PS1

Sam Olson

1.

In Chapter 11.1.2 of the Stat 520 notes we gave the basic form of a Monte Carlo approximation as

$$E_M\{g(X_m)\} = \frac{1}{M}\sum_{m=1}^{M} g(X_m^*), \tag{1}$$

where X_m^* were values sampled from the distribution of X, call it f.

Chapter 11.2 contained an example which compared coverage rates of approximate and exact confidence intervals for the difference in means. For this question, consider only the approximate interval given in expression (11.14) of the Stat 520 notes, and use M_1 from expression (11.13) as the pertinent model. Table 11.1 on page 476 of the Stat 520 notes reports observed coverage rates under the column headed *MC Approx* for various Monte Carlo sample sizes M.

The coverage rates of concern are called Monte Carlo Approximations, and hence must have the form of (??) above. Explicitly identify $g(X_m)$ and f for this use of Monte Carlo.

Answer

2.

In Chapter 11.3.2 of the Stat 520 notes, a $(1-\alpha)100\%$ confidence interval for a parameter θ based on subsampling is represented as (L, U) where these quantities are given in expression (11.25) as,

$$L = \hat{\theta}_M - \tau_M^{-1} q_{M,\,1-\alpha/2}$$

$$U = \hat{\theta}_M - \tau_M^{-1} q_{M,\,\alpha/2}.$$

Here, $q_{M,\nu}$ is the νth quantile of $L_{M,b}(y)$, the empirical distribution function of $\tau_b(\hat{\theta}_{b,j} - \hat{\theta}_M)$ as in expression (11.24) of the Stat 520 notes.

Verify that the intervals given in (11.25) of the Stat 520 notes are correct.

Hint:

(a) The subsampling principle is that the distribution function of $\tau_M(\hat{\theta}_M - \theta)$ is approximated by $L_{M,b}(y)$, the empirical distribution function of

$$\{\tau_b(\hat{\theta}_{b,j} - \hat{\theta}_M) : j = 1, \ldots, k\}.$$

Take this as being true. That is, the difference between the sampling distribution of $\tau_M(\hat{\theta}_M - \theta)$ and the empirical distribution of $\{\tau_b(\hat{\theta}_{b,j} - \hat{\theta}_M) : j = 1, \ldots, k\}$ is not the key for this question.

(b) Begin with what is desired, finding quantities L and U such that

$$\Pr(\theta < L) = \Pr(\theta > U) = \alpha/2.$$

Answer