MATH 392 Problem Set 6

Sam D. Olson

Exercise from the book

9.1 #1

Q: Let X have the exponential distribution with parameter β. Suppose that we wish to test the hypotheses $H_0 : \beta \geq 1$ versus $H_1 : \beta < 1$. Consider the test procedure δ that rejects H_0 if $X \geq 1$.

(A)

Determine the power function of the test.

(B)

Compute the size of the test.

9.1 #2

Q: Suppose that $X_1, ..., X_n$ form a random sample from the uniform distribution on the interval $[0, \theta]$, and that the following hypotheses are to be tested:

$H_0 : \theta \geq 2$

$H_1 : \theta < 2$

Let $Y_n = max\,(X_1, ..., X_n)$ and consider a test procedure such that the critical region contains all the outcomes for which $Y_n \leq 1.5$.

(A)

Determine the power function of the test.

(B)

Determine the size of the test.

9.1 #14

Plus: plot power functions in R

Q: Let $X_1, ..., X_n$ be i.i.d. with exponential distribution with parameter θ. Suppose that we wish to test the hypotheses:

$H_0 : \theta \geq \theta_0$

$H_1 : \theta < \theta_0$

Let $X = \sum\limits_{i=1}^{n} X_i$. Let δ_c be the test that rejects H_0 if $X \geq c$.

(A)

Show that $\pi\,(\theta \mid \delta_c)$ is a decreasing function of θ

(B)

Find c in order to make δ_c have size α_0.

(C)

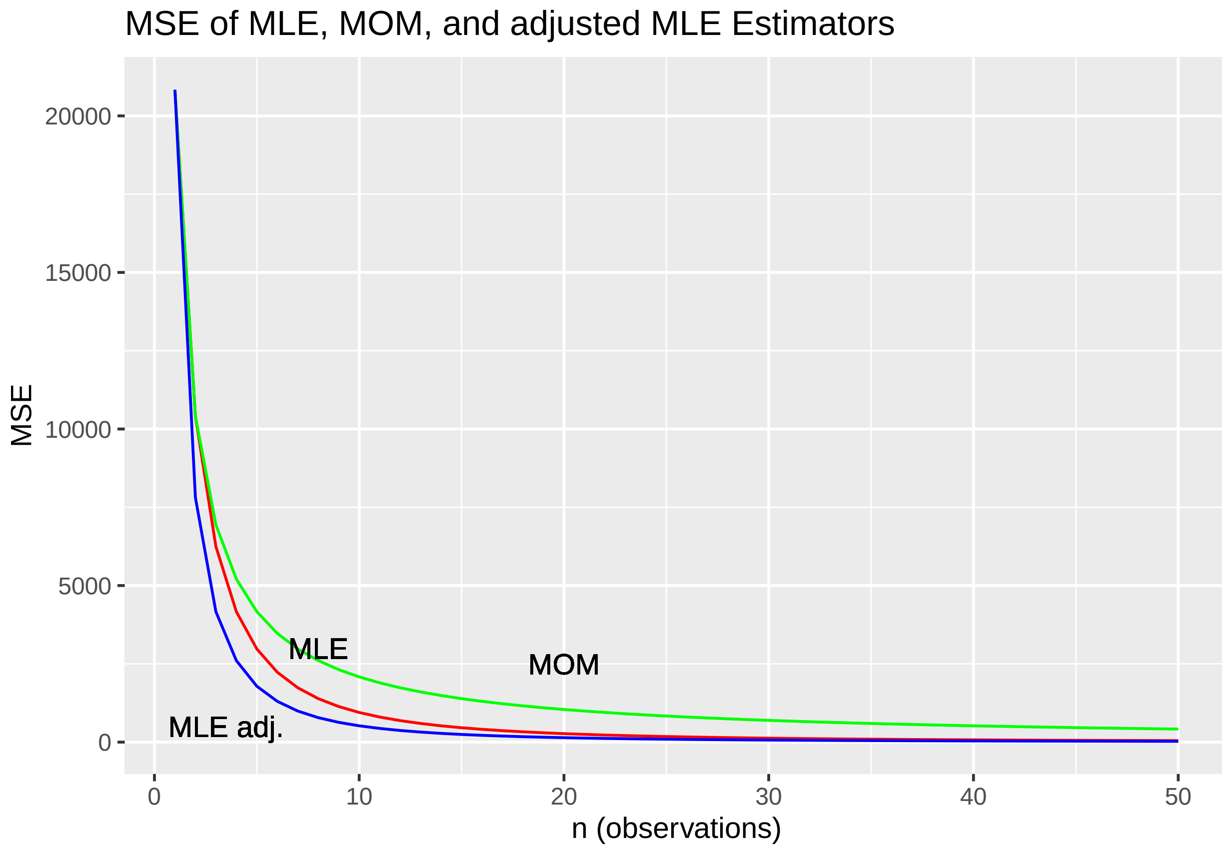
Let $\theta_0 = 2$, $n = 1$, and $\alpha_0 = 0.1$. Find the precise form of the test δ_c and sketch its power function.

```r
theta <- 250
n <- (1:50)

mle_mse <- rep(NA, 50)
mom_mse <- rep(NA, 50)
mle_adj_mse <- rep(NA, 50)
for (i in 1:50){
  mle_mse[i] <- ((theta ^ 2) * 2 )/ ((n[i] + 1) * (n[i] + 2))
  mom_mse[i] <- (theta ^ 2) / (3 * n[i])
  mle_adj_mse[i] <- (theta ^ 2) / ((n[i] * (n[i] +2)))
}

df <- data.frame(n,mle_mse,mom_mse, mle_adj_mse)

ggplot(df, aes(n)) +
  geom_line(aes(y=mle_mse), colour="red") +
  geom_line(aes(y=mom_mse), colour="green") +
  geom_line(aes(y=mle_adj_mse), colour="blue") +
  geom_text(x=3.5, y=500, label="MLE adj.") +
  geom_text(x=20, y=2500, label="MOM") +
  geom_text(x=8, y=3000, label="MLE") +
  ggtitle("MSE of MLE, MOM, and adjusted MLE Estimators") +
  labs(y="MSE", x = "n (observations)")
```



9.2 #2

Q: Consider two p.d.f.'s $f_0(x)$ and $f_1(x)$ that are defined as follows:

$$f_0(x)) = \begin{Bmatrix} 1 \; for \; 0 \le x \le 1 \\ 0 \;\; otherwise \end{Bmatrix}$$

and

$$f_1(x) = \begin{Bmatrix} 2x \; for \; 0 \le x \le 1 \\ 0 \;\; otherwise \end{Bmatrix}$$

Suppose that a single observation X is taken from a distribution for which the p.d.f. $f(x)$ is either $f_0(x)$ or $f_1(x)$, and the following simple hypotheses are to be tested:

$H_0 : f(x) = f_0(x)$

$H_1 : f(x) = f_0(x)$

(A)

Describe a test procedure δ for which the value of $\alpha(\delta) + 2\beta(\delta)$ is a minimum.

(B)

Determine the minimum value of $\alpha(\delta) + 2\beta(\delta)$ attained by that procedure.

9.2 #3

Q: Consider again the conditions of Exercise 2 (9.2.2), but suppose now that it is desired to find a test procedure for which the value of $3\alpha(\delta) + \beta(\delta)$ is a minimum.

(A)

Determine the procedure.

(B)

Determine the minimum value of $3\alpha(\delta) + \beta(\delta)$ attained by the procedure.

9.2 #10

Q: Suppose that $X_1, ..., X_n$ form a random sample from the Poisson distribution with unknown mean λ. Let λ_0 and λ_1 be specified values such that $\lambda_1 > \lambda_0 > 0$, and suppose that it is desired to test the following simple hypotheses:

$H_0 : \lambda = \lambda_0$

$H_1 : \lambda = \lambda_1$

(A)

Show that the value of $\alpha(\delta) + \beta(\delta)$ is minimized by a test procedure which rejects H_0 when $\bar{X}_n > c$.

(B)

Find the value of c.

(C)

For $\lambda_0 = \frac{1}{4}$, $\lambda_1 = \frac{1}{2}$, and $n = 20$, determine the minimum value of $\alpha(\delta) + \beta(\delta)$ that can be attained.

9.3 #1

Q: Suppose that $X_1, ..., X_n$ form a random sample from the Poisson distribution with unknown mean λ $(\lambda > 0)$. Show that the joint p.f. of $X_1, ..., X_n$ has a monotone likelihood ratio in the statistic $\sum\limits_{i=1}^{n} X_i$.

9.3 #2

Q: Suppose that $X_1, ..., X_n$ form a random sample from the normal distribution with known mean μ and unknown variance σ^2 $\left(\sigma^2 > 0\right)$. Show that the joint p.d.f. of $X_1, ..., X_n$ has a monotone likelihood ratio in the statistic $\sum\limits_{i=1}^{n} (X_i - \mu)^2$.

9.3 #13

Q: Suppose that four observations are taken at random from the normal distribution with unknown mean μ and known variance 1. Suppose also that the following hypotheses are to be tested:

$H_0 : \mu \geq 10$

$H_1 : \mu < 10$

(A)

Determine a UMP test at the level of significance $\alpha_0 = 0.1$

(B)

Determine the power of this test when $\mu = 9$.

(C)

Determine the probability of not rejecting H_0 if $\mu = 11$.