CSCE 222 (Carlisle), Honors Discrete Structures for Computing

Fall 2020

Homework 2

Type your name below the pledge to sign

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

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Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Tuesday, September 8, 2020.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- https://www.math24.net/set-identities/
- https://www.math-only-math.com/difference-of-two-sets.html
- https://www.ntg.nl/doc/biemesderfer/ltxcrib.pdf

Exercises for Section 1.7:

20(a-b): (2 points)

a) Proof by Contraposition

Original statement: ”If n is an integer and $3n + 2$ is even, then n is even

Contraposition: ”If $n$is odd then $3n + 2$ is odd”

So we assume n is odd: Let k be an integer where $n = 2k + 1$

Substituting: $3n + 2 = 3(2k + 1) + 2 = 6k + 3 + 2 = 2(3k + 2) + 1$

So that means we can find an integer x where $x = 3k + 2$ such that $3n + 2 = 2x + 1$

This means that $3n + 2$ is odd. And by the fact that the contraposition is true then the original is also true.

b) Proof by Contradiction

We suppose than $3n+2$ is even and n is odd. Let k be an integer where: $n = 2k+1$ Substituting n into $3n + 2$: $3n + 2 = 3(2k + 1) + 2 = 6k + 3 + 2 = 2(3k + 2) + 1$ we can find an integer x where $x = 3k + 2$ $3n + 2 = 2x + 1$ This is clear that $3n + 2$ is odd The assumption that n is odd allows us to see the contradiction that $3n + 2$ is even and odd which it can't both be.

Therefore, the statement ”n is odd” is false and it must be an even number.

26: (1 points) Show that at least 3 of any 25 days chosen must fall in the same month of the year

Proof by contradiction: Contradict ”at least 3” is ”at most 2”

If there were at most 2 days: $2 * 12 = 24$ days for 12 months which contradicts a choice from 25 days.

Therefore, at least 3 days out of any 25 must be chosen on the same month of the year.

Exercises for Section 1.8:

8: (2 points)
Without loss of generality, assume x is odd and y is even Suppose a and b are integers,
Let $x = 2a + 1$ and $y = 2b$

$5x + 5y = 5(2a + 1) + 5(2b)$
$5x + 5y = 5(2a + 1) + 5(2b)$
$5x + 5y = 10a + 10b + 4 + 1$
$5x + 5y = 2(5a + 5b + 2) + 1$, which is odd

20: (2 points) If r is an irrational number there is a unique number n where distance = 1/2 Given an integer r and m:
$|r - n| < 1/2$ and $|r - m| < 1/2$
So then by that logic,
$m < r < m + 1$
$|n - m| \leq |r - n| + |r - m| < 1/2 + 1/2$
$= 1$

46: (2 points)

You CANNOT use dominoes to tile 5 x 5 checkerboard with three corners removed.

A domino takes up a black and a white square. Without removing corners, there are 22 squares and you can use 11 dominoes to tile it. But with 3 corners removed, there are either
12 black and 10 white squares or 10 black and 12 white squares. This contradicts how dominoes tile (one square of each) Therefore, you cannot use dominoes to tile this form of checkerboard with dominoes.

Exercises for Section 2.1:

24: (2 points). (Prove your answer is correct.)
Yes, you can conclude that $A = B$ if they're in the same power set because if you let A and B be two sets Then, set A, P(A), contains all subsets of A. And suppose that set A and set B are different. Given an element x that exists in power set of A but is not an element in power set B. That means, different sets (A and B) will have different power sets so by that logic if A and B were to have the same power set they would also have to have the same set as each other. So, $A = B$

34(a-d): (2 points)
$A = \{a, b, c\}$, $B = \{x, y\}$, $C = \{0, 1\}$
a)
$A \times B \times C$ = $\{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), (b, x, 0),$
$(b, x, 1), (b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\}$
b)
$C \times B \times A$ = $\{(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b),$
$(0, y, c), (1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)\}$
c)
$C \times A \times B$ = $\{(0, a, x), (0, a, y), (0, b, x), (0, b, y), (0, c, x),$
$(0, c, y), (1, a, x), (1, a, y), (1, b, x), (1, b, y), (1, c, x), (1, c, y)\}$
d)
$B \times B \times B$ = $\{(x, x, x), (x, x, y), (x, y, x), (y, x, x), (y, x, y),$
$(y, y, x), (y, y, y)\}$

46(a-d): (2 points)
a) $\exists x \in R(x^3 = -1)$
"There exists a real number whos cube is -1"
This is true as $x = -1$ is a real number that cubes to itself.

b) $\exists x \in Z(x + 1 > x)$
"There exists an integer smaller than itself plus 1"
This is true for every integer x, for example if $x = 1$ then it becomes $x + 1 = 1 + 1 = 2 > 1 = x$

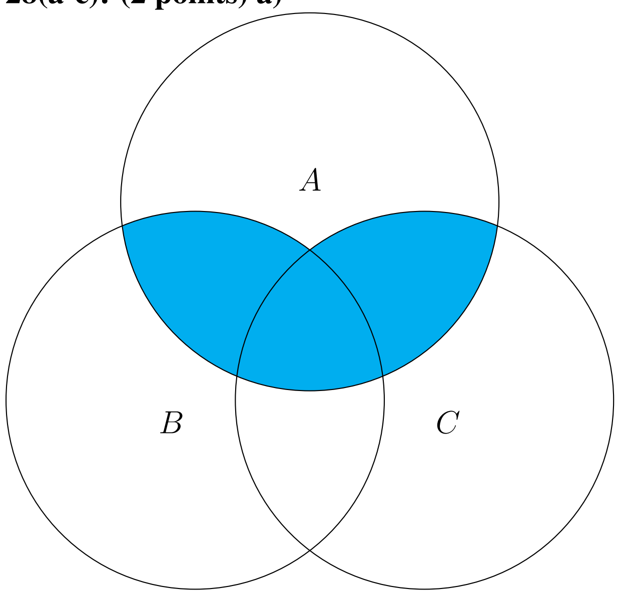
c) $\forall x \in Z(x - 1 \in Z)$
"An integer minus 1 is also an integer"
This is true for every integer x, for example $x - 1$ is also an integer just like 1.

d) $\forall x \in Z(x^2 \in Z)$
"The square of an integer is also an integer"
This is true for every integer x, for example x^2 is also an integer as well.

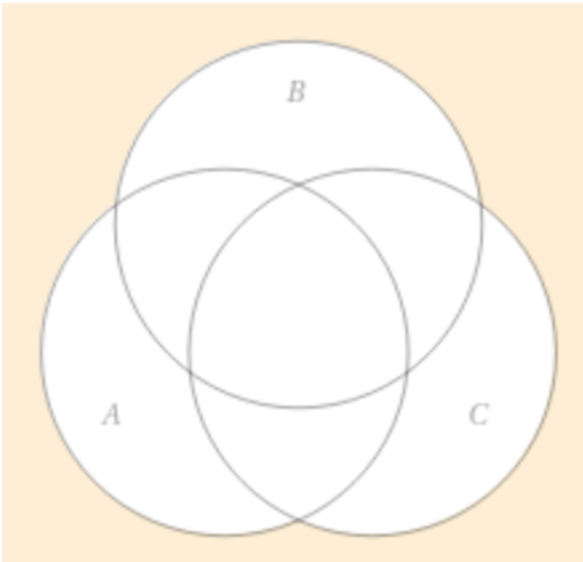
Exercises for Section 2.2:

20d: (1 points)
$(A - C) \cap (C - B) = \emptyset$
Let $x \in (A - C) \cap (C - B)$
x is in intersection when it's in both sets and in $A - C$ that means x in A but not C. x is thus still in a set. The statement $x \in \emptyset$ is false. But the statement is also a subset of the empty set.
$\emptyset \subseteq (A - C)$
Since $(A - C) \cap (C - B) \subseteq \emptyset$ and $\emptyset \subseteq (A - C)$ then $(A - C) \cap (C - B) = \emptyset$

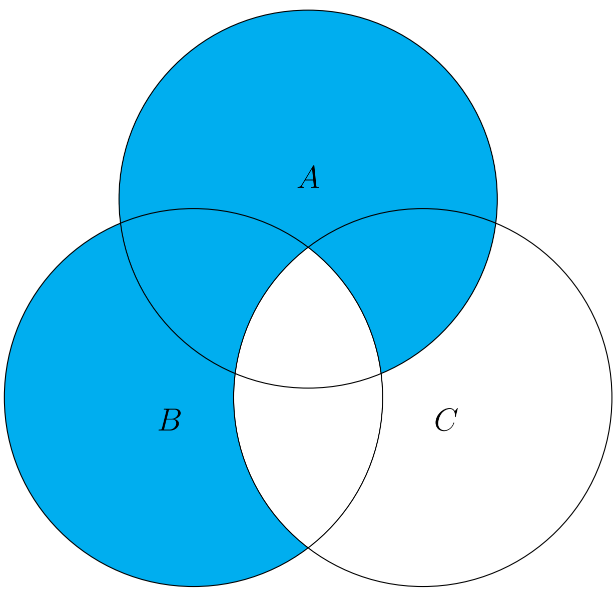
28(a-c): (2 points) a)



b)



c)



46: (2 points)

A symmetric difference $A \oplus B$ means elements in A or B but not both.

We can use set identities to prove this true.

$A \oplus (B \oplus C)$ can be written using x as an element within these sets.
$x \in (A \cup B) \wedge (x \in C)$
This is equal to:
$[(x \in A) \vee (x \in B)] \wedge (x \in C)$
Which means that:
$(x \in (A \cap C)) \vee (x \in (B \cap C)$
So $x \in ((A \cap C) \cup (B \cap C)$
Therefore, this property is associative and
$A \oplus (B \oplus C) = (A \oplus B) \oplus C$

This could have also been proven using a membership table or a Venn diagram.