



Computer Vision

(Course Code: 4047)

Module-2:Lecture-8: Affine Transformations

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Affine and Projective Transformations

❖ An affine transformation is any transformation that preserves collinearity, parallelism as well as the ratio of distances between the points (e.g. midpoint of a line remains the midpoint after transformation).

- All the geometric transformations such as translation, rotation, scaling, etc are all affine transformations as all the above properties are preserved in these transformations.
- In simple terms, one can think of the affine transformation as a composition of rotation, translation, scaling, and shear.
- Affine transformations preserve parallelism
- It doesn't necessarily preserve distances and angles.

❖ A projective transformation shows how the perceived objects change as the observer's viewpoint changes.

- These transformations allow the creating of perspective distortion.
- Projective transformations do not preserve parallelism, length, and angle.
- A projective transformation can be represented as the transformation of an arbitrary quadrangle (that is a system of four points) into another one.

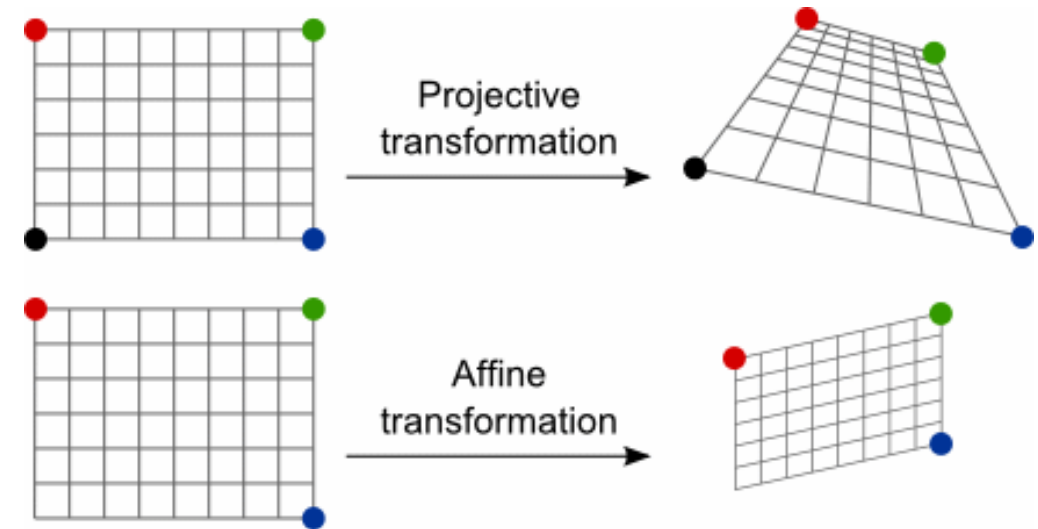


Image Manipulation

Image Filtering: Change range (brightness)

$$g(x, y) = T_r(f(x, y))$$

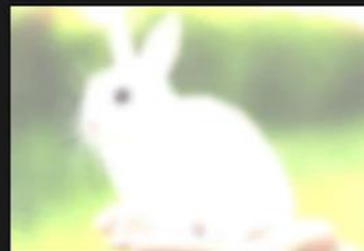


Image Warping: Change domain (location)

$$g(x, y) = f(T_d(x, y))$$

Transformation T_d is a coordinate changing operator



Global Warping/Transformation



Translation



Rotation



Scaling and Aspect

$$g(x, y) = f(T(x, y))$$



Affine



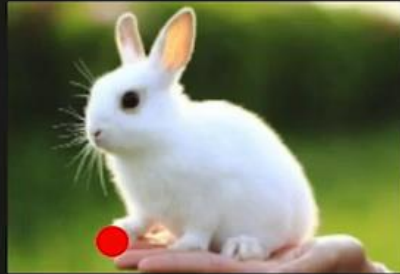
Projective



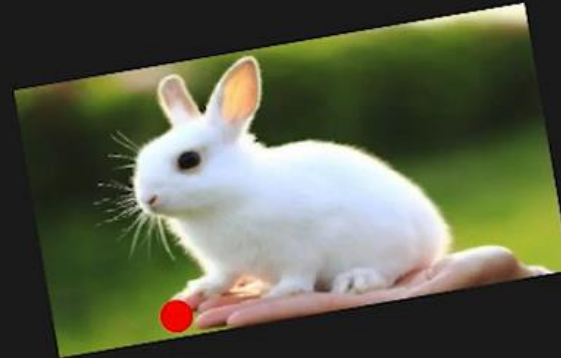
Barrel

Transformation T is the same over entire domain
Often can be described by just a few parameters

2x2 Linear Transformations



$$\mathbf{p}_1 = (x_1, y_1)$$



$$\mathbf{p}_2 = (x_2, y_2)$$

T can be represented by a matrix.

$$\mathbf{p}_2 = T\mathbf{p}_1$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = T \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Linear Transformations

- ❖ A linear transformation is a function that maps one vector space into another and these transformations are often implemented by a matrix.
- ❖ A transformation is considered to be linear if it preserves vector addition and scalar multiplication.

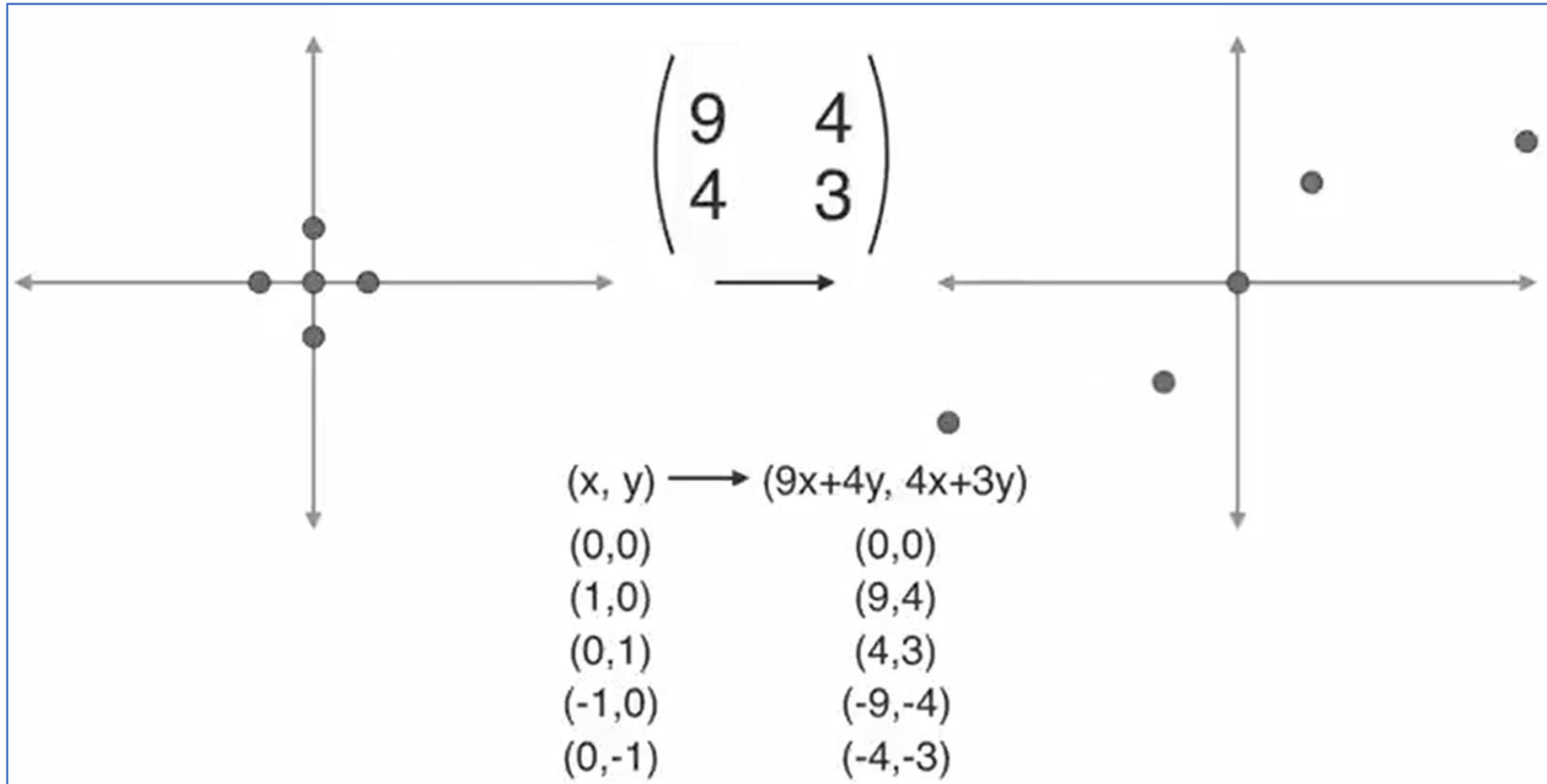
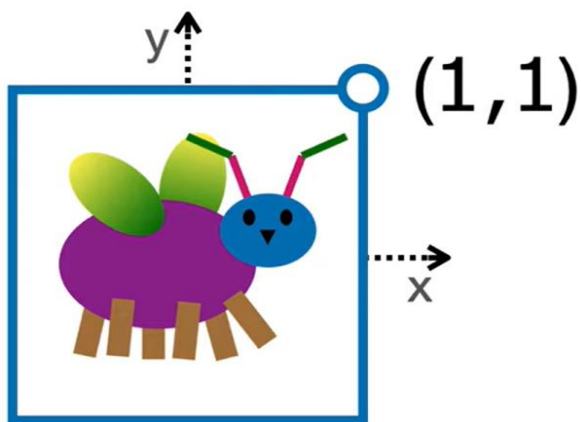


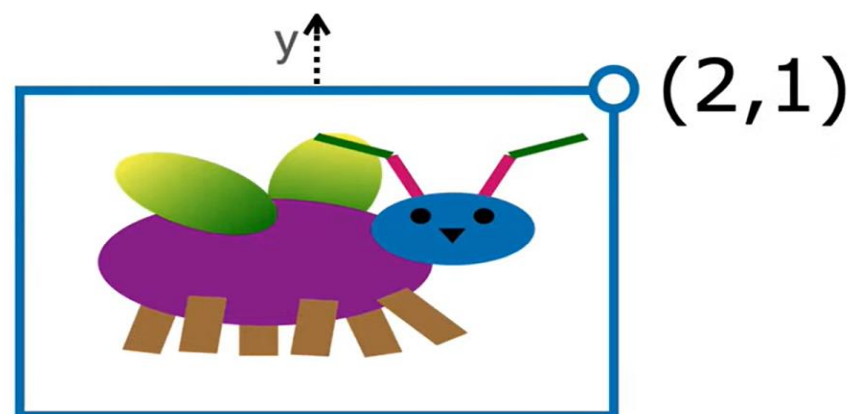
Image Transformation Example

The following matrix helps the scale the image in X-direction

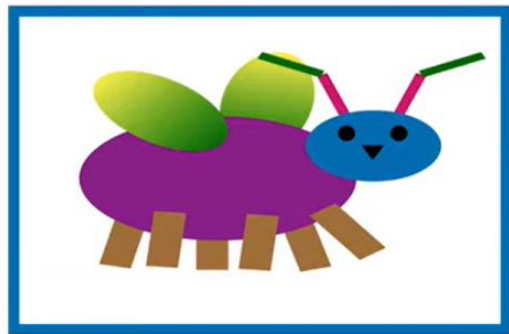
$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$



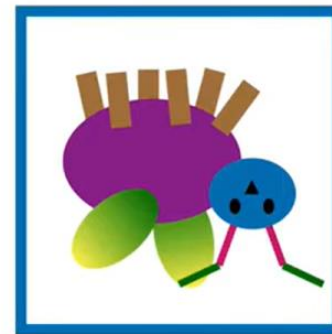
$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$



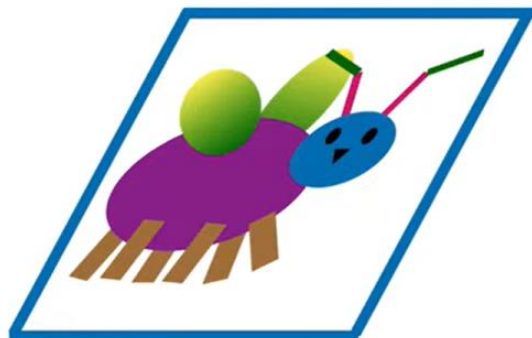
Affine Transformations



scale



reflection

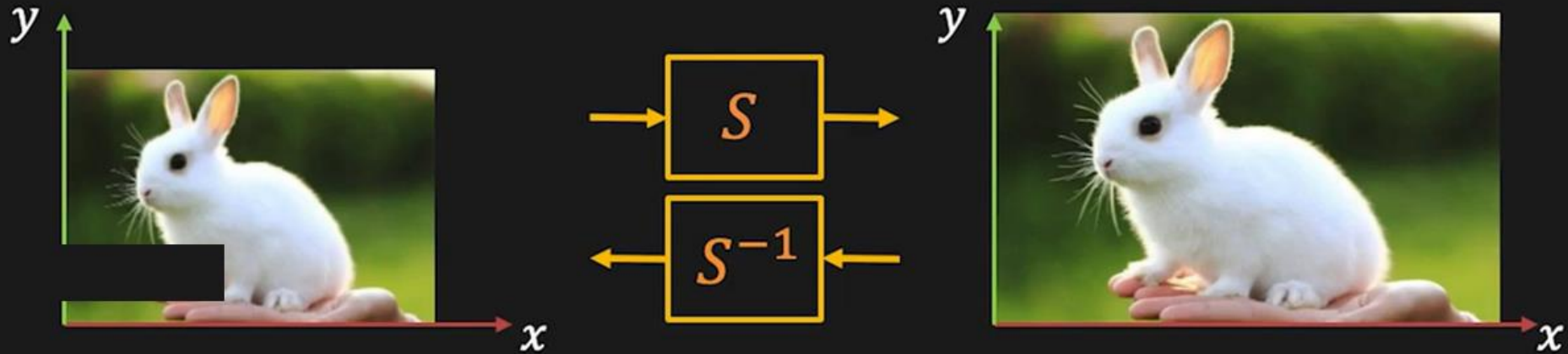


shear



rotation

Scaling (Stretching or Squishing)



Forward:

$$x_2 = ax_1 \quad y_2 = by_1$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

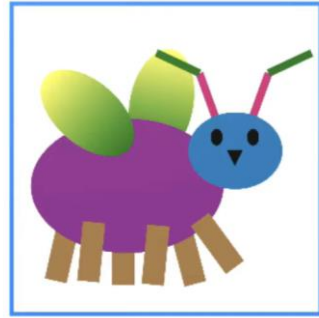
Inverse:

$$x_1 = \frac{1}{a}x_2 \quad y_1 = \frac{1}{b}y_2$$

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = S^{-1} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1/a & 0 \\ 0 & 1/b \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

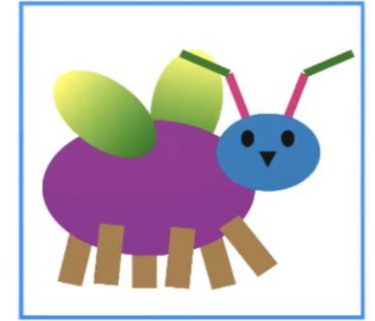
Horizontal and Vertical Scaling

$$\begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$



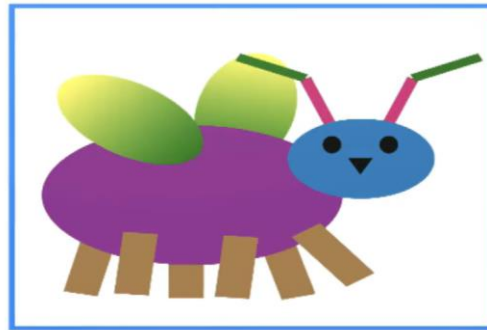
No Scaling

$$\begin{bmatrix} 1.1 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$



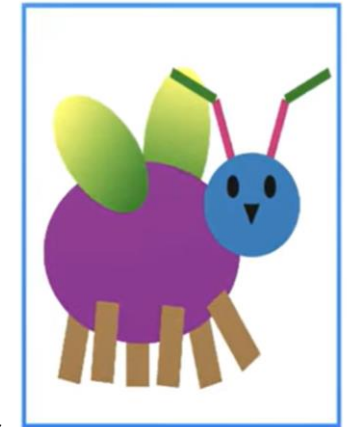
Horizontal Scaling = 1.1x

$$\begin{bmatrix} 1.5 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$



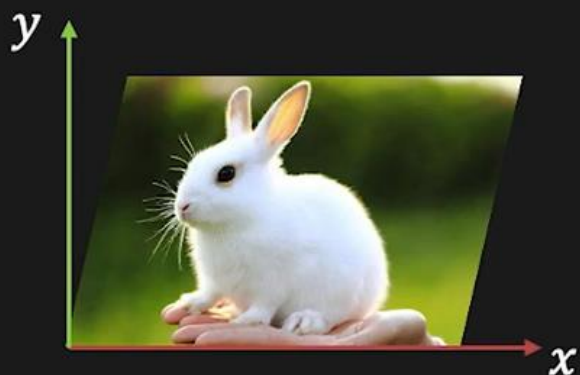
Horizontal Scaling = 1.5x

$$\begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.3 \end{bmatrix}$$



Vertical Scaling = 1.3x

Skew

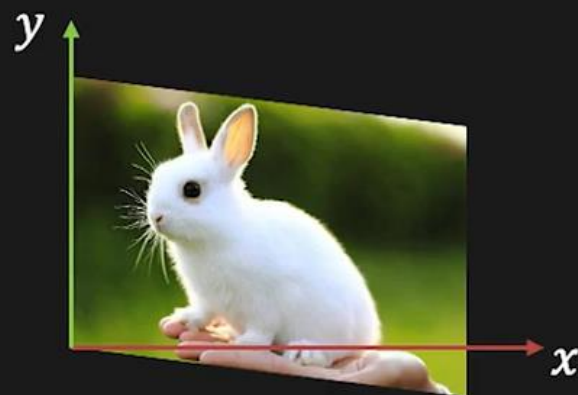


Horizontal Skew:

$$x_2 = x_1 + m_x y_1$$

$$y_2 = y_1$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S_x \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 & m_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$



Vertical Skew:

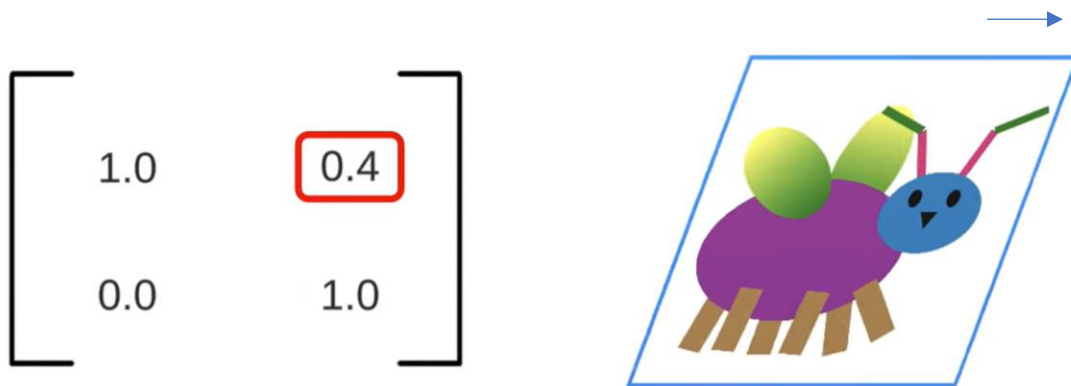
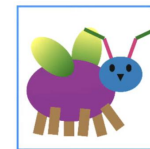
$$x_2 = x_1$$

$$y_2 = m_y x_1 + y_1$$

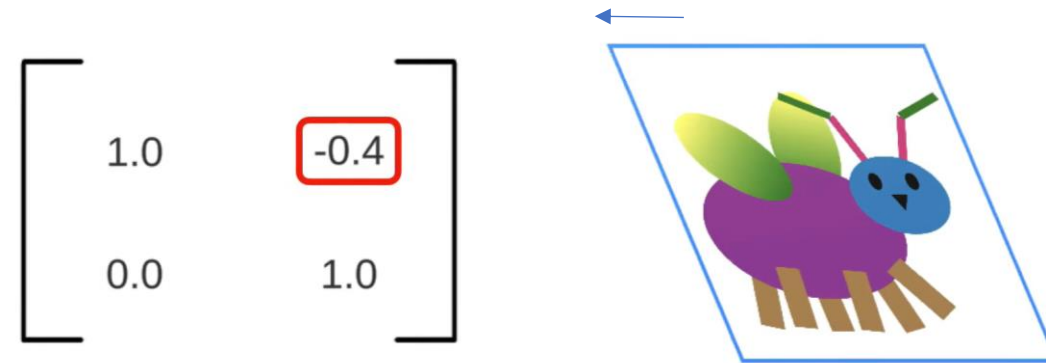
$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = S_y \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ m_y & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Shear

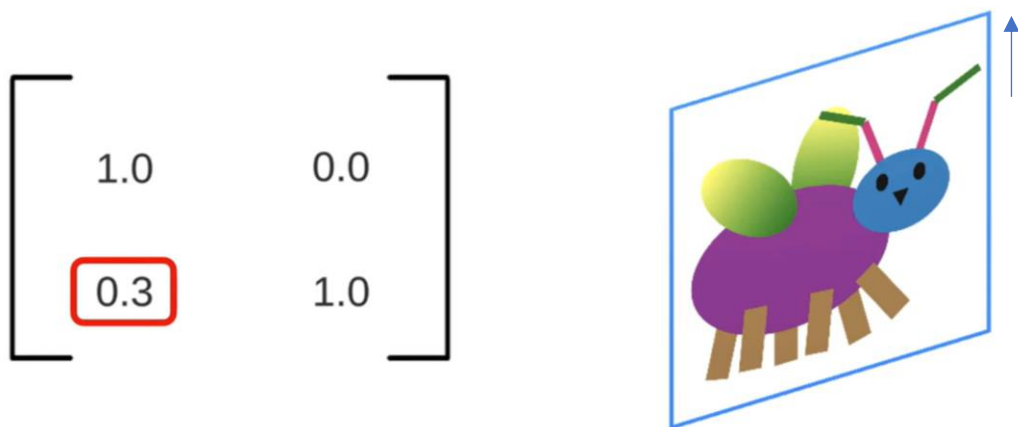
$$\begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$



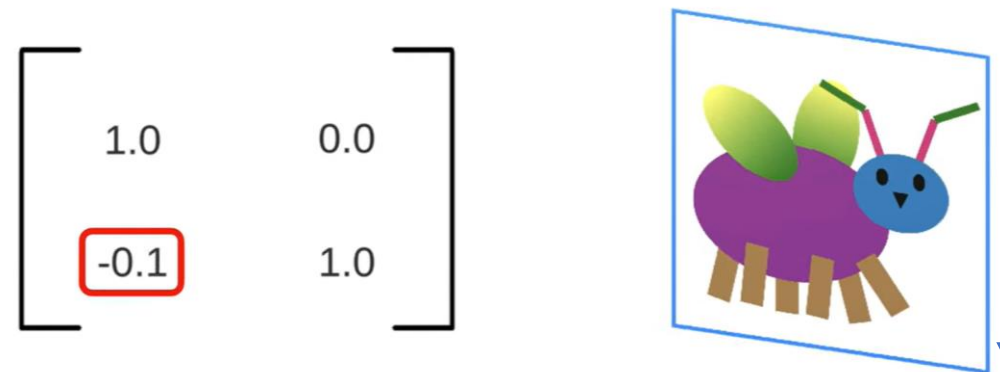
Horizontal Shear = 0.4



Horizontal Shear = -0.4



Vertical Shear = 0.3



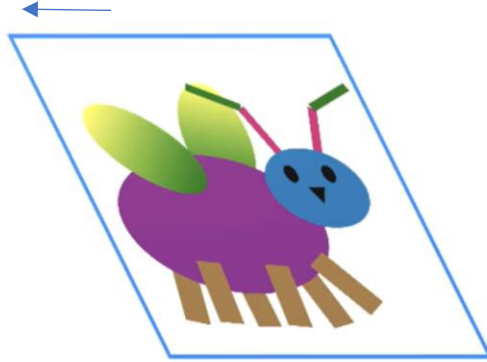
Vertical Shear = -0.1

Rotation



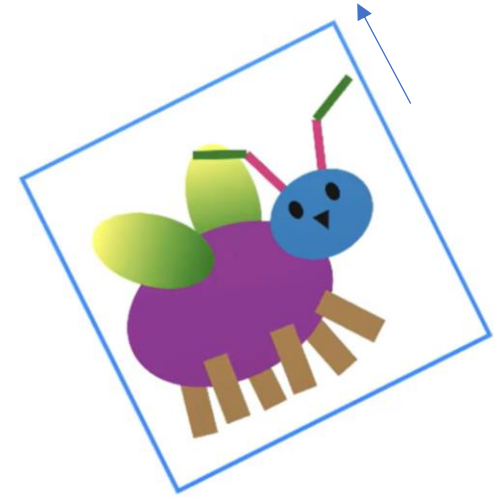
❖ As two shear operations ($\leftarrow + \nearrow$)

$$\begin{bmatrix} 1.0 & -0.5 \\ 0.0 & 1.0 \end{bmatrix}$$



+

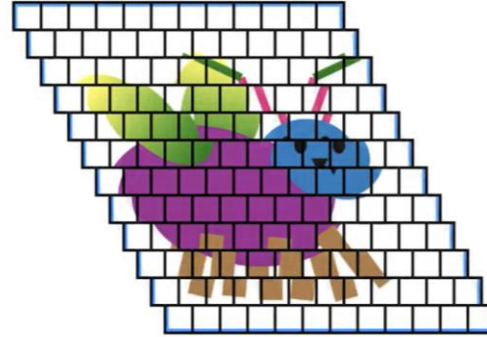
$$\begin{bmatrix} 1.0 & -0.5 \\ 0.5 & 1.0 \end{bmatrix}$$



Rotation: 2d rotation with 1d translation

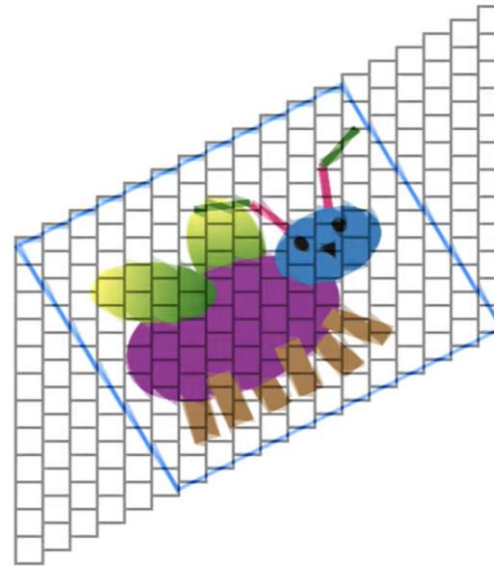
Translate each row by different amount

$$\begin{bmatrix} 1.0 & -0.5 \\ 0.0 & 1.0 \end{bmatrix}$$

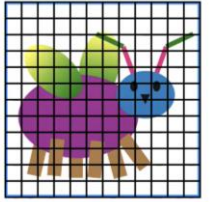


Then by Translate each column by different amount

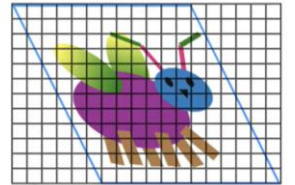
$$\begin{bmatrix} 1.0 & 0.0 \\ 0.5 & 1.0 \end{bmatrix}$$



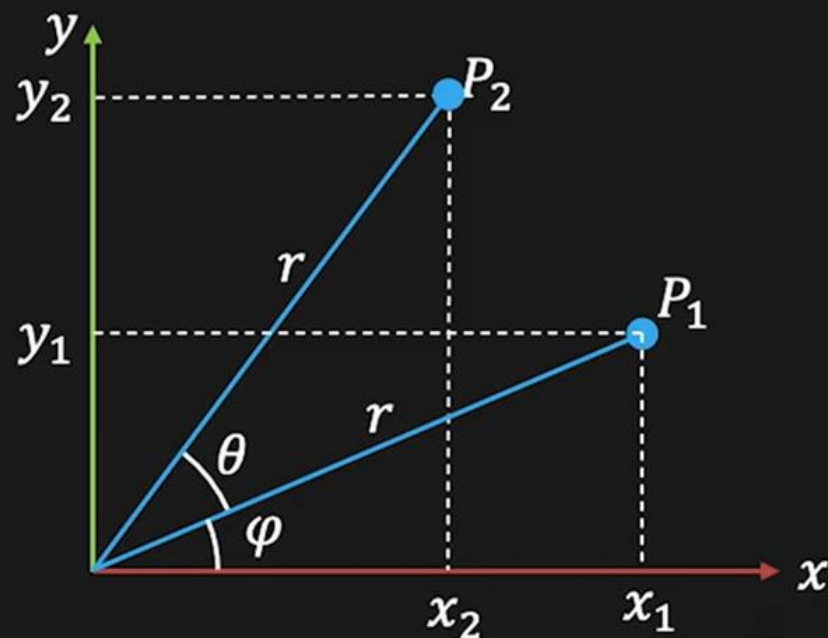
$$\begin{bmatrix} 1.0 & -0.5 \\ 0.0 & 1.0 \end{bmatrix}$$



$$\begin{bmatrix} 1.0 & -0.0 \\ 0.5 & 1.0 \end{bmatrix}$$



2D Rotation



$$x_1 = r \cos(\varphi)$$

$$y_1 = r \sin(\varphi)$$

$$x_2 = r \cos(\varphi + \theta)$$

$$x_2 = r \cos \varphi \cos \theta - r \sin \varphi \sin \theta$$

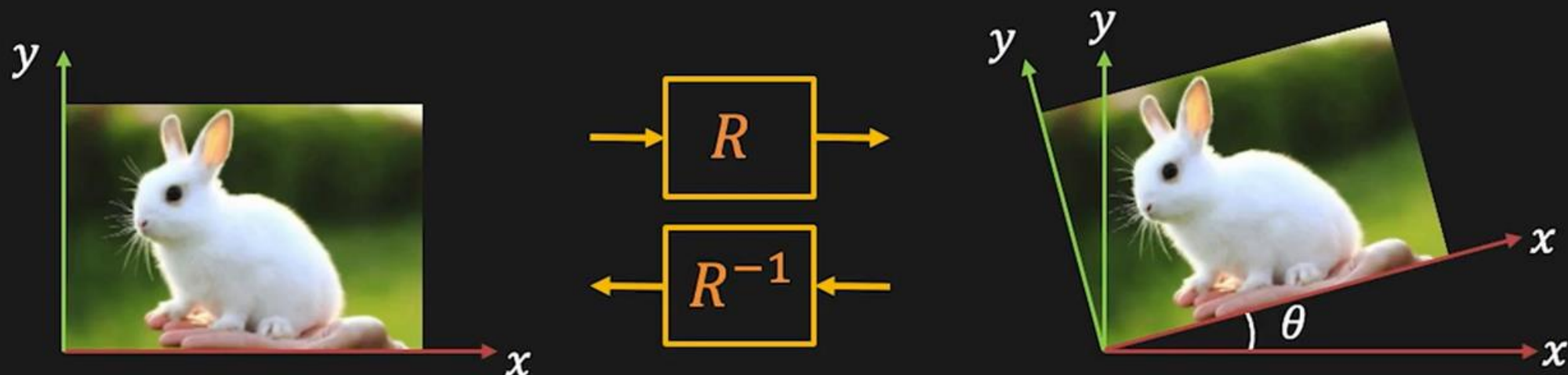
$$x_2 = x_1 \cos \theta - y_1 \sin \theta$$

$$y_2 = r \sin(\varphi + \theta)$$

$$y_2 = r \cos \varphi \sin \theta + r \sin \varphi \cos \theta$$

$$y_2 = x_1 \sin \theta + y_1 \cos \theta$$

2D Rotation



Forward:

$$x_2 = x_1 \cos \theta - y_1 \sin \theta$$

$$y_2 = x_1 \sin \theta + y_1 \cos \theta$$

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = R \begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

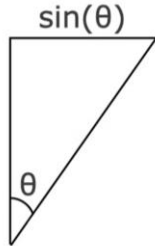
Inverse:

$$x_1 = x_2 \cos \theta + y_2 \sin \theta$$

$$y_1 = -x_2 \sin \theta + y_2 \cos \theta$$

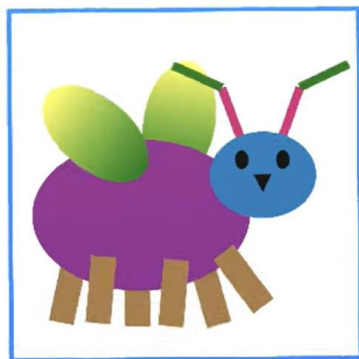
$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = R^{-1} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \end{bmatrix}$$

Rotation using Sine and Cosine

$$\begin{bmatrix} 1.0 & -a \\ a & 1.0 \end{bmatrix}$$


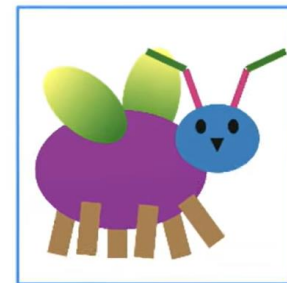
$$\begin{bmatrix} 1.0 & -\sin(\theta) \\ \sin(\theta) & 1.0 \end{bmatrix}$$

$$\begin{bmatrix} 1.0 & -\sin(0.0\pi) \\ \sin(0.0\pi) & 1.0 \end{bmatrix}$$



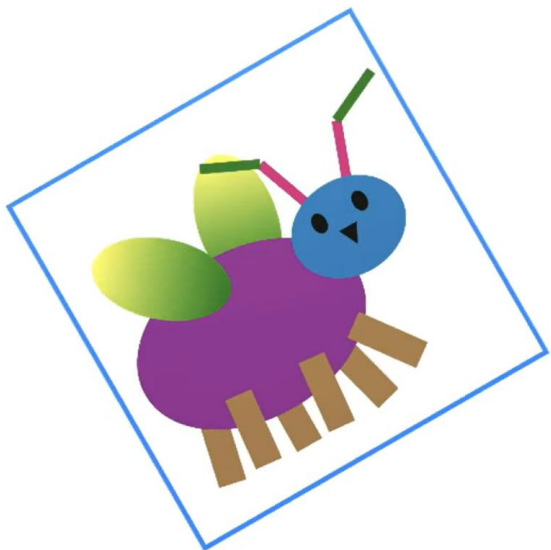
Original Image

$$\begin{bmatrix} \cos(0.2\pi) & 0.0 \\ 0.0 & \cos(0.2\pi) \end{bmatrix}$$



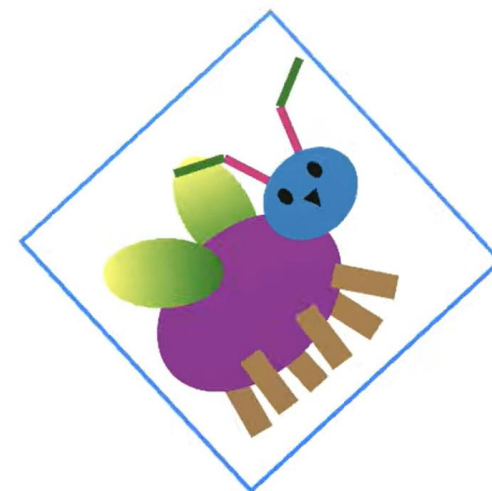
Scales by a factor $1/x$

$$\begin{bmatrix} 1.0 & -\sin(0.2\pi) \\ \sin(0.2\pi) & 1.0 \end{bmatrix}$$



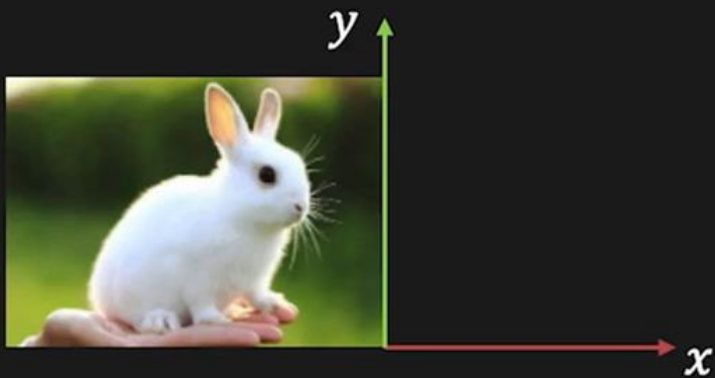
Rotates by 0.2π + Scales by a factor x

$$\begin{bmatrix} \cos(0.2\pi) & -\sin(0.2\pi) \\ \sin(0.2\pi) & \cos(0.2\pi) \end{bmatrix}$$



Rotates by 0.2π

Mirror



Mirror about Y-axis:

$$x_2 = -x_1$$

$$y_2 = y_1$$

$$M_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$



Mirror about line $y = x$:

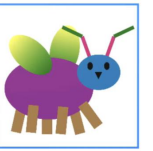
$$x_2 = y_1$$

$$y_2 = x_1$$

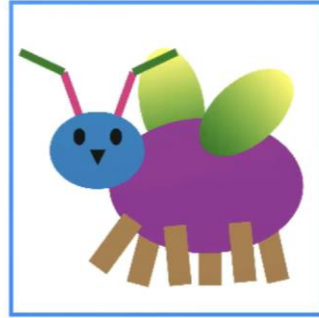
$$M_{xy} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Reflection

$$\begin{bmatrix} 1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$



$$\begin{bmatrix} -1.0 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$



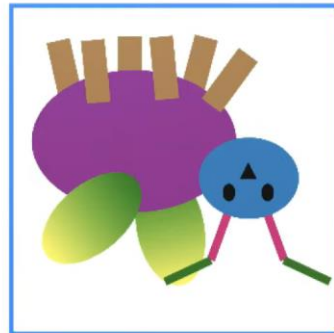
Reflection (around y axis)

$$\begin{bmatrix} -0.5 & 0.0 \\ 0.0 & 1.0 \end{bmatrix}$$



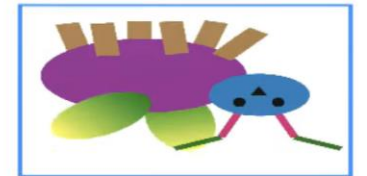
Reflection(around y-axis) + [Horizontal Scaling = 0.5x]

$$\begin{bmatrix} 1.0 & 0.0 \\ 0.0 & -1.0 \end{bmatrix}$$



Reflection (around x axis)

$$\begin{bmatrix} 1.0 & 0.0 \\ 0.0 & -0.5 \end{bmatrix}$$



Reflection (around x-axis) + [Vertical Scaling = 0.5x]

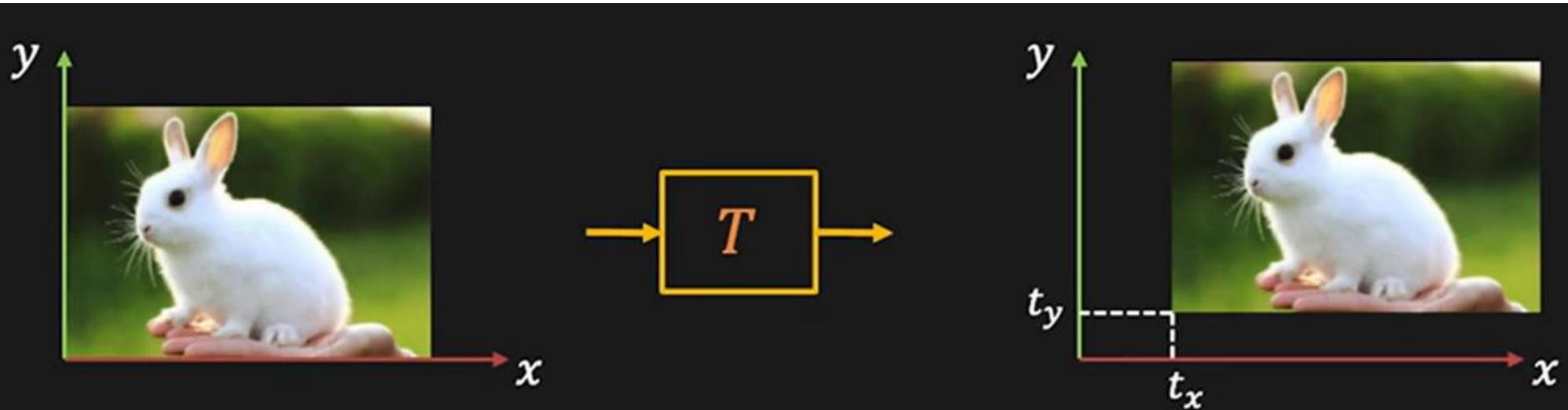
2x2 Matrix Transformations

Any transformation of the form:

$$\begin{bmatrix} x_2 \\ y_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

- Origin maps to the origin
- Lines map to lines
- Parallel lines remain parallel
- Closed under composition

Translation



$$x_2 = x_1 + t_x \quad y_2 = y_1 + t_y$$

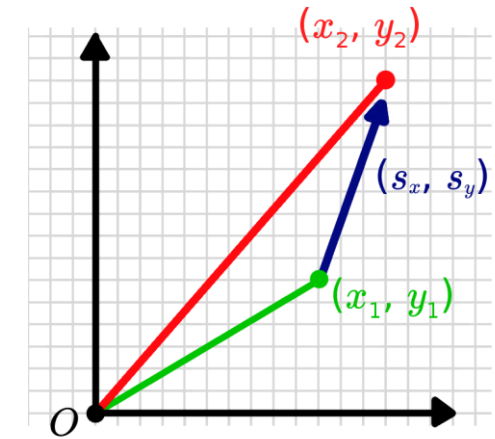
Can translation be expressed as a 2x2 matrix? **No.**

Translation Problem

- ❖ We simply need to add the appropriate amount to the x and y coordinates. Say, for example, that we had the point (x,y) and we wanted to shift it by s_x units in the x direction and s_y units in the y direction. We simply perform the following addition:

$$\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} s_x \\ s_y \end{bmatrix} = \begin{bmatrix} x + s_x \\ y + s_y \end{bmatrix}$$

- ❖ The problem is that this operation is *non-linear*.
- ❖ To solve this problem we're going to introduce a slightly modified representation of our coordinates. This new system is called homogeneous coordinates.
- ❖ The first thing we have to do is modify our coordinates, which simply involves taking a "1" onto the end of our point vector. we are looking for a 3x3 matrix
- ❖ Firstly, we need to guarantee a 1 in the bottom element of the result.
- ❖ Secondly, we know that for the first element of our result, there is one x_1 and no y_1 , and vice versa for the second element.
- ❖ Lastly, the top of the right column of our matrix will contain the column vector we want to translate by.



$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} = \begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ ? & ? & ? \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 + s_x \\ y_1 + s_y \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} ? & ? & ? \\ ? & ? & ? \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & ? \\ 0 & 1 & ? \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 + ? \\ y_1 + ? \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & s_x \\ 0 & 1 & s_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = \begin{bmatrix} x_1 + s_x \\ y_1 + s_y \\ 1 \end{bmatrix}$$

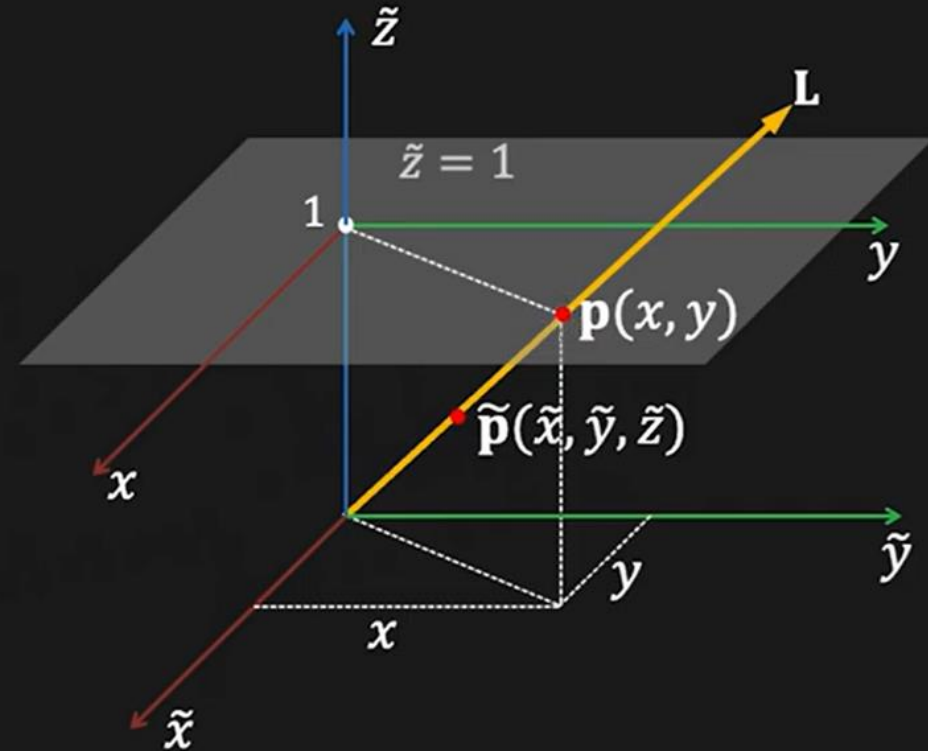
By using homogeneous coordinates, we can represent our non-linear translation as a linear transformation.

Homogeneous Representation

The **homogenous** representation of a 2D point $\mathbf{p} = (x, y)$ is a 3D point $\tilde{\mathbf{p}} = (\tilde{x}, \tilde{y}, \tilde{z})$. The third coordinate $\tilde{z} \neq 0$ is fictitious such that:

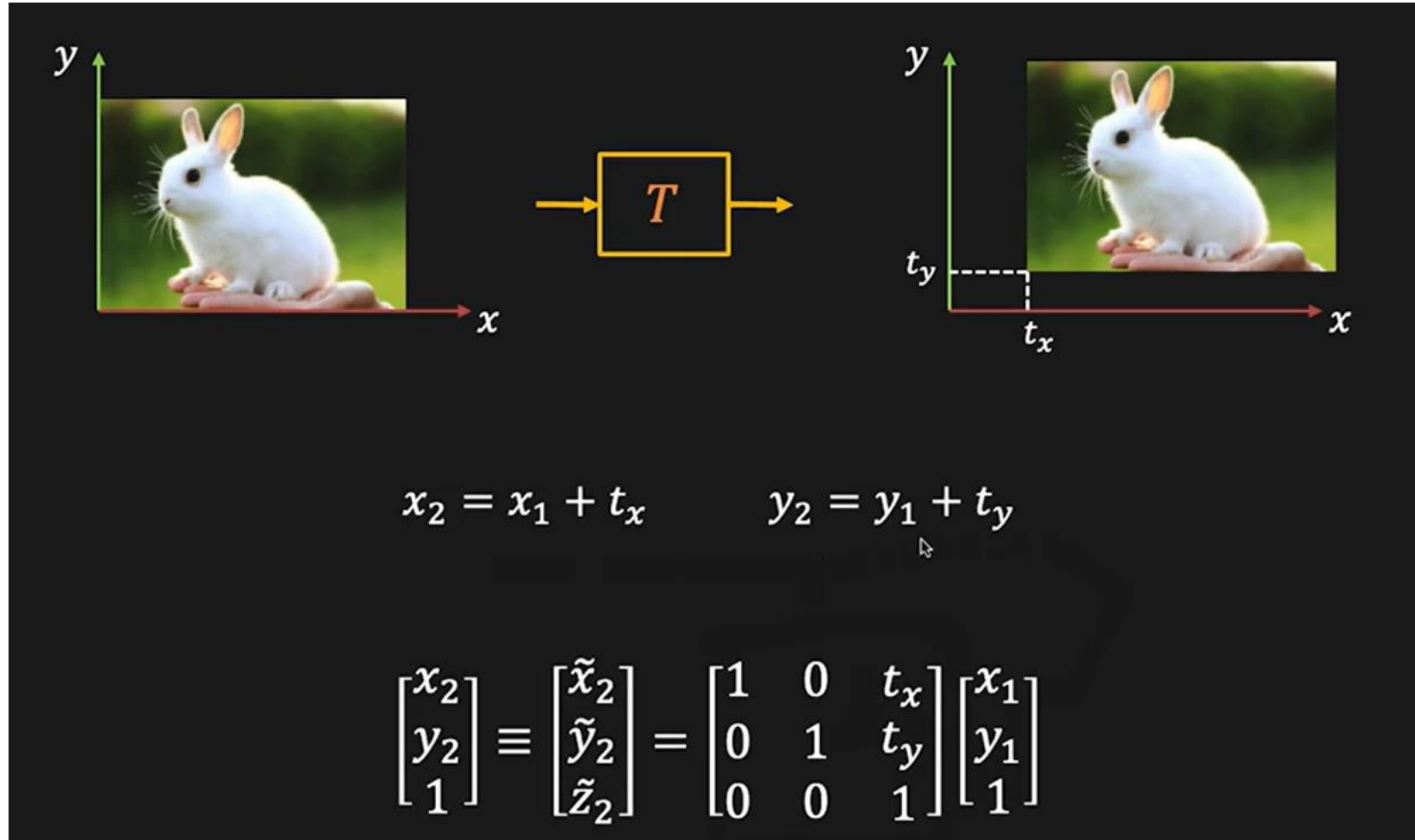
$$x = \frac{\tilde{x}}{\tilde{z}} \quad y = \frac{\tilde{y}}{\tilde{z}}$$

$$\mathbf{p} \equiv \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{z}x \\ \tilde{z}y \\ \tilde{z} \end{bmatrix} \equiv \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \end{bmatrix} = \tilde{\mathbf{p}}$$



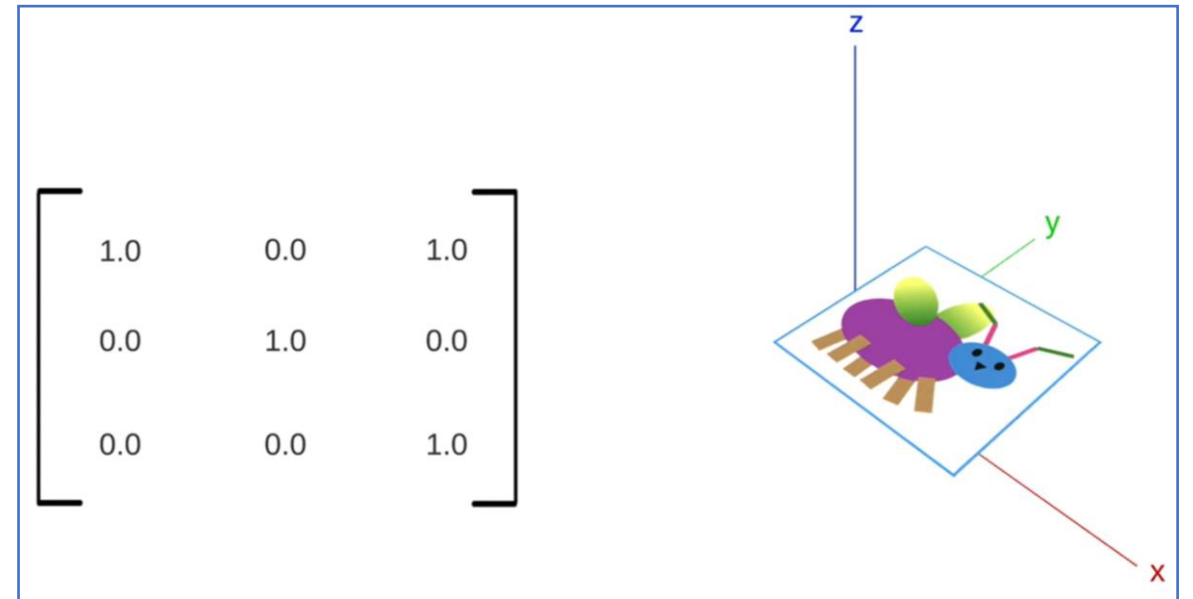
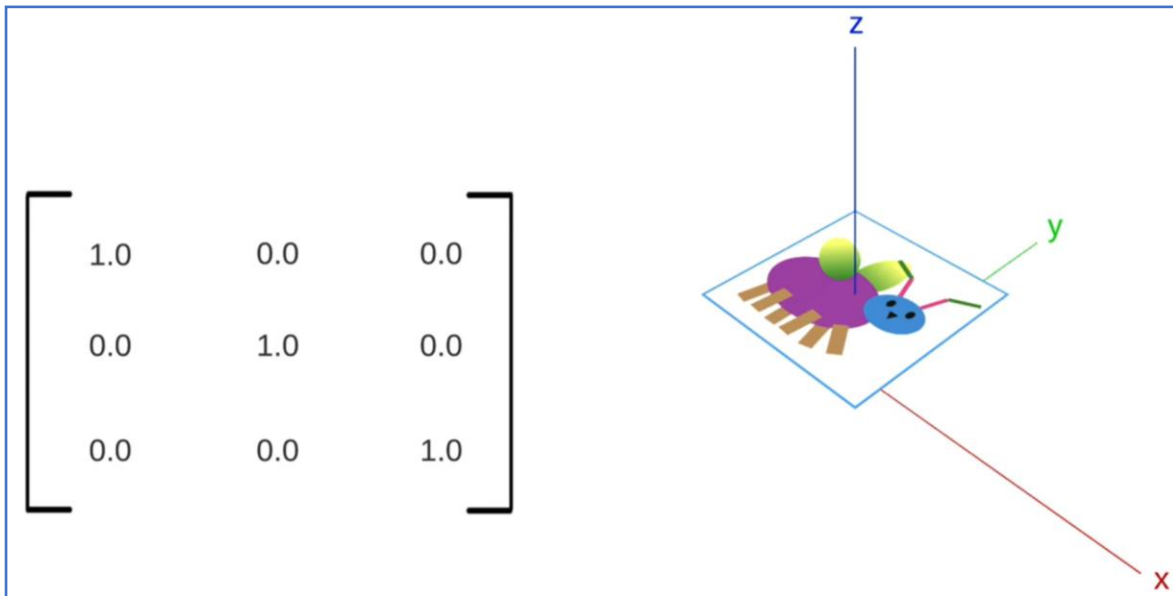
Every point on line L (except origin) represents the homogenous coordinate of $\mathbf{p}(x, y)$

Translation



Translation

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x+1 \\ y \\ 1 \end{bmatrix}$$



Scaling, Rotation, Skew and Translation

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Scaling

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} 1 & m_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Skew

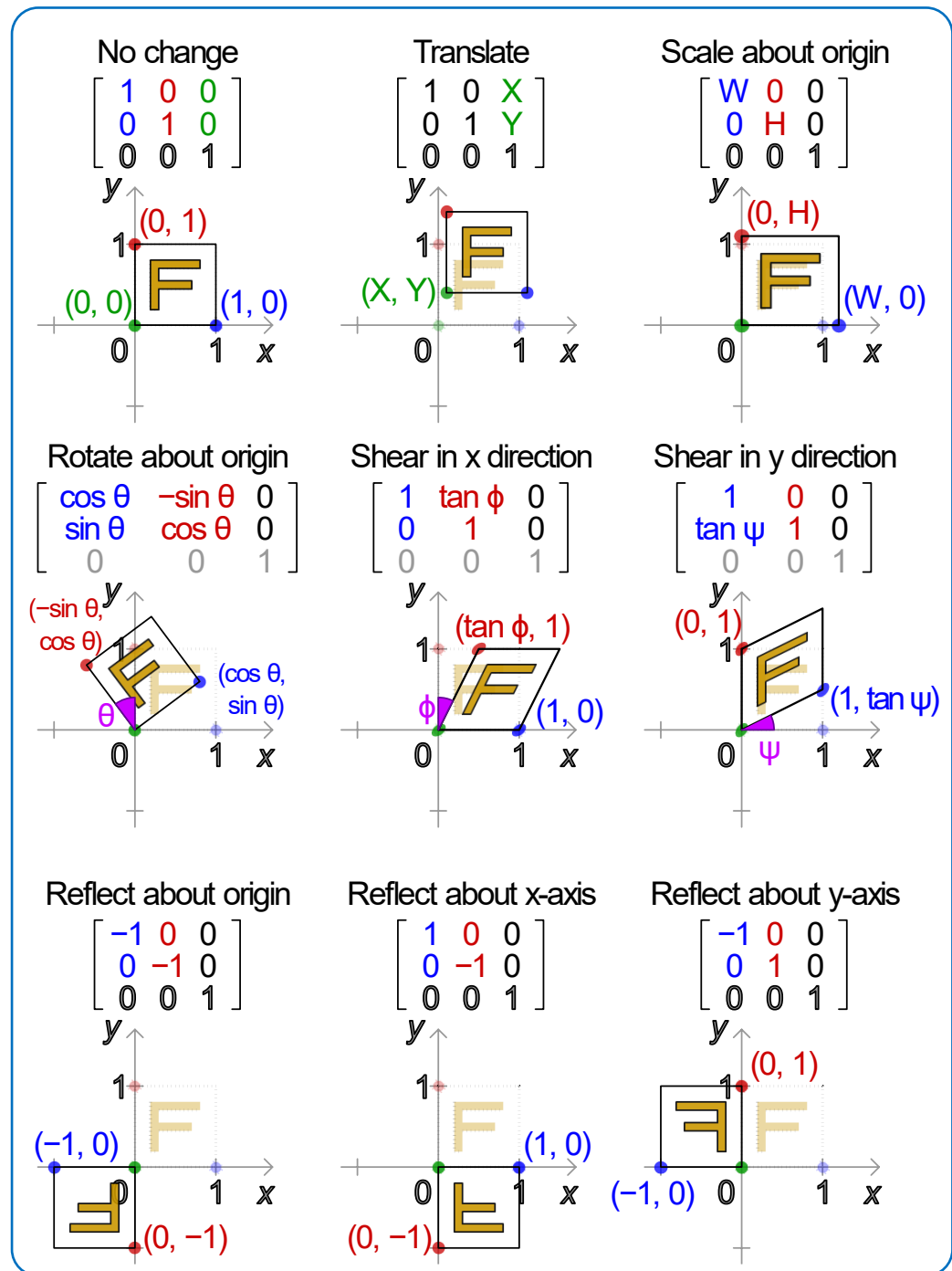
$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Translation

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

Rotation

Affine Transformation Matrices



Affine Transformation

Any transformation of the form:

$$\begin{bmatrix} x_2 \\ y_2 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$



Transformation Matrix: Take a closer look

We can simplify the representation of this matrix as shown below (note the use of a bold **0** to indicate it is a zero vector).

$$\mathbf{T} = \left[\begin{array}{cc|c} \cos(\theta) & -\sin(\theta) & s_x \\ \sin(\theta) & \cos(\theta) & s_y \\ \hline 0 & 0 & 1 \end{array} \right] = \begin{bmatrix} \mathbf{R} & \mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix}$$

❖ Take a look at the Transformation matrix:

- The top left corner contains the original rotation matrix
- The top right hand corner contains the translation offset as a column vector
- The very bottom right hand corner contains a 1
- The rest of the bottom row (to the left of the 1) is all zeros.

❖ Properties of transformation matrices:

➤ Chainability

- Affine transformation matrix is *linear* (using homogeneous coordinates), we are able to chain them together using multiplication, just like with the rotation matrices
- A rotation matrix multiplied by another rotation matrix produces a rotation matrix, in the same way a transformation matrix multiplied by a transformation matrix will always result in a transformation matrix.

$$\begin{aligned} \mathbf{T}_1 \mathbf{T}_2 &= \begin{bmatrix} \mathbf{R}_1 & \mathbf{b}_1 \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}_2 & \mathbf{b}_2 \\ \mathbf{0} & 1 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{R}_1 \mathbf{R}_2 & \mathbf{R}_1 \mathbf{b}_2 + \mathbf{b}_1 \\ \mathbf{0} & 1 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{R}_3 & \mathbf{b}_3 \\ \mathbf{0} & 1 \end{bmatrix} \end{aligned}$$

➤ Inverse

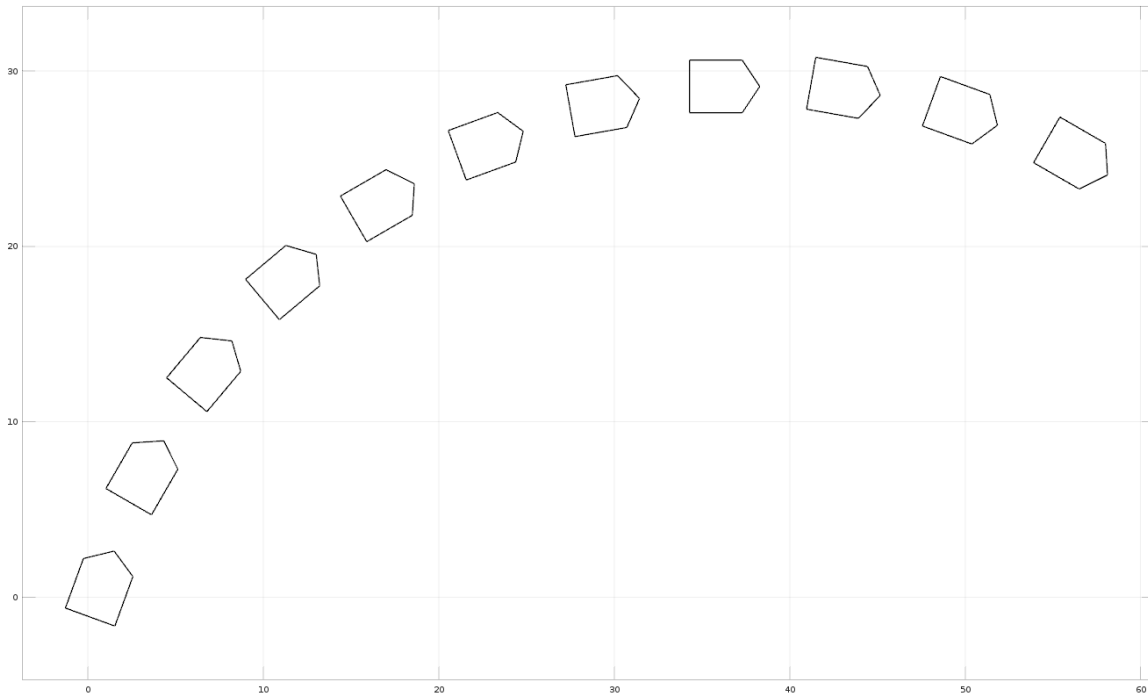
- The second useful property of the transformation matrix is Inverse and taking inverses is relatively easy. Inverse of the rotation matrix is its transpose.
- The formula for the inverse of a transformation matrix **T** and its verification is given below (without the derivation):

$$\begin{aligned} \mathbf{T}^{-1} &= \begin{bmatrix} \mathbf{R} & \mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix}^{-1} \\ &= \begin{bmatrix} \mathbf{R}^{-1} & -\mathbf{R}^{-1}\mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{R}^T & -\mathbf{R}^T\mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \mathbf{T} \mathbf{T}^{-1} &= \begin{bmatrix} \mathbf{R} & \mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \mathbf{R}^{-1} & -\mathbf{R}^{-1}\mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{R} \mathbf{R}^{-1} & \mathbf{R}(-\mathbf{R}^{-1})\mathbf{b} + \mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{I} & -\mathbf{I}\mathbf{b} + \mathbf{b} \\ \mathbf{0} & 1 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix} \end{aligned}$$

Note that if you use this structure to represent other affine transformations (e.g. a shear matrix instead of a rotation), you can use the middle result, but you have to take inverse for R.

Affine Transformation Trajectory Demo



```
%% TRANSFORMATION TRAJECTORY DEMO  
% Demonstrates the use of affine transformation matrices  
% to plot an object moving along a trajectory
```

```
% Set up an array of points  
x_points = [2, 2, 0.5, -1, -1, 2];  
y_points = [-1, 2, 3, 2, -1, -1];  
points = [x_points; y_points; ones(1, length(x_points))];
```

```
% Initial Conditions  
sx = 0; sy = 0; theta = -20;
```

```
clf;  
for t = 0:10  
    % Compute the transformation matrix  
    transf_mat = [cosd(theta), -sind(theta), sx; ...  
                 sind(theta), cosd(theta), sy; ...  
                 0, 0, 1];
```

```
% Compute the new points  
transf_pts = transf_mat*points;
```

```
% Plot the points  
plot(transf_pts(1,:), transf_pts(2,:), '-k');  
hold on;
```

```
% Update the state for the next plot  
sx = sx + 7*cosd(theta+90);  
sy = sy + 7*sind(theta+90);  
theta = theta - 10;  
end
```

```
axis equal; grid on;
```

Projective Transformation

Any transformation of the form:

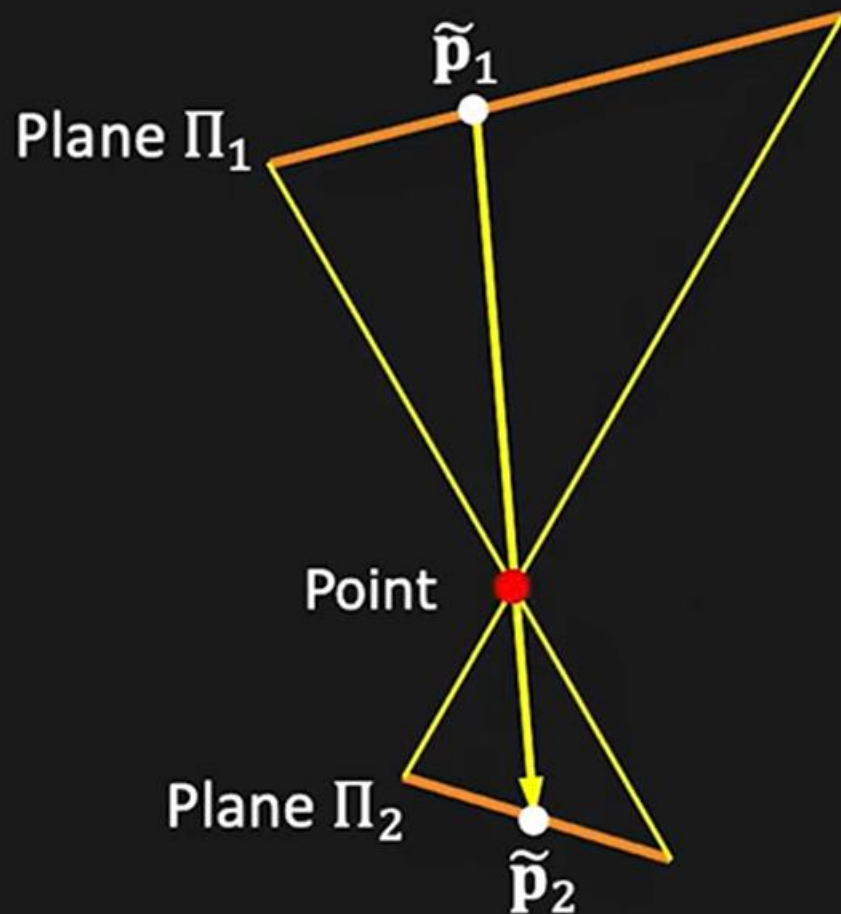
$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix} \quad \tilde{\mathbf{p}}_2 = H\tilde{\mathbf{p}}_1$$



Also called **Homography**

Projective Transformation

Mapping of one plane to another through a point



$$\tilde{\mathbf{p}}_2 = H\tilde{\mathbf{p}}_1$$

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

Projective Transformation

Homography can only be defined up to a scale.

$$\begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix} \equiv \begin{bmatrix} \tilde{x}_2 \\ \tilde{y}_2 \\ \tilde{z}_2 \end{bmatrix} \equiv k \begin{bmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{y}_1 \\ \tilde{z}_1 \end{bmatrix}$$

If we fix scale such that $\sqrt{\sum (h_{ij})^2} = 1$ then 8 free parameters

- Origin does not necessarily map to the origin
- Lines map to lines
- Parallel lines do not necessarily remain parallel
- Closed under composition

References

- ❖ [Affine transformations in 5 minutes](#)
- ❖ [Image Stitching | Face Detection](#)
- ❖ [What are affine transformations?](#)
- ❖ <https://articulatedrobotics.xyz/tutorials/coordinate-transforms/transformation-matrices/>
- ❖ <https://articulatedrobotics.xyz/tutorials/coordinate-transforms/translations>
- ❖ https://www.algorithm-archive.org/contents/affine_transformations/affine_transformations.html