VIT-AP UNIVERSITY

Computer Vision

(Course Code: 4047)

Module-2:Lecture-9: Gabor Filters

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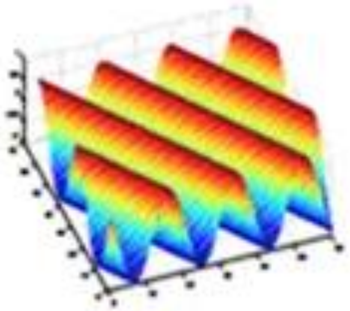
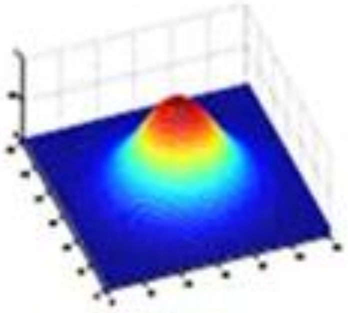
What is Gabor Filter?

- A Gabor filter can be viewed as a sinusoidal signal of particular frequency and orientation, modulated by a Gaussian wave.
- Gabor Filter is named after Dennis Gabor, a brilliant Nobel prize (Physics, 1971, Holography) winning physicist.
- Gabor filters are orientation-sensitive filters.
- Gabor filters are special classes of bandpass filters, i.e., they allow a certain 'band' of frequencies and reject the others.
- The Localized frequency and orientation representation of Gabor filters is very similar to the human visual cortex's representation and discrimination of texture
- A Gabor filter analyzes a specific frequency content at a specific direction in a local region of an image.
- Gabor filters possess optimal localization properties in both spatial and frequency domains
- Used in image processing for edge detection, texture analysis, feature extraction etc. Widely used in Signal processing.



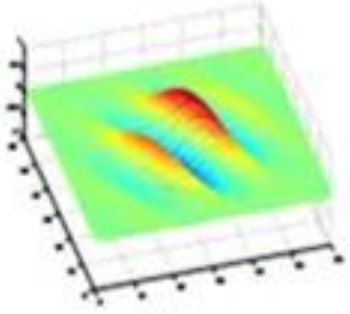
(a)

A Sinusoid oriented 30^0 with X-axis



(b)

A 2-D Gaussian



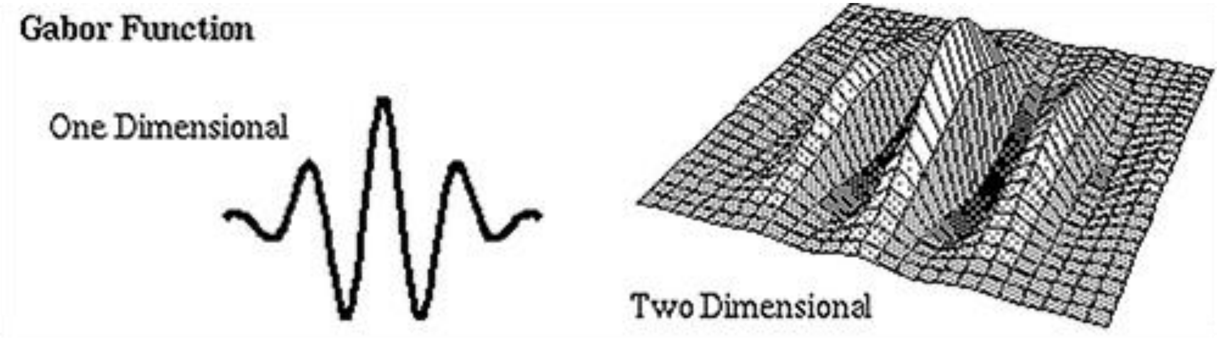
(c)

The corresponding 2-D Gabor filter

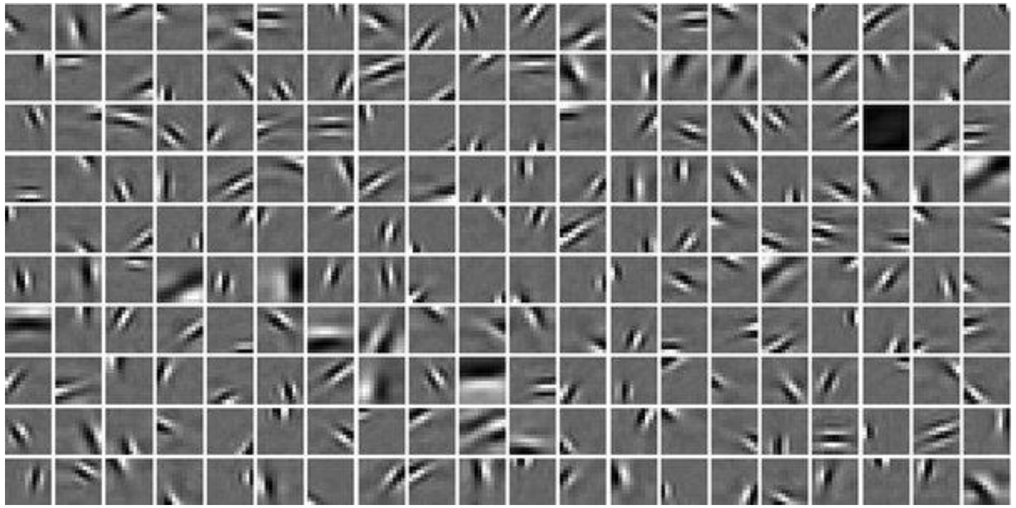
Gabor filters in implemented in Visual area of the Brain

- ❖ It is believed that the first stage of visual processing in the brain, called area V1, implements a "filter bank" of "Gabor filters":
- ❖ These "edge detectors" (or orientation-tuned neurons) fire when an edge is present at a certain location and orientation in the visual receptive field.

Here is what a Gabor function looks like when plotted:

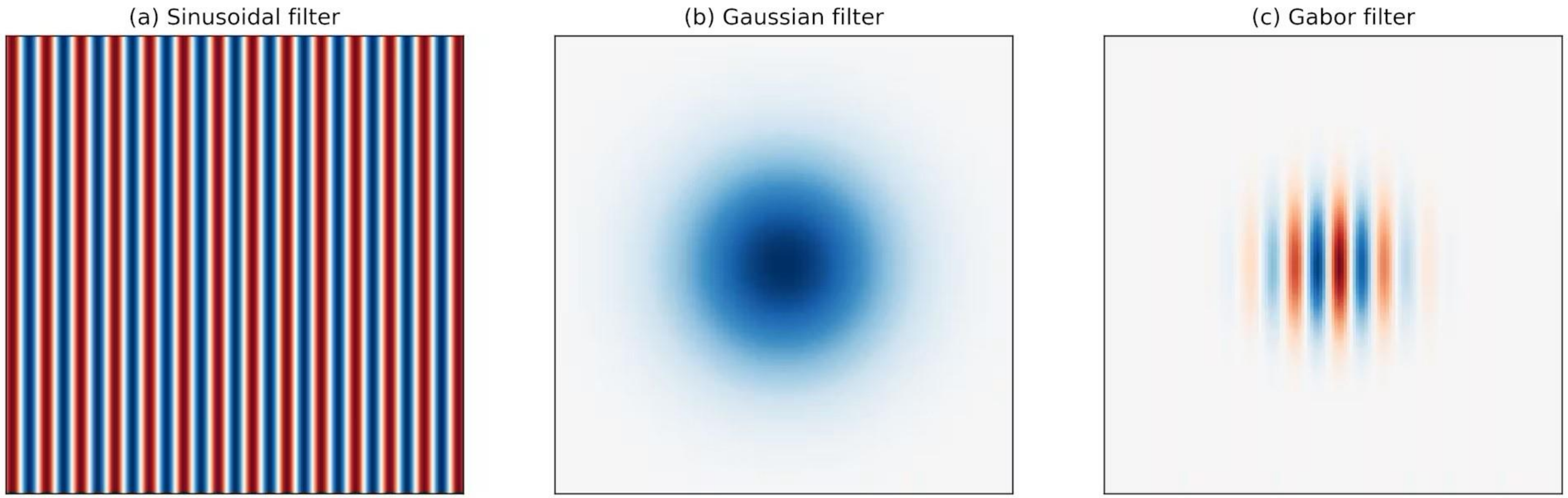


Gabor filters, believed to exist in V1 (Olshausen & Field, 1997)



- ❖ The Gabor filter model helps explain several important visual processing phenomena:
- **Edge Detection**: The ability of V1 neurons to extract edges and contours is critical for object recognition and scene understanding.
- **Texture Perception**: Gabor filters are effective for analyzing textures, which is crucial for visual perception in complex scenes.
- **Visual Attention**: The orientation and frequency selectivity of V1 neurons contribute to how visual attention is directed in the presence of competing stimuli.

Gabor Filter



Constructing a Gabor filter is straightforward. The product of
(a) a complex sinusoid signal and
(b) a Gaussian filter produces
(c) a Gabor filter.

A Gabor filter can be viewed as a sinusoidal signal of particular frequency and orientation modulated by a Gaussian wave. Mathematically, it can be defined as

$$G_{\lambda,\theta,\phi,\sigma,\gamma}(x,y) = \exp\left(-\frac{x'^2+\gamma^2 y'^2}{2\sigma^2}\right)\exp\left(j\left(2\pi\frac{x'}{\lambda}+\phi\right)\right)$$

with

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

λ — Wavelength of the sinusoidal component.

Θ — The orientation of the normal to the parallel stripes of the Gabor function.

Ψ — The phase offset of the sinusoidal function.

σ — The sigma/standard deviation of the Gaussian envelope

ɣ — The spatial aspect ratio and specifies the ellipticity of the support of the Gabor function

The above-mentioned five parameters control the shape and size of the Gabor function.

Starting Values: Lambda (λ) = 30; Theta (*Θ)* = 00, Gamma (ɣ) = 0.25; Sigma (σ) = 10, Psi (*Ψ*) = 0

How does a Gabor filter work?

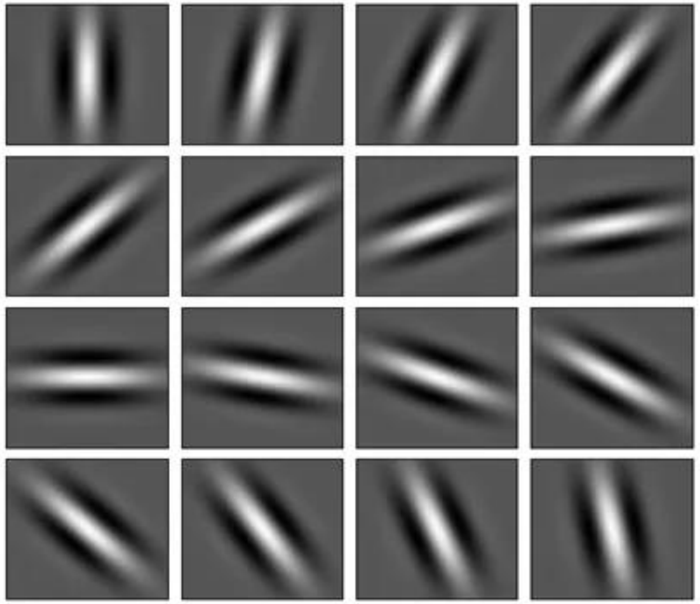
- Gabor filter is mathematically structured in such a way that it can take care of different shapes, sizes and smoothness levels in the image.
- A Gabor filter oriented in a particular direction gives a strong response for locations of the target images that have structures in this given direction.
- For example, if your target image is made up of edges in the diagonal direction, a Gabor filter set will give you a strong response only if its direction matches with the direction of the edges.
- It’s like pouring a solidifying liquid onto a unknown rigid structure. Once it solidifies and you take a look at the solidified form, it looks like the surface of the structure. Gabor filter does something similar for images!

Gabor filter on Images

- Think of Gabor Filters as a sine wave imposed on a bell curve (Gaussian) in 2D space.
  - In 2D space, the sine wave is directional.
  - Meanwhile the bell curve peaks at a point and tapers off in all directions.
  - Combining the two results in a directional wave that focuses on a point in space within a local area.

- When applying a Gabor filter to an image, we see several favorable properties:
  - It emphasizes edge pixels.
  - Pixels immediately adjacent become suppressed.
  - It is directional
  - Pixels farther away have less influence (Local area)

Extracting features on Elephant skin



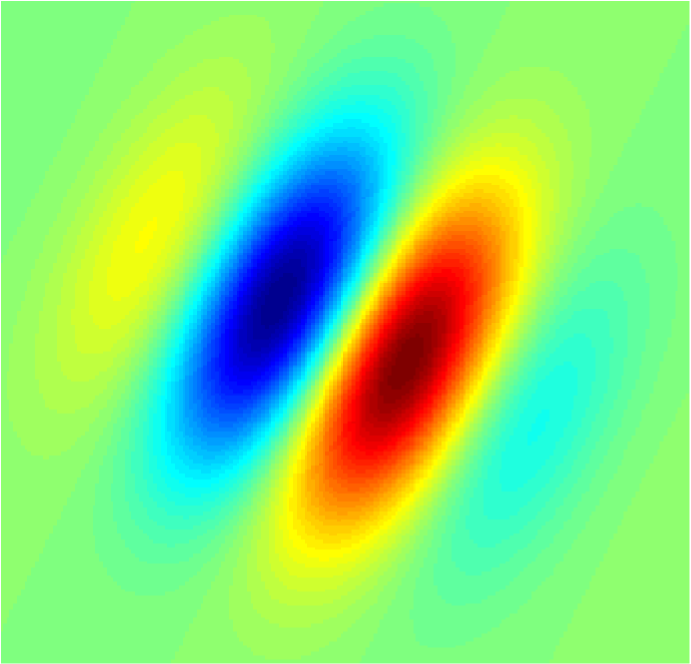
A bank of 16 Gabor filters oriented at an angle of 11.250 (i.e. if the first filter is at 00, then the second will be at 11.250, the third will be at 22.50, and so on.)

- Consider an example of an elephant that has patterns or stripes on its skin at different orientations.
- To highlight or extract out all those patterns, a bank of 16 Gabor filters at different orientation are used.
- When the input image is convolved with all the Gabor filters the patterns are easily highlighted as shown in Figure below.
- When a Gabor filter is applied to an image, it gives the highest response at edges and at points where texture changes. When we say that a filter responds to a particular feature, we mean that the filter has a distinguishing value at the spatial location of that feature.

a b

Figure 3: (a) The input image of an Elephant and (b) the output image after passing it through the Gabor filter bank

Gabor Filter parameters



- The parameters λ, θ, ψ are parameters for the sinusoidal part (or factor).
- λ and ψ are basic parameters of a sine wave.
- **λ** - controls the Wavelength of this sinusoid. A higher λ would "widen" the ellipses you see in the image above, while a lower λ would have the opposite.
- **θ** - controls the rotation of the ellipses you see in the image above. These ellipses are the "parallel stripes". A value of θ = 0 indicates no rotation and would make the ellipses vertical.
- **ψ** - is the phase shift of the sinusoid i.e., how much the ellipses need to be shifted with respect to the center. It's a parameter that has to be specified.
- **σ** - is the standard deviation (i.e., the spread) of the Gaussian part.
- **γ** - controls the aspect ratio of the ellipses (roughly, how elongated the ellipses need to be).

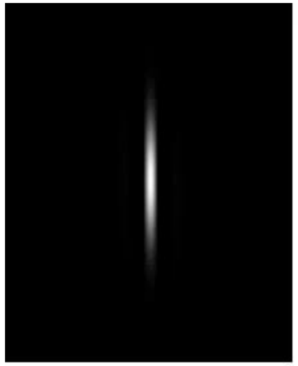
The various parameters specified above are the inputs.

The output value of the calculation is simply the co-efficient (or weight) of the filter at the (x, y) location.

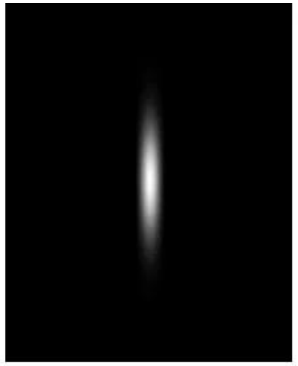
Understanding Gabor filter parameters

Keeping other parameters unchanged

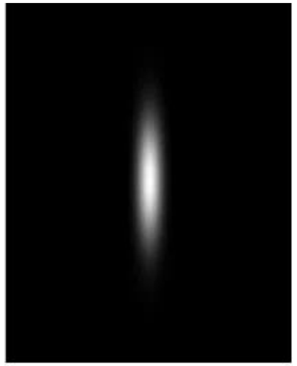
The wavelength governs the width of the strips



Lambda (λ) = 30



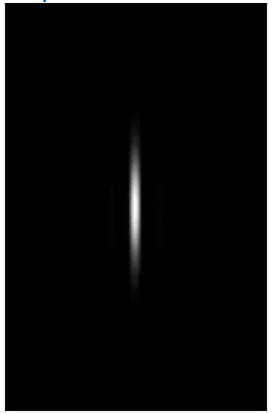
Lambda (λ) = 60



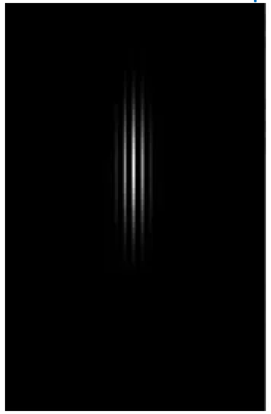
Lambda (λ) = 100

($\Theta = 0^0$, $\gamma = 0.25$, $\sigma = 10$, $\Psi = 0$), and on changing the lambda from 30 to 60 and 100 the Gabor function gets thicker

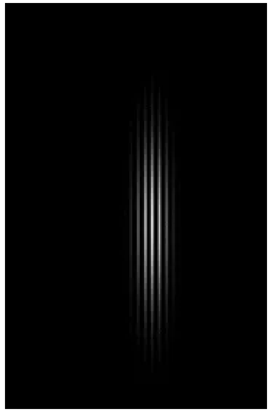
The bandwidth or sigma controls the overall size of the Gabor envelope. For larger bandwidth the envelope increase allowing more stripes and with small bandwidth the envelope tightens.



Sigma (σ) = 10



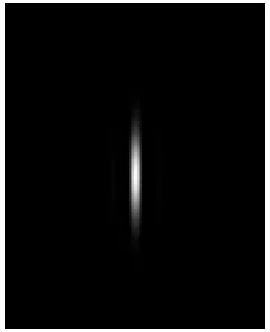
Sigma (σ) = 30



Sigma (σ) = 45

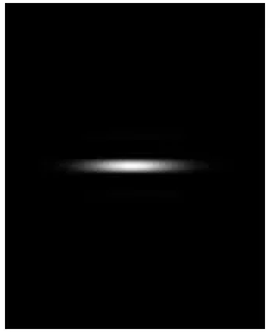
($\lambda = 30$, $\Theta = 0^0$, $\gamma = 0.25$, $\Psi = 0$), and on changing the sigma from 10 to 30 and 45 the number of stripes in the Gabor function increases

The theta controls the orientation



Theta (Θ) = 0

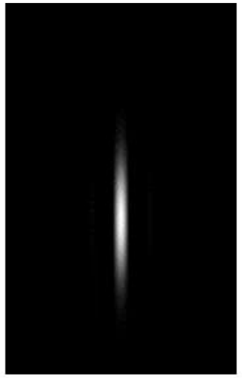
Theta (Θ) = 45



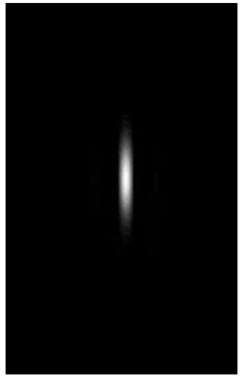
Theta (Θ) = 90

($\lambda = 30$, $\gamma = 0.25$, $\sigma = 10$, $\Psi = 0$), and on changing the theta from 0^0 to 45^0 and 90^0 the Gabor function rotates

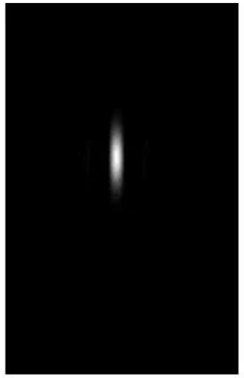
The aspect ratio or gamma controls the height



Gamma (γ) = 0.25



Gamma (γ) = 0.5



Gamma (γ) = 0.75

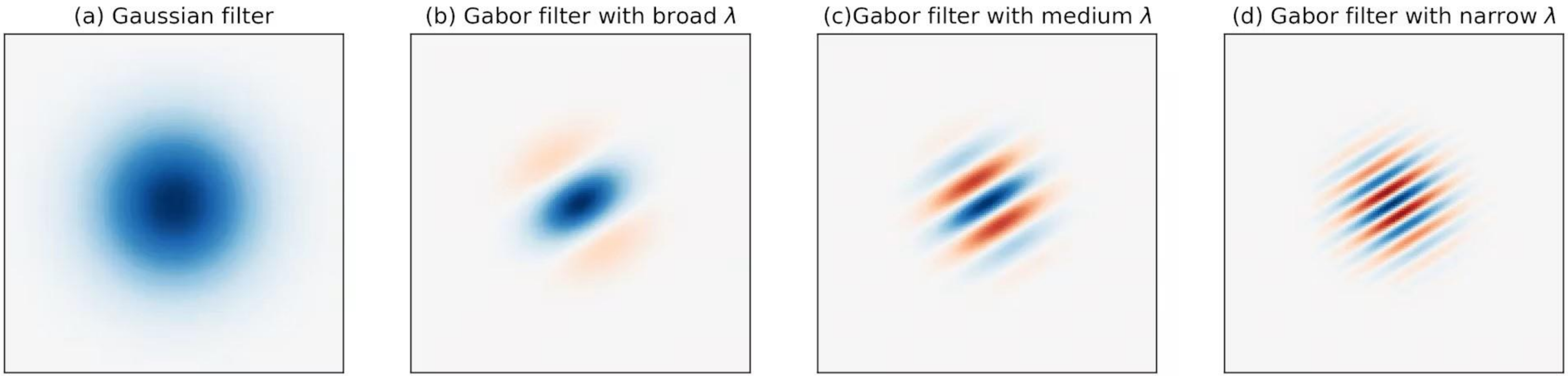
($\lambda = 30$, $\Theta = 0^0$, $\sigma = 10$, $\Psi = 0$), and on changing the gamma from 0.25 to 0.5 and 0.75, the Gabor function gets shorter.

Gabor Filter example with varying wavelength

The following figure shows a series of Gabor filters at a 45^0 angle in increasing spectral selectivity.

While the leftmost Gabor wavelet resembles a Gaussian, the rightmost Gabor wavelet follows a harmonic function and selects a very narrow band from the spectrum.

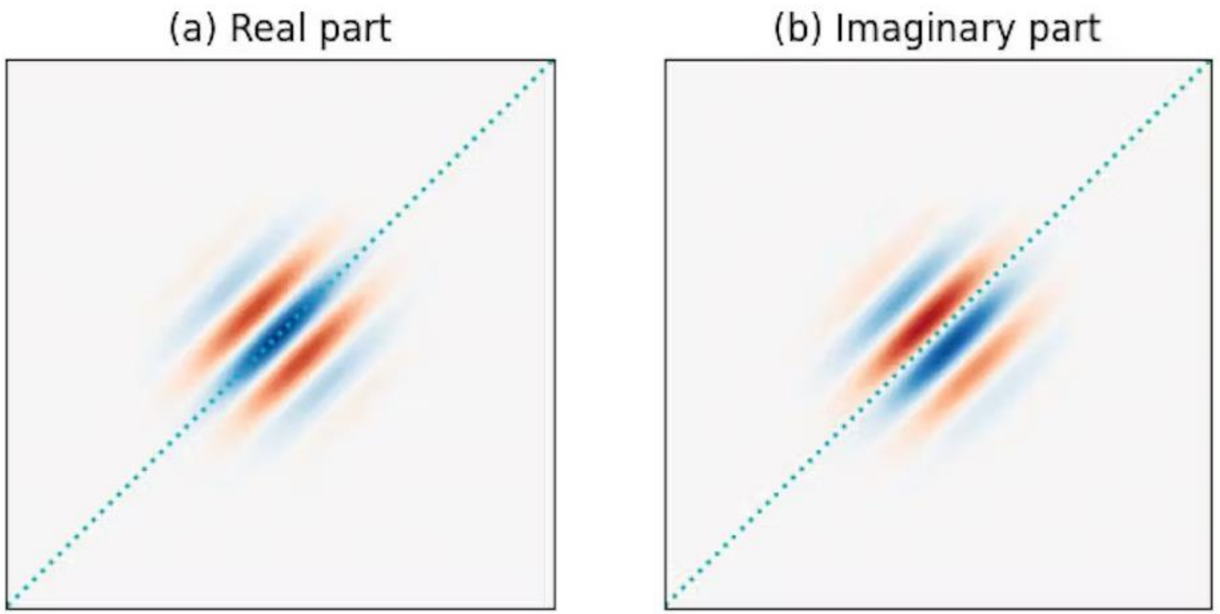
Best for iris feature extraction are the ones in the middle between the two extremes.



Varying wavelength (a-d) from large to small can change the spectral selectivity of Gabor filters from broad to narrow.

Gabor filter: Real and imaginary parts

Gabor filter is a complex filter, the real and imaginary parts act as two filters in quadrature



(a) The real part is even-symmetric and will give a strong response to features such as lines
(b) The imaginary part is odd-symmetric and will give a strong response to features such as edges

Complex

$$g(x,y;\lambda,\theta,\psi,\sigma,\gamma)=\exp\left(-\frac{x'^2+\gamma^2y'^2}{2\sigma^2}\right)\exp\left(i\left(2\pi\frac{x'}{\lambda}+\psi\right)\right)$$

Real

$$g(x,y;\lambda,\theta,\psi,\sigma,\gamma)=\exp\left(-\frac{x'^2+\gamma^2y'^2}{2\sigma^2}\right)\cos\left(2\pi\frac{x'}{\lambda}+\psi\right)$$

Imaginary

$$g(x,y;\lambda,\theta,\psi,\sigma,\gamma)=\exp\left(-\frac{x'^2+\gamma^2y'^2}{2\sigma^2}\right)\sin\left(2\pi\frac{x'}{\lambda}+\psi\right)$$

where

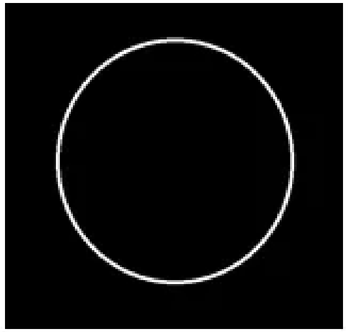
$$x' = x\cos\theta + y\sin\theta$$

and

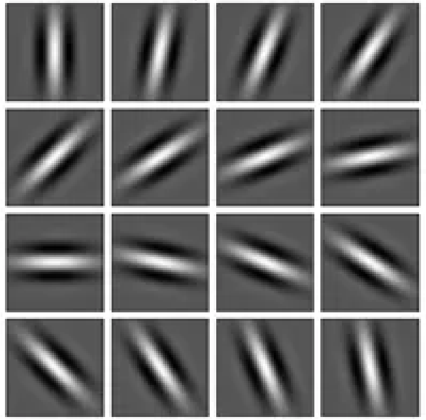
$$y' = -x\sin\theta + y\cos\theta$$

Gabor Filter Example

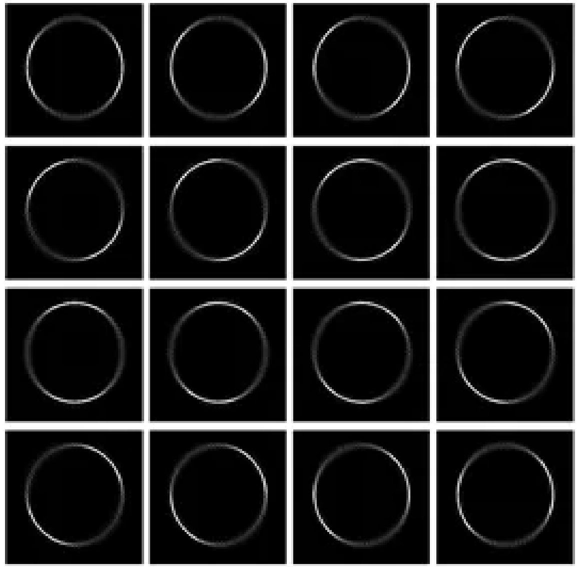
Consider a simple white circle on black background. When this image is passed through each filter in the filter bank, the edge of the circle which gets detected is the edge oriented at an angle at which the Gabor filter is oriented.



Input Image of a circle



A bank of 16 Gabor Filters

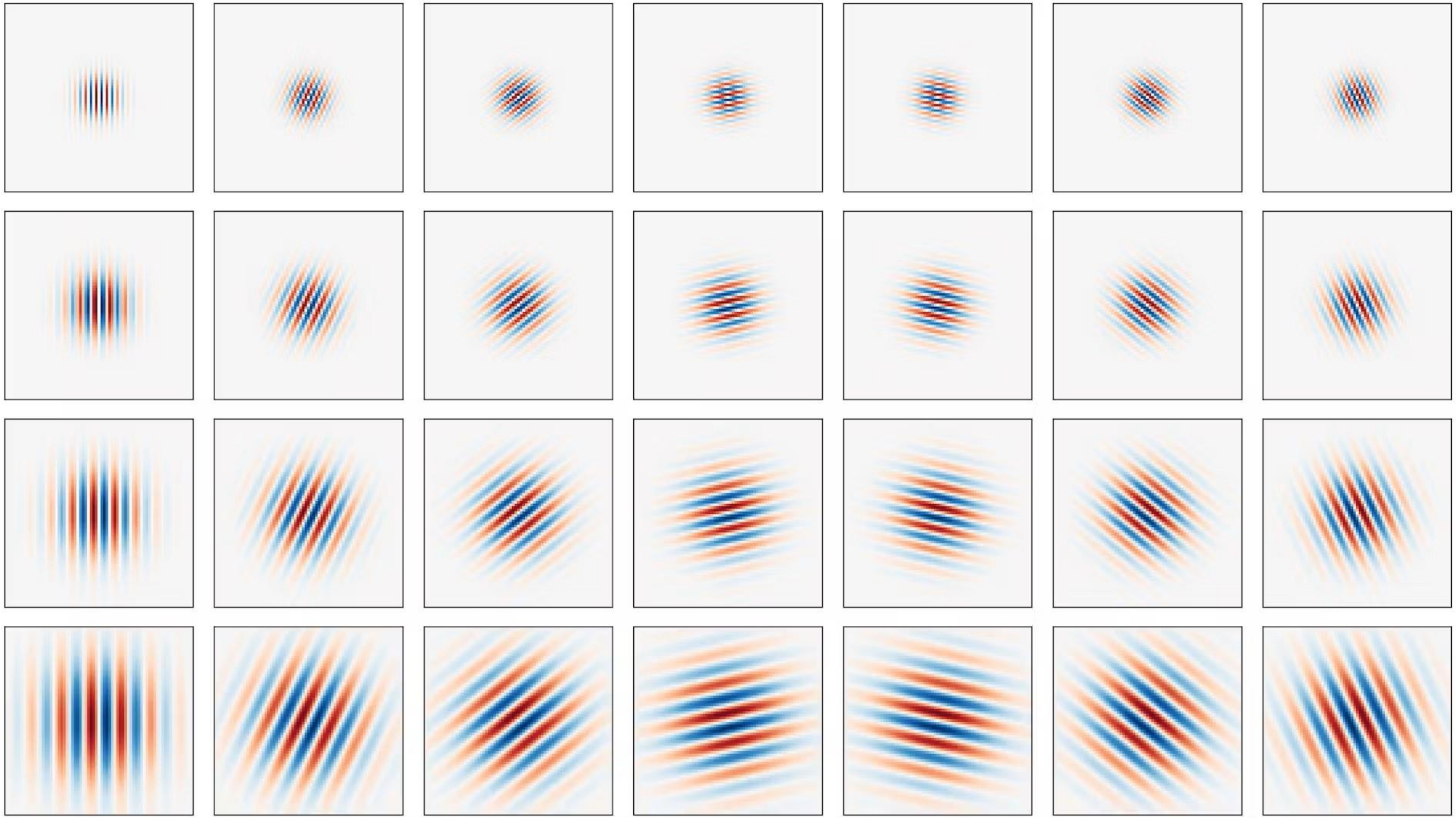


The output circle as seen when pass through individual Gabor filter

The corresponding oriented edge features being detected when passed through individual oriented Gabor filters

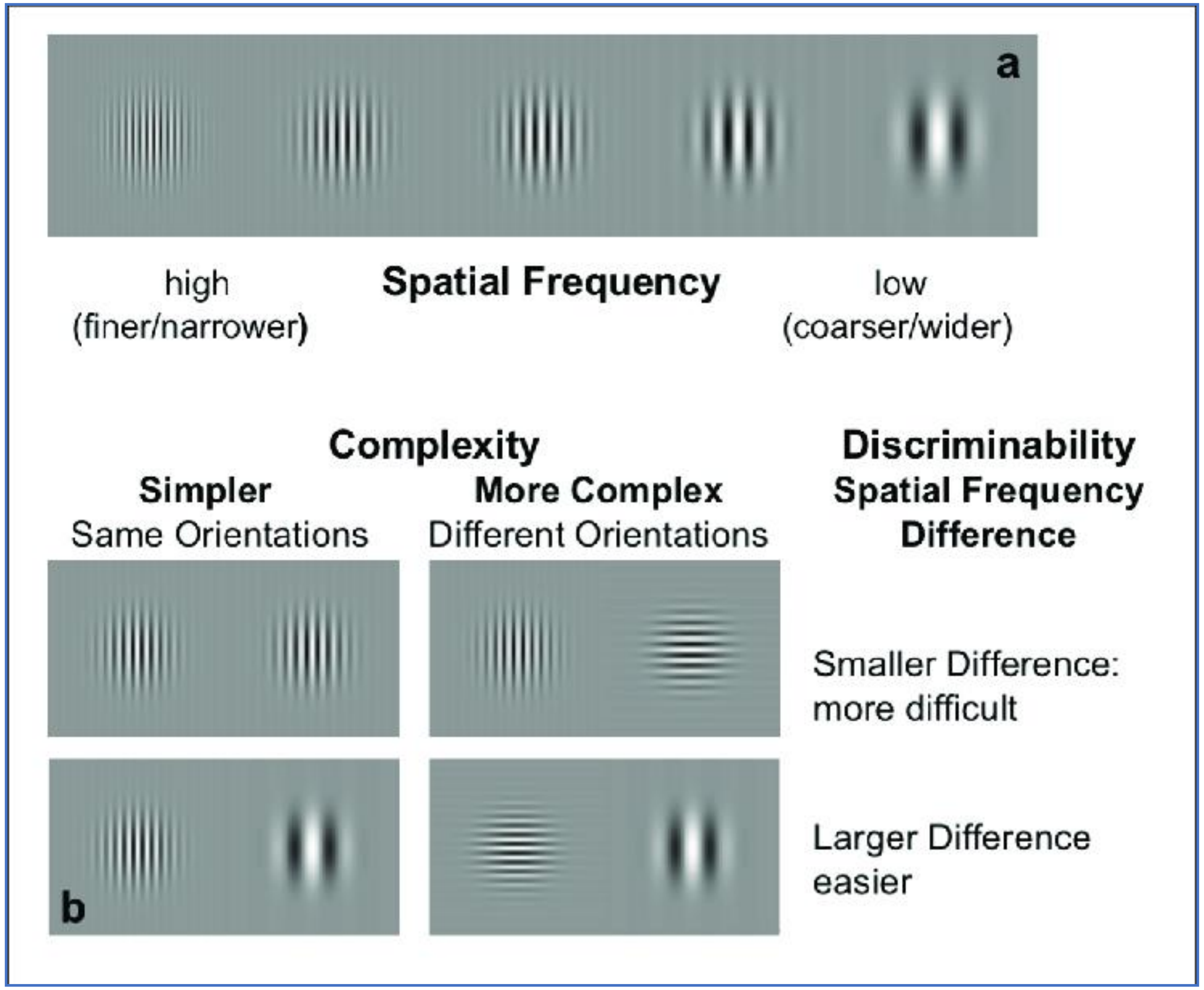
Multi-scale Gabor filtering

Gabor wavelets can be tuned to support multiple scales (controlled by σ) and multiple orientations (specified by θ).



Constructing Gabor wavelets with multiple scales (vertically) and orientations (horizontally) to extract texture features with various frequencies and directions

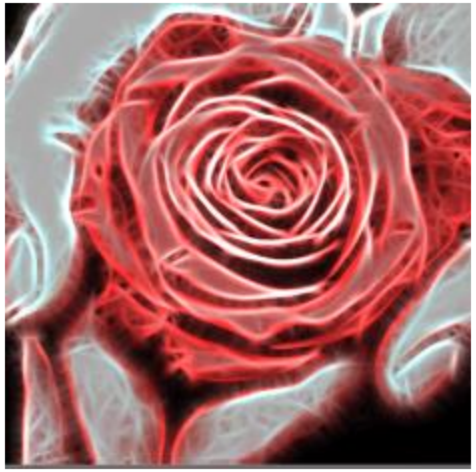
Gabor filters: Spatial Frequency Examples



Stimuli and design. (a) Gabor patch stimuli (sine-wave carrier in a Gaussian window) with examples of different sine-wave frequencies. (b) Example of discriminability levels: participants were asked to specify which pattern of the pair had the highest spatial frequency (finer/narrower lines) for various levels of difference. Top pairs: illustrate low-discriminability (less difference, more difficult), bottom pairs: illustrate high discriminability (more difference, easier). Left panel pairs: same orientation condition (lower complexity), right panel pairs: different orientation conditions (higher complexity)

Gabor filter example

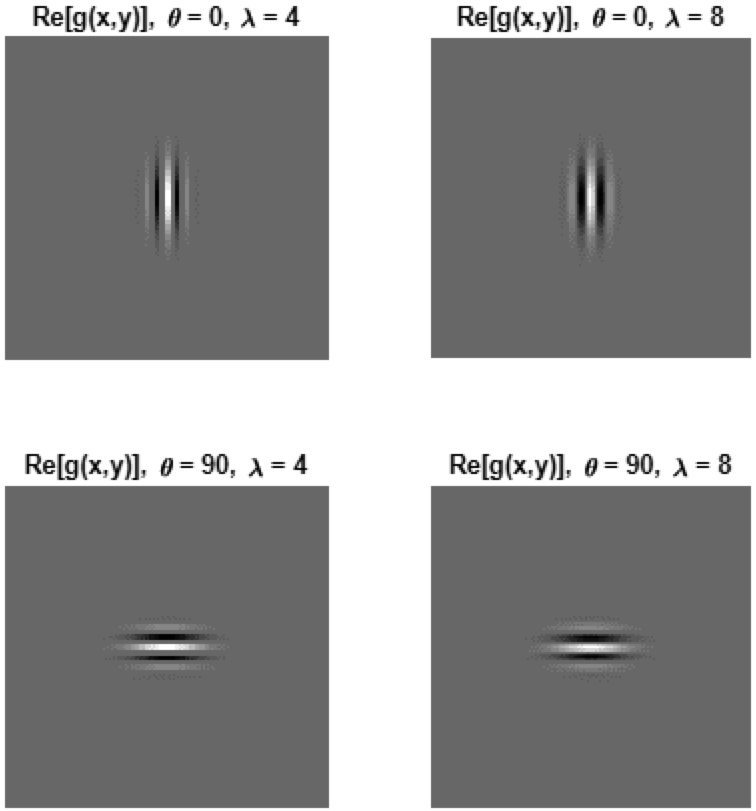
- Gabor filter responds to edges and texture changes.
- “A filter responds to a particular feature” means that the filter has a distinguishing value at the spatial location of that feature
- This is similar to applying convolution kernels in spatial domain on images
- The same holds for other domains, such as frequency domains, as well).



Gabor filter example 2

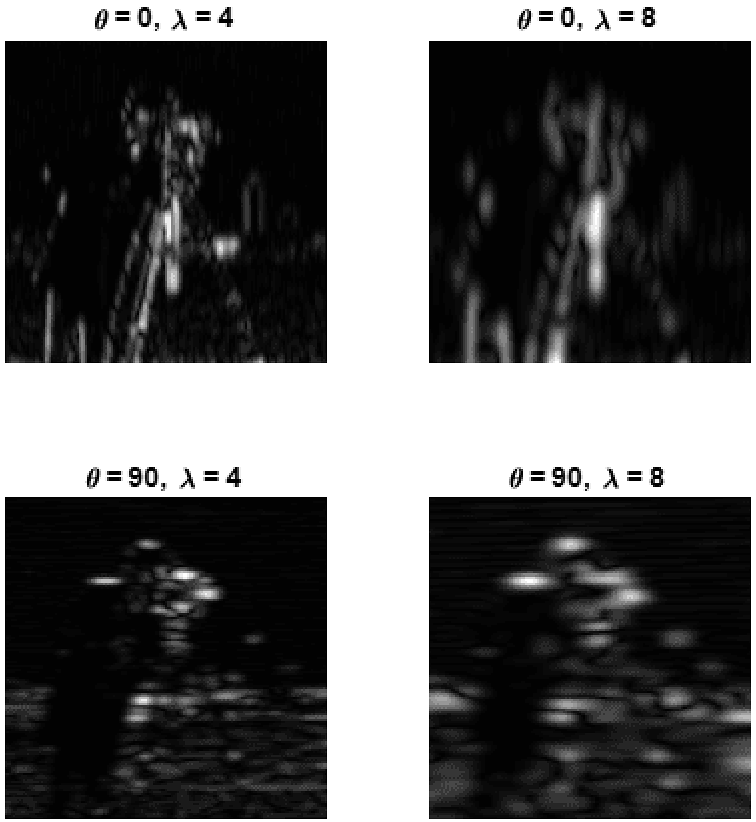
Original image

*



Gabor filters

=



Filtered images

We can see that horizontal and vertical features are extracted with θ=0 and θ=90, respectively. Furthermore, we can extract finer details with lower wavelength values.

Gabor Filter Edge Detection Example

a) Input Image

b) Applying a bank of 16 Gabor Filters (distributed over 180^0)

c) Canny Edge Detection

d) Canny Edge Detection (post Gabor)

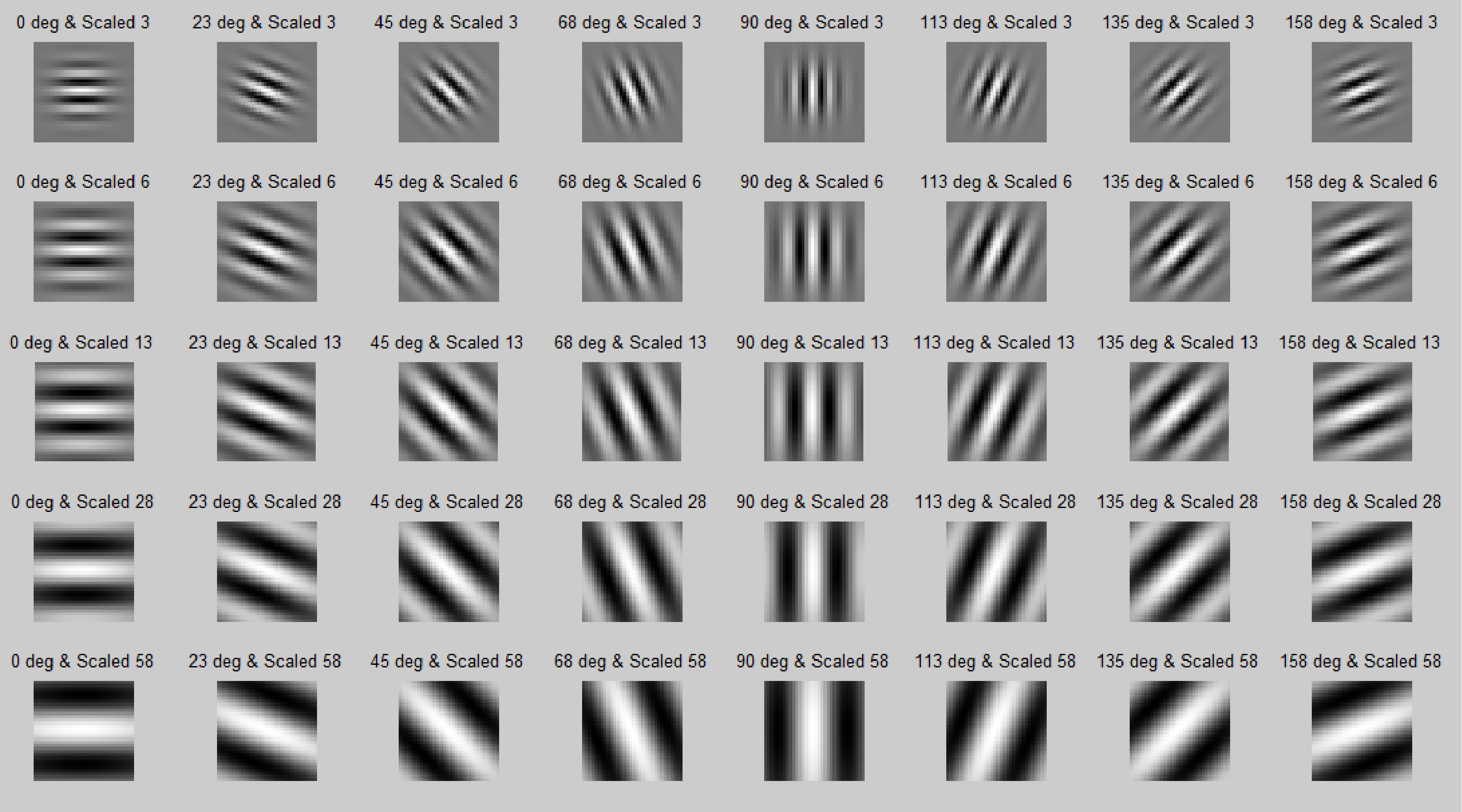
- In fig b), notice how edges have a "glow" to them. The edges of windows seem highlighted as well as edges of houses and the pier.
- Comparatively, applying Gabor Filters in Edge Detection made significant improvements.
- By and large lines are cleaner and object edges are better detected.
- Explicitly, the wooden piers and building edges are better highlighted.
- Straightaway, by applying Gabor Filter, we achieve better results than directly applying Canny.

https://www.freedomvc.com/index.php/2021/10/16/gabor-filter-in-edge-detection/

Gabor feature extraction example 1/2

Input Image

Calculate Gabor features at 5 scales and 8 orientations (will get filters like)



Now convolve each filters with the image to get 40 (8*5=40) different representation(response matrices) of same image where each image gives you a feature vector.

Gabor feature extraction example (2/2)

Response Matrices after convolution

Local Energy = summing up the squared value of each matrix value from a response matrix

Mean Amplitude = sum of absolute values of each matrix value from a response matrix

Thus at the end you will get two matrix that will be [1x40] each.
You can append one matrix to the other to create a [1x80] feature matrix for One image and thus create a [nx80] vector for n images for further training purpose

Now you need to convert those Response Matrices to feature vector.
So feature vector may consist of : Local Energy, Mean Amplitude, Phase Amplitude or Orientation whose local has maximum Energy.

Applications

Gabor wavelets are used here to detect edges, corners and blobs

5 APPLICATIONS

The following are some applications in which derivatives can be obtained by using the Gabor wavelet. The used notation is in accordance with [9]. The first order partial derivative of image I with respect to variable x is denoted by I_x. Analogously, I_{xx} denotes the second order partial derivative with respect to x and I_{xy} is the second order mixed derivative. Furthermore, $I_x(\mathbf{x}, \sigma_D)$ denotes a partial derivative obtained at the location of an point $\mathbf{x}$ and calculated by using a Gabor wavelet with scale $a \propto \sigma_D$.

5.1 Edge detection

For the edge detection, the convolution in two perpendicular directions is performed with variously dilated wavelets (e.g., separately in row and column directions). It is necessary to use a wavelet which serves as the first order partial differential operator (e.g., a first derivative of a Gaussian function [8]). Consequently, local maxima of moduli

$$M(\mathbf{x}, \sigma_D) = \sqrt{I_x^2(\mathbf{x}, \sigma_D) + I_y^2(\mathbf{x}, \sigma_D)} \tag{4}$$

are found. Only the maxima above a given threshold are considered (due to noise and slight edges). As a result, the edges for each scale are obtained.

Another approach for the edge detection is to use a wavelet which serves as the second order partial differential operator. In this case, edges are located in a zero-crossing points (sign changes, in the discrete case) [4]. This approach is used in detectors based on Laplacian, Laplacian of Gaussian (LoG) and Difference of Gaussians (DoG), see [4].

5.2 Corner detection

The key idea is to obtain the partial derivatives needed for a construction of an autocorrelation matrix (second moment matrix) [9]

$$\mu(\mathbf{x}, \sigma_I, \sigma_D) = \sigma_D^2 g(\sigma_I) * \begin{bmatrix} I_x^2(\mathbf{x}, \sigma_D) & I_x I_y(\mathbf{x}, \sigma_D) \\ I_x I_y(\mathbf{x}, \sigma_D) & I_y^2(\mathbf{x}, \sigma_D) \end{bmatrix} \tag{5}$$

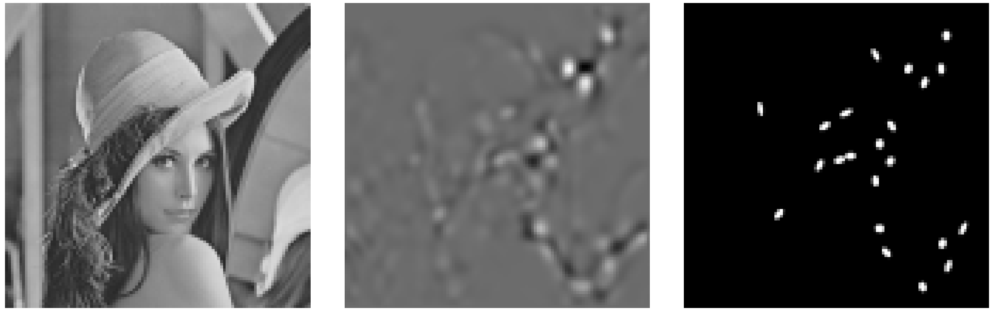
by using the convolution with the Gabor wavelets. A Gaussian window of σ_I scale is used for averaging of the derivatives. On this matrix, e.g., Harris & Stephens [3], Förstner and Shi-Tomasi detectors are based, see [5]. Also here, it is necessary to use such a Gabor wavelet which serves as the first order partial differential operator.

5.3 Blob detection

Following the same principle, blobs can be detected [9] from the second order partial derivatives using a Hessian matrix

$$H(\mathbf{x}, \sigma_D) = \begin{bmatrix} I_{xx}(\mathbf{x}, \sigma_D) & I_{xy}(\mathbf{x}, \sigma_D) \\ I_{xy}(\mathbf{x}, \sigma_D) & I_{yy}(\mathbf{x}, \sigma_D) \end{bmatrix}. \tag{6}$$

An example of such a blob detection is shown in Figure 3. Here, the complex Gabor wavelet serving as the first order partial differential operator has been used.



(a) (b) (c)

Figure 3: Detection of affine covariant features: (a) input image, (b) determinant of the Hessian, (c) the detected affine blobs.

https://www.fit.vut.cz/research/publication-file/9598/postprint.pdf

References

- https://medium.com/@anuj_shah/through-the-eyes-of-gabor-filter-17d1fdb3ac97
- Tutorial on Gabor Filters
- https://www.freedomvc.com/index.php/2021/10/16/gabor-filter-in-edge-detection/
- https://www.quora.com/How-are-Gabor-filters-implemented-in-visual-area-V1-in-the-brain
- https://kgeorge.github.io/2016/02/04/understanding-gabor-filter
- https://www.cogsci.nl/pages/gabor-generator.php
- https://answers.opencv.org/question/1066/how-to-intuitively-interpret-gabor-lambda-param/
- https://prateekvjoshi.com/2014/04/26/understanding-gabor-filters/
- https://cvtuts.wordpress.com/2014/04/27/gabor-filters-a-practical-overview/
- https://www.baeldung.com/cs/ml-gabor-filters