Stat 5000Lab #3
Fall 2024 Due Tue Sep 17th Name: Sam Olson

Assignment

1. Conduct the t-test for the SMS speed example in SAS and complete the following exercises:

    1. Using the formula from the notes, calculate by hand a 95% confidence interval for the difference in the two treatment means. Use $t_{28,0.975} = 2.0484$.

Formula:

$$95\% \text{ Confidence Interval} = (\bar{Y_1} - \bar{Y_2}) \pm t_{28,0.975} S_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

Where:

$$S_p = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$$

Calculation

```
# Load Data
library(readr)
smsspeed_1 <- read_csv("C:/Users/samue/OneDrive/Desktop/Iowa_State_PS/STAT 5000/Labs/Lab 3/smsspeed-1.c
```

```
## Rows: 30 Columns: 4
## -- Column specification --------------------------------------------------------
## Delimiter: ","
## chr (1): AgeGroup
## dbl (3): Age, Own Phone, Control
##
## i Use `spec()` to retrieve the full column specification for this data.
## i Specify the column types or set `show_col_types = FALSE` to quiet this message.
```

```
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

```
data1 <- smsspeed_1 %>%
  filter(smsspeed_1$AgeGroup == "Over30")

data2 <- smsspeed_1 %>%
  filter(smsspeed_1$AgeGroup == "Teens")
```

```r
sampleMean1 <- mean(data1$`Own Phone`)
sampleMean2 <- mean(data2$`Own Phone`)
difference <- sampleMean1 - sampleMean2
difference
```

```
## [1] 44.012
```

```r
#Step 2: Finding standard deviation
s1 <- sd(data1$`Own Phone`)
s2 <- sd(data2$`Own Phone`)

#Step 3: Finding sample size
n1 <- length(data1$`Own Phone`)
n2 <- length(data2$`Own Phone`)

numerator <- (n1-1)*(s1^2) + (n2-1)*(s2^2)
denom <- n1 + n2 - 2
pooled <- sqrt( numerator / denom )
sqrtFactor <- sqrt(1/n1 + 1/n2)

tStatDf <-  2.0484

rightSide <- tStatDf*pooled*sqrtFactor

pooled
```

```
## [1] 18.51069
```

```r
rightSide
```

```
## [1] 13.84544
```

```r
lb <- difference - rightSide
ub <- difference + rightSide

lb
```

```
## [1] 30.16656
```

```r
ub
```

```
## [1] 57.85744
```

This gives a 95% Confidence Interval for the Difference to be between (30.167, 57.857), where units are the difference in the amount of time it took Over 30-year olds compared to teens (I believe in seconds, but that would be wild, wouldn't it?)

2. Provide a screenshot of the SAS output and use it to verify your calculation.

Default: 95% confidence interval

The TTEST Procedure

Variable: Own

AgeGroup	Method	N	Mean	Std Dev	Std Err	Minimum	Maximum
Over30		15	85.2393	23.3959	6.0408	51.9000	122.2
Teens		15	41.2273	11.7440	3.0323	21.2000	65.5800
Diff (1-2)	Pooled		44.0120	18.5107	6.7591		
Diff (1-2)	Satterthwaite		44.0120		6.7591		

AgeGroup	Method	Mean	95% CL Mean		Std Dev	95% CL Std Dev	
Over30		85.2393	72.2831	98.1956	23.3959	17.1288	36.8977
Teens		41.2273	34.7237	47.7310	11.7440	8.5981	18.5215
Diff (1-2)	Pooled	44.0120	30.1665	57.8575	18.5107	14.6897	25.0348
Diff (1-2)	Satterthwaite	44.0120	29.9404	58.0836			

Method	Variances	DF	t Value	Pr > |t|
Pooled	Equal	28	6.51	<.0001
Satterthwaite	Unequal	20.634	6.51	<.0001

Equality of Variances				
Method	Num DF	Den DF	F Value	Pr > F
Folded F	14	14	3.97	0.0145

Figure 1: 95% Confidence Interval SAS

3. Interpret the confidence interval in the context of the problem.

A 95% Confidence Interval can be interpreted as a calculated range within which we can be 95% certain the true effect (true difference between two groups) lies.

Within the context of this particular question, it can be interpreted as: We are 95% Confident that the true difference between people Over 30 and Teens texting speeds is between 30.167 and 57.857 seconds; or is would take people Over 30 30.167 to 57.857 more seconds to type a specified message compared to Teens (within the context of being 95% confident).

This may also be interpreted as a commentary on the procedure of calculating the Confidence Interval: If we repeated this procedure of experimentation and calculation and constructed their respective 95% confidence intervals, these confidence intervals would contain the true difference between Over 30 and Teens texting times 95% of the time.

2. Use SAS to explore sample size determinations for the bone loss example using the **hypothesis testing method** and complete the following exercises:

   1. Explore the effect of changing just the significance level - For $\alpha = 0.01, 0.05, 0.1$, what are the resulting sample sizes? Summarize your findings in one concise sentence.

The POWER Procedure
Two-Sample t Test for Mean Difference

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Mean Difference	4
Standard Deviation	5
Nominal Power	0.8
Number of Sides	2
Null Difference	0

Computed N per Group			
Index	Alpha	Actual Power	N per Group
1	0.01	0.810	39
2	0.05	0.807	26
3	0.10	0.817	21

Figure 2: Alpha

Results: The resulting sample sizes for a given "Alpha" are as follows, in the format of Sample Size (resp. Alpha): 39 (0.01), 26 (0.05), and 21 (0.10).

Findings: For greater significance levels we require an increasing number of samples per Group, and the amount these sample sizes increase by is non-linear, i.e. each decrease of 0.01 in α (greater significant level) requires a larger number of samples to be added per Group compared to its prior significance level.

2. Explore the effect of changing just the power - For $1-\beta = 0.99, 0.95, 0.9, 0.8, 0.7$, what are the resulting sample sizes? Summarize your findings in one concise sentence.

The POWER Procedure
Two-Sample t Test for Mean Difference

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Alpha	0.05
Mean Difference	4
Standard Deviation	5
Number of Sides	2
Null Difference	0

Computed N per Group			
Index	Nominal Power	Actual Power	N per Group
1	0.99	0.991	59
2	0.95	0.952	42
3	0.90	0.902	34
4	0.80	0.807	26
5	0.70	0.716	21

Figure 3: Beta

Results: The resulting sample sizes for a given "Nominal Power" are as follows, in the format of Sample Size (resp. Nominal Power): 50 (0.99), 42 (0.95), 34 0.90, 26 (0.80), and 21 (0.70).

Findings: When changing just the power in relation to the required sample size, we see that greater power (smaller β) requires larger sample sizes, and the increase in sample sizes between power levels becomes larger and larger for smaller and smaller β's.

3. Explore the effect of changing just the true effect size - For $\delta = 1, 2, 3, 4, 5, 6$, what are the resulting sample sizes? Summarize your findings in one concise sentence.

The POWER Procedure
Two-Sample t Test for Mean Difference

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Alpha	0.05
Standard Deviation	5
Nominal Power	0.8
Number of Sides	2
Null Difference	0

Computed N per Group			
Index	Mean Diff	Actual Power	N per Group
1	1	0.801	394
2	2	0.804	100
3	3	0.804	45
4	4	0.807	26
5	5	0.807	17
6	6	0.802	12

Figure 4: Effect Size

Results: The resulting sample sizes for a given “Mean Difference in Effect” are as follows, in the format of Sample Size (resp. Mean Diff): 394 (1), 100 (2), 45 (3), 26 (4), 17 (5), and 12 (6).

Findings: We require larger sample sizes to detect smaller effect sizes, i.e. the larger the effect size, the smaller the calculated sample size required, holding all else equal.

4. Explore the effect of changing just the estimated population variance - For $S_p^2 = 1, 4, 9, 16, 25, 36$, what are the resulting sample sizes? Summarize your findings in one concise sentence.

The POWER Procedure
Two-Sample t Test for Mean Difference

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Alpha	0.05
Mean Difference	4
Nominal Power	0.8
Number of Sides	2
Null Difference	0

Computed N per Group			
Index	Std Dev	Actual Power	N per Group
1	1	0.948	3
2	2	0.876	6
3	3	0.805	10
4	4	0.807	17
5	5	0.807	26
6	6	0.808	37

Figure 5: 95% Confidence Interval SAS

Results: The resulting sample sizes for a given "Std Dev" are as follows, in the format of Sample Size (resp. Std Dev): 3 (1), 6 (2), 10 (3), 17 (4), 26 (5), and 37 (6).

Findings: As the estimated population variance increases, we need increasing larger sample sizes, and the rate at which these sample sizes increase is increasing, e.g. the difference (change in sample size) between 5 and 6 Std Dev is larger than the difference between 1 and 2 Std Dev.

3. Use SAS to explore sample size determinations for the bone loss example using the **confidence interval method** and complete the following exercises:

    1. Explore the effect of changing just the significance level - For $\alpha = 0.01, 0.05, 0.1$, what are the resulting sample sizes? Summarize your findings in one concise sentence.

The POWER Procedure
Two-Sample t Test for Mean Difference

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Mean Difference	4
Standard Deviation	5
Number of Sides	2
Null Difference	0

Computed N per Group				
Index	Alpha	Nominal Power	Actual Power	N per Group
1	0.01	0.995	0.995	85
2	0.01	0.975	0.975	66
3	0.01	0.950	0.952	58
4	0.05	0.995	0.995	66
5	0.05	0.975	0.977	50
6	0.05	0.950	0.952	42
7	0.10	0.995	0.995	57
8	0.10	0.975	0.977	42
9	0.10	0.950	0.952	35

Results: The resulting sample sizes for a given "Alpha" are as follows, with a fixed power level, in the format of Sample Size (resp. Alpha for "Nominal Power" of 0.975): 66 (.01), 50 (0.05), and 42 (0.10).

Findings: Consistent with the findings of Q2, albeit via a different method: Higher significance levels (lower α) require significantly larger sample sizes per group.

2. Explore the effect of changing just the true effect size - For $\delta = 1, 2, 3, 4, 5, 6$, what are the resulting sample sizes? Summarize your findings in one concise sentence.

The POWER Procedure
Two-Sample t Test for Mean Difference

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Alpha	0.05
Standard Deviation	5
Nominal Power	0.975
Number of Sides	2
Null Difference	0

Computed N per Group			
Index	Mean Diff	Actual Power	N per Group
1	1	0.975	770
2	2	0.976	194
3	3	0.976	87
4	4	0.977	50
5	5	0.976	32
6	6	0.978	23

Results: The resulting sample sizes for a given "Mean Difference Effect" are as follows, in the format of Sample Size (resp. Mean Diff): 770 (1), 194 (2), 87 (3), 50 (4), 32 (5), and 23 (6).

Findings: Consistent with the findings of Q2, 3. albeit for a different method: We require much larger sample sizes per Group for smaller differences between groups, and smaller sample sizes per Group for larger differences between groups.

3. Explore the effect of changing just the estimated population variance - For $S_p^2 = 1, 4, 9, 16, 25, 36$, what are the resulting sample sizes? Summarize your findings in one concise sentence.

The POWER Procedure
Two-Sample t Test for Mean Difference

Fixed Scenario Elements	
Distribution	Normal
Method	Exact
Alpha	0.05
Mean Difference	4
Nominal Power	0.975
Number of Sides	2
Null Difference	0

Computed N per Group			
Index	Std Dev	Actual Power	N per Group
1	1	0.996	4
2	2	0.978	9
3	3	0.979	19
4	4	0.976	32
5	5	0.977	50
6	6	0.976	71

Results: The resulting sample sizes for a given "Std Dev" are as follows, in the format of Sample Size (resp. Std Dev): 4 (1), 9 (2), 19 (3), 32 (4), 50 (5), and 71 (6).

Findings: Consistent with the findings of Q2, 4. albeit for a different method: Increasing estimated population variance results in much larger required sample sizes, and the rate of increase for these samples grows larger as the variance increases.

4. Think about how the sample size determination using the confidence interval method relates to the standard error method. Summarize your findings in one concise sentence.

Results:

Confidence Interval Method

$$n_0 = 8(\frac{z_{1-\frac{\alpha}{2}} S_p}{w})^2$$
$$n = 8(\frac{t_{2(n_0-1),1-\frac{\alpha}{2}} S_p}{w})^2$$

Standard Error Method

$$n = \frac{2S_P^2}{(s.e.)^2}$$

Findings: One may consider the confidence interval method as a modification of the standard error method, in that both methods require and take as input the **pooled estimate of the population variance** to estimate the required sample size, the standard error is closely related to the "width" of the confidence interval method, and furthermore, the confidence interval method distinguishes itself through use of the z and t distributions as well (takes additional inputs compared to the standard error method).

Total: 50 points **# correct: %:**