University of Cambridge
Department of Engineering

Masters Project Report

Propulsion Systems for e-VTOL Aircraft

Jordan Eriksen

Supervised by: **Dr Sam Grimshaw**

Abstract

Abstract here..

Nomenclature

Control Volume Analysis

Control volume numbering as below.

1. Upstream flow conditions ($V = 0$; $p_{01} = p_{atm}$; A_1 *large*)
2. Exit flow conditions ($V_2 = V_x$; $p_{02} = p_{atm}$; $A_2 = A$)

2.1 Assumptions

- Upstream velocity ($V_1 \approx 0$) and pressure ($p_1 = p_{01} = p_{atm}$)
- Flow velocity small, hence assume incompressible and Bernoulli applies
- Control volume (CV) large enough such that $\dot{m}_1 V_1 \approx 0$
- Jet leaves propulsor straight and parallel meaning jet exits CV through A_2 with p_2 but all other CV surfaces experience p_{atm}
- Assuming flow area is equal throughout, exit velocity V_2 must be equal to axial velocity V_x, hence stage loading can be written as $\phi = V_2/U$

2.2 Calculations

Steady flow momentum equation (SFME) gives

$$T_p = \dot{m}_2 V_2 - \dot{m}_1 V_1 + A_2 p_2 - A_1 p_1 \tag{1}$$

Assumptions therefore give

$$T_p = \dot{m}_2 V_2 + A_2(p_2 - p_1) \tag{2}$$

Considering a near isentropic process

$$\cancelto{0}{Tds} = dh - \frac{dp}{\rho} \tag{3}$$

$$\therefore \Delta h_0 = \frac{\Delta p_0}{\rho} \tag{4}$$

Now using Bernoulli to define Δp_0, given $V_1 \approx 0$

$$\Delta p_0 = p_{02} - \cancelto{p_1}{p_{01}} \tag{5}$$

$$\Delta p_0 = \frac{1}{2}\rho V_2^2 + (p_2 - p_1) \tag{6}$$

Peak jet efficiency when $p_2 = p_{atm}$

$$\therefore \Delta p_0 = \frac{1}{2}\rho V_2^2 \tag{7}$$

As $p_1 = p_2$, SFME becomes

$$T_p = \dot{m}_2 V_2 = \rho A_2 V_2^2 \tag{8}$$

Hence design exit velocity can be determined by fan geometry and required thrust

$$V_2 = \sqrt{\frac{T_p}{\rho A_2}} \tag{9}$$

2.3 Geometry

Fan geometry can be varied using a sensitivity analysis to obtain desirable values of flow coefficient (ϕ) and stage loading (ψ). From previous design these are approximately (at mean line)

$$\phi = \frac{V_x}{U} \approx 0.7 \qquad\qquad \psi = \frac{\Delta h_0}{U^2} \approx 0.3 \tag{10}$$

Fan speed Ω is predetermined by the choice of motor assembly (electric motor, electric speed controller (ESC) and battery voltage)

Fixed quantities	*Variable quantities*
ρ, Ω, T_p	r_c, r_h

Relating the above coefficients to the geometric parameters r_c and r_h

$$\phi = \sqrt{\frac{T_p}{\rho\pi(r_c^2 - r_h^2)}} \cdot \frac{2}{\Omega(r_c + r_h)} \tag{11}$$

$$\psi = \frac{T_p}{\rho\pi(r_c^2 - r_h^2)} \cdot \frac{2}{\Omega^2(r_c + r_h)^2} \tag{12}$$

These sensitivity analyses, along with consideration of available space and a desire to reduce weight as much as possible (minimize fan radius), resulted in approximate fan radii being determined.

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This is not an optimum this is to design within the constraints applied to the problem.

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$$\mathbf{r_c = 55mm} \qquad\qquad \mathbf{r_h = 25cm} \tag{13}$$