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Chapter - 2

Find the Z-transform of given sequence

1) $x[n] = (1/2)^n u[n] + 1/3 u[n]$

we have

$$a^n u[n] \xrightarrow{Z} \frac{z}{z-a}, \quad |z| > |a|.$$

$$(1/2)^n u[n] \xleftrightarrow{Z} \frac{z}{z-1/2}, \quad |z| > 1/2$$

$$\left(\frac{1}{3}\right)^n u[n] \xleftrightarrow{Z} \frac{z}{z-1/3}, \quad |z| > 1/3$$

Since the is common ROC between the sequences

$$X(z) = \frac{z}{z-1/2} + \frac{z}{z-1/3}$$

$$= \frac{z(z-1/3) + z(z-1/2)}{(z-1/2)(z-1/3)} \quad \text{ROC } |z| > \frac{1}{6}$$

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$$= \frac{2z\,(z - 5/12)}{(z-1/2)\,(z-1/3)}$$

Zeros : 0, 5/12

Poles = 1/2, 1/3

ii) $x[n] = \left(\frac{1}{3}\right)^n u[n] + (1/2)^n u[-n-1]$

we have,

$$a^n u[n] \xleftrightarrow{z} \frac{z}{z-a}, \quad |z| > |a|$$

$$(1/3)^n u[n] \xleftrightarrow{z} \frac{z}{z-1/3}, \quad |z| > 1/3$$

$$(1/2)^n u[-n-1] \xleftrightarrow{z} \frac{z}{z-1}, \quad |z| < 1.$$

Since, the ROC's ~~dont~~ overlap there is ~~no~~ z transform

Now

$$X(z) = \frac{z}{z-1/3} + \frac{z}{z-1}$$

b)

(b)

$$\frac{z(z-1) + z(z-1/3)}{(z-1/3)(z-1)}$$

$$= z^2 - z + z^2 - 1/3\, z \,/\, (z-1/3)(z-1)$$

$$= 2z^2 - z(1+1/3) \,/\, (z-1/3)(z-1)$$

$$= 2z^2 - \frac{4}{3}z \,/\, (z-1/3)(z-1)$$

$$= \frac{z(2z-4/3)}{(z-1/3)(z-1)}$$

$\frac{4}{3} +$ $(4/3)$

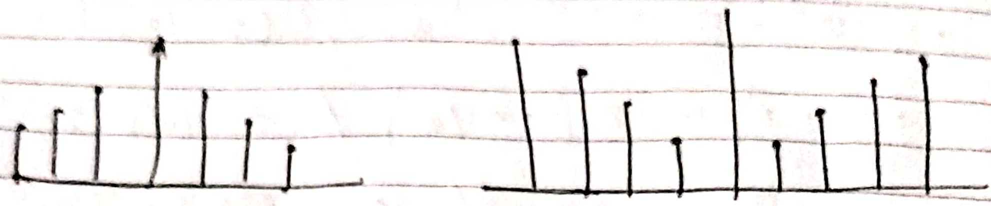
ROC $z > |4/3|$

Zeros $= 0, 2/3$

Poles $= 1/3, 1$

iii) $x[n] = \left(-\frac{1}{3}\right)^n \cancel{u[n]} + \left(\frac{1}{\,}\right.$ $a^{|n|}$.

for $0 < a < 1$



$$x[n] = \begin{cases} a^n & \text{for } n \geq 0 \\ a^{-n} & \text{for } n < 0 \end{cases}$$

So $x[n] = a^n u[n] + a^n u[-n-1]$.

we have

$$a^n u[n] \xleftrightarrow{Z} \frac{z}{z-a}, \quad |z| > a.$$

$$-a^n u[-n-1] \xleftrightarrow{Z} \frac{z}{z-a}, \quad |z| < |a|$$

$$-a^n u[-n-1] = \left(\frac{1}{a}\right)^n u[-n-1].$$

$$= \frac{z}{z-1} \qquad z < 1.$$

for $|a|>1$ there is no common ROC between the two sequences so the Z transform doesn't exist

for $|a|<1$ there is common ROC so. Z transform exists.

$$X(z) = \frac{z}{z-a} - \frac{z}{z-1/a}, \quad |a| < |z| < 1/|a|$$

$$= \frac{z(z-1/a) - z(z-a)}{(z-a)(z-1/a)}$$

$$= z(z-1/a \cdot z^2 - z/a - z^2 + az.$$

$$= \frac{-z}{a} + az$$

$$= \frac{z(-1/a + a)}{(z-a)(z-1/a)}$$

Zero at, $z=0$.

Poles at $z = a, 1/a$.

Find Inverse Z-transform

$$X(z) = \frac{z}{z^2 - z - 6}$$

(a)

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for ROC $|z| < 2$

From Table

$$-a^n u[-n-1] \xleftrightarrow{z} \frac{z}{z-a}.$$

$$= 5\left(-(3)^n u[-n-1] + (2)^n u[-n-1]\right).$$

c) for ROC $2 < |z| < 3$

from table.

$$-a^n u[-n-1] \longleftrightarrow \frac{z}{z-a} \quad |z| < a.$$

$$a^n u[n] \xleftrightarrow{z} \frac{z}{z-a} \quad |z| > a.$$

So,
inverse z-transform of $X(z)$ is

$$5\left(-(3)^n u[-n-1] + 2^n u[n] \right).$$

Q.N.

Find inverse z-transform of

$$X(z) = \frac{z}{(z-1)(z-2)^2}$$

$$\frac{X(z)}{z} = \frac{1}{(z-1)(z-2)^2}$$

$$= \frac{c_0}{(z-1)} + \frac{A_1}{(z-2)} + \frac{A_2}{(z-2)^2}$$

where.

$$c_0 = (z-1)\frac{X(z)}{z}\bigg|_{z=1}$$

$$= (z-1) \times \frac{1}{(z-1)(z-2)^2}\bigg|_{z=1}$$

$$= 1.$$

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$$\lambda_2 = \frac{1}{0!} \frac{d^0}{dz^0} (z-2)^2 \frac{1}{(z-1)(z-2)^2} \Bigg|_{z=2}$$

$$= \frac{1}{z-1} \Bigg|_{z=2} = 1.$$

$$\lambda_1 = \frac{1}{1!} \frac{d}{dz} (z-2)^2 \frac{X(z)}{z} \Bigg|_{z=2}$$

$$= \frac{d}{dz} \frac{1}{(z-1)} \Bigg|_{z=2}$$

$$= -(z-1)^{-2} \Big|_{z=2}$$

$$= -1$$

$$X(z) = \frac{z}{z-1} - \frac{z}{z-2} + \frac{z}{(z-2)^2} \quad \text{ROC } |z|>2$$

we have.

from Table.

$$a^n u[n] \xleftrightarrow{z} \frac{z}{z-a}.$$

$|z| > a.$

So

$$2^n u[n] \xleftrightarrow{z} \frac{z}{z-2}$$

$$u[n] \xleftrightarrow{z} \frac{z}{z-1}$$

from Table.

$$u[n] \xleftrightarrow{z} \frac{z}{z-1}$$

$$n a^n u[n] \xleftrightarrow{z} \frac{az}{(z-a)^2}$$

inverse z transform of $\frac{z}{(z-1)(z-2)^2}$

is

$u[n] - 2^n u[n] + n 2^{n-1} u[n].$

Find the outputs of LTI systems having the following inputs

a) $x[n] = \{3, 5, -2, 4\}$
$h[n] = \{6, 2, -3, 5\}$

b) $x[n] = 2^n u[n]$.
$h[n] = 3^n u[-n-1]$.

c) $x[n] = (1/2)^n u[n]$
$h[n] = (1/3)^n u[n]$

a) soln:
The z transform of sequence $x[n]$ is,

$$x(z) = \sum_{n=\infty}^{\infty} x[n] z^{-n}$$

$$= \cdots + x[0] + x[1] z^{-1} + x[2] z^{-2} + x[3] z^{-3} + \cdots$$

$$= 3 + \frac{5}{z} - \frac{2}{z^2} + \frac{4}{z^3}$$

ROC: All except $z = 0$ & $z = \infty$

The z transform of h[n] is,

$$H(z) = \sum_{n=\infty}^{\infty} h[n] z^{-n}$$

$$= + h[0] + h[1] z^{-1} + h[2] z^{-2} + h[3] z^{-3} + \cdots$$

$$= 6 + \frac{2}{z} - \frac{3}{z^2} + \frac{5}{z^3}.$$

ROC All except $z=0$ and $z=\infty$

Output of the LTI system,

$$y[n] = x[n] * h[n].$$

In z domain,

$$Y(z) = X(z) H(z) \quad \text{ROC } R_1 \cap R_2$$

$$= \left(3 + \frac{5}{z} - \frac{2}{z^2} + \frac{4}{z^3}\right)\left(6 + \frac{2}{z} - \frac{3}{z^2} + \frac{5}{z^3}\right)$$

$$= 18 + \frac{6}{z} - \frac{9}{z^2} + \frac{15}{z^3} + \frac{30}{z} + \frac{10}{z^2} - \frac{15}{z^3}$$

$$+ \frac{25}{z^4} - \frac{12}{z^2} - \frac{4}{z^3} + \frac{6}{z^4} - \frac{10}{z^5}$$

$+ \frac{24}{z^3} + \frac{8}{z^4} - \frac{12}{z^5} + \frac{20}{z^6}$

$= 18 + \frac{36}{z} - \frac{11}{z^2} + \frac{20}{z^3} - \frac{14}{z^4} - \frac{22}{z^5} + \frac{20}{z^6}$

Comparing with $y(z) = \sum_{n=-\infty}^{\infty} y[n] z^{-n}$

$y[n] = \{18, 36, -11, 20, -14, -22, 20\}$

b) $x[n] = 2^n u[n]$

$h[n] = 3^n u[-n-1]$

Taking Z transform

$x[n] \xleftrightarrow{Z} \frac{z}{z-2}$ ROC $|z| > |2|$

$h[n] \xleftrightarrow{Z} \frac{-z}{z-3}$ $|z| < 3.$

$Y[z] = X(z) H(z)$

$$Y(z) = \frac{z}{z-2}\left(\frac{-z}{z-3}\right) \quad \text{ROC } 2<|z|<3.$$

$$\frac{Y(z)}{z} = \frac{-z}{(z-2)(z-3)}, \quad 2<|z|<3$$

$$\frac{Y(z)}{z} = \frac{C_1}{(z-2)} + \frac{C_2}{(z-3)}$$

where

$$C_1 = (z-2) \times \frac{Y(z)}{z}\bigg|_{z=2}$$

$$= \frac{-z}{(z-3)}\bigg|_{z=2}$$

$\therefore$ ~~for~~ $C_1 = 2.$

$$C_2 = (z-3)\frac{Y(z)}{(z)}\bigg|_{z=3}$$

$$= \frac{-z}{z-2}\bigg|_{z=3} \quad \therefore C_2 = -3$$

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$\therefore Y(z) = \frac{2}{z-2} - \frac{3}{z-3} \quad 2<|z|<3$

Taking inverse Z transform

$y[n] = 2^n u[n] + (3)^n u[-n-1]$

c) $x[n] = \left(1/2\right)^n u[n] \qquad h[n] = \left(1/3\right)^n u[n]$

Taking Z transform of inputs.

$x[n] \xleftrightarrow{z} z/z-1/2 \qquad \text{Roc } |z|>1/2$

$h[n] \xleftrightarrow{z} z/z-1/3 \qquad \text{Roc } |z|>1/3$

$Y(z) = X(z)\,H(z) \quad \text{Roc } R_1 \cap R_2$

$= \left(\frac{z}{z-1/2}\right)\left(\frac{z}{z-1/3}\right) \qquad \text{Roc } z>1/3$

$\frac{Y(z)}{z} = \frac{c_1}{z-1/2} + \frac{c_2}{z-1/3}$

$c_1 = (z-1/2)\,Y(z)/z$
$= 1/2 \,/ (1/2-1/3)$
$= 3$

$c_2 = (z-1/3) \frac{Y(z)}{z}\Big|_{z=1/3}$
$= 1/3 \,/ (1/3-1/2)$
$= -2.$

$Y(z) = \frac{3}{z-1/2} - \frac{2}{z-1/3} \qquad \text{Roc } z>1/3$

Taking inverse Z-transform

$y[n] = 3(1/2)^n u[n] - 2(1/3)^n u[n].$