Computer Vision

(Course Code: 4047)

Module-6: 3D Reconstruction

Lecture-2: 3D Surface Representations

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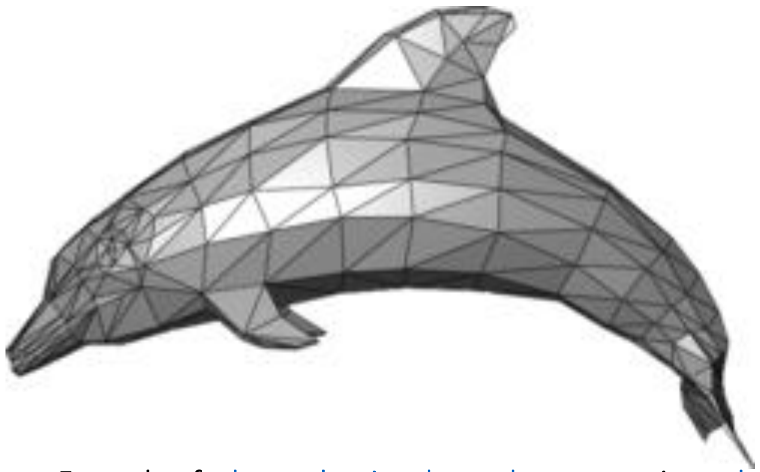
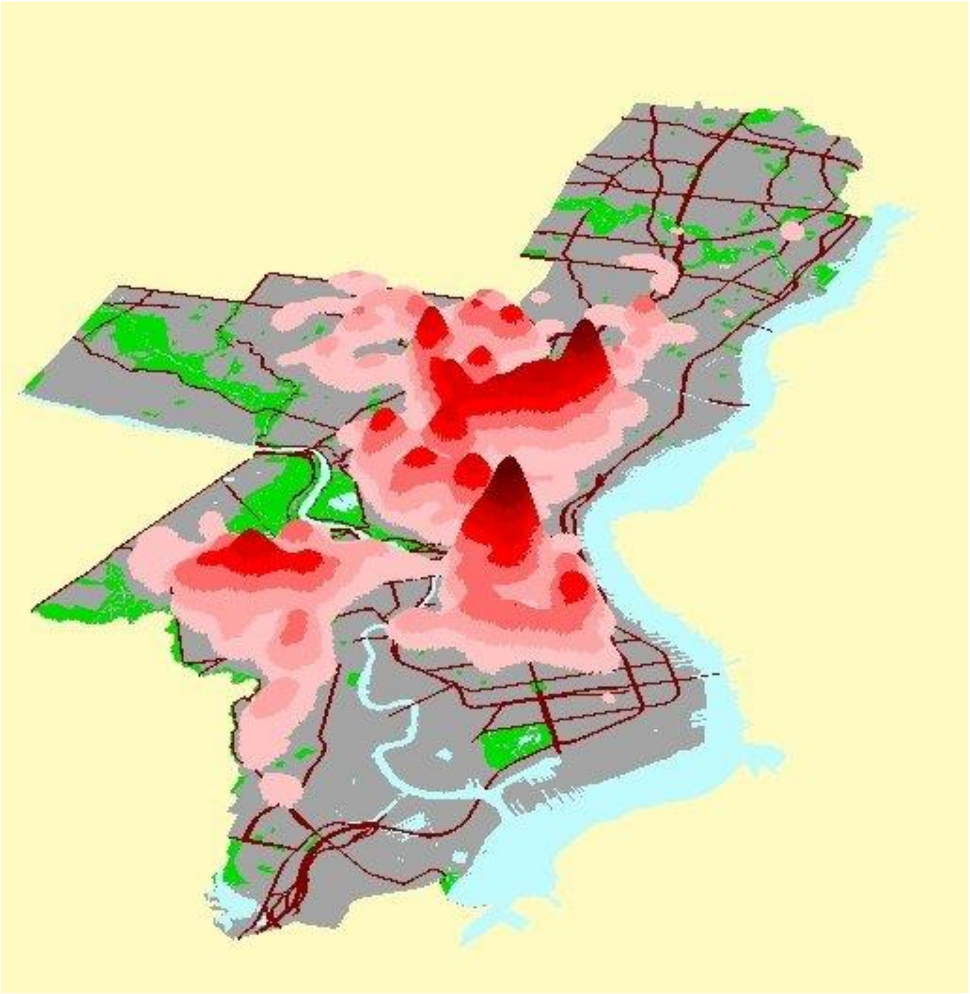
Surfaces

- Surfaces involve a third 'z' dimension (height/elevation/magnitude, quantity) in addition to x,y planimetric location.
- Any type of continuous data can be represented as a surface, whether it be ground elevation, barometric pressure, rainfall, crop yield, noise levels, population density, sales intensity, land value, income, crime rates, etc.

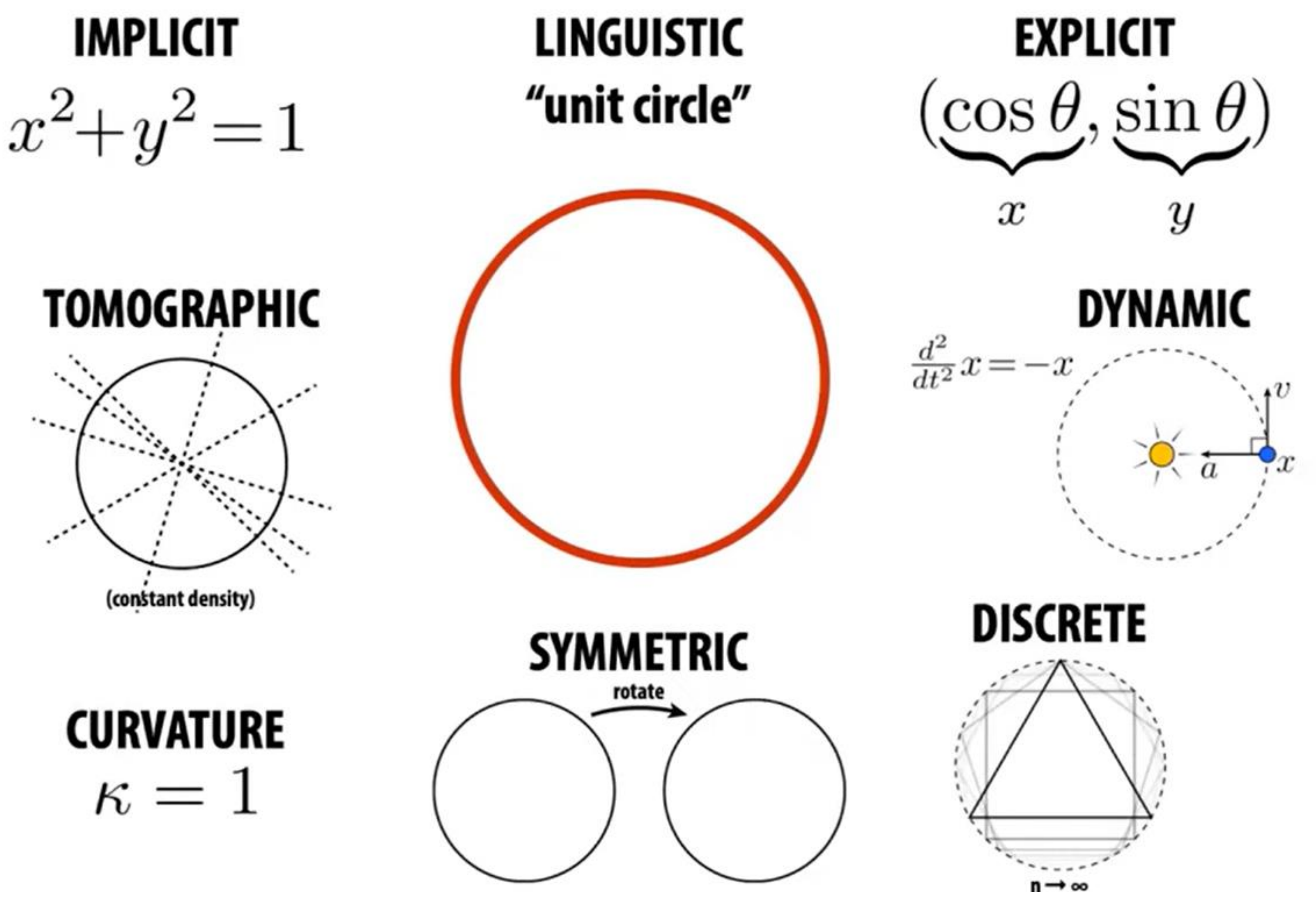


Example of a low poly triangle mesh representing a dolphin



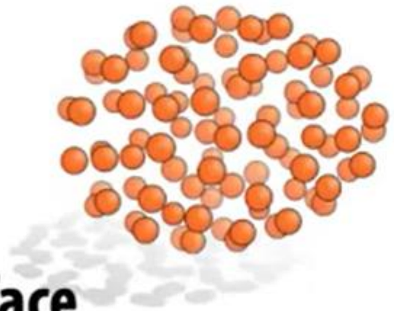
3D Model of Crime Density in Boston, MA
http://gis.mit.edu/classes/11.521/lectures/Lecture_14/Lect14.htm

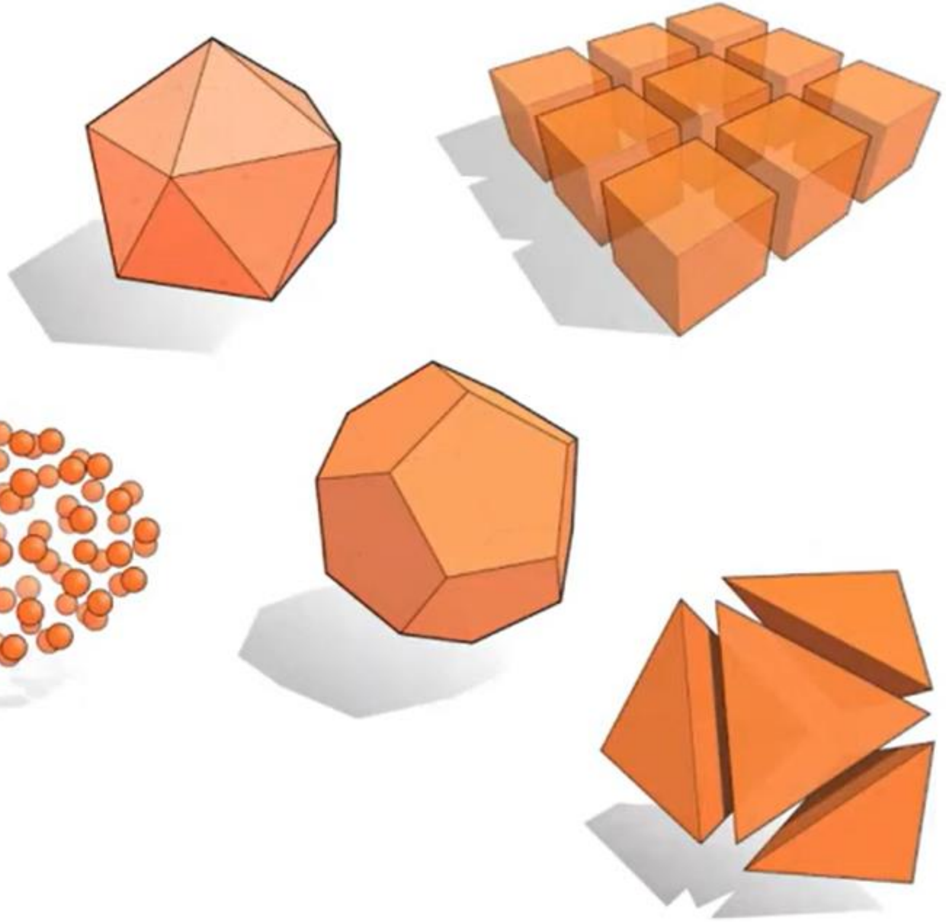
How we can describe Surface Geometry?



Many ways to digitally encode geometry

- **EXPLICIT**
  - point cloud
  - polygon mesh
  - subdivision, NURBS
  - ...
- **IMPLICIT**
  - level set
  - algebraic surface
  - L-systems
  - ...
- **Each choice best suited to a different task/type of geometry**

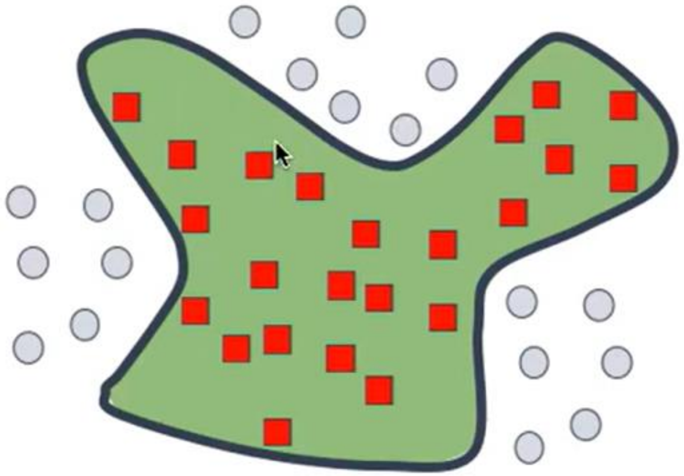


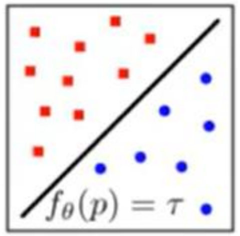


Surfaces as an Implicit Function

$$\mathbf{p} = (x, y, z) \in \mathbb{R}^3$$

$$f(\mathbf{p}) = \begin{cases} 0, & \text{if} \quad \mathbf{p} \in \text{outside} \\ 1, & \text{if} \quad \mathbf{p} \in \text{inside} \end{cases}$$





$$\mathcal{S} = \{\mathbf{p},\ f(\mathbf{p}) = \tau\}$$

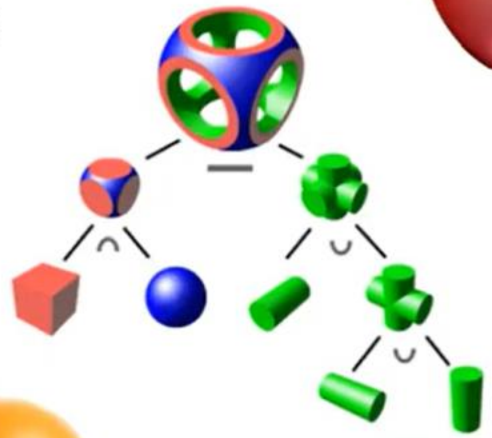
Implicit Representation of Surface Geometry

- **Points aren't known directly, but satisfy some relationship**
- **E.g., unit sphere is all points such that $x^2+y^2+z^2=1$**
- **More generally, $f(x,y,z) = 0$**

$f(x,y)$

Many Implicit Representation in Graphics

- algebraic surfaces
- constructive solid geometry
- level set methods
- blobby surfaces
- fractals
- ...



(Will see some of these a bit later.)

Check if the point is inside the unit sphere

I have a new surface $f(x,y,z) = x^2 + y^2 + z^2 - 1$.

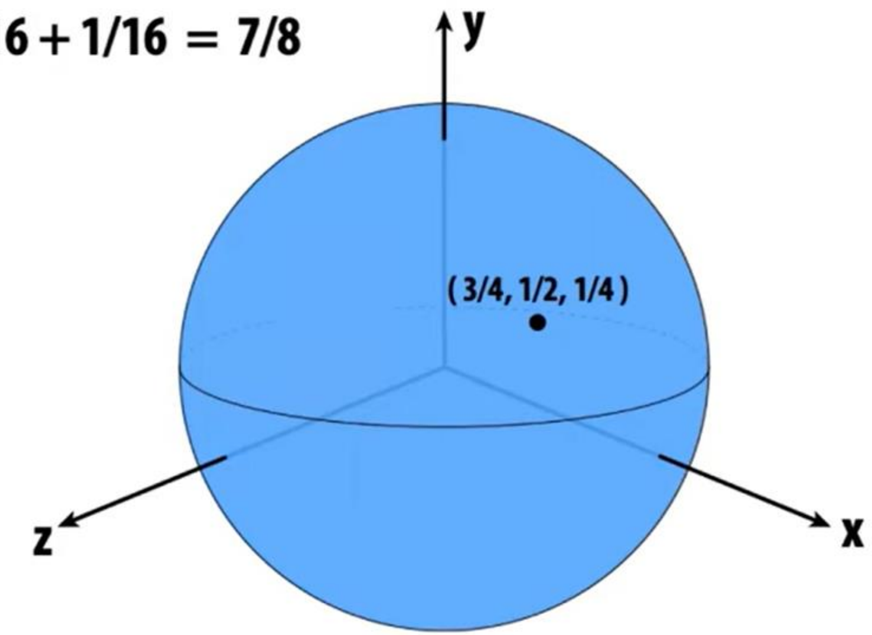
I want to see if a point is *inside* it.

How about the point (3/4, 1/2, 1/4)?

$9/16 + 4/16 + 1/16 = 7/8$

$7/8 < 1$

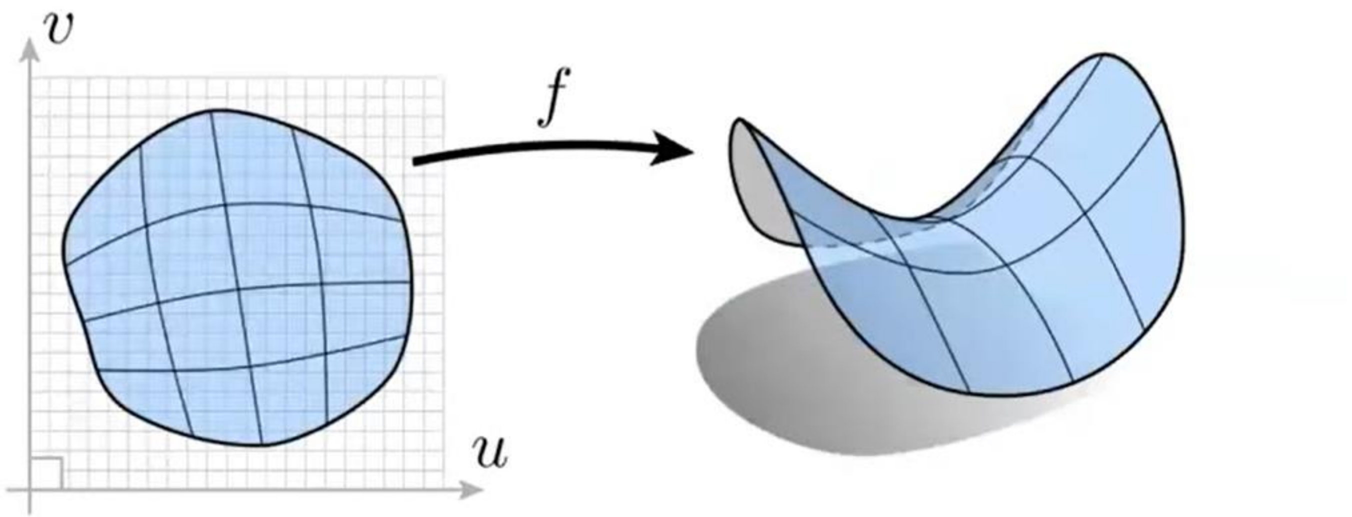
YES.



Implicit surfaces make other tasks easy (like inside/outside tests).

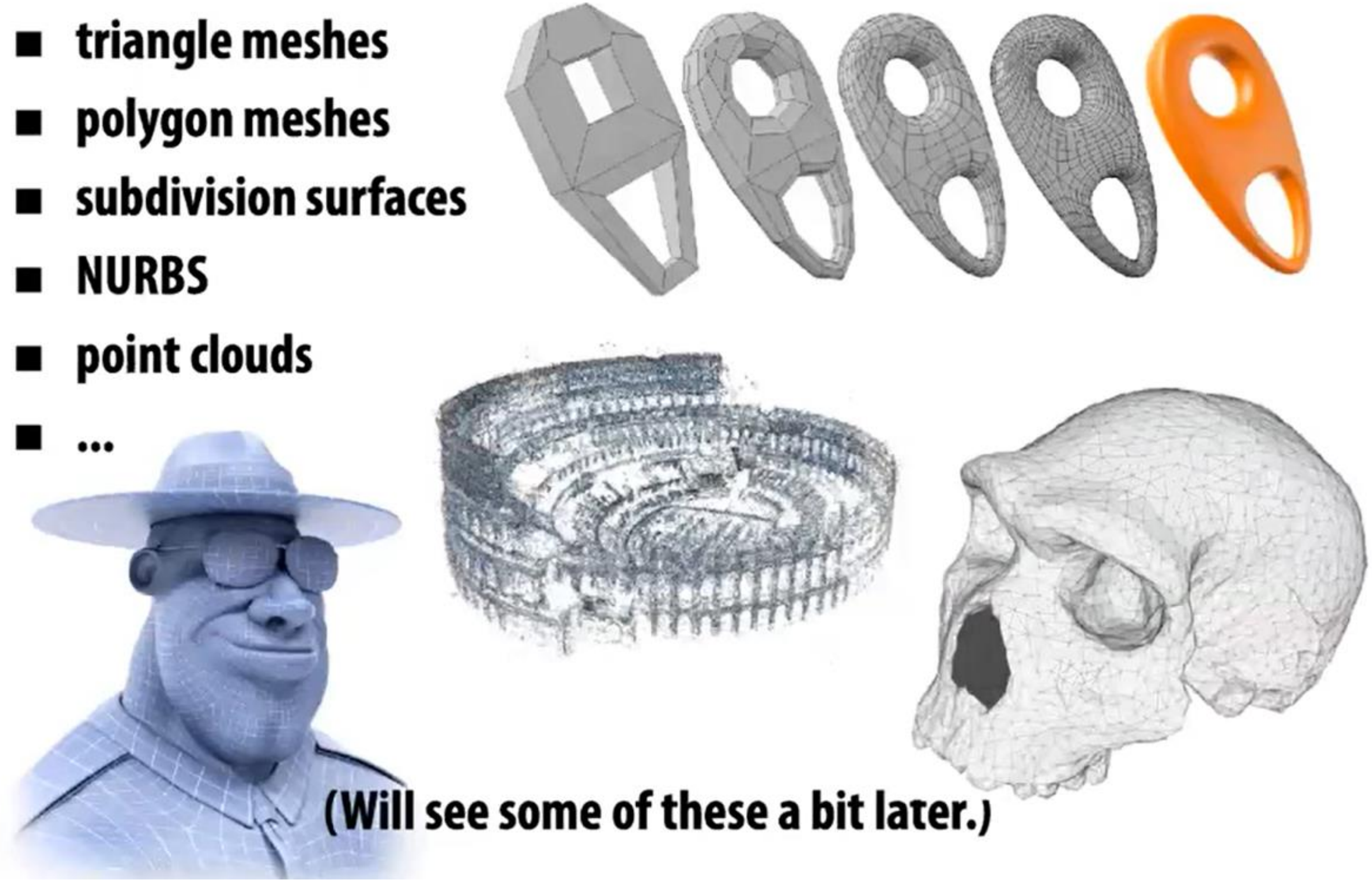
Explicit Representations of the Surface Geometry

- **All points are given directly**
- **E.g., points on sphere are** $(\cos(u)\sin(v), \sin(u)\sin(v), \cos(v)),$
  $\text{for } 0 \le u < 2\pi \text{ and } 0 \le v \le \pi$
- **More generally:** $f : \mathbb{R}^2 \to \mathbb{R}^3; (u, v) \mapsto (x, y, z)$



- **(Might have a bunch of these maps, e.g., one per triangle!)**

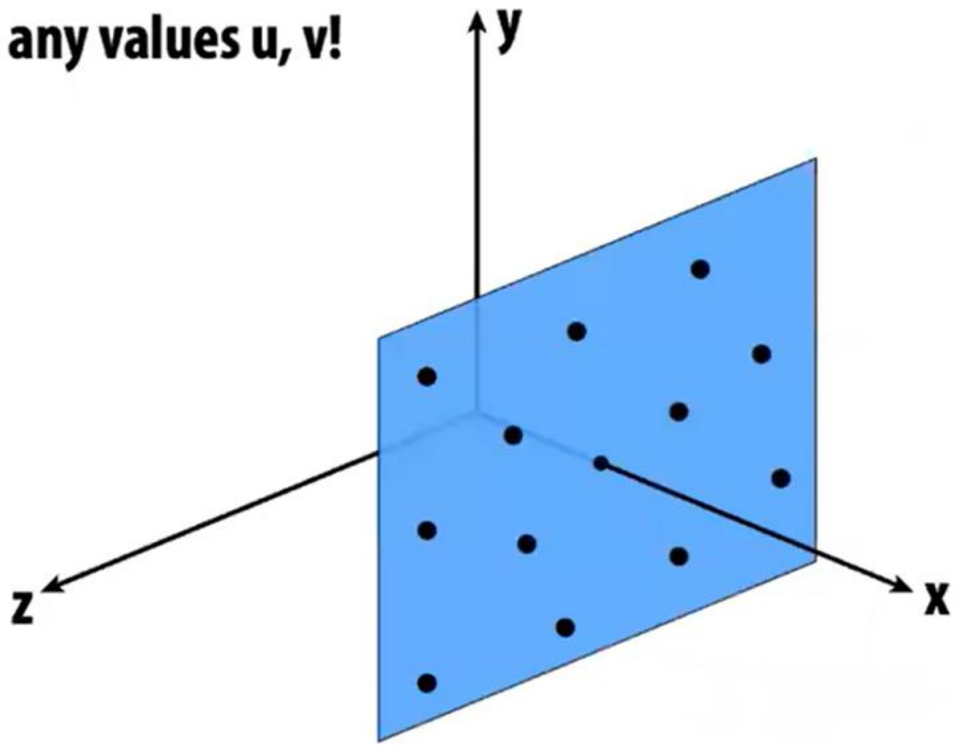
Many explicit representations in Graphics



Sampling an explicit Surface

My surface is f(u, v) = (1.23, u, v).

Just plug in any values u, v!



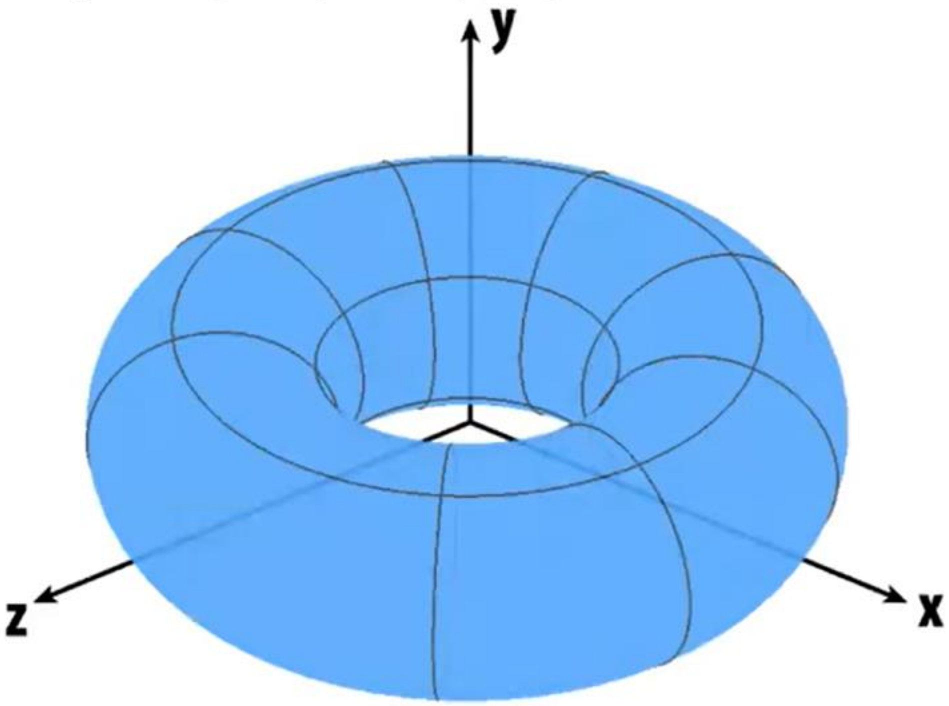
Explicit surfaces make some tasks easy (like sampling).

Explicit Functions: Check if this point inside the Torus

My surface is $f(u,v) = ((2+\cos u)\cos v, (2+\cos u)\sin v, \sin u)$

How about the point (1.96, -0.39, 0.9)?

...NO!



Explicit surfaces make other tasks hard (like inside/outside tests).

Algebraic Surfaces (Implicit)

- **Surface is zero set of a polynomial in x, y, z**
- **Examples:**

$$x^2+y^2+z^2=1$$

$$(R-\sqrt{x^2+y^2})^2+z^2=r^2$$

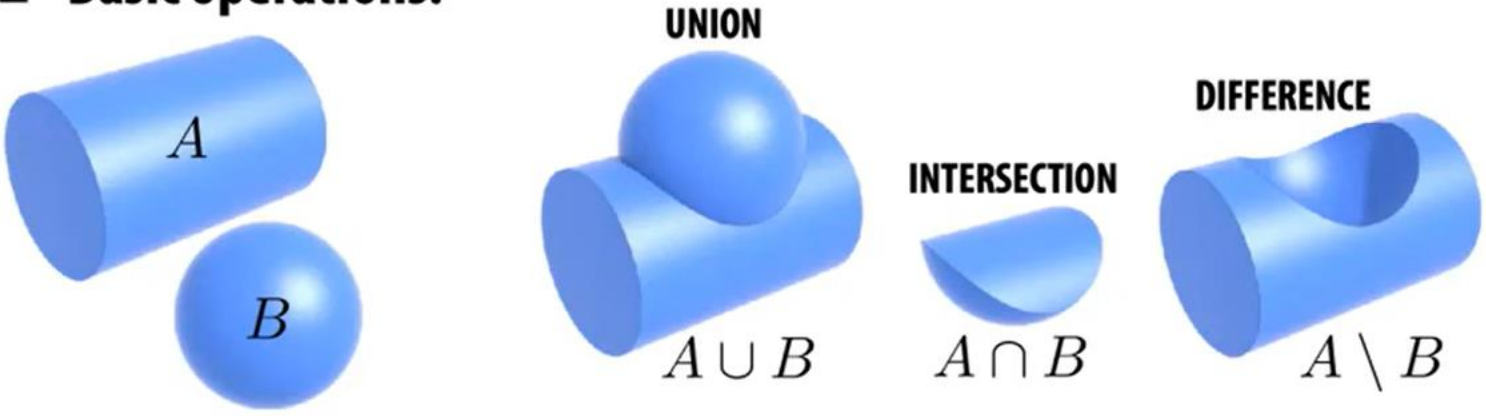
$$(x^2+\frac{9y^2}{4}+z^2-1)^3 = x^2z^3+\frac{9y^2z^3}{80}$$

- **What about more complicated shapes?**

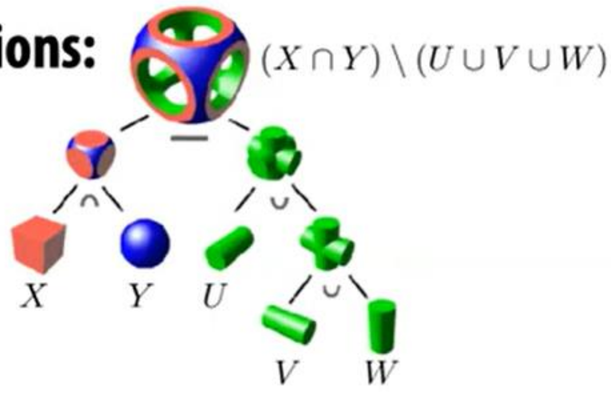
- **Very hard to come up with polynomials!**

Constructive Solid Geometry (Implicit)

- **Build more complicated shapes via Boolean operations**
- **Basic operations:**



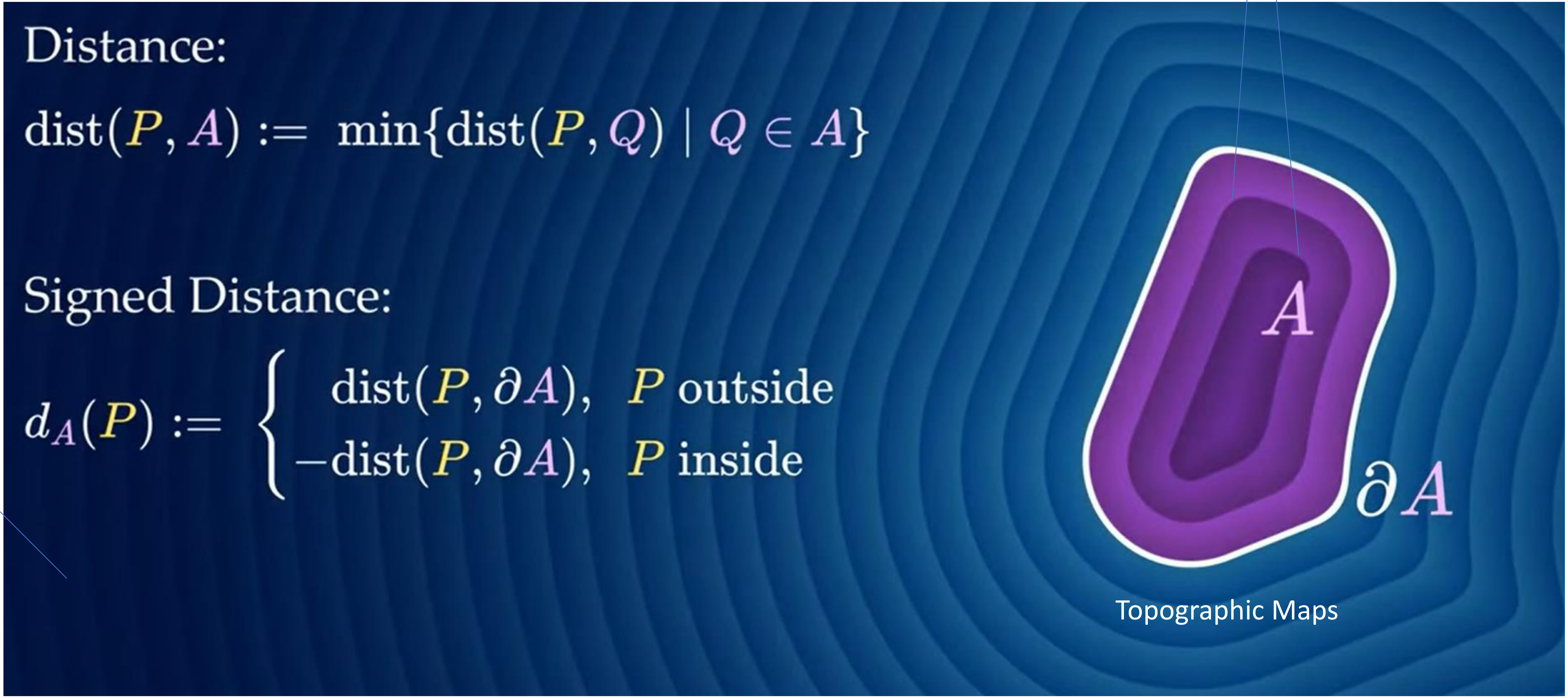
- **Then chain together expressions:**



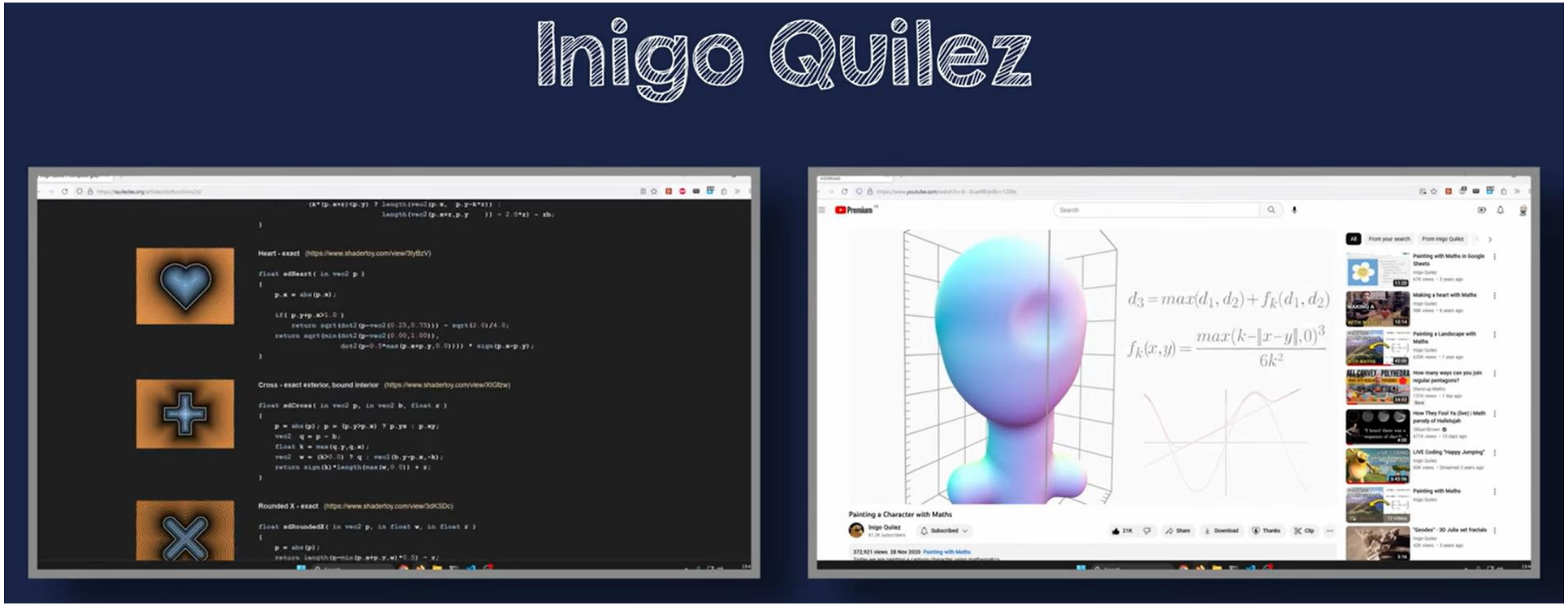
Some Representations work better than others – depends on the task

Signed Distance Function

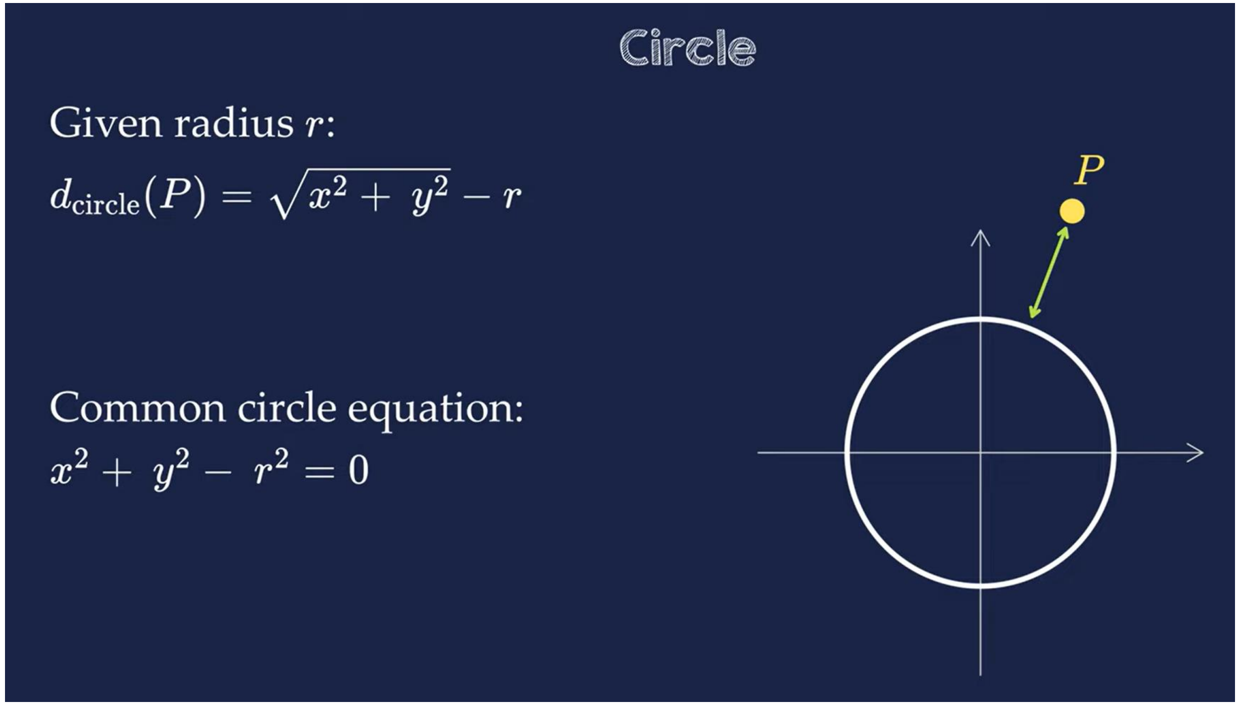
Level Sets - ISO Lines (same distance away from our shape)
Contour Lines (with same Signed Distance)

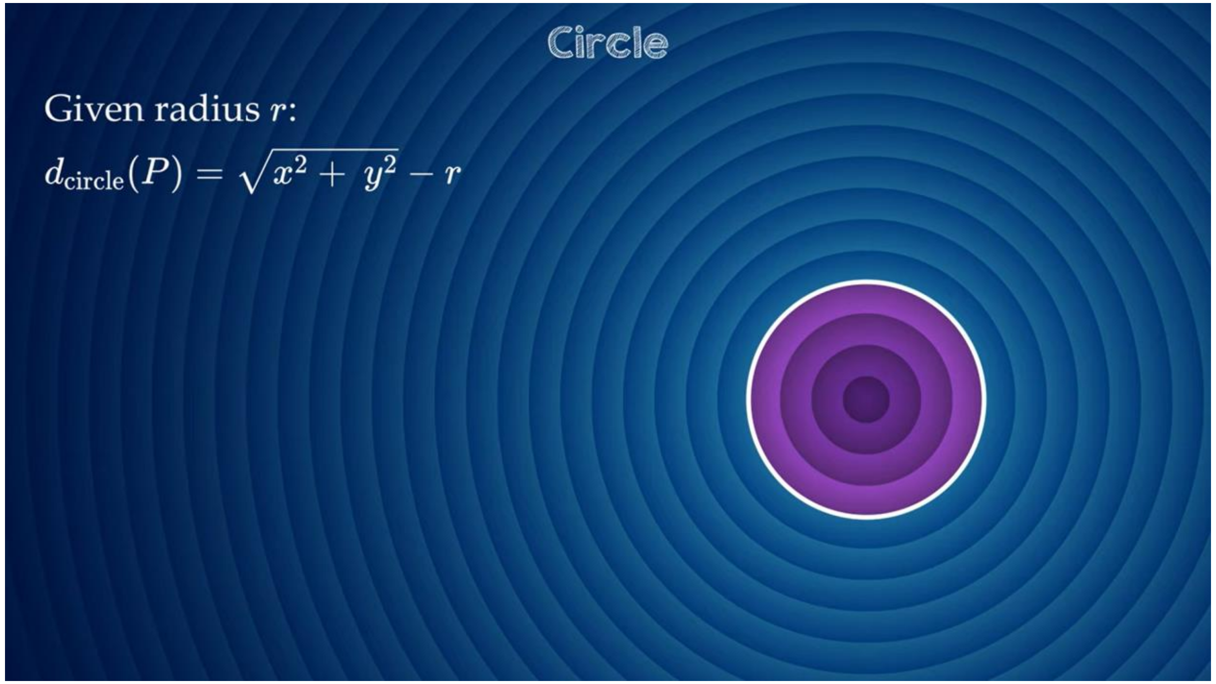


Signed Distance Function Resources



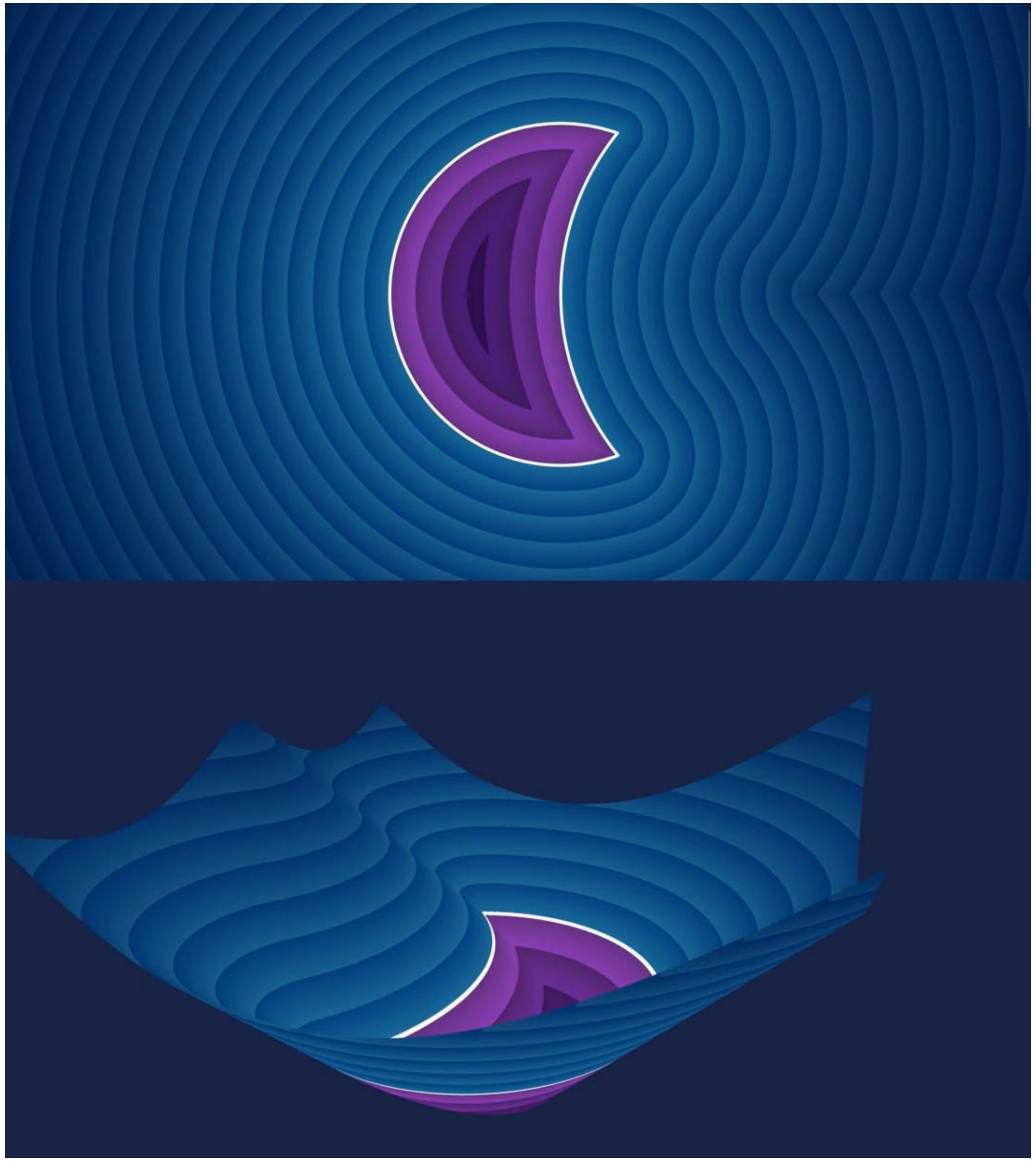
SDF for a Circle





Level Sets

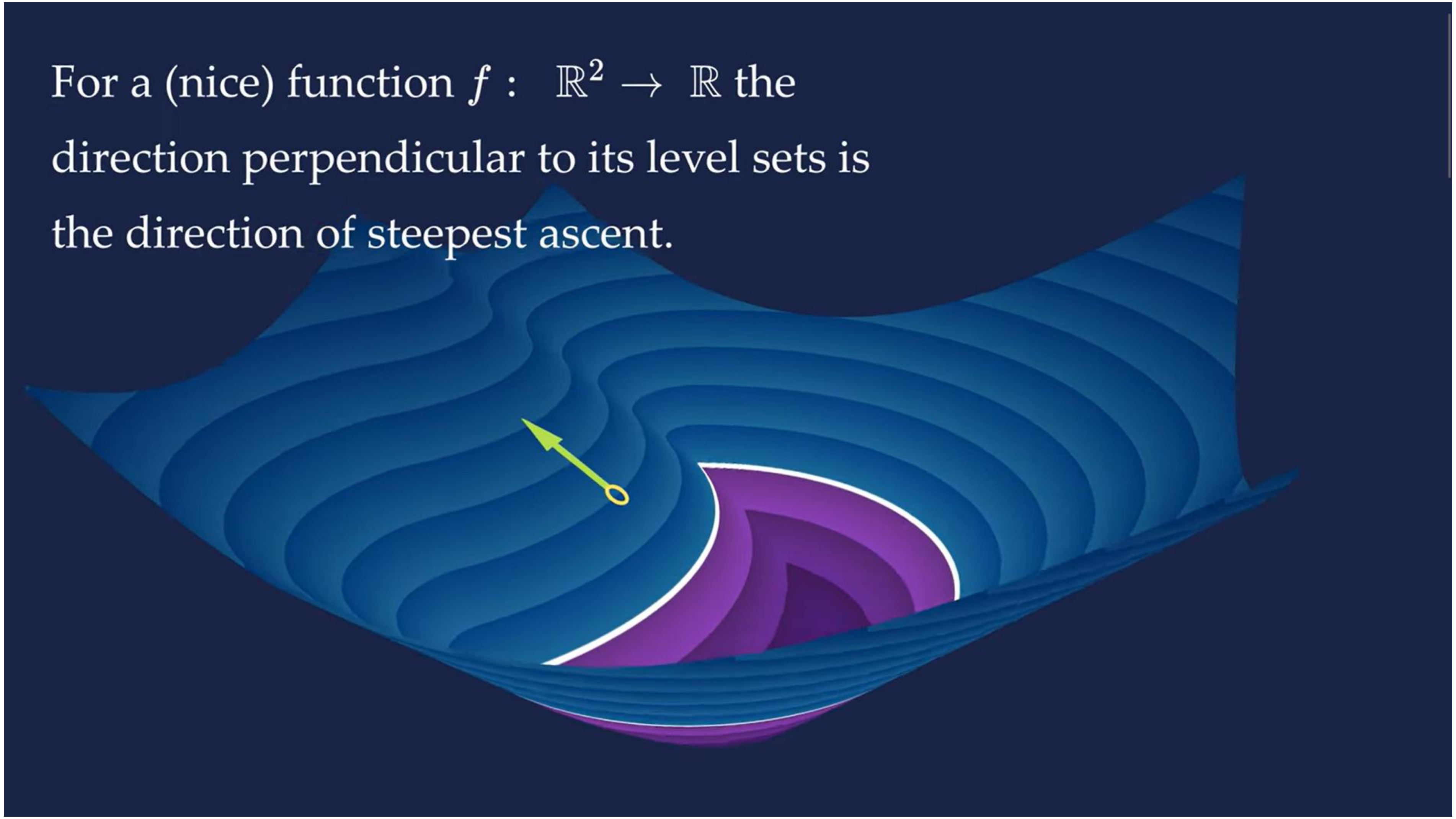
A set of points that all map to the same value of SDF is called a Level Set



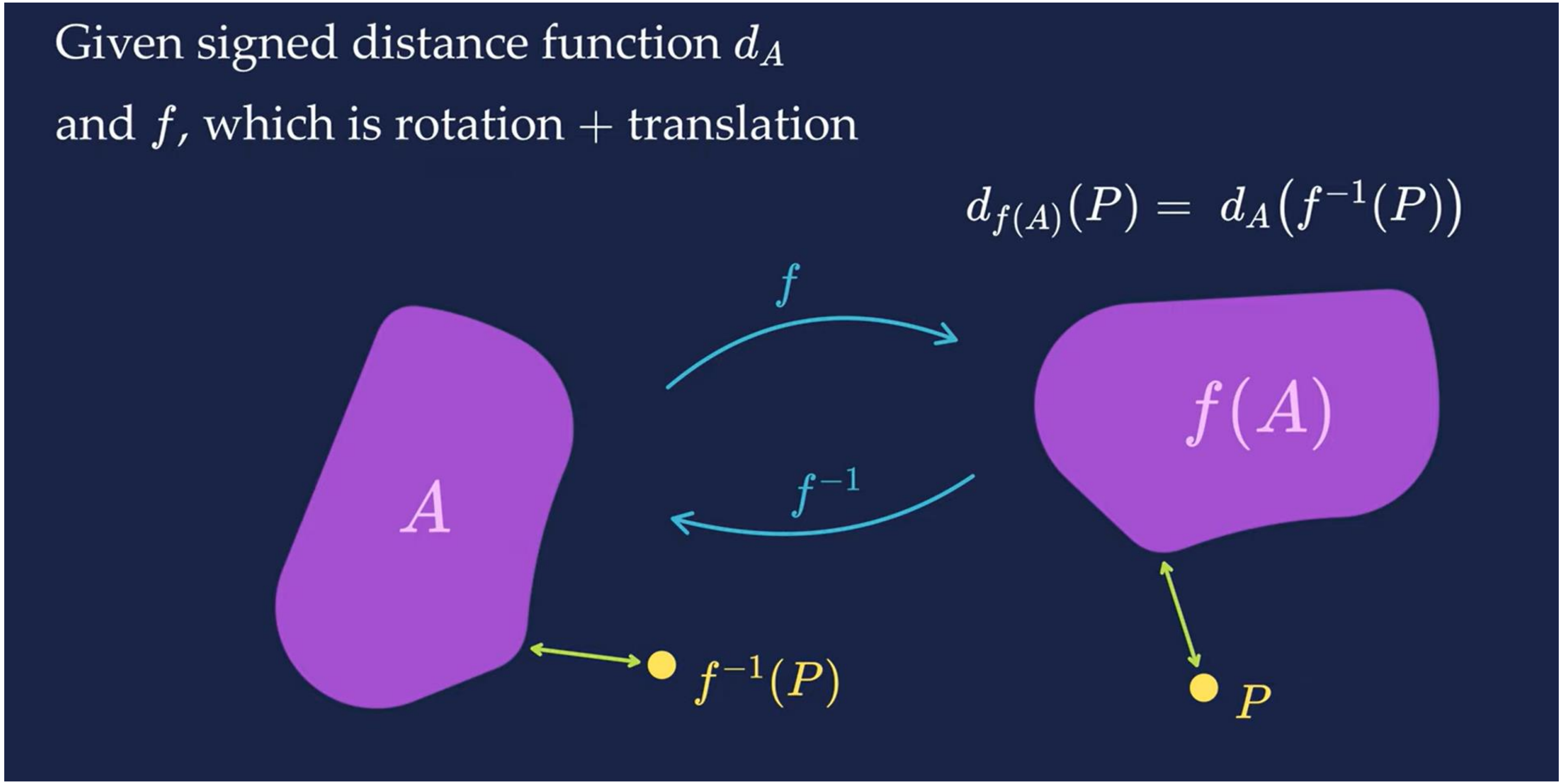
Input: X, Y coordinates
Output Z = SDF (X,Y)

2D plane is converted to a 3D Surface

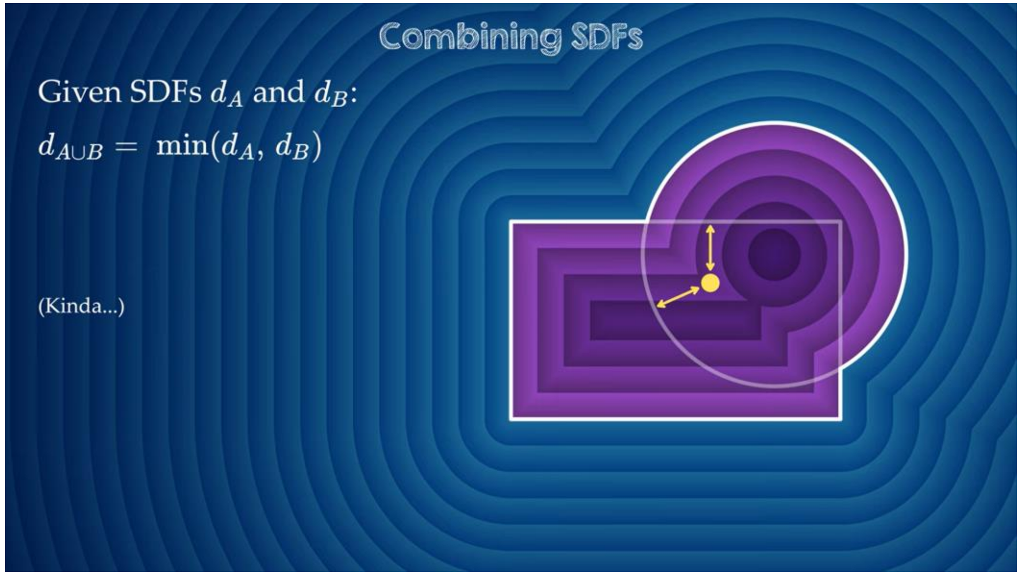
Gradient of Level Sets

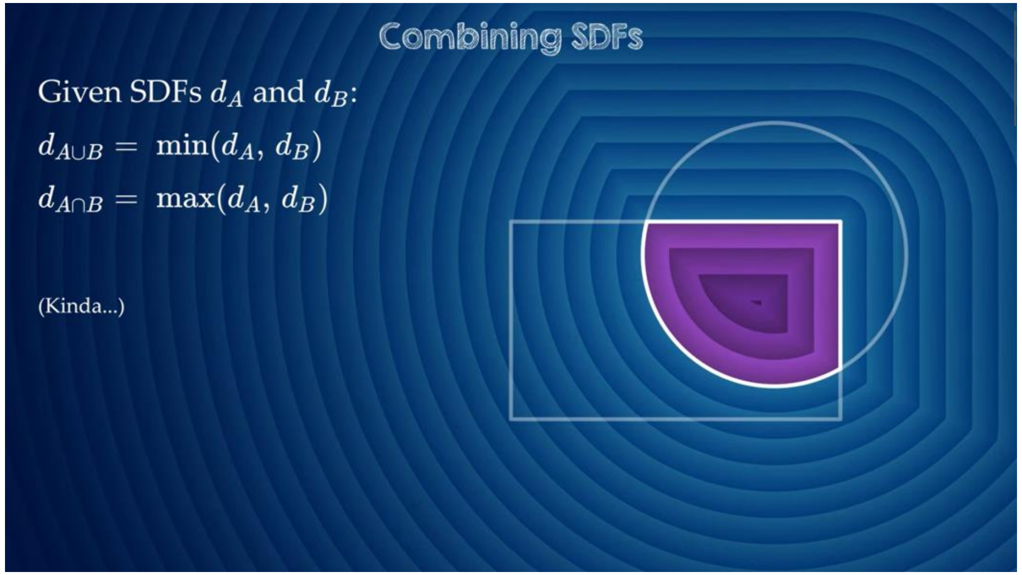


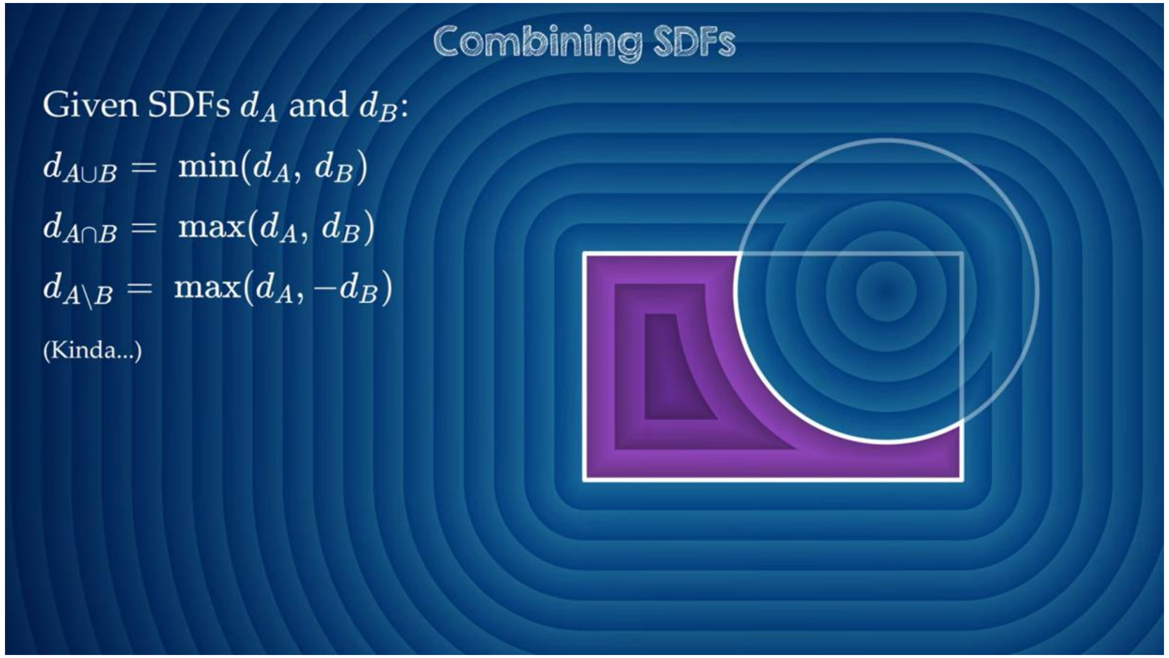
SDF and Rigid Transformation



Combining SDFs

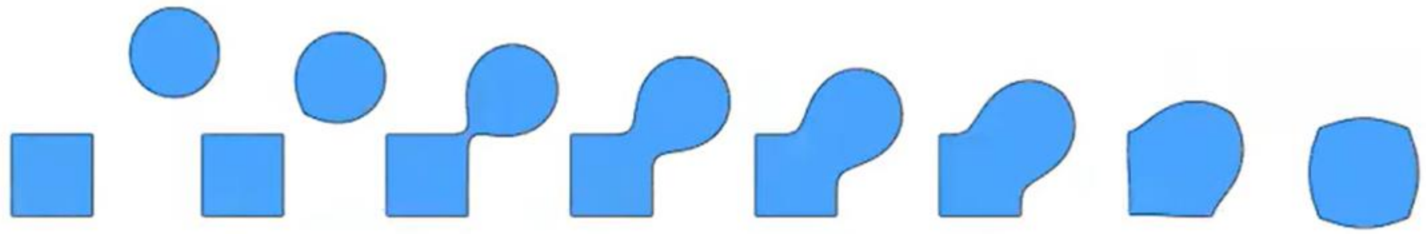






Blending Distance functions (Implicit)

- A *distance function* gives distance to closest point on object
- Can blend any two distance functions d_1, d_2:



- Similar strategy to points, though many possibilities. E.g.,

$$f(x) := e^{d_1(x)^2} + e^{d_2(x)^2} - \frac{1}{2}$$

- Appearance depends on how we combine functions

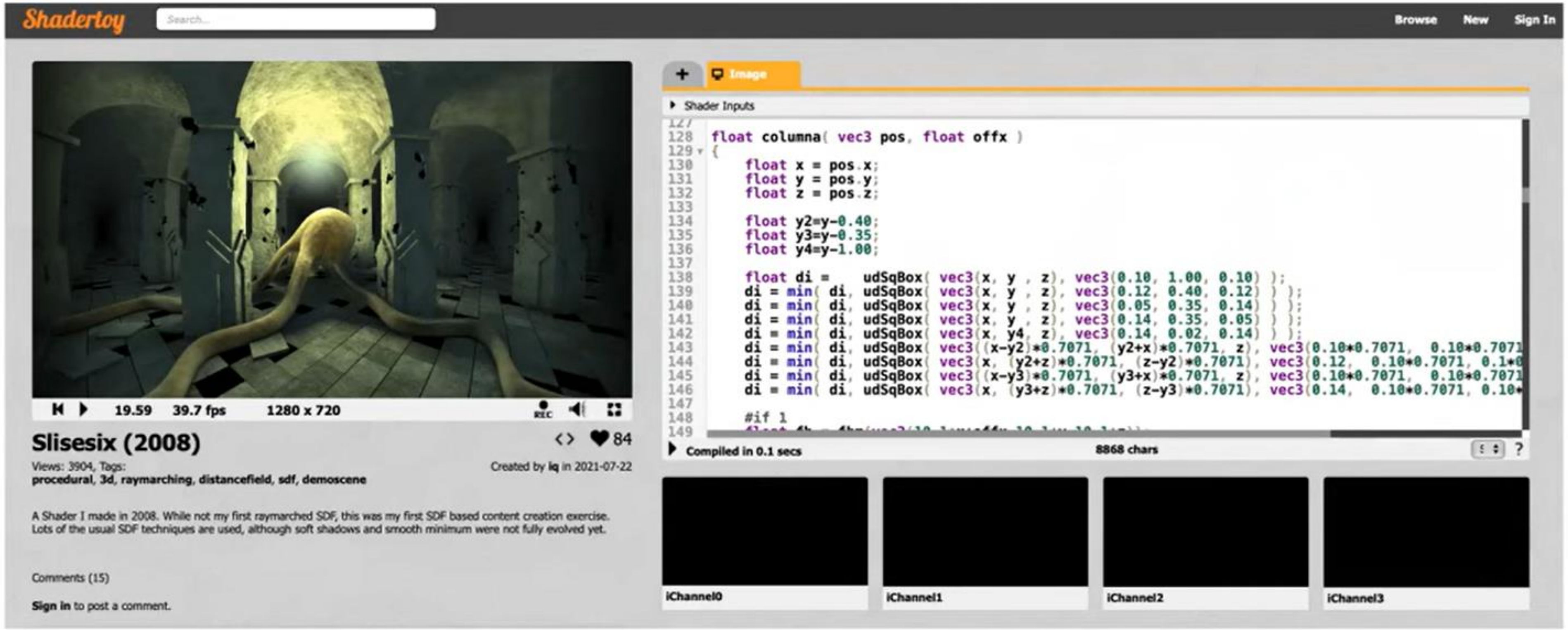
Q: How do we implement a Boolean union $d_1(x)$, $d_2(x)$?

A: Just take the minimum: $f(x) = \min(d_1(x), d_2(x))$

Scene with purely Algebraic functions

Scene made of pure signed distance functions

Art with math -- really hard!



https://www.shadertoy.com/view/NtlSDs

3 basic methods for representing a surface:

- **DEM (digital elevation model):** A Digital Elevation Model, or DEM, is a 3D representation of a topographic surface of the Earth, excluding trees, buildings, and any other surface objects. It provides accurate and detailed information about the terrain, which can be used to make better-informed decisions.
- DEM is a set of regularly spaced sampled ground points in the x and y dimensions (although spacing not necessarily the same in each) accompanied by an elevation measure (z dimension).
- The DEM terminology was introduced by USGS (United States Geological Survey). Two concepts used for determining elevation at points within the grid cells:
  - Lattice: each point represents a value on the surface only at the center of the grid cell
  - Surface grid considers each sample as a square/rectangular cell with a constant surface value.

- **TIN (Triangulated Irregular Network)** a set of adjacent, non-overlapping triangles with x, y coordinates and z vertical elevations for their vertices, along with topological relationship between the triangles and their adjacent neighbors.

- **Contour lines:** lines of equal elevation, drawn at a given interval (e.g. every 6 or 25 feet)

The general term ***digital terrain model*** (DTM) may be used to refer to any of the above surface representations when in digital form.

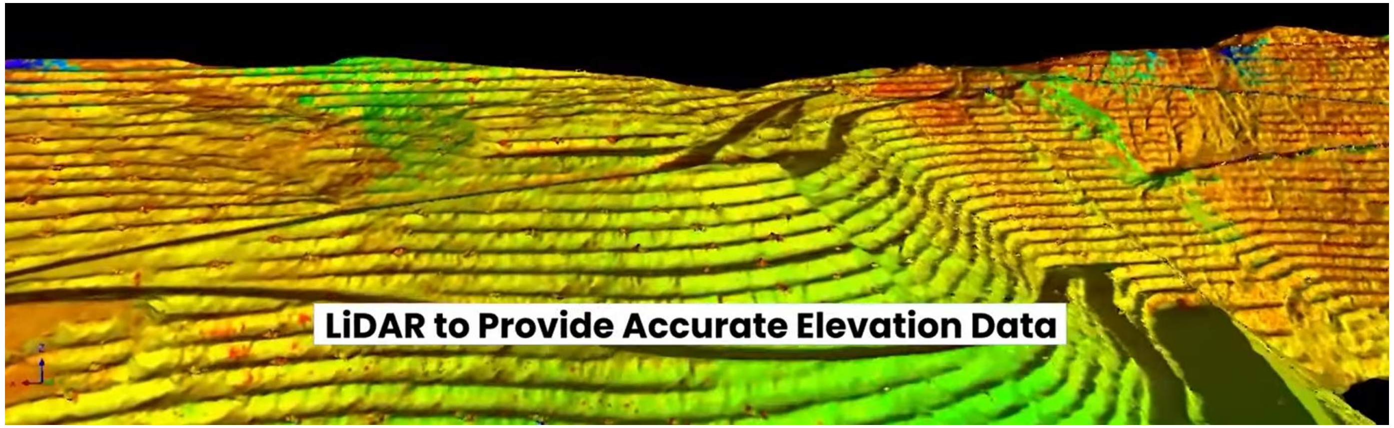
DEM sometimes used synonymously with DTM—don’t.

Storing & Converting Surface Data

- **3-D surfaces** are normally stored in one of two forms within ArcGIS
  - as a **GRID**, which is ArcInfo's general raster format
  - as a **TIN** which is a vector format for surfaces
- **However, when you download data from the Internet, surface data may be in other formats, such as**
  - DEM format, as originally developed by USGS (United States Geological Survey)
  - SDTS (Spatial Data Transfer Standard) format, which is an FGDC (Federal Geographic Data Committee) standard
  - E00 which is ESRI's text formatted for distributing coverages and GRIDS
  - Points and breaklines
- **Conversion to GRID or TIN is generally required for display or analysis within the ArcGIS system**
  - Generally, ArcToolbox has capabilities for converting these formats to GRIDs or TINs
- **Contour lines can be stored as vector lines in a coverage, shapefile, or geodatabase,**
  - can only be used for map display but not analysis, so this is not a recommended format for surface storage.

Digital Elevation Model

- A Digital Elevation Model, or DEM, is a 3D representation of a topographic surface of the Earth, excluding trees, buildings, and any other surface objects. It provides accurate and detailed information about the terrain, which can be used to make better-informed decisions.
- DEM data is created using LIDAR or Stereo Photogrammetry
- a sampled array of elevations (z) that are at regularly spaced intervals in the x and y directions.
- two approaches for determining the surface z value of a location between sample points.
  - In a **lattice**, each mesh point represents a value on the surface only at the center of the grid cell. The z-value is approximated by interpolation between adjacent sample points; it does not imply an area of constant value.
  - A **surface grid** considers each sample as a square cell with a constant surface value.



Advantages

- Simple conceptual model
- Data cheap to obtain
- Easy to relate to other raster data
- Irregularly spaced set of points can be converted to regular spacing by interpolation

Disadvantages

- Does not conform to variability of the terrain
- Linear features not well represented

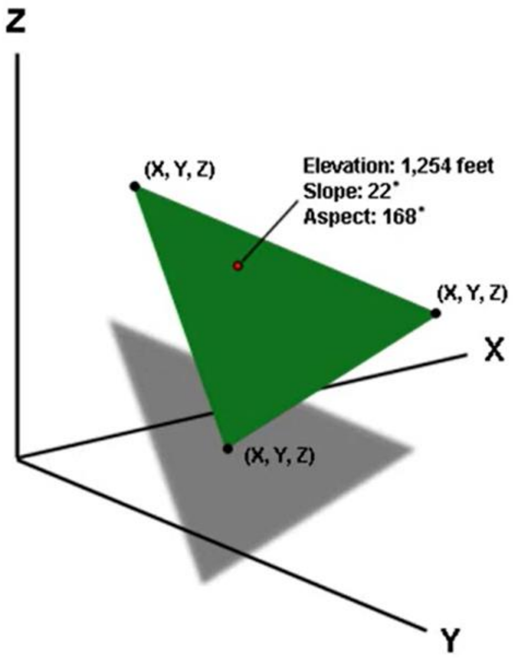
GRID as a Storage Method

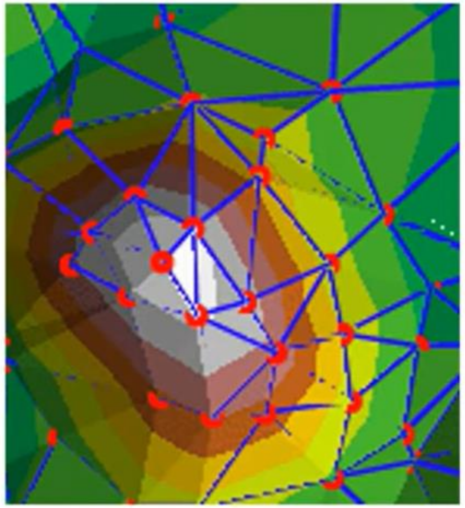
- **GRIDs are Environmental Systems Research Institute (ESRI)'s raster data format**
  - **Use for storing DEMS or other data in raster format**
- **GRID** stores data as either**:**
  - **Integer:** in which case there is an associated *value attribute table* (VAT) which contains one record for each *different* value in the raster (thus there are normally substantially fewer records in the VAT table than there are cells in the raster);
  - this record stores the value itself, a count of the number of cells with that value, and any additional attributes the user wishes to to attach.
  - Thus, the values could be codes for soil type and the VAT could contain fertility measures, soil name, construction suitability codes, etc.
  - If you select a record in the VAT, all cells with that value will highlight in the View or Scene.
  - **Floating point**: (number with a decimal point) in which case there is no VAT table, and simply one decimal value per cell
- Integer GRIDS are generally substantially faster to process.

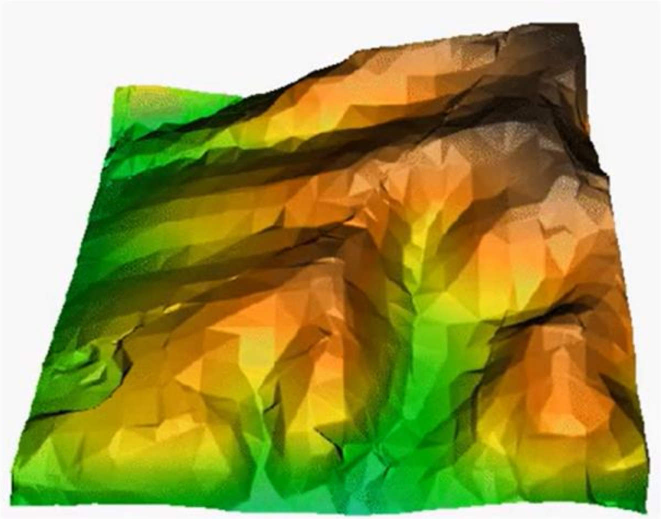
What is a Triangulated Irregular Network (TIN)?

TIN is a vector representation of continuous spatial data

-A list of X,Y,Z nodes connected by edges
-Ideal for representing 3D surfaces

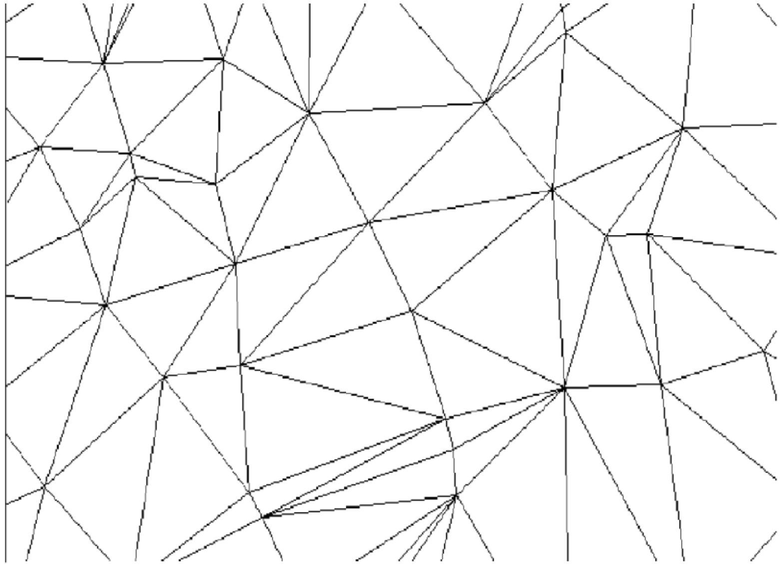






Triangulated Irregular Network

- TIN is a set of adjacent, non-overlapping triangles computed from irregularly spaced points, with x, y horizontal coordinates and z vertical elevations.
- Quick and easy way to represent irregularly spaced datasets
- Useful to represent Topography, Elevation, Contours



- Advantages
  - Can capture significant slope features (ridges, etc)
  - Efficient since require few triangles in flat areas
  - Easy for certain analyses: slope, aspect, volume
- Disadvantages
  - Analysis involving comparison with other layers difficult

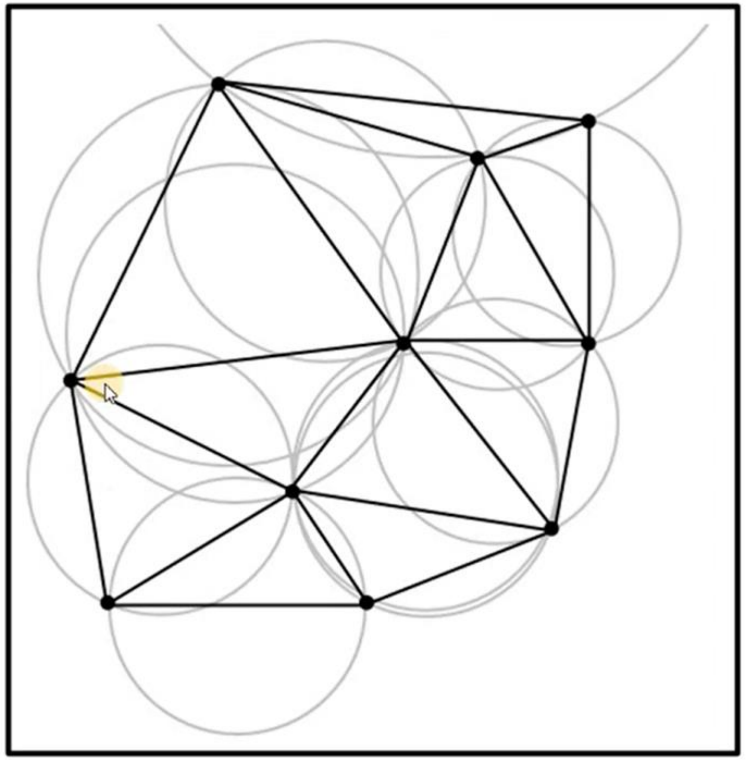
-Ideal for irregularly spaced datasets such as LiDAR or drone-based photogrammetry

-Fewer nodes can be used in areas that don't change much

TIN: Delauney Triangulation

TINs are based on Delauney Triangulation

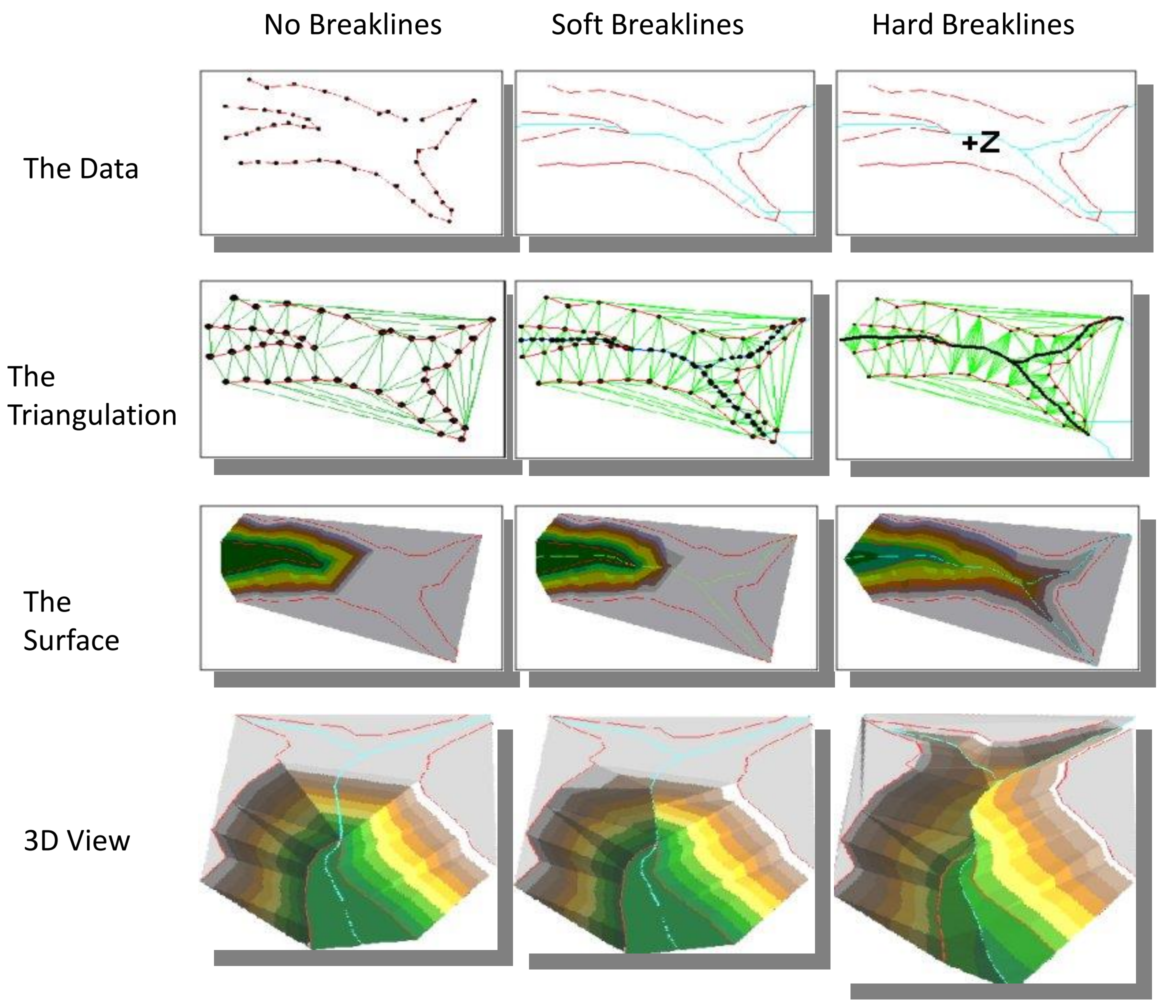
-No node can fall within the circle circumscribed by any triangle



TIN as a Storage Method

- TINs
  - are the most useful method for representing a continuous surface in a vector GIS system.
  - data sets comprising any combination of contours, breaklines and point elevations (either DEM or massed points) can be combined as input to create a TIN
- TINS are especially useful for analytical purposes
  - Good model for representing surfaces
  - slope and aspect easily derived
  - simplify the calculation of surface area and volume

Creating a TIN

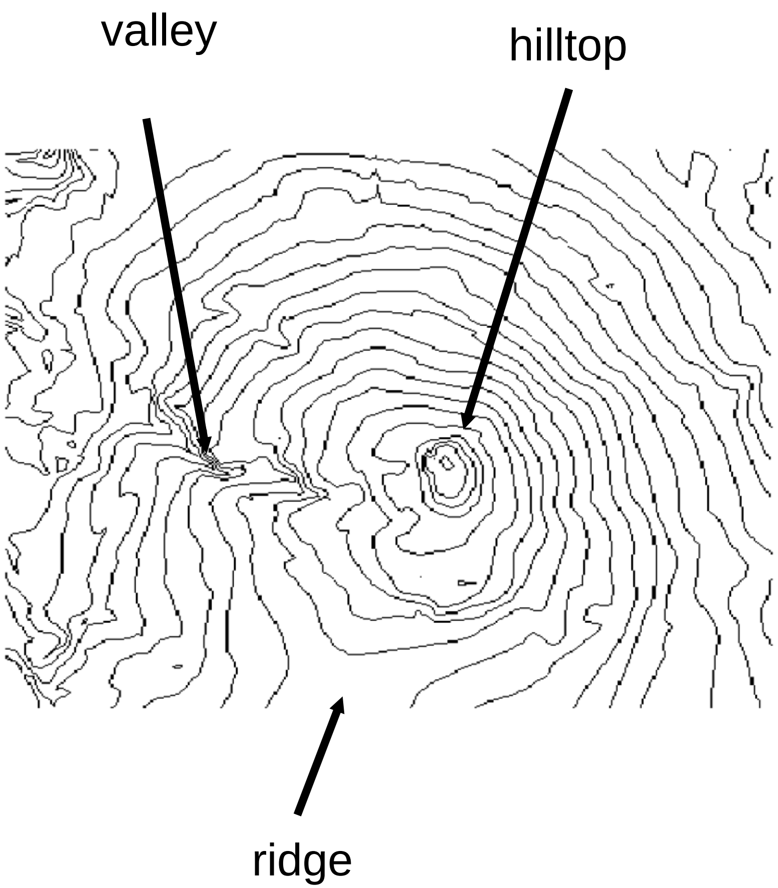


Break lines
Linear features which define and control surface behavior in terms of smoothness and continuity.

http://www.ian-ko.com/resources/triangulated_irregular_network.htm

Contour (isolines) Lines

Contour lines, or isolines, of constant elevation at a specified interval,



Advantages

- ❖ Familiar to many people
- ❖ Easy to obtain mental picture of surface
  - ➢ Close lines = steep slope
  - ➢ Uphill V = stream
  - ➢ Downhill V or bulge = ridge
  - ➢ Circle = hill top or basin

Disadvantages

- ❖ Poor for computer representation: no formal digital model
- ❖ Must convert to raster or TIN for analysis
- ❖ Contour generation from point data requires sophisticated interpolation routines, often with specialized software such as *Surfer* from Golden Software, Inc., or ArcView Spatial Analyst extension

References

- Virtual Humans -- Lecture 03.1 Surface Representations: https://www.youtube.com/watch?v=5uE7Pc5mr1I
- The SDF of a box: https://www.youtube.com/watch?v=62-pRVZuS5c&list=PL0EpikNmjs2CYUMePMGh3IjjP4tQlYqji
- https://iquilezles.org/articles/distfunctions/ (Multiple Example 3D distance functions)
- https://iquilezles.org/articles/distfunctions2d/
- What is Digital Elevation Model or DEM?
- Signed Distance Functions & Ray-Marching (very good)
- What is a Triangular Irregular Network (TIN)?
- https://medium.com/@sim30217/truncated-signed-distance-function-f765a0f1d432