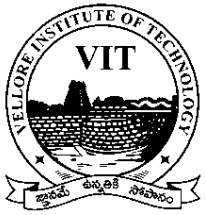


VIT-AP UNIVERSITY

Computer Vision

(Course Code: 4047)

Module-2:Lecture-7: Image Pyramids

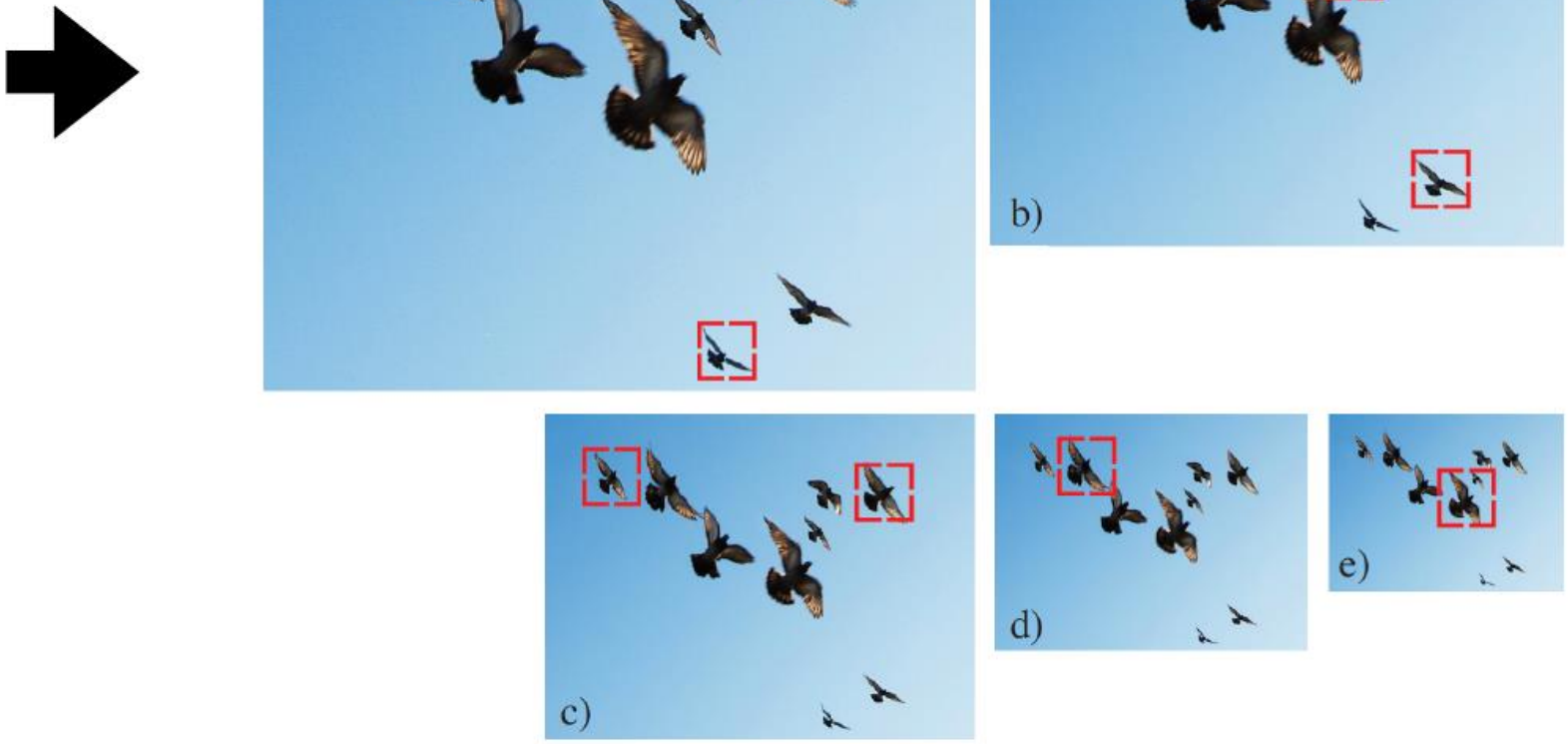
Gundimeda Venugopal, Professor of Practice, SCOPE

Scale Invariance

We want **scale** invariance

Source: Torralba, Freeman, Isola

Image Pyramids



Source: Torralba, Freeman, Isola

Image Pyramid

What are Image Pyramids used for?

Image compression

Multi-scale texture mapping

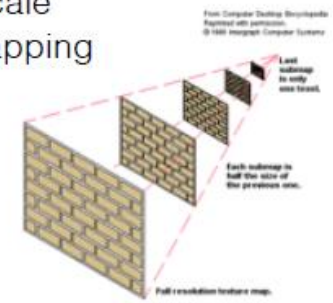


Image blending

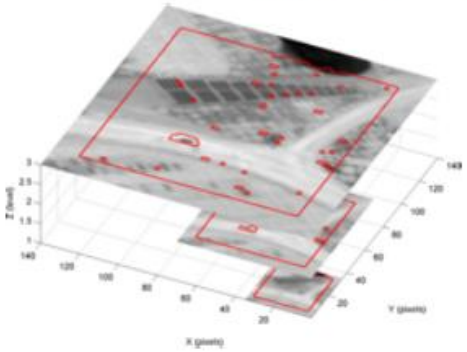
Multi-focus composites

Noise removal

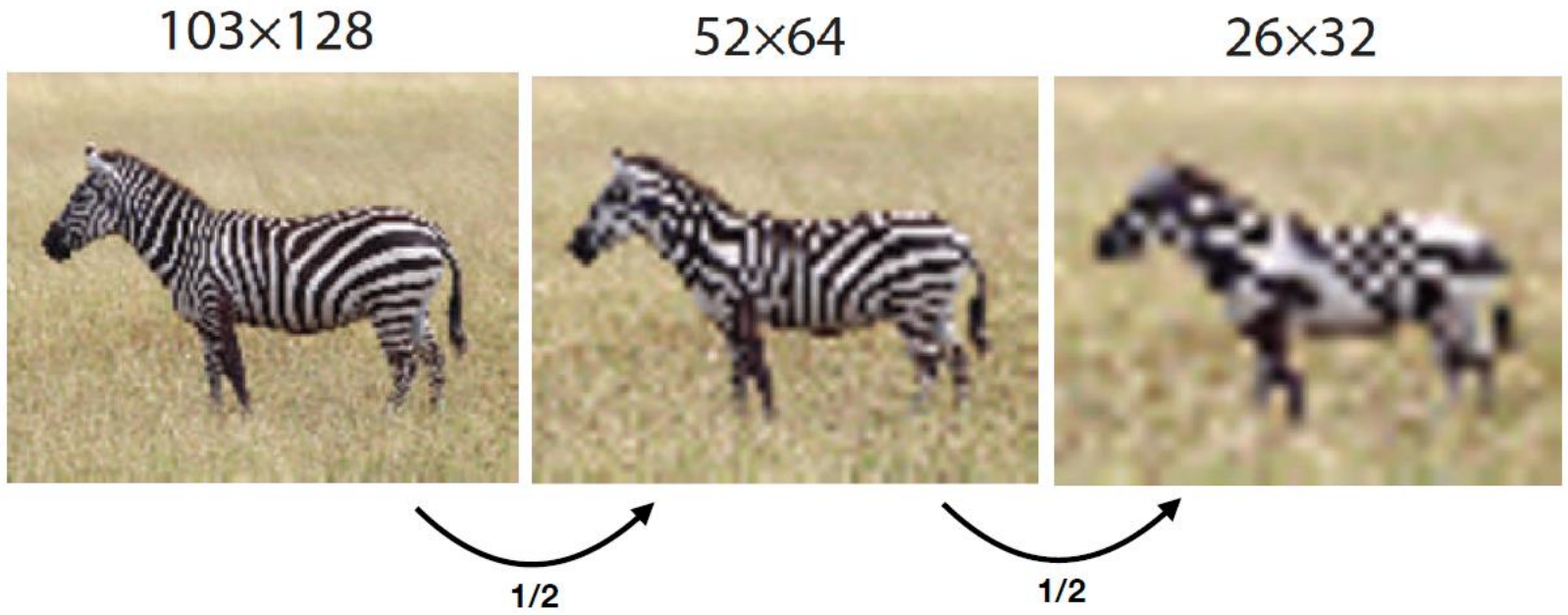
Hybrid images

Multi-scale detection

Multi-scale registration



Subsampling and Aliasing

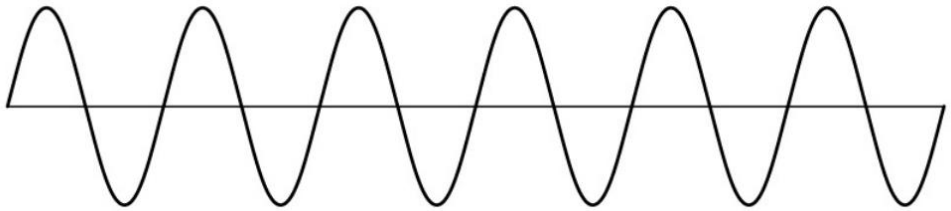


Idea #1: Throw away every other pixel.

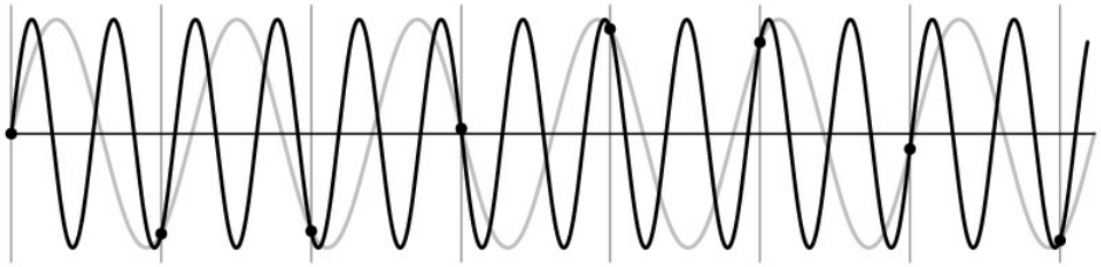
Source: Torralba, Freeman, Isola

Sinusoid and Undersampling

Consider a sinusoid:



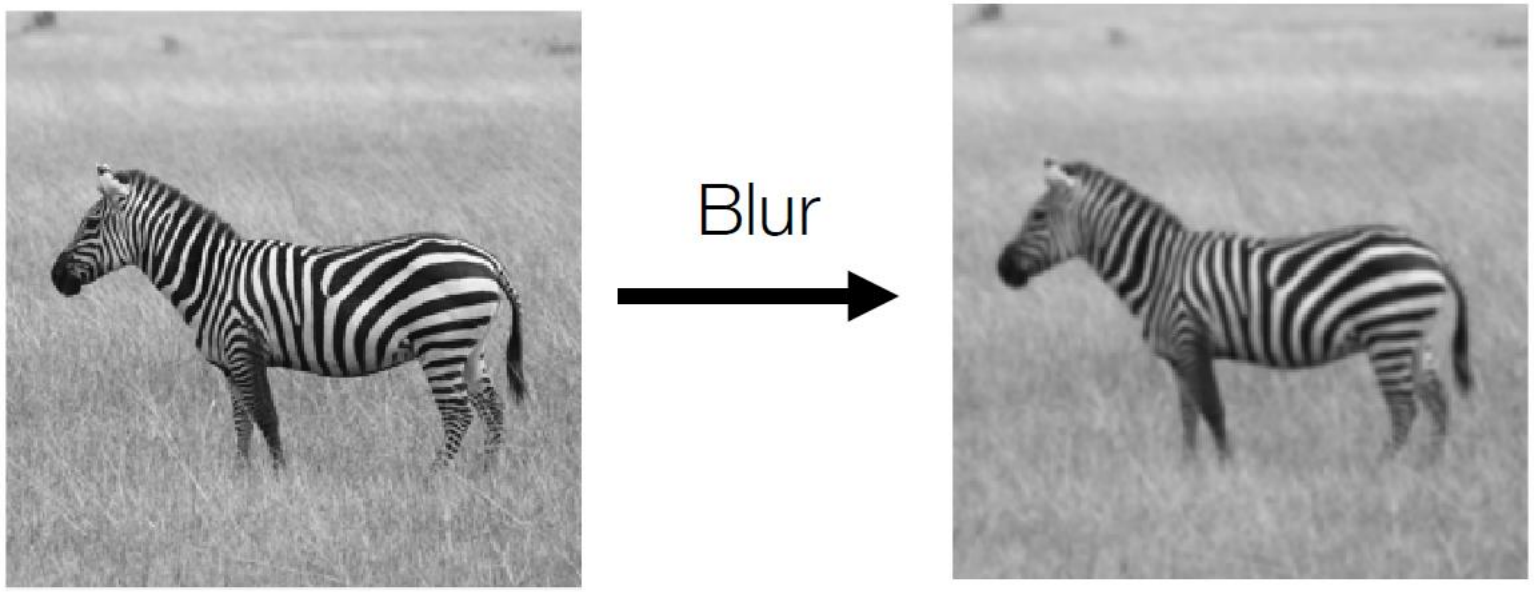
- What if we “missed” things between the samples?
- As expected, information is lost
- Unexpectedly: indistinguishable from low-frequency sinusoid!
- Also indistinguishable from higher frequencies
- **Aliasing:** signals “traveling in disguise” as other frequencies



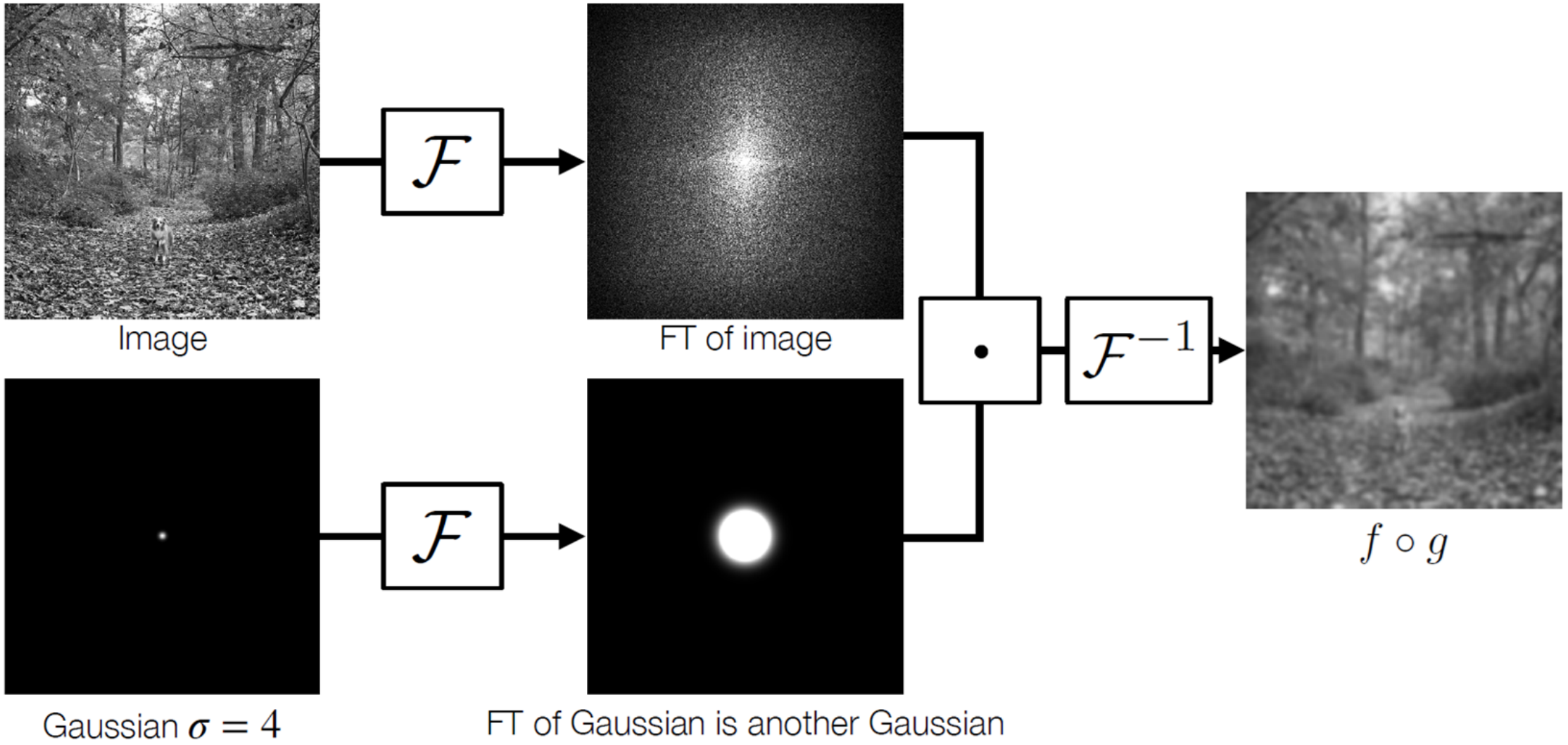
Source: S. Marschner

Remove Aliasing

- Remove the high frequencies first!
- Blur the image before downsampling



Recall: Blurring removes high frequencies



Note: The convolution of two functions in real space is the same as the product of their respective Fourier transforms in Fourier space

Constructing a Gaussian Pyramid

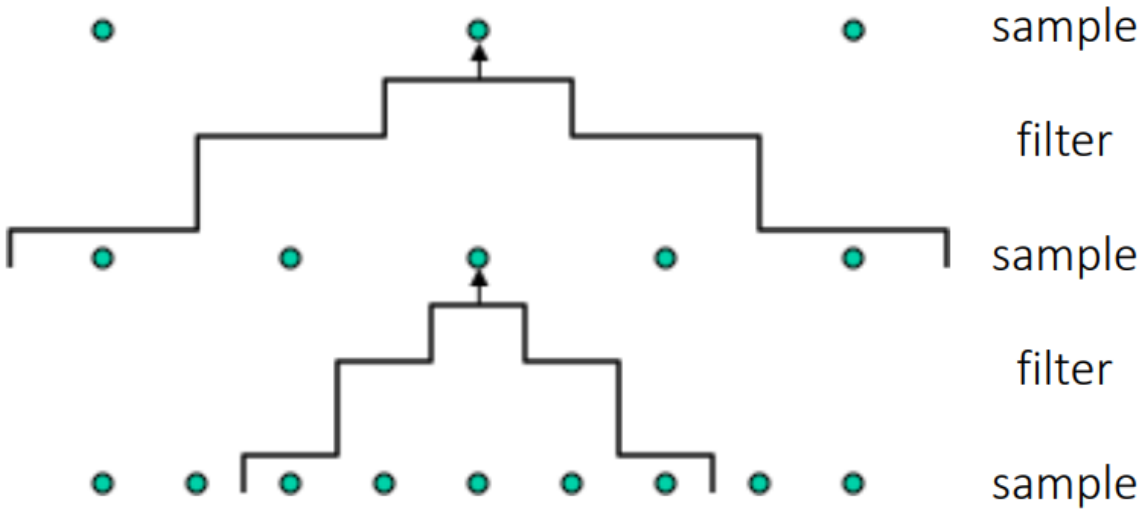
Algorithm

repeat:

filter

subsample

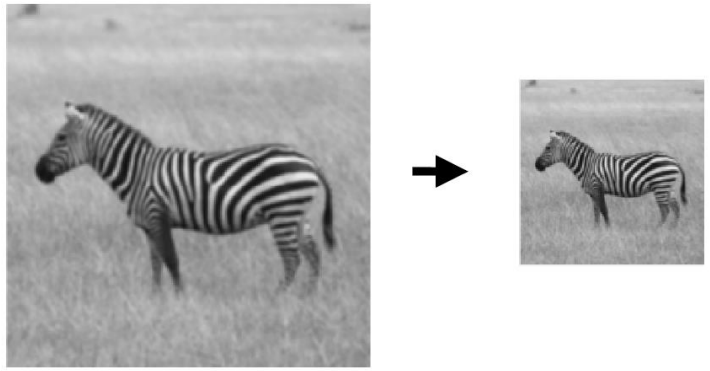
until min resolution reached



Gaussian Pyramid

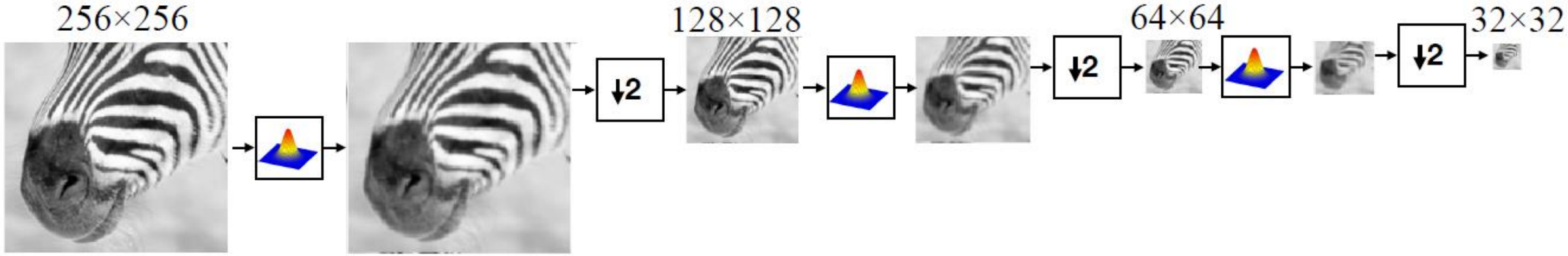
For each level:

1. Blur input image with a Gaussian (or binomial) filter
2. Downsample (throw away every other pixel)



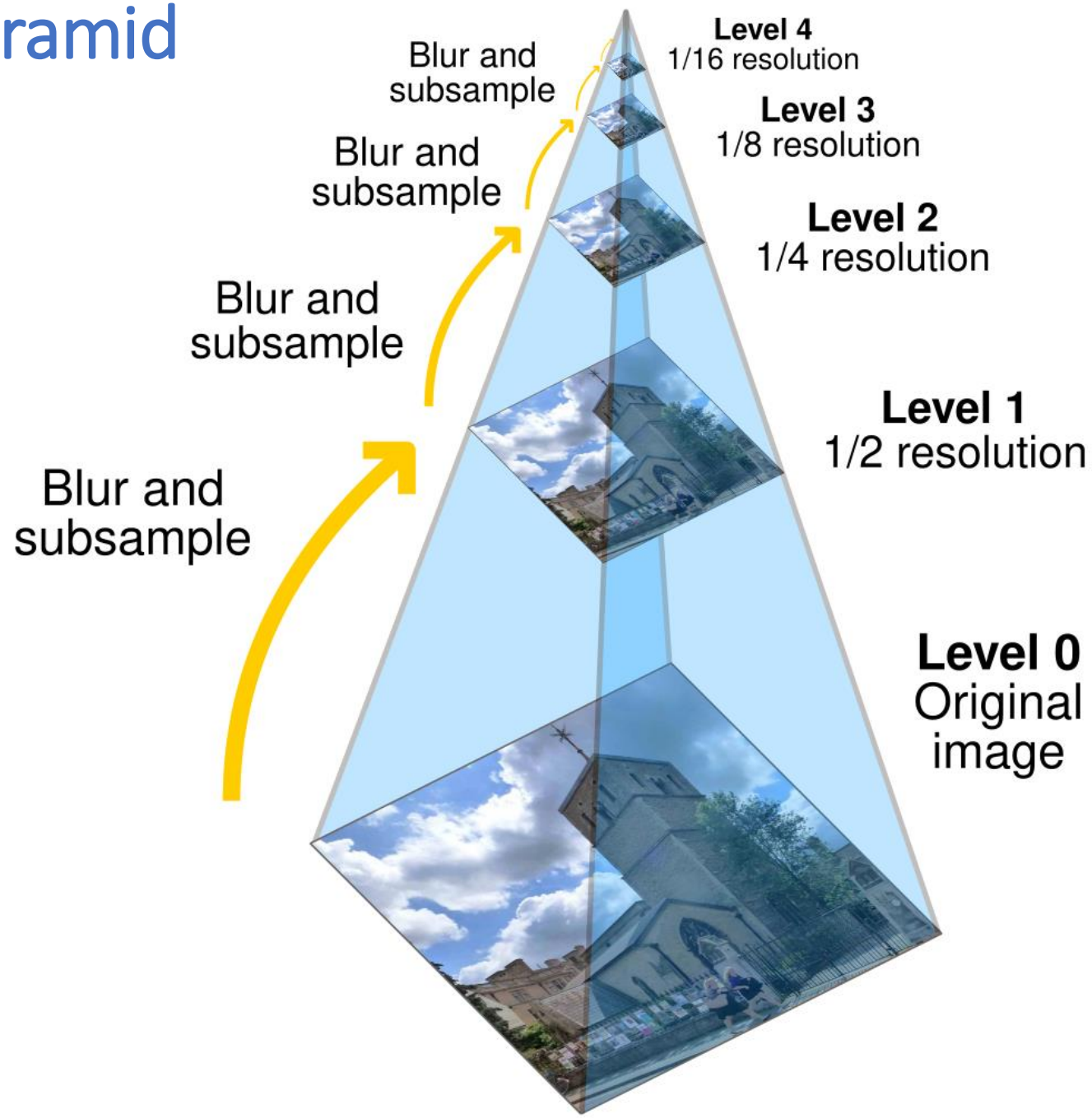
512×512 256×256 128×128 64×64 32×32

(original image)

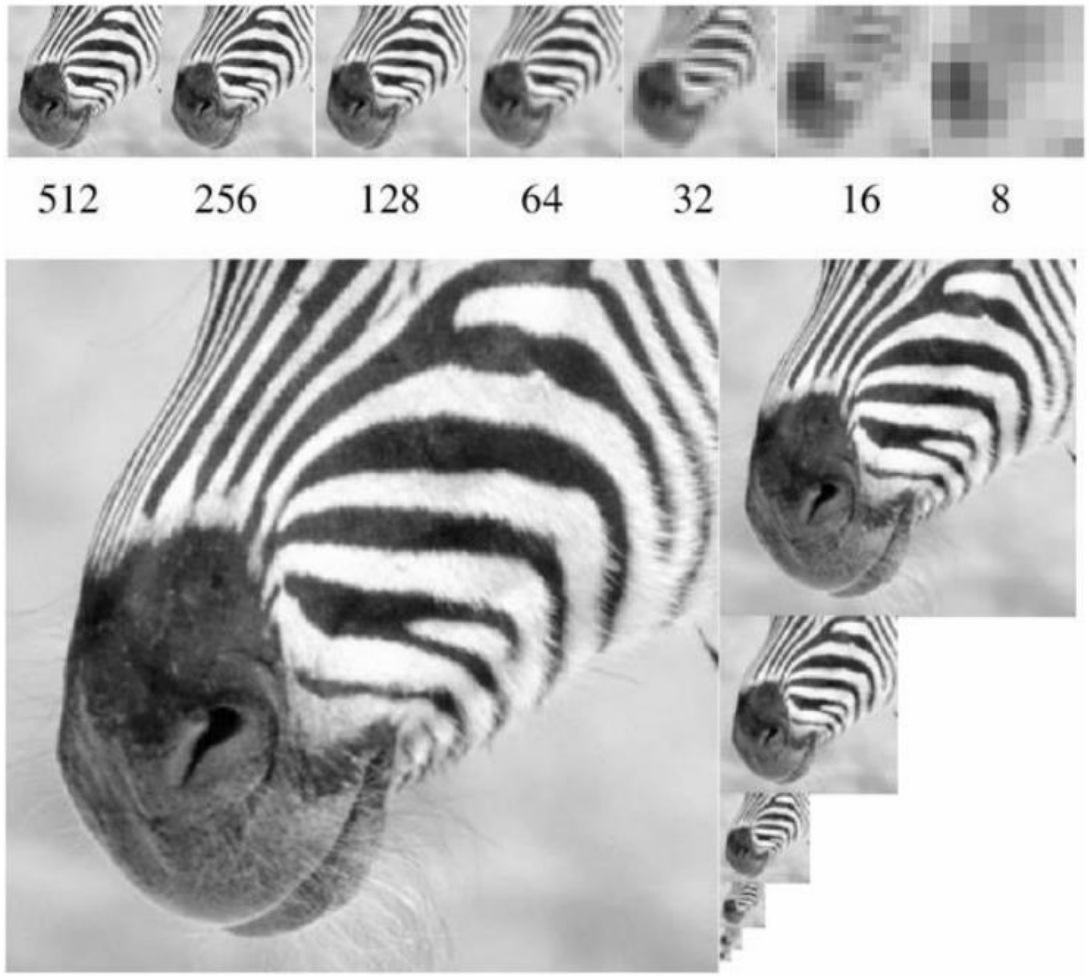


For subsampling, we can use any interpolation algorithm such as the nearest neighbor, bilinear, bicubic, etc.

Gaussian Pyramid



Some properties of Gaussian Pyramid



What happens to the details of the image?

- They get smoothed out as we move to higher levels.

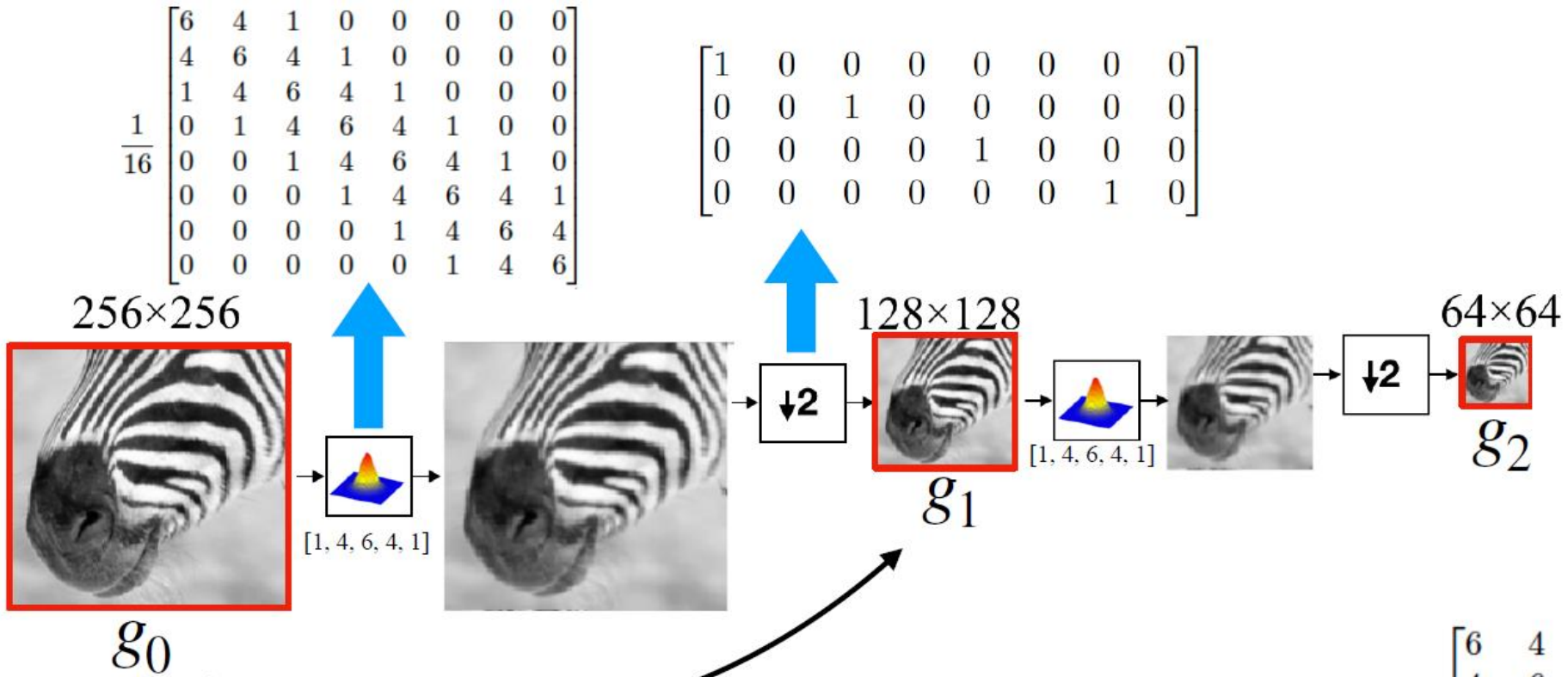
What is preserved at the higher levels?

- Mostly large uniform regions in the original image.

How would you reconstruct the original image from the image at the upper level?

- That's not possible.

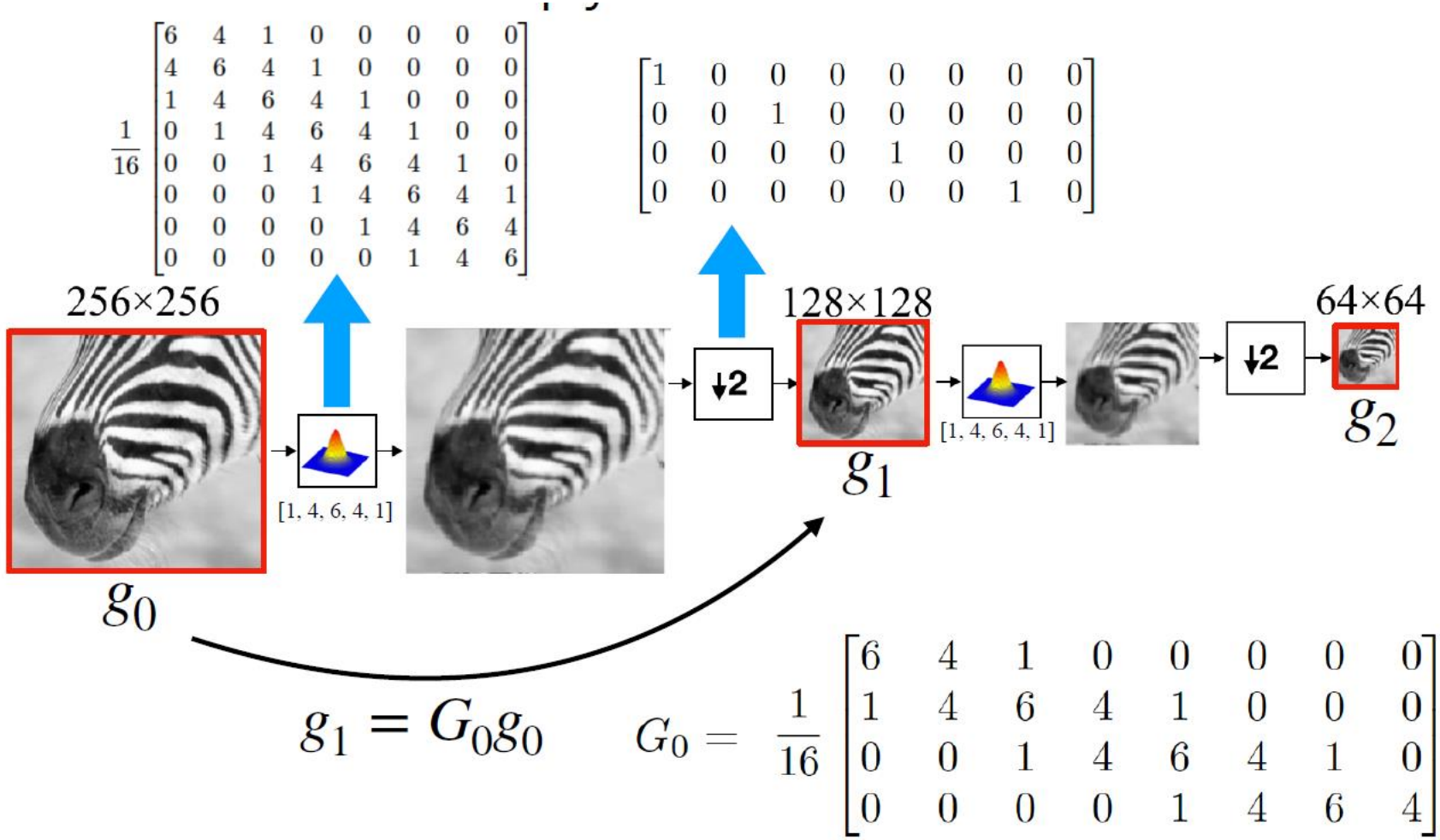
Gaussian Pyramid as a linear transformation



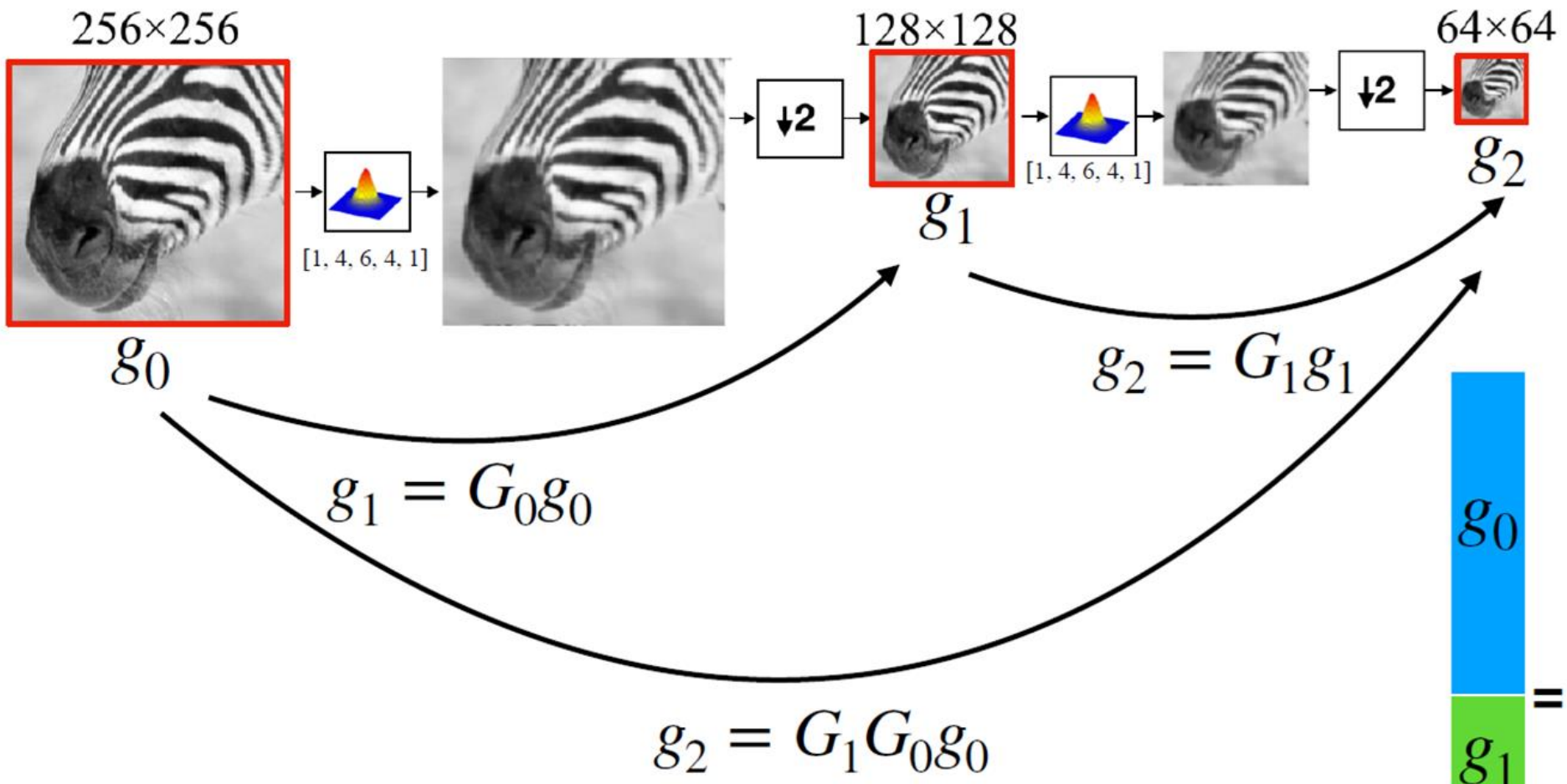
$$\frac{1}{16}\begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

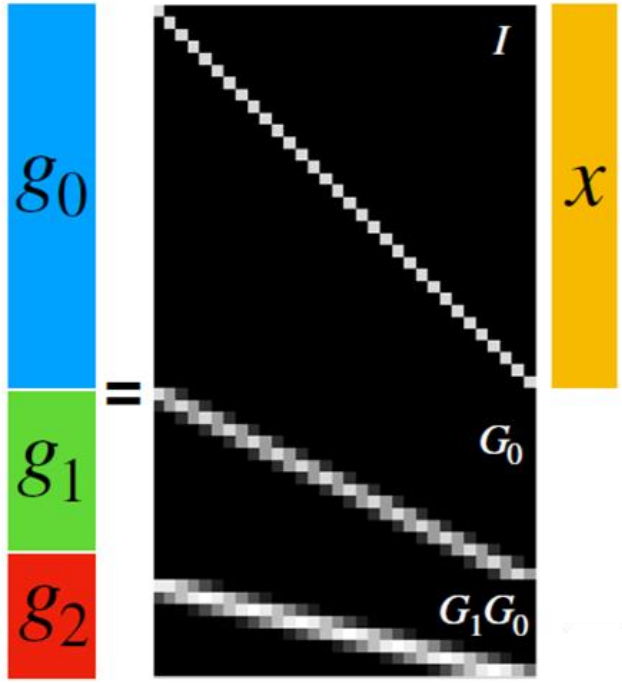
$$g_1 = G_0 g_0 \qquad G_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \frac{1}{16} \begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix}$$

Gaussian Pyramid as a linear transformation

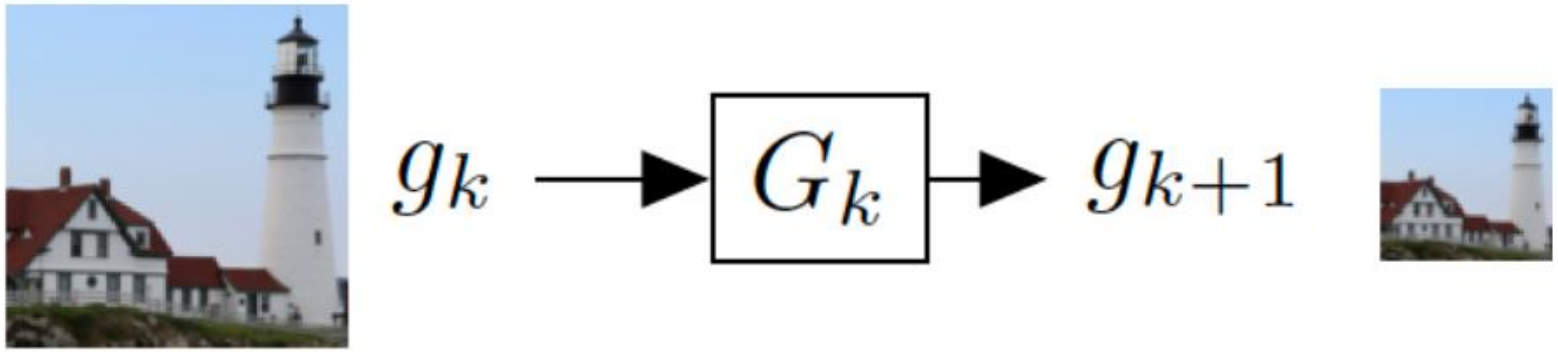


Gaussian Pyramid as a linear transformation





Gaussian Pyramid



For each level

1. Blur input image with a Gaussian filter
2. Downsample image

Gaussian Pyramid: what does blurring take away?

Level 0 Level 1

What is lost between levels?
What does blurring take away?

Level 0 Level 1 Residual

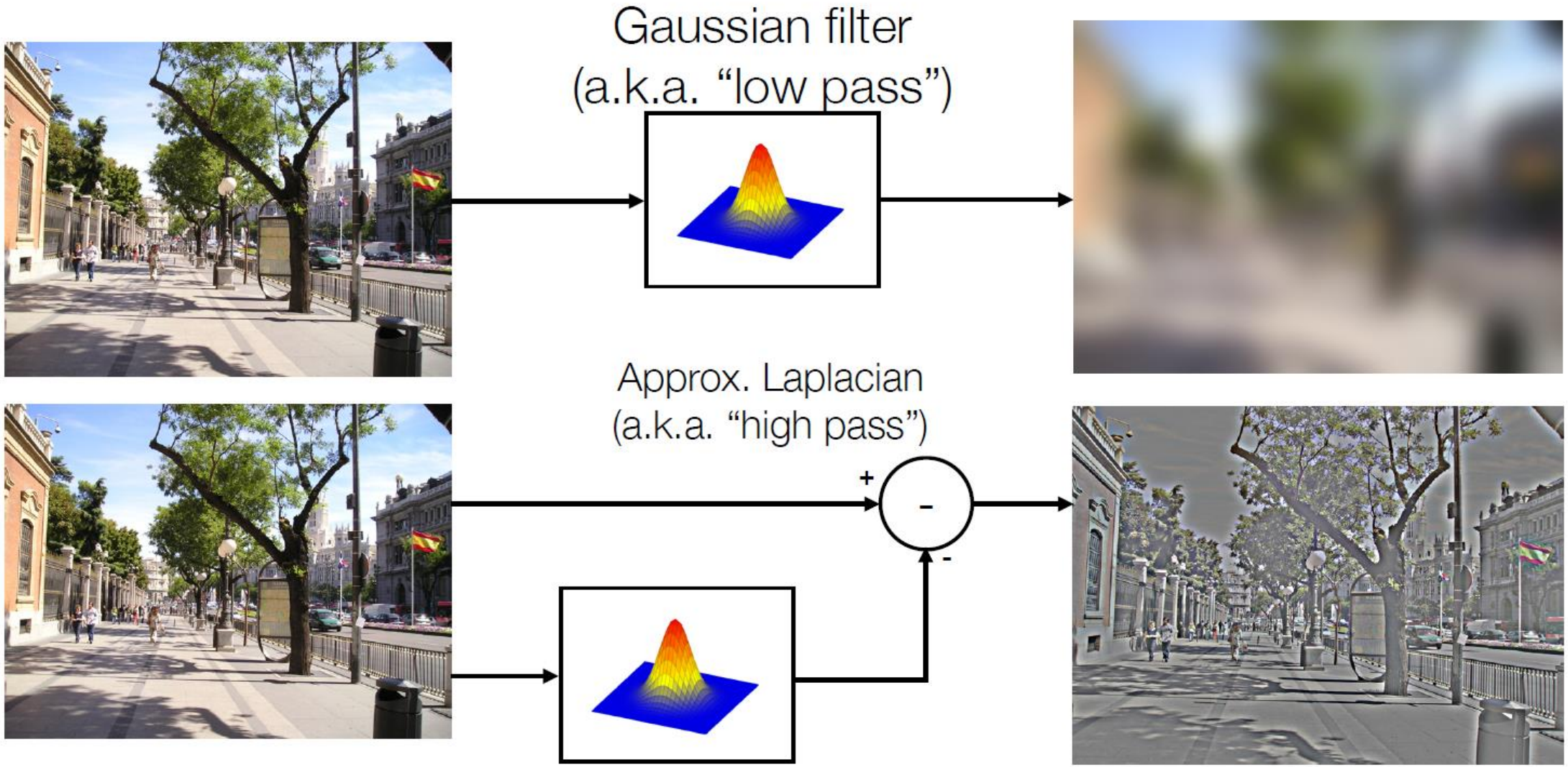
(thrown away by blurring)

(band-pass filter)

We can retain the residuals with a ...

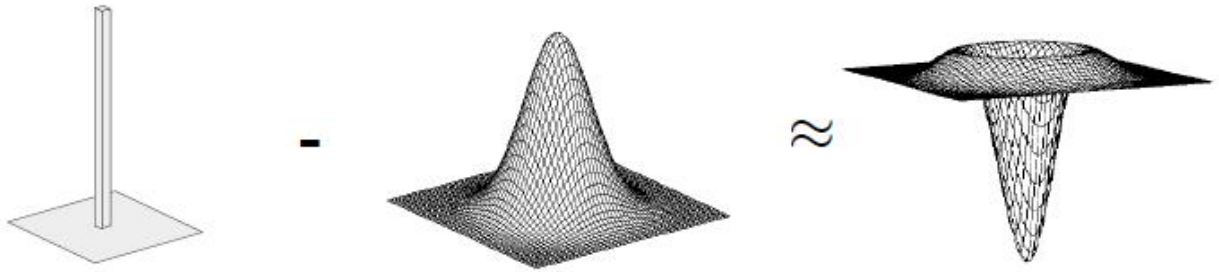
Can we make a pyramid that is lossless?

Recall: Laplacian



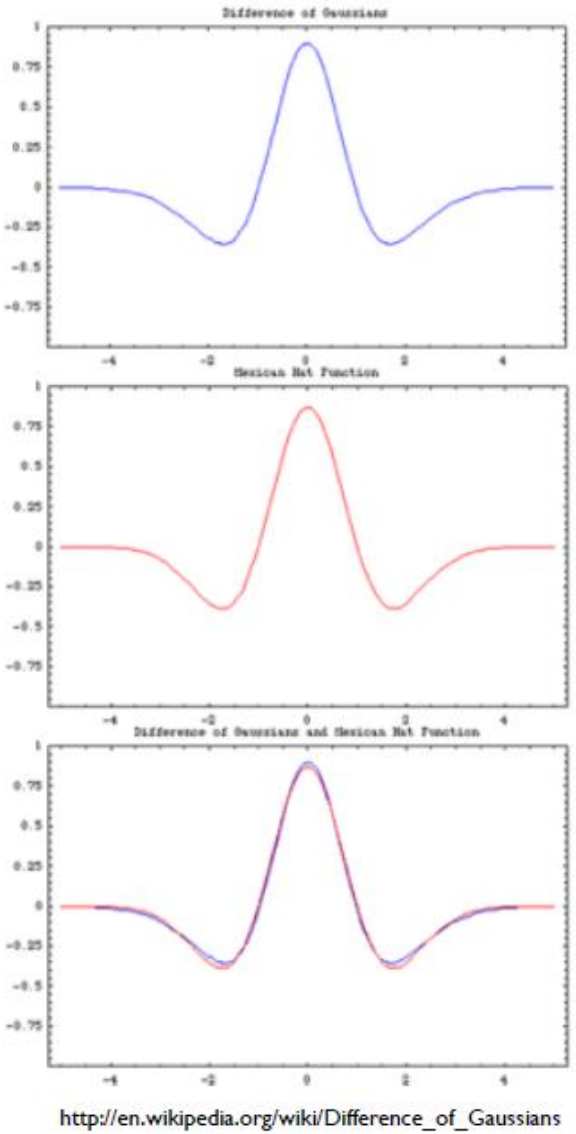
Laplacian Pyramid

Why is it called the Laplacian Pyramid?



unit Gaussian Laplacian

Difference of Gaussians approximates the Laplacian

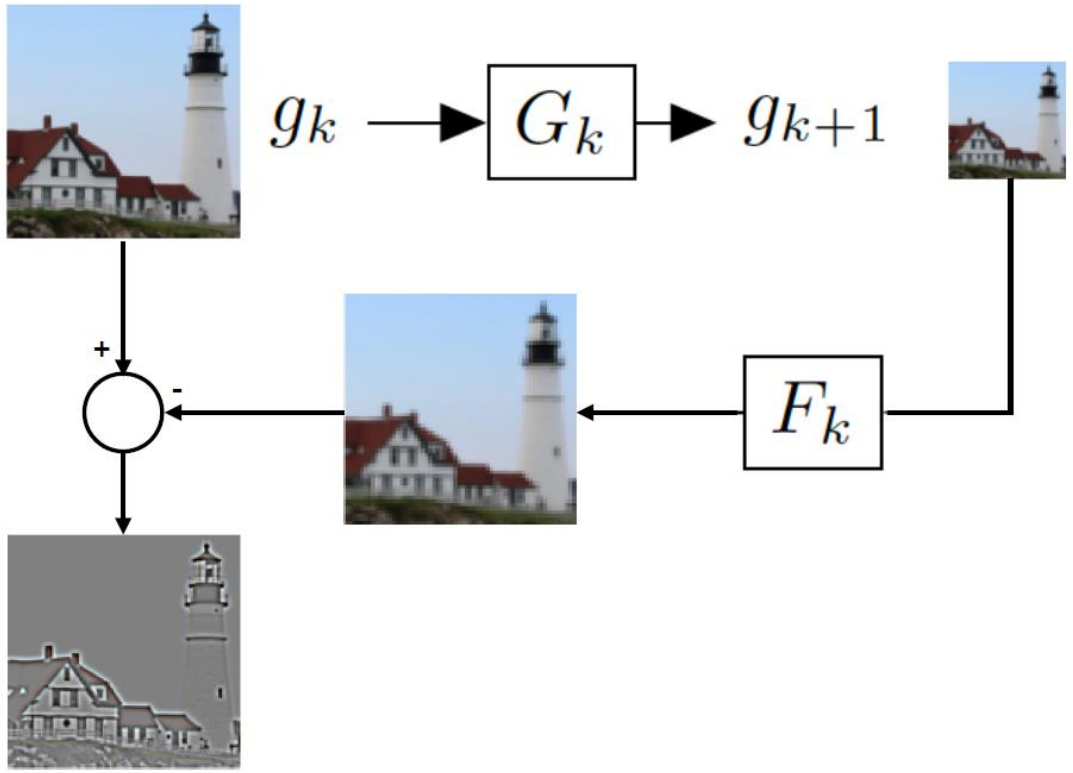


http://en.wikipedia.org/wiki/Difference_of_Gaussians

Laplacian Pyramid (1)

A Laplacian pyramid is a hierarchical image representation that decomposes an image into frequency bands, or a set of band-pass images, and a low-frequency residual. Each band can be manipulated independently, and the final image is reconstructed by adding the bands and the residual together.

Compute the difference between upsampled Gaussian pyramid level k+1 and Gaussian pyramid level k. Recall that this approximates the blurred Laplacian.



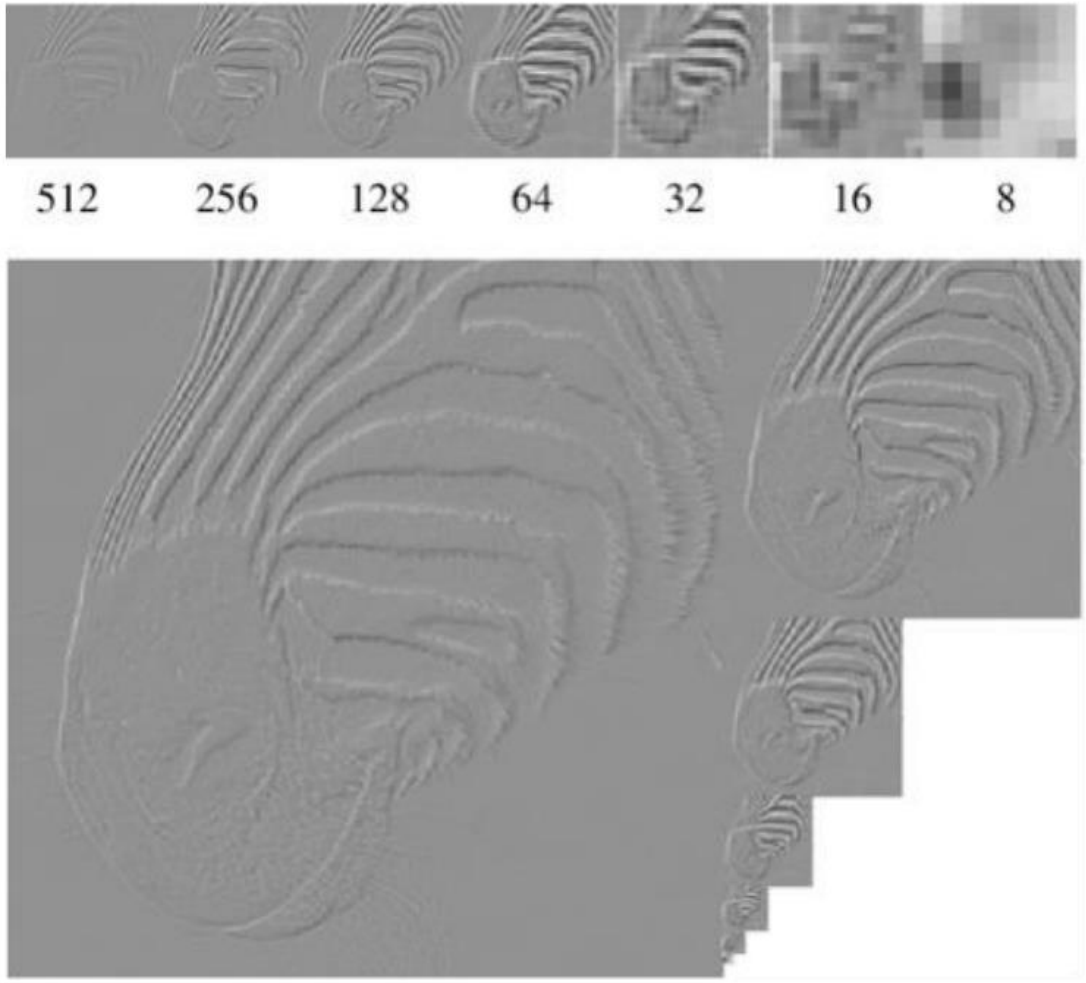
Source: Torralba, Freeman,

The Laplacian pyramid is derived from the Gaussian pyramid, and is obtained by taking the difference between adjacent layers in the Gaussian pyramid.

Laplacian pyramid Characteristics:

- Compact representation
- Robust in the presence of noise
- Enhances image features
- Relatively simple computation

Laplacian Image Pyramid



At each level, retain the residuals instead of the blurred images themselves.

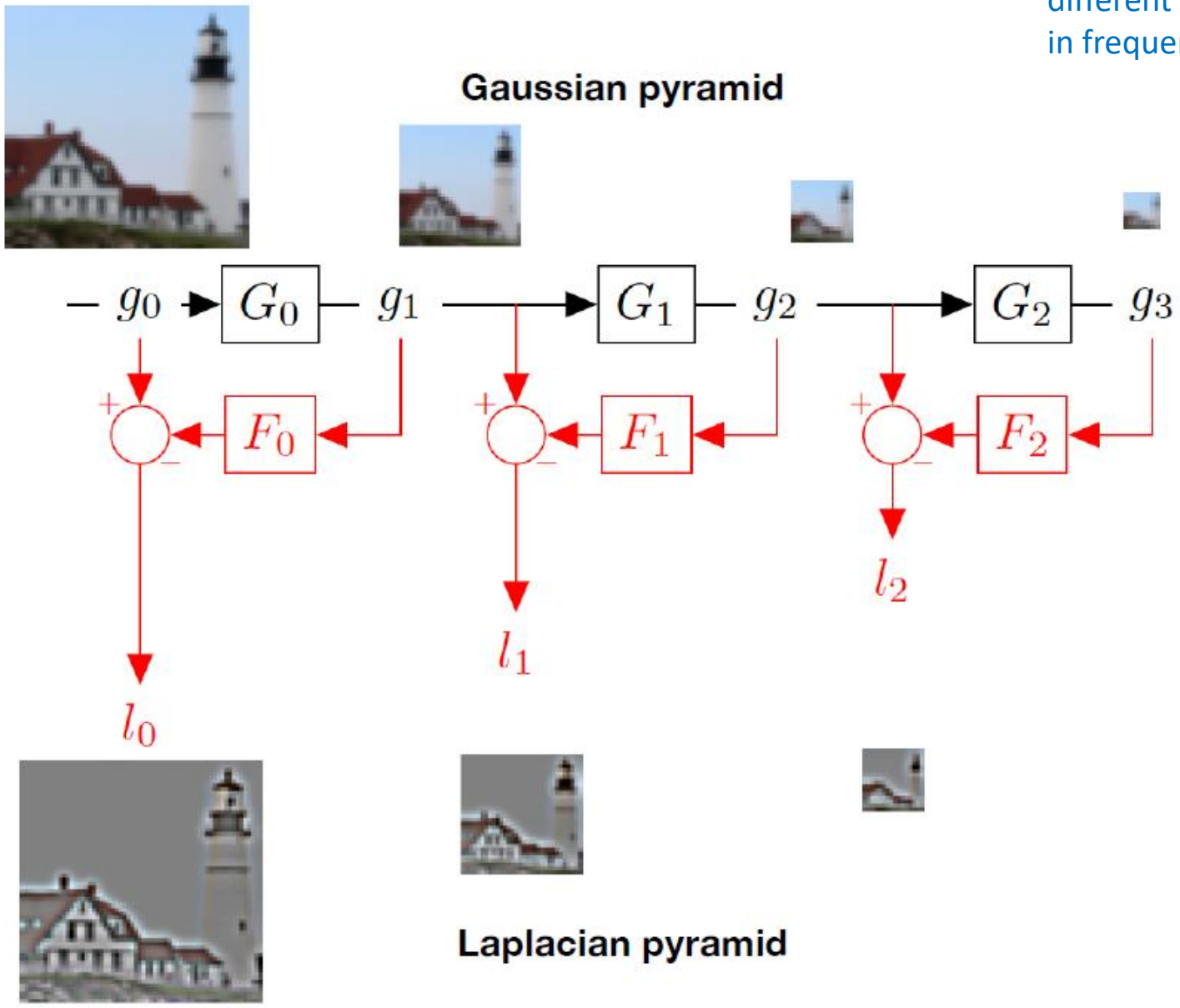
Can we reconstruct the original image using the pyramid?

- Yes we can!

What do we need to store to be able to reconstruct the original image?

Laplacian Pyramid (2)

In Laplacian Pyramid, each spatial scale corresponds to a different frequency band. As we go lower in scale we go lower in frequency until we get to the final Low Pass component.

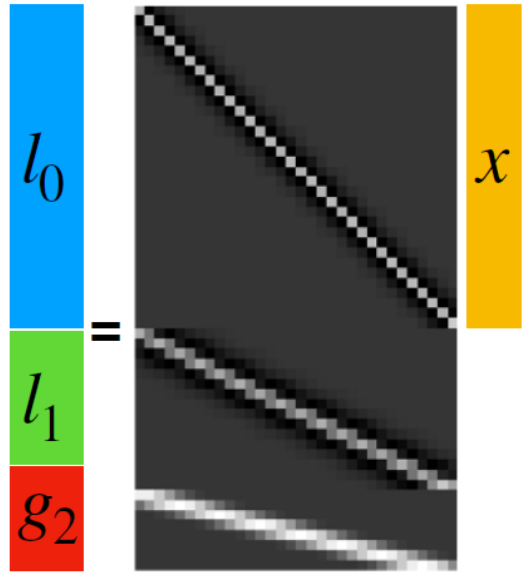


Algorithm

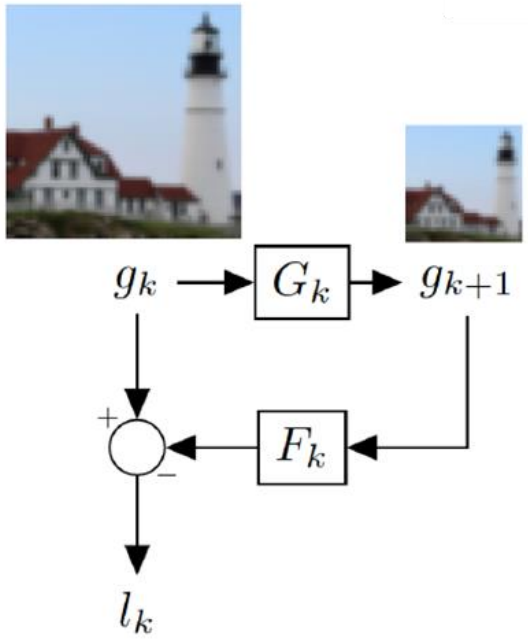
repeat:

- filter
- compute residual
- subsample

until min resolution reached



Laplacian Pyramid (3)



Blurring and downsampling:

$$G_0 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \frac{1}{16} \begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix}$$

(Downsampling by 2) (blur)

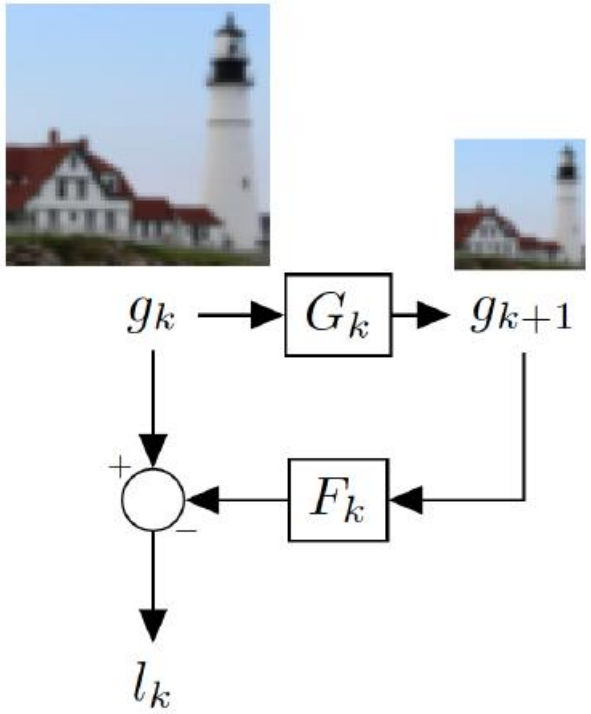
Upsampling and blurring:

$$F_0 = \frac{1}{8} \begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 4 & 6 & 4 & 1 & 0 & 0 & 0 & 0 \\ 1 & 4 & 6 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 4 & 6 & 4 & 1 & 0 & 0 \\ 0 & 0 & 1 & 4 & 6 & 4 & 1 & 0 \\ 0 & 0 & 0 & 1 & 4 & 6 & 4 & 1 \\ 0 & 0 & 0 & 0 & 1 & 4 & 6 & 4 \\ 0 & 0 & 0 & 0 & 0 & 1 & 4 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(blur) (Upsampling by 2)

$$l_0 = (I_0 - F_0 G_0) g_0$$

Laplacian Pyramid (4)



$$l_0 = (I_0 - F_0 G_0) g_0$$

$$= \frac{1}{256} \begin{bmatrix} 182 & -56 & -24 & -8 & -2 & 0 & 0 & 0 \\ -56 & 192 & -56 & -32 & -8 & 0 & 0 & 0 \\ -24 & -56 & 180 & -56 & -24 & -8 & -2 & 0 \\ -8 & -32 & -56 & 192 & -56 & -32 & -8 & 0 \\ -2 & -8 & -24 & -56 & 180 & -56 & -24 & -8 \\ 0 & 0 & -8 & -32 & -56 & 192 & -56 & -32 \\ 0 & 0 & -2 & -8 & -24 & -56 & 182 & -48 \\ 0 & 0 & 0 & 0 & -8 & -32 & -48 & 224 \end{bmatrix} x$$

Can we invert the Laplacian Pyramid?

Gaussian residual

Laplacian pyramid

Can we invert the Laplacian Pyramid?

Let us look at just one level

level 0

=

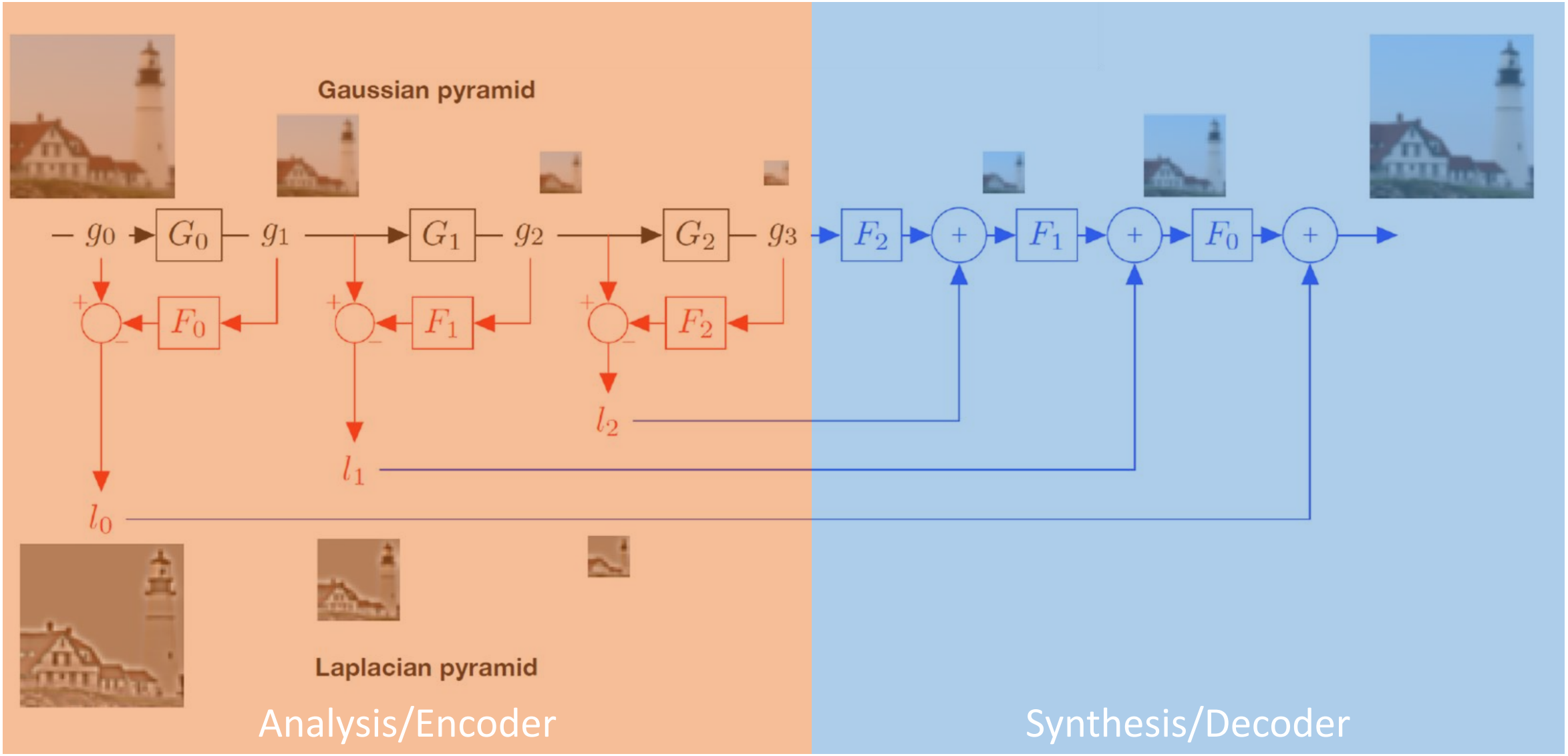
level 1 (upsampled)

+

residual

Does this mean we need to store both residuals and the blurred copies of the original?

Invert the Laplacian Pyramid



Laplacian Pyramid applications

The Laplacian pyramid is similar to a Gaussian pyramid, but it saves the difference image between each level. The base of the pyramid has the highest resolution, while the apex has the lowest resolution. The difference images on higher levels can be used to reconstruct the high resolution image.

- Texture synthesis
- Image compression
  - The Laplacian pyramid can be used to compress images by representing them as a set of band-pass images and a low-frequency residual.
  - Compression is achieved by quantizing the pixel values in the band pass images
  - The image can be reconstructed by adding together all the bands and the residual.
- Noise removal
  - Dehazing: A multi-scale dehazing algorithm uses Laplacian and Gaussian pyramids to break down a hazy image into different levels. Different noise reduction and haze removal techniques are applied to each level of the pyramid, and the pyramid is then collapsed to restore a haze-free image
- Computing image “keypoints”

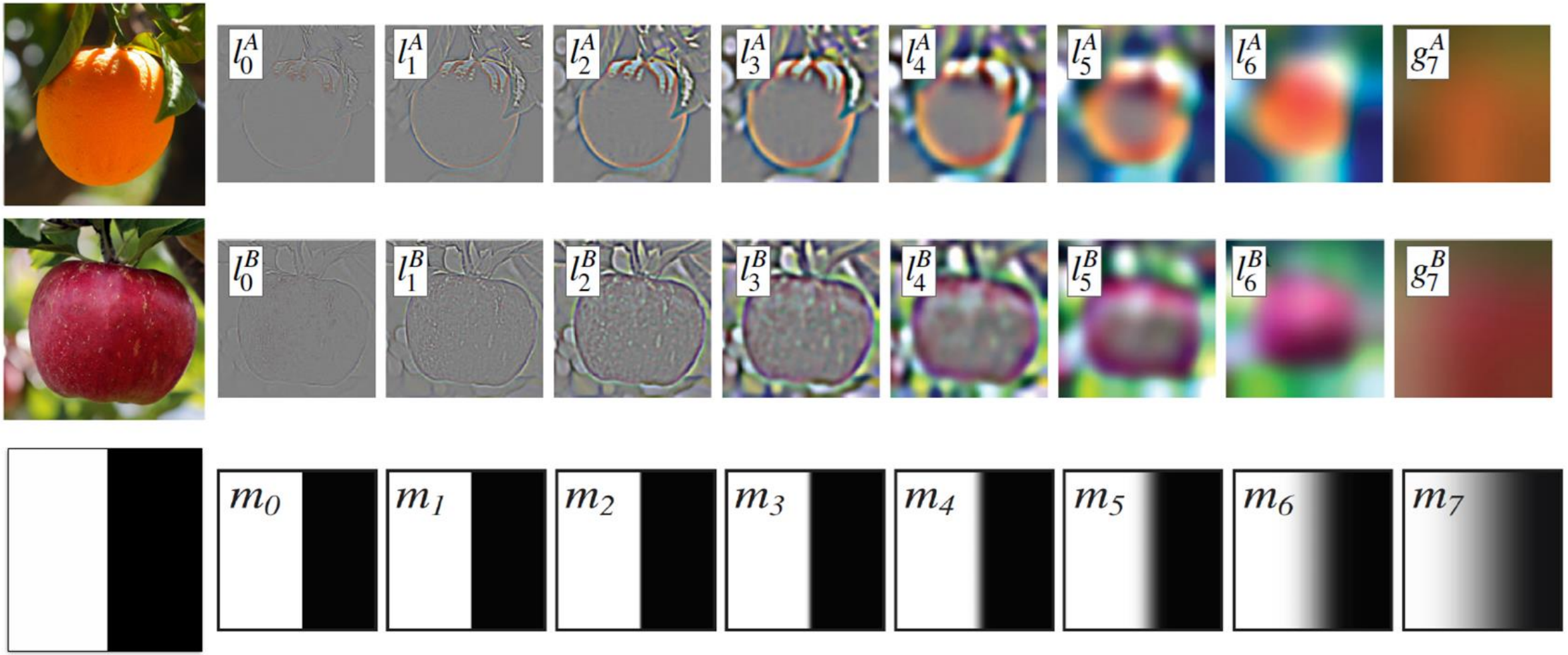
Image Blending

Image blending is a popular technique in computer vision and image processing that allows us to combine two or more images to create a seamless and visually appealing result. This technique finds its applications in various areas, such as creating panoramas, special effects in movies, and more.

I^A I^B m I

$$I = m * I^A + (1 - m) * I^B$$

Image Blending with the Laplacian Pyramid



$$l_k = l_k^A * m_k + l_k^B * (1 - m_k)$$

Image Blending

- Build Laplacian pyramid for both images: L_A, L_B
- Build Gaussian pyramid for mask: G
- Build a combined Laplacian pyramid
- Collapse L to obtain the blended image

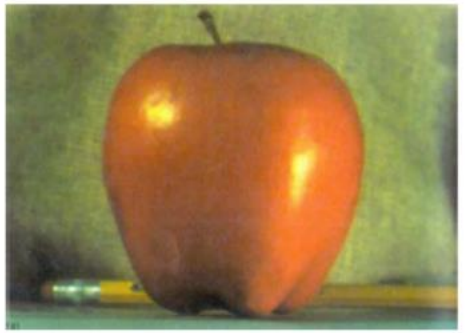
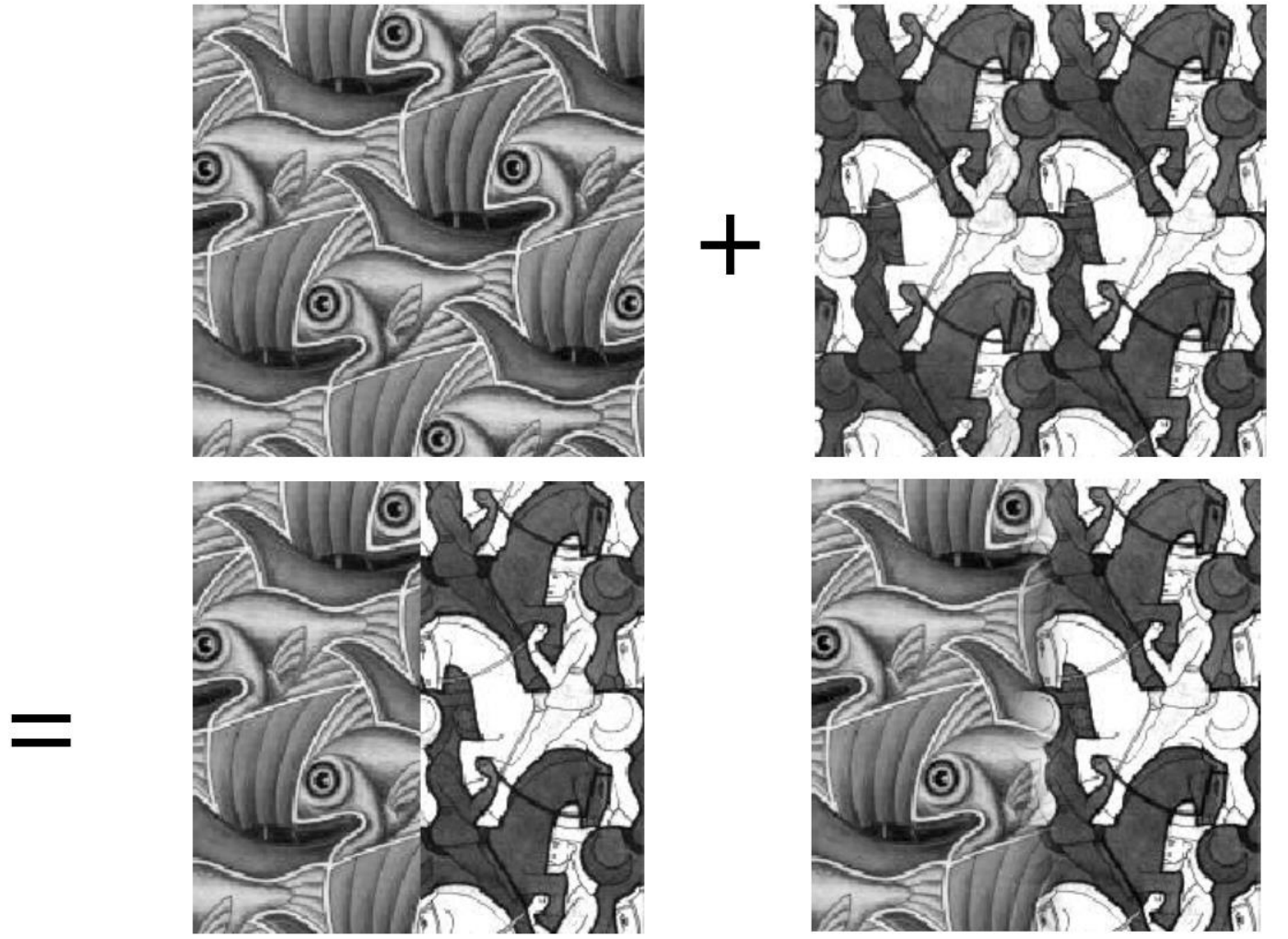


Image Blending with the Laplacian Pyramid

Image Blending: Simple Blend vs Laplacian Blend



Simple blend With Laplacian pyr.

Image Blending Example 2

Excited Federer

Angry Tiger

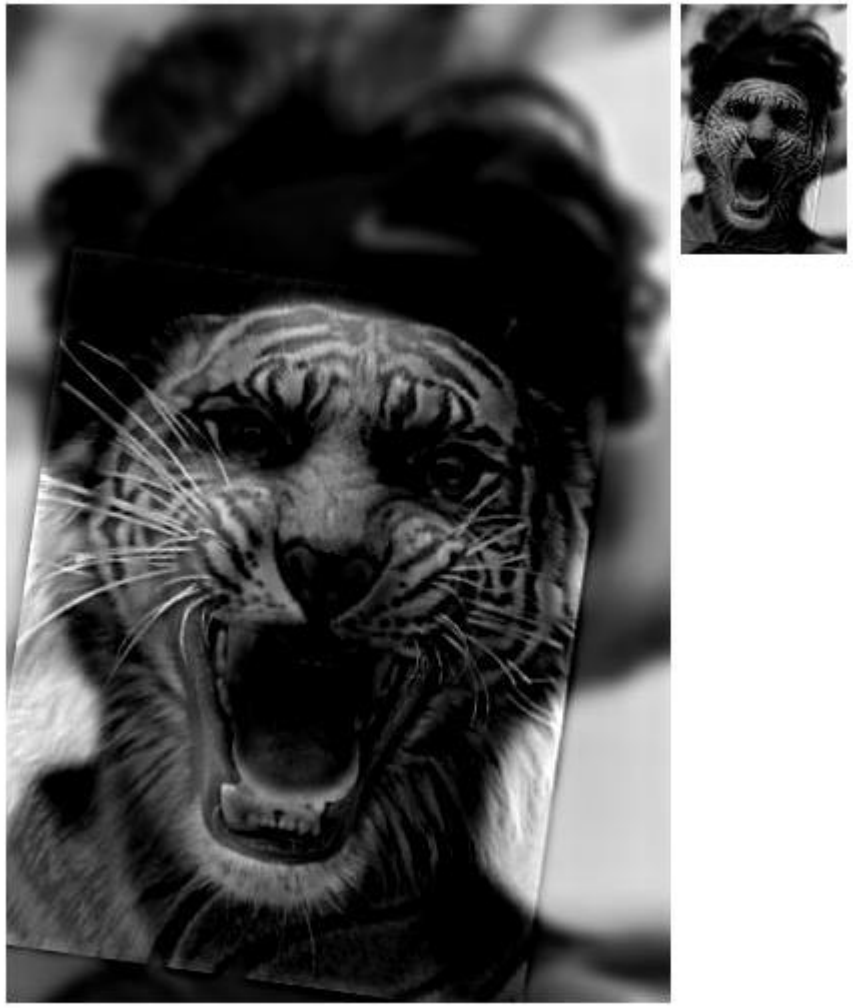
The algorithm of the program:

1)Two images are loaded into memory
2) transformed to grey scale
3) Aligned relative to each other
4) number of layers in Gaussian pyramid is set to a constant
5) pyramids function is called for both images
    Calculate a matrix of Gaussian filter (used in project is 5x5 with omega=8.0)
    For each layer:
        Apply Gaussian filter to image
        Get Laplacian filtered image by subtracting Gaussian filtered image from original
        Reduce size of image with both frequencies by 2
    Set last level of Laplacian pyramid equal to last level of Gaussian to preserve low frequencies

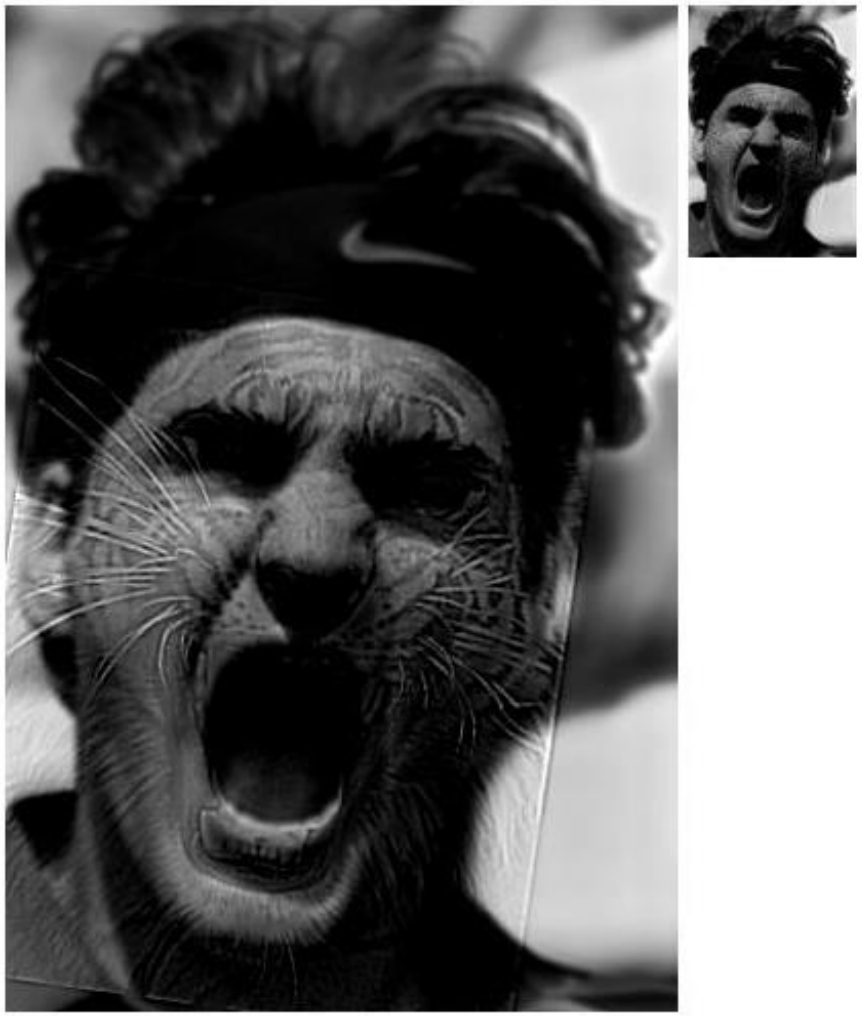
6) Output of pyramids function is Laplacian and Gaussian pyramid for each image
7) Setting the cutoff value (the bigger the value the less low frequency and more high frequency is in the final hybrid)
8) hybridImage function is called (parameters are two Laplacian pyramids & cutoff value)
    start creating a hybrid pyramid
    for levels up to cutoff copy first low frequencies from first image
    after cutoff add high frequencies from second image
    when hybrid pyramid is filled with filtered images for every layer do:
        resize result image (we start with the lowest level) to size of current Laplacian layer
        sum them into a resulting image
    result contains all layers resized and combined
9) The image is shown (see result images in next slide)
10) If needed cropped and rescaled

Reference: https://cs.brown.edu/courses/csci1430/2011/results/proj1/georgem/

Image Blending Example 2 (output images)

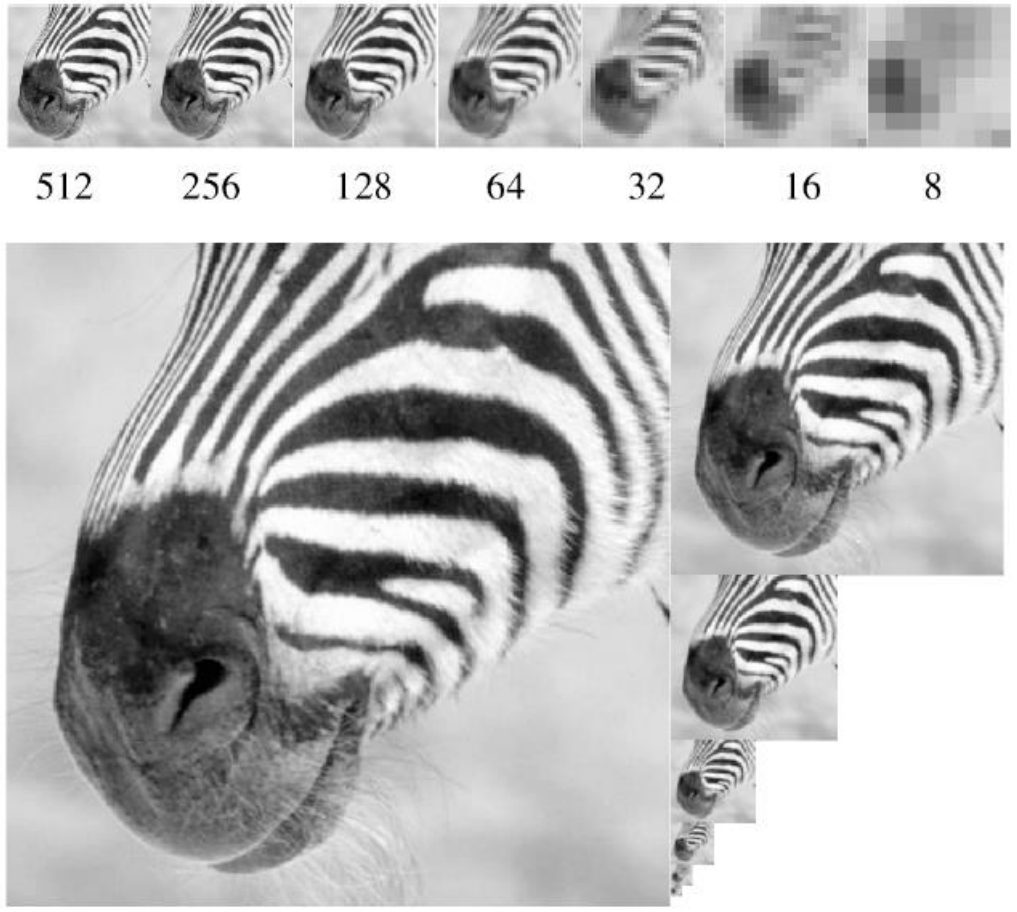


Cutoff = 2. A lot of high frequency details from one source and a low frequency canvas from another

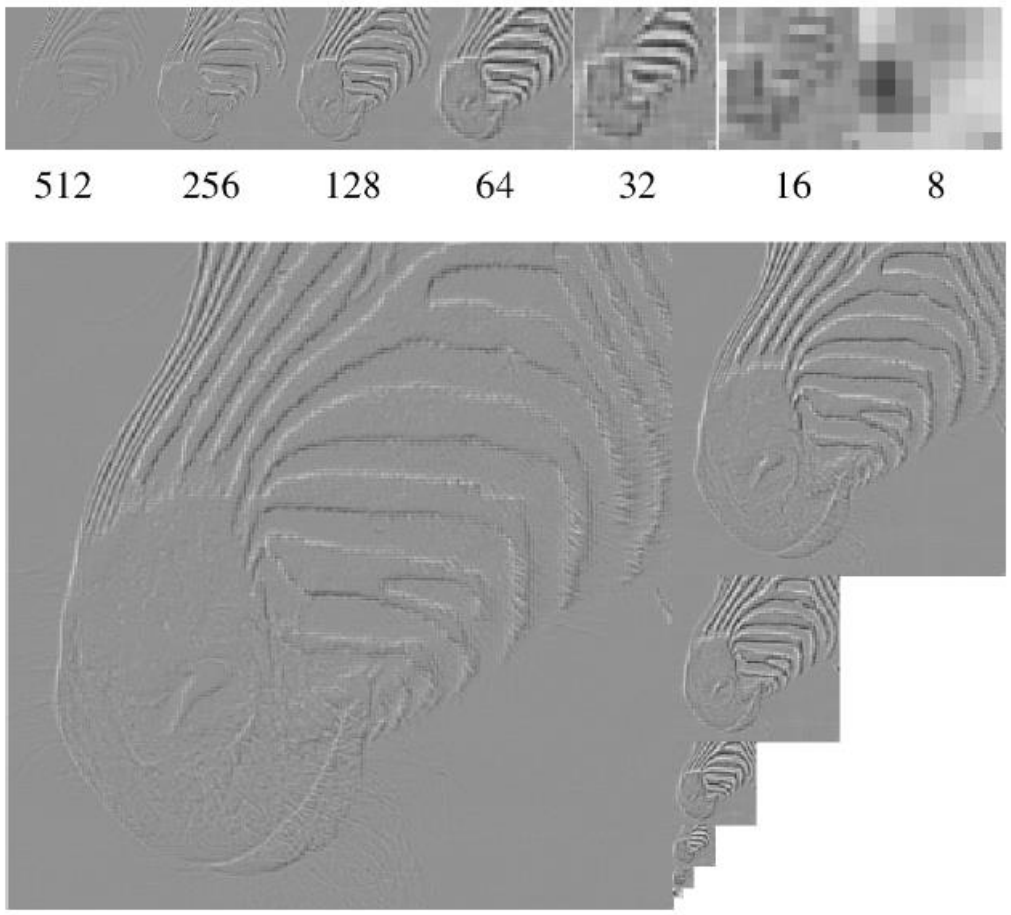


Cutoff = 1. Image looks sharper with less high frequency details from second source

Image Pyramids



Gaussian Pyramid



Laplacian Pyramid

And many more: steerable filters, wavelets, … (and later) convolutional networks!

Steerable filters

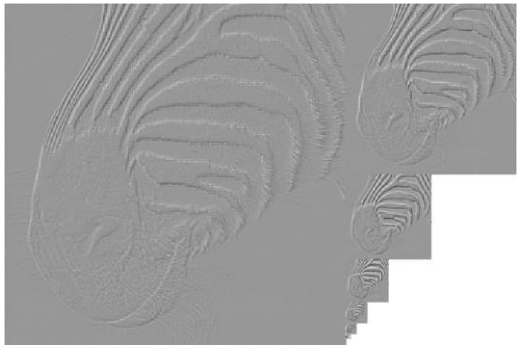
- A Steerable Filter is able to provide an Image Gradient in any orientation/direction that we choose. We can steer the filter in this direction by taking a linear combination of filters that produce a gradient in a known direction.

Image pyramids

- Gaussian

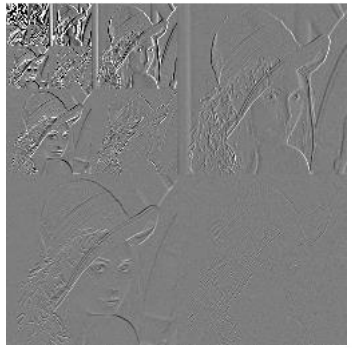
Progressively blurred and subsampled versions of the image. Adds scale invariance to fixed-size algorithms.

- Laplacian



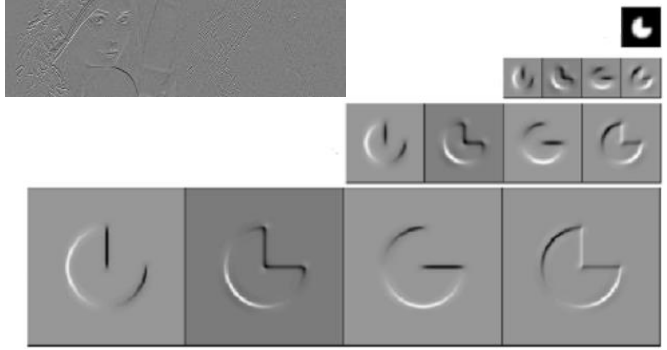
Shows the information added in Gaussian pyramid at each spatial scale. Useful for noise reduction & coding.

- Wavelet/QMF



Bandpassed representation, complete, but with aliasing and some non-oriented sub-bands.

- Steerable pyramid



Shows components at each scale and orientation separately. Non-aliased subbands. Good for texture and feature analysis. But overcomplete and with HF residual.

References

- https://www.cs.cmu.edu/~16385/lectures/lecture3.pdf
- http://6.869.csail.mit.edu/fa19/lectures/notes_lecture_7.pdf
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