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Masters Project Report

Propulsion Systems for e-VTOL Aircraft

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Abstract

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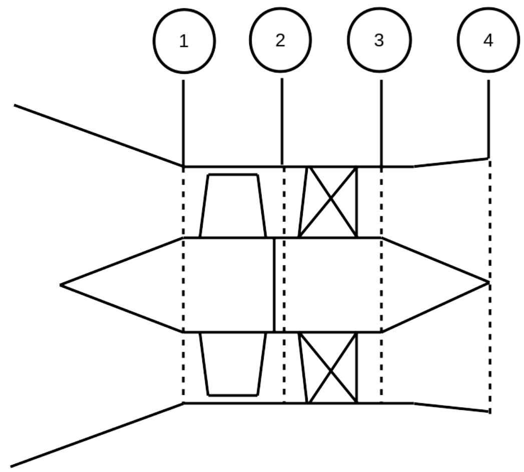
Nomenclature

Control Volume Analysis



Control volume numbering as below:

Upstream flow conditions ($V_0 = 0$; $p_{00} = \eta_{inlet} \cdot p_{atm}$; A_0 *large*)

1. Rotor inlet ($V_1 = V_x$; $p_{01} = p_{atm}$; $A_1 = A_x$)
2. Stator inlet ($V_2 = V_x$; $p_{02} = p_{01} + (\Delta p_0)_{rotor}$; $A_2 = A_x$)
3. Stage exit ($V_3 = V_x$; $p_{03} = p_{01} + (\Delta p_0)_{stage}$; $A_3 = A_x$)
4. Diffuser exit ($V_4 \neq V_x$; $p_{04} = \eta_{diff} \cdot p_{03}$; $A_4 = A_3\sigma$)

2.1 Assumptions

- $M = 0.069$, hence incompressible and Bernoulli applies
- Control volume (CV) large enough such that $\dot{m}V_0 \approx 0$
- Jet leaves propulsor straight and parallel meaning jet exits CV through A_4 with $p_4 = p_{atm}$
- Assuming flow area is equal throughout stage, exit velocity V_3 must be equal to axial velocity V_x, hence flow coeff can be written as $\phi = V_3/U$

- Diffusion factor defined (in terms of exit and axial quantities), considering continuity and incompressibility $\frac{A_4}{A_x} = \frac{V_x}{V_4} = \sigma$
- No accumulation or leakage in stage so $\dot{m}$ constant throughout

2.2 Calculations

Steady flow momentum equation (SFME) gives

$$T_T = \dot{m}V_4 - \dot{m}V_0 + A_4(p_4 - p_{atm}) \tag{1}$$

Assumptions therefore give

$$T_T = \dot{m}V_4 = \rho A_4 V_4^2 \tag{2}$$

Relating to axial quantities

$$T_T = \frac{\rho A_x V_x^2}{\sigma} \tag{3}$$

Hence design axial velocity can be determined by fan geometry and required thrust

$$V_x = \sqrt{\frac{T_T \sigma}{\rho A_x}} \tag{4}$$

2.2.1 Flow Coefficient and Stage Loading

Flow coefficient (ϕ) and stage loading (ψ) can be related to propulsor geometry and required thrust by considering diffuser aspect ratio limitations and the resulting impact on stage exit pressure.

$$\phi = \frac{V_x}{U} \approx 0.7 \qquad\qquad \psi = \frac{\Delta h_0}{U^2} \approx 0.3 \tag{5}$$

Considering the thermodynamic relation between enthalpy and pressure, in an isentropic pressure rise

$$T\cancelto{0}{ds} = dh - \frac{dp}{\rho} \tag{6}$$

$$\Delta h_0 = \frac{\Delta p_0}{\rho} \qquad\qquad \therefore \psi = \frac{\Delta p_0}{\rho U^2} \tag{7}$$

Need ψ in terms of velocity

$$\Delta p_0 = p_{03} - p_{01} = p_3 + \frac{1}{2}\rho V_x^2 - p_{atm} \tag{8}$$

Relating p_3 to p_4 by Bernoulli

$$\Delta p_0 = \left[p_{atm} + \frac{1}{2}\rho\left(\frac{V_x^2}{\sigma^2} - V_x^2\right)\right] + \frac{1}{2}\rho V_x^2 - p_{atm} = \frac{\rho V_x^2}{2\sigma^2} \tag{9}$$

Giving ψ in terms of V_x and σ

$$\psi = \frac{V_x^2}{2U^2\sigma^2} \tag{10}$$

Now inserting equation 4 into 5(a) and 5(b)

$$\phi = \sqrt{\frac{T_T\sigma}{\rho A_x U^2}} \qquad \psi = \frac{T_T}{2\rho A_x U^2 \sigma} \tag{11}$$

Solve for T_T as a function of σ and ϕ.

$$T_T = \frac{\phi^2 \rho A_x U^2}{\sigma} \tag{12}$$

In terms of geometric parameters

$$T_T = \frac{\phi^2 \rho\pi(r_c^2 - r_h^2)\cdot\Omega^2\left(\frac{r_h+r_c}{2}\right)^2}{\sigma} \tag{13}$$

In order to determine the critical value of casing radius for which the thrust produced is equal to the weight of the vehicle, a mass model is required. This is described in section 2.3 below. At each design point, the critical casing radius required is determined numerically.

Equation 11- (a) and (b) can be combined to determine a relationship between the flow coefficient, stage loading and limiting diffusion factor. Selecting σ and ϕ sets ψ by

$$\sigma^2 = \frac{\phi^2}{2\psi} \tag{14}$$

This reflects the desire for the flow to exit straight and parallel. This condition requires the exit static pressure to equal that of the atmosphere, hence there is a restriction on the ND flow coefficients with sigma for this to be achieved. This is only valid when all the assumptions stated above are considered. In practice there will be some deviation from design and some variation from the model will be due to this assumption being away from the desired value.

2.2.2 Work Analysis

In order to consider the relative merits of the propulsor, the power output must be considered. An energy balance gives the output power as

$$P = \frac{1}{2}\dot{m}V_4^2 = \frac{\rho A_x V_x^3}{2\sigma^2} \tag{15}$$

Inserting equation (4) into the above yields an expression for power in terms of the required thrust

$$P = \frac{T_T^{3/2}}{\sqrt{4\sigma\rho A_x}} \tag{16}$$

Hence, using equation (12), this can be expressed solely in terms of geometric and flow quantities

$$P = \frac{\phi^3 \rho A_x U^3}{2\sigma^2} \tag{17}$$

2.2.3 Electrical Analysis

The lowest limit of operation of the motors driving the fan is the current limit. This limit arises from heating within the motor. The implications this has on the operation of the fan can be determined by a simple power analysis.

Assuming the power in (electrical) is equal to the shaft power of the motor (mechanical)

$$(VI)_{electrical} = (T\omega)_{mechanical} \tag{18}$$

Rearranging

$$\frac{\omega}{V} = \frac{I}{T} \tag{19}$$

The motor performance is characterised by the constant $k_v \cdot \frac{\pi}{30} = \frac{\omega}{V}$, the RPMs per volt. Hence

$$k_v \left(\frac{\pi}{30}\right) = \frac{\omega}{V} = \frac{I}{T} \tag{20}$$

The motor datasheet gives $I_{max} = 21.5A$, hence we can express the maximum power output as

$$P_{max} = \frac{30 I_{max}}{\pi k v} \cdot \omega \tag{21}$$

Using the chosen design speed of 7500 RPM, the maximum power contour can be plotted corresponding to an electrical motor limit.

2.3 Diffuser Geometry and Thrust vs Mass Optimisation

A simple mass model is employed to determine variation in mass as r_c is changed. The limiting aspect ratio of the diffuser is governed by separation behaviour on the diffuser walls. ESDU 75026 data gives a separation limit estimate that can be approximated by the following relation

$$\sigma \leq 0.3313 \cdot \left(\frac{r_c - r_h}{L_{diff}}\right)^{0.5275} + 1 \tag{22}$$

In the limit of separation, a minimum aspect ratio, θ_{min}, can be found for a given σ. This limit is to be explored and tested with respect to a small scale real-world propulsion system.

$$\theta_{min} = \frac{r_c - r_h}{L_{diff}} = \left(\frac{\sigma - 1}{0.3313}\right)^{\frac{1}{0.5275}} \tag{23}$$

2.3.1 Mass Model

The diffuser is defined as having a starting annulus area of A_x with hub radius r_h and casing radius r_c. The exit area is taken as annulus as well, in order to reduce the angle through which the flow must turn in the diffuser, hence reducing likelihood of separation at both hub and casing. Hence, the exit annulus hub radius r_{eh} and casing radius r_{ec} can be determined in terms of r_c, r_h and the diffusion ratio σ

$$r_{ec} = \frac{1}{2}(r_c(1 + \sigma) + r_h(1 - \sigma)) \tag{24}$$

$$r_{eh} = \frac{1}{2}(r_c(1 - \sigma) + r_h(1 + \sigma)) \tag{25}$$

The volumes of the diffuser and inlet are calculating by modelling concentric truncated cones with equal gradients and radii differing by the design thickness, t. This results in an approximate geometry of the required parts, providing a reasonable estimate for the volume of the final parts.

$$Vol_{cone} = \frac{h}{3}\pi r^2 \tag{26}$$

The height of these cones is determined using the expressions above relating to the aspect ratio (gradient) of the part. These are used to determine the length, and equations (17) and (18) to determine the radii.

$$m_d = \rho_{PLA}\Big(Vol_{hub} + Vol_{case}\Big) \tag{27}$$

The mass of the rest of the propulsor is made up approximately of a hub, casing, stator and rotor. This can be approximated as

$$m_p = \Bigg(\rho\pi l_c((r_c + t)^2 - r_c^2)\Bigg)_{casing} + \Bigg(\rho\pi l_h(r_h^2 - (r_h - 0.01)^2)\Bigg)_{hub} + \Bigg(0.056\Bigg)_{motor} \tag{28}$$

This propulsor mass can therefore be determined as a function of geometric properties r_c, r_h and t. Combining with diffuser mass means the total engine mass is determined by the above geometric properties as well as flow conditions ϕ, ψ and the ESDU separation limit, θ.

2.4 Figure of Merit

The Figure of Merit, M_f, is a non-dimensional descriptor of a given propulsor in terms of the thrust delivered, power output and flow area.

$$M_f = \frac{T_T}{P}\sqrt{\frac{T_T}{2\rho A_x}} \tag{29}$$

For an standard propeller, this has a value of 1. An ideal ducted fan with a diffusion ratio σ can be shown to have a figure of merit of $\sqrt{2\sigma}$ (figure 1 shows value of $M_f = 1.414$ for $\sigma = 1$). However, the increase in M_f comes at the cost of weight. This can be accounted for by assuming the mass model described in section 2.4.1.

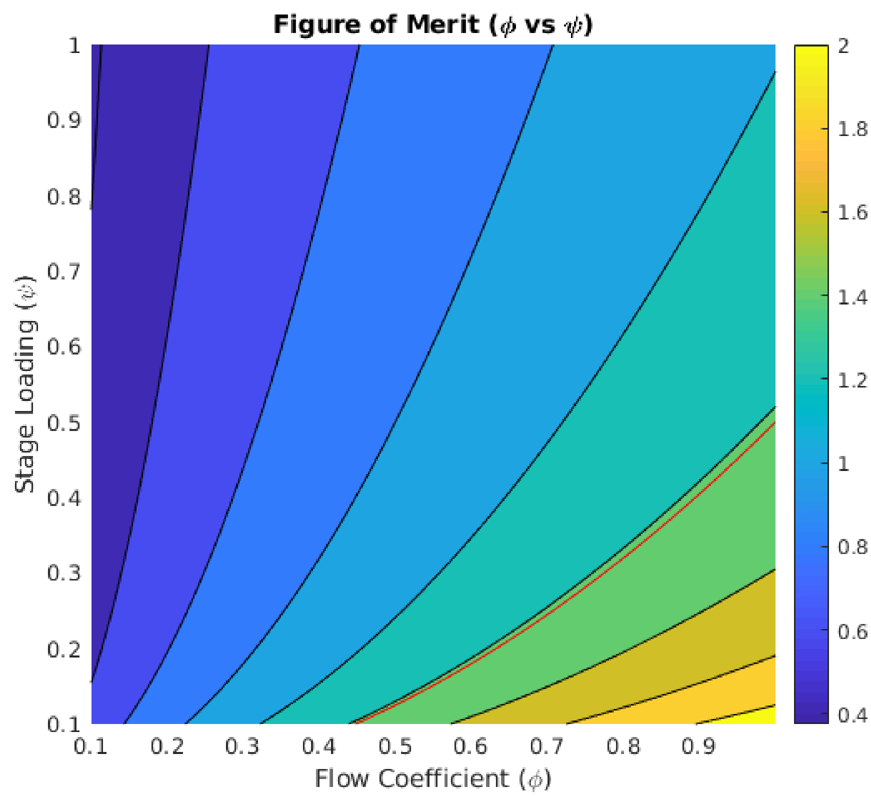


Figure 1: Plot of Figure of Merit for a ducted fan. Red line indicates location of ducted fan with $\sigma = 1$.

As this has been produced assuming ideal operating conditions (such as no loss), suggests a design that maximises flow coefficient whilst minimising stage loading. In an ideal world this is logical as stage loading has the largest influence on the power term. From (22) and (14)

$$P = \phi\psi\rho A_x U^3 \tag{30}$$

Hence both stage loading and flow coefficient have equal impact on power. Thrust can also be expressed in terms of stage loading and flow coefficient. Similarly to above, taking (12) and (14)

$$T_T = \phi\sqrt{2\psi}\rho A_x U^2 \tag{31}$$

Hence thrust increases with $\phi\sqrt{\psi}$, meaning more dependence on flow coefficient than stage loading. Combining these two (noting that the power term is larger than the thrust term, and so it's effect is dominant) results in the maximum M_f being located in the bottom right corner of figure 1. M_f is a measure of effectiveness of the propulsor but does not take into account either efficiency of the system or the mass increase associated with changing the design. This sets a lower limit on the figure of merit plot, found by considering that the thrust output needs to be greater than the total mass the propulsor is required to lift, M. The ratio of T_T to M is defined as the thrust excess, $\tilde{T}_E$.

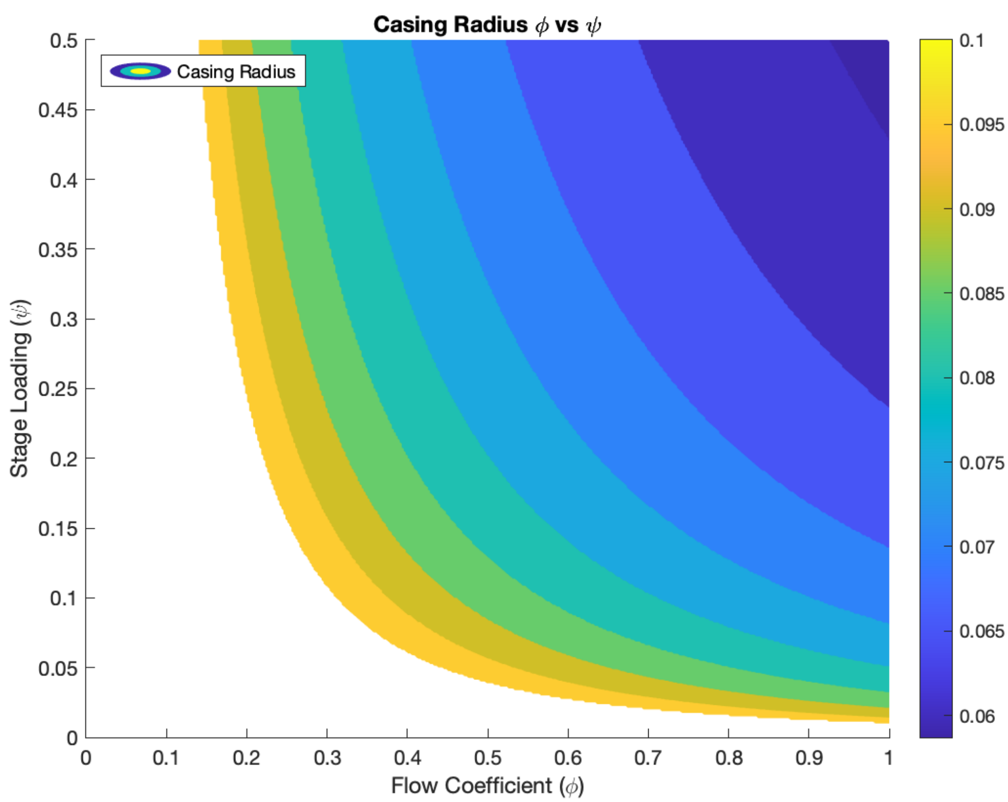


Figure 2: Critical casing radius at each design point. South-West 'NaN' region represents designs with casing radius greater than $100mm$.

The line of $\tilde{T}_E = 1$ therefore operates as a lower bound for ψ at a given ϕ in order to satisfy the minimum thrust condition. Hence figure 2 considers each design point determined at it's critical radius, with a physical limit of $100mm$ applied. The region of missing data in the south-western corner of the plot reflects this limit.

Consideration of Smith charts from machines with similar flow Re suggests the propulsor should limit the maximum value of flow coefficient so as not to obtain an efficiency penalty. Therefore a flow condition operating point of $\phi = 0.85$, $\psi = 0.25$ and a nominal motor RPM of 7500 is chosen. With this information, the propulsor size can be optimised to reduce the mass as much as possible. This analysis fixes the variables mentioned above as well as the hub radius, r_h, floating the casing radius, r_c, to determine the minimum radius possible whilst maintaining sufficient thrust. The above analysis provides a value of $r_c = 0.0551m$. The operating point is summarised below.

Flow coefficient	ϕ	0.85
Stage loading	ψ	0.25
Diffusion ratio	σ	1.2
Figure of Merit	M_f	1.515
Diffuser Length	L_D	12.0 mm
Casing radius	r_c	55.1 mm

2.5 Power

All the above considerations can be represented on a power contour plot across the design space. These analyses ensure all constraints are satisfied, and an optimisation can be done by evaluating the variation in total power requirement across the design space.

2.5.1 Current Limit

Explained in detail in section 2.2.3. Current limit arises due to electrical limitation of the motors used. As this is an operational limit and does NOT arise from aerodynamic considerations, this will vary as the choice of motor varies. A suitable motor has been selected to ensure that the range of the fan is within a suitable operating limit.

2.5.2 Diffusion Limit

In order to reduce waste kinetic energy from the fan, a diffuser at the exit can be used to improve propulsive efficiency. The exit only behaves as a diffuser when the exit area is greater than the turbomachinery flow area, hence when $\sigma > 1$.

2.5.3 Geometric Diffuser Hub Limit

The diffuser has been set to have a fixed gradient at both the hub and casing. This results in a symmetric geometry. However, for larger diffusion factors, the exit hub radius will reach 0 and so cannot decrease it's radius further. At this point the diffuser hub is a closed cone.

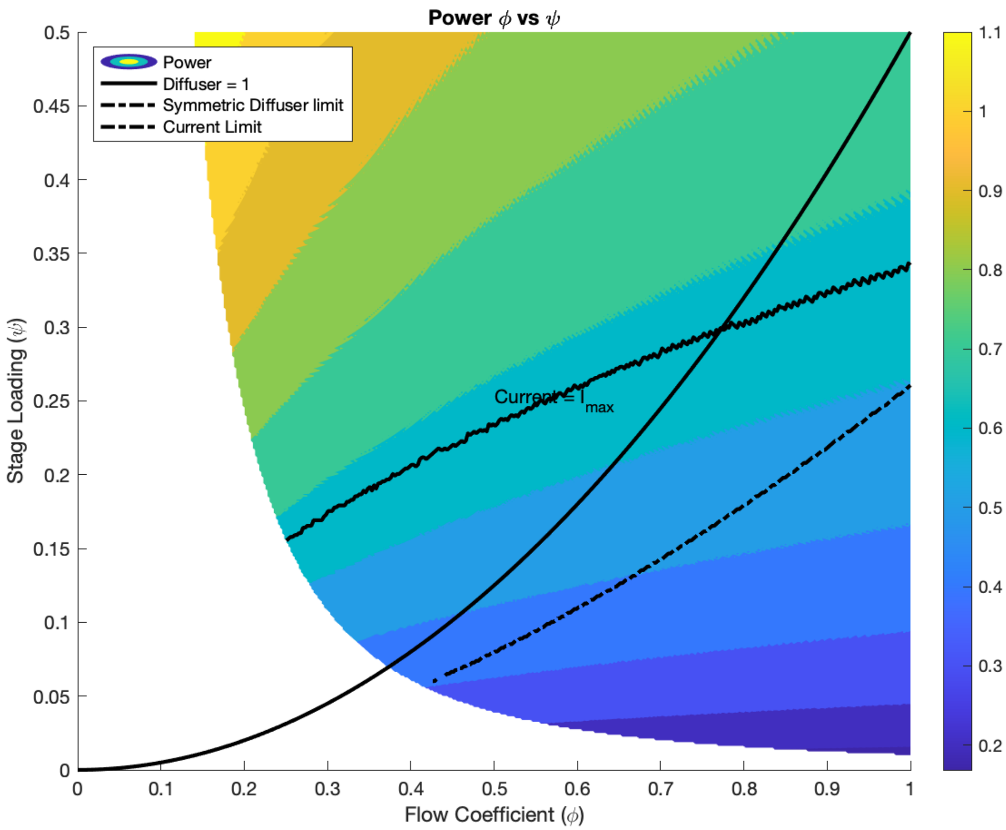


Figure 3: Power domain of the design space with operating limits shown