Stat 5000 Homework #11
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1

Berkeley Guidance Study

The dataset is located in `BGSgirls2.txt`. It contains one line of data for each of 70 girls with the following variables:

- ID: Girl identification number
- WT2: Weight (kg) at 2 years
- HT2: Height (cm) at 2 years
- WT9: Weight (kg) at 9 years
- HT9: Height (cm) at 9 years
- LG9: Leg circumference (cm) at 9 years
- ST9: Strength (kg) at 9 years
- WT18: Weight (kg) at 18 years
- HT18: Height (cm) at 18 years
- LG18: Leg circumference (cm) at 18 years
- ST18: Strength (kg) at 18 years
- BMI: Body Mass Index at 18 years
- SOMA: Somatotype (SOMA), on a scale from 1 (very thin) to 7 (very obese)

USE SAS TO COMPLETE THE FOLLOWING EXERCISES:

(a)

Fit a multiple regression model:

$$BMI_i = \beta_0 + \beta_1 \text{WT2}_i + \beta_2 \text{HT2}_i + \beta_3 \text{WT9}_i + \beta_4 \text{HT9}_i + \beta_5 \text{ST9}_i + \epsilon_i$$

for i=1, . . . , 70.

And use the following diagnostics to assess model assumptions. (Do not submit the output; just examine the results and briefly describe the insight provided by each).

i.

Normal Q-Q plot of residuals and the related Shapiro-Wilk test.

The QQ plot closely aligns with the reference line within the first theoretical quantile, but there are deviations past the first (positive/negative) quantile. Furthermore, the QQ plot appears to have a slight “S” shape/curve, further suggesting deviation from normality.

The Shapiro-Wilk test provides a small p-value (<0.0001) such that we would have evidence to reject the null hypothesis that the residuals are normally distributed.

Overall, we have reason to suspect our normality assumption is being violated.

ii.

Plot of the residuals versus the estimates of the conditional means for BMI

We generally observe a random spread of residual values across fitted values. However, there are two negative residuals around predicted BMI 25+, such that we'd consider these points to either be candidates for removal (due to being outliers) or that we may in fact be violating our assumption of form of the model.

This notwithstanding, we generally have reason to believe our form of the model and constant variance assumptions are not being violated.

iii.

Individual plots of the residuals versus each of the five explanatory variables

Plots of residuals versus each of the five explanatory variables are generally consistent with the depictions present from the residual v. fitted values graph, insomuch as we may have a few problematic points to address but generally do not have reason to suspect our assumptions are being violated.

(b)

Given that an outlier should be detected from part (a), refit the model and recheck the diagnostics listed in (a) to assess whether model assumptions are violated or not. (HINT: You can filter observations from the dataset using the where statement inside the reg procedure in SAS.)

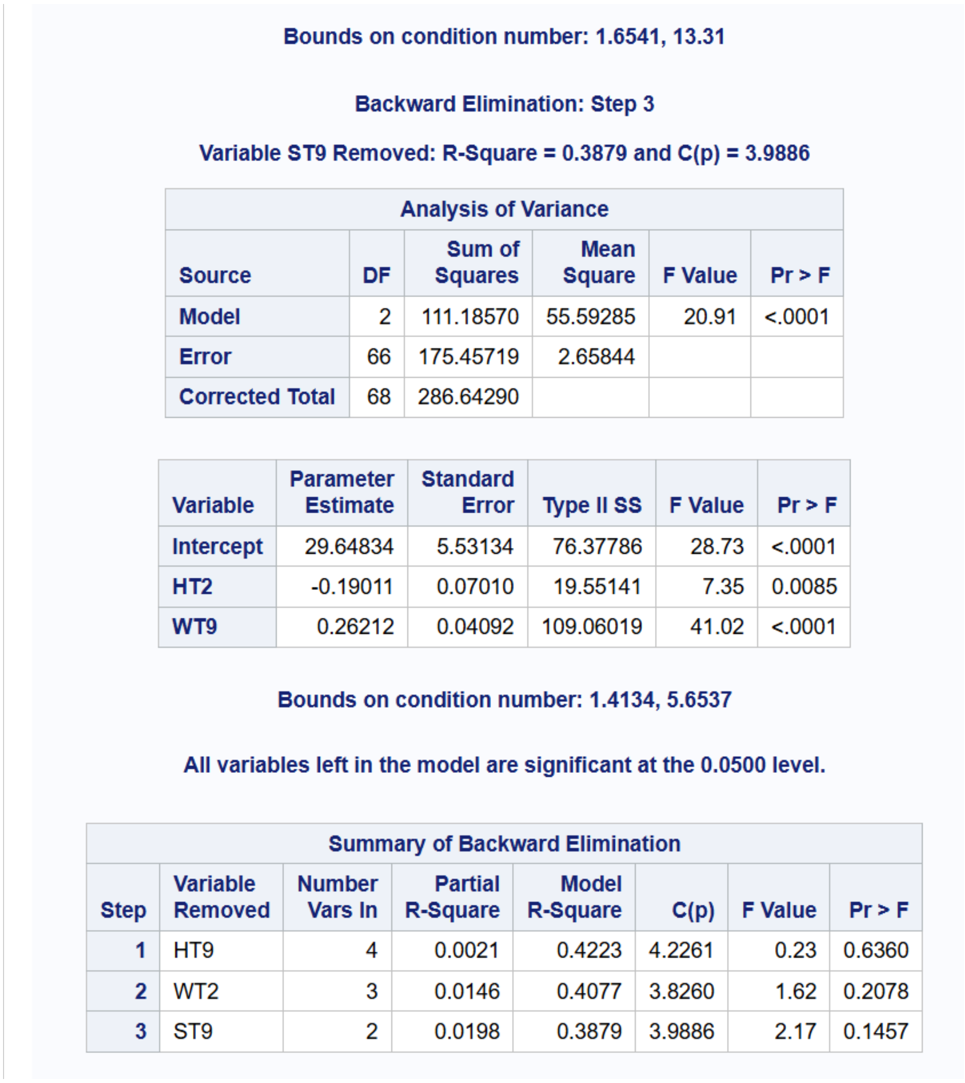
The QQ plot looks better, insomuch as it more closely tracks with the reference line, and this is consistent with a larger Shapiro-Wilk test statistics, such that we would not have evidence to reject the null hypothesis that the residuals are normally distributed. As such we have reason not to suspect the normality assumption is being violated as it was in part (a).

Furthermore, the residual plots consistently (across the x-axis of fitted values as well as individually across the explanatory variables) appear to be randomly spread, such that our constant variance and form of the model assumptions are likely not being violated either.

However, it is worth noting that we still appear to have potential outliers in our diagnostic plots, including both the residual plot as well as the QQ plot. So we see improvements but still have reason to be concerned our assumptions are violated.

(c)

For the 69 observations (without the outlier that was detected from part (a)), use a backward selection procedure to search for a model using $\alpha_{stay} = 0.05$. For this question, just consider the five variables mentioned in part (a): WT2, HT2, WT9, HT9, ST9. For your final model, report the estimated coefficients and their standard errors.



Bounds on condition number: 1.6541, 13.31

Backward Elimination: Step 3

Variable ST9 Removed: R-Square = 0.3879 and C(p) = 3.9886

Analysis of Variance					
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Value**	**Pr > F**
Model	2	111.18570	55.59285	20.91	<.0001
Error	66	175.45719	2.65844		
Corrected Total	68	286.64290			

Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept	29.64834	5.53134	76.37786	28.73	<.0001
HT2	-0.19011	0.07010	19.55141	7.35	0.0085
WT9	0.26212	0.04092	109.06019	41.02	<.0001

Bounds on condition number: 1.4134, 5.6537

All variables left in the model are significant at the 0.0500 level.

Summary of Backward Elimination							
Step	**Variable Removed**	**Number Vars In**	**Partial R-Square**	**Model R-Square**	**C(p)**	**F Value**	**Pr > F**
1	HT9	4	0.0021	0.4223	4.2261	0.23	0.6360
2	WT2	3	0.0146	0.4077	3.8260	1.62	0.2078
3	ST9	2	0.0198	0.3879	3.9886	2.17	0.1457

Figure 1: CocoMelon

In the final model from the method described, we have:

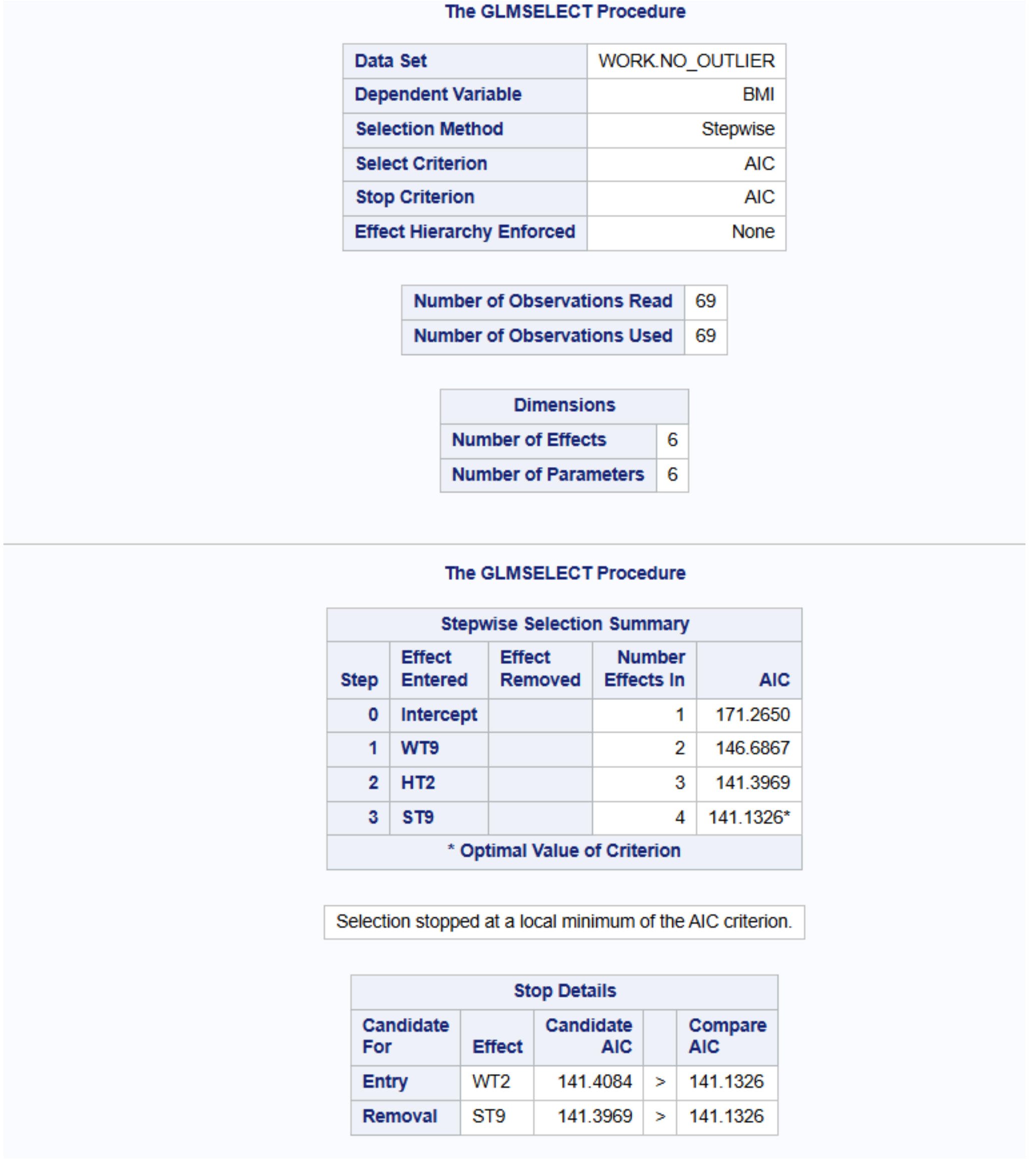
Variable: Intercept Coefficient: 29.648 Standard Error: 5.531

Variable: HT2 Coefficient: -0.190 Standard Error: 0.070

Variable: WT9 Coefficient: 0.262 Standard Error: 0.0409

(d)

For the 69 observations (without the outlier that was detected from part (a)), check all possible models that could be constructed using at most the five variables WT2, HT2, WT9, HT9, ST9 and then give the best one that you recommend. Justify your choice.



The GLMSELECT Procedure

Data Set	WORK.NO_OUTLIER
Dependent Variable	BMI
Selection Method	Stepwise
Select Criterion	AIC
Stop Criterion	AIC
Effect Hierarchy Enforced	None

Number of Observations Read	69
Number of Observations Used	69

Dimensions	
Number of Effects	6
Number of Parameters	6

The GLMSELECT Procedure

Stepwise Selection Summary

Step	Effect Entered	Effect Removed	Number Effects In	AIC
0	Intercept		1	171.2650
1	WT9		2	146.6867
2	HT2		3	141.3969
3	ST9		4	141.1326*

* Optimal Value of Criterion

Selection stopped at a local minimum of the AIC criterion.

Stop Details

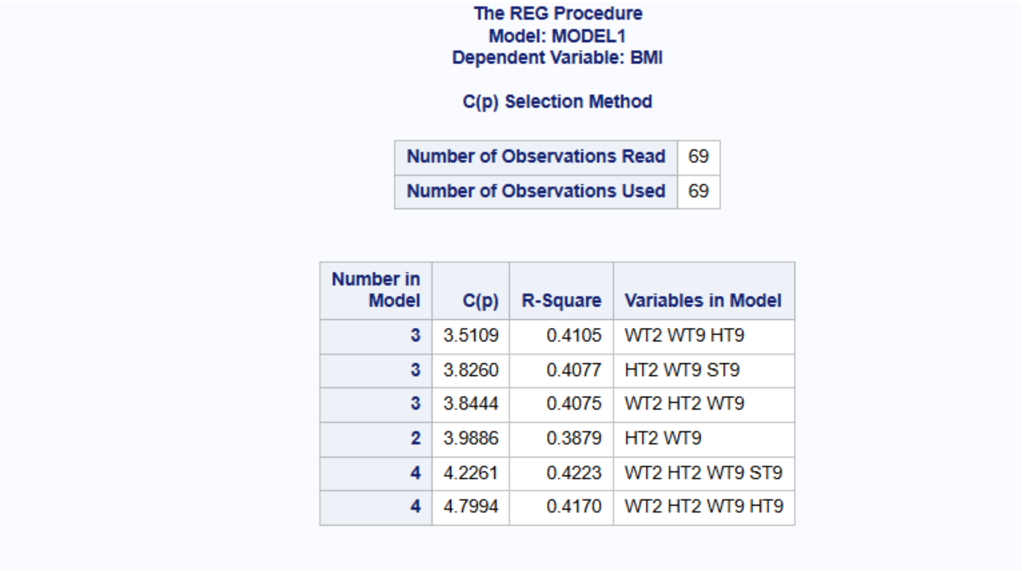
Candidate For	Effect	Candidate AIC		Compare AIC
Entry	WT2	141.4084	>	141.1326
Removal	ST9	141.3969	>	141.1326

Figure 2: CocoMelon

After much back and forth, I ultimately decided on recommending the following model, which includes an intercept term: HT2, WT9, ST9 explanatory variables/predictors

Descriptively, we have some reason to believe that multicollinearity would be minimized compared to models with similar variables, i.e. that use WT2 and WT9 or HT2 and HT9. Furthermore, we want to avoid overfitting, such that we have a preference for a simpler model (Occam's), which means a preference for less explanatory variables.

Bearing that in mind, there were 3 statistics used as criteria used for evaluating models: AIC, Adjusted R-Squared, and Mallow's C_p.



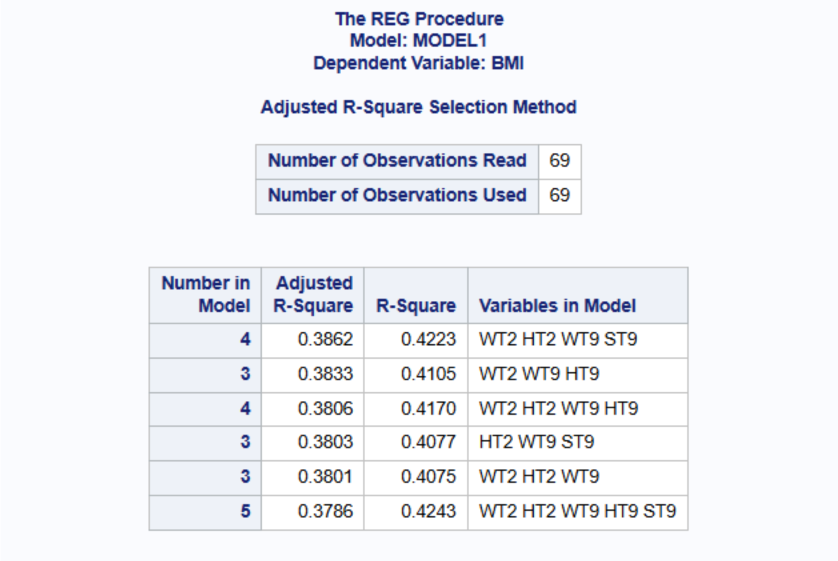
The REG Procedure
Model: MODEL1
Dependent Variable: BMI

C(p) Selection Method

Number of Observations Read	69
Number of Observations Used	69

Number in Model	C(p)	R-Square	Variables in Model
3	3.5109	0.4105	WT2 WT9 HT9
3	3.8260	0.4077	HT2 WT9 ST9
3	3.8444	0.4075	WT2 HT2 WT9
2	3.9886	0.3879	HT2 WT9
4	4.2261	0.4223	WT2 HT2 WT9 ST9
4	4.7994	0.4170	WT2 HT2 WT9 HT9

Figure 3: CocoMelon



The REG Procedure
Model: MODEL1
Dependent Variable: BMI

Adjusted R-Square Selection Method

Number of Observations Read	69
Number of Observations Used	69

Number in Model	Adjusted R-Square	R-Square	Variables in Model
4	0.3862	0.4223	WT2 HT2 WT9 ST9
3	0.3833	0.4105	WT2 WT9 HT9
4	0.3806	0.4170	WT2 HT2 WT9 HT9
3	0.3803	0.4077	HT2 WT9 ST9
3	0.3801	0.4075	WT2 HT2 WT9
5	0.3786	0.4243	WT2 HT2 WT9 HT9 ST9

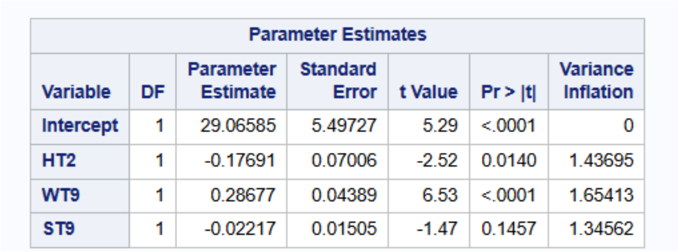
Figure 4: CocoMelon

The model I recommend has the best AIC value (minimized), and it also has a “pretty good” Adjusted R-Squared and Mallow’s C_p.

For Adjusted R-Squared, it has the second best for models with 3 explanatory variables, and 4th overall across all models. Marginally, the difference in Adjusted R-Squared is in the hundreths of decimal places. Also, when comparing values of Mallow (looking at k+1 closest to C_p value), we see that the model recommended has the second best C_p value, beaten only by WT2, HT2, and WT9. However, the difference between these two is less than 0.02, so the difference is marginal. And the model we recommend doesn’t “reuse” the same variable WT, such that we potentially avoid multicollinearity. To that end…

(e)

Are there concerns about multicollinearity for the explanatory variables of the model you picked in part (d)?

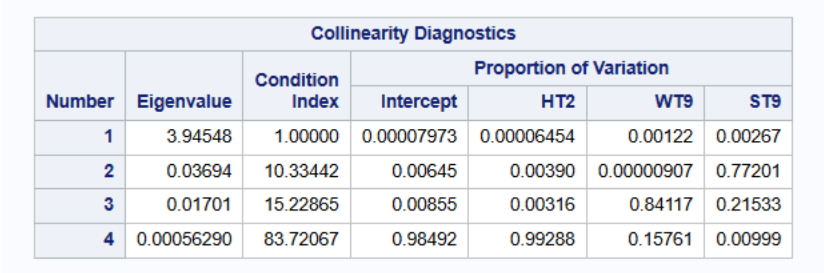


Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|	Variance Inflation
Intercept	1	29.06585	5.49727	5.29	<.0001	0
HT2	1	-0.17691	0.07006	-2.52	0.0140	1.43695
WT9	1	0.28677	0.04389	6.53	<.0001	1.65413
ST9	1	-0.02217	0.01505	-1.47	0.1457	1.34562

Figure 5: CocoMelon

VIF values are not especially large (less than cutoff for "moderate" of value of 3 or5), so minimal issue with multicollinearity based on VIF criteria.



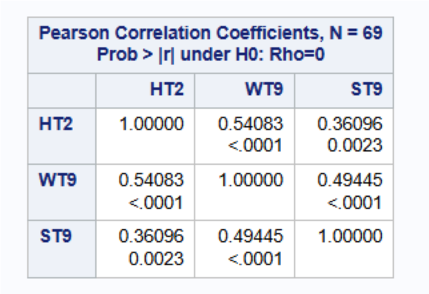
Collinearity Diagnostics

			Proportion of Variation			
Number	Eigenvalue	Condition Index	Intercept	HT2	WT9	ST9
1	3.94548	1.00000	0.00007973	0.00006454	0.00122	0.00267
2	0.03694	10.33442	0.00645	0.00390	0.00000907	0.77201
3	0.01701	15.22865	0.00855	0.00316	0.84117	0.21533
4	0.00056290	83.72067	0.98492	0.99288	0.15761	0.00999

Figure 6: CocoMelon

However, when looking at the Condition Index for the Eigenvalues, we do observe a rather high value (larger than 30), with the 83.72 corresponding to a significant proportion of variation for Interccept, HT2, and WT9, or an indication of potential extreme multicollinearity between HT2 and WT9. Given this, we look to the correlations to confirm.

However, this is something that I don't think has been discussed in-depth at the moment, so I believe for all intents and purposes we are good to go (the above is for considering extra-dimensional multicollinearity, if I recall correctly).



Pearson Correlation Coefficients, N = 69
Prob > |r| under H0: Rho=0

	HT2	WT9	ST9
HT2	1.00000	0.54083 <.0001	0.36096 0.0023
WT9	0.54083 <.0001	1.00000	0.49445 <.0001
ST9	0.36096 0.0023	0.49445 <.0001	1.00000

Figure 7: CocoMelon

We additionally double check our observation of potential multicollinearity by looking at the correlation between our explanatory variables. To that end: The above is further corroborated by the correlation coefficient between HT2 and WT9 being greater than 0.5 (0.54083); however, the correlation is still less than our "typical significance threshold" of $|r| > 0.7$, so this is a good thing (and motivation for choosing this over another hypothetical model like, say, WT2, WT9, and HT9.)

So overall, yes we do have some concerns about multicollinearity for the model we chose in part (d), though not especially extreme.

2

Ames Housing (+25)

A dataset (introduced in the previous homework assignment) was collected from home sales in Ames, Iowa between 2006 and 2010. The variables collected are:

- Year Built: The year the house was built
- Basement Area (in sq. ft): The amount of area in the house below ground level
- Living Area (in sq. ft): The living area in the home (includes Basement Area)
- Total Room: The number of rooms in the house
- Garage Cars: The number of cars that can be placed in the garage
- Year Sold: The year the home was sold
- Sale Price: The sale price of the home (the response variable)
- Garage Size: S = Small (Garage Cars = 0,1) or L = Large (Garage Cars = 2+)
- Age (in yrs.): Age of house = Year Sold - Year Built

Use SAS to complete the following exercises:

The data from 999 sales can be found in the file housing train.csv and for the remaining 1,924 sales in the file housing eval.csv in our course's shared folder in SAS Studio. You will determine a final multiple linear regression model for predicting sale price from the explanatory variables: Basement Area, Living Area, Total Room, Garage Size, and Age.

(a)

Fit the full model using all 5 explanatory variables listed above to the training data (housing train.csv).

And so it was fit.

i.

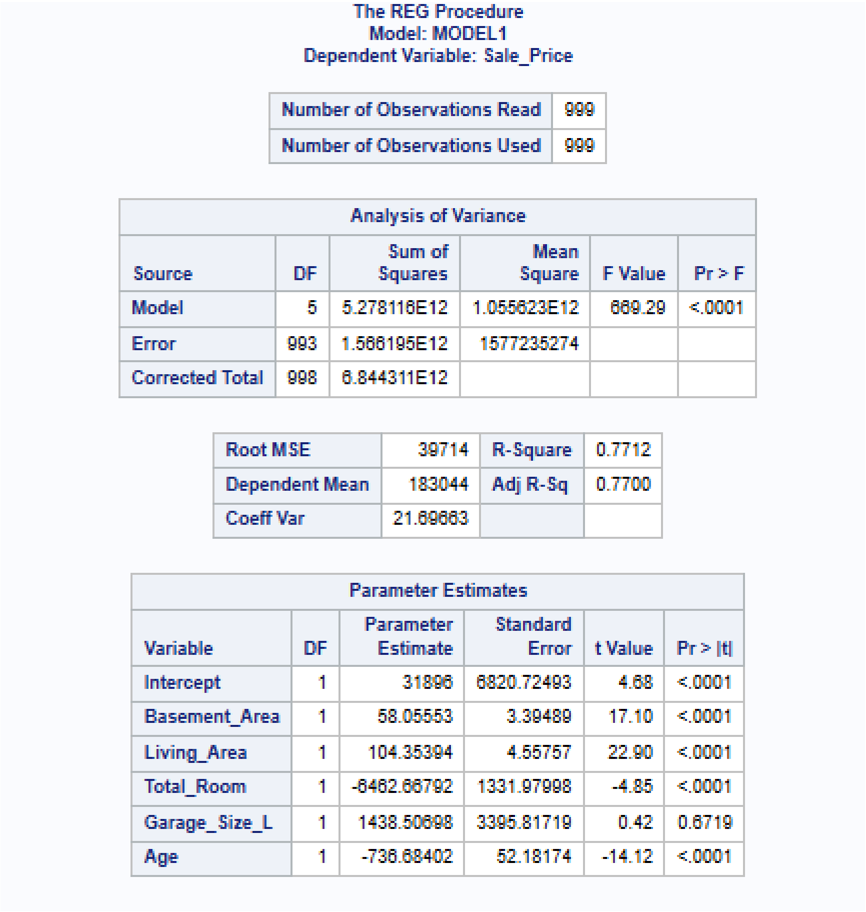
Find and interpret the R^2 value for the full model.

77.12% of variability in Sales price can be explained using the multiple linear regression using Basement Area, Living Area, Total Room, Garage Size, and Age as explanatory variables (and including an intercept term).

ii.

Interpret the value of the estimated regression coefficient corresponding to the Garage Size variable for the full model.

Increasing Garage Size to "Large" is associated with an increased Sales Price of $1,438.507 compared to a Garage Size of "Small", all else (all other explanatory variables) being equal.



The REG Procedure
Model: MODEL1
Dependent Variable: Sale_Price

Number of Observations Read	999
Number of Observations Used	999

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	5	5.278116E12	1.055623E12	669.29	<.0001
Error	993	1.566195E12	1577235274		
Corrected Total	998	6.844311E12			

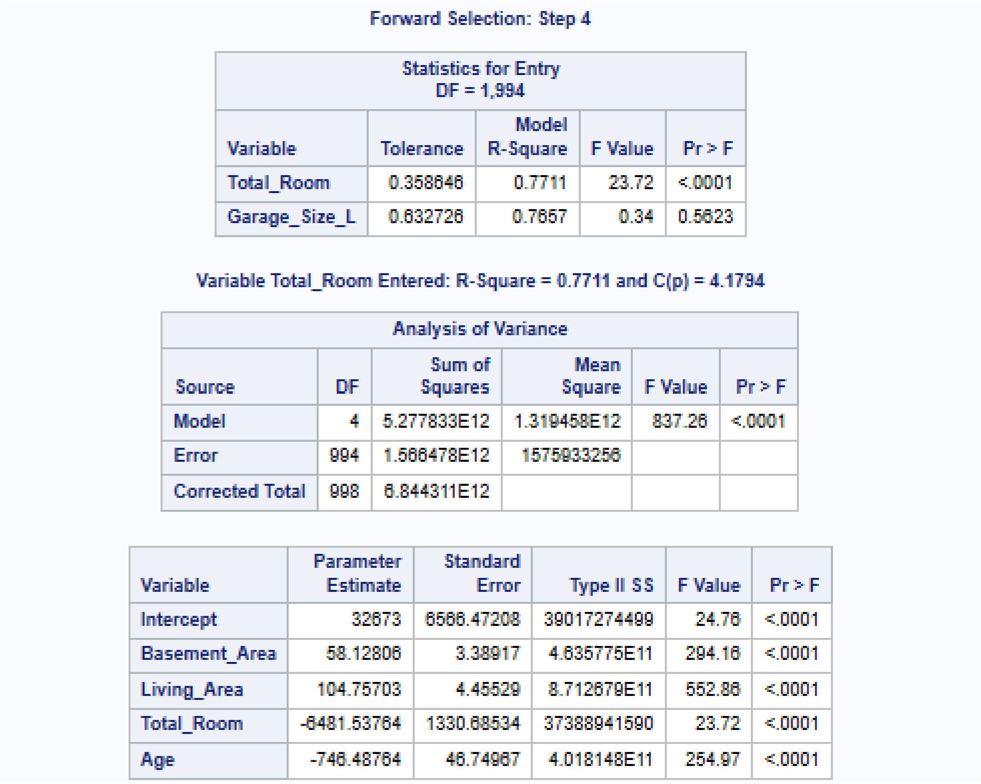
Root MSE	39714	R-Square	0.7712
Dependent Mean	183044	Adj R-Sq	0.7700
Coeff Var	21.69663		

Parameter Estimates					
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	31896	6820.72493	4.68	<.0001
Basement_Area	1	58.05553	3.39489	17.10	<.0001
Living_Area	1	104.35394	4.55757	22.90	<.0001
Total_Room	1	-6462.66792	1331.97998	-4.85	<.0001
Garage_Size_L	1	1438.50698	3395.81719	0.42	0.6719
Age	1	-736.68402	52.18174	-14.12	<.0001

Figure 8: CocoMelon

(b)

Use forward selection to fit a reduced model to the training data using some subset of the 5 explanatory variables listed above. Provide an equation for the estimated MLR model.



Forward Selection: Step 4

Statistics for Entry DF = 1,994				
Variable	Tolerance	Model R-Square	F Value	Pr > F
Total_Room	0.358646	0.7711	23.72	<.0001
Garage_Size_L	0.632726	0.7657	0.34	0.5623

Variable Total_Room Entered: R-Square = 0.7711 and C(p) = 4.1794

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	5.277833E12	1.319458E12	837.26	<.0001
Error	994	1.566478E12	1575933256		
Corrected Total	998	6.844311E12			

Variable	Parameter Estimate	Standard Error	Type II SS	F Value	Pr > F
Intercept	32873	6566.47208	39017274499	24.76	<.0001
Basement_Area	58.12806	3.38917	4.635775E11	294.16	<.0001
Living_Area	104.75703	4.45529	8.712679E11	552.86	<.0001
Total_Room	-6481.53764	1330.68534	37388941590	23.72	<.0001
Age	-746.48764	46.74967	4.018148E11	254.97	<.0001

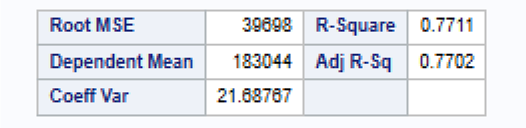
Figure 9: CocoMelon

For the preceding problems, I used the model with “Garage Size” removed, as it didn’t get added for Step 5 of forward selection. The formula corresponding to the above output and the following equation:

$$\widehat{\text{Sale Price}} = 32,873 + 58.128 * \text{Basement Area} + 104.75703 * \text{Living Area} - 6,481.53784 * \text{Total Room} - 746.48764 * \text{Age}$$

(c)

How does the adjusted R^2 value for the reduced model compare to the full model?



Root MSE	39698	R-Square	0.7711
Dependent Mean	183044	Adj R-Sq	0.7702
Coeff Var	21.68767		

Figure 10: CocoMelon

Full: 0.7700 Reduced: 0.7702 Difference: 0.0002 (2e-04) Note: For Adjusted R^2

The difference of 0.0002 (2e-04) corresponds to a difference of 0.02% between the two models. So the reduced model has a very marginally larger adjusted R^2 value.

(d)

Using the reduced model, check for:

For reference:

Training Data observations:

$$n = 999$$

$$k = 4$$

i.

outliers

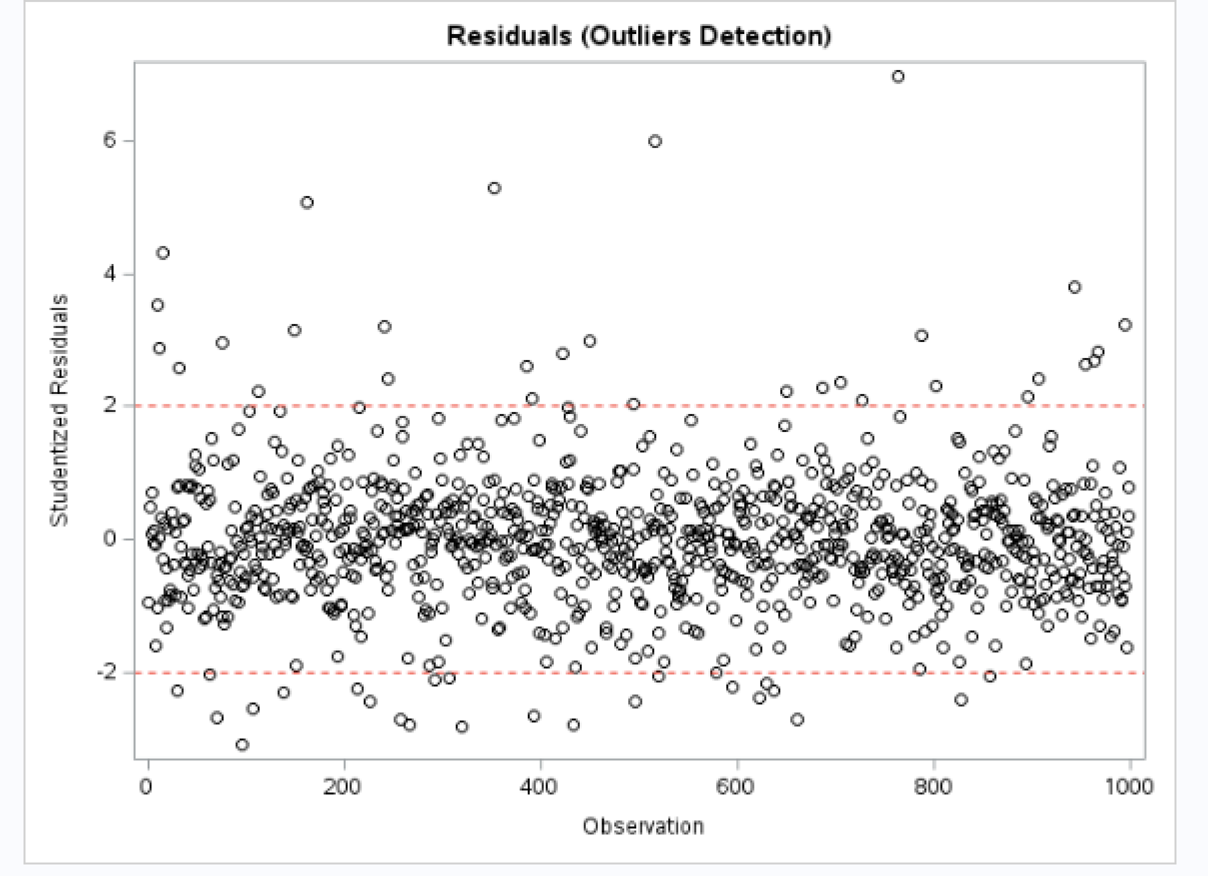


Figure 11: CocoMelon

We do observe there being some potential outliers in the training data, where an outlier is a studentized residual value greater in magnitude to 2, i.e. |r| > 2 where r is a residual.

ii.

high leverage points

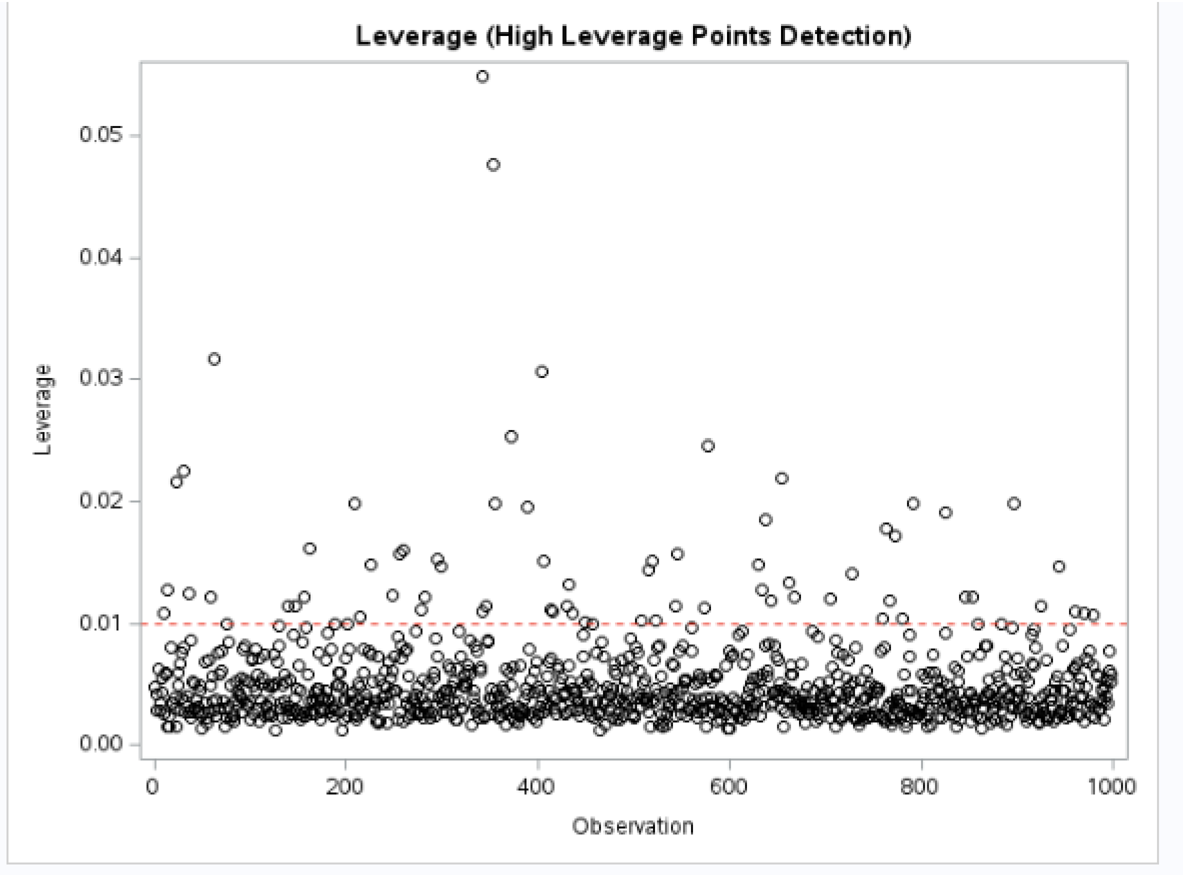


Figure 12: CocoMelon

Leverage threshold: $2(k+1)/n = 2(4+1)/999 = 0.01001001$

We do observe there being some leverage points in the training data, where our leverage threshold value is calculated to be ≈ 0.01.

iii.

potential influence points

Influence threshold: Cook's D $D > 2\sqrt{(2/n)} = 2\sqrt{(2/999)} = 0.08948747$

DDFITS: $2\sqrt{(k+1/n)} = 2\sqrt{(4+1/999)} = 0.1414921$

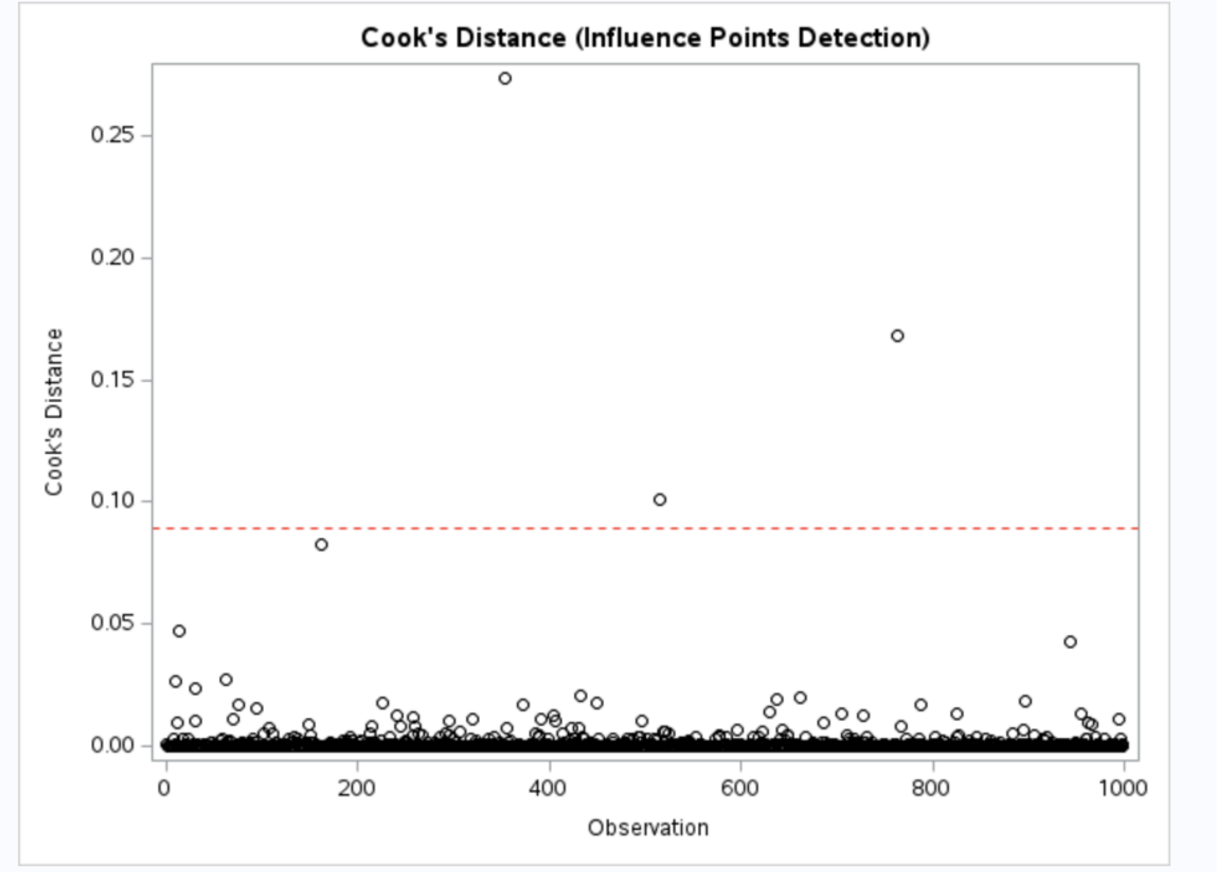


Figure 13: CocoMelon

We do observe there being some potential influence points in the training data, particularly when evaluating based on the Cook's Distance method and a threshold value ≈ 0.08949, as illustrated above. (I also did DDFITS and gave a calculation of the threshold above, but results are consistent with there being some influential points.)

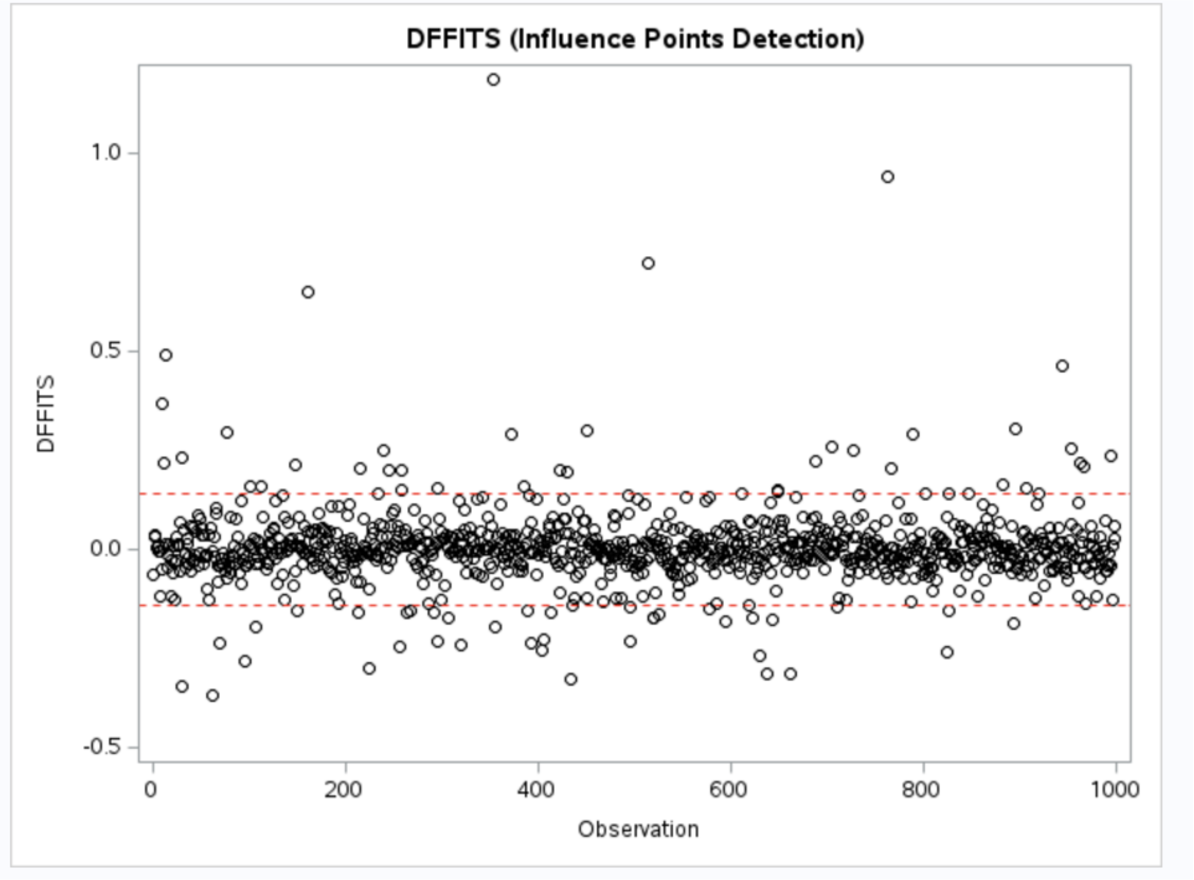
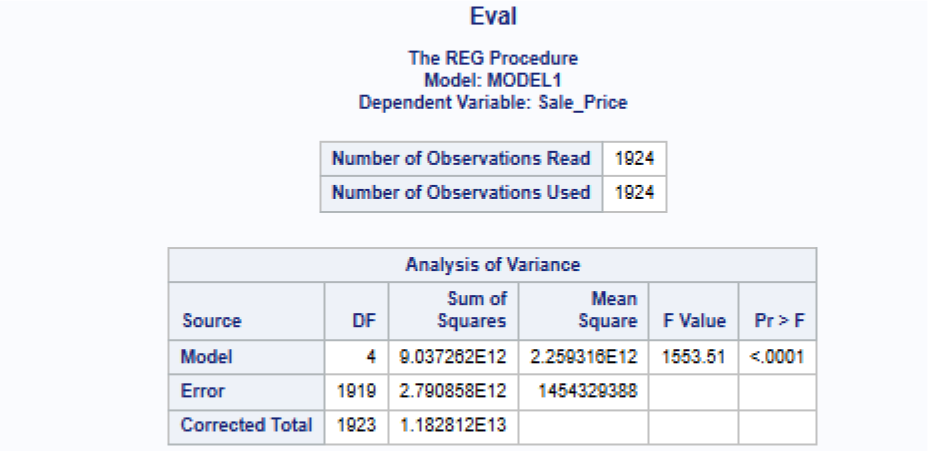


Figure 14: CocoMelon

(e)

Fit the reduced model from part (b) to the evaluation data (housing eval.csv). Compare the mean squared error from fitting the model to the testing data to the mean squared error from fitting the model to the evaluation data. What does this imply?



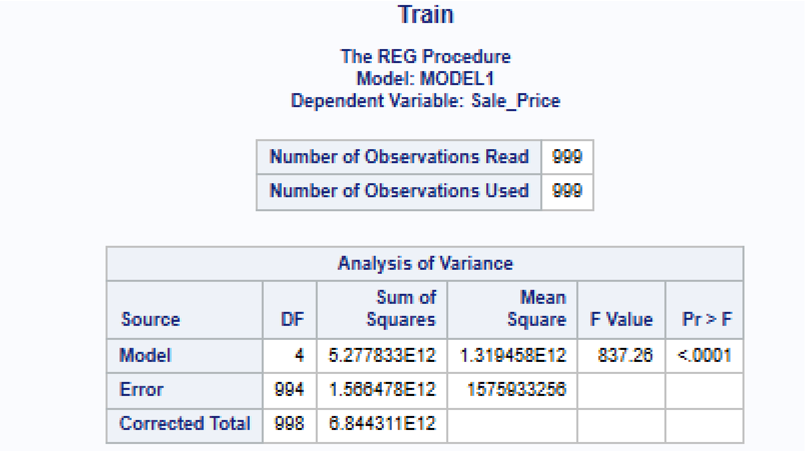
Eval

The REG Procedure
Model: MODEL1
Dependent Variable: Sale_Price

Number of Observations Read	1924
Number of Observations Used	1924

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	9.037262E12	2.259316E12	1553.51	<.0001
Error	1919	2.790858E12	1454329388		
Corrected Total	1923	1.182812E13			

Figure 15: CocoMelon



Train

The REG Procedure
Model: MODEL1
Dependent Variable: Sale_Price

Number of Observations Read	999
Number of Observations Used	999

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	5.277833E12	1.319458E12	837.26	<.0001
Error	994	1.566478E12	1575933256		
Corrected Total	998	6.844311E12			

Figure 16: CocoMelon

$$\text{MSE}_{\text{train}} = 1.5759 \times 10^9$$

$$\text{MSE}_{\text{eval}} = 1.454329 \times 10^9$$

$$\text{Difference in MSE} = \text{MSE}_{\text{train}} - \text{MSE}_{\text{eval}} \approx 1.21571 \times 10^8$$

Importantly, we see that $\text{MSE}_{\text{eval}} < \text{MSE}_{\text{train}}$, though the difference is not especially large! This is when looking specifically at the relative difference in MSE, and not the absolute difference in MSE. The relative difference is:

$$\text{Relative Difference} = \frac{\text{Difference in MSE}}{\text{MSE}_{\text{eval}}} * 100 = \frac{1.21571 \times 10^8}{1.454329 \times 10^9} \times 100 \approx 8.359\%$$

Or a roughly 8.48 percent decrease in MSE when going to the eval dataset.

This tells us that: When the MSE (Train) is slightly smaller than the MSE (Eval), meaning the model we chose performs marginally better on the evaluation dataset than the training dataset, and corresponding to a decrease of 8.36%! Generally this marginal relative difference indicates that our model is not overfitting, and that it is generalizing well from the training to the evalution dataset. It is performing “ok”! (We want MSE to be lower for the eval compared to training, otherwise we’d suspect overfitting based on the training dataset).

3

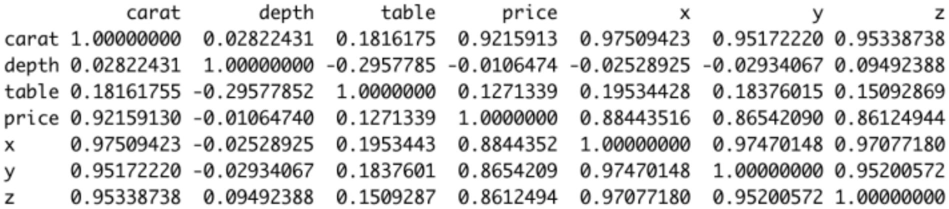
The dataset for this exercise is called diamonds and it is available directly in the ggplot2 package in R. The data set contains prices (response variable – in US dollars) of over 50,000 diamonds, which we will try to explain using the quantitative size measurements:

- carat – weight,
- x – length in mm,
- y – width in mm,
- z – depth in mm,
- depth – total depth percentage = z / mean(x, y),
- table – width of top of diamond relative to widest point)

And categorical quality (cut, color, and clarity) of the diamonds. The R code used to create the figures below is provided in the diamonds Hmwk 11.R file posted in Canvas.

(a)

Summarize your findings from examining the pairwise scatterplots (on the next page) and correlation matrix (shown below).



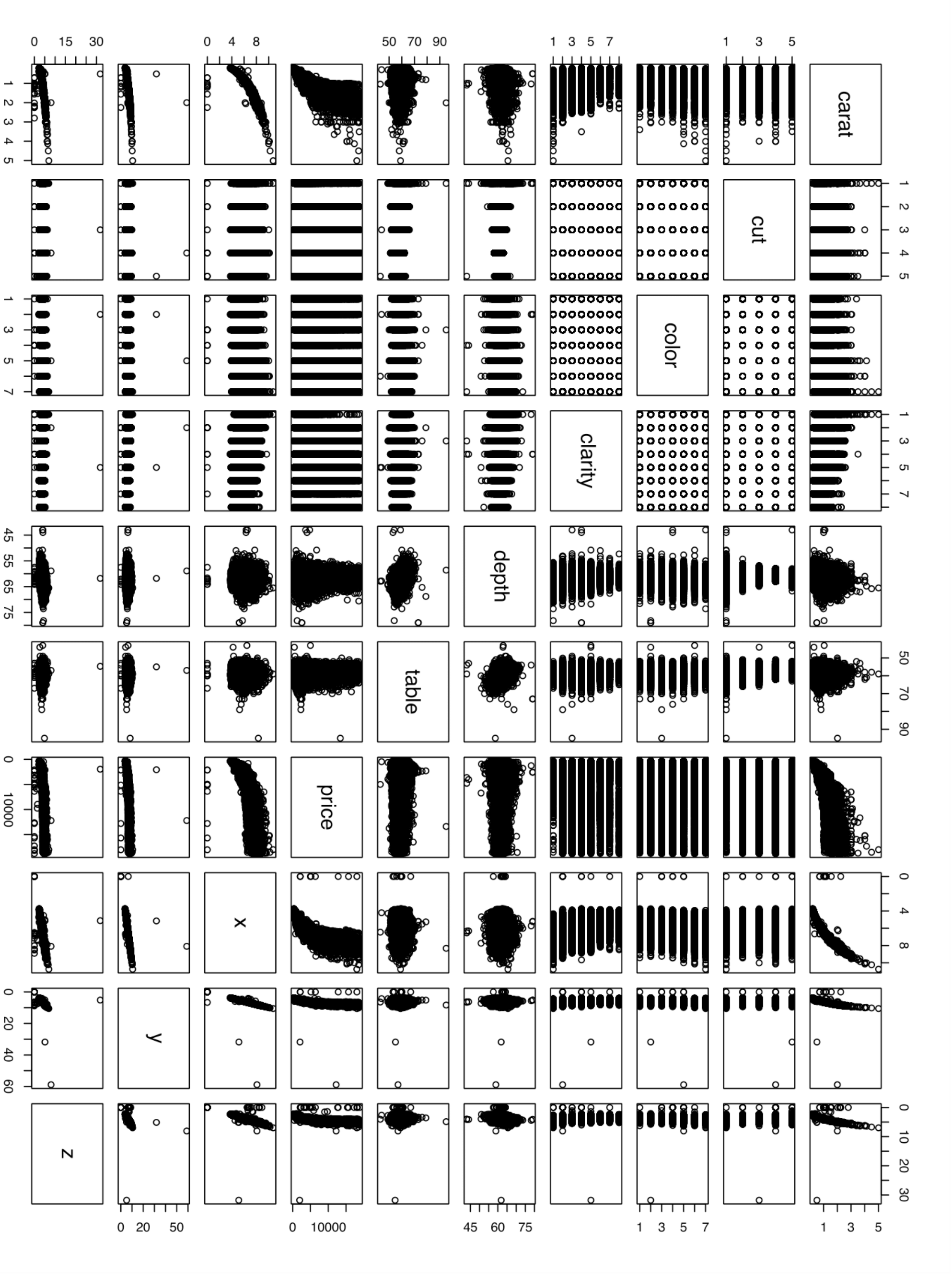
```
           carat       depth      table      price           x           y          z
carat 1.00000000  0.02822431  0.1816175  0.9215913  0.97509423  0.95172220 0.95338738
depth 0.02822431  1.00000000 -0.2957785 -0.0106474 -0.02528925 -0.02934067 0.09492388
table 0.18161755 -0.29577852  1.0000000  0.1271339  0.19534428  0.18376015 0.15092869
price 0.92159130 -0.01064740  0.1271339  1.0000000  0.88443516  0.86542090 0.86124944
x     0.97509423 -0.02528925  0.1953443  0.8844352  1.00000000  0.97470148 0.97077180
y     0.95172220 -0.02934067  0.1837601  0.8654209  0.97470148  1.00000000 0.95200572
z     0.95338738  0.09492388  0.1509287  0.8612494  0.97077180  0.95200572 1.00000000
```

Figure 17: CocoMelon

Many of the variables are highly correlated with each other; this holds for combinations of variables with the response variable (price) as well as between explanatory variables. Using the "significance" threshold of |r| > 0.7, we have the following "signficant" correlations: Carat and price, carat and x, carat and y, carat and z, price and x, price and y, price and z, x and y, x and z, y and z. Of note is that all of the "significant" correlations are positive.

Overall, we do corroborate these findings when looking at the pairwise scatter plots, insomuch as positive correlations generally show a positive linear relationship when looking at the respective graph. On the flip side, we see "small" correlations (|r| < 0.20, such as x and depth), exhibit a rather large "blob" of points, or for other pairs a general overall coverage of the plot area. This is understandable, as a small-in-magnitude correlation coefficient is evidence that there is not a significant linear relationship between the two variables.

Generally, we see that our response variable is highly correlated with some of the explanatory variables ("significant" being correlation coefficients greater in magnitude than 0.7). However, we also observe some explanatory variables appear highly correlated with one another, which is problematic and an indication of possible multicollinearity (using the same magnitude threshold of 0.7 for correlation coefficient).



10

Figure 18: CocoMelon

(b)

Discuss whether the VIFs, shown in the plot below, indicate any explanatory variables exhibiting moderate or extreme multicollinearity.

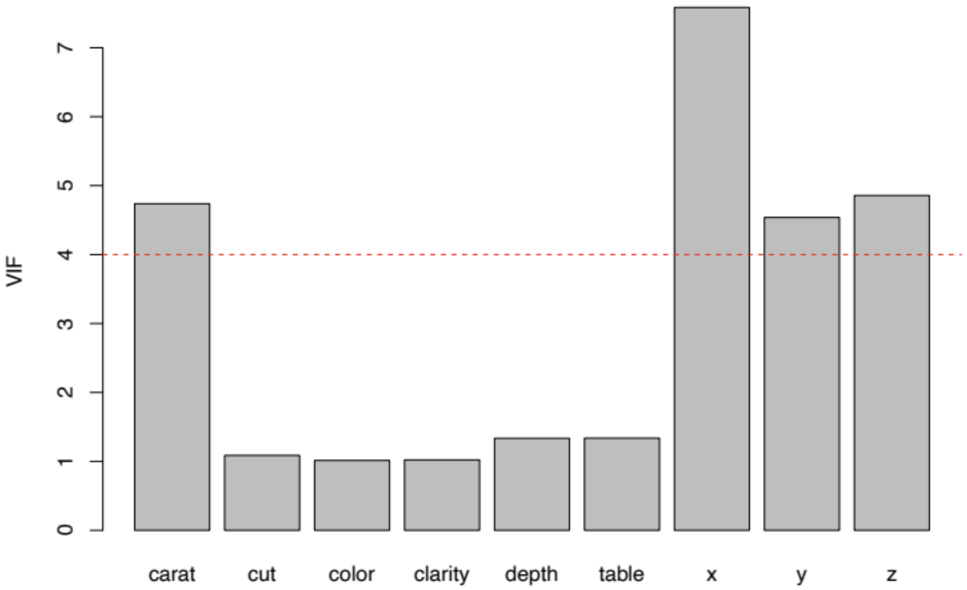


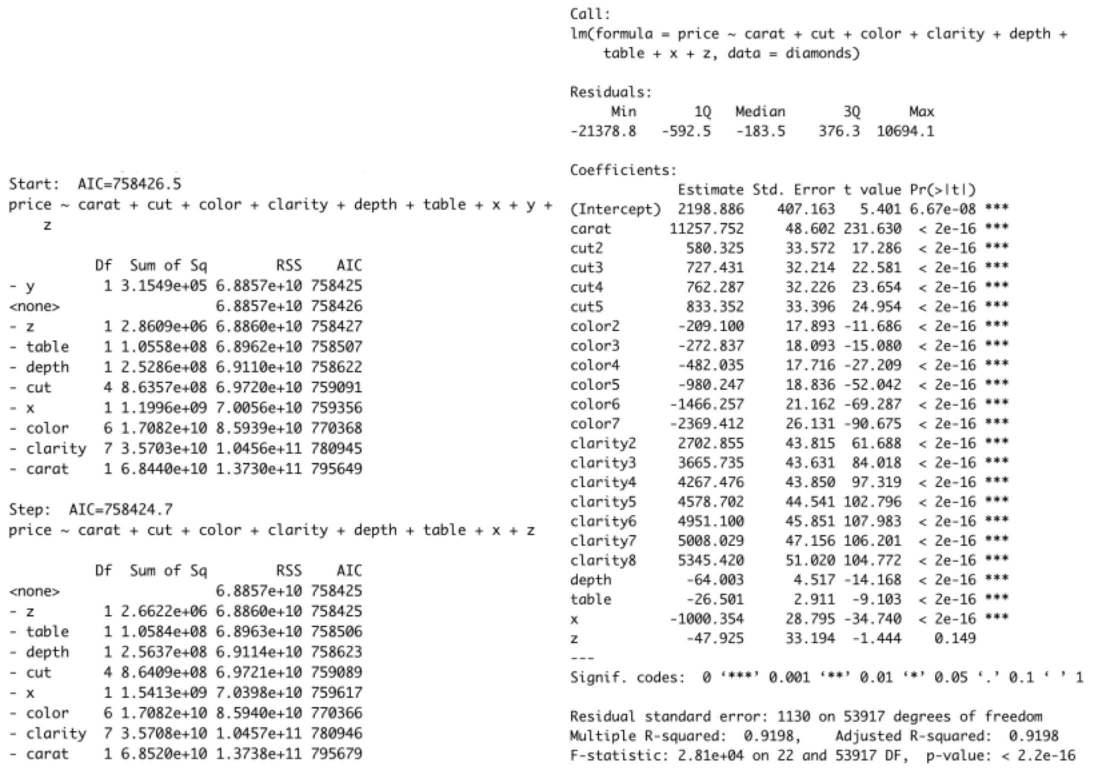
Figure 19: CocoMelon

Carat, x, y, and z all have VIF $>$ 4, indicating moderate multicollinearity.

The output does not explicitly state if any of the VIF values are greater than 10, as values are cut off at the 7. So for "extreme multicolinearity" corresponding to a VIF greater than 10, we cannot determine explicitly if the "x" explanatory variable exhibits extreme multicollinearity (which is the only explanatory variable that *may* meet that criteria).

(c)

Summarize the backward elimination method of model selection by providing:



```
Start:  AIC=758426.5
price ~ carat + cut + color + clarity + depth + table + x + y +
    z

           Df  Sum of Sq        RSS    AIC
- y         1 3.1549e+05 6.8857e+10 758425
<none>                   6.8857e+10 758426
- z         1 2.8609e+06 6.8860e+10 758427
- table     1 1.0558e+08 6.8962e+10 758507
- depth     1 2.5286e+08 6.9110e+10 758622
- cut       4 8.6357e+08 6.9720e+10 759091
- x         1 1.1996e+09 7.0056e+10 759356
- color     6 1.7082e+10 8.5939e+10 770368
- clarity   7 3.5703e+10 1.0456e+11 780945
- carat     1 6.8440e+10 1.3730e+11 795649

Step:  AIC=758424.7
price ~ carat + cut + color + clarity + depth + table + x + z

           Df  Sum of Sq        RSS    AIC
<none>                   6.8857e+10 758425
- z         1 2.6622e+06 6.8860e+10 758425
- table     1 1.0584e+08 6.8963e+10 758506
- depth     1 2.5637e+08 6.9114e+10 758623
- cut       4 8.6409e+08 6.9721e+10 759089
- x         1 1.5413e+09 7.0398e+10 759617
- color     6 1.7082e+10 8.5940e+10 770366
- clarity   7 3.5708e+10 1.0457e+11 780946
- carat     1 6.8520e+10 1.3738e+11 795679
```

```
Call:
lm(formula = price ~ carat + cut + color + clarity + depth +
    table + x + z, data = diamonds)

Residuals:
     Min       1Q   Median       3Q      Max
-21378.8   -592.5   -183.5    376.3  10694.1

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  2198.886    407.163   5.401 6.67e-08 ***
carat       11257.752     48.602 231.630  < 2e-16 ***
cut2          580.325     33.572  17.286  < 2e-16 ***
cut3          727.431     32.214  22.581  < 2e-16 ***
cut4          762.287     32.226  23.654  < 2e-16 ***
cut5          833.352     33.396  24.954  < 2e-16 ***
color2       -209.100     17.893 -11.686  < 2e-16 ***
color3       -272.837     18.093 -15.080  < 2e-16 ***
color4       -482.035     17.716 -27.209  < 2e-16 ***
color5       -980.247     18.836 -52.042  < 2e-16 ***
color6      -1466.257     21.162 -69.287  < 2e-16 ***
color7      -2369.412     26.131 -90.675  < 2e-16 ***
clarity2     2702.855     43.815  61.688  < 2e-16 ***
clarity3     3665.735     43.631  84.018  < 2e-16 ***
clarity4     4267.476     43.850  97.319  < 2e-16 ***
clarity5     4578.702     44.541 102.796  < 2e-16 ***
clarity6     4951.100     45.851 107.983  < 2e-16 ***
clarity7     5008.029     47.156 106.201  < 2e-16 ***
clarity8     5345.420     51.020 104.772  < 2e-16 ***
depth         -64.003      4.517 -14.168  < 2e-16 ***
table         -26.501      2.911  -9.103  < 2e-16 ***
x           -1000.354     28.795 -34.740  < 2e-16 ***
z             -47.925     33.194  -1.444    0.149
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1130 on 53917 degrees of freedom
Multiple R-squared:  0.9198,    Adjusted R-squared:  0.9198
F-statistic: 2.81e+04 on 22 and 53917 DF,  p-value: < 2.2e-16
```

Figure 20: CocoMelon

i.

an ordered list of which variable was removed from the model at each step;

Step 1: variable “y” was removed.

There are no more explanatory variables eliminated.

ii.

a list of which variables remained in the final model;

Explanatory variables that remained: Carat, cut, color, clarity, depth, and table.

iii.

a summary of the partial regression coefficients effects tests for the final model.

Of note, all explanatory variables except for “z” meet the significance threshold of an “overwhelming” or “strong” effect (which is whether we have “overwhelming” or “strong” evidence in favor of rejecting the null hypothesis). This appears to be the type of language used in the lab solutions.

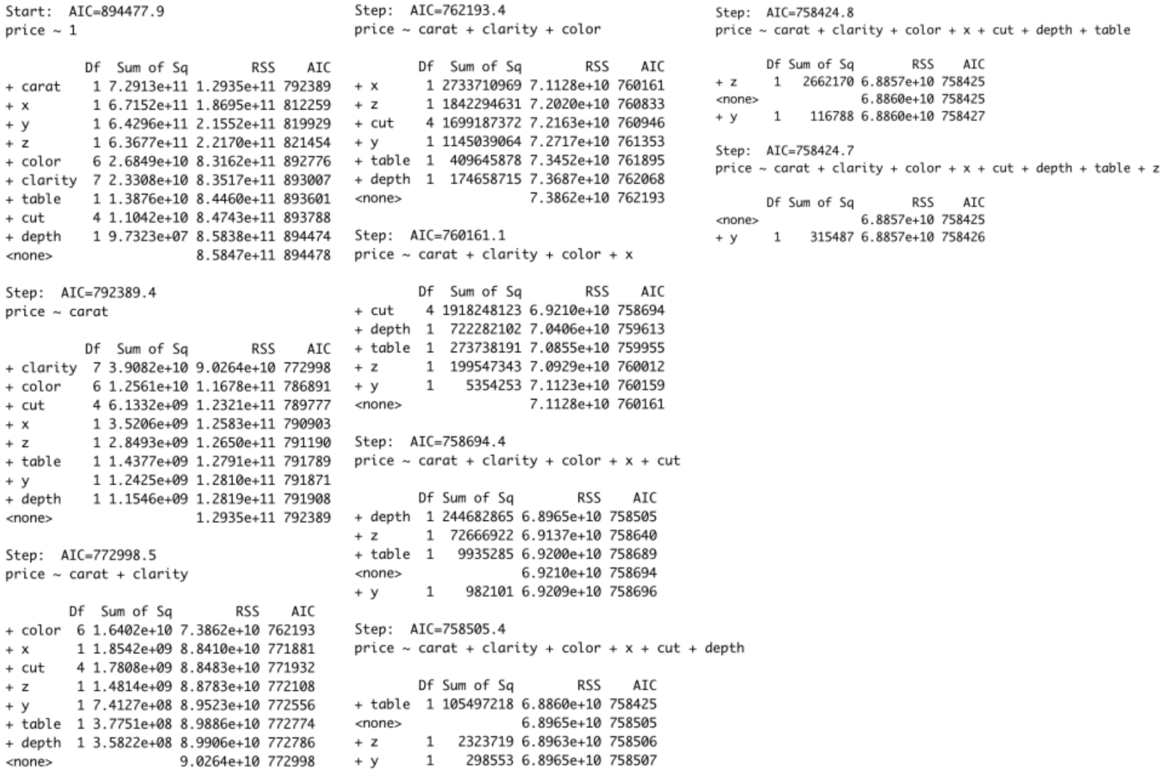
The explanatory variables used that have an “overwhelming” or “strong” effect are: carat, cut 2 through 5, color 2 through 7, clarity 2 through 8, depth, table , and “x”. This model also includes an intercept term significant at the level specified. The explanatory variable with “little to no effect” is the “z” variable.

All the final model partial regression coefficients meet statistical significance to reject null hypothesis at the $\alpha = 0.05$ level except for “z”. There is only one partial regression coefficients in the final model that does not meet statistical significance to reject null hypothesis at the $\alpha = 0.05$ level, “z”.

The statistical significance test is to determine whether there is evidence to reject the null hypothesis that the estimated beta coefficient is equal to zero (statistical significance referring to being statistically significant from zero) in the model that is composed of all the explanatory variables used in the model. This is a test of whether be have reason to suspect the estimated slope parameter of the particular explanatory variable is equal to zero in the multiple linear regression that includes the explanatory variables noted previously.

(d)

Summarize the forward selection method of model selection by providing:



```
Start:  AIC=894477.9
price ~ 1

           Df  Sum of Sq        RSS    AIC
+ carat     1 7.2913e+11 1.2935e+11 792389
+ x         1 6.7152e+11 1.8695e+11 812259
+ y         1 6.4296e+11 2.1552e+11 819929
+ z         1 6.3677e+11 2.2170e+11 821454
+ color     6 2.6849e+10 8.3162e+11 892776
+ clarity   7 2.3308e+10 8.3517e+11 893007
+ table     1 1.3876e+10 8.4460e+11 893601
+ cut       4 1.1042e+10 8.4743e+11 893788
+ depth     1 9.7323e+07 8.5838e+11 894474
<none>                   8.5847e+11 894478

Step:  AIC=792389.4
price ~ carat

           Df  Sum of Sq        RSS    AIC
+ clarity   7 3.9082e+10 9.0264e+10 772998
+ color     6 1.2561e+10 1.1678e+11 786891
+ cut       4 6.1332e+09 1.2321e+11 789777
+ x         1 3.5206e+09 1.2583e+11 790903
+ z         1 2.8493e+09 1.2650e+11 791190
+ table     1 1.4377e+09 1.2791e+11 791789
+ y         1 1.2425e+09 1.2810e+11 791871
+ depth     1 1.1546e+09 1.2819e+11 791908
<none>                   1.2935e+11 792389

Step:  AIC=772998.5
price ~ carat + clarity

         Df  Sum of Sq        RSS    AIC
+ color   6 1.6402e+10 7.3862e+10 762193
+ x       1 1.8542e+09 8.8410e+10 771881
+ cut     4 1.7808e+09 8.8483e+10 771932
+ z       1 1.4814e+09 8.8783e+10 772108
+ y       1 7.4127e+08 8.9523e+10 772556
+ table   1 3.7751e+08 8.9886e+10 772774
+ depth   1 3.5822e+08 8.9906e+10 772786
<none>                 9.0264e+10 772998

Step:  AIC=762193.4
price ~ carat + clarity + color

        Df  Sum of Sq        RSS    AIC
+ x      1 2733710969 7.1128e+10 760161
+ z      1 1842294631 7.2020e+10 760833
+ cut    4 1699187372 7.2163e+10 760946
+ y      1 1145039064 7.2717e+10 761353
+ table  1  409645878 7.3452e+10 761895
+ depth  1  174658715 7.3687e+10 762068
<none>                7.3862e+10 762193

Step:  AIC=760161.1
price ~ carat + clarity + color + x

        Df  Sum of Sq        RSS    AIC
+ cut    4 1918248123 6.9210e+10 758694
+ depth  1  722282102 7.0406e+10 759613
+ table  1  273738191 7.0855e+10 759955
+ z      1  199547343 7.0929e+10 760012
+ y      1    5354253 7.1123e+10 760159
<none>                7.1128e+10 760161

Step:  AIC=758694.4
price ~ carat + clarity + color + x + cut

        Df Sum of Sq        RSS    AIC
+ depth  1 244682865 6.8965e+10 758505
+ z      1  72666922 6.9137e+10 758640
+ table  1   9935285 6.9200e+10 758689
<none>               6.9210e+10 758694
+ y      1    982101 6.9209e+10 758696

Step:  AIC=758505.4
price ~ carat + clarity + color + x + cut + depth

        Df Sum of Sq        RSS    AIC
+ table  1 105497218 6.8860e+10 758425
<none>               6.8965e+10 758505
+ z      1   2323719 6.8963e+10 758506
+ y      1    298553 6.8965e+10 758507

Step:  AIC=758424.8
price ~ carat + clarity + color + x + cut + depth + table

       Df Sum of Sq        RSS    AIC
+ z     1   2662170 6.8857e+10 758425
<none>              6.8860e+10 758425
+ y     1    116788 6.8860e+10 758427

Step:  AIC=758424.7
price ~ carat + clarity + color + x + cut + depth + table + z

       Df Sum of Sq        RSS    AIC
<none>              6.8857e+10 758425
+ y     1    315487 6.8857e+10 758426
```

Figure 21: CocoMelon

i.

an ordered list of which variable was added to the model at each step;

First Step: carat, Second Step: clarity, Third Step: color, Fourth Step: x, Fifth Step: cut, Sixth Step: depth, Seventh Step: table, Eighth Step: z

ii.

a list of which variables never entered the final model;

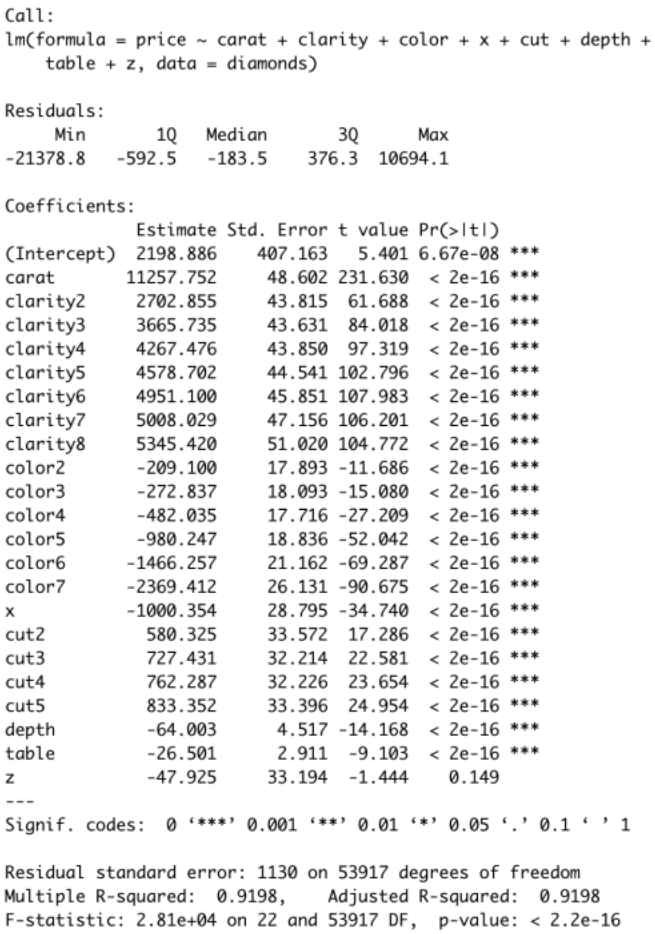
The explanatory variable "y" never entered the final model.

iii.

a summary of the partial regression coefficients effects tests for the final model.

Consistent with the prior model,

Of note, all explanatory variables except for "z" meet the significance threshold of an "overwhelming" or "strong" effect (which is whether we have "overwhelming" or "strong" evidence in favor of rejecting the null hypothesis). This appears to be the type of language used in the lab solutions.



```
Call:
lm(formula = price ~ carat + clarity + color + x + cut + depth +
    table + z, data = diamonds)

Residuals:
     Min       1Q   Median       3Q      Max 
-21378.8   -592.5   -183.5    376.3  10694.1 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  2198.886    407.163   5.401 6.67e-08 ***
carat       11257.752     48.602 231.630  < 2e-16 ***
clarity2     2702.855     43.815  61.688  < 2e-16 ***
clarity3     3665.735     43.631  84.018  < 2e-16 ***
clarity4     4267.476     43.850  97.319  < 2e-16 ***
clarity5     4578.702     44.541 102.796  < 2e-16 ***
clarity6     4951.100     45.851 107.983  < 2e-16 ***
clarity7     5008.029     47.156 106.201  < 2e-16 ***
clarity8     5345.420     51.020 104.772  < 2e-16 ***
color2       -209.100     17.893 -11.686  < 2e-16 ***
color3       -272.837     18.093 -15.080  < 2e-16 ***
color4       -482.035     17.716 -27.209  < 2e-16 ***
color5       -980.247     18.836 -52.042  < 2e-16 ***
color6      -1466.257     21.162 -69.287  < 2e-16 ***
color7      -2369.412     26.131 -90.675  < 2e-16 ***
x           -1000.354     28.795 -34.740  < 2e-16 ***
cut2          580.325     33.572  17.286  < 2e-16 ***
cut3          727.431     32.214  22.581  < 2e-16 ***
cut4          762.287     32.226  23.654  < 2e-16 ***
cut5          833.352     33.396  24.954  < 2e-16 ***
depth         -64.003      4.517 -14.168  < 2e-16 ***
table         -26.501      2.911  -9.103  < 2e-16 ***
z             -47.925     33.194  -1.444    0.149    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1130 on 53917 degrees of freedom
Multiple R-squared:  0.9198,	Adjusted R-squared:  0.9198 
F-statistic: 2.81e+04 on 22 and 53917 DF,  p-value: < 2.2e-16
```

Figure 22: CocoMelon

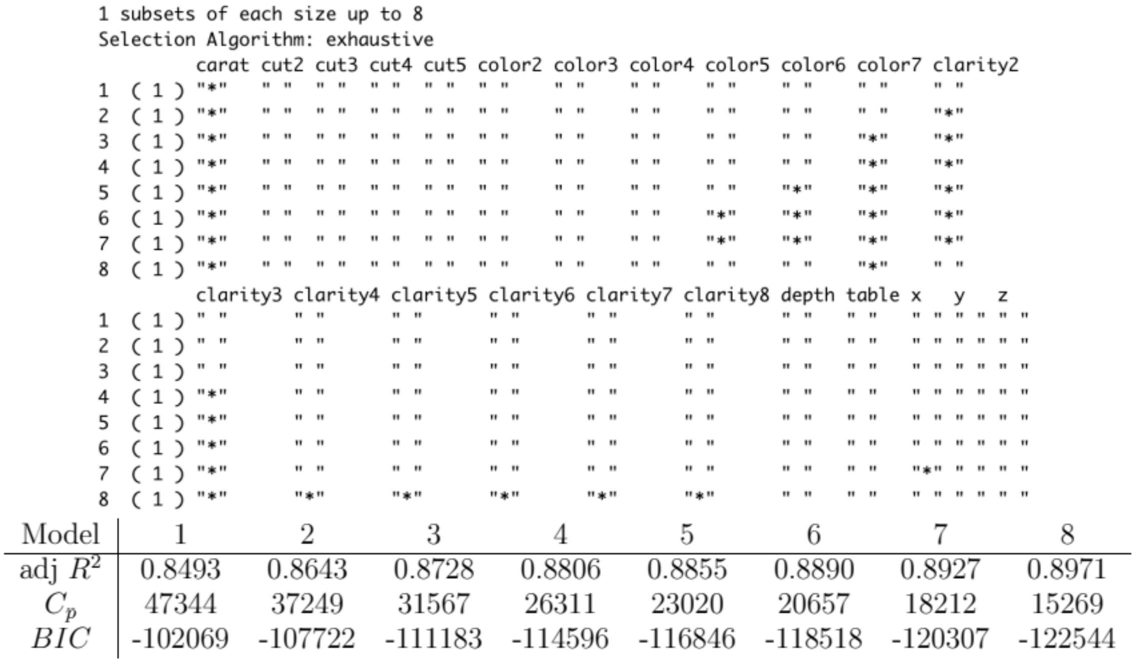
All the final model partial regression coefficients meet statistical significance to reject null hypothesis at the $\alpha = 0.05$ level except for "z". There is only one partial regression coefficients in the final model that does not meet statistical significance to reject null hypothesis at the $\alpha = 0.05$ level, "z".

The explanatory variables used that have an "overwhelming" or "strong" effect are: carat, cut 2 through 5, color 2 through 7, clarity 2 through 8, depth, table , and "x". This model also includes an intercept term significant at the level specified. The explanatory variable with "little to no effect" is the "z" variable.

The statistical significance test is to determine whether there is evidence to reject the null hypothesis that the estimated beta coefficient is equal to zero (statistical significance referring to being statistically significant from zero) in the model that is composed of all the explanatory variables used in the model. This is a test of whether be have reason to suspect the estimated slope parameter of the particular explanatory variable is equal to zero in the multiple linear regression that includes the explanatory variables noted previously.

(e)

Summarize the all-possible-subsets method of model selection by providing:



```
1 subsets of each size up to 8
Selection Algorithm: exhaustive
         carat cut2 cut3 cut4 cut5 color2 color3 color4 color5 color6 color7 clarity2
1  ( 1 ) "*"   " "  " "  " "  " "  " "    " "    " "    " "    " "    " "    " "
2  ( 1 ) "*"   " "  " "  " "  " "  " "    " "    " "    " "    " "    " "    "*"
3  ( 1 ) "*"   " "  " "  " "  " "  " "    " "    " "    " "    " "    "*"    "*"
4  ( 1 ) "*"   " "  " "  " "  " "  " "    " "    " "    " "    " "    "*"    "*"
5  ( 1 ) "*"   " "  " "  " "  " "  " "    " "    " "    " "    "*"    "*"    "*"
6  ( 1 ) "*"   " "  " "  " "  " "  " "    " "    " "    "*"    "*"    "*"    "*"
7  ( 1 ) "*"   " "  " "  " "  " "  " "    " "    " "    "*"    "*"    "*"    "*"
8  ( 1 ) "*"   " "  " "  " "  " "  " "    " "    " "    " "    " "    "*"    " "
         clarity3 clarity4 clarity5 clarity6 clarity7 clarity8 depth table x   y   z
1  ( 1 ) " "      " "      " "      " "      " "      " "      " "   " "   " " " " " "
2  ( 1 ) " "      " "      " "      " "      " "      " "      " "   " "   " " " " " "
3  ( 1 ) " "      " "      " "      " "      " "      " "      " "   " "   " " " " " "
4  ( 1 ) "*"      " "      " "      " "      " "      " "      " "   " "   " " " " " "
5  ( 1 ) "*"      " "      " "      " "      " "      " "      " "   " "   " " " " " "
6  ( 1 ) "*"      " "      " "      " "      " "      " "      " "   " "   " " " " " "
7  ( 1 ) "*"      " "      " "      " "      " "      " "      " "   " "   "*" " " " "
8  ( 1 ) "*"      "*"      "*"      "*"      "*"      "*"      " "   " "   " " " " " "
```

Model	1	2	3	4	5	6	7	8
adj R^2	0.8493	0.8643	0.8728	0.8806	0.8855	0.8890	0.8927	0.8971
C_p	47344	37249	31567	26311	23020	20657	18212	15269
BIC	-102069	-107722	-111183	-114596	-116846	-118518	-120307	-122544

Figure 23: CocoMelon

i.

Which model would you choose based on the adjusted R2 values?

Model 8, using the explanatory variables of carat, color 7, and clarity 3 through 8 (possibly with an intercept term as well). Generally we would say we include “carat”, “color”, and “clarity” explanatory variables, but we only see a subset of possible categorical variable values being significant in the above model.

ii.

Which model would you choose based on the Mallow’s Cp criteria?

Model 8, using the explanatory variables of carat, color 7, and clarity 3 through 8 (possibly with an intercept term as well). Generally we would say we include “carat”, “color”, and “clarity” explanatory variables, but we only see a subset of possible categorical variable values being significant in the above model.

iii.

Which model would you choose based on the BIC values?

Model 8, using the explanatory variables of carat, color, and clarity (possibly with an intercept term as well). Generally we would say we include “carat”, “color”, and “clarity” explanatory variables, but we only see a subset of possible categorical variable values being significant in the above model.

(f)

Interpret the values of the estimated regression coefficients for the final model selected:

i.

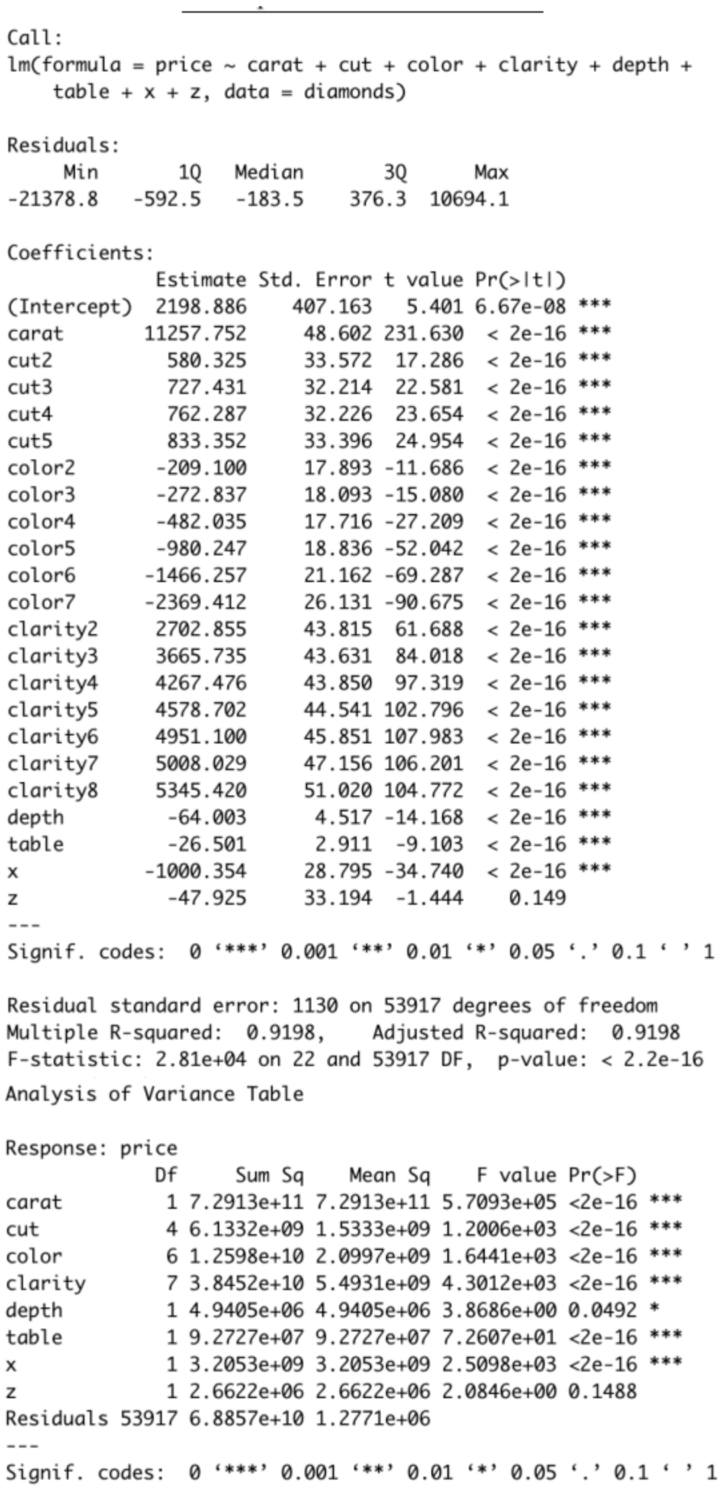
one of the values corresponding to the categorical variable of your choice;

The mean price of diamonds that are cut 2 are priced $580.325 more than diamonds that are cut 1, holding all other variables constant (given the other variables in the model).

ii.

one of the values corresponding to the quantitative variable of your choice.

For every 1 carat increase in weight, the mean price of diamonds will increase by $11,257.752 (associated with an increase of), holding all other variables constant (given all other variables in the model).



```
Call:
lm(formula = price ~ carat + cut + color + clarity + depth +
    table + x + z, data = diamonds)

Residuals:
     Min       1Q   Median       3Q      Max
-21378.8   -592.5   -183.5    376.3  10694.1

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  2198.886    407.163   5.401 6.67e-08 ***
carat       11257.752     48.602 231.630  < 2e-16 ***
cut2          580.325     33.572  17.286  < 2e-16 ***
cut3          727.431     32.214  22.581  < 2e-16 ***
cut4          762.287     32.226  23.654  < 2e-16 ***
cut5          833.352     33.396  24.954  < 2e-16 ***
color2       -209.100     17.893 -11.686  < 2e-16 ***
color3       -272.837     18.093 -15.080  < 2e-16 ***
color4       -482.035     17.716 -27.209  < 2e-16 ***
color5       -980.247     18.836 -52.042  < 2e-16 ***
color6      -1466.257     21.162 -69.287  < 2e-16 ***
color7      -2369.412     26.131 -90.675  < 2e-16 ***
clarity2     2702.855     43.815  61.688  < 2e-16 ***
clarity3     3665.735     43.631  84.018  < 2e-16 ***
clarity4     4267.476     43.850  97.319  < 2e-16 ***
clarity5     4578.702     44.541 102.796  < 2e-16 ***
clarity6     4951.100     45.851 107.983  < 2e-16 ***
clarity7     5008.029     47.156 106.201  < 2e-16 ***
clarity8     5345.420     51.020 104.772  < 2e-16 ***
depth         -64.003      4.517 -14.168  < 2e-16 ***
table         -26.501      2.911  -9.103  < 2e-16 ***
x           -1000.354     28.795 -34.740  < 2e-16 ***
z             -47.925     33.194  -1.444    0.149
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1130 on 53917 degrees of freedom
Multiple R-squared:  0.9198,    Adjusted R-squared:  0.9198
F-statistic: 2.81e+04 on 22 and 53917 DF,  p-value: < 2.2e-16
Analysis of Variance Table

Response: price
             Df     Sum Sq    Mean Sq    F value Pr(>F)
carat         1 7.2913e+11 7.2913e+11 5.7093e+05 <2e-16 ***
cut           4 6.1332e+09 1.5333e+09 1.2006e+03 <2e-16 ***
color         6 1.2598e+10 2.0997e+09 1.6441e+03 <2e-16 ***
clarity       7 3.8452e+10 5.4931e+09 4.3012e+03 <2e-16 ***
depth         1 4.9405e+06 4.9405e+06 3.8686e+00 0.0492 *
table         1 9.2727e+07 9.2727e+07 7.2607e+01 <2e-16 ***
x             1 3.2053e+09 3.2053e+09 2.5098e+03 <2e-16 ***
z             1 2.6622e+06 2.6622e+06 2.0846e+00 0.1488
Residuals 53917 6.8857e+10 1.2771e+06
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Figure 24: CocoMelon

(g)

Summarize your findings from examining all the residual plots used to diagnose the MLR model assumptions. Are there any assumptions that aren't met for this analysis? Briefly justify your response.

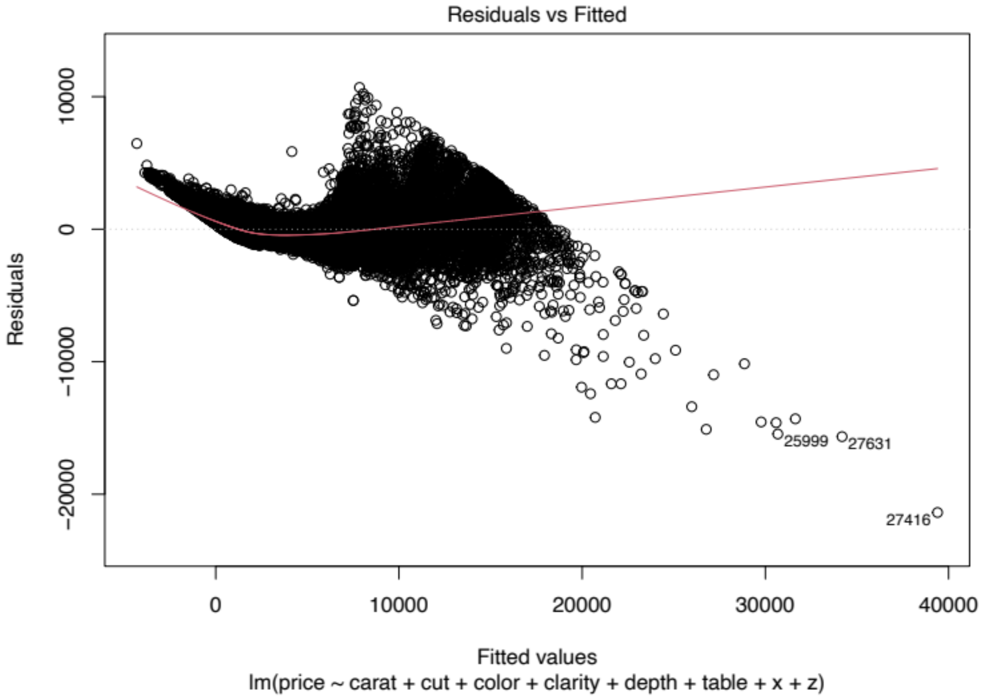


Figure 25: CocoMelon

The key model assumptions we evaluate with the given plots and graphs are: Equal variance, linearity (form of the model), and normality. That being said:

The residual plot has a clearly obvious non-random-scattering trend, suggesting the linearity and equal variance assumptions may be violated. (We observe a trend in the residual plot, particularly indicating a violation of linearity).

Additionally, the QQ is roughly linear as it follows the reference line closely in the middle of the plot. However, there are deviations in the left tail and right of the middle, suggesting the normality assumption is violated.

Taken together, we have reason to be concerned that our key assumptions are being violated, with regards to the residuals.

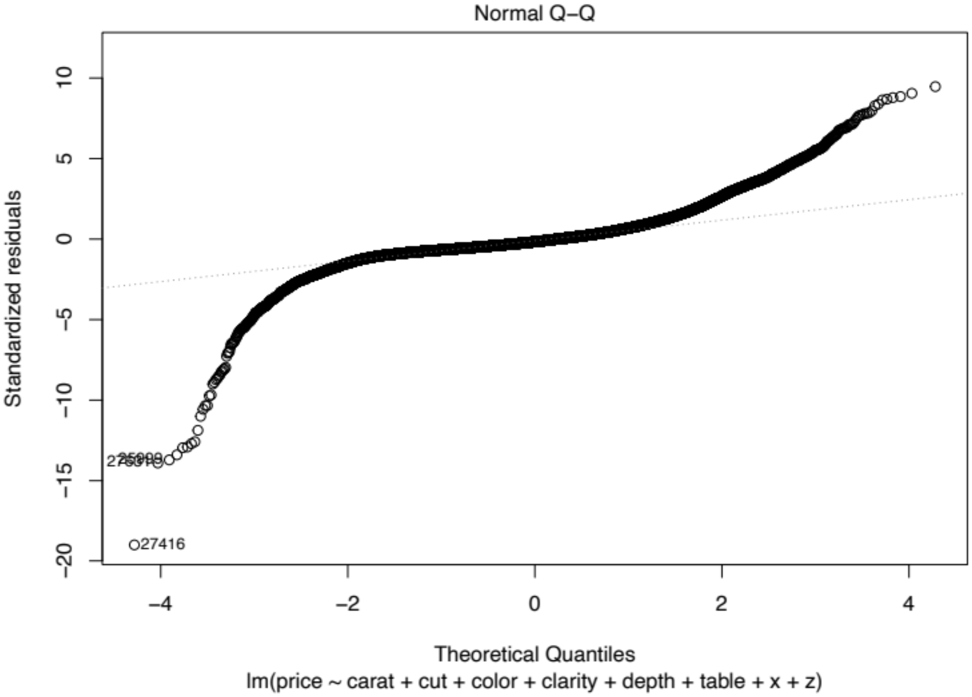


Figure 26: CocoMelon

(h)

Summarize your findings from examining the case diagnostic values/plots. Are there any outliers, leverage points, or influential observations?

Outliers: Looking at the studentized residual plot, there appear to be many outliers (looking for observations where the studentized residuals exceed 2, based on magnitude, i.e. |r| for r residuals, as indicated by the dashed red lines).

Leverage: From the leverage plot, there appears to be many high leverage points (leverage values exceeding $2(8+1)/50000 \approx 0.00036$, as indicated by the red line).

From the cook's D plot, there appear to be several high influence points (influence value exceeding $2\sqrt{2/50000} \approx 0.01264911$, as indicated by the red line).

Overall, we have reason to believe there are outliers, leverage points, and influential points, such that we may recommend considering other models or undergoing a transformation of our data and reevaluating our assumptions again. As-is, we have reason to be suspect.

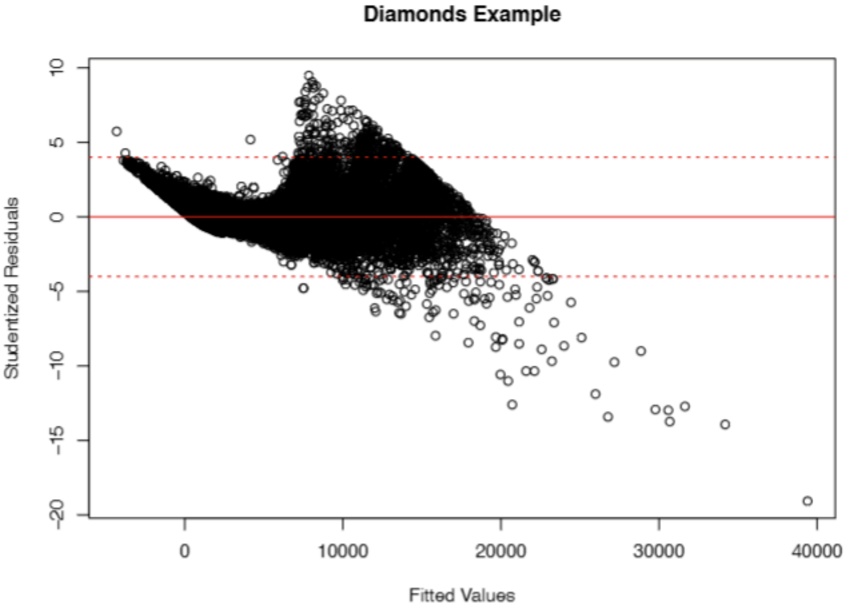


Figure 27: CocoMelon

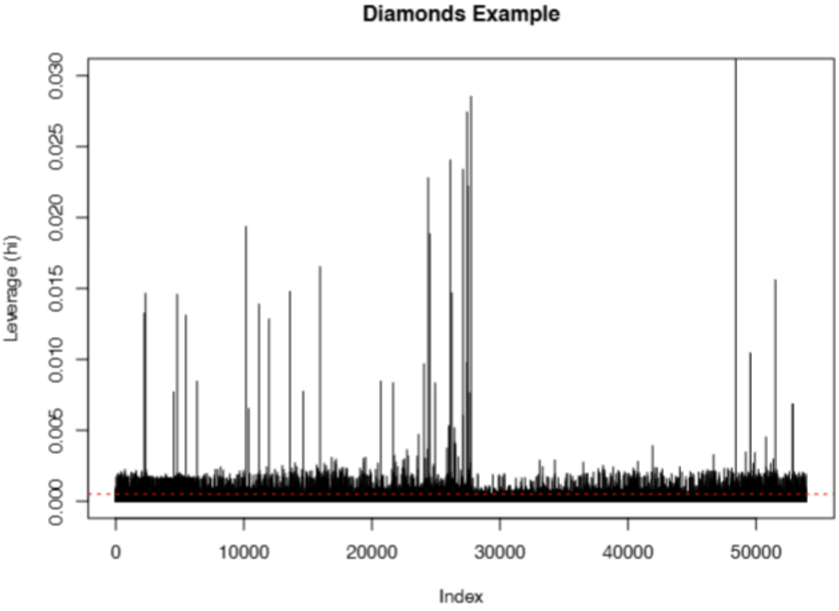


Figure 28: CocoMelon

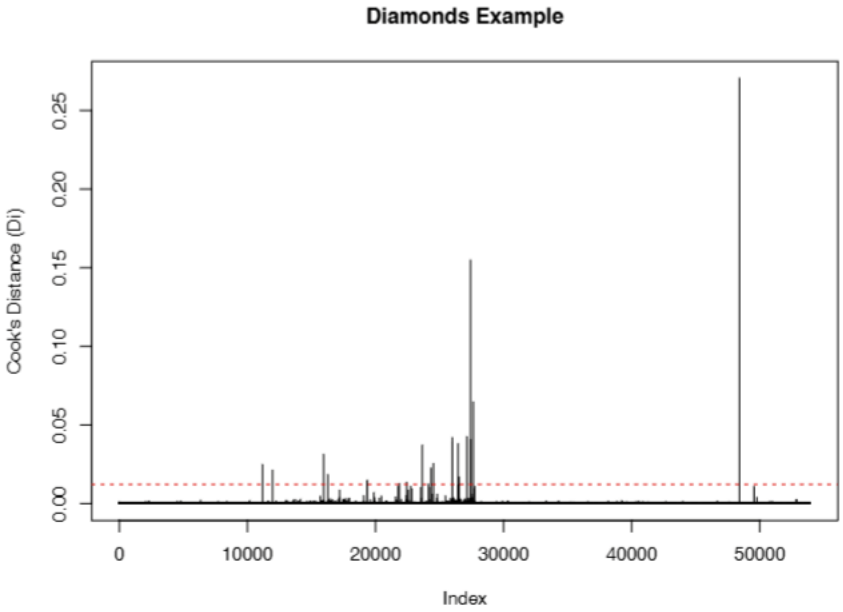


Figure 29: CocoMelon