Computer Vision

(Course Code: 4047)

Module-3:Lecture-4: Spatio-Temporal Analysis

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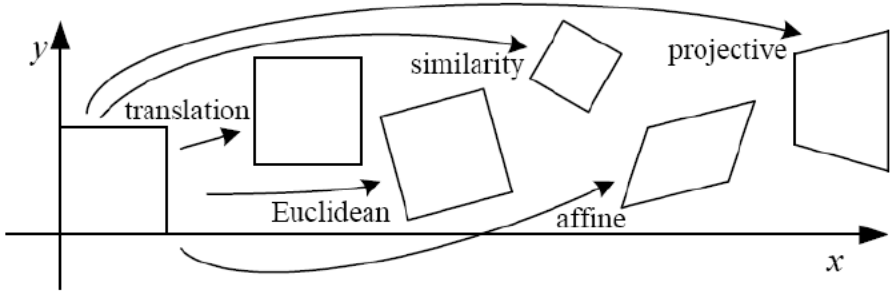
Spatio-Temporal Analysis

- Spatiotemporal models arise when data are collected across time as well as space and has at least one spatial and one temporal property.
- An event in a spatiotemporal dataset describes a spatial and temporal phenomenon that exists at a certain time t and location x.
- Typical examples of spatiotemporal data mining include
  - Video Sequence Analysis
  - Action Detection
- Other examples
  - Discovering the evolutionary history of cities and lands
  - Uncovering weather patterns
  - Predicting earthquakes and hurricanes and determining global warming trends.

Common 2D Transformations



- Translation

$$x' = x + a_1$$
$$y' = y + a_2$$

- Euclidean

$$x' = x\cos(a_3) - y\sin(a_3) + a_1$$
$$y' = x\sin(a_3) + y\cos(a_3) + a_2$$

- Affine

$$x' = a_1x + a_3y + a_5$$
$$y' = a_2x + a_4y + a_6$$

- Projective (homography)

$$x' = \frac{a_1x + a_2y + a_3}{a_7x + a_8y + 1}$$

$$y' = \frac{a_4x + a_5y + a_6}{a_7x + a_8y + 1}$$

Summary of Displacement Models (2-D Transformations)

Translation

$$x' = x + b_1$$
$$y' = y + b_2$$

Rigid

$$x' = x\cos\theta - y\sin\theta + b_1$$
$$y' = x\sin\theta + y\cos\theta + b_2$$

Affine

$$x' = a_1x + a_2y + b_1$$
$$y' = a_3x + a_4y + b_2$$

Projective

$$x' = \frac{a_1x + a_2y + b_1}{c_1x + c_2y + 1}$$
$$y' = \frac{a_3x + a_4y + b_2}{c_1x + c_2y + 1}$$

Bi-quadratic

$$x' = a_1 + a_2x + a_3y + a_4x^2 + a_5y^2 + a_6xy$$
$$y' = a_7 + a_8x + a_9y + a_{10}x^2 + a_{11}y^2 a_{12}xy$$

Bi-Linear

$$x' = a_1 + a_2x + a_3y + a_4xy$$
$$y' = a_5 + a_6x + a_7y + a_8xy$$

Pseudo-Perspective

$$x' = a_1 + a_2x + a_3y + a_4x^2 + a_5xy$$
$$y' = a_6 + a_7x + a_8y + a_4xy + a_5y^2$$

Displacement Models Parameterizations

Translation
$$x' = x + b_1$$
$$y' = y + b_2$$
$$W(\mathbf{x};\mathbf{p}) = (x + b_1, y + b_2)$$

Rigid
$$x' = x\cos\theta - y\sin\theta + b_1$$
$$y' = x\sin\theta + y\cos\theta + b_2$$
$$W(\mathbf{x};\mathbf{p}) = (x\cos\theta - y\sin\theta + b_1, x\sin\theta + y\cos\theta + b_2)$$

Affine
$$x' = a_1x + a_2y + b_1$$
$$y' = a_3x + a_4y + b_2$$
$$W(\mathbf{x};\mathbf{p}) = (a_1x + a_2y + b_1, a_3x + a_4y + b_2)$$

Homogenous coordinates

Translation
$$W(\mathbf{x};\mathbf{p}) = \begin{bmatrix} 1 & 0 & b_1 \\ 0 & 1 & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Rigid
$$W(\mathbf{x};\mathbf{p}) = \begin{bmatrix} c\theta & -s\theta & b_1 \\ s\theta & c\theta & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$
$$W(\mathbf{x};\mathbf{p}) = [R \mid t]_{2X3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Affine
$$W(\mathbf{x};\mathbf{p}) = \begin{bmatrix} a_1 & a_2 & b_1 \\ a_3 & a & b_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$
$$W(\mathbf{x};\mathbf{p}) = A_{2X3} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Displacement Models (Parameterizations)

Projective
$$x' = \frac{a_1 x + a_2 y + b_1}{c_1 x + c_2 y + 1}$$
$$y' = \frac{a_3 x + a_4 y + b_2}{c_1 x + c_2 y + 1}$$

$$W(\mathbf{x};\mathbf{p}) = \left(\frac{a_1 x + a_2 y + b_1}{c_1 x + c_2 y + 1}, \frac{a_3 x + a_4 y + b_2}{c_1 x + c_2 y + 1}\right)$$

Projective

$$W(\mathbf{x};\mathbf{p}) = \begin{bmatrix} a_1 & a_2 & b_1 \\ a_3 & a_4 & b_2 \\ c_1 & c_2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Common 2D Transformations (using Matrices)

We denote the transformation $W(\mathbf{x}, \mathbf{p})$ and $\mathbf{p}$ the set of parameters $p = (a_1, a_2, \dots, a_n)$

- Translation

$$W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} x + a_1 \\ y + a_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & a_1 \\ 0 & 1 & a_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Homogeneous coordinates

- Euclidean

$$W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} x\cos(a_3) - y\sin(a_3) + a_1 \\ x\sin(a_3) + y\cos(a_3) + a_2 \end{bmatrix} = \begin{bmatrix} \cos(a_3) & -\sin(a_3) & a_1 \\ \sin(a_3) & \cos(a_3) & a_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

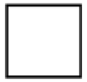
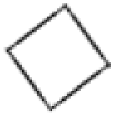
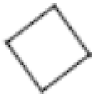
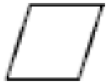
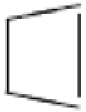
- Affine

$$W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} a_1 x + a_3 y + a_5 \\ a_2 x + a_4 y + a_6 \end{bmatrix} = \begin{bmatrix} a_1 & a_3 & a_5 \\ a_2 & a_4 & a_6 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Projective (homography)

$$W(\tilde{\mathbf{x}}, \mathbf{p}) = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Common 2D Transformations (using Matrices)

Name	Matrix	# D.O.F.	Preserves:	Icon	
translation	$\begin{bmatrix} \boldsymbol{I} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	2	orientation $+ \cdots$		$W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} 1 & 0 & a_1 \\ 0 & 1 & a_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
rigid (Euclidean)	$\begin{bmatrix} \boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	3	lengths $+ \cdots$		$W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} \cos(a_3) & -\sin(a_3) & a_1 \\ \sin(a_3) & \cos(a_3) & a_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
similarity	$\begin{bmatrix} s\boldsymbol{R} \mid \boldsymbol{t} \end{bmatrix}_{2\times 3}$	4	angles $+ \cdots$		$W(\mathbf{x}, \mathbf{p}) = a_4 \begin{bmatrix} \cos(a_3) & -\sin(a_3) & a_1 \\ \sin(a_3) & \cos(a_3) & a_2 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
affine	$\begin{bmatrix} \boldsymbol{A} \end{bmatrix}_{2\times 3}$	6	parallelism $+ \cdots$		$W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} a_1 & a_3 & a_5 \\ a_2 & a_4 & a_6 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$
projective	$\begin{bmatrix} \tilde{\boldsymbol{H}} \end{bmatrix}_{3\times 3}$	8	straight lines		$W(\tilde{x}, \mathbf{p}) = \begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$

Derivative and Gradient

- Function: $f(x)$
- Derivative: $f'(x) = \frac{df}{dx}$, where x is a scalar
- Function: $f(x_1, x_2, \ldots, x_n)$
- Gradient: $\nabla f(x_1, x_2, \ldots, x_n) = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial x_2}, \ldots, \frac{\partial f}{\partial x_n}\right)$

Jacobian

- $F(x_1, x_2, \ldots, x_n) = \begin{bmatrix} f_1(x_1, x_2, \ldots, x_n) \\ \vdots \\ f_m(x_1, x_2, \ldots, x_n) \end{bmatrix}$ is a vector-valued function

- The derivative in this case is called Jacobian $\frac{\partial F}{\partial \mathbf{x}}$:

$$\frac{\partial F}{\partial \mathbf{x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}, \ldots, \frac{\partial f_1}{\partial x_n} \\ \vdots \\ \frac{\partial f_m}{\partial x_1}, \ldots, \frac{\partial f_m}{\partial x_n} \end{bmatrix}$$

Carl Gustav Jacob Jacobi

10 December 1804—18 February 1851

- Made fundamental contributions to elliptic functions, dynamics, differential equations, and number theory.
- Jacobi was the first Jewish mathematician to be appointed professor at a German university.[2]
- In 1825 he obtained the degree of Doctor of Philosophy.
- He followed immediately with his Habilitation and at the same time converted to Christianity.
  - Now qualifying for teaching University classes, the 21 year old Jacobi lectured in 1825/26 on the theory of curves and surfaces at the University of Berlin.[4][5]
- Jacobi suffered a breakdown from overwork in 1843. He then visited Italy for a few months to regain his health.
- Jacobi died in 1851 from a smallpox infection.
- The crater Jacobi on the Moon is named after him.

Displacement-model Jacobians ∇W_p

p is a set of parameters that control the transformation

$$p = (a_1, a_2, \dots, a_n)$$

- Translation: $W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} x + a_1 \\ y + a_2 \end{bmatrix}$ $\quad \dfrac{\partial W}{\partial \mathbf{p}} = \begin{bmatrix} \dfrac{\partial W_1}{\partial a_1} & \dfrac{\partial W_1}{\partial a_2} \\ \dfrac{\partial W_2}{\partial a_1} & \dfrac{\partial W_2}{\partial a_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

- Euclidean: $W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} x\cos(a_3) - y\sin(a_3) + a_1 \\ x\sin(a_3) + y\cos(a_3) + a_2 \end{bmatrix}$ $\quad \dfrac{\partial W}{\partial \mathbf{p}} = \begin{bmatrix} 1 & 0 & -x\sin(a_3) - y\cos(a_3) \\ 0 & 1 & x\cos(a_3) - y\sin(a_3) \end{bmatrix}$

- Affine: $W(\mathbf{x}, \mathbf{p}) = \begin{bmatrix} a_1 x + a_3 y + a_5 \\ a_2 x + a_4 y + a_6 \end{bmatrix}$ $\quad \dfrac{\partial W}{\partial \mathbf{p}} = \begin{bmatrix} x & 0 & y & 0 & 1 & 0 \\ 0 & x & 0 & y & 0 & 1 \end{bmatrix}$

References