HW3

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HW 3

Name: Sam Olson
Collaborators: **Tne Hatman**

1. 2.23(b)

Question 1

Let X have the pdf

$$f(x) = \frac{1}{2}(1 + x)$$

, $-1 < x < 1$

Define the random variable Y by $Y = X^2$

(b) Find $E(Y)$ and Var(Y).

Answer 1

(b)

2.

Question 2

A family continues to have children until they have one female child. Suppose, for each birth, a single child is born and the child is equally likely to be male or female. The gender outcomes are independent across births. (a): Let X be a random variable representing the number of children born to this family. Find the distribution of X.

(b): Find the expected value $E(X)$

(c): Let X_m denote the number of male children in this family and let X_f denote the number of female children. Find the expected value of X_m and the expected value of X_f

Answer 2

(a):

(b):

(c):

3. 2.30 (a), (b), (c)

Question 3

Find the moment generating function corresponding to:

(a): $f(x) = \frac{1}{c}$, $0 < x < c$

(b): $f(x) = \frac{2x}{c}$, $0 < x < c$

(c): $f(x) = \frac{1}{2\beta} e^{\frac{-|x-\alpha|}{\beta}}$, $-\infty < x < \infty$, $-\infty < \alpha < \infty$, $\beta > 0$

Answer 3

(a):

(b):

(c):

4. 2.31

Question 4

Does a distribution exist for which $M_X(t) = \frac{t}{(1-t)}$, $|t| < 1$? If yes, find it. If no, prove it.

Answer 4

5.

Question 5

Suppose that X has the standard normal distribution with pdf:

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$$

, $-\infty < x < \infty$

Then the random variable Y, $Y = e^X$ has a log-normal distribution.

(a): Find $E(Y^r)$ for any r.

(b): Show the moment generating function of Y does not exist (even though all moments of Y exist).

Answer 5

(a):

(b):

6.

Question 6

Suppose that X has a normal distribution with pdf:

$$f(x)\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{-(x-\mu)^2}{\sigma^2 2}}$$

, $-\infty < x < \infty$

The mean of X is μ. Show that the moment generating function of X satisfies $M_X(t) \geq e^{t\mu}$

Answer 6

7.

Question 7

Suppose that X has pmf $f(x) = p(1-p)^{x-1}$, for $x = 1, 2, 3, ...$ where $0 < p < 1$. Find the mgf $M_X(t)$ and use this to derive the mean and variance of X.

Answer 7

8.

Question 8

Suppose for one month a company purfchases c copies of a software package at a cost of d_1 dollars per copy. The packages are sold to customers for d_2 dollars per copy; any unsold copies are destroyed at the end of the month. Let X represent the demand for this software package in the month. Assume that X is a discrete random variable with pmf $f(x)$ and cdf $F(x)$.

(a): Let $s = \min\{X, c\}$ represent the number of sales during the month. Show that:

$$E(S) = \sum_{x=0}^{c} xf(x) + c(1 - F(c))$$

(b): Let $Y = S * d_2 - cd_1$ represent the profit for the company, the total income from sales minus the total cost of all copies. Find $E(Y)$

(c): As $Y \equiv Y_c$ depends on integer $c \geq 0$, write the expected profit function as $g(c) \equiv E(Y_c)$ from part (b). The company should choose the value of c which maximizes $g(c)$; that is, choose the smallest c such that $g(c+1)$ is less than or equal to $g(c)$. Show that such $c \geq 0$ is the smallest integer with $F(c) \geq \frac{d_2 - d_1}{d_2}$

Answer 8

(a):

(b):

(c):