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Chapter - 3. Assignment.

Q.No.1 ⇒.

Find the output of LTI system having impulse response $h[n] = (1/3)^n u[n]$ and input sequence

$$x[n] = 2e^{j\pi n}, \quad -\infty < n < \infty.$$

~~Ans~~ here

FT of $h[n]$

$$H(e^{jw}) = \sum_{n=-\infty}^{\infty} h[n] e^{-jw_0 n}$$

$$= \sum_{n=-\infty}^{\infty} (1/3)^n u[n] e^{-jwn}$$

$$= \sum_{n=-\infty}^{\infty} \left(\frac{1}{3} e^{-jw}\right)^n$$

$$= \frac{1}{1 - 1/3 e^{-jw}}$$

$$= \left| \frac{1}{3} e^{-jw} \right| < 1$$

$H(e^{j\omega})\Big|_{\omega=\omega_0}$ Here, $\omega_0 = \pi$,

$= H.(e^{j\omega_0})$ $\cos\pi$

$= H e^{j\pi}$

$= \dfrac{1}{1 - \frac{1}{3} e^{-j\pi}}$

$= \dfrac{1}{1 + \frac{1}{3}}$

$= \dfrac{3}{4}$

Output of LTI system due to complex exponential is given by

$y[n] = x[n]\, H(e^{j\omega_0})$

$=$

Input sequence:

$x[n] = 2e^{j\pi n}, \quad -\infty < n < \infty.$

Now,

output of LTI system,

$$y[n] = h[n] * x[n]$$

$$= \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$

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$$= \sum_{k=-\infty}^{\infty} h[k]\, A\, e^{j\omega_0 (n-k)}.$$

Putting value of A and wo.

$$= \sum_{k=-\infty}^{\infty} h[k]\, 2\, e^{j\pi (n-k)}$$

$$= 2 \sum_{k=-\infty}^{\infty} h[k].\, e^{j\pi n} \times e^{-j\pi k}.$$

$$= 2 e^{j\pi n} \sum_{k=-\infty}^{\infty} h[k] e^{-j\pi k}$$

$$= 2A e^{j\pi n} [H(e^{j\pi})]$$

$$= x[n].[H(e^{j\pi})]$$

$$= 2e^{j\pi n} \times |H e^{(j\pi)}|\, e^{j\pi\theta}$$

$$= 2|H(e^{(j\pi)})|.\, e^{j(\pi n + \pi\theta)}$$

$$= 2e^{j\pi n} \times 3/4$$ ~~Ans~~ //

$$= \frac{3}{2} e^{j\pi n}.$$

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Q. No. 1.

Find output of LTI system having impulse response $h[n] = (1/2)^n u[n]$ and inp sequence

$x[n] = 10 - 5\sin \pi/2\, n + 2\cos \pi n$

FT of $H[n]$.

$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} (1/2)^n u[n] e^{-j\omega n}$$

$$= \sum_{n=0}^{\infty} (1/2)^n e^{-j\omega n}$$

$$= \frac{1}{1 - 1/2 e^{-j\omega}}, \quad \left| 1/2\, e^{j\omega} \right| < 1$$

Now,

at $\omega = \pi/2$.

$$H_2(e^{j\pi/2}) = \frac{1}{1 - 1/2 e^{-j\pi/2}}$$

$$= \frac{2}{\sqrt{5}} e^{-j\, 26.56}$$

At $\omega_0 = \pi$

$$H_3(e^{j\pi}) = \frac{1}{1 - \frac{1}{2}e^{-j\pi}} = \frac{1}{1 + \frac{1}{2}}$$

$$= 2/3$$

at $\omega_0 = 0$.

$$H_1(e^{j\omega_0}) = \frac{1}{1 - \frac{1}{2}}$$

$$= 2$$

$$x_2[n] = -5\sin \pi/2\, n$$

$$= \frac{-5}{2j} e^{j\pi/2 n} + \frac{5}{2j} e^{-j\pi/2 n}$$

Output of LTI system due to above input.

$$y_2[n] = \frac{-5}{2j} \left| H(e^{j\pi/2}) \right| e^{j\theta(\pi/2)} e^{j\pi/2 n}$$

$$+ \frac{5}{2j} \left| H(e^{j\pi/2}) \right| e^{-j\theta\pi/2} - e^{-j\pi/2 n}$$

$$= \frac{-5}{2j} \left| H(e^{j\pi/2}) \right| \left\{ e^{j(\pi/2 n + \pi/2 \theta)} - e^{-j(\pi/2 n + \pi/2 \theta)} \right\}$$

$$= \frac{-5}{2j} \left| H(e^{j\pi/2}) \right| \left[\sin(\pi/2 n + \pi/2 \theta) \right]$$

$$= -5 \times \frac{2}{\sqrt{5}} \sin\left(\frac{n\pi}{2} - 26.6\right).$$

Similarly

$x_8[n] = 20 \cos \pi n.$

$y_8[n] = \frac{20}{2} e^{j\pi n} + \frac{20}{2} e^{-j\pi n}$

$= 10 e^{j\pi n} + 10 e^{-j\pi n}.$

Output of LTI system due to above input is:

$y[n] = 10 |H e^{j\pi}| e^{j\theta\pi} \times e^{j\pi n}$

$+ 10 |H(e^{j\pi})| e^{-j\theta\pi} \times e^{-j\pi n}.$

$= 10 |H(e^{j\pi})| \left\{ e^{j(\pi\theta + \pi n)} + e^{-j(\pi\theta + \pi n)} \right\}$

$= 20 |H(e^{j\pi})| \{ \cos(\pi\theta + \pi n) ,,$

$= 20 \times \frac{2}{3} \times \cos \pi n$

$= \frac{40}{3} \cos \pi n.$

Similarly $x_1[n] = 10.$

Thus. $x_1[n] = \frac{A}{2} e^{jw_0 n} + \frac{A}{2} e^{-jw_0 n}.$

we have $A=10$
$w_0 = 0.$

output of LTI system due to following ~~out~~ input is:

$y_1[n] = \frac{A}{2} \left| H e^{(jw_0)} \right| e^{jw_0\theta} \times e^{jw_0 n}$
$+ \frac{A}{2} \left| H e^{(jw_0)} \right| \left[e^{-jw_0\theta} \times e^{-jw_0 n} \right].$

on solving we get.

$= A \left(H e^{(jw_0)} \right| \left[\cos\left(w_0\theta + w_0 n \right) \right].$

putting value of A and w_0 we get. (ie, $A=10$ $w_0=0.$

$y_1[n] = 10. \left(H e^{jw} \right)$

$= 2 \times 10 = 20.$

The final output is

$y[n] = y_1 n + y_2[n] + y_3[n]$

$= 10 - 5|H(e^{j\pi/2})| [\sin(\pi/2 n + \pi/2 \theta)]$

$+ 20(H(e^{j\pi}))[\cancel{\sin} \cos(\pi n + \pi\theta)]$.

Ans//

$= 20 - \frac{10}{\sqrt{5}} \sin\left(\frac{n\pi}{2} - 26.6\right) + \frac{40}{3} \cos(n\pi)$.

for $-\infty < n < \infty$

Q. NO. 3 ⇒.

Show the pole zero in Z-plane and plot frequency response of the graph.

$$y[n] + 0.2y[n-1] + 0.3y[n-2]$$
$$= x[n] + 0.3x[n-1] - 0.04x[n-2].$$

Solution

Taking Z-Transform of given difference equation

$$Y(Z) + 0.2Z^{-1}Y(Z) + 0.3Z^{-2}Y(Z)$$
$$= X(Z) + 0.3Z^{-1}X(Z) - 0.04Z^{-2}X(Z).$$

$$\frac{Y(Z)}{X(Z)} = \frac{1 + 0.3Z^{-1} - 0.04Z^{-2}}{1 + 0.2Z^{-1} + 0.3Z^{-2}}$$

$$= \frac{Z^2 + 0.3Z - 0.04}{Z^2 + 0.2Z + 0.3}$$

$$= \frac{(Z-0.1)(Z+0.4)}{Z^2 + 0.2Z + 0.3}.$$

Rough

$Z^2 + 0.4Z - 0.1Z$
$- 0.04$
$Z(Z+0.4)$
$- 0.1(Z+0.4).$
$(Z-0.1)(Z+0.4)$
$Z^2 - 0.1Z + 0 +$
$0.3Z + 0.3$
$0.3(Z+1)$

system has two poles at z

$$= \frac{-(0.2) \pm \sqrt{(0.2)^2 - 4 \times 0.3 \times 1}}{2 \times 1}$$

$$= \frac{-0.2 \pm \sqrt{(-1.16)}}{2}$$

$$= -0.2 \pm j\,1.08$$

Now,
system has two poles at

$$z = -0.2 \mp 1.08j.$$

$$r_1 = \sqrt{(-0.2)^2 + (1.08)^2}$$

$$= 1.098$$

$$\theta_1 = 1.754 \text{ rad.}$$

~~r_2 =~~ Another pole at

$z = -0.2 - 1.08J$

$r_2 = 1.098$

$\theta_2 = -1.754$ rad

System has 2 zeros.

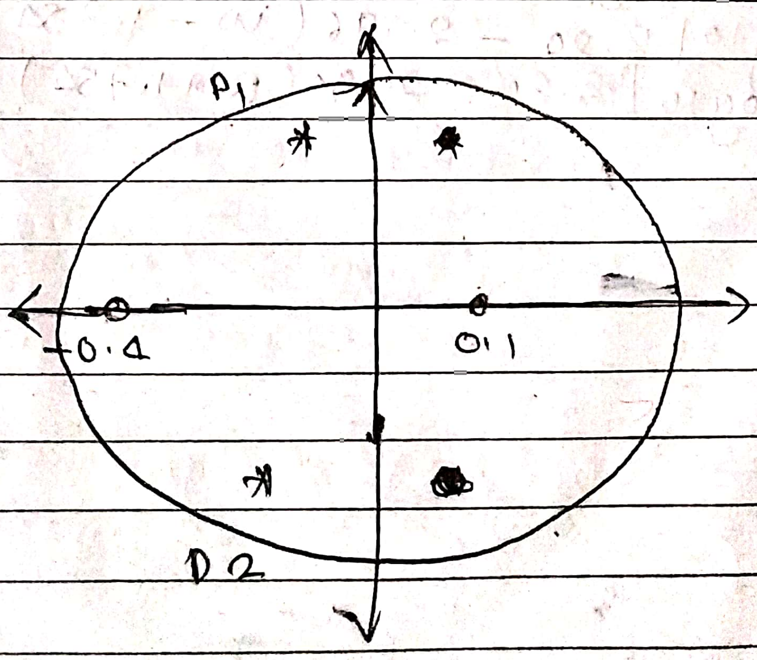
One at $z = 0.1$

another at $z = -0.4$

$r_3 = 0.1$, $\theta_3 = 0$

$r_4 = 0.4$, $\theta_4 = \pi$

Showing in graph,



Magnitude response in dB.

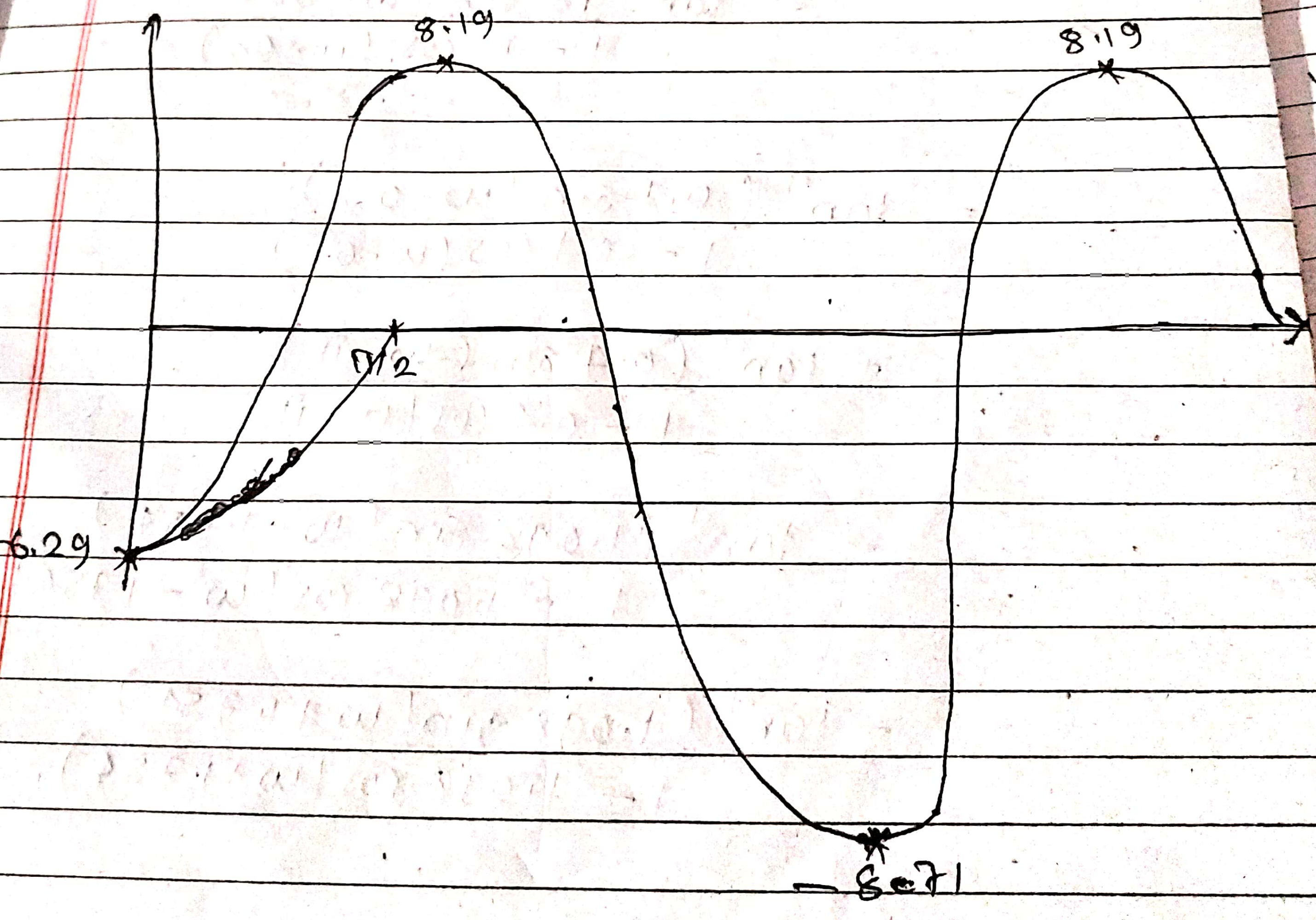
$$20\log_{10}|H(e^{j\omega})| = 10\log_{10}[1 + r_3^2 - 2r_3\cos(\omega-\theta_3)]$$
$$+10\log_{10}(1+r_4^2-2r_4\cos(\omega-\theta_4)]$$
$$-10\log_{10}[1+r_1^2-2r_1\cos(\omega-\theta_1)].$$
$$-10\log_{10}[1+r_2^2-2r_2\cos(\omega-\theta_2)]$$

Putting value of r and θ.

$$= 10\log_{10}[1+(0.1)^2-2\times 0.1\cos(\omega-0)$$
$$+10\log_{10}[1+(0.4)^2-2\times 0.4\cos(\omega-\pi)$$
$$-10\log_{10}[1+(1.098)^2-2\times 1.098\cos(\omega-1.754)$$
$$-10\log_{10}[1+(1.098)^2-2\times 1.098\cos(\omega+1.754)]$$

$$= 10\log_{10}[1.01 - 0.2\cos\omega]$$
$$+10\log_{10}[1.16 - 0.8\cos(\omega-\pi)$$
$$-10\log_{10}[2.20 - 2.196(\omega-1.754)$$
$$-10\log_{10}[2.20 - 2.196(\omega+1.754)$$

ω	Magnitude Response in dB
0	-6.29
$\pi/6$	-5.35
$\pi/4$	-4.07
$\pi/3$	-2.025
$\pi/2$	8.19
$2\pi/3$	2.24
π	-8.71
$3\pi/2$	8.19
2π	-6.29



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Phase Response

$$\angle H(e^{j\omega}) = \tan^{-1} \frac{r_3 \sin(\omega - \theta_3)}{1 - r_3 \cos(\omega - \theta_3)}$$

$$+ \tan^{-1} \frac{r_4 \sin(\omega - \theta_4)}{1 - r_4 \cos(\omega - \theta_4)}$$

$$- \tan^{-1} \frac{r_1 \sin(\omega - \theta_1)}{1 - r_1 \cos(\omega - \theta_1)}$$

$$- \tan^{-1} \frac{r_2 \sin(\omega - \theta_2)}{1 - r_2 \cos(\omega - \theta_2)}$$

$$= \tan^{-1} \left(\frac{0.1 \sin(\omega - 0)}{1 - 0.1 \cos(\omega - 0)} \right)$$

$$+ \tan^{-1} \left(\frac{0.4 \sin(\omega - \pi)}{1 - 0.4 \cos(\omega - \pi)} \right)$$

$$- \tan^{-1} \left(\frac{1.098 \sin(\omega - 1.754)}{1 - 1.098 \cos(\omega - 1.754)} \right)$$

$$- \tan^{-1} \left(\frac{1.098 \sin(\omega + 1.754)}{1 - 1.098 \cos(\omega + 1.754)} \right).$$

ω	$\angle H(e^{j\omega})$	
0	0 + (−7.57)	−7.57
$\pi/6$	0.05 + 6.42	6.47
$\pi/4$	0.075 + 6.642	6.717
$\pi/3$	0.09 + 6.88	6.97
$\pi/2$	0.099 + −1.46	−1.361
$2\pi/3$	0.082 + 1.44	1.522
π	0 + 1.08	1.08
$3\pi/2$	−0.99 + 1.45	0.46
2π	0 + (−7.57)	−7.57