Stat 5100 **Assignment 1**

Due: Wednesday, January 29th 11:59PM in gradescope.

Please note that I will post a template for the assignment, however, its use is not required. When you submit your assignment in gradescope please have every problem set (3–8) start on a new page.

Purpose: The main purpose of this assignment is to familiarize yourself with the *Preliminary Knowledge on Linear Algebra* and *Statistics* posted for Lecture 1. Aside from Question 2, all questions are related to Linear Algebra. Questions on statistical concepts will follow on Homework 2.

Problem 1. *Search the on-line catalog of Parks Library for a Linear Algebra book specifically for Statistics. I found at least one that is available online through your ISU account. Feel free to search elsewhere. I am not asking you to purchase any books, I want you to have access to at least one, however, to serve as a resource.*

Problem 2. *Read through the notes posted for Lecture 1 (15-page document). Post any questions you have on the discussion board in the designated space. Grant and I, or your own peers will answer your questions.*

Problem 3. *Let* $\mathbf{A}$ *be an* $m \times m$ *idempotent matrix. Show that*

a) $\underset{m\times m}{\boldsymbol{I}} - \mathbf{A}$ *is idempotent.*

b) $\mathbf{B}\mathbf{A}\mathbf{B}^{-1}$ *is idempotent, where* $\mathbf{B}$ *is any* $m \times m$ *nonsingular matrix.*

Problem 4. *A matrix* $\mathbf{A}$ *is symmetric if* $\mathbf{A} = \mathbf{A}^\top$*. Which of these are true?*

a) If $\mathbf{A}$ *and* $\mathbf{B}$ *are symmetric then their product* $\mathbf{AB}$ *is symmetric.*

b) If $\mathbf{A}$ *is not symmetric then* $\mathbf{A}^{-1}$ *is not symmetric.*

c) When $\mathbf{A}$ *,* $\mathbf{B}$*,* $\mathbf{C}$ *are symmetric, the transpose of* $\mathbf{ABC}$ *is* $\mathbf{CBA}$.

If $\mathbf{A} = \mathbf{A}^\top$ *and* $\mathbf{B} = \mathbf{B}^\top$*, which of these matrices are certainly symmetric?*

d) $\mathbf{A}^2 - \mathbf{B}^2$

e) $\mathbf{ABA}$

f) $\mathbf{ABAB}$

g) $(\mathbf{A} + \mathbf{B})(\mathbf{A} - \mathbf{B})$

Problem 5. *Consider the matrix*

$$\boldsymbol{X} = \begin{bmatrix} 1 & -3 & 0 & -3 \\ 1 & -2 & -1 & 2 \\ 2 & -5 & -1 & -1 \end{bmatrix}.$$

a) *Show that the columns of $\boldsymbol{X}$ are linearly dependent.*

b) *Find the rank of $\boldsymbol{X}$.*

c) *Use the generalized inverse algorithm in slide set 1 to find a generalized inverse of $\boldsymbol{X}$.*

d) *Use the* `R` *function* `ginv` *in the* `MASS` *package to find a generalized inverse of $\boldsymbol{X}$. (To load the* `MASS` *package into your* `R` *workspace use the command* `library(MASS)`. *If the* `MASS` *package is not already installed, you will need to install it before loading. The command* `install.packages(``MASS'')` *should install the* `MASS` *package if necessary.)*

e) *Provide one matrix $\boldsymbol{X}^*$ that satisfies both of the following characteristics:*

- *$\boldsymbol{X}^*$ has full-column rank (i.e., $\text{rank}(\boldsymbol{X}^*)$ is equal to the number of columns of $\boldsymbol{X}^*$), and*
- *$\boldsymbol{X}^*$ has column space equal to the column space of $\boldsymbol{X}$; i.e., $\mathcal{C}(\boldsymbol{X}^*) = \mathcal{C}(\boldsymbol{X})$.*

Problem 6. *Prove the following result. Suppose the set of $m \times 1$ vectors $\mathbf{x}_1, \ldots, \mathbf{x}_n$ is a basis for the vector space $\mathcal{S}$. Then any vector $\mathbf{x} \in \mathcal{S}$ has a unique representation as a linear combination of the vectors $\mathbf{x}_1, \ldots, \mathbf{x}_n$.*

Problem 7. *Am I a vector space? (The basic question here is whether every linear combination is in the space. If there is no zero, then I'm for sure not a vector space.)*

a) *All vectors in $\mathbb{R}^n$ whose entries sum to 0.*

b) *All matrices in $\mathbb{R}^{m \times n}$ whose entries when squared sum to 1.*

Problem 8. *Let $\mathbf{A}$ represent any $m \times n$ matrix, and let $\mathbf{B}$ represent any $n \times q$ matrix. Prove that for any choices of generalized inverses $\mathbf{A}^-$ and $\mathbf{B}^-$, $\mathbf{B}^-\mathbf{A}^-$ is a generalized inverse of $\mathbf{AB}$ if and only if $\mathbf{A}^-\mathbf{ABB}^-$ is idempotent.*