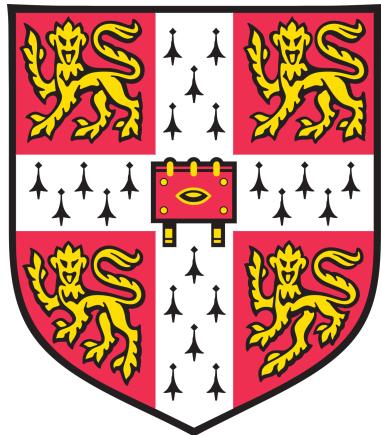


UNIVERSITY OF CAMBRIDGE
DEPARTMENT OF ENGINEERING

MASTERS PROJECT REPORT



Propulsion Systems for [small?] VTOL Electric Vehicles

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Abstract

Abstract here..

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Electric Vertical Take-Off and Landing	e-VTOL
Flying Test-Bed	FTB
Axial Flow Velocity	V_x
Fan Speed	Ω
Fan Hub Radius	r_h
Fan Casing Radius	r_c
Fan Mid-line Radius	r_m
Mid-line Fan Speed	$U = \Omega \cdot r_m$
Flow Coefficient	$\phi = \frac{V_x}{U}$
Stage Loading	$\psi = \frac{\Delta h_0}{U^2}$

Nomenclature

e-VTOL	Test
FTB	Test
V_x	Axial flow velocity
Ω	
r_h	
r_c	
r_m	
U	
ϕ	Flow coefficient
ψ	Stage loading

Introduction

Intro and motivation

Consider Heilmeier when structuring this section.

- Integrate battery into casing, increase payload volume/utilise space

1.1 Literature Review

1.2 Research Questions

1.3 Approach

High level overview of what's been done and how the report is set out.

Flying Test Bed and Propulsor Design

2.1 System Design

2.1.1 UROP – Flying vehicle development

A UROP was conducted in the summer preceding this project during which an axi-symmetric quadcopter flying test bed was designed and tested. The two principal design problems were structural design of the chassis, and the customisation of flight control hardware and software. The UROP concluded with successful hover testing of the flying test bed. Off-the-shelf 10in. propellers were used in the test.

Mechanical design

The test bed was designed to be rapidly 3D printable, light – whilst maintaining structural integrity – and modular to allow various propulsors and instrumentation to be mounted. Instrumentation management and flight hardware are mounted on a central chassis that's

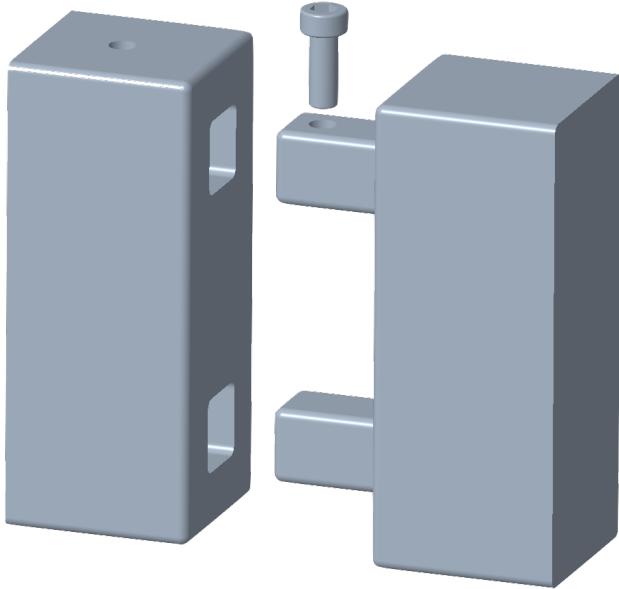


Figure 1: Exploded view of a double mortise and tenon join used to assemble modular drone chassis.

3D printed as one piece for rigidity and to reduce complexity. Cantilevered propulsor arms are attached to the central chassis with a double mortise and tenon style mount, fastened by one M3 cap-head as in fig. 1. Modularity allows for design iteration and rapid part replacement if required.

The following hardware is mounted directly onto the central chassis:

- ***Pixhawk 4 Flight Control hardware*** – Packaged, customizable autopilot hardware
- ***PM07 Power Module*** – Power module managing distribution of power from battery/power supply to flight controllers, other on-board computers, and motors
- ***Raspberry Pi 3 Model A+*** – On-board computer managing data acquisition and assisting flight controller automation
- ***ADCPi ADC Breakout*** – Analogue to digital converter mounted directly to the Raspberry Pi 3
- ***4x Aerostar 50A ESC*** – Electronic Speed Controller driving BLDCM motors
- ***Optional: Turnigy 5000mAh 14.8V (4-cell) Battery*** – Large capacity 14.8V (nominal) battery. Can be replaced by a tethered 12V power supply.

The central chassis has dimensions of approximately $125mm \times 139mm \times 93mm$ (see fig. 9). It has a dry mass (excluding battery) of $0.529kg$ and the battery has a mass of $0.466kg$.

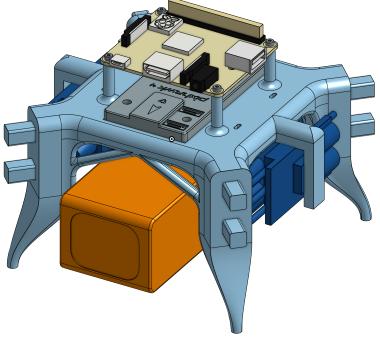


Figure 2: Render of flying test bed chassis
(propulsor arms and mounting points not shown)

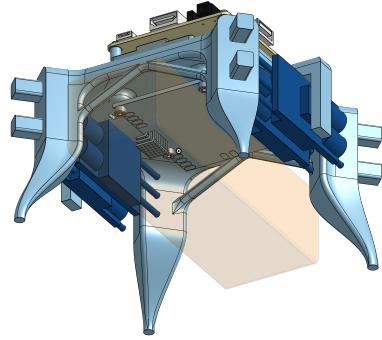


Figure 3: Underside view with battery transparent showing PM07 mounting

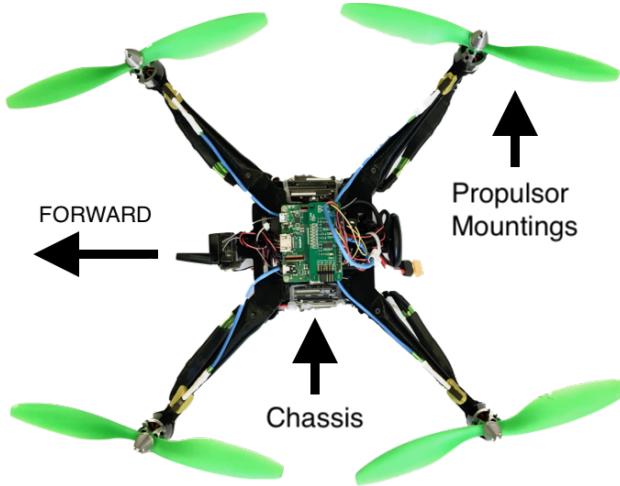


Figure 4: Image of flying test bed with propellers attached

Customisation of *Pixhawk 4* flight hardware/software

Pixhawk 4 is a packaged flight control hardware system running open source PX4 firmware. The system can be interacted with via the GUI QGroundControl and either a wireless telemetry transmitter-receiver pair or a serial connection (such as a USB). QGroundControl allows a predetermined automated flight path to be followed or alternatively the hardware can be configured to allow manual control provided adequate control inputs can be provided (a 16-channel remote with an 8-channel PPM-PWM converter is used here). The PM4 firmware includes various airframe configuration profiles, enabling it to adapt to the vehicle configuration and motor layout. The flying test bed is configured as a symmetric 'X' quadcopter. The profile was tuned to the flying test bed by varying PID gains on the control software, ensuring stable operation in hover. It provides a stable hover mode that is intended to reduce horizontal position deviation and maintain a fixed altitude. This means only small correc-

tive inputs are required from the operator to ensure near stationary hover even in enclosed environments. It was found however that wall effects can be significant and so care must be taken when operating in such environments.

2.1.2 Subsystem Functions

Raspberry Pi 3 Model A+ (RPi3)

The RPi3 is a light-weight, small-footprint, 64-bit quad-core processor with UNIX-based operating system and Wi-Fi connectivity that can be run ‘blind’ through an SSH connection from a remote machine. It provides data acquisition and on-board data processing in Python and is used to pre-process test data before wirelessly transferring it back to the remote machine. RPi3 has 28 general purpose input/output (GPIO) pins as well as I²C and serial interfaces among others.

ADCPi ADC Breakout

ADC breakout board designed specifically for the RPi3. Operating through the I²C interface, each breakout board provides an 8-channel ADC at 11 to 17 bit resolution with sample rates of 240 to 3.75 *samples/sec* respectively.

PM07 Power Module

Designed to operate in tandem with the Pixhawk 4 flight controller, the PM07 provides power control for all the high-power outputs (such as the motors) as well as a regulated 5V output for the flight controller and RPi3. Power input is 7-51V DC and the output current is limited to 120A (approximately 30A per propulsor). The battery selected can operate up to 125A to accommodate this demand. PM07 also provides 2 ADC inputs.

2.1.3 Subsystem Interaction

Pixhawk 4 provides simple subsystem interaction through its input ports. 5 ports connect the flight controller to the power module: 2x power inputs, 1x motor PWM outputs, 1x auxiliary PWM output, and 1x ADC input. The serial port on the Pixhawk 4 is rewired to connect to the RPi3’s UART TXD and RXD terminals (figure 8 shows all connections made to the RPi3 header). The I²C port connects to an I²C bus that allows up to 5 devices to interface with the Pixhawk 4, including 3 ultrasound sensors measuring proximity in the x, y, and z directions. These sensors can optionally be connected via the ADC for use by the RPi3. Figure 5 shows schematically how the subsystems interact.

2.1.4 Telemetry

Pixhawk 4 wireless telemetry

Essentially plug-and-play telemetry module allows wireless connection to the Pixhawk 4 from a ground control station on a remote machine such as QGroundControl. This is used to communicate with the Pixhawk 4 about flight control and to log power usage data (PM07

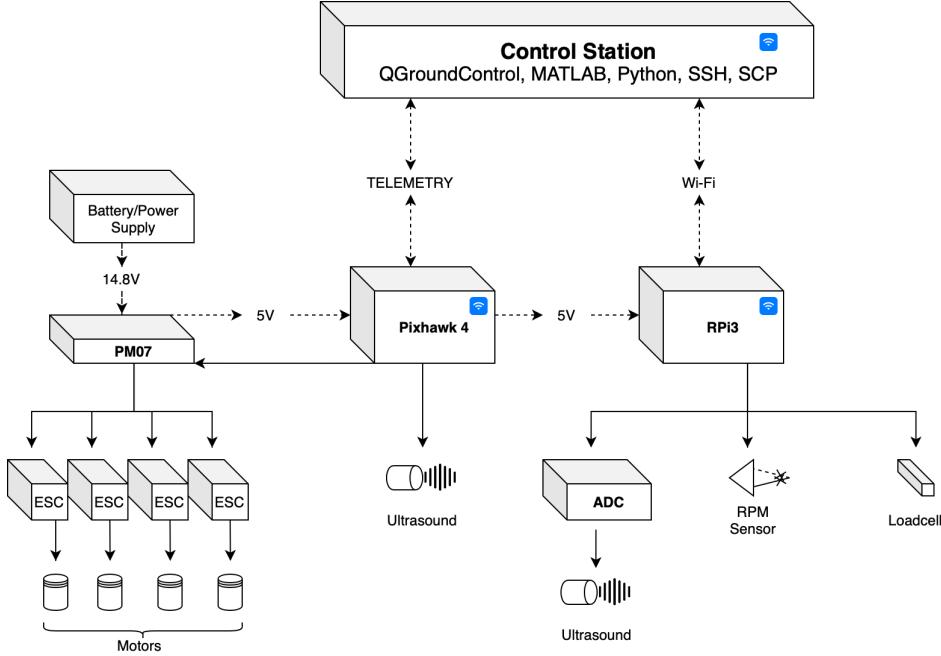


Figure 5: Subsystem schematic of flying test bed

voltage and current output).

SSH/SCP on RPi3

RPi3 is wireless enabled allowing it to be accessed remotely via the SSH (secure shell) protocol. SCP (secure copy protocol) allows files to be copied to and from the RPi3. This is used after data collection to collect all data on the remote machine. A dedicated wireless router is used to connect the RPi3 to a remote machine – done automatically on start-up of the RPi3.

2.2 Aerodynamic Design

2.2.1 Figure of Merit and Design Point

A low-speed (incompressible) ducted fan in hover fundamentally differs from both a propeller and a ducted fan in high-speed free stream velocity. A propellers exit static pressure is approximately equal to atmospheric, however a low-speed ducted fan can vary its exit pressure by varying the diffusion factor, σ , defined as the ratio of exit flow area (A_e) and blade passage flow area (A_x), as in eqn. 1.

$$\sigma = \frac{A_e}{A_x} \quad (1)$$

As the exit static pressure must be approximately atmospheric, assuming the diffuser flow remains attached variation in σ will result in a variation in p_x , the blade passage static

pressure.

A ducted fan in non-zero free stream velocity is required to reduce frontal area in order to decrease aerodynamic drag, however a ducted fan in static hover is not. This means the addition of a diffuser to reduce exit velocity – increasing efficiency – does not come at the cost of drag.

Figure of Merit:¹ The effectiveness of a propulsor in hover can be measured by the figure of merit, M_F , defined as a non-dimensional thrust-to-power ratio (eqn. 2).

$$M_F = \frac{T}{P} \sqrt{\frac{T}{2\rho A_x}} \quad (2)$$

Considering a large control volume ($V_{in} \approx 0$) around a simple ducted fan, treating the blade plane as a stagnation pressure actuator disk and the exit static pressure as atmospheric allows the theoretical thrust to be determined. This can then be written in terms of the blade passage quantities (A_x , V_x etc) and the diffusion factor, σ , using continuity.

$$T = \dot{m}V_e = \rho A_e V_e^2 \quad \text{by continuity...} \quad \therefore T = \frac{\rho A_x V_x^2}{\sigma} \quad (3)$$

Theoretical power requirement can be determined by the rate of transfer of kinetic energy to the flow.

$$P = \frac{1}{2}\dot{m}V_e^2 \quad \text{by continuity...} \quad \therefore P = \frac{\rho A_x V_x^3}{2\sigma^2} \quad (4)$$

Substituting eqn. 3 and eqn. 4 into eqn. 2 gives the result in eqn. 5.

$$M_F = \sqrt{2\sigma} \quad (5)$$

Hence to maximise hover performance the diffusion factor must be maximised. This necessitates a high performing diffuser with a minimised mass. Due to the negative effects of an adverse pressure gradient on flow attachment (and effective flow area) in a diffuser, the required length of a diffuser flow that remains attached will increase non-linearly with diffusion factor². This trade-off between performance and weight limits the design choice of diffusion factor to avoid prohibitively heavy propulsors. Section 2.2.2 discusses this problem. This result also shows that a diffusion factor of 1, that is to say with no diffusion downstream of the blades, the theoretical figure of merit is $M_F = \sqrt{2} \approx 1.414$. Periera 2008³ shows the maximum theoretical figure of merit for a propeller to be 1, therefore provided the diffusion factor remains above 0.5, fan figure of merit should be greater than propeller performance in hover.

¹REFERENCE PERIERA

²REFERENCE ESDU

³REFERENCE PERIERA

Non-dimensional Operating Point: The design of both the rotor and stator fan blades can be determined by the choice of non-dimensional operating point given by flow coefficient (eqn. 6(a)) and stage loading (eqn. 6(b)) where the mid-line blade velocity is $U = \Omega \cdot r_m$ with r_m equal to the mid-line radius.

$$\phi = \frac{V_x}{U} \quad (a) \qquad \psi = \frac{\Delta h_0}{U^2} \quad (b) \quad (6)$$

Assuming isentropic flow, stage loading can be written in terms of the stagnation pressure rise, Δp_0 . Using Bernoulli and assuming exit static pressure is equal to atmospheric therefore gives an expression for Δh_0 in terms of the exit velocity V_e .

$$\begin{aligned} \Delta p_0 &= (p_0)_e - (p_0)_{in} = (p_e + \frac{1}{2}\rho V_e^2) - p_a \\ \Delta p_0 &= \frac{1}{2}\rho V_e^2 \\ \therefore \Delta h_0 &= \frac{\Delta p_0}{\rho} = \frac{1}{2}V_e^2 = \frac{V_x^2}{2\sigma^2} \end{aligned} \quad (7)$$

Therefore ϕ , ψ , and σ are determined by eqn. 8

$$\sigma^2 = \frac{\phi^2}{2\psi} \quad (8)$$

The choice of operating point therefore sets the diffusion factor and consequently the theoretical figure of merit of the propulsor. Section 2.2.2 considers the interaction between the operating point and geometric variables, and the subsequent impact of required performance, whilst section 2.2.3 considers the embodiment of the design point into 3D rotor and stator blades.

Estimation of Operating Fan Speed: Equation 5 shows the effectiveness of a ducted fan in hover is independent of the speed of rotation of the fan. The fan speed however does impact the magnitude of the thrust developed, shown by substituting eqn. 6(a) into eqn. 3.

$$T = \frac{\rho A_x (\Omega r_m \phi_m)^2}{\sigma} \quad (9)$$

Equation 9 shows thrust to vary with both fan speed and fan size, and so the required thrust will vary depending on the choice of these parameters. The effect of fan size is discussed in section 2.2.2.

*****NOTES ON SECTION*****

ESDU diffusion tables?

2.2.2 Mid-line Blade Design

***** NOTES *****

Add preamble about general configuration of propulsor and labelled diagram with key variables as in table 1??

Mid-line Location: Maximising the flow area for a given propulsor radius ensures minimum use of material and therefore a lighter design. This results in a small hub radius ($20mm$) and consequently very low hub-to-tip ratios compared to conventional compressors. As a result, selecting a mid-line at the mean radius introduces large differences in mass flow above and below the mid-line. This results in large variations in loading across the span. The alternative mid-line location is based on balancing the mass flow above and below the mid-line. This sets the mid-line radius at the root-mean-square off the hub and tip radii, as in eqn. 10.

$$r_m = \sqrt{\frac{r_h^2 + r_c^2}{2}} \quad (10)$$

Overall Performance: As shown in section 2.2.1 the overall characteristics such as thrust and power can be determined from the fan geometry and the mid-line non-dimensional quantities ϕ_m and ψ_m . Equation 5 suggested a larger diffusion factor will increase performance, however due to the difficulty of diffusing flow, the length of the diffuser required to enable large diffusion factors can be prohibitive. Further, on transition from static hover to horizontal flight a long diffuser will generate considerable drag due to its large flow facing area. A problem therefore arises when determining the required thrust. For steady hover, thrust must equal the weight of the vehicle. However as the design thrust varies, as does the weight, creating a circular problem. In order to solve this a mass model is required that, given the propulsors operating point, estimates it's weight.

The variables required to define both the weight and performance of a ducted fan are outlined in table 1.

Table 1: Variables defining propulsor weight and performance

r_c	Blade passage casing radius
r_h	Blade passage hub radius
Ω	Rotor angular velocity
σ	Diffusion factor
ϕ_m	Mid-line flow coefficient
ψ_m	Mid-line stage loading

A notable exclusion from this list is the length of the diffuser, however the lower limit of this value is estimated from ESDU 75026:Figure 3⁴ which gives empirical limits of diffuser

⁴REFERENCE ESDU TABLES HERE

lengths for symmetric annular diffusers. Therefore the length of the diffuser is determined by correlating the appropriate values from r_c , r_h , and σ to the ESDU data.

Equations 3 and 4 can be expressed in terms of the variables in tab. 1 as follows

$$T = \frac{\rho\pi\phi_m^2\Omega^2(r_c^4 - r_h^4)}{2\sigma} \quad (11)$$

$$P = \rho\pi\phi\psi\Omega^3(r_c^2 - r_h^2)\left(\frac{r_c^2 + r_h^2}{2}\right)^{\frac{3}{2}} \quad (12)$$

Equation 11 can be equated to the vehicle weight to determine the variables required for hover. A mass model is developed to enable this.

ESDU 75026 Diffuser Data: ESDU 75026 presents the performance of circular annular diffusers in incompressible flow.

$$C_{pr} = \frac{p_e - p_x}{\frac{1}{2}\rho V_x^2} \quad (13)$$

ESDU 75026:Figure 3 shows the line of C_{pr}^{**} representing the length ratio at which C_{pr} (eqn. 13) is maximised for a given area ratio. This allows a minimum diffuser length to be determined for a given area ratio and upstream span ($r_c - r_h$). This line can be approximated with an R-squared value of 0.9999 by

$$\frac{L}{r_c - r_h} = -5.56\sigma^3 + 24.16\sigma^2 - 23.41\sigma + 5.413 \quad (14)$$

Mass Model – Diffuser: The diffuser hub and casing are modelled as a hollow truncated cone 3D printed in PLA with height L (determined by eqn. 14), a base radius r_h (for the hub) and $(r_c)_{EXIT}$ (for the casing), a truncation radius of $(r_h)_{EXIT}$ (for the hub) and r_c for the casing, and a thickness t , as in fig. 6. Equation 15 gives the volume of a hollow truncated cone with the respective values of R , r , S , and s for the hub and casing defined in tab. 2.

$$V = \frac{h\pi}{3}(R^2 + Rr + r^2 - S^2 - Ss - s^2) \quad (15)$$

$$m_{DIFF.} = \rho_{PLA}(V_{HUB} + V_{CASE}) \quad (16)$$

From the ESDU data, a annulus flow with uniform axial velocity is diffused in the shortest distance if the diffuser is symmetric such that the diffuser angle at casing and hub is equal giving

$$\frac{(r_c)_{EXIT} - r_c}{L} = \frac{r_h - (r_h)_{EXIT}}{L} \quad (17)$$

Applying eqn. 17 and expanding eqn. 1 in terms of radius, the diffuser exit hub and casing radii can be determined.

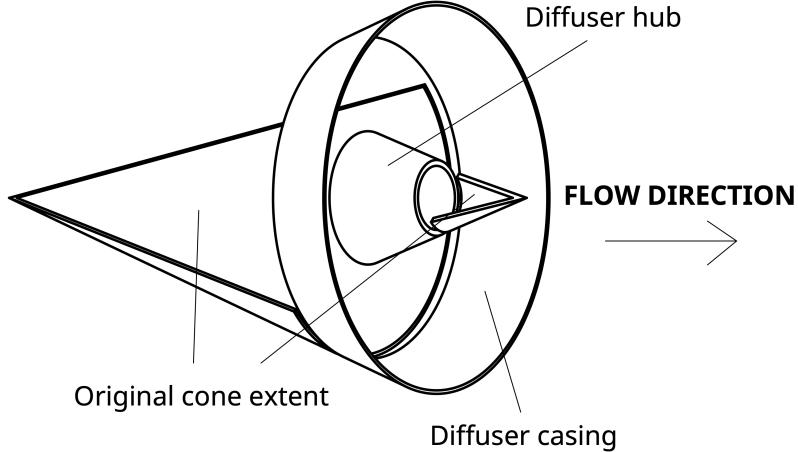


Figure 6: Hollow truncated cone diffuser model showing original extent of cone before truncation

	Hub value	Casing value
R	r_h	$(r_c)_{EXIT} + t$
r	$(r_h)_{EXIT}$	$r_c + t$
S	$r_h - t$	$(r_c)_{EXIT}$
s	$(r_h)_{EXIT} - t$	r_c
h	L	L

Table 2: Parameter values for diffuser hub and casing

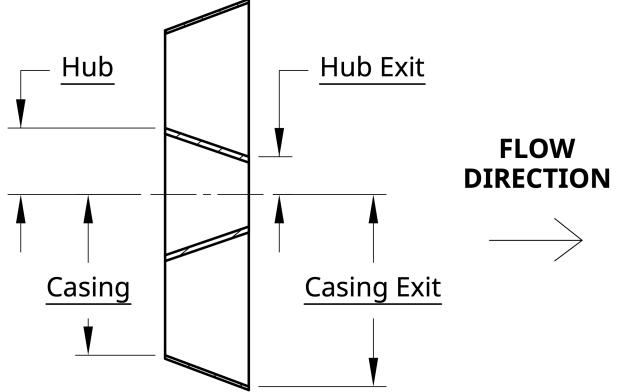


Figure 7: Dimensions of hollow truncated cone diffuser model

$$(r_h)_{EXIT} = \frac{r_c + r_h}{2} - \frac{\sigma}{2}(r_c - r_h) \quad (18)$$

$$(r_c)_{EXIT} = \frac{r_c + r_h}{2} + \frac{\sigma}{2}(r_c - r_h) \quad (19)$$

Substituting eqn. 18 and eqn. 19 into eqn. 15, eqn. 16 can be solved to approximate the mass and therefore the weight of a diffuser at a particular set of design parameters.

Mass Model – Blade Passage: The blade passage (excluding rotor and stator) are modelled as a straight annulus 3D printed in PLA. The passage itself has inner radius r_h and outer radius r_c . The casing is modelled to have thickness of $t_c = 1.5mm$ and the hub section

has a thickness of $t_h = 5mm$. The length of the blade passage is $L_{BP} = 70mm$ giving a volume of

$$V_{BP} = L_{BP}\pi[2r_h t_h - t_h^2 + 2r_c t_c + t_c^2] \quad (20)$$

Multiplying by the density of PLA gives the mass of the blade passage section, which can be written in terms of the unknown variable r_c and r_h

$$m_{BP} = 2.73r_h + 0.818r_c - 0.0062 \quad (21)$$

Mass Model – Intake: For simplicity the intake is also modelled as a hollow truncated cone, an approximation for it's actual ellipsoidal geometry. It has approximately equal length to it's change in radius which is set approximately equal to the blade passage span, $r_c - r_h$. Therefore using eqn. 15, and assuming a thickness of $1mm$, the intake mass can be approximated by

$$m_{IN.} = 3.90(r_c - r_h)(3r_c - r_h + 0.001) \quad (22)$$

Mass Model – Blades: As the number of blades has not yet been determined, the blades are modelled as having the equivalent mass of a annular flat disk of thickness $t_{ROTOR} = 1.5mm$ for the rotor and $t_{STATOR} = 3mm$ for the stator. The resulting mass is given by

$$V_{BLADES} = \pi(r_c^2 - r_h^2)(t_{ROTOR} + t_{STATOR}) \quad (23)$$

$$m_{BLADES} = 17.5(r_c^2 - r_h^2) \quad (24)$$

Thrust = Weight: The mass model described above provides an approximation for the mass of a particular design based on the design parameters listed in tab. 1. In hover, the load carried by each propulsor is equal to the propulsor's weight and its share of the payload weight (4 propulsors). The payload here is the flying test bed chassis and battery, described in section 2.1.1. Using eqn. 11 the thrust developed can be set equal to the required thrust to give the relationship between the design variables when the propulsor is operating in the static hover state.

$$T = g \left[m_{PROPULSOR} + \frac{m_{CHASSIS}}{4} + \frac{m_{BATTERY}}{4} \right] \quad (25)$$

$$\frac{\rho\pi\phi_m^2\Omega^2(r_c^4 - r_h^4)}{2\sigma} = g[m_{DIFF.} + m_{BP} + m_{IN.} + m_{BLADES}] + 2.44N \quad (26)$$

Reduction of Unknown Variables:

In order to solve the design problem, the number of unknowns must be reduced. This is achieved by fixing the propulsor size. The application demonstrated here is for a small quadcopter style vehicle. Furthering work carried out by Jonny 2019⁵ in which the performance of a ducted fan was determined with $M_F > 1$ and $r_c = 120mm$, this design is to be scaled by a half giving

$$r_c = 60mm \quad (27)$$

The value of r_h is determined by the outer diameter of the electric motor used to drive the rotor. This is typically $d < 35mm$, therefore r_h is chosen to be

$$r_h = 20mm \quad (28)$$

Giving a hub-to-tip ratio of 0.333. This leaves diffusion factor, σ , and the rotor angular velocity, Ω . The thrust = weight requirement sets the required value of Ω at each design point, and so this can be used to further aid the choice of design point as it has an impact on the performance of the electric motor. The relationship between the design parameters ϕ , ψ , and σ , as well as the relationship between diffuser length and σ , results in a limit on the design point due to the requirement for diffuser length, $L_{DIFF.}$, to not become prohibitive (as discussed in sec. 2.2.2: *Overall Performance*). This is limited to $L_{DIFF.} \leq 100mm$. Therefore from eqn. 14

$$-5.56\sigma^3 + 24.16\sigma^2 - 23.41\sigma + 5.413 \leq 2.5 \quad (29)$$

$$\therefore \sigma \leq 1.2 \quad (30)$$

It should be noted that this limit on σ arises from the performance of a conventional circular annular diffuser, as presented in ESDU 75026. This limit may be exceeded however through the use of various flow devices, such as splitter vanes. These ideas are discussed further in the future work section (sec. 5) and are not implemented here.

***** NOTES *****

Discuss which variables to fix and define here??

r_h - Minimise, motor diameter limiting factor therefore = 20mm

r_c - Reducing Jonny's design scale by 1/2? Therefore = 60mm

σ - Depends on choice of ϕ and ψ , explore how these values effect performance.

Ω - Use with mass model to guide selection of RPM based on required ϕ and ψ .

Want plots of variation in M_F and variation in Ω at various ϕ and ψ . Discuss electrical efficiency?

⁵Reference this how...?

2.2.3 3D Blade Design

3-dimensional Flow Solutions to Radial Equilibrium: Radial equilibrium states that for axi-symmetric and incompressible annulus flow eqn. 31 must be satisfied.

$$\frac{dh_0}{dr} - T \frac{ds}{dr} = V_x \frac{dV_x}{dr} + \frac{V_\theta}{r} \frac{d(rV_\theta)}{dr} \quad (31)$$

Assuming there is no variation in stagnation enthalpy or entropy across the span, this reduces to

$$V_x \frac{dV_x}{dr} + \frac{V_\theta}{r} \frac{d(rV_\theta)}{dr} = 0 \quad (32)$$

Noting eqn. 6 and applying Euler's work equation, we obtain

$$\phi r \frac{d(\phi r)}{dr} + \psi \frac{d(\psi r^2)}{dr} = 0 \quad (33)$$

Which has general solutions in the form

$$\phi = \phi_m \left(\frac{r}{r_m} \right)^A \quad (a) \qquad \psi = \psi_m \left(\frac{r}{r_m} \right)^B \quad (b) \quad (34)$$

Where ϕ_m and ψ_m are the mid-line quantities. Substituting eqns. 34(a) & 34(b) into eqn. 33 gives⁶

$$\left[\phi_m \left(\frac{r}{r_m} \right)^A \right]^2 (1 + A) = - \left[\psi_m \left(\frac{r}{r_m} \right)^B \right]^2 (2 + B) \\ \therefore \phi^2 (1 + A) = -\psi^2 (2 + B) \quad (35)$$

This solution can be used to determine the values of the exponents, A and B . The constant B is referred to here as the vortex distribution exponent as it determines the tangential velocity variation in the r - θ plane.

Free Vortex: One such distribution arises from the free vortex condition. This results from the solution to eqn. 35 in which both LHS and RHS are equal to zero giving $A = -1$ and $B = -2$. Therefore as $\phi(r) (= V_x(r)/\Omega r) \sim 1/r$, axial velocity across the span must be constant. Similarly it can be shown that as $\psi(r) (= \Delta h_0(r)/\Omega^2 r^2) \sim 1/r^2$, the loading is equal across the span, $\Delta h_0(r) = \text{constant}$, giving a vorticity free (lowest loss) flow⁷. In practice the variation in V_θ across the span results in large variations in blade twist angle that can reduce efficiency.⁸

⁶STICK THE DERIVATION IN AN APPENDIX????

⁷REFERENCE DIXON AND HALL FLUID MECHANICS AND THERMODYNAMICS OF TURBO-MACHINERY, CHAPTER 6.3, P218

⁸AGAIN NEED A REFERENCE FOR THIS?

***** NOTES *****

Reference as to why this isn't always used?

Mixed Vortex/Constant Angle: Another common design is for constant absolute flow angle across the span, such that

$$\frac{d}{dr} \left(\frac{V_\theta}{V_x} \right) = 0 \quad (36)$$

$$\therefore \frac{\psi}{\phi} = \text{constant} \left(= \frac{\psi_m}{\phi_m} \right) \quad (37)$$

This results in the exponents A and B being equal giving

$$A = B = - \left(\frac{2\psi_m^2 + \phi_m^2}{\psi_m^2 + \phi_m^2} \right) \quad (38)$$

Equation 35 can also be used to numerically solve the radial equilibrium equation for any value of the vortex distribution exponent provided ϕ_m and ψ_m are known. A summary of vortex distribution exponent values is shown n tab. 3.

Table 3: Vortex distribution exponents for various boundary conditions

	Free Vortex $V_x(r) = \text{const.}$	Constant Angle $V_x/V_\theta = \text{const.}$	$B = \dots$ see eqn. 35
A	-1	$-\left(\frac{2\psi_m^2 + \phi_m^2}{\psi_m^2 + \phi_m^2}\right)$	<i>Solve numerically</i>
B	-2	$-\left(\frac{2\psi_m^2 + \phi_m^2}{\psi_m^2 + \phi_m^2}\right)$	<i>Solve numerically</i>

Design space exploration
 Blade design code
 Profiles
 Deviation
 Span-wise psi distribution
 Boundary conditions (Constant P exit, Vx, etc)
 Sweep/Lean
 Blade number

2.3 Mechanical Design

2.3.1 Mass Model/Propulsor Dimensions

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*****NOTES ON SECTION*****

Thrust = Weight condition
Mass model to close the problem

2.3.2 Shroud Tip Clearance

Shroud thickness
FEA
3D printer tolerances

2.3.3 Hollow Stators

Power supply
Weight reduction
Tolerance
How to

2.3.4 3D Printing

Printer used
Printer tolerance
Design adjustments to enable good printing
- Straight TRAILING edge
- Raft and reduce air-gap

2.4 Electrical Design

2.4.1 Choice of Electric Motor

Size ($d < 36mm$)

2.4.2 Power vs Torque Design Space

2.4.3 Power Supply

Want to simulate using a battery pack for actual flight conditions but increase run-time.
Choose 12V steady state operating voltage (14.8V nominal Li-Po).

2.4.4 Instrumentation Design

Maybe reference here but give details in the appendix? Described more in section 3.2.2 RPM Thrust

- Calibration
- Power
- Pressures

Methods

3.1 Experiments

3.1.1 Stationary Propulsor Test

- Setup
- Stand
- Test variables
 - Speeds
 - Sigma
 - Rotor design

3.1.2 Flying Test Bed

3.2 Experimental Method & Data Processing

3.2.1 Non-dimensional Quantities

Pressure quantities, FOM, ϕ , ψ .

3.2.2 Instrumentation

Power (DC Current and Voltage), Thrust, FOM, RPM, Pressures

3.2.3 Data Acquisition

Acquisition and integration of systems and software.

3.2.4 Flight Management for Auto Position Hold

System not used.

3.2.5 Cage Design & Tether

Experimental Results

4.1 Stationary Propulsor Test

4.1.1 Version 1.0 EDF

Heavy Blue: 3 exits and 3 rotor vortex designs. Comparison of intake performance (long and short).

- Power (Current and Voltage)
- Thrust
- FOM
- RPM

4.1.2 Version 2.0 EDF

1 exit and 2 rotor vortex designs. Long intake only.

- Power (Current and Voltage)
- Thrust
- FOM
- RPM
- Pressures

4.1.3 Version 3.0 EDF

1 exit and 1 rotor vortex designs. Long intake only.

- Power (Current and Voltage)
- Thrust
- FOM
- RPM
- P1 Only

4.1.4 Propeller

4.2 Hover Tests

4.2.1 Propeller Performance

4.2.2 Hover Test Limitations

Future Work

Future work can be a proper section

Do the hover tests

Manoeuvre tests

- Look at what the Mf vs manoeuvre metric would look like

- What could the manoeuvre metric look like?

Actual theory on contra-rotating

5.1 Propulsor Design

5.1.1 Stationary Propulsor Tests

What else would I have done to finish these off?

5.1.2 Contra-rotating Ducted Fan

Motivation for CR design More smaller motors better? CAD of what it might look like.
Velocity triangles ie Do the design and see how it would work theoretically.

5.1.3 Computational Fluid Dynamics

- Discuss limitations of design method (with respect to previous sections on deviation etc.)
- Reference Megs work? Show that CFD can get close to experimental results ie prove the method.

5.2 Flying Test Bed Experiments

How to quantify manoeuvre stability? The ‘Eriksen’ manoeuvre?

Conclusions

Appendix A: COVID-19

Temporary Appendix Figures

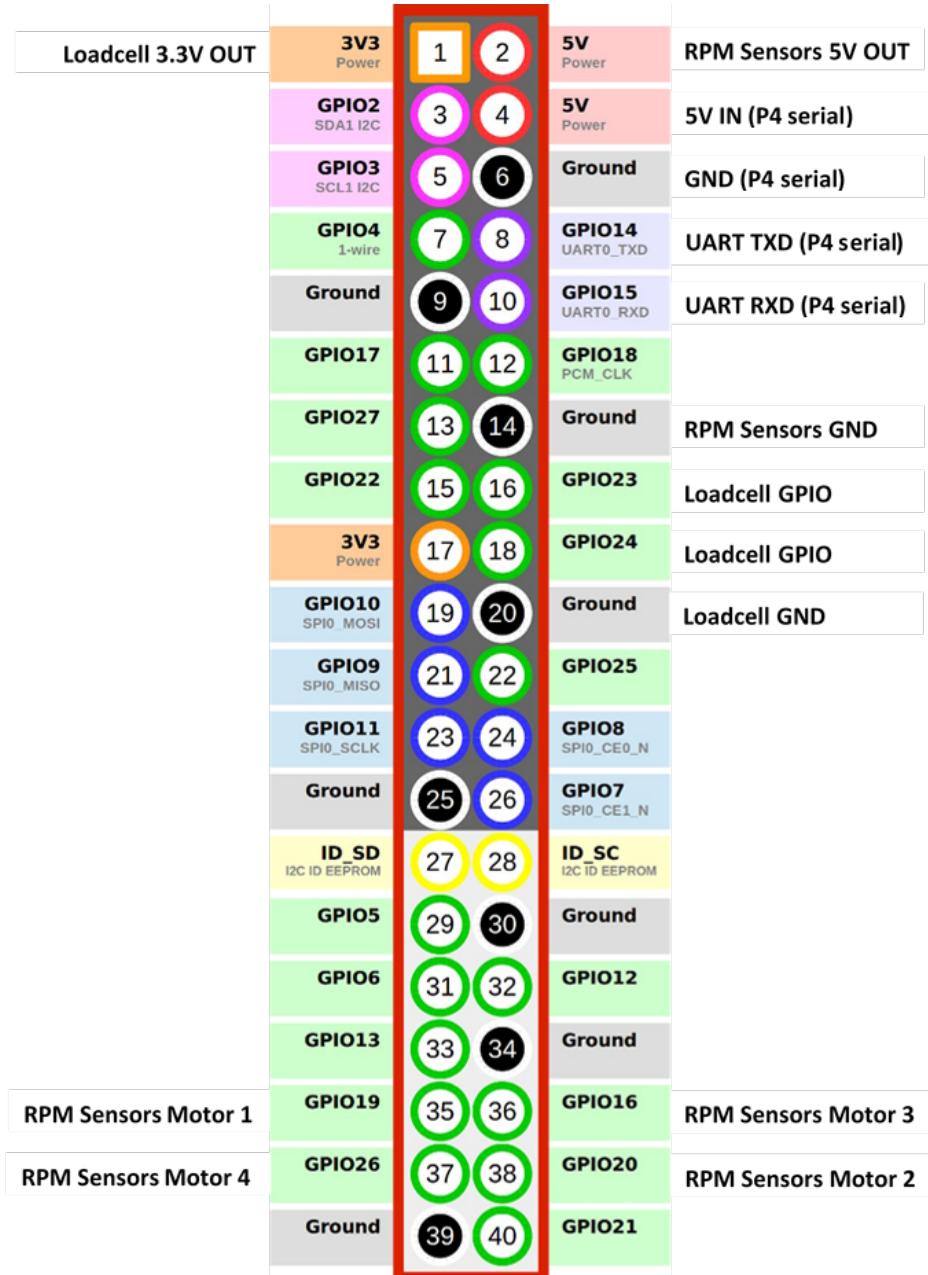


Figure 8: RPi3 header board with labelled used pins

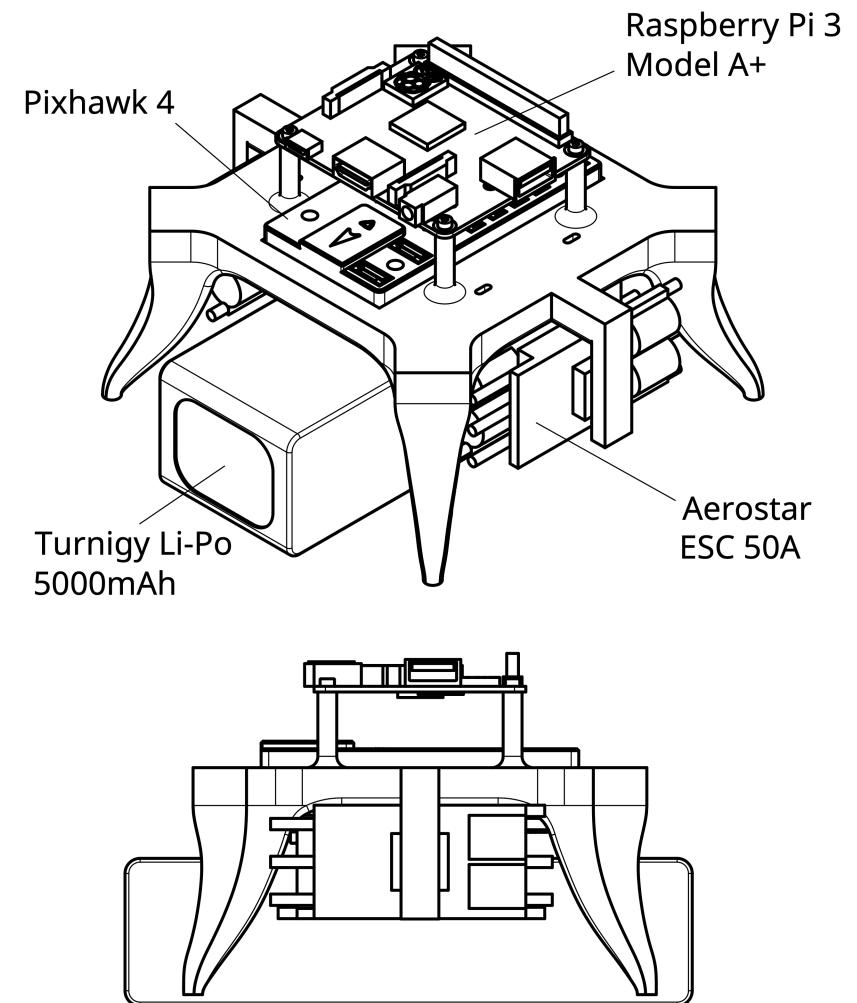
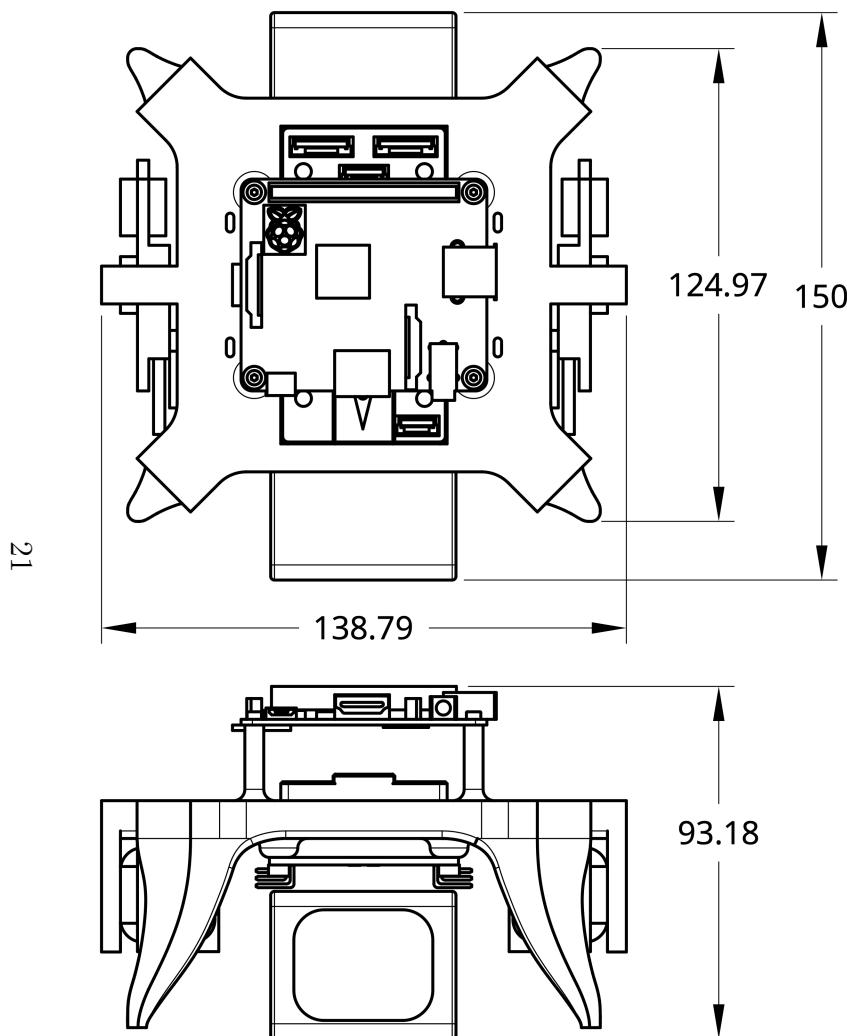


Figure 9: 3rd Angle projection of flying test bed chassis with principal dimensions and an isometric view