Impact of Precision

2024-09-16

Sample Size Calculations Using Hypothesis Tests

Formula

$$n_0 = \frac{(z_{1-\frac{\alpha}{2}} + z_{1-\beta})^2(2S_p^2)}{\delta^2}$$

$$n = \frac{(t_{2(n_0-1),1-\frac{\alpha}{2}} + t_{2(n_0-1),1-\beta})^2(2S_p^2)}{\delta^2}$$

Comparison of Methods

Methods (1) and (2) are thankfully (though perhaps unsurprisingly) consistent. However, the final n obtained through Method (3) is significantly different between the other methods noted.

Overall, I think it may help to note that the t statistic ("critical value") should be obtained through direct computation, and not through the use of a lookup table, though at most in this problem it caused a difference of ≈ 7 in sample size.

Issue

In HW3, Problem 3 we have:

> 3. Given an approximate pooled sample standard deviation of $S_p = 0.16$ and an effect size of $\delta = 0.03$, what sample size is needed in each of two equally-sized treatment groups in order for a level $\alpha = 0.05$ two-sided test to have 80% power?

Where $\delta = 0.03$, $S_p = 0.16$, $\alpha = 0.05$, and $\beta = 0.2 \to 1 - \beta = 0.8$

We three approaches to consider when calculating sample sizes: 1. Round the values of the z and t statistics we obtain 2. Directly compute the z and t statistics within the calculation 3. Reference a z/t statistic table such as the below

Method 1

```r
z1 <- qnorm(p = 0.975)
z2 <- qnorm(p = 0.8)

round(z1, 3)
```

```
## [1] 1.96
```

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.

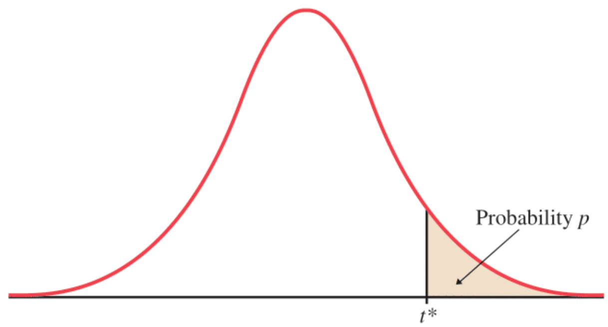

TABLE D ***t* distribution critical values**

	Upper tail probability p											
df	**.25**	**.20**	**.15**	**.10**	**.05**	**.025**	**.02**	**.01**	**.005**	**.0025**	**.001**	**.0005**
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	**60%**	**70%**	**80%**	**90%**	**95%**	**96%**	**98%**	**99%**	**99.5%**	**99.8%**	**99.9%**
	Confidence level C											

Figure 1: t Table Figure

```r
round(z2, 3)
```

```
## [1] 0.842
```

```r
numerator <- (round(z1, 3)  + round(z2, 3))^2 * (2 * 0.16^2)
denom <- 0.03^2
n0 <- numerator/denom
n0
```

```
## [1] 446.6463
```

```r
t1 <- qt(p = .975, df = 446.6463)
t2 <- qt(p = .80, df = 446.6463)
t1
```

```
## [1] 1.965289
```

```r
t2
```

```
## [1] 0.8424267
```

```r
numerator <- (round(t1, 3) + round(t2,3))^2 * (2 * 0.16^2)
denom <- 0.03^2
n <- numerator/denom
n
```

```
## [1] 448.2417
```

Method 2

```r
z1 <- qnorm(p = 0.975)
z2 <- qnorm(p = 0.8)

numerator <- (z1 + z2)^2 * (2 * 0.16^2)
denom <- 0.03^2
n0 <- numerator/denom
n0
```

```
## [1] 446.514
```

```r
t1 <- qt(p = .975, df = 447)
t2 <- qt(p = .80, df = 447)

t1
```

```
## [1] 1.965285
```

```
t2
```

```
## [1] 0.8424261
```

```
numerator <- (t1 + t2)^2 * (2 * 0.16^2)
denom <- 0.03^2
n <- numerator/denom
n
```

```
## [1] 448.4689
```

Method 3

```
z1 <- 1.96
z2 <- qnorm(p = 0.8)

numerator <- (z1 + z2)^2 * (2 * 0.16^2)
denom <- 0.03^2
n0 <- numerator/denom
n0
```

```
## [1] 446.5255
```

```
t1 <- 1.984
t2 <- 0.845

numerator <- (t1 + t2)^2 * (2 * 0.16^2)
denom <- 0.03^2
n <- numerator/denom
n
```

```
## [1] 455.2955
```