DOUBT SESSIONS

1. Prime Number

The expression : $\frac{1}{X^{a_1}} + \frac{1}{X^{a_2}} + \ldots\ldots + \frac{1}{X^{a_n}} = \frac{s}{t}$

Let $p = (a_1 + a_2 + a_3 + \cdots a_n)$

$$\Rightarrow \frac{s}{t} = \frac{X^{p-a_1} + X^{p-a_2} + X^{p-a_3} \ldots X^{p-a_n}}{X^p}$$

Goal is to find $GCD\left(\sum_{i=1}^{n} X^{p-a_i}, X^p\right)$

$\Rightarrow$ ans will be of the form X^{ans}.

We need to find largest value of ans such that (ΣX^{p-a_i}) is divisible by X^{-}.

Core Idea : X^a is present x times, these terms can be merged to form x^{a+1}

Exa : $X = 2$

$S = 2^3 + 2^4 + 2^8 + 2^{11} + 2^3 = 2^3(1 + 2^1 + 2^5 + 2^8 + 1)$
$= 2^3(2 + 2^1 + 2^5 + 2^8)$
$= 2^3 \cdot 2\,(1 + 1 + 2^4 + 2^7)$
$= 2^3 \cdot 2 \cdot 2\,(1 + 2^3 + 2^6) = 2^5$

From here, we know that the largest power that will divide this expression is 5.

Algo :-

① map <power, freq>

② Find the smallest power;
count the frequency
```
if (freq % X == 0) {
    mp[power+1] += freq/x;
    delete power from the map
}
else {
    return power
}
```

2. Find $(A,B)\ \%\ M$ where $A, B, M <= 10^{18}$ and only use long long data type not int_128.

int $a = 1e6$, $b = 1e6$

$\Rightarrow \underbrace{(a * b)}_{\downarrow} \% (10^9+7) \Rightarrow$ Incorrect

int * int $\Rightarrow$ (int)

$1e6 * 1e6 \Rightarrow 1e^{12}$ (over flow)

So, Binary Form:

$B = 2^{a_1} + 2^{a_2} + 2^{a_3} + \cdots + 2^{a_n}$

$A * B = (A * 2^{a_1} + A * 2^{a_2} + \cdots A * 2^{a_n})$

Taking % mod on both sides,

$\Rightarrow (A * B) \% mod = ((A * 2^{a_1}) \% mod + (A * 2^{a_2}) \% mod + \cdots + (A * 2^{a_n}) \% mod) \% mod$

$\Rightarrow A * 2^{a_1}$

$\hookrightarrow$ If this is 4 $\Rightarrow \underbrace{(A+A+A+A)}_{4 \text{ times}}$

So,

$A * 4 \Rightarrow ((A \times A) \% mod + A) \% mod + A) \% mod$

$A * 8 \Rightarrow (A + \cdots A)$ 8 times

Pseudocode :-

```
while (b>0) {
    if (b&1)
        ans = (ans+a)%mod;
    a = (a+a)%mod;     2a, 4a, 8a, 16a, 32a, ---
    b/=2;
}
```

3. Yet Another Broken Keyboard

Given, s = abacaba

letters = a, b (valid, allowed).

Ex:-

Ex :-

a b a c a b a

|1 1 1| 0 |1 1 1|

For any substring to be valid, it should be continous one

No. of substring $= \frac{2(3 \times 4)}{2} = 12$

s a d f a a s d d a

0 1 1 1 1 1 0 1 1 1

$\underbrace{1\ 1\ 1\ 1\ 1}_{\frac{5 \times 6}{2}} \qquad \underbrace{1\ 1\ 1}_{\frac{3 \times 4}{2}}$

$\downarrow \qquad \downarrow$

$15 + 6 = 21$

4. Beautiful String

Suppose we want to find out if the k-th string will have 'a' in the beginning or not.

→ If fix 'a', then how many strings are psbl that will have a in the beginning.

$\boxed{a}$ .... 2B (4 place)

$\Downarrow$

$^4C_2 = 6$

The strings are sorted in lexicographical order, means strings with 'b's in later posn appears first. For $n=5$, 'aaabb' is first and 'bbaaa' is last.

→ We can determine the posn of first 'b' (from left). The no. of strings that begin with 'a' is the no. of ways to place two 'b's in the remaining $n-1$ posn, which is $^{n-1}C_2$.

5. Delta & Function

Total Subset Psbl $= {}^nC_k$

$n = [1, 2 \ldots n]$, length $= k$

$E = \frac{1}{^nC_k} * \Sigma$ (min Value of each subset)

Using the contribution Technique, the sum of min is :

$\Rightarrow \frac{1 * {}^{n-1}C_{k-1} + 2 * {}^{n-2}C_{k-1} + 3 * {}^{n-3}C_{k-1} + \ldots}{^nC_k}$