CSCE 222 (Carlisle), Honors Discrete Structures for Computing
Fall 2020
Homework 1

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
Sameer Hussain

Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Tuesday, September 1, 2020.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- https://www.ntg.nl/doc/biemesderfer/ltxcrib.pdf
- http://discrete.openmathbooks.org/dmoi2/secpropositional.html
- https://www.cs.yale.edu/homes/aspnes/classes/202/notes.pdf

LaTeX hints: Read this .tex file for some explanations that are in the comments.

Math formulas are enclosed in $ signs, e.g., `$x + y = z$` becomes $x + y = z$.

Logical operators: $\neg, \wedge, \vee, \oplus, \rightarrow, \leftrightarrow$.

Here is a truth table using the "tabular" environment:

p	$\neg p$
T	F
F	T

Exercises for Section 1.1:

30c: (1 pt)
Converse - When I sleep until noon, it is necessary that I stay up late
Inverse - When I do not stay up late, it is necessary that I do not sleep until noon.
Contrapositive - When I do not sleep until noon, it is necessary that I do not stay up late.

40: (2 pts)

p	q	r	s	$((p- > q)- > r)- > s$
F	F	F	F	T
F	F	F	T	T
F	F	T	F	F
F	F	T	T	T
F	T	F	F	T
F	T	F	T	T
F	T	T	F	F
F	T	T	T	T
T	F	F	F	F
T	F	F	T	T
T	F	T	F	F
T	F	T	T	T
T	T	F	F	T
T	T	F	T	T
T	T	T	F	F
T	T	T	T	T

Exercises for Section 1.2:

40a: (1 pt) John had the unauthorized access.
This is deduced by Carlos and Diana's explanations. One of the suspects is telling the truth and Carlos and Diana **cannot** BOTH be lying because their statements intersect in the sense that if Carlos is lying, then Diana is truthful and vice-versa. So by that logic, either one of them is the suspect actually telling the truth. Alice's statement does not have anything to do with herself. Therefore, John is lying since John said ”I did not do it”.

42: (2 pts) Hint: you might want to write a program – see e.g. `http://rosettacode.org/wiki/Zebra_puzzle`
Zebra is owned by the Japanese person and the Norwegian drinks mineral water.

English - Red
Spain - Dog
Japan - Painter
Italy - Tea Norway - 1st house from left to right

White is left from the Green house Photographer has pet snail Diplomat in Yellow Milk is 3rd house Green house - coffee Blue - 2nd house Owner of fox next to Physician Horse next to Diplomat

Norway cannot be blue or white or green or red based on the sequencing of house. It has to be yellow.

Orange juice is violinist, tea is Italy, coffee is green, therefore Norway is mineral water.

The fox is next to the physician, its in the first house. Japanese person must be the owner of the zebra.

Country	Pet	Color	Job	Drink	Num
Norway	Fox	Yellow	Diplomat	Mineral Water	1
Italy	Horse	Blue	Physician	Tea	2
English	Snail	Red	Photographer	Milk	3
Spaniard	Dog	White	Violinist	Orange juice	4
Japanese	Zebra	Green	Painter	Coffee	5

Exercises for Section 1.3:

28 (1 pt)

Solution

$p- > (q \vee r) \equiv \neg p \vee (q \vee r)$

$\equiv (\neg p \vee \neg p) \vee (q \vee r)$

$\equiv (p- > q) \vee (p- > r)$

Therefore these statementsa are logically equivalent $(p- > q) \vee (p- > r) \equiv p- > (q \vee r)$

48 (2 pts) Based on De Morgan's laws we know

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

after double negation we get: $p \vee q \equiv \neg(\neg p \wedge \neg q)$

Disjunction can be made using only negation and conjunction so $\neg$ and $\wedge$ form complete collection of logical operators.

Exercises for Section 1.4:

42(b): (1 pt) ”No directories in the file system can be opened and no files can be closed when system errors have been detected.”

Let $Q(x)$ be Directory x in file system cannot be opened and $P(y)$ be File y can be closed and $T(z)$ be system error z detected.

The system specification is:

$\forall z T(z) - > (\forall x(\neg Q(x)) \wedge \forall y(\neg P(y)))$

46: (2 pts)

Not logically equivalent

In the domain for y and z, $P(y)$ and $Q(z)$ are false and $Q(y)$ and $P(z)$ are true.

So then for all x, P(x) if and only if Q(x) is false if x = z.

$\forall x P(x) \iff \forall x Q(x)$ is true since for all $xP(x)$ and for all $xQ(x)$ is false, they are biconditionally true.

The propositions have different truth value here so they are not logically equivalent.

Exercises for Section 1.5:

16(e): (1 pt) Quantifier statement: Let Q(x) be "there is a student in the class in every year of study x"

$\exists x Q(x)$

This is false though, as there is no senior math major or freshman computer science major.

32(d): (1 pt) Negating

$\neg \forall y \exists x \exists z (T(x, y, z) \vee Q(x, y))$

$\equiv \exists y \forall x \forall z \neg (T(x, y, z) \vee Q(x, y))$

$\equiv \exists y \forall x \forall z (\neg T(x, y, z) \wedge \neg Q(x, y))$

44: (2 pt) A quadratic polynomial in $ax^2 + bx + c$ will have at most two real roots if there is a real solution for x and y and they will be different.

Using quantifiers and logical connectives to express this:

$\exists x \exists y (x \neq y \wedge ax^2 + bx + c = 0 \wedge ay^2 + by + c = 0)$

Exercises for Section 1.6:

34 (a-e): (4 pts)
Let x represents logic and m represents mathematics.

$D(x)$ represents x is difficult and $A(x)$ is many students like x.

"Logic is difficult or not many students like logic": $D(x) \lor \neg A(x)$

"If mathematics is easy, then logic is not difficult": $\neg(D(m))- > \neg(D(x))$

a) That mathematics is not easy, if many students like logic: $A(x)- > D(m)$

We can re write the first statement to: $\neg(A(m))- > \neg(D(x)) \equiv \neg A(x) \lor D(x)$

By hypothetical syllogism, $D(x)- > D(x)$ and $D(x)- > D(m) \equiv A(x)- > D(m)$

This conclusion is valid

b) That not many students like logic, if mathematics is not easy:

$D(m)- > \neg A(x)$

$D(x) \lor \neg A(x)$: $\equiv A(x)- > D(x)$

Taking the contrapositive: $\neg(D(m))- > \neg(D(x)) \equiv D(x)- > D(m)$

But here $D(m)$ does not lead to the conclusion so the conclusion is not valid for the assumptions.

c) That mathematics is not easy or logic is difficult: $D(m) \lor D(x)$
$\equiv D(m) \lor D(x) \equiv \neg(\neg D(m)) \lor D(x)$
$\equiv \neg D(m)- > D(x)$

but this does not make sense as the assumption is: $\neg(D(m))- > \neg(D(x))$

So the conclusion is not valid

d) That logic is not difficult or mathematics is not easy: $\neg D(x) \lor D(m) \equiv D(x)- > D(m)$
- by implication This is valid given the assumptions as one of them is: $\neg(D(m))- > \neg(D(x))$

e) If not many students like logic, then either mathematics is not easy or logic is not difficult:

$\neg A(x)- > (D(m) \vee \neg D(x))$

Implication to: $\neg S(x)- > D(m) \vee \neg D(x))$

$\equiv D(x)- > D(m)$

This is one of the assumptions, therefore the conclusion is valid.