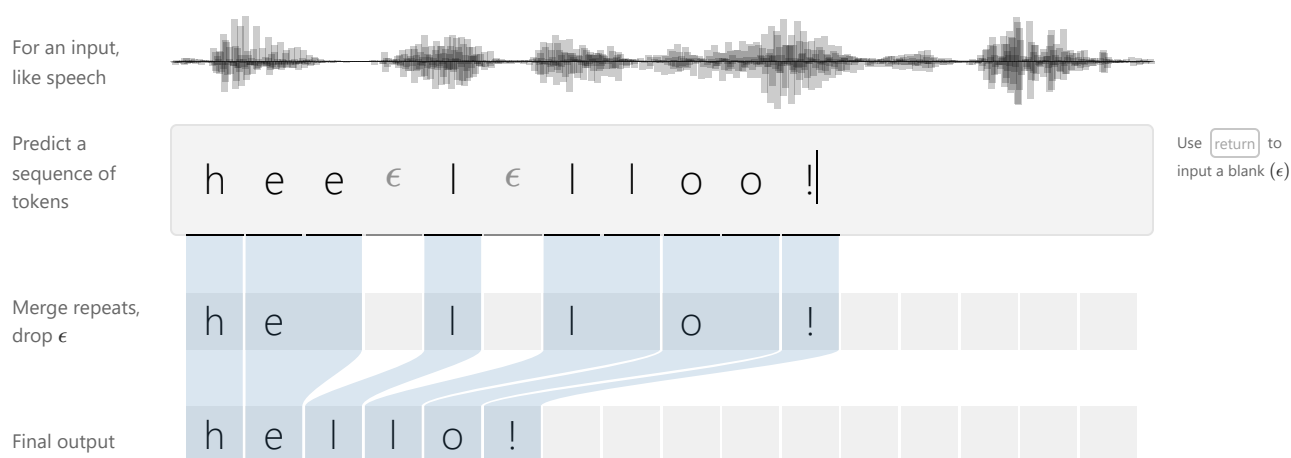


Sequence Modeling With CTC

A visual guide to Connectionist Temporal Classification, an algorithm used to train deep neural networks in speech recognition, handwriting recognition and other sequence problems.

How CTC collapsing works



AUTHORS

Awni Hannun

PUBLISHED

Nov. 27, 2017

AFFILIATIONS

Stanford University

DOI

10.23915/distill.00008

Introduction

Consider speech recognition. We have a dataset of audio clips and corresponding transcripts. Unfortunately, we don't know how the characters in the transcript align to the audio. This makes training a speech recognizer harder than it might at first seem.

Without this alignment, the simple approaches aren't available to us. We could devise a rule like "one character corresponds to ten inputs". But people's rates of speech vary, so this type of rule can always be broken. Another alternative is to hand-align each character to its location in the audio. From a modeling standpoint this works well — we'd know the ground truth for each input time-step. However, for any reasonably sized dataset this is prohibitively time consuming.

This problem doesn't just turn up in speech recognition. We see it in many other places. Handwriting recognition from images or sequences of pen strokes is one example. Action labelling in videos is another.

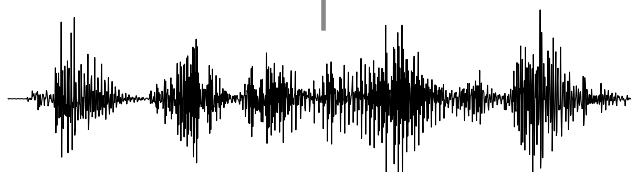
t h e q u i c k b r o w n f o x



The quick brown fox

Handwriting recognition: The input can be (x, y) coordinates of a pen stroke or pixels in an image.

j u m p s o v e r t h e l a z y d o g



Speech recognition: The input can be a spectrogram or some other frequency based feature extractor.

Connectionist Temporal Classification (CTC) is a way to get around not knowing the alignment between the input and the output. As we'll see, it's especially well suited to applications like speech and handwriting recognition.

To be a bit more formal, let's consider mapping input sequences $X = [x_1, x_2, \dots, x_T]$, such as audio, to corresponding output sequences $Y = [y_1, y_2, \dots, y_U]$, such as transcripts. We want to find an accurate mapping from X 's to Y 's.

There are challenges which get in the way of us using simpler supervised learning algorithms. In particular:

- Both X and Y can vary in length.
- The ratio of the lengths of X and Y can vary.
- We don't have an accurate alignment (correspondence of the elements) of X and Y .

The CTC algorithm overcomes these challenges. For a given X it gives us an output distribution over all possible Y 's. We can use this distribution either to *infer* a likely output or to assess the *probability* of a given output.

Not all ways of computing the loss function and performing inference are tractable. We'll require that CTC do both of these efficiently.

Loss Function: For a given input, we'd like to train our model to maximize the probability it assigns to the right answer. To do this, we'll need to efficiently compute the conditional probability $p(Y | X)$. The function $p(Y | X)$ should also be differentiable, so we can use gradient descent.

Inference: Naturally, after we've trained the model, we want to use it to infer a likely Y given an X . This means solving

$$Y^* = \operatorname{argmax}_Y p(Y | X).$$

Ideally Y^* can be found efficiently. With CTC we'll settle for an approximate solution that's not too expensive to find.

The Algorithm

The CTC algorithm can assign a probability for any Y given an X . The key to computing this probability is how CTC thinks about alignments between inputs and outputs. We'll start by looking at these alignments and then show how to use them to compute the loss function and perform inference.

Alignment

The CTC algorithm is *alignment-free* — it doesn't require an alignment between the input and the output. However, to get the probability of an output given an input, CTC works by summing over the probability of all possible alignments between the two. We need to understand what these alignments are in order to understand how the loss function is ultimately calculated.

To motivate the specific form of the CTC alignments, first consider a naive approach. Let's use an example. Assume the input has length six and $Y = [c, a, t]$. One way to align X and Y is to assign an output character to each input step and collapse repeats.

x_1	x_2	x_3	x_4	x_5	x_6	input (X)
c	c	a	a	a	t	alignment
c		a			t	output (Y)

This approach has two problems.

- Often, it doesn't make sense to force every input step to align to some output. In speech recognition, for example, the input can have stretches of silence with no corresponding output.

- We have no way to produce outputs with multiple characters in a row. Consider the alignment [h, h, e, l, l, l, o]. Collapsing repeats will produce “helo” instead of “hello”.

To get around these problems, CTC introduces a new token to the set of allowed outputs. This new token is sometimes called the *blank* token. We'll refer to it here as ϵ . The ϵ token doesn't correspond to anything and is simply removed from the output.

The alignments allowed by CTC are the same length as the input. We allow any alignment which maps to Y after merging repeats and removing ϵ tokens:

h h e ϵ ϵ l l l ϵ l l o

First, merge repeat characters.

h e ϵ l ϵ l o

Then, remove any ϵ tokens.

h e l l o

The remaining characters are the output.

h e l l o

If Y has two of the same character in a row, then a valid alignment must have an ϵ between them. With this rule in place, we can differentiate between alignments which collapse to “hello” and those which collapse to “helo”.

Let's go back to the output [c, a, t] with an input of length six. Here are a few more examples of valid and invalid alignments.

Valid Alignments

ϵ c c ϵ a t

c c a a t t

c a ϵ ϵ ϵ t

Invalid Alignments

c ϵ c ϵ a t

corresponds to
 $Y = [c, c, a, t]$

c c a a t

has length 5

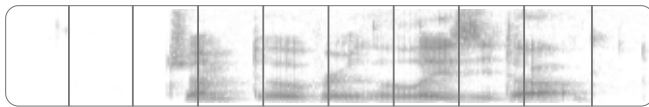
c ϵ ϵ ϵ | t t

missing the 'a'

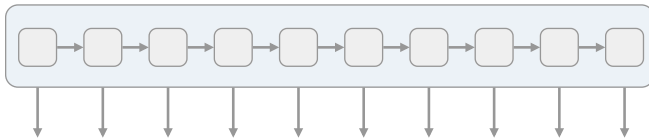
The CTC alignments have a few notable properties. First, the allowed alignments between X and Y are monotonic. If we advance to the next input, we can keep the corresponding output the same or advance to the next one. A second property is that the alignment of X to Y is many-to-one. One or more input elements can align to a single output element but not vice-versa. This implies a third property: the length of Y cannot be greater than the length of X .

Loss Function

The CTC alignments give us a natural way to go from probabilities at each time-step to the probability of an output sequence.



We start with an input sequence, like a spectrogram of audio.



The input is fed into an RNN, for example.

h	h	h	h	h	h	h	h	h	h
e	e	e	e	e	e	e	e	e	e
l	l	l	l	l	l	l	l	l	l
o	o	o	o	o	o	o	o	o	o
€	€	€	€	€	€	€	€	€	€

The network gives $p_t(a | X)$, a distribution over the outputs $\{h, e, l, o, \epsilon\}$ for each input step.

h	e	€	l	l	€	l	l	o	o
h	h	e	l	l	€	€	l	€	o
€	e	€	l	l	€	€	l	o	o

With the per time-step output distribution, we compute the probability of different sequences

h	e	l	l	o
e	l	l	o	
h	e	l	o	

By marginalizing over alignments, we get a distribution over outputs.

To be precise, the CTC objective for a single (X, Y) pair is:

$$p(Y \mid X) = \sum_{A \in \mathcal{A}_{X,Y}} \prod_{t=1}^T p_t(a_t \mid X)$$

The CTC conditional **probability**

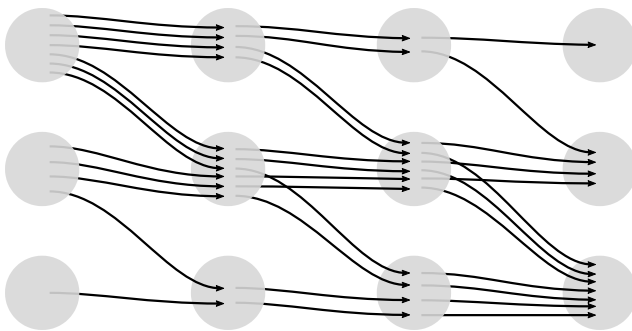
marginalizes over the set of valid alignments

computing the **probability** for a single alignment step-by-step.

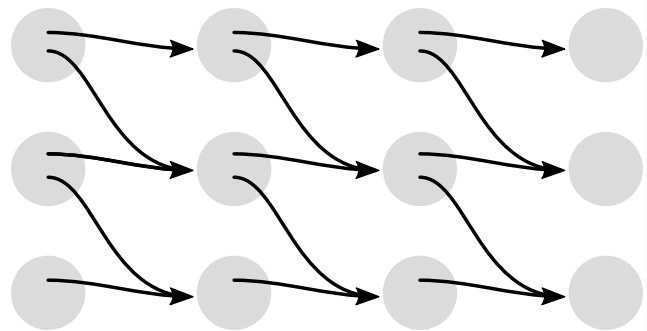
Models trained with CTC typically use a recurrent neural network (RNN) to estimate the per time-step probabilities, $p_t(a_t \mid X)$. An RNN usually works well since it accounts for context in the input, but we're free to use any learning algorithm which produces a distribution over output classes given a fixed-size slice of the input.

If we aren't careful, the CTC loss can be very expensive to compute. We could try the straightforward approach and compute the score for each alignment summing them all up as we go. The problem is there can be a massive number of alignments. ¹ For most problems this would be too slow.

Thankfully, we can compute the loss much faster with a dynamic programming algorithm. The key insight is that if two alignments have reached the same output at the same step, then we can merge them.



Summing over all alignments can be very expensive.



Dynamic programming merges alignments, so it's much faster.

Since we can have an ϵ before or after any token in Y , it's easier to describe the algorithm using a sequence which includes them. We'll work with the sequence

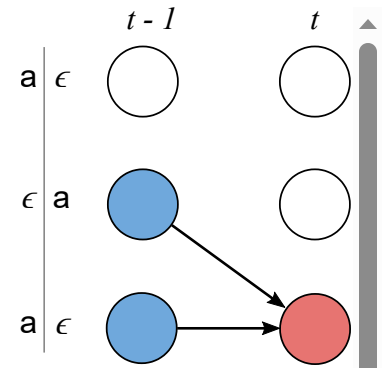
$$Z = [\epsilon, y_1, \epsilon, y_2, \dots, \epsilon, y_U, \epsilon]$$

which is Y with an ϵ at the beginning, end, and between every character.

Let's let α be the score of the merged alignments at a given node. More precisely, $\alpha_{s,t}$ is the CTC score of the subsequence $Z_{1:s}$ after t input steps. As we'll see, we'll compute the final CTC score, $P(Y \mid X)$, from the α 's at the last time-step. As long as we know the values of α at the previous time-step, we can compute $\alpha_{s,t}$. There are two cases.

Case 1:

In this case, we can't jump over z_{s-1} , the previous token in Z . The first reason is that the previous token can be an element of Y , and we can't skip elements of Y . Since every element of Y in Z is followed by an ϵ , we can identify this when $z_s = \epsilon$. The second reason is that we must have an ϵ between repeat characters in Y . We can identify this when $z_s = z_{s-2}$.



To ensure we don't skip z_{s-1} , we can either be there at the previous time-step or have already passed through at some earlier time-step. As a result there are two positions we can transition from.

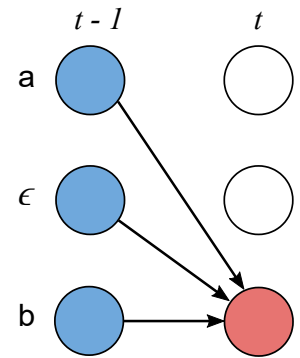
$$\alpha_{s,t} = (\alpha_{s-1,t-1} + \alpha_{s,t-1}) \cdot p_t(z_s | X)$$

The CTC probability of the two valid subsequences after $t - 1$ input steps.

The probability of the current character at input step t .

Case 2:

In the second case, we're allowed to skip the previous token in Z . We have this case whenever z_{s-1} is an ϵ between unique characters. As a result there are three positions we could have come from at the previous step.

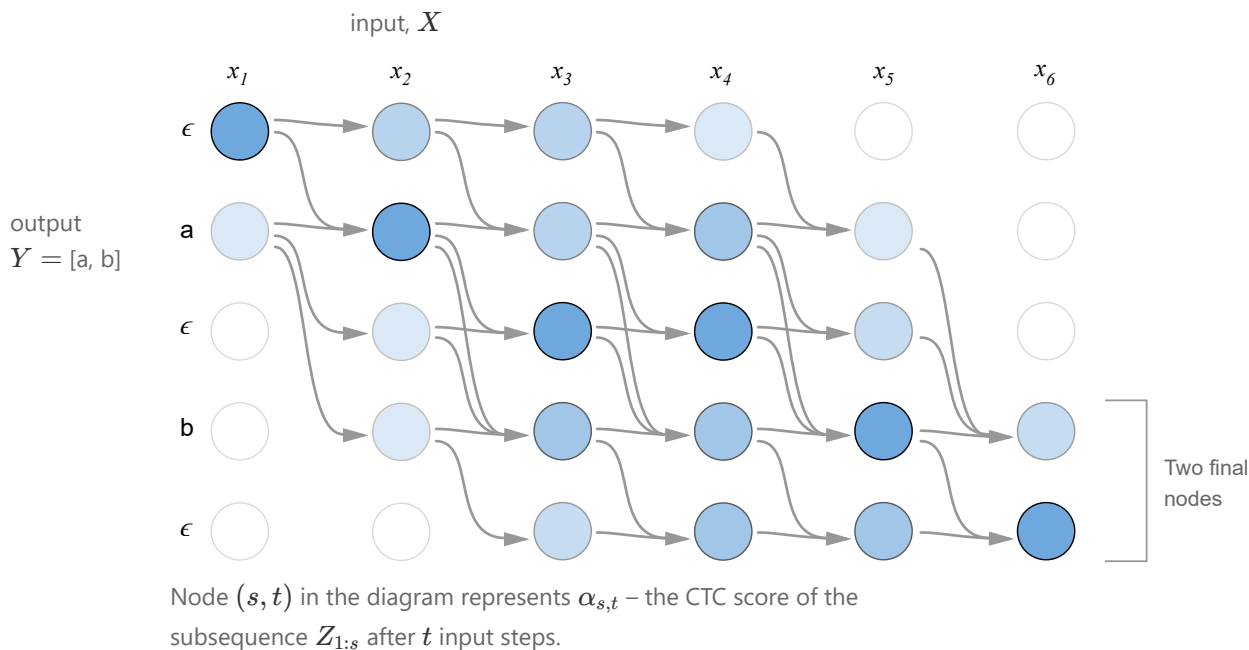


$$\alpha_{s,t} = (\alpha_{s-2,t-1} + \alpha_{s-1,t-1} + \alpha_{s,t-1}) \cdot p_t(z_s | X)$$

The CTC probability of the three valid subsequences after $t - 1$ input steps.

The probability of the current character at input step t .

Below is an example of the computation performed by the dynamic programming algorithm. Every valid alignment has a path in this graph.



There are two valid starting nodes and two valid final nodes since the ϵ at the beginning and end of the sequence is optional. The complete probability is the sum of the two final nodes.

Now that we can efficiently compute the loss function, the next step is to compute a gradient and train the model. The CTC loss function is differentiable with respect to the per time-step output probabilities since it's just sums and products of them. Given this, we can analytically compute the gradient of the loss function with respect to the (unnormalized) output probabilities and from there run backpropagation as usual.

For a training set \mathcal{D} , the model's parameters are tuned to minimize the negative log-likelihood

$$\sum_{(X,Y) \in \mathcal{D}} -\log p(Y | X)$$

instead of maximizing the likelihood directly.

Inference

After we've trained the model, we'd like to use it to find a likely output for a given input. More precisely, we need to solve:

$$Y^* = \operatorname{argmax}_Y p(Y | X)$$

One heuristic is to take the most likely output at each time-step. This gives us the alignment with the highest probability:

$$A^* = \underset{A}{\operatorname{argmax}} \prod_{t=1}^T p_t(a_t | X)$$

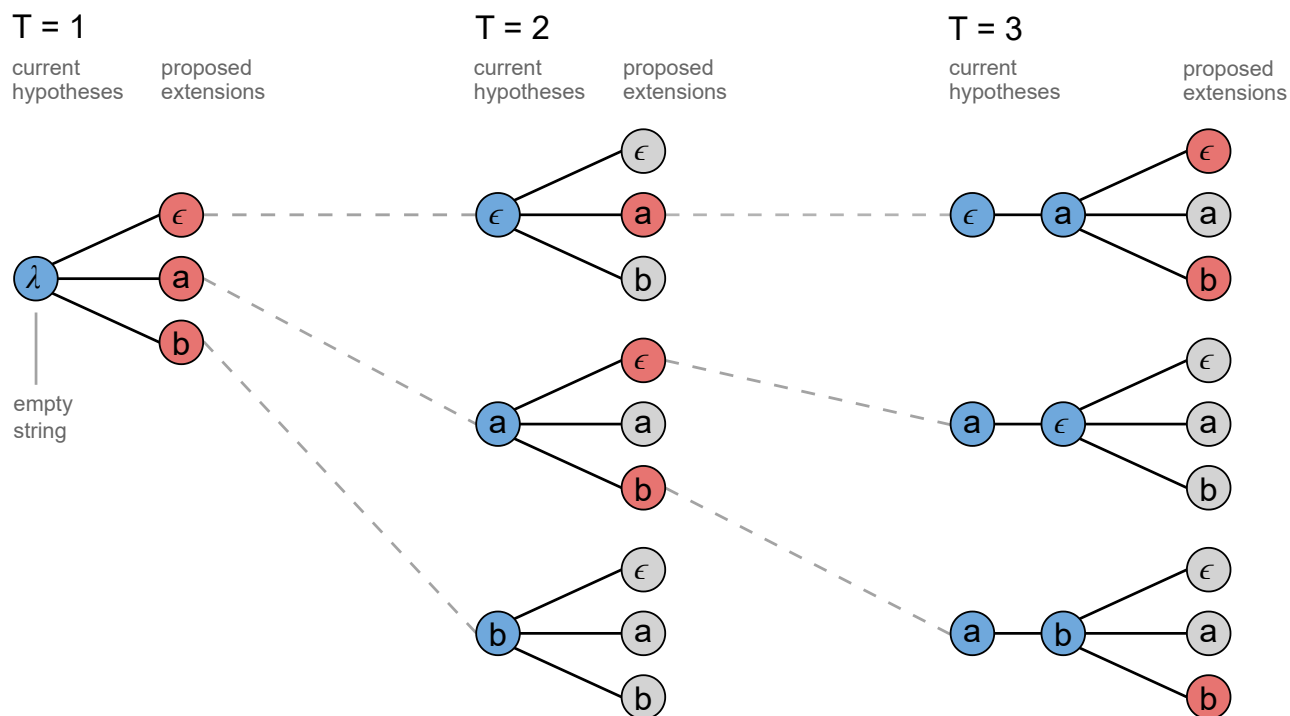
We can then collapse repeats and remove ϵ tokens to get Y .

For many applications this heuristic works well, especially when most of the probability mass is allotted to a single alignment. However, this approach can sometimes miss easy to find outputs with much higher probability. The problem is, it doesn't take into account the fact that a single output can have many alignments.

Here's an example. Assume the alignments $[a, a, \epsilon]$ and $[a, a, a]$ individually have lower probability than $[b, b, b]$. But the sum of their probabilities is actually greater than that of $[b, b, b]$. The naive heuristic will incorrectly propose $Y = [b]$ as the most likely hypothesis. It should have chosen $Y = [a]$. To fix this, the algorithm needs to account for the fact that $[a, a, a]$ and $[a, a, \epsilon]$ collapse to the same output.

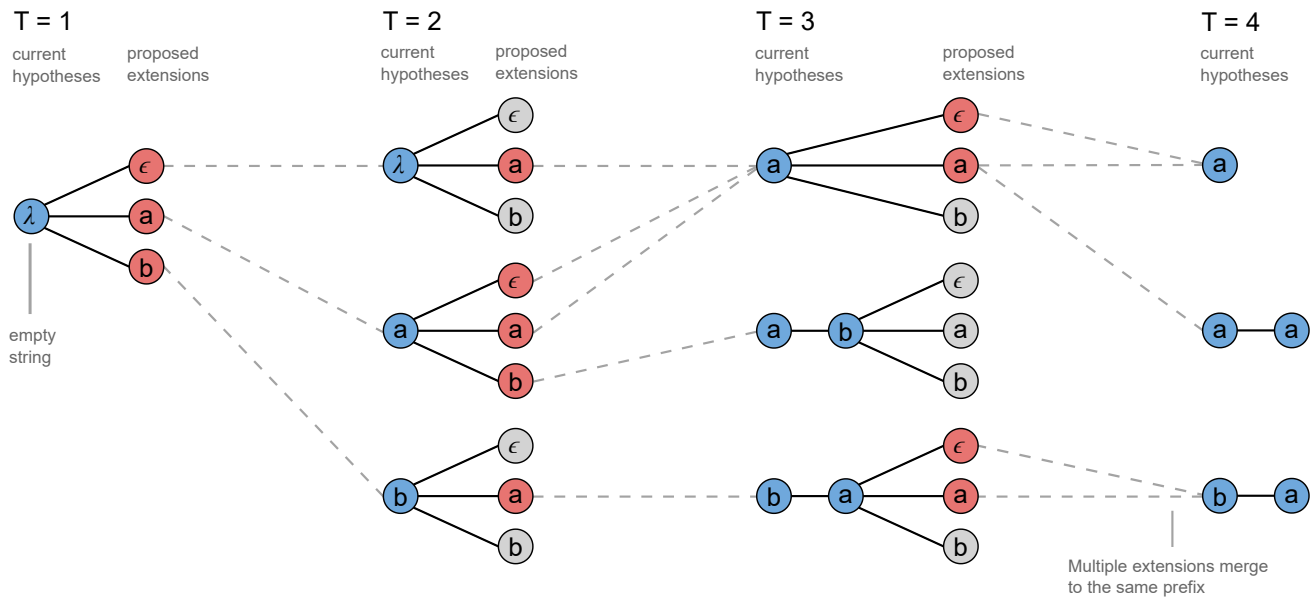
We can use a modified beam search to solve this. Given limited computation, the modified beam search won't necessarily find the most likely Y . It does, at least, have the nice property that we can trade-off more computation (a larger beam-size) for an asymptotically better solution.

A regular beam search computes a new set of hypotheses at each input step. The new set of hypotheses is generated from the previous set by extending each hypothesis with all possible output characters and keeping only the top candidates.



A standard beam search algorithm with an alphabet of $\{\epsilon, a, b\}$ and a beam size of three.

We can modify the vanilla beam search to handle multiple alignments mapping to the same output. In this case instead of keeping a list of alignments in the beam, we store the output prefixes after collapsing repeats and removing ϵ characters. At each step of the search we accumulate scores for a given prefix based on all the alignments which map to it.



The CTC beam search algorithm with an output alphabet $\{\epsilon, a, b\}$ and a beam size of three.

A proposed extension can map to two output prefixes if the character is a repeat. This is shown at $T = 3$ in the figure above where 'a' is proposed as an extension to the prefix [a]. Both [a] and [a, a] are valid outputs for this proposed extension.

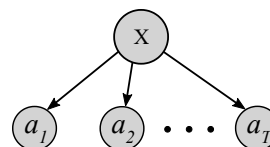
When we extend [a] to produce [a,a], we only want include the part of the previous score for alignments which end in ϵ . Remember, the ϵ is required between repeat characters. Similarly, when we don't extend the prefix and produce [a], we should only include the part of the previous score for alignments which don't end in ϵ .

Given this, we have to keep track of two probabilities for each prefix in the beam. The probability of all alignments which end in ϵ and the probability of all alignments which don't end in ϵ . When we rank the hypotheses at each step before pruning the beam, we'll use their combined scores.

Conditional Independence

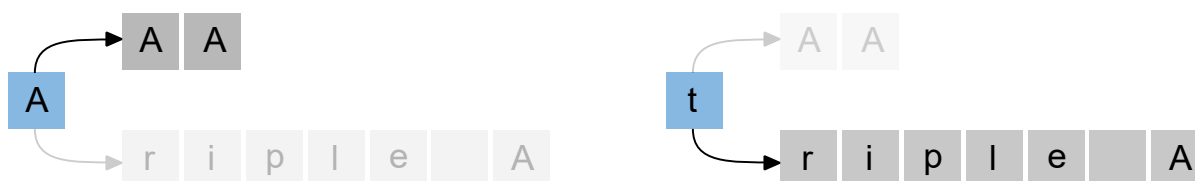
One of the most commonly cited shortcomings of CTC is the conditional independence assumption it makes.

The model assumes that every output is conditionally independent of the other outputs given the input. This is a bad assumption for many sequence to sequence problems.



Graphical model for CTC.

Say we had an audio clip of someone saying “triple A”. ^[1] Another valid transcription could be “AAA”. If the first letter of the predicted transcription is ‘A’, then the next letter should be ‘A’ with high probability and ‘r’ with low probability. The conditional independence assumption does not allow for this.



If we predict an ‘A’ as the first letter then the suffix ‘AA’ should get much more probability than ‘riple A’. If we predict ‘t’ first, the opposite should be true.

In fact speech recognizers using CTC don’t learn a language model over the output nearly as well as models which are conditionally dependent. ^[2] However, a separate language model can be included and usually gives a good boost to accuracy.

The conditional independence assumption made by CTC isn’t always a bad thing. Baking in strong beliefs over output interactions makes the model less adaptable to new or altered domains. For example, we might want to use a speech recognizer trained on phone conversations between friends to transcribe customer support calls. The language in the two domains can be quite different even if the acoustic model is similar. With a CTC acoustic model, we can easily swap in a new language model as we change domains.

Alignment Properties

The CTC algorithm is *alignment-free*. The objective function marginalizes over all alignments. While CTC does make strong assumptions about the form of alignments between X and Y , the model is agnostic as to how probability is distributed amongst them. In some problems CTC ends up allocating most of the probability to a single alignment. However, this isn’t guaranteed. ²

As mentioned before, CTC only allows *monotonic* alignments. In problems such as speech recognition this may be a valid assumption. For other problems like machine translation where a future word in a target sentence can align to an earlier part of the source sentence, this assumption is a deal-breaker.

Another important property of CTC alignments is that they are *many-to-one*. Multiple inputs can align to at most one output. In some cases this may not be desirable. We might want to enforce a strict one-to-one correspondence between elements of X and Y . Alternatively, we may want to allow multiple output elements to align to a single input element. For example, the characters “th” might align to a single input step of audio. A character based CTC model would not allow that.

The many-to-one property implies that the output can’t have more time-steps than the input.³ This is usually not a problem for speech and handwriting recognition since the input is much longer than the output. However, for other problems where Y is often longer than X , CTC just won’t work.

CTC in Context

In this section we’ll discuss how CTC relates to other commonly used algorithms for sequence modeling.

HMMs

At a first glance, a Hidden Markov Model (HMM) seems quite different from CTC. But, the two algorithms are actually quite similar. Understanding the relationship between them will help us understand what advantages CTC has over HMM sequence models and give us insight into how CTC could be changed for various use cases.

Let’s use the same notation as before, X is the input sequence and Y is the output sequence with lengths T and U respectively. We’re interested in learning $p(Y | X)$. One way to simplify the problem is to apply Bayes’ Rule:

$$p(Y | X) \propto p(X | Y) p(Y).$$

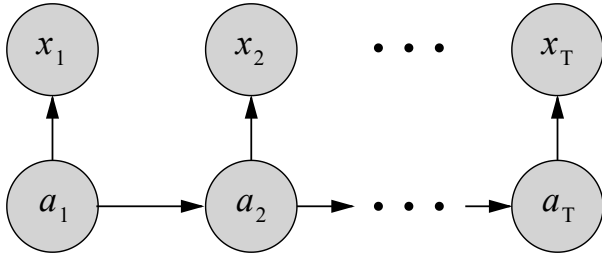
The $p(Y)$ term can be any language model, so let’s focus on $p(X | Y)$. Like before we’ll let \mathcal{A} be a set of allowed alignments between X and Y . Members of \mathcal{A} have length T . Let’s otherwise leave \mathcal{A} unspecified for now. We’ll come back to it later. We can marginalize over alignments to get

$$p(X | Y) = \sum_{A \in \mathcal{A}} p(X, A | Y).$$

To simplify notation, let’s remove the conditioning on Y , it will be present in every $p(\cdot)$. With two assumptions we can write down the standard HMM.

$p(X)$	=	$\sum_{A \in \mathcal{A}} \prod_{t=1}^T$	$p(x_t a_t)$	·	$p(a_t a_{t-1})$
The probability of the input		Marginalizes over alignments	The emission probability		The transition probability

The first assumption is the usual Markov property. The state a_t is conditionally independent of all historic states given the previous state a_{t-1} . The second is that the observation x_t is conditionally independent of everything given the current state a_t .



The graphical model for an HMM.

Now we can take just a few steps to transform the HMM into CTC and see how the two models relate. First, let's assume that the transition probabilities $p(a_t | a_{t-1})$ are uniform. This gives

$$p(X) \propto \sum_{A \in \mathcal{A}} \prod_{t=1}^T p(x_t | a_t).$$

There are only two differences from this equation and the CTC loss function. The first is that we are learning a model of X given Y as opposed to Y given X . The second is how the set \mathcal{A} is produced. Let's deal with each in turn.

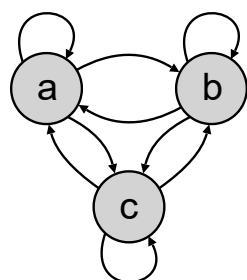
The HMM can be used with discriminative models which estimate $p(a | x)$. To do this, we apply Bayes' rule and rewrite the model as

$$\begin{aligned} p(X) &\propto \sum_{A \in \mathcal{A}} \prod_{t=1}^T \frac{p(a_t | x_t) p(x_t)}{p(a_t)} \\ &\propto \sum_{A \in \mathcal{A}} \prod_{t=1}^T \frac{p(a_t | x_t)}{p(a_t)}. \end{aligned}$$

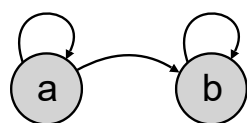
If we assume a uniform prior over the states a and condition on all of X instead of a single element at a time, we arrive at

$$p(X) \propto \sum_{A \in \mathcal{A}} \prod_{t=1}^T p(a_t | X).$$

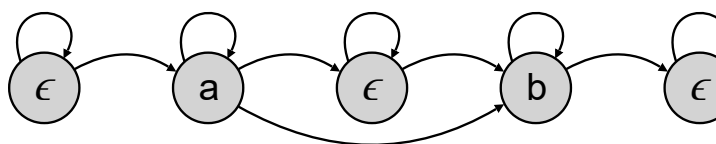
The above equation is essentially the CTC loss function, assuming the set \mathcal{A} is the same. In fact, the HMM framework does not specify what \mathcal{A} should consist of. This part of the model can be designed on a per-problem basis. In many cases the model doesn't condition on Y and the set \mathcal{A} consists of all possible length T sequences from the output alphabet. In this case, the HMM can be drawn as an *ergodic* state transition diagram in which every state connects to every other state. The figure below shows this model with the alphabet or set of unique hidden states as $\{a, b, c\}$.



Ergodic HMM: Any node can be either a starting or final state.



Linear HMM: Start on the left, end on the right.



CTC HMM: The first two nodes are the starting states and the last two nodes are the final states.

In CTC we augment the alphabet with ϵ and the HMM model allows a subset of the left-right transitions. The CTC HMM has two start states and two accepting states.

One possible source of confusion is that the HMM model differs for any unique Y . This is in fact standard in applications such as speech recognition. The state diagram changes based on the output Y . However, the functions which estimate the observation and transition probabilities are shared.

Let's discuss how CTC improves on the original HMM model. First, we can think of the CTC state diagram as a special case HMM which works well for many problems of interest. Incorporating the blank as a hidden state in the HMM allows us to use the alphabet of Y as the other hidden states. This model also gives a set of allowed alignments which may be a good prior for some problems.

Perhaps most importantly, CTC is discriminative. It models $p(Y | X)$ directly, an idea that's been important in the past with other discriminative improvements to HMMs.^[3] Discriminative training let's us apply powerful learning algorithms like the RNN directly towards solving the problem we care about.

Encoder-Decoder Models

The encoder-decoder is perhaps the most commonly used framework for sequence modeling with neural networks. These models have an encoder and a decoder. The encoder maps the input sequence X into a hidden representation. The decoder consumes the hidden representation and produces a distribution over

the outputs. We can write this as

$$\begin{aligned} H &= \text{encode}(X) \\ p(Y | X) &= \text{decode}(H). \end{aligned}$$

The `encode(·)` and `decode(·)` functions are typically RNNs. The decoder can optionally be equipped with an attention mechanism. The hidden state sequence H has the same number of time-steps as the input, T . Sometimes the encoder subsamples the input. If the encoder subsamples the input by a factor s then H will have T/s time-steps.

We can interpret CTC in the encoder-decoder framework. This is helpful to understand the developments in encoder-decoder models that are applicable to CTC and to develop a common language for the properties of these models.

Encoder: The encoder of a CTC model can be just about any encoder we find in commonly used encoder-decoder models. For example the encoder could be a multi-layer bidirectional RNN or a convolutional network. There is a constraint on the CTC encoder that doesn't apply to the others. The input length cannot be sub-sampled so much that T/s is less than the length of the output.

Decoder: We can view the decoder of a CTC model as a simple linear transformation followed by a softmax normalization. This layer should project all T steps of the encoder output H into the dimensionality of the output alphabet.

We mentioned earlier that CTC makes a conditional independence assumption over the characters in the output sequence. This is one of the big advantages that other encoder-decoder models have over CTC — they can model the dependence over the outputs. However in practice, CTC is still more commonly used in tasks like speech recognition as we can partially overcome the conditional independence assumption by including an external language model.

Practitioner's Guide

So far we've mostly developed a conceptual understanding of CTC. Here we'll go through a few implementation tips for practitioners.

Software: Even with a solid understanding of CTC, the implementation is difficult. The algorithm has several edge cases and a fast implementation should be written in a lower-level programming language. Open-source software tools make it much easier to get started:

- Baidu Research has open-sourced [warp-ctc](#). The package is written in C++ and CUDA. The CTC loss function runs on either the CPU or the GPU. Bindings are available for Torch, TensorFlow and [PyTorch](#).
- TensorFlow has built in [CTC loss](#) and [CTC beam search](#) functions for the CPU.
- Nvidia also provides a GPU implementation of CTC in [cuDNN](#) versions 7 and up.

Numerical Stability: Computing the CTC loss naively is numerically unstable. One method to avoid this is to normalize the α 's at each time-step. The original publication has more detail on this including the adjustments to the gradient. [4] In practice this works well enough for medium length sequences but can still underflow for long sequences. A better solution is to compute the loss function in log-space with the log-sum-exp trick. [4] Inference should also be done in log-space using the log-sum-exp trick.

Beam Search: There are a couple of good tips to know about when implementing and using the CTC beam search.

The correctness of the beam search can be tested as follows.

1. Run the beam search algorithm on an arbitrary input.
2. Save the inferred output \bar{Y} and the corresponding score \bar{c} .
3. Compute the actual CTC score c for \bar{Y} .
4. Check that $\bar{c} \approx c$ with the former being no greater than the latter. As the beam size increases the inferred output \bar{Y} may change, but the two numbers should grow closer.

A common question when using a beam search decoder is the size of the beam to use. There is a trade-off between accuracy and runtime. We can check if the beam size is in a good range. To do this first compute the CTC score for the inferred output c_i . Then compute the CTC score for the ground truth output c_g . If the two outputs are not the same, we should have $c_g < c_i$. If $c_i \ll c_g$ then the ground truth output actually has a higher probability under the model and the beam search failed to find it. In this case a large increase to the beam size may be warranted.

Bibliographic Notes

The CTC algorithm was first published by Graves et al. in 2006. [4] The first experiments were on TIMIT, a popular phoneme recognition benchmark. [5] Chapter 7 of Graves' thesis [6] also gives a detailed treatment of CTC.

One of the first applications of CTC to large vocabulary speech recognition was by Graves et al. in 2014. [7] They combined a hybrid DNN-HMM and a CTC trained model to achieve state-of-the-art results. Hannun et al. subsequently demonstrated state-of-the-art CTC based speech recognition on larger benchmarks. [8] A CTC model outperformed other methods on an online handwriting recognition benchmark in 2007. [9]

CTC has been used successfully in many other problems. Some examples are lip-reading from video [10], action recognition from video [11] and keyword detection in audio. [12, 13]

Many extensions and improvements to CTC have been proposed. Here are a few. The *Sequence Transducer* discards the conditional independence assumption made by CTC. [14] As a consequence, the model allows the output to be longer than the input. The *Gram-CTC* model generalizes CTC to marginalize over n-gram output classes. [15] Other works have generalized CTC or proposed similar algorithms to account for segmental structure in the output. [16, 17]

The Hidden Markov Model was developed in the 1960's with the first application to speech recognition in the 1970's. For an introduction to the HMM and applications to speech recognition see Rabiner's canonical tutorial. [18]

Encoder-decoder models were developed in 2014. [19, 20] *Distill* has an in-depth guide to attention in encoder-decoder models. [21]

Acknowledgments

I'm especially grateful to the Distill team for *dramatically* improving the quality of this article. Thanks to Chris Olah's suggestions and feedback, both the written and visual content of the article are substantially better. Thanks to Shan Carter for substantial improvements to the figures, and thanks to Ludwig Schubert for help with the Distill template.

Thanks to Sanjeev Satheesh, Chris Lengerich, Dan Jurafsky and the anonymous reviewers for their feedback. I'm also very grateful to Andrew Ng for feedback on the article and his support.

Discussion and Review

[Review-1 Anonymous](#)

[Review-2 Anonymous](#)

Footnotes

1. For a Y of length U without any repeat characters and an X of length T the size of the set is $\binom{T+U}{T-U}$. For $T = 100$ and $U = 50$ this number is almost 10^{40} . [↩]

2. We could force the model to choose a single alignment by replacing the sum with a max in the objective function,

$$p(Y | X) = \max_{A \in \mathcal{A}_{X,Y}} \prod_{t=1}^T p(a_t | X).$$

[↩]

3. If Y has r consecutive repeats, then the length of Y must be less than the length of X by $2r - 1$. [↩]

4. When computing the sum of two probabilities in log space use the identity

$$\log(e^a + e^b) = \max\{a, b\} + \log(1 + e^{-|a-b|})$$

Most programming languages have a stable function to compute $\log(1 + x)$ when x is close to zero. [↩]

References

1. Listen, Attend and Spell: A Neural Network for Large Vocabulary Conversational Speech Recognition [PDF]
Chan, W., Jaitly, N., Le, Q.V. and Vinyals, O., 2016. ICASSP.
2. Exploring Neural Transducers for End-to-End Speech Recognition [PDF]
Battenberg, E., Chen, J., Child, R., Coates, A., Gaur, Y., Li, Y., Liu, H., Satheesh, S., Seetapun, D., Sriram, A. and Zhu, Z., 2017.
3. Large scale discriminative training of hidden Markov models for speech recognition
Woodland, P. and Povey, D.. Computer Speech & Language, pp. 25--47. Academic Press. DOI: 10.1006/csla.2001.0182

4. **Connectionist Temporal Classification : Labelling Unsegmented Sequence Data with Recurrent Neural Networks** [\[PDF\]](#)
Graves, A., Fernandez, S., Gomez, F. and Schmidhuber, J., 2006. Proceedings of the 23rd international conference on Machine Learning, pp. 369--376. DOI: 10.1145/1143844.1143891
5. **Phone recognition on the TIMIT database** [\[link\]](#)
Lopes, C. and Perdigão, F., 2011. Speech Technologies, Vol 1, pp. 285--302. DOI: 10.5772/17600
6. **Supervised Sequence Labelling with Recurrent Neural Networks** [\[link\]](#)
Graves, A., 2012. Springer, Vol 385. DOI: 10.1007/978-3-642-24797-2
7. **Towards End-To-End Speech Recognition with Recurrent Neural Networks** [\[PDF\]](#)
Graves, A. and Jaitly, N., 2014. Proceedings of the 31st International Conference on Machine Learning (ICML-14), Vol 32(1), pp. 1764--1772. DOI: 10.1145/1143844.1143891
8. **Deep Speech: Scaling up end-to-end speech recognition** [\[PDF\]](#)
Hannun, A.Y., Case, C., Casper, J., Catanzaro, B., Diamos, G., Elsen, E., Prenger, R., Satheesh, S., Sengupta, S., Coates, A. and Ng, A.Y., 2014. , Vol abs/1412.5.
9. **A novel approach to on-line handwriting recognition based on bidirectional long short-term memory networks** [\[PDF\]](#)
Liwicki, M., Graves, A., Bunke, H. and Schmidhuber, J., 2007. Proceedings - 9th Int. Conf. on Document Analysis and Recognition, Vol 1, pp. 367--371. DOI: 10.1.1.139.5852
10. **LipNet: End-to-End Sentence-level Lipreading** [\[PDF\]](#)
Assael, Y.M., Shillingford, B., Whiteson, S. and de Freitas, N., 2016.
11. **Connectionist Temporal Modeling for Weakly Supervised Action Labeling** [\[PDF\]](#)
Huang, D., Fei-Fei, L. and Niebles, J.C., 2016. European Conference on Computer Vision, pp. 137--153. DOI: 10.1007/978-3-319-46493-0
12. **An application of recurrent neural networks to discriminative keyword spotting** [\[link\]](#)
Fernández, S., Graves, A. and Schmidhuber, J., 2007. The 17th international conference on Artificial neural networks, pp. 220--229. DOI: 10.1007/978-3-540-74695-9_23
13. **An End-to-End Architecture for Keyword Spotting and Voice Activity Detection** [\[PDF\]](#)
Lengerich, C. and Hannun, A., 2016. NIPS 2016 End-to-End Learning for Speech and Audio Processing Workshop.
14. **Sequence Transduction with Recurrent Neural Networks** [\[PDF\]](#)
Graves, A., 2012. DOI: 10.1145/2661829.2661935
15. **Gram-CTC: Automatic Unit Selection and Target Decomposition for Sequence Labelling** [\[PDF\]](#)
Liu, H., Zhu, Z., Li, X. and Satheesh, S., 2017. Proceedings of the 34th International Conference on Machine Learning.
16. **Sequence Modeling via Segmentations** [\[PDF\]](#)
Wang, C., Wang, Y., Huang, P., Mohamed, A., Zhou, D. and Deng, L., 2017.
17. **Segmental Recurrent Neural Networks** [\[PDF\]](#)
Kong, L., Dyer, C. and Smith, N.A., 2016. ICLR. DOI: 10.21437/Interspeech.2016-40
18. **Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition** [\[link\]](#)
Rabiner, L.R., 1989. Proceedings of the IEEE, Vol 77(2), pp. p257--286. DOI: 10.1109/5.18626
19. **Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation** [\[PDF\]](#)
Cho, K., van Merriënboer, B., Gulcehre, C., Bahdanau, D., Bougares, F., Schwenk, H. and Bengio, Y., 2014. EMNLP. DOI: 10.3115/v1/D14-1179
20. **Sequence to Sequence Learning with Neural Networks** [\[PDF\]](#)
Sutskever, I., Vinyals, O. and Le, Q.V., 2014. Advances in neural information processing systems.

21. Attention and Augmented Recurrent Neural Networks [\[link\]](#)

Olah, C. and Carter, S., 2016. Distill. DOI: 10.23915/distill.00001

Updates and Corrections

If you see mistakes or want to suggest changes, please [create an issue on GitHub](#).

Reuse

Diagrams and text are licensed under Creative Commons Attribution [CC-BY 4.0](#) with the [source available on GitHub](#), unless noted otherwise. The figures that have been reused from other sources don't fall under this license and can be recognized by a note in their caption: "Figure from ...".


Citation

For attribution in academic contexts, please cite this work as

Hannun, "Sequence Modeling with CTC", Distill, 2017.

BibTeX citation

```
@article{hannun2017sequence,  
  author = {Hannun, Awni},  
  title = {Sequence Modeling with CTC},  
  journal = {Distill},  
  year = {2017},  
  note = {https://distill.pub/2017/ctc},  
  doi = {10.23915/distill.00008}  
}
```

 Distill is dedicated to clear explanations of machine learning

[About](#) [Submit](#) [Prize](#) [Archive](#) [RSS](#) [GitHub](#) [Twitter](#) [ISSN 2476-0757](#)