1), (1, -1), (-1,1), and (-1, -1). That is, the joint pdf is $f(x, y) = \frac{1}{4}$ on the square. Determine the probabilities of the following events.

Note: The joint pdf is

$$f(x, y) = \frac{1}{4}$$

constant for -1<x<1, -1<y<1 here; probability (integrals under joint pdf) will be determined as "an (x,y) subregion of (-1, 1) X (-1, 1)" multiplied by $\frac{1}{4}$.

Q2: (a)

$$X^2 + Y^2 < 1$$

A2: (a)

Q2: (b)

$$2X - Y > 0$$

A2: (b)

Q2: (c)

$$|X + Y| < 2$$

A2: (c)

Q3: 4.4, Casella & Berger, (a), (b), (c), (d)

A pdf is defined by:

$$f(x,y) = \begin{cases} C(x+2y) & \text{if } 0 < y < 1 \text{ and } 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

Q3: (a)

Find the value of C.

A3: (a)

Q3: (b)

Find the marginal distribution of X.

A3: (b)

Q3: (c)

Find the joint pdf of X and Y.

A3: (c)

Q3: (d)

Find the pdf of the random variable

$$Z = \frac{9}{(X+1)^2}$$

A3: (d)

Q4: 4.5, Casella & Berger, (a), (b)

Q4: (a)

Find $P(X > \sqrt{Y})$ if X and Y are jointly distributed with pdf:

$$f(x, y) = x + y, 0 \leq x \leq 1, 0 \leq y \leq 1$$

A4: (a)

Q4: (b)

Find $P(X^2 < Y < X)$ if X and Y are jointly distributed with pdf:

$$f(x, y) = 2x, 0 \leq x \leq 1, 0 \leq y \leq 1$$

A4: (b)

Q5: (a), (b), (c), (d)

Suppose that an urn contains 4 balls: 1, 3, 5, 8. We choose two balls at random from the urn without replacement. Let X be the number on the first ball chosen and let Y represent the larger of the numbers appearing on the two balls.

Q5: (a)

State the joint pmf for the random vector (X, Y).

Hint: It might be easier to start with the joint distribution of (X, Z) where X is the number on the first ball chosen and Z is the number on the second ball chosen. There are 12 outcomes possible for (X, Z), each with probability 1/12; these outcomes and probabilities determine the distribution of (X, Y).

A5: (a)

Q5: (b)

Give the marginal distribution of X and the marginal distribution of Y.

A5: (b)

Q5: (c)

Find the expected value of X and the expected value of Y - X.

A5: (c)

Q5: (d)

Give the covariancne of X and Y.

A5: (d)

Q6

Q6

Consider (X, Y) have the distriution in problem 2 above. Find the mean μ_X and variance σ_X^2 of X and the mean μ_Y and variance σ_Y^2 of Y, and determine

$$E[(X-\mu_X)^2(Y-\mu_Y)^2]$$

A6

Q7: (a), (b), (c)

Prove or disprove the following.

Q7: (a)

If EX > EY then P(X > Y) = 0.

Hint: Consider that P(X > Y) = 0 means that $Y \geq X$ with probability 1.

A7: (a)

Q7: (b)

Suppose that $F_X(x)$ and $F_Y(y)$ are univariate cdfs. Is $F(x, y) \equiv max\{F_X(x), F_Y(y)\}$ a legitimate bivariate cdf?

A7: (b)

Q7: (c)

Suppose that $F_X(x)$ and $F_Y(y)$ are univariate cdfs. Is $F(x, y) \equiv min\{F_X(x), F_Y(y)\} = [F_X(x) + F_Y(y) - |F_X(x) - F_Y(y)|]/2$ a legitimate bivariate cdf?

A7: (c)