University of Cambridge
Department of Engineering

Masters Project Report

Propulsion Systems for e-VTOL Aircraft

Jordan Eriksen

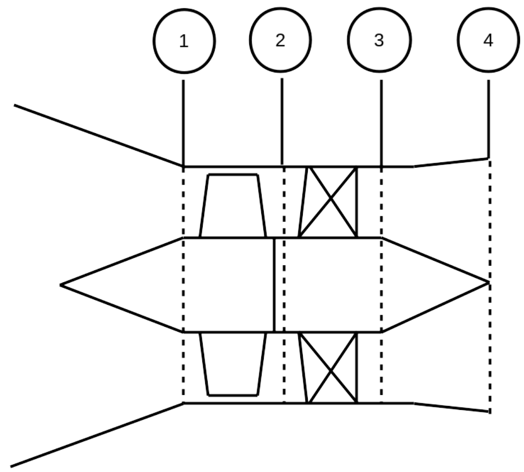
Supervised by: **Dr Sam Grimshaw**

Abstract

Abstract here..

Nomenclature

Control Volume Analysis



Control volume numbering as below:

Upstream flow conditions ($V_0 = 0$; $p_{00} = \eta_{inlet} \cdot p_{atm}$; A_0 large)

1. Rotor inlet ($V_1 = V_x$; $p_{01} = p_{atm}$; $A_1 = A_x$)
2. Stator inlet ($V_2 = V_x$; $p_{02} = p_{01} + (\Delta p_0)_{rotor}$; $A_2 = A_x$)
3. Stage exit ($V_3 = V_x$; $p_{03} = p_{01} + (\Delta p_0)_{stage}$; $A_3 = A_x$)
4. Diffuser exit ($V_4 \neq V_x$; $p_{04} = \eta_{diff} \cdot p_{03}$; $A_4 = A_3\sigma$)

2.1 Assumptions

- $M = 0.069$, hence incompressible and Bernoulli applies
- Control volume (CV) large enough such that $\dot{m}V_0 \approx 0$
- Jet leaves propulsor straight and parallel meaning jet exits CV through A_4 with $p_4 = p_{atm}$
- Assuming flow area is equal throughout stage, exit velocity V_3 must be equal to axial velocity V_x, hence flow coeff can be written as $\phi = V_3/U$

- Diffusion factor defined (in terms of exit and axial quantities), considering continuity and incompressibility $\frac{A_4}{A_x} = \frac{V_x}{V_4} = \sigma$
- No accumulation or leakage in stage so $\dot{m}$ constant throughout

2.2 Calculations

Steady flow momentum equation (SFME) gives

$$T_T = \dot{m}V_4 - \dot{m}V_0 + A_4(p_4 - p_{atm}) \tag{1}$$

Assumptions therefore give

$$T_T = \dot{m}V_4 = \rho A_4 V_4^2 \tag{2}$$

Relating to axial quantities

$$T_T = \frac{\rho A_x V_x^2}{\sigma^2} \tag{3}$$

Hence design axial velocity can be determined by fan geometry and required thrust

$$V_x = \sqrt{\frac{T_T \sigma^2}{\rho A_x}} \tag{4}$$

2.3 Flow Coefficient and Stage Loading

Flow coefficient (ϕ) and stage loading (ψ) can be related to propulsor geometry and required thrust by considering diffuser aspect ratio limitations and the resulting impact on stage exit pressure.

$$\phi = \frac{V_x}{U} \approx 0.7 \qquad\qquad \psi = \frac{\Delta h_0}{U^2} \approx 0.3 \tag{5}$$

Considering the thermodynamic relation between enthalpy and pressure, in an isentropic pressure rise

$$T\cancelto{0}{ds} = dh - \frac{dp}{\rho} \tag{6}$$

$$\Delta h_0 = \frac{\Delta p_0}{\rho} \qquad\qquad \therefore \psi = \frac{\Delta p_0}{\rho U^2} \tag{7}$$

Need ψ in terms of velocity

$$\Delta p_0 = p_{03} - p_{01} = p_3 + \frac{1}{2}\rho V_x^2 - p_{atm} \tag{8}$$

Relating p_3 to p_4 by Bernoulli

$$\Delta p_0 = \left[p_{atm} + \frac{1}{2}\rho\left(\frac{V_x^2}{\sigma^2} - V_x^2\right)\right] + \frac{1}{2}\rho V_x^2 - p_{atm} = \frac{\rho V_x^2}{2\sigma^2} \tag{9}$$

Giving ψ in terms of V_x and σ

$$\psi = \frac{V_x^2}{2U^2\sigma^2} \tag{10}$$

Now inserting equation 4 into 5(a) and 5(b)

$$\phi = \sqrt{\frac{T_T\sigma^2}{\rho A_x U^2}} \qquad \psi = \frac{T_T}{2\rho A_x U^2} \tag{11}$$

Solve for T_T as a function of σ and ϕ.

$$T_T = \frac{\phi^2 \rho A_x U^2}{\sigma^2} \tag{12}$$

In terms of geometric parameters

$$T_T = \frac{\phi^2\rho\pi(r_c^2 - r_h^2)\cdot\Omega^2\left(\frac{r_h+r_c}{2}\right)^2}{\sigma^2} \tag{13}$$

Which is solved numerically to obtain the required value of r_c for a given thrust.

Equation 11- (a) and (b) can be combined to determine a relationship between the flow coefficient, stage loading and limiting diffusion factor. Selecting σ and ϕ sets ψ by

$$\sigma^2 = \frac{\phi^2}{2\psi} \tag{14}$$

This reflects the desire for the flow to exit straight and parallel. This condition requires the exit static pressure to equal that of the atmosphere, hence there is a restriction on the ND flow coefficients with sigma for this to be achieved. This is only valid when all the assumptions stated above are considered.

2.4 Diffuser Geometry and Thrust vs Mass Optimisation

A simple mass model is employed to determine variation in mass as r_c is changed. The limiting aspect ratio of the diffuser is governed by separation behaviour on the diffuser walls. ESDU 75026 data gives a separation limit estimate that can be approximated by the following relation

$$\sigma \leq 0.3313 \cdot \left(\frac{r_c - r_h}{L_{diff}}\right)^{0.5275} + 1 \tag{15}$$

In the limit of separation, a minimum aspect ratio, θ_{min}, can be found for a given σ. This limit is to be explored and tested with respect to a small scale real-world propulsion system.

$$\theta_{min} = \frac{r_c - r_h}{L_{diff}} = \left(\frac{\sigma - 1}{0.3313}\right)^{\frac{1}{0.5275}} \tag{16}$$

2.4.1 Mass Model

The diffuser is defined as having a starting annulus area of A_x with hub radius r_h and casing radius r_c. The hub is modelled as a cone with length L, base radius r_h and therefore a volume of $\frac{L}{3}\pi r_h^2$. The casing has initial radius r_c, thickness of t, final radius r_e and a length $L = \sqrt{L_{diff}^2 + (r_e - r_c)^2}$. Using the above expression for aspect ratio the volumes can be written in terms of known geometric and non-dimensional flow quantities. Assuming a uniform density of $1240 kgm^{-3}$ (for PLA) the approximate diffuser mass can be determined

$$m_d = \rho_{PLA}\Big(V_{cone} + V_{casing}\Big) \tag{17}$$

$$m_d = \rho_{PLA}\left[\frac{\pi r_h^2(r_c - r_h)}{3\theta} + 2\pi t\sqrt{\frac{\phi^2(r_c^2 - r_h^2)}{2\psi}} \cdot \sqrt{\left(\frac{r_c - r_h}{\theta}\right)^2 + \left(\frac{\phi^2(r_c^2 - r_h^2)}{2\psi} - r_c\right)^2}\right] \tag{18}$$

The mass of the rest of the propulsor is made up approximately of a hub, casing, stator and rotor. This can be approximated as

$$m_p = \Big(\rho\pi l_c((r_c + t)^2 - r_c^2)\Big)_{casing} + \Big(\rho\pi l_h(r_h^2 - (r_h - 0.01)^2)\Big)_{hub} + \Big(0.08\Big)_{rest} \tag{19}$$

This propulsor mass can therefore be determined as a function of geometric properties r_c, r_h and t. Combining with diffuser mass means the total engine mass is determined by the above geometric properties as well as flow conditions ϕ, ψ and the ESDU separation limit, θ.

2.5 Work Analysis

In order to consider the relative merits of the propulsor, the power output must be considered. An energy balance gives the output power as

$$P = \frac{1}{2}\dot{m}V_4^2 = \frac{\rho A_x V_x^3}{2\sigma^2} \tag{20}$$

Inserting equation (4) into the above yields an expression for power in terms of the required thrust

$$P = \frac{T_T^{3/2}}{\sqrt{4\sigma\rho A_x}} \tag{21}$$

Hence, using equation (12), this can be expressed solely in terms of geometric and flow quantities

$$P = \frac{\phi^3 \rho A_x U^3}{2\sigma^{7/2}} \tag{22}$$

2.6 Figure of Merit

The Figure of Merit, M_f, is a non-dimensional descriptor of a given propulsor in terms of the thrust delivered, power output and flow area.

$$M_f = \frac{T_T}{P}\sqrt{\frac{T_T}{2\rho A_x}} \tag{23}$$

For an standard propeller, this has a value of 1. An ideal ducted fan with a diffusion ratio σ can be shown to have a figure of merit of $\sqrt{2\sigma}$ (figure 1 shows value of $M_f = 1.414$ for $\sigma = 1$). However, the increase in M_f comes at the cost of weight. This can be accounted for by assuming the mass model described in section 2.4.1.

As this has been produced assuming ideal operating conditions (such as no loss), suggests a design that maximises flow coefficient whilst minimising stage loading. In an ideal world this is logical as stage loading has the largest influence on the power term. From (22) and (14)

$$P = \frac{2\psi^2 \rho A_x U^3}{\phi} \tag{24}$$

Hence stage loading has an order larger impact on power than flow coefficient. Thrust can be expressed purely in terms of stage loading. Similarly to above, taking (12) and (14)

$$T_T = 2\psi\rho A_x U^2 \tag{25}$$

Hence thrust increases with ψ, having no dependence on ϕ and σ. Combining these two (noting that the power term is much larger than the thrust term, and so it's effect is dominant) results in the maximum M_f being located in the bottom right corner of figure 1. M_f is a measure of effectiveness of the propulsor but does not take into account either efficiency of the system or the mass increase associated with changing the design. This sets a lower limit on the figure of merit plot, found by considering that the thrust output needs to be greater than the total mass the propulsor is required to lift, M. The ratio of T_T to M is defined as the thrust excess, $\tilde{T}_E$

The line of $\tilde{T}_E = 1$ therefore operates as a lower bound for ψ at a given ϕ in order to satisfy the minimum thrust condition. Hence figure 3 shows the region of the M_f in which the design flow conditions can be chosen.

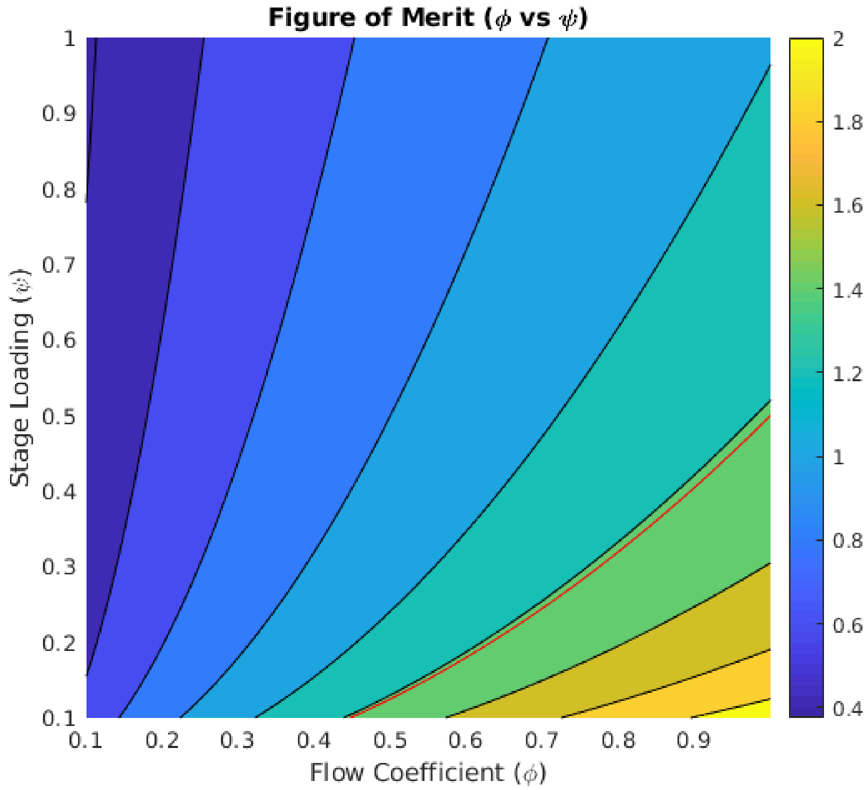


Figure 1: Plot of Figure of Merit for a ducted fan. Red line indicates location of ducted fan with $\sigma = 1$.

Consideration of Smith charts from machines with similar flow Re suggests the propulsor should limit the maximum value of flow coefficient so as not to obtain an efficiency penalty. Therefore a flow condition operating point of $\phi = 0.85$, $\psi = 0.25$ and a nominal motor RPM of 7500 is chosen. With this information, the propulsor size can be optimised to reduce the mass as much as possible. This analysis fixes the variables mentioned above as well as the hub radius, r_h, floating the casing radius, r_c, to determine the minimum radius possible whilst maintaining sufficient thrust. The above analysis provides a value of $r_c = 0.0556m$. The operating point is summarised below.

Flow coefficient	ϕ	0.85
Stage loading	ψ	0.25
Diffusion ratio	σ	1.202
Ideal Thrust	T_T	4.740 N
Ideal Power	P	48.350 W
Figure of Merit	M_f	1.515
Diffuser Length	L_D	120 mm
Casing radius	r_c	55.6 mm

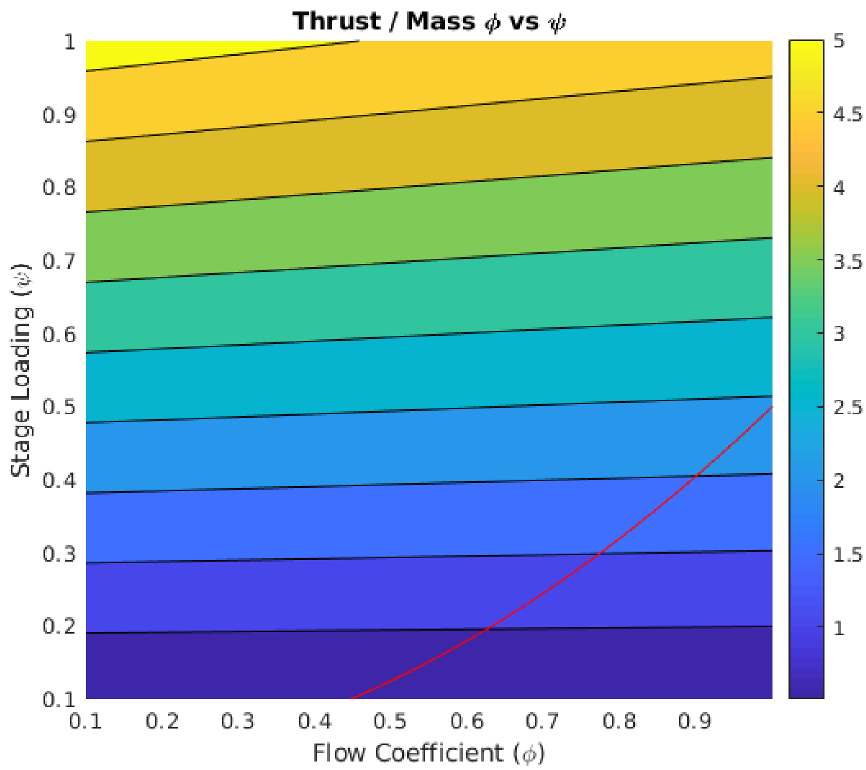


Figure 2: Thrust excess for a ducted fan with fixed hub and casing radius. Red line indicates location of ducted fan with $\sigma = 1$.

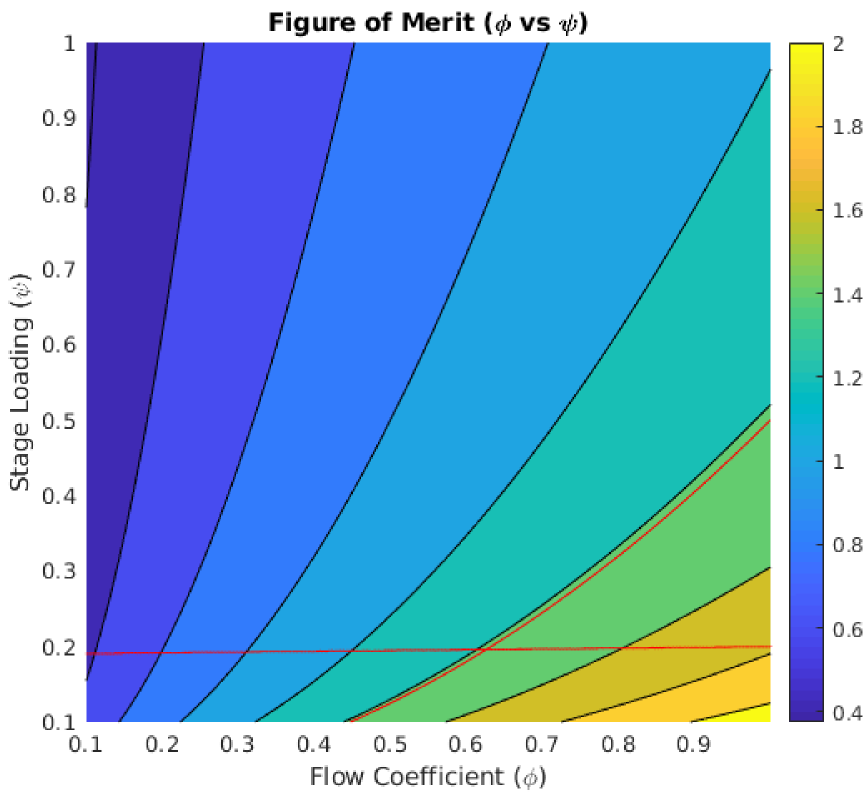


Figure 3: Plot of Figure of Merit for a ducted fan. Red lines indicate operating limits.