Test Two Review

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Trees

 doesn't depend on actual values of features – just ordering within each feature

Bagging

- T/F: In bagging, as we increase the number of bootstrap samples, we expect that we will eventually overfit.
 - False.
- With bagging, how can we get an estimate of test perfomance while still using all our data for training?
 - "out-of-bag" error

Random Forest

- T/F: Random forest is **just** bagging with trees.
 - The tree growing algorithm is modified by features selecting randomly to consider for splitting splitting
- T/F: Generating too many trees in a random forest will probably lead to overfitting.
 - False

AdaBoost

- T/F: We can use regression trees as the base classifier for AdaBoost.
 - False. AdaBoost is for hard classifiers. (Well, you can use sign of the output of regression trees.)
- T/F: We can use SVM as a base classifier for AdaBoost.
 - True, if you map the output to $\{-1,1\}$ with a modified sign function.
- T/F: We can view AdaBoost a method for minimizing the exponential loss using forward stagewise additive modeling.
 - True

Gradient Boosting

 Know how to do gradient boosting with a new loss function and a black box regression algorithm.

Multiclass Classification

- Understand the key pieces
 - class sensitive loss function $\Delta(y, y')$
 - feature map: $\Psi: \mathfrak{X} \times \mathfrak{Y} \to \mathbf{R}^d$
 - linear score function $(x,y) \mapsto \langle w, \Psi(x,y) \rangle$, parameterized by $w \in \mathbf{R}^d$
 - final prediction function $x \mapsto \arg\max_{y \in \mathcal{Y}} \langle w, \Psi(x, y) \rangle$
 - loss functions:
 - multiclass hinge / SVM loss
 - multinomial regression loss

Likelihood, MLE, Conditional Likelihood

- go through examples in slides (Poisson regression, Gaussian regression, binomial, multinomial)
- note that you can use these same likelihoods as losses for gradient boosting (take negative log of likelihood)

Conditional Exponential Distribution

- Input: x gives location and time
- Output: y gives waiting time for taxi pickup
- Exponential distributions are a natural candidate:

$$\mathsf{ExpDists} = \left\{ p_{\lambda}(y) = \lambda e^{-\lambda y} \mathbb{1}(y \in [0, \infty)) \mid \lambda \in (0, \infty) \right\}.$$

- For input x, we want to give back λ , the exponential distribution parameter.
- Let's make a generalized linear model.
- So we'll predict $x \mapsto f(w^T x)$ for some x.
- What can we use for f?

Conditional Exponential Distribution

- Taking $w^T x \mapsto \exp(w^T x)$ does the trick. Maps into $(0, \infty)$.
- The likelihood for observation $y \ge 0$ for

$$p_{\lambda}(y) = \lambda e^{-\lambda y}$$

- For input x, predicted parameter is $\lambda = \exp(w^T x)$.
- Likelihood of $y \mid x$ is then

$$p_w(y \mid x) = \exp(w^T x) e^{-\exp(w^T x)y}$$

Conditional Exponential Distribution

• Log-likelihood of $y \mid x$ is then

$$p_{w}(y|x) = \exp(w^{T}x) e^{-\exp(w^{T}x)y}$$

$$\implies \log p_{w}(y|x) = w^{T}x - y \exp(w^{T}x)$$

• Log-likelihood of $(x_1, y_1), \ldots, (x_n, y_n)$ is

$$\sum_{i=1}^{n} \left[w^{T} x_{i} - y_{i} \exp \left(w^{T} x_{i} \right) \right]$$

MLE is then

$$\hat{w}_{\mathsf{MLE}} = \underset{w \in \mathbf{R}^d}{\mathsf{arg\,max}} \sum_{i=1}^n \left[w^T x_i - y_i \exp\left(w^T x_i\right) \right]$$

Conditional Exponential Distribution with GBM?

• For linear version, we take parameter to be $f(w^Tx)$.

$$\log p_w(y|x) = w^T x - y \exp(w^T x)$$

- Replace $w^T x$ by a general function g(x) that we will learn with GBM.
- Log-likelihood objective function is

$$J(g) = \sum_{i=1}^{n} [g(x_i) - y_i \exp(g(x_i))].$$

(we want to maximize it)

• Differentiate w.r.t. $g(x_i)$... etc...