CSCE 222 (Carlisle), Honors Discrete Structures for Computing
Fall 2020
Homework 4

Type your name below the pledge to sign
On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.
Sameer Hussain

Instructions:

- The exercises are from the textbook. You are encouraged to work extra problems to aid in your learning; remember, the solutions to the odd-numbered problems are in the back of the book.
- Grading will be based on correctness, clarity, and whether your solution is of the appropriate length.
- Always justify your answers.
- Don't forget to acknowledge all sources of assistance in the section below, and write up your solutions on your own.
- *Turn in .pdf file to Gradescope by the start of class on Tuesday, September 29, 2020.* It is simpler to put each problem on its own page using the LaTeX clearpage command.

Help Received:

- https://en.wikipedia.org/wiki/Wikipedia:LaTeX_symbolsArrows
- https://www.tutorialspoint.com/discrete_mathematics/discrete_mathematics_sets

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Exercises for Section 2.5:

2(a-f): (6 points)

a)

We are considering the set of integers greater than 10.
The set would be $S = \{11, 12, 13, 14...\}$
This is countably infinite. Function $f : \mathbb{N} \to S$
$f(x) = x + 10$ which is one-to-one correspondence.

b)

We are considering the set of odd negative integers.
$S = \{-(2x - 1)$ where $x \in \mathbb{N}$
This set is countably infinite.
Define $f : \to S$ where $f(x) = 1 - 2x$
There is a one-to-one correspondence between set of positive integers and set S

c)

This is the set of integers with absolute value less than 1,000,000.
This is finite set.

d)

This is the set of real numbers between 0 and 2
It is uncountable

e)

This is the set $A \times \mathbb{Z}^+$ where $A = \{2, 3\}$
This is countably infinite

f)

Now the set of integers that are multiples of 10.
This is countably infinite. We have a function $f(n) = (1 - n) * 10$ (odd numbers) and $f(n) = 5n$ (even numbers) that has a one-to-one correspondence to set of positive integers.

4(a-d): (4 points)

a)

Set of integers not divisible by 3.

In form $3n + 1$ or $3n + 2$

This set is countable and is part of the set of integers. $f(n) = \begin{cases} 3n + 1, \text{if n is even} \\ 3n + 1, \text{if n is odd} \end{cases}$

b)

Set of integers divisible by 5 but not by 7.

It can be mapped to the set of real numbers (0 to 5, 1 to -5, 2 to 10, 3 to -10, etc)

This is a countable set.

c)

Set of real numbers with decimal representations consisting of all 1s

$S = \{n.\overline{1} | n \in Z\}$

Bijection of $Z \to S$

$n = n + 0.\overline{1}$

These integers are countable and the set is countable.

d)

Set of the real numbers with decimal representations of all 1s or 9s.

This set is uncountable.

Can't list all real numbers with decimal representations of 1s and 9s.

6: (2 points)
The hotel closes even number rooms so now we move guests in even number to the next room and odd number guest to next odd number.
Move guest in room $2n$ to room $2n + 1$
Move guest in room $2n + 1$ to room $2n + 3$

8: (3 points)
A countable infinite number of guests arriving at Hilbert's Hotel. The Hotel has infinite rooms. You can shift current members into a different numbered room, for example shift all into odd numbered rooms and then put new guests into even number rooms that are also infinite so everyone can stay at the Hotel.

10(a-c): (3 points)
a)
Finite set: $A = \mathbb{R} = 0$
$B = \mathbb{R} > 0$
A and B are equal to all real numbers. So $A - B = \emptyset$ which is finite

b)
Countably infinite
$A = (0, 1) \cup \mathbf{N}$ all real numbers between 0 and 1 and non-negative numbers $B = (0, 1)$ all real numbers between 0 and 1
$A - B = ((0, 1) \cup N) - (0, 1) = N$ Set N of non-negative integers is countably infinite.

c)
Uncountable
$A = \mathbb{R}$ real numbers and is uncountable
$B = \mathbb{R}^-$ and 0 which is also uncountable.

$$A - B = R - (R^- \cup \{0\}) = R^+$$
The set R^+ of positive real numbers is uncountable.

30: (2 points)

Proof that: the set of real numbers that are solutions of quadratic equations are countable.

The set (a,b,c) is countable and the roots are finite and the pair (b,a) is countable as well as (a,c). So for the quadratic equation with values for a,b,c there are at most two solutions for the countable tuple (a,b,c) so the solution set is also countable.