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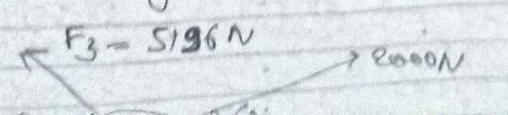
→ the result of four force show in fig. is along y-axis the magnitude force F_1, F_2, F_3, F_4 are 10, 20N, 40N respectively. the angle made by 10°, 20°, 40° with x axis are 30°, 90°, 120° respectively by the magnitude in axis is resultant is 72.

$\Sigma H = 0$ —(i)

$\Sigma V = R$ —(ii) $\Sigma V = 72$ —(ii)

$F_2 = 16.07\,kN$ & $\theta = 47.76°$

(2) The forces a place an active in show in fig. Determine the result & magnitude & direction.



$\lim_{z\to 0} \frac{f(z)-f(0)}{z-0} \Rightarrow \lim_{z\to 0} \frac{|xy|^{1/2}-0}{x+iy}$

Along $y=mx$.

$\lim_{x\to 0} \Rightarrow \frac{|mx^2|^{1/2}}{(1+im)x} = \frac{\sqrt{m}}{1+im}$

Hence, function is not analytic, so the function is not differentiable.

→ Harmonic function:-

A function $f(x,y)$ is called harmonic if it satisfies Laplace eqn. That is f is harmonic if $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 0$.

or $\nabla^2 f = 0$ (in short)

→ Show that the function $u = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ is harmonic

$\Rightarrow \frac{\partial u}{\partial x} \Rightarrow 3x^2 - 3y^2 + 6x$

$\frac{\partial^2 u}{\partial x^2} \Rightarrow 6x + 6$ —(i)

It means both eqn satisfies & function is harmonic.

$\frac{\partial u}{\partial y} \Rightarrow -6xy - 6y$

$\frac{\partial^2 u}{\partial y^2} = -6x + 6$ —(ii)

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$$\lim_{z \to 0} \frac{f(z) - f(0)}{z - 0}$$

$$\lim_{z \to 0} \frac{x^3(1+i) - y^3(1-i)}{(x+iy)(x^2+y^2)}$$

Along $y = mx$. $z \Rightarrow x+iy$

$$\lim_{z \to 0} \frac{(1+i) - m^3(1-i)}{(1+im)(1+m^2)}$$

Since value of limit depends on m, so it is not unique. That is function is not differentiable at the origin. $\therefore$, the given function is not analytic at the origin.

Q → Show that the function $f(z) = |xy|^{1/2}$ is not regular at the origin. Also C.R. equ^n are satisfied there.

Analytic = Holomorphic = Meromorphic = Regular

→ $\left(\frac{\partial u}{\partial x}\right)_{(0,0)} \Rightarrow (0,0)$

$\left(\frac{\partial u}{\partial y}\right)_{(0,0)} = 0, \quad \frac{\partial v}{\partial x} \Rightarrow 0, \quad \frac{\partial v}{\partial y} = 0.$

$f(z) = |xy|^{1/2}$

$u = |xy|^{1/2}, \; v = 0$

It is obivous that C.R. equ^n is satisfie

$$\left(\frac{\partial v}{\partial x}\right)_{(a,b)} = \lim_{h\to 0} \frac{v(a+h,b) - v(a,b)}{h}$$

$$\left(\frac{\partial u}{\partial y}\right)_{(a,b)} = \lim_{k\to 0} \frac{v(a,b+k) - v(a,b)}{k}$$

Q → Prove that for the $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}, z \neq 0$

$f(0) = 0$, C.R. equ^n are satisfied at the origin but function is not analytic there.

→ $u \Rightarrow \frac{x^3 - y^3}{x^2+y^2}$, $v = \frac{x^3+y^3}{x^2+y^2}$

$$\left(\frac{\partial u}{\partial x}\right)_{(0,0)} = \lim_{h\to 0} \frac{u(h,0) - u(0,0)}{h}$$

$$= \lim_{h\to 0} \frac{h-0}{h} = 1.$$

$$\left(\frac{\partial u}{\partial y}\right)_{(0,0)} = \lim_{k\to 0} \frac{u(0,k) - u(0)}{k} = -1$$

$$\left(\frac{\partial v}{\partial x}\right)_{(0,0)} = 1, \quad \left(\frac{\partial v}{\partial y}\right)_{(0,0)} = 1$$

Since $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ & $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

So, the C.R. equ^n is satisfied.

$$\frac{\partial v}{\partial y} = \cos x \cosh y$$

Polynomial, a^x, $\log x$, trignometric function of sin, cos. are every where continous including exponential function.
If 2 functions are continous then there product will also be continous.

Q → $f(z) = e^z$

$$= e^x \cos y + i e^x \sin y$$

$$u = e^x \cos y \qquad v = e^x \sin y$$

$$\frac{\partial u}{\partial x} = e^x \cos y \qquad \frac{\partial v}{\partial x} = e^x \sin y$$

$$\frac{\partial u}{\partial y} = -e^x \sin y \qquad \frac{\partial v}{\partial x} = e^x \cos y$$

C.R. equ^n is satisfied.

→ Partial Derivative at a point :-

$$\left(\frac{\partial u}{\partial x}\right)_{(a,b)} = \lim_{h \to 0} \frac{u(a+h, b) - u(a,b)}{h}$$

$$\left(\frac{\partial u}{\partial x}\right)_{(a,b)} = \lim_{k \to 0} \frac{u(a, b+k) - u(a,b)}{k}$$

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ (for Real)

$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$ (for imaginary part)

⇒ Sufficient condition for function u+iv to analytic:-

A $f(z) = u + iv$ is analytic if and only if

(i) CR equations are satisfied

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

(ii) $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial x} \;\&\; \frac{\partial v}{\partial y}$ are continous functions of x & y.

$$f(z) = \sin z$$
$$= \sin x \cosh y + i \cos x \sinh y.$$

$u = \sin x \cosh y$, $\frac{\partial u}{\partial x} = \cos x \cosh y.$

$v = \cos x \sinh y$ $\frac{\partial u}{\partial y} = \sin x \sin y.$

$$\frac{\partial v}{\partial x} = -\sin x \sinh y$$

Necessary Condition for a function to be analytic :- (Cauchy Reimann Equ^n) (CR Equ^n)

→ Show that for a function $f(z) = u + iv$ to be analytic, the necessary conditions are

$$\frac{\partial u}{\partial x} = \frac{-\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$$ These equ^n

should be satisfied. These equ^n are known as CR equ^n.

Proof:- Let $\omega = f(z) = u + iv$ be analytic

We have $$\frac{d\omega}{dz} = \lim_{\delta z \to 0} \frac{\delta \omega}{\delta z} \quad -(i)$$

Along real axis i.e. $y = 0$.

$$\frac{d\omega}{dz} = \lim_{\delta x \to 0} \frac{\delta \omega}{\delta x} = \frac{\partial \omega}{\partial x} = \frac{\partial u}{\partial x} + \frac{i\partial v}{\partial x} \quad -(ii)$$

Along imaginary axis i.e $x = 0$.

from (i) $$\frac{d\omega}{dz} = \lim_{i\delta y \to 0} \frac{\delta \omega}{i\delta y} = \frac{1}{i} \frac{\partial \omega}{\partial y}$$

$$= -i\left(\frac{\partial u}{\partial y} + \frac{i\partial v}{\partial y}\right) = -i\frac{\partial u}{\partial y} + \frac{\partial v}{\partial y} \quad - (i$$

Analytic Function

A function $f(z)$ is said to be analytic at a point $z = z_0$ if $f(z)$ is differentiable at every point in a neighbourhood of $|z - z_0| < \delta$ (nbd. of z_0) of z_0.

Function $f(z)$ is analytic in a domain D if it is analytic at each point in D except sum finite number of points.
And these exceptions points are called **singular points** or *singularities* of the function $f(z)$.

$$f(z) = \frac{z^2}{z(z-1)}$$ $= 0, 1$ are the singular points

→ $f(z) = |z|^2$ is differentiable at the origin $(z=0)$ but it is not analytic there.

Continuity is the necessary condition for differentiabilty but not sufficient.

$f(x) = |x|$
$f(x) = x \sin(1/x)$ {Continous at 0).

Since, limit depends on the path that is not independent so the function is not differen-tiable.

Q → Discuss the differentiabilty of the function

$$f(z) = \begin{cases} \dfrac{xy^2(x+iy)}{x^2+y^4} & z \neq 0 \\ 0 & z = 0 \end{cases}$$

at $z=0$.

$$\lim_{z\to 0} \frac{f(z)-f(0)}{z-0}$$

$$\lim_{z\to 0} \frac{xy^2}{x^2+y^4}$$

Along $y = mx$

$$\lim_{x\to 0} \frac{m^2x^3}{x^2+m^4x^4}$$

$$= \lim_{x\to 0} \frac{m^2x}{1+m^4x^2} = 0$$

Along $y^2 = x$

$f(z) = e^z$

$z = 0$.

$$= \lim_{z\to 0} \frac{f(z)-f(0)}{z-0}$$

$$= \lim_{z\to 0} \frac{e^z-1}{z} = 1$$

A function $f(z)$ is said to be differentiable at $z = z_0$ if, the limit $\frac{f(z) - f(z_0)}{z - z_0}$ exist in that condition the limit $f'(z_0) = h$.

→ Discuss

$$f(z) = \frac{x^2y^5(x+iy)}{x^4+y^{10}} \quad z \neq 0$$

$$= 0 \quad z = 0$$

$$\lim_{z\to 0} \frac{f(z) - f(0)}{z - 0}$$

$$\lim_{z\to 0} \frac{x^2y^5(x+iy)}{x^4+y^{10}} - 0$$

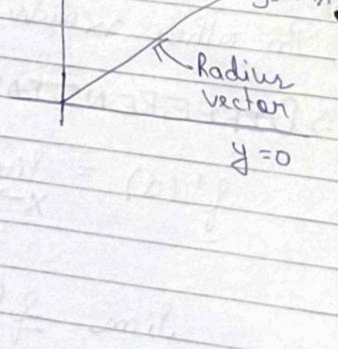
$z = x+iy$

$z = (1+i)x$

$$= \lim_{z\to 0} \frac{x^2y^5}{x^4+y^{10}}$$

Along $y = x$

$$\lim_{x\to 0} \frac{x^7}{x^4+x^{10}}$$



$$\lim_{x\to 0} \frac{x^3}{1+x^6} = 0$$

$$y^5 = x^2$$

$$\frac{x^2 \cdot x^2}{x^4+x^4} \Rightarrow \frac{1}{2}$$

⇒ LIMIT OF COMPLEX FUNCTION :-

Every complex number can be represented in a point of plane.

A limit $f(z)$ $z \to z_0$ is said to be exist if it is independent on the path alon which $z \to z_0$

⇒ CONTINUITY OF COMPLEX FUNCTION:

A function $f(z)$ is said to be continou at $z = z_0$ if,

$$\lim_{z \to z_0} f(z) = f(z_0)$$

In other words $|z - z_0| < \delta \Rightarrow |f(z) - f(z_{0}$

⇒ DIFFERENTIABILTY OF THE COMPLEX

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}.$$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$