HW7

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Q1

Problem 8.6 a) - b), Casella and Berger (2nd Edition)

Suppose that we have two independent random samples: $X_1, \ldots, X_n$ are exponential(θ), and $Y_1, \ldots, Y_m$ are exponential(μ).

a)

Find the LRT of

$$H_0 : \theta = \mu \quad \text{versus} \quad H_1 : \theta \neq \mu.$$

Answer

b)

Show that the test in part a) can be based on the statistic

$$T = \frac{\sum X_i}{\sum X_i + \sum Y_i}.$$

Answer

Q2

Problem 8.28, Casella and Berger (2nd Edition)

Let $f(x|\theta)$ be the logistic location probability density function:

$$f(x|\theta) = \frac{e^{(x-\theta)}}{(1+e^{(x-\theta)})^2}, \quad -\infty < x < \infty, \quad -\infty < \theta < \infty.$$

a)

Show that this family has an MLR.

Answer

b)

Based on one observation X, find the most powerful size α test of

$$H_0 : \theta = 0 \quad \text{versus} \quad H_1 : \theta = 1.$$

For $\alpha = 0.2$, find the size of the Type II error.

Answer

c)

Show that the test in part b) is UMP size α for testing

$$H_0 : \theta \leq 0 \quad \text{versus} \quad H_1 : \theta > 0.$$

What can be said about UMP tests in general for the logistic location family?

Answer

Q3

Problem 8.29 a) - b), Casella and Berger (2nd Edition)

Let X be one observation from a Cauchy(θ) distribution.

The Cauchy(θ) density is given by:

$$f(x|\theta) = \frac{1}{\pi} \cdot \frac{1}{1 + (x - \theta)^2}, \quad x \in \mathbb{R}, -\infty < \theta < \infty.$$

a)

Show that this family does not have an MLR.

Hint:

Show that the Cauchy(θ) family $\{f(x|\theta) : \theta \in \mathbb{R} = \Theta\}$, based on one observation X, does not have monotone likelihood ratio (MLR) in $t(X) = X$ or $t(X) = -X$. That is, the ratio

$$\frac{f(x|\theta_2)}{f(x|\theta_1)}$$

might not be monotone (either increasing or decreasing) in x.

Answer

b)

Show that the test

$$\phi(x) = \begin{cases} 1 & \text{if } 1 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

is most powerful of its size for testing

$$H_0 : \theta = 0 \quad \text{versus} \quad H_1 : \theta = 1.$$

Calculate the Type I and Type II error probabilities.

Hint:

Show that the test given is equivalent to rejecting H_0 if

$$f(x|\theta = 1) > 2f(x|\theta = 0)$$

and not rejecting otherwise. Conclude that this must be the most powerful (MP) test for its size. Justify why.

Answer

Q4

Consider one observation X from the probability density function

$$f(x|\theta) = 1 - \theta^2 \left(x - \frac{1}{2}\right), \quad 0 \leq x \leq 1, \quad 0 \leq \theta \leq 1.$$

We wish to test:

$$H_0 : \theta = 0 \quad \text{vs.} \quad H_1 : \theta > 0$$

a)

Find the UMP test of size $\alpha = 0.05$ based on X. Carefully justify your answer.

Answer

b)

Find the likelihood ratio test statistic $\lambda(X)$ based on X, expressed as a function of X.

Answer

c)

Find the likelihood ratio test (LRT) of size $\alpha = 0.05$ for the above hypotheses.

Answer