RPI 3 A+ Header Board

	+3.3V	+5V	RPM +Ve (Red)
	SDA	+5V	Telem 2 +Ve (Red)
	SCL	⏚	Telem 2 Gnd (Black)
	GPIO 4	TXD	Telem 2 TXD (Green)
(Gnd) RPM	⏚	RXD	Telem 2 RXD (Blue)
	GPIO 17	18	RPM (Yellow)
	GPIO 27	⏚	RPM GND (Purple)
	GPIO 22	23	LC (Yellow)
	+3.3V	24	LC (Orange)
	10	⏚	LC (Purple)
	9	25	
	11	8	
	⏚	7	
	ID_SD	ID_SC	
	5	⏚	
	6	12	
	13	⏚	
RPM Motor 1	19	16	RPM Motor 3
RPM Motor 4	26	20	RPM Motor 2
	⏚	21	

RPi — Function index

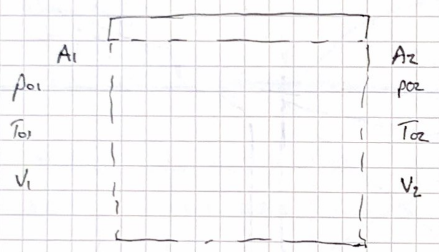
~/drone/

rpm-countlow.py	Use rpm sensors, output either rpm at each motor & time stamp, or print sensor states
rpm-sensor.py	Use gap measure~~me~~ to determine rev #. Replaced by rpm-countlow.py.
poshold.py	test script for MAVProxy module mavproxy_cmdlong.py

$\phi = \frac{V_{x}}{U}$ $\psi = \frac{\Delta h_0}{U^2}$ $\Lambda = 0.5$

$\dot{m} = \rho A V_x$

Assume 1:1 diffuser $\therefore A_1 = A_2 = A$



Assume $T_{01} = T_{02}$ ie isentropic expansion

$\therefore \Delta h_0 \sim \frac{\Delta P_0}{\rho}$

Due to:

$$\cancel{Tds}^{\,0} = dh - \frac{dp}{\rho}$$

$$dh = \frac{dp}{\rho}$$

$$\Delta h_0 = U \Delta V_\theta = \frac{W_x}{\dot{m}}$$

$$\Delta h_0 \dot{m} = W_x \qquad W_x =$$

SFME

p_1 far field $= p_{01}$

$$F = \dot{m}_2 V_2 - \dot{m}_1 V_1 + (p_2 - p_1) A$$

$$= \dot{m}(V_2 - \cancel{V_1}^{\,0}) + (p_2 - p_{01}) A$$

$$= \dot{m} V_2 + \cancel{g}(p_2 - p_{01}) A$$

$$T = \frac{Mg}{4} \approx \frac{2 \cdot 9.81}{4}$$

$$= 4.905$$

$$T = \rho A V_2^2 + (p_2 - \overset{p_a}{p_{01}}) A$$

$$= A\left(\rho V_2^2 + \ldots\right)$$

Assume incompressible

$$p_2 = p_a + \tfrac{1}{2}\rho V_2^2$$

$$\therefore T = A\rho V_2^2 + \tfrac{1}{2} A\rho V_2^2$$

$$= \tfrac{3}{2} A\rho V_2^2$$

$$\therefore V_2^2 = \frac{2T}{3A\rho}$$

$$V_2 = \sqrt{\frac{2T}{3A\rho}}$$

WRONG

$$\phi = \frac{V_x}{U}$$

$$\psi = \frac{\Delta h_0}{U^2}$$

$$\Lambda = \frac{\Delta h_{rotor}}{\Delta h_{stage}} = 0.5$$

$$\Omega = 4000 \text{ rpm}$$

$$= \frac{4000}{60} \cdot 2\pi$$

$$\Delta h_0 \sim \frac{\Delta P_0}{\rho}$$

$$\therefore \Delta h_0 \simeq \frac{1}{2} V_2^2$$

$$\therefore \psi \sim \frac{\frac{1}{2} V_2^2}{U^2}$$

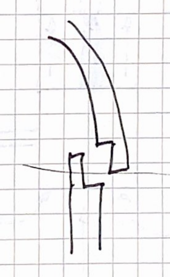
PROJECT MEETING

- → Blade count
- → Flow coeff. ? Lower?
  Hows the efficiency? → UROP.
- → Jonny's report
  Fritz
  - Check thrust.

BL write IBL Jonny

THIS WEEK

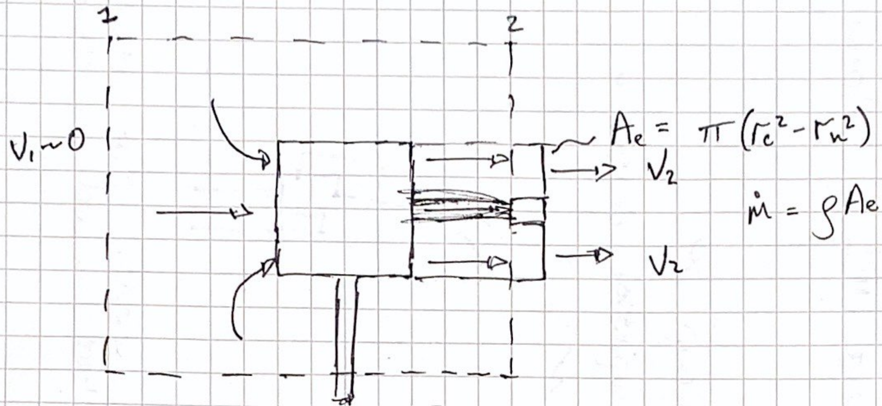
- → Turbo Comp. ~
- → Latex compressor design ✓
- → Scripts. ~



// Free Vortex const. $r V_\theta$ at exit
~~See~~ See where in code & change.

CONTROL VOLUME ANALYSIS

Upstream velocity & pressure ~ zero, atmospheric respectively.



$\rightarrow T_T$ ~ on Arm ∴ equal opposite on fluid

ρ = const (Low V, $M \ll 1$)

∴ $p_0 = p_2 + \frac{1}{2}\rho V^2$

CV large enough such that $\dot{m}_n V_n \simeq 0$ for all circumference excluding A_e.

∴ $T_T = \dot{m}(V_2 - \cancel{V_1}^{\approx 0}) + (p_2 - p_{01})A_e$ $\qquad p_2 + \frac{1}{2}\rho V_2^2 = p_1 + \frac{1}{2}\rho V_1^2$

$T_T = \dot{m}(V_2 - V_1) + A_e(p_2 - p_1)$ $\qquad p_{02} - \frac{1}{2}\rho V_2^2 - (p_{01} - \frac{1}{2}\rho V_1^2)$

$= \Delta h_0 \rho - \frac{1}{2}\rho(V_2^2 - V_1^2)$

$V_1 \approx 0$; $p_1 \simeq p_{01} \simeq p_a$; $\cancel{p_2 = p_{02} - \frac{1}{2}\rho V^2}$

$= \frac{1}{2}V_2^2\rho - \frac{1}{2}\rho V_2^2 + \frac{1}{2}\rho V_1^2$

$p_2 + \frac{1}{2}\rho V_2^2 = p_1 + \cancel{\frac{1}{2}\rho V_1^2}^{\,0}$

$\Delta h_0 = \frac{\Delta p_0}{\rho}$ $\qquad \Delta p_0 = p_{02} - p_{01}$

$= \cancel{p_2} + \frac{1}{2}\rho V_2^2 - \cancel{p_{01}}$

$p_{02} = p_2 + \frac{1}{2}\rho V_2^2$

$p_2 = p_{01} = p_a$

$\Delta h_0 = \frac{1}{2}V_2^2$

If assume exit static = inlet static

$p_2 - p_1 = 0$

$p_{02} - \frac{1}{2}\rho V_2^2$

$$T_p = \dot{m}(V_2 - V_1) + A_e(p_2 - p_1)$$

$$T\cancel{ds}^{\to 0} = dh - \frac{dp}{\rho}$$

$$dh = \frac{dp}{\rho}$$

$$\therefore \Delta h_0 = \frac{\Delta p_0}{\rho} \qquad \Delta p_0 = p_{02} - \cancel{p_{01}}^{\to p_1}$$

$$= p_2 + \tfrac{1}{2}\rho V_2^2 - p_1$$

$$= \tfrac{1}{2}\rho V_2^2 + (p_2 - p_1)$$

and accelerate

$\therefore$ Max p_{02} + ~~max~~ max V_2

$\therefore$ To maximise thrust, maximise $p_2 - p_1$ and V_2 ie diffuse flow downstream of fan. Most efficient when flow leaves $=$ ie parallel and pressure matched

$$\Downarrow$$

$$p_2 = p_1$$

$$\therefore \Delta p_0 = \tfrac{1}{2}\rho V_2^2 \Rightarrow \Delta h_0 = \tfrac{1}{2}V_2^2$$

$$\therefore T_p = \dot{m}(V_2 - \cancel{V_1}^{\to 0})$$

$$= \rho A_e V_2^2$$

$$\therefore V_2^* = \sqrt{\frac{T_p}{\rho A_e}}$$

$$\phi = \frac{V_2}{U}$$

$$\psi = \frac{\tfrac{1}{2}V_2^2}{U^2}$$

$$\phi = \frac{V_2}{U} \qquad V_2 = \sqrt{\frac{T_p}{\rho A_2}} = \sqrt{\frac{T_p}{\rho\pi(r_c^2 - r_h^2)}}$$

$$U = \Omega\left(\frac{r_c + r_h}{2}\right)$$

$$\phi = \sqrt{\frac{T_p}{\rho\pi(r_c^2 - r_h^2)}} \cdot \frac{2}{\Omega(r_c + r_h)}$$

$$\psi = \frac{T_p}{\rho\pi(r_c^2 - r_h^2)} \cdot \frac{2}{\Omega^2(r_c + r_h)^2}$$

Main

r_c
r_h
r_m
span
r.radius
HTR
AR_rotor
AR-stator
tip_gap-percentage
r.11
s.12
ac_r

$$DF \approx 1 - \frac{V_2}{V_1} + \frac{1}{2}\frac{\Delta V_\theta}{V_1}\frac{s}{c}$$

$$\Delta V_{\theta\_m} = |V_{\theta_2} - V_{\theta_1}|$$

Jonny's CV Analysis:

$$p_0 = p_{00} - \frac{1}{2}\rho V_0^2$$

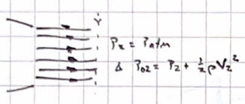
$$p_4 = p_{04} - \frac{1}{2}\rho V_4^2$$

$$p_4 - p_1 = \frac{1}{2}\rho(V_1^2 - V_4^2)$$

CV Analysis → fan design

$P_2 = P_{atm}$

Work in' ∴ $P_{02} \neq P_{01}$



2.1 "M < ..."

Vectorized graphics drawing program

Coral draw
Illustrator (mac OS

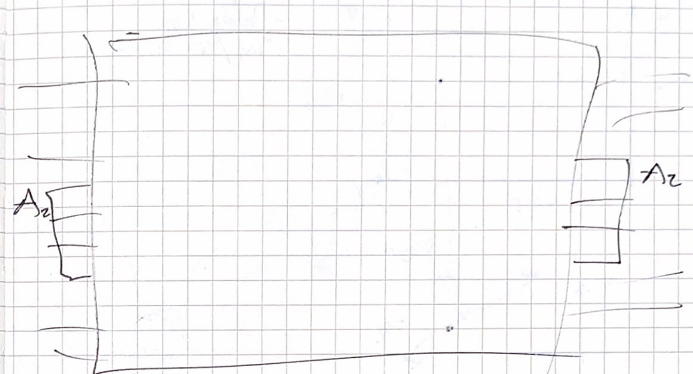
(11) & (12) Mean line

Define How (12)

$$\Delta h_0 = \frac{\Delta P_0}{\rho} \Rightarrow \frac{\frac{1}{2}\rho V^2}{\rho}$$

$$\Delta h_0 = \frac{\frac{1}{2}\rho V^2}{\rho}$$

$$\psi = \frac{\frac{1}{2}V^2}{U^2}$$

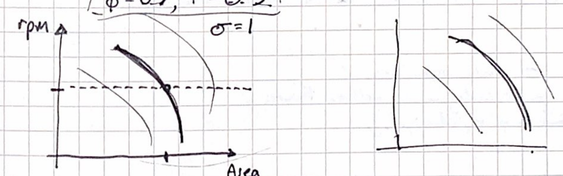


$$\psi = \frac{\Delta P_0}{\frac{1}{2}\rho U_{mid}^2}$$

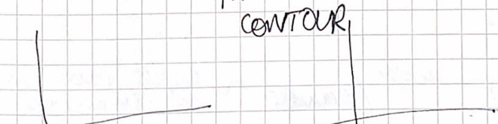
- Flip dependent variables so ϕ + ϕ are pre-set
- Get contours of the below.

$\boxed{\phi = 0.7,\ \psi = 0.3}$

$\sigma = 1$

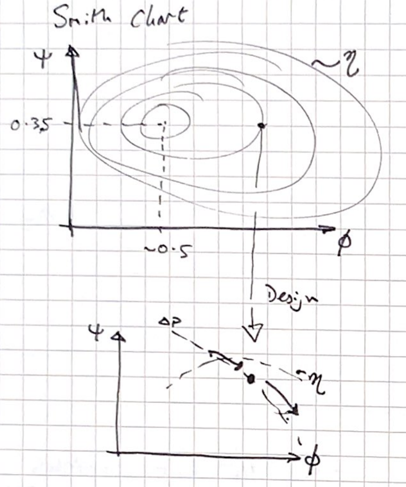


THRUST CONTOUR



+ Power to analysis. SFEE

Smith Chart



- Profile loss
- Tip/corner loss

$\sigma = 1.5 \Rightarrow P_{02} - P_{01}$ across turbomachinery

- Get σ into equations. ✓
- Do power analysis as well as thrust ✓
- Contour maps of design. ✓

Simple mass model of diffuser

Diffusers

L/D length/diameter ~ Aspect ratio increases as DF increases

How does that trade off with weight?

Optimising weight vs pressure force from diff.

$$T_p = \frac{\rho A V_2^2}{\cancel{\ \ }} = \frac{\rho A V_\infty^2}{\sigma}$$

$V_2 =$

How does σ effect triangles?

Simple mass model relates σ to weight of diffuser. ⇝ AR limited by BL seperatation.

σ related to thrust by pressure change

~~ΔM~~

$H = 1.8 + 3.75B$

$H =$ shape factor δ^*/θ

$B = \frac{2\delta^*}{R}$

Jonnies

Ref: [14]

[16]

Fixed

RPM (or Power...) → Torque vs rpm → Worke eqn. (Matlab)

ϕ ~ } Linked as fn (σ) once it is included.

ψ

T_T → {Intake, Fan, Exit} See sam's email

r_h

Vary

σ ~

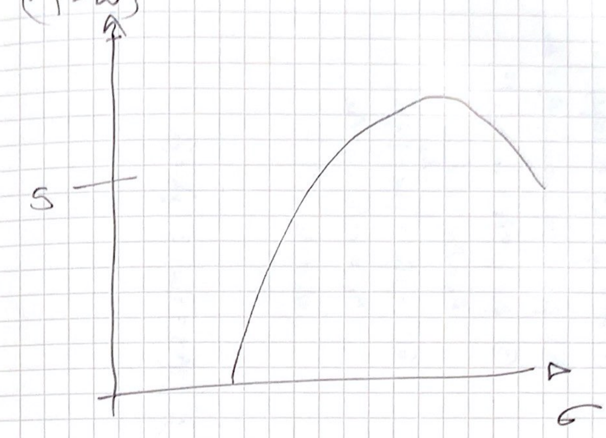
Put in σ, spits out r_c (via area)

Then from ESDU (expressed analytically)

σ → length from r_c & r_h as above.

→ length modelled to weight

(T-W)

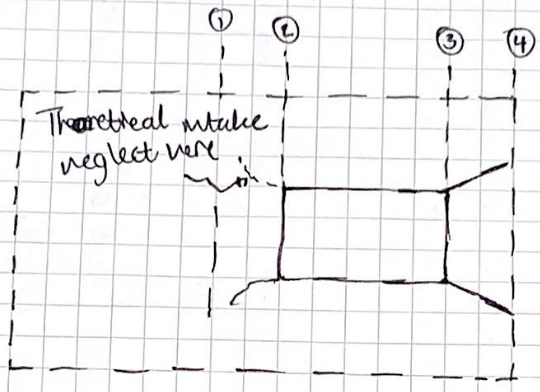


→ Want POWER optimised with constraint on (T-W)

→ ~~Then fix~~ L & r ∴ σ limited by mass model

∴ Vary r (hence σ) to optimise power.

CV Analyses with σ, OPTIMISE FOR MIN POWER wrt $r_c, (r_h)$



① ~~Atte~~ $p_1 = p_a = p_{01}$, A_1 large, $V_1 = 0$

② $p_2 = p_a$ (no intake) $A_2 = A_x$

③ $p_3 = p_3 \cancel{\neq p_a} = p_{03} - \frac{1}{2}\rho V_3^2$

$A_3 = A_x$, $V_3 = V_x$

④ $p_4 = p_a = \overset{= p_{03}}{p_{04}} - \frac{1}{2}\rho V_4^2$

$A_4 = \sigma A_3 = \sigma A_x$

$V_4 \neq V_x$

Relating ③ & ④

$p_3 = p_{03} - \frac{1}{2}\rho V_3^2$; $p_4 = p_{04} - \frac{1}{2}\rho V_4^2$; $p_{03} = p_{04}$; $p_4 = p_a$

$p_3 + \frac{1}{2}\rho V_3^2 = p_4 + \frac{1}{2}\rho V_4^2$

By continuity

$A_3 V_3 = A_4 V_4$ ~~ρ~~

$\therefore \left(\frac{V_3}{V_4}\right)^2 = \sigma = \left(\frac{V_x}{V_4}\right)^2$ $\therefore V_x^2 = \sigma V_4^2$

SFME

$T_T = \dot{m}_4 V_4 \overset{-\dot{m}_1 V_1}{+} A_4 p_4 - A_4 p_1$ $\quad p_4 = p_1 \quad \therefore p_4 - p_1 = 0$

$= \dot{m}_4 V_4$

$= \rho A_4 V_4^2$

$= \rho A_4 \frac{V_x^2}{\sigma}$ $\quad A_x = \frac{A_4}{\sigma}$

$= \frac{\rho A_x V_x^2}{\sigma}$

$\therefore V_x = \sqrt{\frac{T_T \sigma}{\rho A_x}}$ ✓

$\rho A_4 V_4^2 = \frac{\rho A_x V_x^2}{\sigma}$

$\dot{m} V_4 = \rho A_x V_x V_4$

$= \frac{\rho A_x V_x^2}{\sqrt{\sigma}}$

Flow coefficients; stage loading

$\phi = \frac{v_x}{u}$; $\varphi = \frac{\Delta h_0}{u^2}$

Near isentropic pressure rise

$\therefore T\,ds^{\approx 0} = dh - \cancel{\rho}\,dp\,v$

$\therefore dh_0 = \frac{dp}{\rho}$

$\Delta h_0 = \frac{\Delta P_0}{\rho} \quad \therefore \quad \psi = \frac{\Delta P_0}{\rho U^2}$

$\Delta P_0 = P_{03} - P_{02}$

$\Delta P_0 = P_3 + \frac{1}{2}\rho V_3^2 - P_a$

$\begin{cases} P_{02} = P_a \\ P_{03} = P_3 + \frac{1}{2}\rho V_3^2 \end{cases}$

$P_3 = P_4 + \frac{1}{2}\rho\left(V_4^2 - V_3^2\right)$ // from bernoulli

$= P_a + \frac{1}{2}\rho\left(\frac{V_3^2}{\sigma^2} - V_3^2\right) \qquad V_3 = V_x$

$\therefore \Delta P_0 = \frac{1}{2}\rho V_x^2\left(\frac{1}{\sigma^2} - 1\right) + \frac{1}{2}\rho V_x^2$

$= \cancel{\ldots} = \frac{1}{2}\rho V_x^2\left(\frac{1}{\sigma^2}\right)$

$\therefore \psi = \cancel{\ldots} = \frac{\frac{1}{2}\rho V_x^2}{\rho U^2\sigma^2}$

$P_3 = P_a + \frac{1}{2}\rho\left(\frac{V_x^2}{\sigma} - V_x^2\right)$

$\Delta P_0 = \frac{1}{2}\rho\left(\frac{V_x^2}{\sigma} - V_x^2\right) + \frac{1}{2}\rho V_x^2$

$= \frac{1}{2}\rho\frac{V_x^2}{\sigma}$

$\therefore \psi = \frac{V_x^2}{2\sigma \cdot U^2}$

$\therefore \psi = \frac{V_x^2}{2U^2\sigma^2}; \quad \phi = \frac{V_x}{U}; \quad V_x = \sqrt{\frac{T_T}{\rho}\frac{\sigma}{A_x}}$

$\frac{\phi^2}{2\sigma} = \psi$

$\frac{\phi^2}{\psi} = 2\sigma$

Redefine in terms of A_x & T_T

$\phi = \sqrt{\frac{T_T\sigma}{\rho A_x U^2}}; \quad \psi = \frac{T_T}{2\rho A_x U^2\sigma}$

Rearrange for σ

$$\left.\begin{aligned} \phi^2 \cdot \frac{\rho A_x U^2}{T_T} &= \sigma \\ \frac{T_T}{2\rho A_x U^2\psi} &= \sigma \end{aligned}\right\} \quad \cancel{\frac{\phi^2}{\psi} = 2\sigma^2}$$

$A_x = \pi\left(r_c^2 - r_h^2\right)$

$\therefore$ Limit of diffusion factor can be determined by flow coeff & stage loading

$$\sigma = \frac{\phi}{\sqrt{2\psi}}; \quad M = \rho\left[\frac{r_h(r_c - r_h)}{DF} + 2t\sqrt{\left(\frac{r_c - r_h}{DF}\right)^2 + \left(\frac{A_x\phi}{\pi\sqrt{2\psi}} - r_c\right)^2}\,\right]$$

Optimise for mass wrt r_c Assume fixed: r_h, φ, ϕ

$$DF = \left(\frac{\sigma - C}{A}\right)^{-1/B} = \left(\frac{\phi}{2A\sqrt{2\varphi}} - \frac{C}{A}\right)^{-1/B}$$

$A = 0.2863$
$B = 0.5627$
$C = 1.06$
} Empirically from ESDU 75026

$$\cancel{\therefore M = \rho_{PLA}\left[\frac{r_h(r_c - r_h)}{DF} + 2t\sqrt{\left(\frac{r_c - r_h}{DF}\right)^2 + \left(\frac{(r_c^2 - r_h^2)\phi}{\sqrt{2\varphi}} - r_c\right)^2}\,\right]}$$

Total Diffuser Volume

$$\left(V_{cone} + \underbrace{V_{thin\ shell}}_{\Downarrow\ t \times 2\pi r \times L}\right)$$

$$V_{total} = \frac{\pi r_h^2 (r_c^2 - r_h^2)}{3DF} + 2\pi t\sqrt{\frac{\phi(r_c^2 - r_h^2)}{\sqrt{2\varphi}}} \cdot \sqrt{\left(\frac{r_c - r_h}{DF}\right)^2 + \left(\frac{\phi(r_c^2 - r_h^2)}{\sqrt{2\varphi}} - r_c\right)^2}$$

$\therefore$ Mass $= V_{total} * \rho_{PLA}$

$$\cancel{\phi}\ \varphi = \frac{T_T}{2\rho A_x U^2 \sigma} \qquad \sigma = \frac{\phi}{\sqrt{2\varphi}}$$

$$= \frac{T_T\sqrt{2\varphi}}{2\rho A_x U^2 \phi}$$

$A_x = \pi(r_c^2 - r_h^2)$
$U = \Omega\frac{(r_c + r_h)}{2}$
$U^2 = \frac{\Omega^2 (r_c + r_h)^2}{4}$

$$\phi\sqrt{\varphi} \cdot \sqrt{2}\rho A_x U^2 = T_T$$

$$\frac{\phi\rho\pi(r_c^2 - r_h^2)\Omega^2(r_h + r_c)^2\sqrt{\varphi}}{\sqrt{2}} = T_T$$

$$\phi\sqrt{\varphi}\,\rho\pi\Omega\,\frac{(r_c^2 - r_h^2)(r_c + r_h)}{\sqrt{2}} = T_T$$

$$\phi\sqrt{\varphi}\,\rho\pi\Omega\,\frac{(r_c - r_h)(r_c + r_h)^3}{\sqrt{2}} = T_T$$

$$\therefore \varphi = \frac{T_T \cdot \sqrt{2\varphi} \cdot 4}{2\rho\pi(r_c^2 - r_h^2)\Omega^2(r_c + r_h)^2\phi}$$

$$\therefore T_T = \frac{\varphi\, 2\rho\pi(r_c^2 - r_h^2)\Omega^2(r_c + r_h)^2\phi}{4\sqrt{2\varphi}}$$

$$= \frac{\sqrt{\varphi}\,\phi\,\rho\pi(r_c^2 - r_h^2)\Omega^2(r_c + r_h)^2}{2\sqrt{2}}$$

$$L = \frac{r_c - r_h}{DF} \qquad \frac{\frac{\phi}{\sqrt{2\varphi}} - C}{A}$$

SPEE

$$\dot m \left(h_0 + \cancel{gz_0}^{\to 0} + \frac{V_0^2}{2}\right) + \dot Q = \dot m \left(h_4 + \cancel{gz_4}^{\to 0} + \frac{V_4^2}{2}\right) + \dot W \qquad \cancel{\dot m} = 1$$

$$h_4 - h_0 + \tfrac{1}{2}(V_4^2 - V_0^2) = \cancel{\dot Q}^{\to 0} - \dot W \qquad \text{Assume } \dot Q = 0$$

$$\Delta h_0 + \tfrac{1}{2} V_4^2 = -\dot W$$

+ve + ve = $-\dot W$ $\qquad$ $W = -ve$ $\qquad$ ie work done on fluid

$$\frac{\Delta P_0}{\rho} = \frac{1}{2}\frac{V_x^2}{\sigma} \qquad V_4^2 = \frac{V_x^2}{\sigma}$$

$$P = \frac{\tfrac{1}{2}\rho A V_x^3}{\sigma^2} = \tfrac{1}{2}\rho A T \qquad P = \frac{T^{3/2}}{\sqrt{4\sigma\rho A}} \quad W$$

$$P_{sp} = \frac{\tfrac{1}{2} V_x^2}{\sigma^2} = \frac{\tfrac{1}{2} T \sigma}{\rho A \sigma^x} \quad W/kg$$

$$\therefore \Delta h_0 + \tfrac{1}{2} V_4^2 = -\dot W$$

$$-\dot W = \frac{V_x^2}{2\sigma} + \frac{V_x^2}{2\sigma} = \frac{V_x^2}{\sigma} = \frac{T_T}{\rho A_x \sigma} = \frac{T_T}{2\rho A_x \sigma}$$

$$\therefore -\dot W = \frac{\sqrt{2\varphi}\,\phi\, U^2}{\cancel{2\sigma}} = 2\varphi U^2 = \frac{\cancel{2}\varphi\Omega^2(r_c + r_h)^2}{\cancel{4}\,2}$$

$$\therefore |\dot W| = \frac{\varphi\Omega^2(r_c+r_h)^2}{2}$$

Omega vs r_c ✓

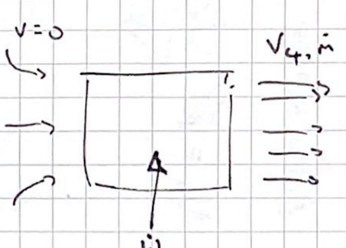
Fix omega, r_c vs r_h ?

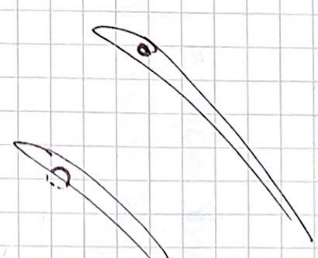
HTR ~ 0.4 as a maximum

effects ?

119.4

59.57





$$\dot W = \tfrac{1}{2}\dot m V_4^2 = \tfrac{1}{2}\dot m \frac{V_x^2}{\sigma}$$

MISC

Using loss models
produce S&S of ϕ vs ϕ for loss.

Lo Smith chart.

A. Dicksen Ch 9 → Check out appendix as well.

→ well written

A bit beyond 4A3

→ Check against notes sam sent!!

3 control volumes.

$\sigma \Rightarrow$ efficiency.

2. ΔP_0 rotor
3. ΔP_0 stator ✓ $A_3 = A_x$

5 $V_4 = \frac{V_3}{\sqrt{\sigma}}$

$\sigma \rightarrow \sigma^2$

2.1 Mach # < ... ✓

2.2 (1) P_{atm}

2.3

2.4 (18)/(17) replace constants with a or b

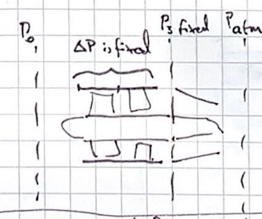
2.4.1 (21) Model rotors & stators as flat plates & relate to N

∴ relates to P/c - pitch chord.

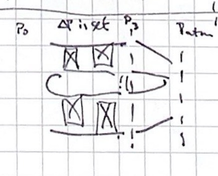
Trade off between weight + N

(22) Want thickness of diffuser also defined as predetermined variable.

(21) ~~Compare~~ Combine motor weight with rest of drone instead of propulsion



⇒ Gives σ

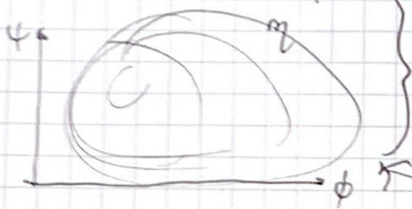


Ω $\Rightarrow$ r_c

r_h, r_c σ

t

θ



See if combining M_f with η gives any optimum ~~terms~~ in OP.

Use existing data for η, to prove concept before using Loss model.

$$M_f = \frac{M}{P}\sqrt{\frac{M}{2\rho A}}$$

$$\frac{1}{2}\frac{\rho A V_x^3}{\sigma^2} = \frac{1\,\rho A\, T^{3/2}\,\sigma^{3/2}}{4^{1/2}\rho^{3/2}A_x^{3/2}\,\sigma^{2}}$$

$$= \frac{T^{3/2}}{\sqrt{4\rho A_x \sigma}} = \frac{M^{3/2}}{\sqrt{4\rho A_x \sigma}}$$

$$\omega = \frac{T_T}{2\rho A_x \sigma} \qquad T_T = \sqrt{2\psi}\,\phi\,\rho A_x U^2$$

$$= \frac{\sqrt{2\psi}\,\phi\,\rho A_x U^2}{\sqrt{2}\rho A_x \sigma} = \frac{\sqrt{\psi}\,\phi\, U^2}{\sqrt{2}\,\sigma} \qquad \sigma = \frac{\phi^2}{2\psi}$$

$$= \frac{\sqrt{2}\,\psi^{3/2}\,\theta\, U^2}{\phi}$$

$$P = \eta\frac{\sqrt{2}\,\psi^{3/2}U^2}{\phi} \cdot \rho A_x V_x$$

$$M_f = \frac{T}{P}\sqrt{\frac{T}{2\rho A}}$$

$\phi\ \psi\ \sigma\ r_c\ r_h\ \Omega\ \theta$

$$P = \frac{T^{3/2}}{\sqrt{4\rho A_x \sigma}} \qquad T = \sqrt{2\varphi}\,\phi \rho A_x U^2$$

$$= \frac{(2\varphi)^{3/4} \phi^{3/2} \rho^{3/2} A_x^{3/2} U^{3}}{2 \rho^{1/2} A_x^{1/2} \sigma^{1/2}}$$

$$\sigma^{1/2} = \frac{\phi}{\sqrt{2\varphi}} \qquad \frac{\phi^2}{2\varphi}$$

$$= \frac{\sqrt{2}\,\varphi^{3/4} \phi^{3/2} \rho A_x U^3}{2^{1/4} \phi}$$

$$P = \frac{\sqrt{2\varphi . T^3}}{\sqrt{4\rho A_x \phi}}$$

$$= \frac{2^{3/4} \varphi^{5/4} \rho A_x U^3 \phi^{1/2}}{\cancel{\phi}}$$

$$= \sqrt{\frac{\varphi . T^3}{\rho \phi \cancel{(r_c + r_h)} \pi (r_c^2 - r_h^2)}}$$

$$= \sqrt{\sqrt{2^3 \varphi^5} \cdot \phi} \cdot \rho A_x U^3$$

$$= \sqrt{\frac{\varphi T^3}{2\rho \phi \pi (r_c^2 - r_h^2)}}$$

$$= \sqrt{\sqrt{2^3 \varphi^5} \cdot \phi} \cdot \rho A_x U^3 = \sqrt{\sqrt{2^3 \varphi^5}\,\phi} \cdot \rho \pi (r_c^2 - r_h^2) \frac{\Omega^3}{8} (r_c + r_h)^3$$

4A3 Turbomachinery - Turbine Cascade

1.1 Free-stream $\Delta\rho$

$$p_2 - p_1 = (p_{01} - p_1) - (p_{01} - p_{02}) - (p_{02} - p_2)$$

$$\approx -200 \text{ mm } H_2O$$

$$\frac{p_1}{\rho_1^\gamma} = \frac{p_2}{\rho_2^\gamma}$$

$$\frac{p_1}{p_2} = \left(\frac{\rho_1}{\rho_2}\right)^\gamma$$

$p_2 = p_{atm} = \cancel{100126}\ 100\,130$

$|p_2 - p_1| = 200 \times 10^{-3} (1000)(9.81)$

$= 200 (9.81)$ Pa

$\therefore p_1 = 100130 + 200(9.81)$

$= 102\,092$

$\therefore \frac{p_1}{p_2} = 1.0196$

$\gamma = 1.4$ # ~~at~~ Air, uncombusted

$\therefore \frac{\rho_1}{\rho_2} = 1.0140$

$\therefore$ change in $\rho \sim 1\%$

In the context of turbomachinery, 1% changes should NOT be neglected.

$$\cancel{\rho A}\, \rho A_x V_x = \rho A_4 V_4 \qquad A_x = \frac{A_4}{\sigma} \quad \therefore$$

σA_x

$$\frac{A_x}{A_4} = \frac{V_4}{V_x} = \frac{1}{\sigma} \quad \therefore \quad \frac{V_x}{V_4} = \sigma$$

$$T_T = \frac{\rho A_x V_x^2}{\sigma^2}$$

$$\therefore V_x^2 = \sqrt{\frac{T_T \sigma^2}{\rho A_x}}$$

$$p_3 + \tfrac{1}{2}\rho V_x^2 = p_4 + \tfrac{1}{2}\rho V_4^2 \quad p_4 \to p_a$$

$$\cancel{p_a + \tfrac{1}{2}\rho\left(V_x^2 - \frac{V_x^2}{\sigma^2}\right)} \quad p_3 = p_4 + \tfrac{1}{2}\rho\left(\frac{V_x^2}{\sigma^2} - V_x^2\right)$$

$$\Delta p_0 = p_3 + \tfrac{1}{2}\rho V_x^2 - p_a$$

$$= \left[\cancel{p_a} + \tfrac{1}{2}\rho\left(\frac{V_x^2}{\sigma^2} - \cancel{V_x^2}\right)\right] + \tfrac{1}{2}\rho \cancel{V_x^2} - \cancel{p_a}$$

$$= \frac{\rho V_x^2}{2\sigma^2}$$

$$\varphi = \frac{\Delta p_0}{\rho U_0^2} = \frac{V_x^2}{2U^2\sigma^2}$$

$$\sigma^2 = \frac{\phi^2 \rho A_x U^2}{T_T} \qquad \sigma^{-2} = \frac{T_T}{2\rho A_x U^2 \varphi}$$

$$2\rho^2 A_x^2 U^4 \phi^2 \varphi = T_T^2$$

$$\rho A_x \phi U^2 \sqrt{2\varphi} = T_T$$

$$\phi^2 = \frac{T_T \sigma^{-2}}{\rho A_x U^2} \qquad \varphi = \frac{T_T}{2\rho A_x U^2 \sigma^{-2}} \qquad 2\varphi\sigma^{-4} = \phi^2$$

$$\sigma^{-4} = \left(\frac{\phi^2}{2\varphi}\right)$$

$$\frac{\varphi}{\phi^2} = \frac{1}{\sigma^4}$$

$$\sigma^4 = \frac{\phi^2}{\varphi} \qquad \sigma = \sqrt{\frac{\phi}{\sqrt{\varphi}}} = \left(\frac{\phi^2}{\varphi}\right)^{1/4}$$

$$\phi^2 = \frac{T_T 6^2}{\rho A_x V^2} \qquad \psi = \frac{T_T}{2\rho A_x V^2}$$

$$\phi^2 = \frac{2\psi}{\cancel{M}} \frac{6^2}{V^2} V^2 \qquad \frac{T_T}{\rho A_x} = \psi 2\rho V^2 = \frac{\phi^2 V^2}{6^2}$$

$$6^2 = \frac{\phi^2 \cancel{M}}{2\psi}$$

$$T_T = \frac{\phi^2 \rho A_x V^2}{6^2}$$

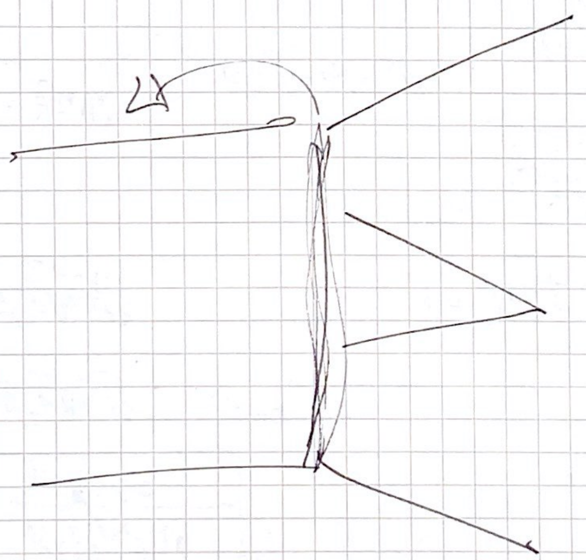
$$\phi^2 = \frac{T_T}{\rho A_x U^2} \cdot \frac{\phi^2}{2\psi}$$

$$T_T = 2\psi \rho A_x U^2 \qquad \phi \quad 6$$

$$T_T = \frac{\phi^2 \rho A_x U^2}{6^2}$$

sigh

faigh



SAM'S EQUATIONS

$$P_i = \frac{T_{TOT}^{3/2}}{\sqrt{4\sigma}\,\rho A_x}$$

$$T_T = \frac{\phi^2 \rho A_x U^2}{\sigma^2} = \underbrace{\phi^2 \rho A_x U^2}\,.\,\sigma^{-2}$$

$$kg\,m^{-3}\,m^2\,m^2 s^{-2} = kg\,m\,s^{-2}$$

$$= \left[\frac{\phi^6 \rho^{\cancel{3}2} A_x^{\cancel{3}2} U^6}{4\cancel{\sigma}\,\cancel{\rho}\,\cancel{A_x}}\right]^{1/2}$$

$$\frac{(kg\,m\,s^{-2})^{3/2}}{(kg\,m^{-3}\,m^2)^{1/2}} = kg\,m^2 s^{-3}$$

$$= \frac{\phi^3 \rho A_x U^3}{2\sqrt{\sigma}}\,.\,\sigma^{-3}$$

$$= \frac{\phi^3 \rho A_x U^3}{2\sigma^{7/2}}$$

$$\therefore P_i = \frac{T_T\,.\,\phi\,\cancel{U}}{2\sigma^{3/2}}$$

$$= \frac{T_T\,.\,\phi U}{2\sigma^{3/2}}$$

$$P_f = \tfrac{1}{2}\dot{m}V_4^2$$

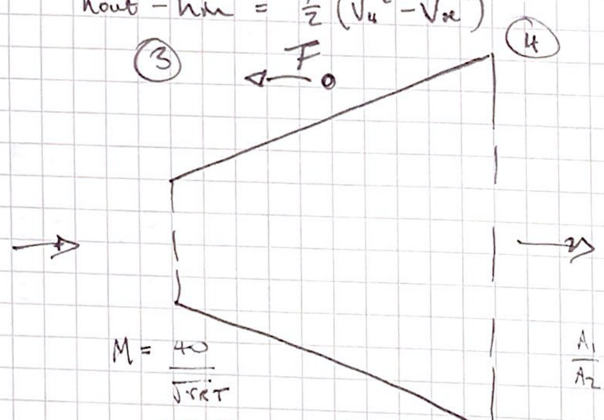
$$\phi^3 \rho^{3/2} A_x^{3/2} U^3$$

$$M_c = \frac{T}{P}\sqrt{\frac{T}{2\rho A_x}}$$

$$M_f = \cancel{\sigma^{3/2}}\sqrt{\frac{2A_x\sigma}{A_x}} = \sqrt{2\sigma}$$

$$= \frac{2\sigma^{3/2}}{\phi U}\sqrt{\frac{\phi^2 A_x U^2}{2\sigma^{-2}}} = \sqrt{2}A_x\sigma^{1/2}$$

$$h_{out} - h_{in} = \tfrac{1}{2}(V_4^2 - V_x^2)$$



~~Energy:~~

Force/mom

$$F + \dot{m}V_4 = \dot{m}V_3$$

Mass

$$\rho A_3 V_3 = \rho A_4 V_4$$

Energy

$$\tfrac{1}{2}\dot{m}V_3^2 = \tfrac{1}{2}\dot{m}V_4^2 + \int \frac{F}{L} V\,dx$$

$$M = \frac{40}{\sqrt{\gamma R T}} = 0.12$$

adiabatic ⇒ T_0 = const

isentropic ⇒ p_0 = const

$$\frac{A_1}{A_2} = 2$$

$$\frac{\dot{m}\sqrt{c_pT_0}}{Ap_0} = 0.2633$$

$$T = 293 \qquad \frac{T}{T_0} = 0.9971$$

$$T_0 = 293.85$$

$$\frac{\dot{m}\sqrt{c_pT_{02}}}{A_2p_{02}} = \frac{\dot{m}\sqrt{c_pT_{01}}}{A_1p_{01}}\cdot\frac{A_1}{A_2} = 0.316$$

$$M = 0.06$$

$$\frac{T}{T_0} = 0.9993$$

$$= \frac{\cancel{293.9}}{293.65}$$

$$V(x) = V_3 - \frac{(V_3 - V_4)}{L}x$$

$$= V_3 - (V_3 - V_4)\,.\,\frac{x}{L}$$

$$V_3\left(1 - \left(1 - \frac{1}{\sigma}\right)\frac{x}{L}\right)$$

$$\frac{L}{V_3 F}\therefore \tfrac{1}{2}\dot{m}(V_3^2 - V_4^2) = \left[x - \left(1 - \frac{1}{\sigma}\right)\frac{x^2}{2L}\right]_0^L$$

$$= L - \left(1 - \frac{1}{\sigma}\right)\frac{L}{2}$$

$$= L\left(\frac{1}{2} + \frac{1}{2\sigma}\right) = \frac{L}{2\sigma}(\sigma + 1)$$

$$\therefore \frac{\dot{m}(V_3^2 - V_4^2)}{2V_3 F} = \left(\frac{\sigma + 1}{2\sigma}\right)$$

$$\frac{\dot m\,(V_3^2 - V_4^2)}{2V_3F} = \frac{\sigma+1}{2\sigma}$$

$$\frac{\dot m}{2F}\left(V_3 - \frac{V_3}{\sigma^2}\right) = \frac{\sigma+1}{2\sigma}$$

$$\frac{\dot m V_3}{2\sigma F}(\sigma - 1) = \frac{\sigma+1}{2\sigma}$$

$$\frac{\dot m V_3}{F} = \frac{\sigma+1}{\sigma-1} \qquad \therefore \sigma > 1$$

$$F + \dot m V_4 = \dot m V_3$$

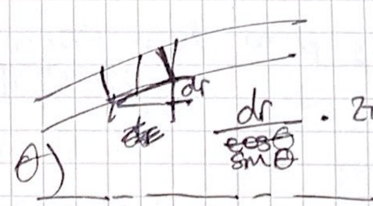
$$F + \dot m\left(\frac{V_3}{\sigma} - V_3\right) = 0$$

$$F + \frac{\dot m V_3}{\sigma}(1-\sigma) = 0$$

$$F = \frac{\dot m V_3}{\sigma}(\sigma - 1)$$

∴ More diffusion, more resistive force on propulsor from flow deceleration

Now assuming $p_4 = p_a$



$$\frac{dr}{\sin\theta}\cdot 2\pi r = dA$$

$$\text{Total Pressure force} = \int p\,dA$$

$$= \int_{r_h}^{r_e} p(r)\cdot\frac{2\pi r}{\sin\theta}\cdot dr$$

$$p_4 + \tfrac{1}{2}\rho V_4^2 = p_r + \tfrac{1}{2}\rho V_r^2$$

$$\rho A_r V_r = \rho A_4 V_4$$

$$V_r = \frac{A_4 V_4}{A_r}$$

$$A_r =$$

QUESTIONS FOR SAM

- Having corrected the σ issue I've obtained an expression for thrust. which has correct dimensions.

$$\phi^3 = (2\varphi\sigma^{-2})^{3/2}$$
$$= (2\varphi)^{3/2}\sigma^{3}$$
$$P = \frac{(2\varphi)^{3/2}\sigma^{-3}\rho A x U^3}{2\sigma^{3/2}}$$
$$= \frac{(2\varphi)^{3/2}\rho A x U^3}{2\sqrt{\sigma}}$$
$$= \frac{\sqrt{2\varphi^3}\cdot\sqrt{2\varphi}}{\phi}\rho A x U^3 = \frac{2\varphi^3\rho A x}{\phi}$$

$$\sigma^2 = \frac{\phi^2}{2\varphi}$$

→ Tip velocity limit wrt Stress →

→ What geometry & features drive figure of merit.

"Reasons its happening is A B C

Don't want to go there because

→ Efficiency changes across map and I've assumed $\eta = 0.9$.

→ Flip to Smith chart explain why not in bottom right

→ Ideal prop $M_f = 1$

→ Ideal fan $M_f = \sqrt{2}$ // FOR $\sigma = 1$

we are $\sigma > 1$ ∴ M_f can be greater.

→ Weight UROP chat towards 4th Yr & UROP / Proj.

UROP	Proj.
Build-drive-cage	- Instrument - Get flying - Get data.

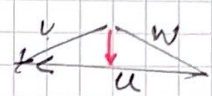
→ Presentation

→ Mass model, outline EXACTLY what the set-up looks like.

Replace V_0 with V_m

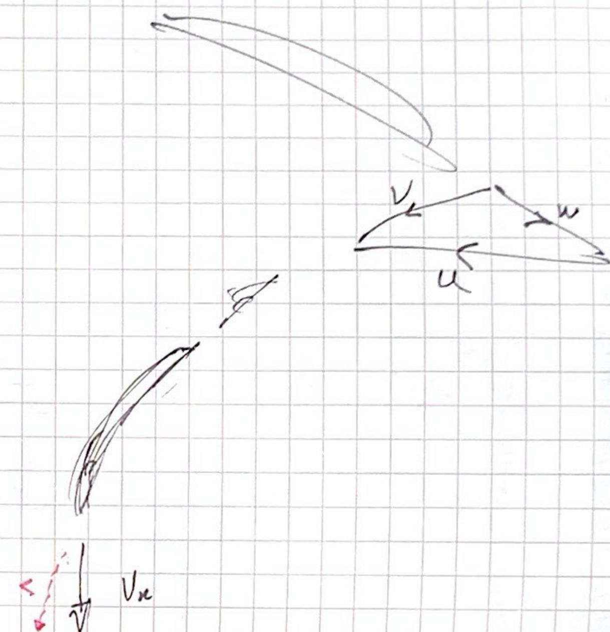
2. Leave out $(\cdots)$

3. Consider exit swirl



$V_3 \cos\alpha_3 = V_x$

Maybe talk later about how exit deviation effects thrust re how well are stators working.



Mass Model

→ Solid cone?

(19) look at how payload changes

$\tilde{T}_E$ lie on M_f.

$$\sigma^2 = \frac{\phi^2}{2\psi}$$

$$\sigma^{\frac{1}{2}} = \frac{\phi^{\frac{1}{2}}}{(2\psi)^{\frac{1}{4}}}$$

$$P = \frac{\rho A V_x^3}{2\sigma^2} \qquad \sigma^2 = \frac{\phi^2}{2\psi}$$

$$= \frac{\rho A V_x^3 \cdot 2\psi}{2\phi^2}$$

$$= \frac{\rho A V_x^3 \psi}{\frac{V_x^2}{U^2}} = \rho A V_x \psi U^2$$

$$= \rho A \phi \psi U^3$$

$$\phi^3 = (\sigma^2 \, 2\psi)^{3/2}$$

$$= \sigma^3 (2\psi)^{3/2}$$

$$\frac{\sigma^3}{\sigma^{7/2}} = \sigma^{\frac{6}{2}-\frac{7}{2}} = \sigma^{-\frac{1}{2}} = \frac{1}{\sqrt{\sigma}}$$

$$P = \frac{\phi^3 \rho A U^3}{2\sigma^{7/2}} = \frac{\sigma^3 (2\psi)^{3/2} \rho A U^3}{2\sigma^{7/2}}$$

$$= \frac{(2\psi)^{3/2} \rho A U^3}{2\sqrt{\sigma}}$$

$$= \frac{\sqrt{2}\, \psi^{3/2} \rho A U^3}{\sqrt{\sigma}}$$

$$= \frac{2\psi^2 \rho A U^3}{2\phi}$$

$\dot m = \rho A_x^e V_x$

$\rho A_x \sigma V_{\infty}$

$\dot m$?

$\frac{\rho A_x V_x V_x}{\rho A_x c V_x^2}$

$$V_x = \sqrt{\frac{T_T\,\sigma}{\rho A_x}}$$

$$\phi = \sqrt{\frac{T_T\,\sigma}{\rho A U^2}} \qquad \varphi = \frac{T_T}{2\rho A_x U^2 \sigma} \qquad \frac{\phi^2}{\sigma} = \frac{\phi^2\sqrt{2\varphi}}{\phi} = \phi\sqrt{2\varphi}$$

$$T_T = \frac{\phi^2 \rho A U^2}{\sigma} \quad (12)$$

$$\frac{T_T}{\rho A U^2} = \frac{\phi^2}{\sigma} \qquad 2\varphi\sigma = \frac{T_T}{\rho A_x U^2}$$

$$2\varphi\sigma = \frac{\phi^2}{\sigma}$$

$$\sigma^2 = \frac{\phi^2}{2\varphi} \qquad M_f = \sqrt{2\sigma}$$

$$P = \frac{\rho A_x V_x^3}{2\sigma^2} \qquad V_x^3 = \frac{T_T^{3/2}\,\sigma^{3/2}}{\rho^{3/2}\,A_x^{3/2}}$$

$$P = \frac{\rho A_x}{2\sigma^2} \cdot \frac{T_T^{3/2}\,\sigma^{3/2}}{\rho^{3/2} A_x^{3/2}}$$

$$= \frac{T_T^{3/2}}{\sqrt{4\sigma\rho A_x}}$$

$$= \frac{\phi^3 \rho^{3/2} A_x^{3/2} U^3}{2\sigma^{3/2}\sigma^{1/2}\rho^{1/2} A_x^{1/2}}$$

$$= \frac{\phi^3 \rho A_x U^3}{2\sigma^2}$$

$$T_T = \frac{\phi^2 \rho A_x U^2}{\sigma}$$

$$0.4 = \frac{\Delta P_0}{\frac{1}{2}\rho U^2}$$

$$\frac{1}{2}\,0.4 = \frac{\Delta P_0}{\rho U^2}$$

$$P\,\frac{U^3}{U^3} = \text{same}$$

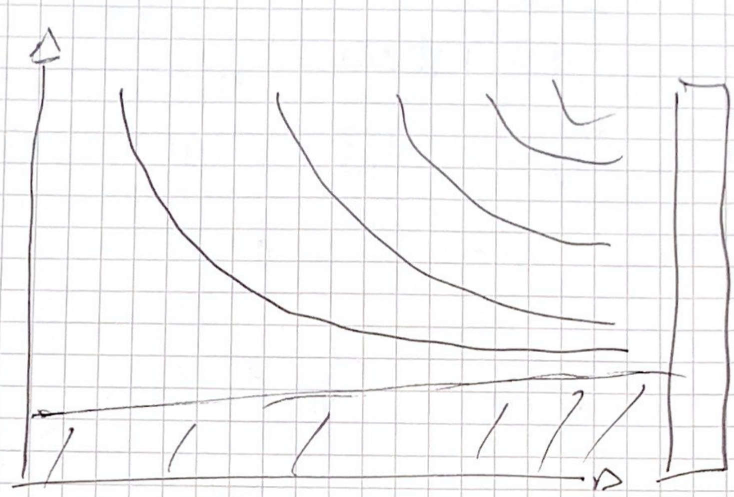
$$T_T = \phi\sqrt{2\varphi}\,\rho A_x U^2$$

$$P = \rho A U^3 \phi\varphi$$

Ask about $\varphi = \frac{\Delta P_0}{\frac{1}{2}\rho U^2}$ vs $\frac{\Delta P_0}{\rho U^2}$ → Have 3 plots each at a different speed. How does it effect payload

Plots range y 0:0.5

x 0:1



$P = \quad T_T \sim M$

$T_J = M$

M_p

- Make rotor lighter
- Reduce AR a bit.
- Be more generous with fillets.
- Change connector to motor.

- Get flying, get measurements.

F1 Jobs !
Apply?

- What does it look like?
- Bar chart of component weights
- Thrust vs RPM at design
- RPM required for Thrust = ~~Ma~~ Weight?
- Max payload for theoretical max RPM?
- Thrust vs Power at design
- Power required for Thrust = weight
- Max payload at max power?
- Sensitivity with efficiency.