Statistics 520

Five Minute Quiz 4

Fall 2025

1. (2 pts.) We have briefly come across the right-skew version of an extreme value distribution which has density, for $-\infty < \xi < \infty$ and $\phi > 0$,

$$f(y|\xi,\phi) = \exp\left\{\frac{-(y-\xi)}{\phi}\right\}\exp\left[-\exp\left\{\frac{-(y-\xi)}{\phi}\right\}\right];\ -\infty < y < \infty. \quad (1)$$

Suppose that $Y_1, \ldots, Y_n$ are independent and identically distributed random variables having common probability density function (1) with known $\xi = 0$. Given a realization of this model $\boldsymbol{y} = (y_1, \ldots, y_n)$, the maximum likelihood estimate of ϕ was found to be $\hat{\phi} = 1.16$, and 90% approximate confidence intervals were computed using (1) Wald theory, and (2) inversion of a likelihood ratio test statistic. Identify which of the following intervals was computed using which approach:

Interval A: $(-0.02, 2.34)$

Interval B: $(0.03, 2.89)$

Answer:

Interval A: Wald Theory

Interval B: Inversion of Likelihood Ratio Statistic

2. (2 pts.) How did you determine your answer to question 1?

Answer: Interval A has a lower endpoint outside of the parameter space of ϕ which would not happen for inversion of likelihood ratio statistics, and interval B is not symmetric about the maximum likelihood estimate which cannot happen for Wald theory intervals.