

Lab5

2024-10-03

Assignment - Lab 5

Due October 8th

1.

Refer to the ANOVA results for the donut example to complete the following exercises:

(a)

Construct a scatter plot of the absorption to visualize potential differences between oil treatment groups. Include a screenshot of the plot.

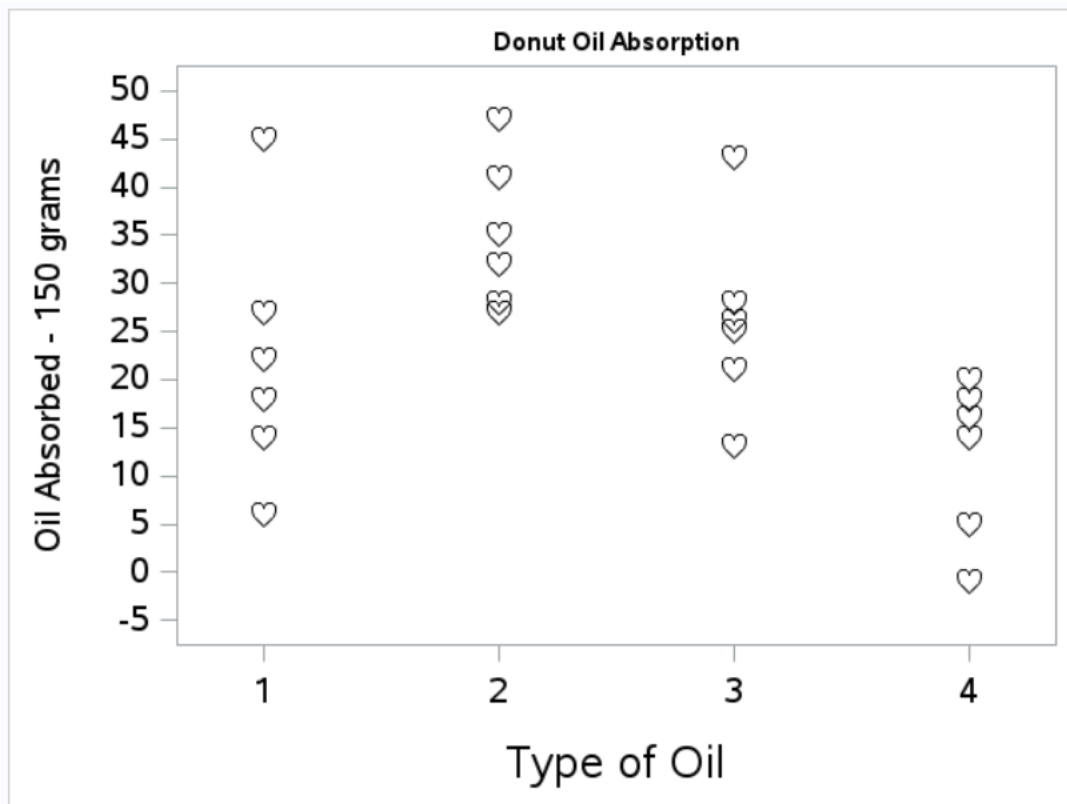


Figure 1: Scatter Plot of Absorption

(b)

Compute the relevant summary statistics to compare the amount of oil absorbed for each type. Include a screenshot or record the means and standard deviations.

Donut Oil Absorption						
The MEANS Procedure						
oil=1						
Analysis Variable : absorbed						
N	Mean	Std Dev	Variance	Std Error	Minimum	Maximum
6	22.0000000	13.3416641	178.0000000	5.4467115	6.0000000	45.0000000
oil=2						
Analysis Variable : absorbed						
N	Mean	Std Dev	Variance	Std Error	Minimum	Maximum
6	35.0000000	7.7717437	60.4000000	3.1728011	27.0000000	47.0000000
oil=3						
Analysis Variable : absorbed						
N	Mean	Std Dev	Variance	Std Error	Minimum	Maximum
6	26.0000000	9.8792712	97.6000000	4.0331956	13.0000000	43.0000000
oil=4						
Analysis Variable : absorbed						
N	Mean	Std Dev	Variance	Std Error	Minimum	Maximum
6	12.0000000	8.2219219	67.6000000	3.3565856	-1.0000000	20.0000000

Figure 2: Summaries of Absorption

(c)

Conduct the F-test for the equality of the four means. Provide a screenshot of the full ANOVA table for the different types of cooking oil and absorption measurements.

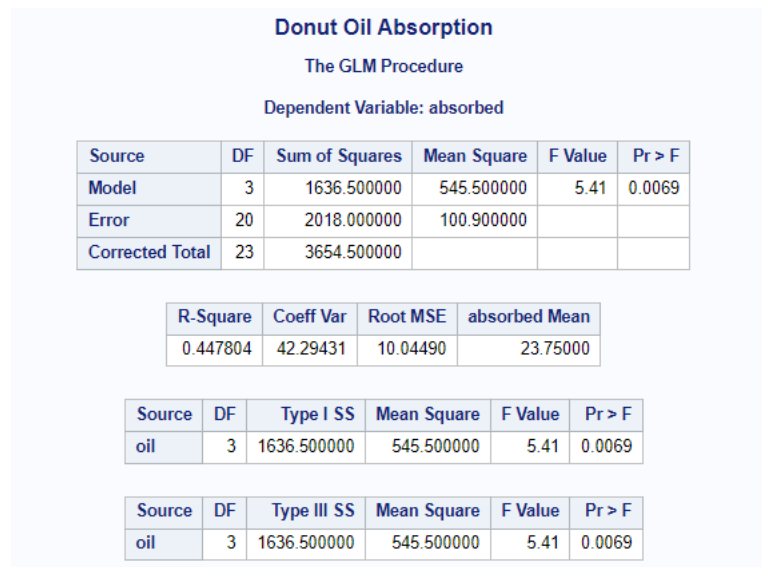


Figure 3: F-Test

(d)

Given the results in part (c), state the null and alternative hypotheses, test statistic, p-value, and interpretation of the result in the context of the study.

H_0 : Null Hypothesis: $\mu_1 = \mu_2 = \mu_3 = \mu_4$; the mean absorption of donuts is the same across all groups of oils.

H_A : Alternative Hypothesis: At least one μ_i is different for $i = 1, 2, 3, 4$; at least one mean absorption of a group of donuts is different than the mean absorption of the other donuts.

Test Statistic: F-Value: 5.41

p-value: $\text{Pr} > F$: 0.0069

Interpretation:

We have evidence to reject the null hypothesis that mean donut absorption of oil is the same for all groups of oil treatments, specifically at the $\alpha = 0.05$ level. We can be more confidence as well though given the p-value of 0.0069, which is a low probability that we observed the differences in mean absorption between groups that we did under the null hypothesis being true. This lends evidence in support of the alternative hypothesis that at least one group has a mean absorption of oil different from the mean absorption of oil in the other groups.

2.

Refer to the process of diagnosing the ANOVA assumptions for the donut example to complete the following exercises:

(a)

Discuss the independence assumption in the context of the study. Is it satisfied for the donut example?

It is reasonable to assume that the assumption of independence between groups is not violated, as different oils were used and each type of oil was distinct (presumably) from the others. The within group independence is a bit trickier by comparison, though I would say we have reason to believe this assumption is also not violated.

My reasoning is as follows: We are cooking in batches, so we guarantee that our observational units (a batch of donuts) is not repeated (used again for frying). Because we go from raw dough to cooked donuts, and because we only cook one batch of donuts at a time, we avoid any cross contamination of either mixing batches or of mixing oils, meaning independence is satisfied within and between groups.

However, the above does rest upon batches being cooked separately, which is a jump of logic; though the contrapositive (that multiple batches were cooked at once) was not noted.

(b)

Check the assumption of equal variances and provide a screenshot of any relevant tables and/or figures that you used. Is it satisfied for the donut example?

Overall Methods:

- Different plots of the residuals, including residual plots (Fit Diagnostics)
- Standard side-by-side boxplot of the responses within each group
- Brown-Forsythe test for equal variances
- Summary statistics

Overall Assessment: Overall we have reason to believe our equal variances assumption is not violated, though there is some visual evidence otherwise when assessing groupwise box plots.

The Brown-Forsythe test provides a large p-value which provides evidence to not reject the null hypothesis of equal variances between groups. Though visually, we see some differences in the variability/spread of absorption values between groups, such as those visually displayed in the box plots, we must take these with a grain of salt given we only have $n = 6$, 6 batches per oil type group. It is also worth noting that the reported summary statistics for each group (particularly the sample standard deviations) are all in the same orders of magnitude, ranging from approx. 7.8 to approx 13.3. If we were to construct pairwise comparisons of the ratio of standard deviations, they would all be between 1 and 2, (max of $13.3/7.8 = 1.705$), which using a heuristic would be support that there is “little impact” regarding differences of standard deviation.

When we then turn our attention to the residual plots, we notice a number of things to further support the notion that our equal variances assumption not being violated. Particularly, the residual plot by group does not show a general pattern, meaning we have constant variance in the residuals.

I also added a Levene test for homogeneity of variances and reported it below. This test affirms the above assessment, inasmuch as it provides a large(ish) p-value which provides further evidence not to reject the null hypothesis of homogenous (equal) variances between (all) groups.

Figures:

The GLM Procedure					
Levene's Test for Homogeneity of absorbed Variance ANOVA of Squared Deviations from Group Means					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
oil	3	36268.5	12089.5	0.75	0.5351
Error	20	322365	16118.3		

Figure 4: Extra

Level of oil	N	absorbed	
		Mean	Std Dev
1	6	22.0000000	13.3416641
2	6	35.0000000	7.7717437
3	6	26.0000000	9.8792712
4	6	12.0000000	8.2219219

Figure 5: Observed Variances

(c)

Check the assumptions of normality and provide a screenshot of any relevant tables and/or figures that you used. Is it satisfied for the donut example?

Overall Methods:

- Different plots of the residuals, including residual plots
- Standard side-by-side boxplot of the responses within each group
- Summary statistics
- Univariate procedure on the residuals, including:
 - Summary statistics, which includes skewness, excess kurtosis, mean, and median
 - Normality tests
- QQ Plot

Overall Assessment: Overall, normality in the data does not to be violated, at least at the overall (non-grouped) level, which similarly carries over to the residuals not violating normality.

For the sake of brevity, the below plots are only **new** plots for the purpose of diagnosing potential normality violations. The assessment was also done using a number of the plots given above and used in part (b) for assessing the equal variance assumption.

That being said:

Observations: From above figures, the residuals average to zero, which we also see when assessing the summary statistics (which also happens to equal the median value of 0!) The overall distribution of the observations appear normally distributed visually, with regards to the histogram. However, when looking at individual groups, the shape of the distributions appear somewhat non-normal, e.g. box plots by group have means not equal to medians, unequal spreads in variability such that there is not symmetry in variability about the mean. Again, this may be more of a byproduct of small sample sizes of groups, as the remaining evidence detailed above and below does not provide evidence that we are violating the normality assumption.

Residuals: Mean and median of the residuals are both zero, which are two details in favor of normality not being violated. We see a little amount of skewness (below 1) and a smidgen of excess kurtosis (also below 1 in magnitude). The Shapiro-Wilk test provides a large p-value that would lead us not to reject the null hypothesis of normality for the residuals (which are further supported by the other named statistical tests of normality). Additionally, the QQ Plot of the residuals closely follows along the reference line with minor deviations around the tails of the distribution.

Figures:

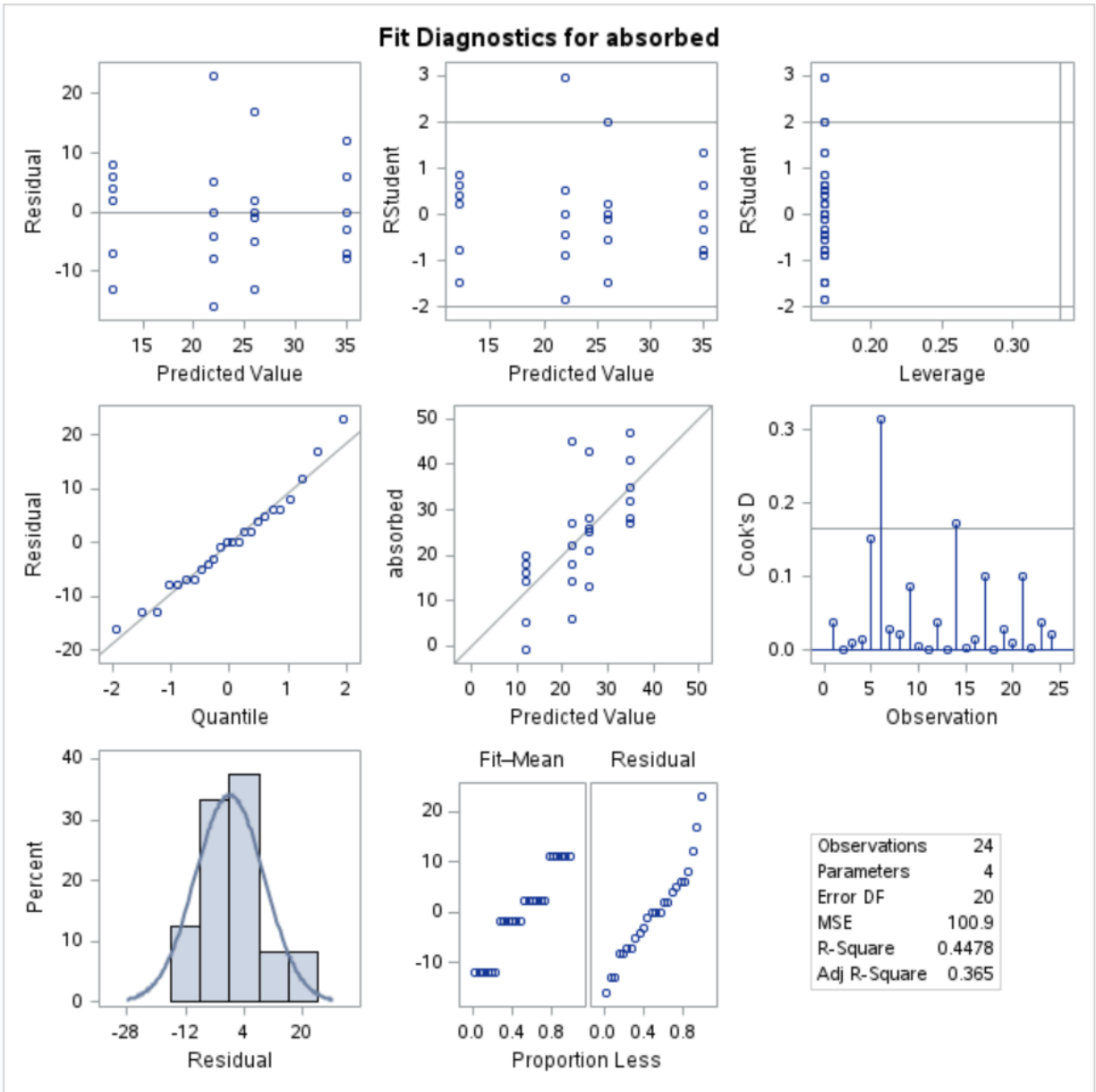


Figure 6: Lots of Plots

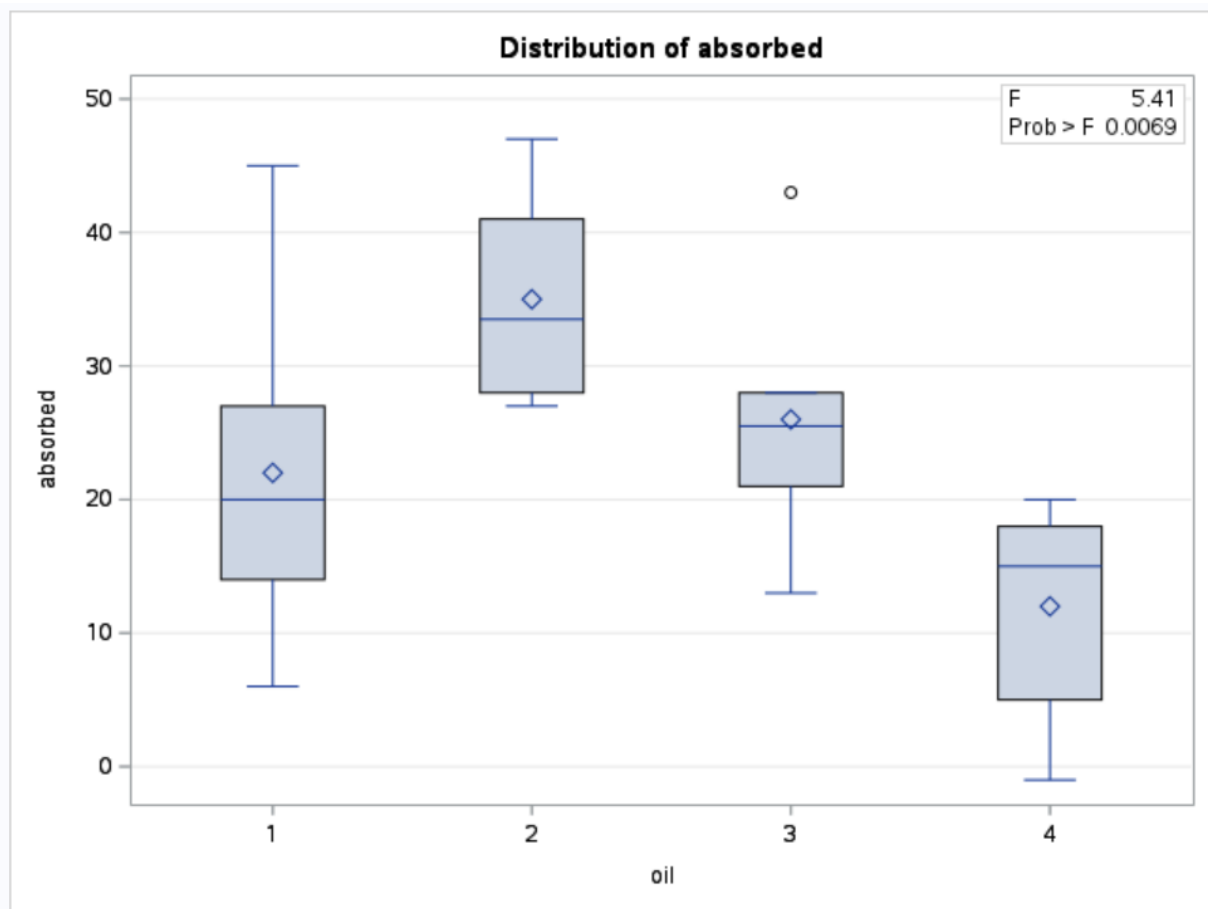


Figure 7: BoxPlots

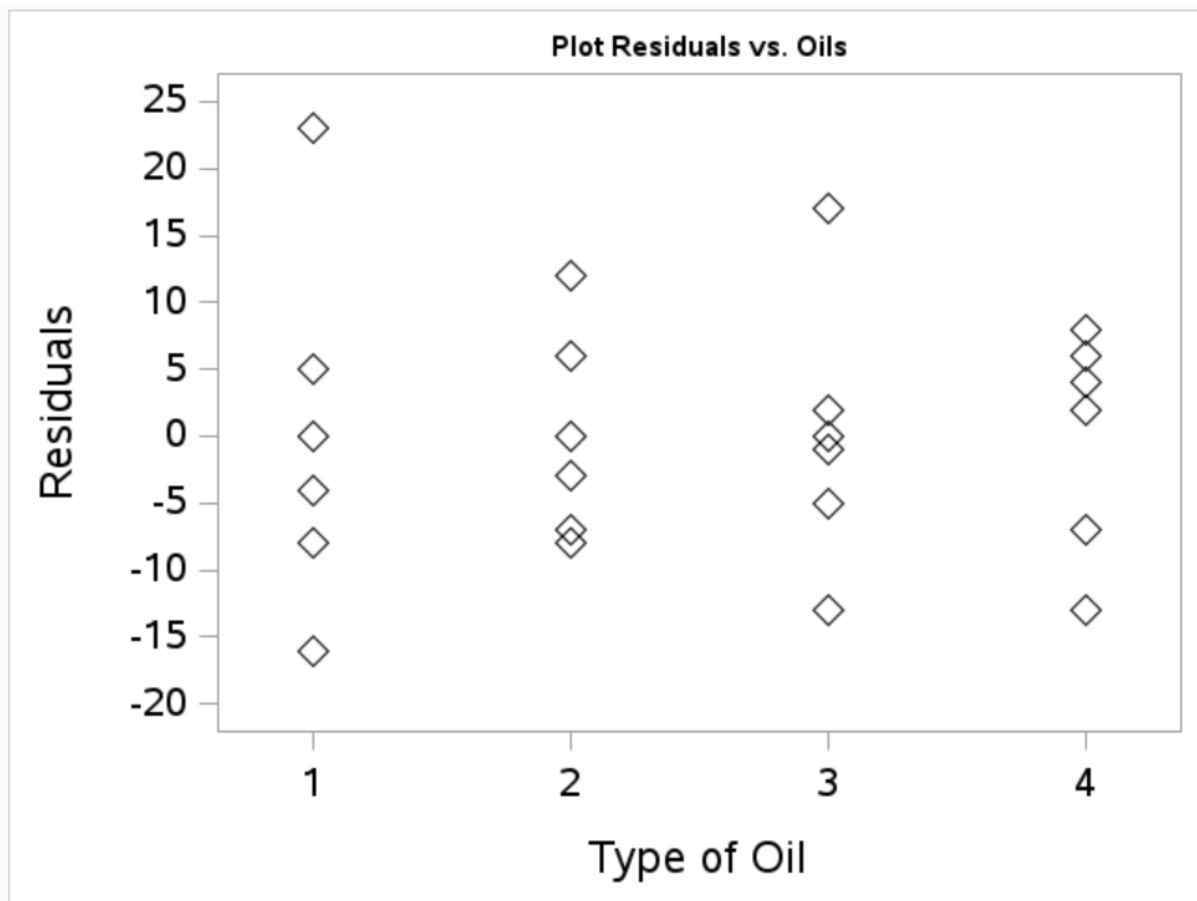


Figure 8: Residual Plot By Group



Figure 9: Normality Pt1

Plot Residuals vs. Oils

The UNIVARIATE Procedure

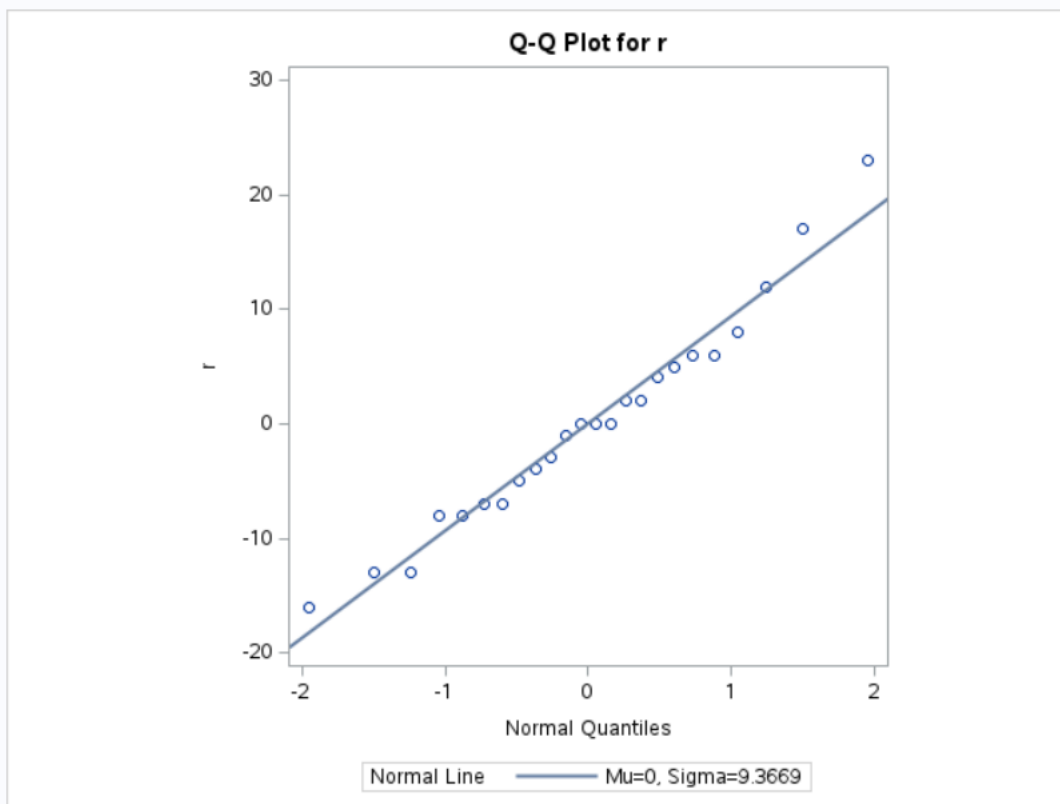


Figure 10: Normality Pt2

3.

Refer to the results of the Kruskal-Wallis test for the donut example to complete the following exercises:

(a)

Conduct the Kruskal-Wallis test to analyze the different types of cooking oil on the oil absorbed. Provide a screenshot of the results obtained.

Kruskal-Wallis test					
The NPAR1WAY Procedure					
Wilcoxon Scores (Rank Sums) for Variable absorbed Classified by Variable oil					
oil	N	Sum of Scores	Expected Under H0	Std Dev Under H0	Mean Score
1	6	67.50	75.0	14.988951	11.250000
2	6	117.00	75.0	14.988951	19.500000
3	6	81.50	75.0	14.988951	13.583333
4	6	34.00	75.0	14.988951	5.666667
Average scores were used for ties.					

Kruskal-Wallis Test		
Chi-Square	DF	Pr > ChiSq
11.8322	3	0.0080

Monte Carlo Estimates for the Exact Test					
Probability	Estimate	99% Confidence Limits		Samples	Seed
Pr >= ChiSq	0.0027	0.0021	0.0033	50000	118308878

Figure 11: Kruskal-Wallis test

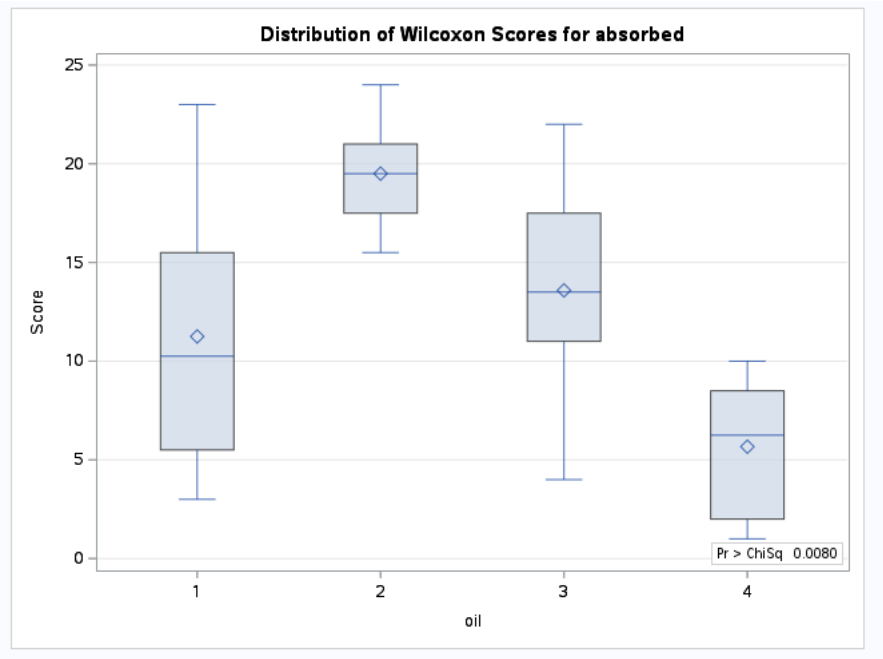


Figure 12: Kruskal-Wallis test Distribution

(b)

Given the results in part (a), state the null and alternative hypotheses, test statistic, and p-value.

H_0 : Null Hypothesis: r populations have the same distribution; each oil group (group of 4 different types of oil) have the same distributions.

H_A : Alternative Hypothesis: At least one group of oils (from the 4 different types of oil used) has a different distribution; at least one oil group of the four different types of frying oil has a different distribution than the others groups.

Test Statistic: Chi-Square: 11.8322

p-value: $\text{Pr} > \text{ChiSq}$: 0.0080

(c)

Interpret the result from part (b) in the context of the study.

We have evidence to reject the null hypothesis the distributions of oil absorption between the four groups of frying oil are all the same. We have evidence to reject the null hypothesis at the $\alpha = 0.05$ level, and have evidence to support the alternative hypothesis that at least one of the types of oil has a distribution that is different from the distributions of the other types of oil used in the study.