Statistics 520, Fall 2025

Assignment 2

In contrast to inverse gamma distributions, an inverse Gaussian distribution is NOT the distribution of the reciprocal of a random variable having a Gaussian distribution. It is an entirely different distribution. There are any number of ways that people have parameterized inverse Gaussian probability density functions. One way is, for parameters $\mu > 0$ and $\lambda > 0$,

$$f(y|\mu,\lambda) = \left(\frac{\lambda}{2\pi y^3}\right)^{1/2} \exp\left[-\frac{\lambda}{2\mu^2 y}(y-\mu)^2\right]; \quad y > 0. \tag{1}$$

1. (5 pts.) Write the density (1) in exponential dispersion family form. Identify the natural parameter θ and dispersion parameter ϕ in terms of the original parameters μ and λ.

2. (5 pts.) Using the result from question 1, find the expected value and variance of a random variable Y that follows an inverse Gaussian distribution. Write these moments in terms of θ and ϕ and then also in terms of μ and λ. How is the variance related to the expected value for this distribution?

3. (5pts.) To get a feel for this distribution with the same location but different values of the dispersion parameter, produce a graph with three overlaid density functions having $\mu = 1$ and $\lambda = 4, 8$ and 16.

4. (10pts.) Now compare the inverse Gaussian distributions from question 3 with the corresponding gamma distributions, which means gamma distributions having the same mean and variance as the inverse Gaussian distributions. Write a gamma density with parameters $\alpha > 0$ and $\beta > 0$ as,

$$f(y|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \exp(-\beta y); \;\; y > 0$$

The expected value and variance for a random variable Y having this distribution are,

$$E(Y) = \frac{\alpha}{\beta}$$
$$var(Y) = \frac{\alpha}{\beta^2}.$$

To find the corresponding gamma distribution for an inverse Gaussian distribution with parameter values μ and λ, you would first determine the expected value and variance that result from your answer to question 2, equate these with the expected value and variance for a gamma(α, β) distribution as just given, and solve for α and β.

Finally, produce three graphs with the three inverse Gaussian distributions already computed in question 3 and overlay the corresponding gamma distribution. For the three cases of question 3, do you notice any systematic difference in inverse Gaussian and gamma distributions?