CSCE 222 (Carlisle), Honors Discrete Structures for Computing
Fall 2020
Homework 3

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
Sameer Hussain

Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Tuesday, September 15, 2020.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- https://artofproblemsolving.com/wiki/index.php/LaTeX:Symbols

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Exercises for Section 2.3:

20(a-d): (2 points)

a)

A function from **N** to **N** that is one-to-one but not on-to is $f(n) = n^2$

b)

A function from **N** to **N** that is on-to but not one-to-one is $f(n) = (n/4)$

c)

A function from **N** to **N** that is both one-to-one and on-to is
$f(n) = n + 1$, if n is even
$f(n) = n - 1$, if n is odd

d)

A function from **N** to **N** that is neither one-to-one nor onto. $f(n) = 5$ It has the same image for 5 and all inputs.

34(a): (2 points) a)
If f o g is onto then let $m \in C$
Then because f o g is onto, $\exists x \in A$ where $(f \text{ o } g)(x) = f(g(x)) = m$
Let $y = g(x) \in B$ then $f(y) = m$ therefore, f is also an onto.

38: (2 points) Given $g : R- > R$ and $f : R- > R$
$f(x) = x^2 + 1$
$g(x) = x + 2$
Since f and g are both functions from R to R.
$(f \text{ o } g)(x) = f(g(x))$
$(g \text{ o } f)(x) = g(f(x))$
$(f \text{ o } g)(x) = f(x + 2) = (x + 2)^2 = x^2 + 4x + 5$
$(g \text{ o } f)(x) = g(x^2 + 1) = x^2 + 3$

58: (2 points)
Finding integers n to satisfy the inequality $a \leq n \leq b$
From this inequality we know that the number of integers that satisfy is the difference between boundaries, a and b plus 1.
We can use floor and ceiling functions to show this.
Lets say we have a number i, we know that $i \leq n \iff \lceil i \rceil \leq n$
Using floor and ceiling commands, the number of integers can be expressed as the floor of b minus ceiling of a and that expression plus 1.
$\lceil a \rceil \leq n \leq \lfloor b \rfloor$
And so number, n, of integers that satisfy are:
$(\lfloor b \rfloor - \lceil a \rceil) + 1$

74: (2 points)
Given that $f : A- > B$ and $|A| = |B|$ we can prove that f is one-to-one if and only if it is onto function.

We can prove by contradiction and to do so assume that f is one-to-one first.
Then, $\exists b \in B$ where:
$\forall a \in A$: $f(a) \neq b$

Because $|A| = |B|$ then two elements in A would have the same image.

$\exists n \in A$, $\exists m \in A : f(n) = f(m)$ where $n \neq m$.

But because f is a one-to-one function then $f(n)$ should equal $f(m)$ and $n = m$

This contradicts the statement that these values are different. So this means that f cannot be a one-to-one and then be an onto

Now let's assume that f is an onto. Since $|A| = |B|$ then there's an element in B that's not in the image of element in A.
$\exists n \in A$ and $\exists m \in A : f(n) = f(m) \wedge n \neq m$.

This means that f is not onto and we have a contradiction. So, f is one-to-one $\iff$ (if and only if) f is onto

Exercises for Section 2.4:

14f: (1 points)
Finding the recurrence relation to:
$a_n = n^2 + n$
we know for some values n:
= $0^2 + 0, 1^2 + 1, 2^2 + 2, 3^3 + ... + n^2 + n$
a_{n-1} would be equal to $(n-1)^2 + n - 1$

18(a-c): (2 points)
a)
$a_n = a_{n-1} + 0.09a_{n-1}$ plus initial amount:
$a_n = (1.09)a_{n-1}$, $a_0 = 1000$
b)
Now we find an explicit formula for the amount in the account at end of n years:
$a_n = (1.09)^n 1000$
c)
Money after 100 years:
$a_1 00 = (1.09)^{100} 1000$
$= \$5529040.8$

22(a-c): (2 points)
a)
$A_n = A_{n-1} + (0.05)A_{n-1} + 1000$, if $n > 2017$
$\$50,000$, if $n = 2017$

b)
Salary in 2025: $A(2025) = A(2024) + (0.05)(A(2024)) + 1000$
$= \$83421.88$

c)
An explicit formula for this for n years after 2017 is:
$1.05^n * 50000$
where n is years after 2017.

24(a-b): (2 points)

a)

The recurrence relation is:

$B(k) = (1 + \frac{r}{12} * B(k-1) - P$

b)

Loan paid off in T months in terms of monthly payment P:

$P = \frac{(\frac{r}{12}+1)^T * B(0) * \frac{r}{12}}{(\frac{r}{12}+1)^T - 1}$

where $B(0)$ is initial amount of loan.

26e: (2 points)

Looking at this sequence, the simple formula is:

$a_{n+1} = 3a + 2$ where $a_1 = 0$

The next three terms:

$a_{11} = 3(19682) + 2 = 59048$

$a_{12} = 3(29048) + 2 = 177146$

$a_{13} = 3(177146) + 2 = 531440$

40: (1 points)

Using the Table 2:

$\sum_{k=99}^{200} k^3 = (\sum_{k=1}^{200} k^3) - (\sum_{k=1}^{98} k^3)$

$= \frac{200^2 * 201^2 - 98^2 * 99^2}{4}$

$= 380477799$