Example 71: *Obtain disjunctive normal form of*

$$p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim r)))$$

Solution: $p \vee (\sim p \rightarrow (q \vee (q \rightarrow \sim \vee r))) = p \vee (\sim p \rightarrow (- q \vee \cdots r)))$

$$= p \vee (p \vee (q \vee (\sim p \vee \sim r)))$$

$$= p \vee p \vee q \vee \sim q \vee \sim r$$

$$= p \vee q \vee \sim q \vee \sim r$$

which is required disjunctive normal form.

2.10.5 Conjunctive Normal Form

Let A be a given formula, another formula B which is equivalent to A is called ***Conjunctive*** normal form if it is a product of an elementary sum.

Example 72: *Obtain conjunctive normal form:*

$$\sim (p \vee q) \leftrightarrow (p \wedge q)$$

Solution: Let $P = \sim (p \vee q)$ and $Q = (p \wedge q)$

Now, $P \leftrightarrow Q$

$\Rightarrow$ $(P \rightarrow Q) \wedge (Q \rightarrow P)$

$\Rightarrow$ $\sim (p \vee q) \leftrightarrow (p \wedge q)$

$\Rightarrow$ $[\sim (p \vee q) \rightarrow (p \wedge q)] \wedge [(p \wedge q) \rightarrow \sim (p \vee q)]$

$\Rightarrow$ $[(p \vee q) \vee (p \wedge q)] \wedge [\sim (p \wedge q) \vee \sim (p \vee q)]$

$\Rightarrow$ $[(p \vee q) \vee (p \wedge q)] \wedge [(\sim p \vee \sim q) \vee (\sim p \wedge \sim q)]$ [By Demorgan's laws]

$\Rightarrow$ $[(p \vee q \vee p) \wedge (p \vee q \vee q) \wedge (\sim p \vee \sim q \vee \sim p) \wedge (\sim p \vee \sim q \vee \sim q)$

which is required conjunctive normal form.

Example 73: *Obtain DNF of:*

(i) $(p \rightarrow q) \wedge (\sim p \wedge q)$ (ii) $(p \wedge (p \rightarrow q)) \rightarrow q$

Solution: (i) $p \rightarrow q$ is logical equivalent to $(\sim p \vee q)$

$\therefore$ $(p \rightarrow q) \wedge (\sim p \wedge q)$

$= (\sim p \vee q) \wedge (\sim p \wedge q)$

$= (\sim p \vee \sim p \wedge q) \vee (q \wedge \sim p \wedge q)$

$= (\sim p \wedge q) \vee (q \wedge \sim p)$

(ii) $p \wedge (p \rightarrow q) \rightarrow q = \sim (\sim p \wedge \sim p \vee q) \vee q$

$= \sim p \vee \sim (\sim p \vee q) \vee q$

$= \sim p \vee (p \wedge \sim q) \vee q$