University of Cambridge
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Masters Project Report

# Propulsion Systems for e-VTOL Aircraft

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**Abstract**

Abstract here..

# Nomenclature

# Control Volume Analysis

Control volume numbering as below:

1. Upstream flow conditions ($V_0 = 0;\ \ p_{00} = \eta_{inlet} \cdot p_{atm};\ \ A_0\ \ large$)
2. Rotor inlet ($V_1 = V_x;\ \ p_{01} = p_{atm};\ \ A_1 = A_x$)
3. Stator inlet ($V_2 = V_x;\ \ p_{02} \neq p_{01};\ \ A_2 = A_x$)
4. Stage exit ($V_3 = V_x;\ \ p_{03} = p_{01} + \Delta p_0;\ \ A_2 = A_x$)
5. Diffuser exit ($V_4 \neq V_x;\ \ p_{04} = \eta_{diff} \cdot p_{03};\ \ A_4 = A_3\sigma$)

## 2.1 Assumptions

- Flow velocity small, hence assume incompressible and Bernoulli applies
- Control volume (CV) large enough such that $\dot{m}V_0 \approx 0$
- Jet leaves propulsor straight and parallel meaning jet exits CV through $A_4$ with $p_4 = p_{atm}$
- Assuming flow area is equal throughout stage, exit velocity $V_3$ must be equal to axial velocity $V_x$, hence flow coeff can be written as $\phi = V_3/U$
- Diffusion factor defined (in terms of exit and axial quantities), considering continuity and incompressibility $\frac{A_4}{A_x} = (\frac{V_x}{V_4})^2 = \sigma$
- No accumulation or leakage in stage so $\dot{m}$ constant throughout

## 2.2 Calculations

Steady flow momentum equation (SFME) gives

$$T_T = \dot{m}V_4 - \dot{m}V_0 + A_4(p_4 - p_atm) \tag{1}$$

Assumptions therefore give

$$T_T = \dot{m}V_4 = \rho A_4 V_4^2 \tag{2}$$

Relating to axial quantities

$$T_T = \frac{\rho A_x V_x^2}{\sqrt{\sigma}} \tag{3}$$

Hence design axial velocity can be determined by fan geometry and required thrust

$$V_x = \sqrt{\frac{T_T\sqrt{\sigma}}{\rho A_x}} \tag{4}$$

## 2.3 Flow Coefficient and Stage Loading

Flow coefficient ($\phi$) and stage loading ($\psi$) can be related to propulsor geometry and required thrust by considering diffuser aspect ratio limitations and the resulting impact on stage exit pressure.

$$\phi = \frac{V_x}{U} \approx 0.7 \qquad\qquad \psi = \frac{\Delta h_0}{U^2} \approx 0.3 \tag{5}$$

Considering the thermodynamic relation between enthalpy and pressure, in an isentropic pressure rise

$$T\cancelto{0}{ds} = dh - \frac{dp}{\rho} \tag{6}$$

$$\Delta h_0 = \frac{\Delta p_0}{\rho} \qquad\qquad \therefore \psi = \frac{\Delta p_0}{\rho U^2} \tag{7}$$

Need $\psi$ in terms of velocity

$$\Delta p_0 = p_{03} - p_{01} = p_3 + \frac{1}{2}\rho V_x^2 - p_{atm} \tag{8}$$

Relating $p_3$ to $p_4$ by Bernoulli

$$\Delta p_0 = \left[p_{atm} + \frac{1}{2}\rho\left(\frac{V_x^2}{\sigma} - V_x^2\right)\right] + \frac{1}{2}\rho V_x^2 - p_{atm} = \frac{\rho V_x^2}{2\sigma} \tag{9}$$

Giving $\psi$ in terms of $V_x$ and $\sigma$

$$\psi = \frac{V_x^2}{2U^2\sigma} \tag{10}$$

Now inserting equation 4 into 5(a) and 5(b)

$$\phi = \sqrt{\frac{T_T\sqrt{\sigma}}{\rho A_x U^2}} \qquad\qquad \psi = \frac{T_T}{2\rho A_x U^2\sqrt{\sigma}} \tag{11}$$

Rearranging for $\sigma$

$$\sigma = \left(\frac{\phi^2\rho A_x U^2}{T_T}\right)^2 \tag{12}$$

$$\sigma = \left( \frac{T_T}{2\rho A_x U^2 \psi} \right)^2 \tag{13}$$

Equate and solve for $T_T$ as a function of $\psi$ and $\phi$

$$T_T = \sqrt{2\psi}\phi\rho A_x U^2 \tag{14}$$

In terms of geometric parameters

$$T_T = \sqrt{2\psi}\phi\rho \cdot \pi(r_c^2 - r_h^2) \cdot \Omega^2 \left( \frac{r_h + r_c}{2} \right)^2 \tag{15}$$

Which is solved numerically to obtain the required value of $r_c$ for a given thrust.

Equation 12- (a) and (b) can be combined to determine a relationship between the flow coefficient, stage loading and limiting diffusion factor.

$$\sigma = \frac{\phi^2}{2\psi} \tag{16}$$

*** ADD SECTION ON FIXED AND VARIABLE PARAMETERS ***

## 2.4 Diffuser Geometry and Thurst vs Mass Optimisation

A simple mass model is employed to determine variation in mass as $r_c$ is changed. The limiting aspect ratio of the diffuser is governed by separation behaviour on the diffuser walls. ESDU 75026 data gives a separation limit estimate that can be approximated by the following relation

$$\sigma \leq 0.3313 \cdot \left( \frac{r_c - r_h}{L_{diff}} \right)^{0.5275} + 1 \tag{17}$$

In the limit of separation, a minimum aspect ratio, $\theta_{min}$, can be found for a given $\sigma$

$$\theta_{min} = \frac{r_c - r_h}{L_{diff}} = \left( \frac{\sigma - 1}{0.3313} \right)^{\frac{1}{0.5275}} = \left( \frac{\phi^2 - 1}{0.6626 \cdot \psi} \right)^{\frac{1}{0.5275}} \tag{18}$$

### 2.4.1 Mass Model

The diffuser is defined as having a starting annulus area of $A_x$ with hub radius $r_h$ and casing radius $r_c$. The hub is modelled as a cone with length $L$, base radius $r_h$ and therefore a volume of $\frac{L}{3}\pi r_h^2$. The casing has initial radius $r_c$, thickness of $t$, final radius $r_e$ and a length $L = \sqrt{L_{diff}^2 + (r_e - r_c)^2}$. Using the above expression for aspect ratio the volumes can be written in terms of known geometric and non-dimensional flow quantities. Assuming a uniform density of $1240 kgm^{-3}$ (for PLA) the approximate diffuser mass can be determined

$$m_d = \rho_{PLA}\Big(V_{cone} + V_{casing}\Big) \tag{19}$$

$$m_d = \rho_{PLA}\left[\frac{\pi r_h^2(r_c - r_h)}{3\theta} + 2\pi t\sqrt{\frac{\phi^2(r_c^2 - r_h^2)}{2\psi}} \cdot \sqrt{\left(\frac{r_c - r_h}{\theta}\right)^2 + \left(\frac{\phi^2(r_c^2 - r_h^2)}{2\psi} - r_c\right)^2}\right] \tag{20}$$

The mass of the rest of the propulsor is made up approximately of a hub, casing, stator, rotor and electric motor. This can be approximated as

$$m_p = \Big(\rho\pi l_c((r_c + t)^2 - r_c^2)\Big)_{casing} + \Big(\rho\pi l_h(r_h^2 - (r_h - 0.01)^2)\Big)_{hub} + \Big(0.08\Big)_{rest} \tag{21}$$

Therefore total mass, $M$, can be determined as a function of $r_c$. By subtracting the total mass from the thrust output, the thrust excess/deficit can be determined as a function of $r_c$ for a predetermined set of variables

| *Parameter* | *Value* |
|---|---|
| $\phi$ | 0.8 |
| $\psi$ | 0.2 |
| $r_h$ | $25mm$ |
| $\Omega$ | $838rads^{-1}$ ($8000rpm$) |

$$T_T - M = 0 \tag{22}$$

$$\mathbf{(r_c)_{critical} = 52.4mm} \qquad \mathbf{L_{diff} = 94.8mm} \tag{23}$$

## 2.5 Work Analysis

Considering how to rank designs in terms of work input, thrust and mass, the following metric is considered

$$METRIC = \frac{W \cdot (|T_T - M| + 1)}{T_T} \tag{24}$$

For an optimum design (minimum work input for a given thrust), this metric is to be minimised. Figure 1 below shows the variation in $METRIC$ for various motor speeds and radii.

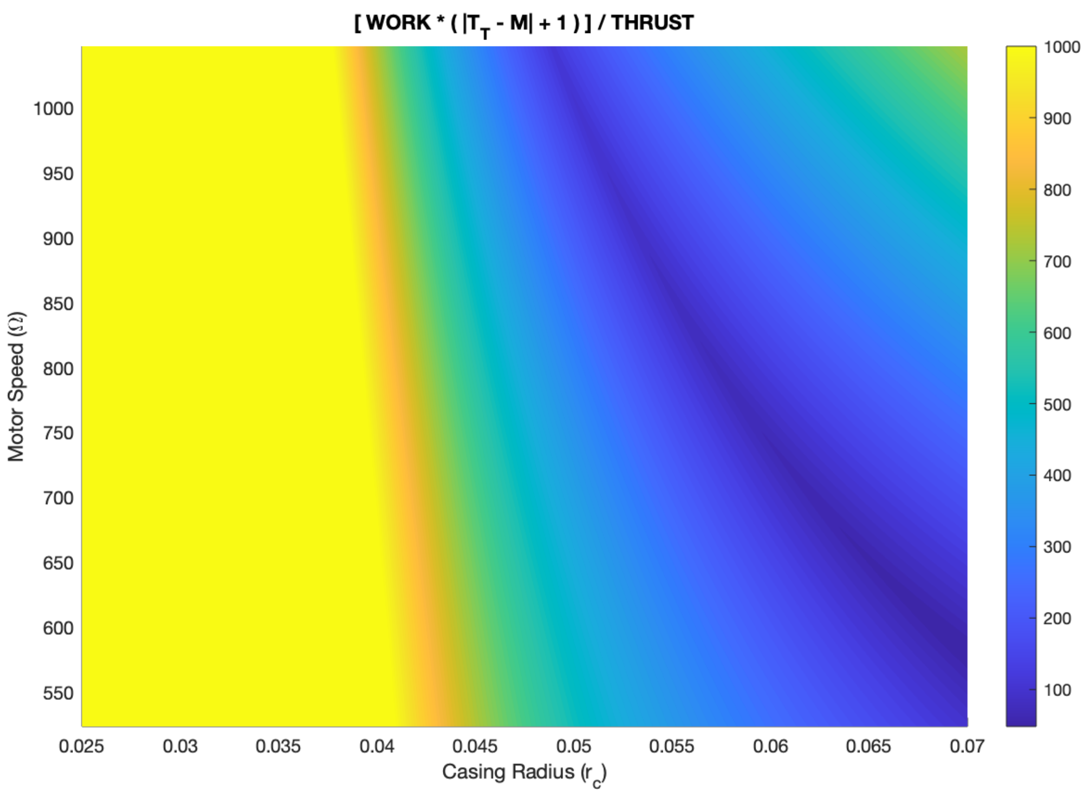


Figure 1: Plot of (ALMOST) ND work performance ch'ic