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Q. No. 1 =)

Solution

Taking z transform of given difference equation. we get

$$Y(z) - 0.3z^{-1}Y(z) + 0.12\, z^{-2} Y(z) = X(z) + 0.2z^{-1}X(z) - 0.15\, z^{-2}X(z).$$

$$\therefore \frac{Y(z)}{X(z)} = \frac{1+0.2z^{-1} - 0.15z^{-2}}{1 - 0.3z^{-1} + 0.12z^{-2}}$$

$$\therefore H(z) = \frac{z^2 + 0.2z - 0.15}{z^2 - 0.3z + 0.12}.$$

$$H(z) = \frac{(z-0.3)\ (z+0.5)}{z^2 - 0.3z + 0.12}.$$

System has two zero at.

$z = 0.3$

and $z = -0.5$.

Similarly.

Two pole at

$$= \frac{-(-0.3) \pm \sqrt{(-0.3)^2 - 4\times 0.12 \times 1}}{2\times 1}$$

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~~Therefore one~~ pol

Poles at.

$$\frac{0.3 \pm \sqrt{0.09 - 0.48}}{2}$$

$$= \frac{0.3 \pm \sqrt{0.624}}{2}$$

$$= 0.15 \pm j0.31.$$

Therefore:

one pole.

$Z_1 = 0.15 - 0.31j.$

$\therefore r_1 = 0.344. \quad \theta_1 = -1.120 \text{rad}$

Similarly

$r_2 = 0.344, \quad \theta_2 = +1.120 \text{ rad}$

Now.

Poles are at — zeros at

r	θ	r	θ
0.344.	-1.12	0.3	0
0.344.	1.120	-0.5	π

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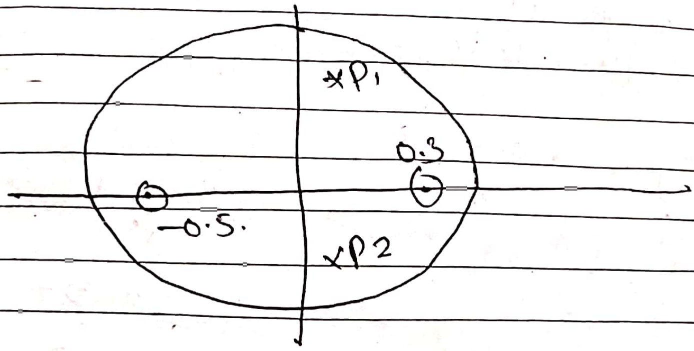
∴ Plotting



magnitude plot.

Magnitude Response.

$20\log_{10}|H(e^{j\omega})|$

$= 10\log_{10}[1+r_3^2-2r_3\cos(\omega-\theta_3)]$
$+ 10\log_{10}[1+r_4^2-2r_4\cos(\omega-\theta_4)]$
$- 10\log_{10}[1+r_1^2-2r_1\cos(\omega-\theta_1)]$
$- 10\log_{10}[1+r_2^2-2r_2\cos(\omega-\theta_2)]$.

Putting value of r and θ.

$= 10\log_{10}[1+0.3^2-2.0.3\cos(\omega-0)]$
$+ 10\log_{10}[1+0.5^2+2\times0.5\cos(\omega-\pi)]$
$- 10\log_{10}[1+0.344^2-2\times0.344\times\cos(\omega+1.12)$
$- 10\log_{10}[1+0.340^2-2\times0.344\times\cos(\omega\mp1.12c)$

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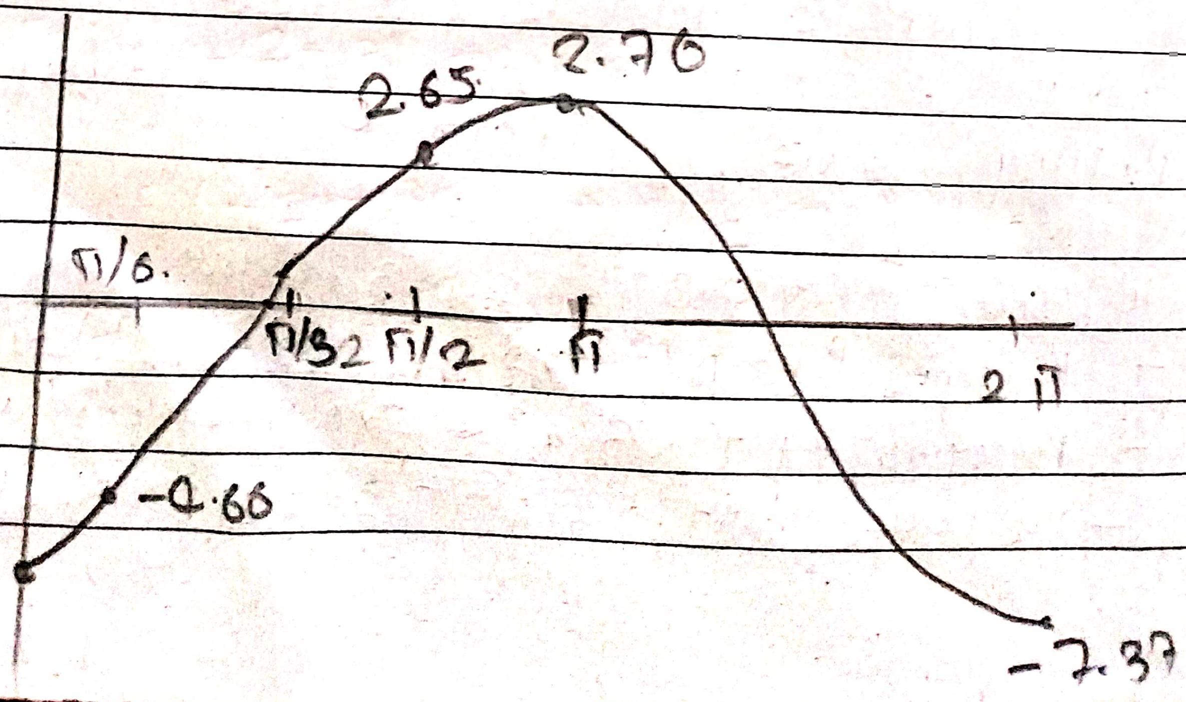
$$= 10\log_{10}(1.09 - 0.6\cos\omega)$$
$$+ 10\log_{10}(1.25 + 1\cos(\omega - \pi))$$
$$- 10\log_{10}(1.118 - 0.688\cos(\omega + 1.120))$$
$$- 10\log_{10}(1.116 - 0.688\cos(\omega - 1.120)).$$

Solving we get.

ω	mg response
0	-7.37
$\pi/6$	-4.66
$\pi/4$	-2.40.
$\pi/3$	-0.39
$\pi/2$	1.96
$2\pi/3$	2.65.
π	2.76.
$3\pi/2$	1.96.
2π	-7.37

Plotting Magnitude



$$= \left(-2+3/\sqrt{2}\right) + j\left(3 + 1/\sqrt{2}\right).$$

Q. No. 2 =)

$$X(z) = \frac{1}{z^2+0.3z-0.1}$$

i) ROC: $0.2<|z|<0.5$

ii) ROC: $|z|<0.2$

iii) ROC: $|z|>0.5$

Using partial fraction expansion metho

$$\frac{X(z)}{z} = \frac{1}{z(z^2+0.3z-0.1)}$$

$$= \frac{1}{z(z+0.5)(z-0.2)}$$

$$= \frac{c_1}{z} + \frac{c_2}{z+0.5} + \frac{c_3}{z-0.2}$$

where

$$C_1 = \frac{z \quad 1}{z(z^2+0.3z-0.1)}\Bigg|_{z=0}$$

$$= \frac{1}{0.3-0.1}$$

$$= 1/0.2$$

$$C_2 = (z+0.5)\,\frac{1}{z(z+0.5)(z-0.2)}\Bigg|_{z=-0.5}$$

$$= \frac{1}{-0.5(-0.5-0.2)}$$

$$= 1/0.35$$

$$C_3 = (z-0.2) \times \frac{1}{z(z+0.5)(z-0.2)}\Bigg|_{z=0.2}$$

$$= \frac{1}{0.14}.$$

Now

$$X(z) = \frac{1}{0.2z} + \frac{1}{0.35} \times \frac{z}{z+0.5} + \frac{1}{0.14} \times \frac{z}{z-0.2}$$

i) ROC $|z| < 0.2$.

Taking inverse of Z-Transform

$$x[n] = \frac{1}{0.2} \delta[n-1] + \frac{1}{0.35} (-0.5)^n u[-n-1] + \frac{1}{0.14} (0.2)^n u[-n-1]$$

ii) Roc $|z| > 0.5$

Taking inverse Z Transform

$$x[n] = \frac{1}{0.2} \delta(n-1) + \frac{1}{0.35} (-0.5)^n u[n] + \frac{1}{0.14} (0.2)^n u[n]$$

iii) Roc $0.2 < |z| < 0.5$.

Taking inverse Z Transform

$$x[n] = \frac{1}{0.2} \delta[n-1] + \frac{1}{0.35} [-0.5]^n + \frac{1}{0.14} (0.2)^n u[n$$

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Q.No.3 ⇒

Solution:

The properties of Region of convergence are

i) ROC doesn't contain any poles of $x(z)$.

ii) If $x[n]$ is right sided finite length sequence & its z transform exist for some value of z, then ROC will be entire z plane except $z=0$.

eg $x[n]=\{1,3,5,2,7\}$.

on solving we get

$$X(z) = 1 + 3z^{-1} + 5z^{-2} + 2z^{-3} + 7z^{-4}.$$

$\therefore$ ROC = All except $z=0$

iii) left sided finite length sequence and its z transform $X(z)$ exists for some value of z, then ROC will be entire z plane except $z=\infty$.

eg.

$x[n]=\{1,3,5,2,7,0\}$.

we get.

$$X(z) = z^5 + 3z^4 + 5z^3 + 2z^2 + 7z \ldots$$

$\therefore$ ROC all except $z=\infty$

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iv) Both sided finite length sequence and z transform $X(z)$ exist for some value of z. Then ROC is entire z-plane possibly except $z=0$ and $z=\infty$.

eg.

$x[n] = \{1, 3, \underset{\uparrow}{5}, 2, 7\}$

Solving

$X(z) = z^2 + 3z + 5 + 2z^{-1} + 7z^{-2}.$

$\therefore$ ROC will be all except $z=0$ & $z=\infty$.

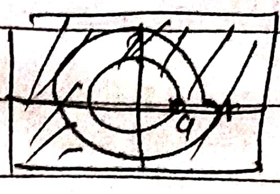
v) Right sided infinite length

if $x[n]$ is Right sided infinite length sequence and its z transform $X(z)$ exist then ROC has the form

$|z| > r_{max}$. (largest magnitude).

$x[n] = a^n . u[n]$.

$\therefore X(z) = \frac{z}{z-a}$, ROC: $|z|>|a|$.



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vi) Left Sided infinite length.

if $x[n]$ is left sided sequence, $X(z)$ exist then ROC will be in form

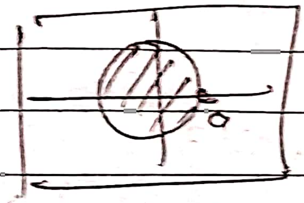
$|z| < r_{min}$. (smallest magnitude)

example

$x[n] = -a^n u[-n-1]$.

solving

$X(z) = \frac{z}{z-a}$, ROC: $|z| < a$.

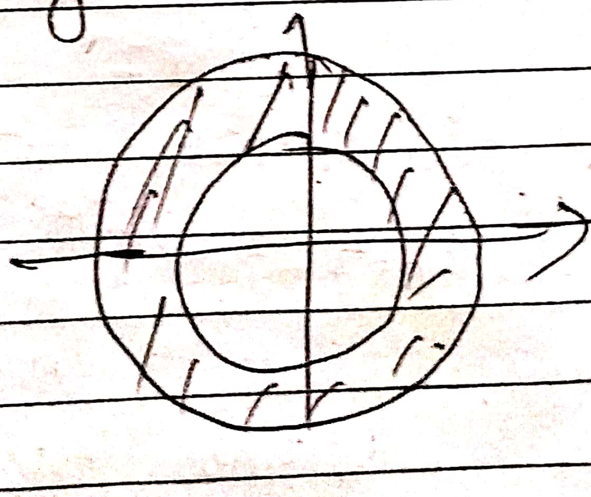


vii) Both sided infinite length.

if $x[n]$ then if exist $X(z)$ then ROC will be.

$r_1 < |z| < r_2$.

angular ring between 2 circles



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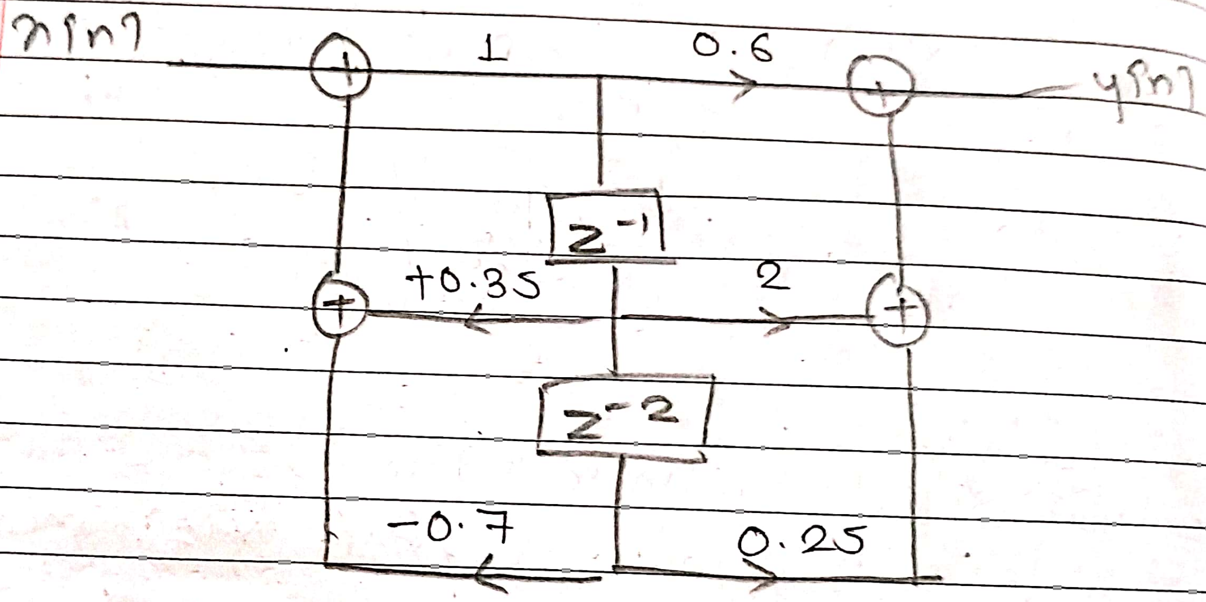
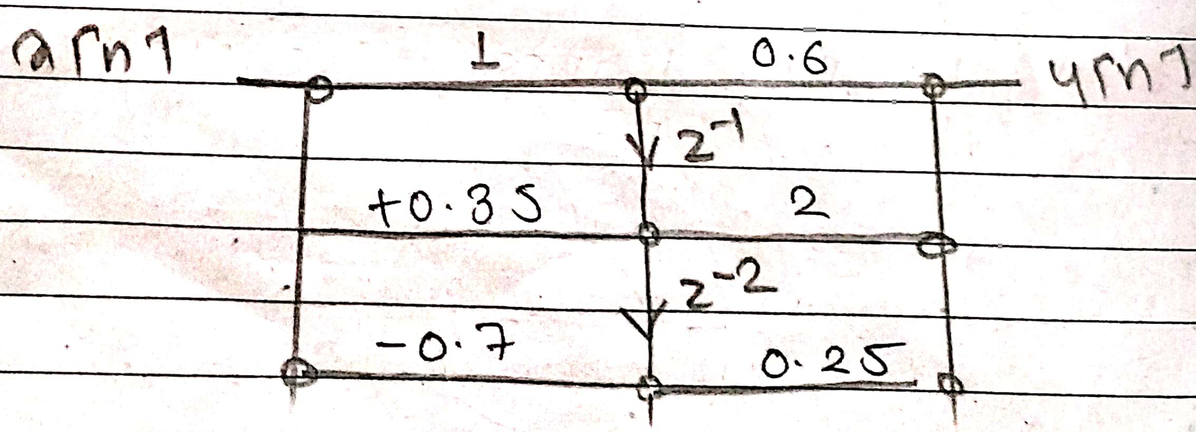
Draw Direct form II.

$$H(z) = \frac{(0.6 + 2z^{-1} + 0.25z^{-2})}{(1 - 0.35z^{-1} + 0.7z^{-2})}.$$

Thus:
Direct form II.



Block Diagram



Signal flow graph

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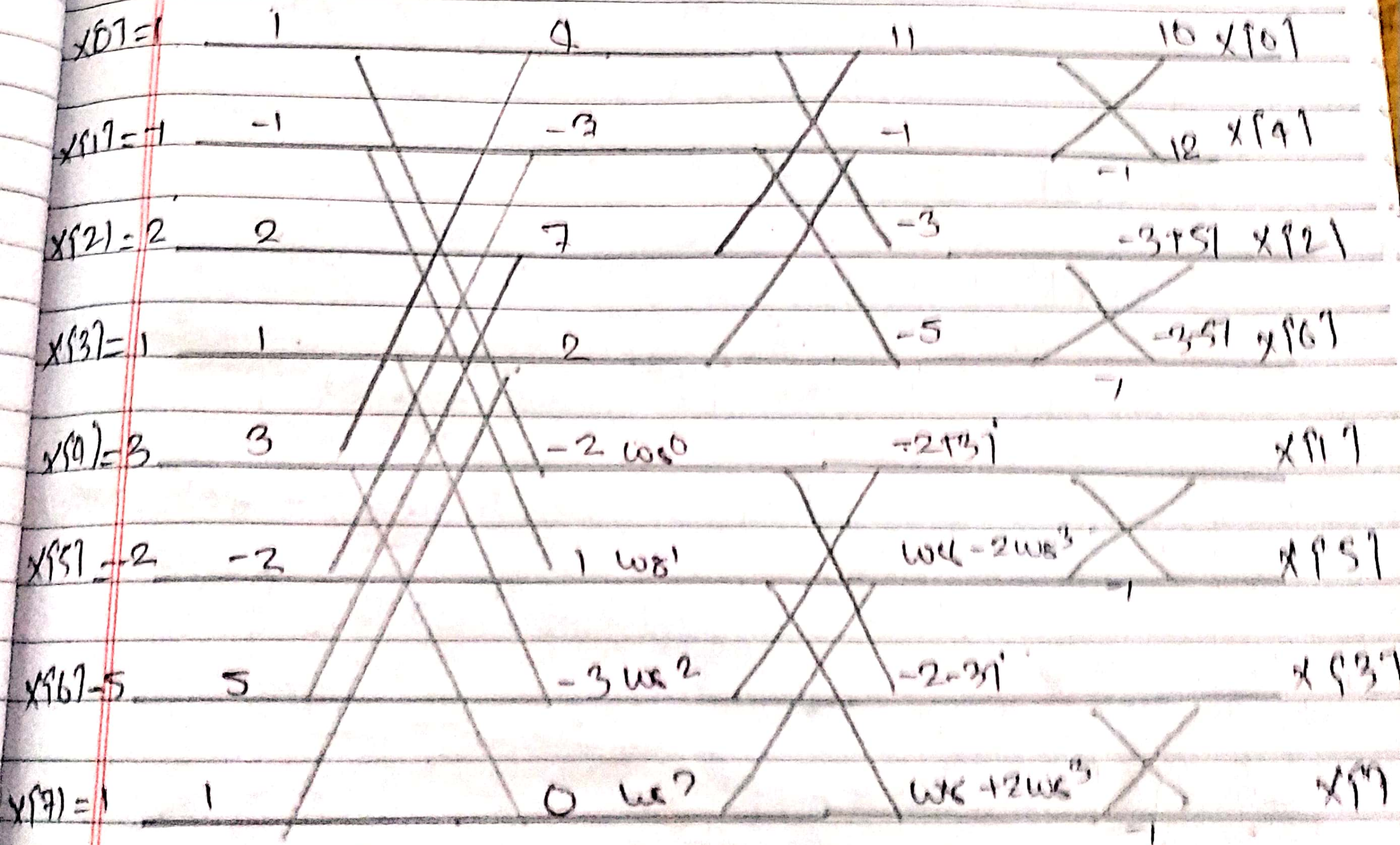
Q. No 4 =>

$x[n] = \{1, -1, 2, 1, 3, -2, 5, 1\}$

Soln:



X[0]

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$X[0] = 10$

$X[1] = (-2+3j) + w_8 - 2w_8^3$

$= -2+3j + 1/\sqrt{2} - j\,1/\sqrt{2} - 2(-1/\sqrt{2} - 3/\sqrt{2})$

$= (-2+1/\sqrt{2}) + j(3-1/\sqrt{2})$

$X[2] = 3+5j$

$X[4] = 12.$

$X[5] = (-2+3j) - (w_8 - 2w_8^3)$

$= -2+3j - 1/\sqrt{2} + j/\sqrt{2} - 2(-1/\sqrt{2} - 3/$

$= (-2-3/\sqrt{2}) + j(3+1/\sqrt{2})$

$X[6] = -3 \cancel{+} -5j$

$X[3] = -2-3j + w_8$

$= (-2-3j) + 1/\sqrt{2} - j\,1/\sqrt{2}$

$= (-2 + 3/\sqrt{2}) - j(3+1/\sqrt{2}).$

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$\therefore X[7] = -2-3j - W_8$

$= \left(-2 + 3/\sqrt{2}\right) + j\left(3 + 1/\sqrt{2}\right).$

Q. No. 2 =)

$X(2) = \frac{1}{\quad}$

Q. No. 5 =>.

Ans. Difference equation

$$\sum_{k=0}^{N} a_k\, y[n-k] = \sum_{k=0}^{M} b_k\, x[n-k]$$

Taking Z - Transform

$$\sum_{k=0}^{N} a_k z^{-k}\, Y(z) = \sum_{k=0}^{M} b_k z^{-k}\, X(z)$$

$$\therefore \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{M} a_k z^{-k}}.$$

$$\therefore H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$$

Transpose fn of system

$\therefore H(z) = \dfrac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_N z^{-N}}$

i.e $H(z) = \left(\dfrac{b_0}{a_0}\right) \dfrac{\prod_{k=1}^{M} (1 - c_k z^{-1})}{\prod_{k=1}^{N} (1 - d_k z^{-1})}$.

where.

$H(z)$ contain M zeros at $z = c_k$

N poles at $z = d_k$

Casuality and stability of LTI system in terms of ROC of its Transfer function are:

i) casuality.

LTI system casual

if $h[n] = 0$ for $n < 0$.

If $H(z)$, then ROC: $|z| > r_{max}$.

at least $|z| > 0$.

ii) Stable
LTI system is stable if

$$\left[\sum_{n=-\infty}^{\infty} h[n] \right] < \infty$$

if $H(z)$ exists.

ROC must include unit circle
i.e $|z| = 1$.

iii) Both causal and stable
For LTI system to be both causal and stable: all poles of LTI system must be inside unit circle in z-plane.

Examples:

i) $h[n] = 3^n u[n] \xleftrightarrow{z} H(z) = \frac{z}{z-3}$, $|z|>3$.

- causal, unstable

ii) $h[n] = (1/3)^n u[-n-1] \xleftrightarrow{z} H(z) = \frac{-z}{z-}$

Non causal stable

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iii) $h[n] = 3^n u[-n-1] \xleftrightarrow{Z} H(z) = \frac{-z}{z-3}, |z| < 3$

Non causal, stable.

iv) $h[n] = (1/3)^n u[n] \xleftrightarrow{Z} H(z) = \frac{z}{z-1/3}$ $|z| > 1/3$

Causal, stable