DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

DISCREE MATHEMATICS

UNIT -3 : *Theory of Logics*

Lecture - 01

Today's Target

- *Propositions*
- Logical Connectives
- PYQ
- DPP

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -3 : Theory of Logics

- *Propositions*
- *Truth tables*
- *Tautology*
- *Satisfiability*
- *Contradiction*
- *Algebra of Proposition*
- *Theory of inference*
- *Predicate logic*
- *First order Predicate*
- *Well Formed Formula of predicate*
- *Quantifiers*
- *Inference Theory of predicate logic*

Propositions (Statement):

A proposition or statement is a declarative sentence that is either true or false, but not both simultaneously.

- If any proposition is true then its truth value is denoted by T or 1 and if the proposition is false then its truth value is denoted by F or 0.
- Questions, commands, order, exclamations, wish etc. are not proposition
- T , F are called truth value
- T , F are also called propositional constant

Following are the propositions

Propositions	Truth Value
(1) Two Plus Two is Four	T
(2) Two Plus Two is Six	F
(3) Paris is in India	F
(4) $x = 2$ is a solution of $x^2 = 4$	T
(5) The Sun rises in the west	F
(6) $6 < 7$	T
(7) $3 > 5$	F

Following are not propositions

(1) Where are you going ? (Question)

(2) Close the door (Command)

(3) What a hot day ! (Exclamations)

(4) May god held you (Wish)

(5) $4 - x = 8$

Propositions Variables

A proposition or statement is denoted by the letters p, q, r ….. called propositional variables.

Example

$p =$ Two plus Two is four

$q =$ Delhi is the capital of India.

Q.1. Which of the following sentences are propositions. What are the truth values of those that are proposition?

(a) Is this true ?

(b) Ram is a name

(c) Please submit your proposal as soon as possible.

(d) Four is even.

(e) $5 \in \{1, 6, 7\}$.

(f) What a hit !

(g) Answer this question

(h) $5 + 6 = 12$.

(i) Buy two cinema tickets for Friday.

(j) May God bless you.

Answer:

(a) Not a proposition
(b) Proposition (T)
(c) Not a proposition
(d) Proposition (T)
(e) Proposition (F)
(f) Not a proposition
(g) Not a proposition
(h)Proposition (F)
(i) Not a proposition
(j) Not a proposition

Proposition

Simple Proposition	Compound Proposition
OR	OR
Proposition	Molecular Proposition
OR	OR
Atomic Proposition	Composite Proposition

Simple Proposition :

A proposition consisting of only a single propositional Variable or single propositional constant is called an simple proposition or proposition

Example

1. The sun rises in the west

   $p =$ The sun rises in the west

2. Delhi is in India

   $q =$ Delhi is in India

Compound Proposition

A proposition obtained from the combinations of two or more propositions by means of logical connectives or negating a single proposition.

Example

p q

1. John is intelligent or studies every night

$p \vee q$ = John is intelligent or studies every night

p q

2. The earth is round and revolves around the sun

$p \wedge q$ = The earth is round and revolves around the sun

Logical Connectives or Logical operators

The words or symbols used to form compound propositions are called logical connectives.

There are five basic logical connectives called negation, conjunction, Disjunction, conditional and bi conditional.

Table of Logical connectives with their symbols

S.No.	Name	Symbol	Connective Word
1	Negation	$\sim$ or $\neg$	Not
2	Conjunction	$\wedge$	And
3	Disjunction	$\vee$	Or
4	Conditional	$\rightarrow$ or $\Rightarrow$	If …… then
5	Bi-conditional	$\leftrightarrow$ or $\Leftrightarrow$	Iff or if and only if

Q.2. Consider *the following*

p : *This computer is good*
q : *This computer is cheap*

Write each of the following statements in symbolic forms

$p \quad \wedge \quad q$

(a) *This computer is good and cheap.*

$\sim p \quad \wedge \quad q$

(b) *This computer is not good but cheap.*

$\sim q \quad \wedge \quad p$

(c) *This computer is costly but good.*

(d) *This computer is neither good nor cheap.*

$\sim p \wedge \sim q$

$p \quad q$

(e) *This computer is good or cheap.*

Answer:

(a) $p \wedge q$

(b) $(\sim p) \wedge q$

(c) $(\sim q) \wedge p$

(d) $(\sim p) \wedge (\sim q)$

(e) $p \vee q$

Q.3. *If* $p \equiv$ *It is 4 o'clock*, $q \equiv$ *the train is late, then state in words the following results:*

(a) $p \vee q$ (b) $p \wedge q$ (c) $p \wedge (\sim q)$

(d) $q \vee \sim p$ (e) $(\sim p) \wedge q$ (f) $(\sim p) \wedge (\sim q)$

(g) $(\sim p) \vee (\sim q)$ (h) $\sim (p \wedge q)$ (i) $\sim p \Rightarrow q$

Answer:-

(a) It is 4 O'clock, or the train is late. (b) It is 4 O'clock, and the train is late.

(c) It is 4 O'clock, and the train is not late. **Or** It is 4 O'clock, but the train is not late.

(d) The train is late or it is not 4 O'clock. (e) It is not 4 O'clock, and the train is late.

(f) It is not 4 O'clock, and the train is not late **Or** Neither It is 4 O'clock, nor the train is late.

(g) It is not 4 O'clock, or the train is not late **Or** Either It is not 4 O'clock, or the train is not late.

(h) It is not true that it is 4 O'clock, and the train is late.

(i) If it is not 4 O'clock, then the train is late.

Q.4. *Let p = It is cold, q = It is raining. Give a simple verbal sentence which describes each of the following statements:*

(a) $\sim p$ (b) $p \wedge q$ (c) $p \vee q$

(d) $q \Leftrightarrow p$ (e) $p \Rightarrow \sim q$ (f) $q \vee \sim p$

(g) $\sim p \wedge \sim q$ (h) $p \Leftrightarrow \sim q$ (i) $\sim(\sim q)$

(j) $(p \wedge \sim q) \Rightarrow p$ (k) $\sim(\sim p)$ (l) $(p \wedge \sim q) \Rightarrow q$

Answer:-

(a) It is not cold.

(b) It is cold and raining.

(c) It is cold or it is raining.

(d) It is raining if and only if it is cold.

(e) If It is cold, then it is not raining.

(f) It is raining or it is not cold.

(g) ~~If~~ It is not cold and it is not raining.

(h) It is cold if and only if it is not raining.

(i) It is not true that it is not raining.

(j) If It is cold and not raining, then it is cold.

(k) it is not true that it is not cold.

(l) If is raining and not cold, then it is raining.

DPP- 1

Topic : *Propositions and* Logical Connectives

Q.1. Consider *the following*

p : *It is cold day*
q : *The Temperature is* 5°C

Write in simple sentences the meaning of the following

(a) $\sim q$ (b) $p \vee q$ (c) $p \wedge q$ (d) $\sim (p \vee q)$

(e) $\sim (p \wedge q)$ (f) $\sim p \wedge \sim q$ (g) $\sim (\sim p \vee \sim q)$

Answer:-

(a) *It is not a cold day*

(b) *It is a cold day or temperature is* 5°C

(c) *It is a cold day and temperature is* 5°C

(d) *It is false that it is cold day or temperature is* 5°C

(e) *It is not true that it is cold day and temperature is* 5°C

(f) *It is neither a cold day nor temperature is* 5°C

(g) *It is false that it is not a cold day or temperature is not* 5°C

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UNIT -3 : *THEORY OF LOGICS*

Lecture - 02

Today's Target

- **Truth Table**
- **Negation** ($\sim$ *or* $\neg$)
- **Conjunction** ($\wedge$)
- **Disjunction** ($\vee$)
- PYQ
- DPP

Logical Connectives or Logical operators

The words or symbols used to form compound Statement (or propositions) are called logical connectives.

Table of Logical connectives with their symbols

S.No.	Name	Symbol	Connective Word
1	Negation	$\sim or \neg$	Not
2	Conjunction	$\wedge$	And
3	Disjunction	$\vee$	Or
4	Conditional	$\rightarrow or \Rightarrow$	If …… then
5	Bi-conditional	$\leftrightarrow or \Leftrightarrow$	Iff or if and only if

Truth Table

A truth table is a table that shows the truth value of a compound Statement (or proposition) for all possible cases.

(1) Negation ($\sim$ *or* $\neg$)

If p is any proposition, the negation of p, denoted by $\sim p$ and read as not p, is a proposition which is false when p is true and true when p is false.

Truth table for negation

p	$\sim p$
T	F
F	T

Example: What is the negation of each of the following propositions ?

(i) $p:$ *Paris is in france.*

(ii) $q:$ *All students are intelligent.*

(iii) $r:$ *Today is tuesday.*

(iv) p : Ramesh is a good player of Hockey

Solution

(i) $\sim p:$ *Paris is not in france.*

(ii) $\sim q:$ *Some students are not intelligent.*

(iii) $\sim r:$ *Today is not tuesday.*

(iv) $\sim p$: Ramesh is not a good player of Hockey

(2) Conjunction ($\wedge$)

If p and q are two statements, then conjunction of p and q is the compound statement denoted by $p \wedge q$ and read as "p and q"

The compound statement $p \wedge q$ is true when both p and q are true otherwise it is false.

Truth table for Conjunction

T T $\longrightarrow$ T otherwise F

(i) When compound statement have 2 components p and q

No. of rows $= 2^2 = 4.$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(ii) When compound statement have 3 components p , q and r

No. of rows $= 2^3 = 8$.

p	q	r	$p \wedge q$	$p \wedge q \wedge r$
T	T	T	T	T
T	T	F	T	F
T	F	T	F	F
T	F	F	F	F
F	T	T	F	F
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

Example : From the Conjunction of p and q for each of the following.

(i) p : *Ram is healthy* q : *He has blue eyes*

(ii) p : *It is cold* q : *It is raining*

(iii) $p : 5x + 6 = 26$ $q : x > 3$

Solution

(i) $p \wedge q$: *Ram is healthy and he has blue eyes.*

(ii) $p \wedge q$: *It is cold and raining.*

(iii) $p \wedge q : 5x + 6 = 26$ and $x > 3$.

(3) Disjunction ($\vee$)

If p and q is are two statement, then disjunction of p and q is the compound statement denoted by $p \vee q$ and read as "p or q"

The compound statement $p \vee q$ is false when both p and q are false otherwise it is true

Truth table for Disjunction

F F $\longrightarrow$ F otherwise T

(i) When compound statement have 2 components p *and* q

No. of rows $= 2^2 = 4$.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

(ii) When compound statement have 3 components p, q, r

No. of rows $= 2^3 = 8$.

p	q	r	$p \vee q$	$p \vee q \vee r$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

Example : From the Disjunction of p and q for each of the following.

(i) p : *It is cold* q : *It is raining*

(ii) p : *He will go to Delhi* q : *He will go to Kolkata*

Solution

(i) $p \vee q$: *It is cold or raining.*

(ii) $p \vee q$: *He will go to Delhi or kolkata.*

Q.1: Find the truth table for the statement $(q \wedge r) \wedge (p \vee \sim r)$

p	q	r	$\sim r$	$q \wedge r$	$p \vee \sim r$	$(q \wedge r) \wedge (p \vee \sim r)$
T	T	T	F	T	T	T
T	T	F	T	F	T	F
T	F	T	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	T	F	T	F	T	F
F	F	T	F	F	F	F
F	F	F	T	F	T	F

Q.2:- Find the truth table of the compound proposition $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (q \wedge r)$ $= A$

p	q	r	$\sim p$	$\sim q$	$(\sim q \wedge r)$	$(\sim p \wedge (\sim q \wedge r))$	$q \wedge r$	$p \wedge r$	A
T	T	T	F	F	F	F	T	T	T
T	T	F	F	F	F	F	F	F	F
T	F	T	F	T	T	F	F	T	T
T	F	F	F	T	F	F	F	F	F
F	T	T	T	F	F	F	T	F	T
F	T	F	T	F	F	F	F	F	F
F	F	T	T	T	T	T	F	F	T
F	F	F	T	T	F	F	F	F	F

Q.3:- Find the truth table of the statement $(p \wedge q) \vee (\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q) = A$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim p \wedge q$	$p \wedge \sim q$	$\sim p \wedge \sim q$	A
T	T	F	F	T	F	F	F	T
T	F	F	T	F	F	T	F	T
F	T	T	F	F	T	F	F	T
F	F	T	T	F	F	F	T	T

DPP- 1

Topic : Truth table for compound statement

Q.1. Make a truth table for the statement $(p \wedge q) \vee (\sim p)$

Q.2. Construct a truth table for the compound proposition $p^\wedge(\sim q \vee q)$

Q.3. Construct a truth table for the compound Statement $\sim(p \vee q) \vee (\sim p^\wedge \sim q)$

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

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UNIT -3 : *THEORY OF LOGICS*

Lecture - 03

Today's Target

- **Conditional ($\rightarrow$ *or* $\Rightarrow$)**
- Bi-conditional ($\leftrightarrow$ ***or*** $\Leftrightarrow$)
- PYQ
- DPP

Logical Connectives or Logical operators

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Table of Logical connectives with their symbols

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4	Conditional	$\rightarrow$ or $\Rightarrow$	If …… then
5	Bi-conditional	$\leftrightarrow$ or $\Leftrightarrow$	Iff or if and only if

(4) Conditional statement or Direct implication or implication ($\Rightarrow$ *or* $\rightarrow$)

If p *and* q are statements, then the compound statement "if p *then* q ", denoted by $p \rightarrow q$ or $p \Rightarrow q$ is called conditional statement   T   F $\longrightarrow$ F   otherwise T

The compound statement $p \rightarrow q$ is false when p is true and q is false otherwise it is true.

$p \rightarrow q$

Antecedent or hypothesis

Consequent or Conclusion

Truth Table

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Example :

(i) p : *Tomorrow is sunday*

q : *Today is saturday*

$p \to q$: If tomorrow is Sunday then today is Saturday.

(ii) p : *It rains.*

q : *I will carry an umbrella*

$p \to q$: If it rains then I will carry an umbrella .

Kinds of conditional

If $p \rightarrow q$ is a conditional statement or Direct implication or implication.

(1) *Converse Implication* : $q \longrightarrow p$

(2) *Inverse Implication* : $\sim p \longrightarrow \sim q$

(3) *Contrapositive Implication* : $\sim q \longrightarrow \sim p$

Truth Table for kinds of conditional

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$q \rightarrow p$	$\sim p \rightarrow \sim q$	$\sim q \rightarrow \sim p$
T	T	F	F	T	T	T	T
T	F	F	T	F	T	T	F
F	T	T	F	T	F	F	T
F	F	T	T	T	T	T	T

Example :

p : *It rains*

q : *The crops will grow*

$p \to q$: If it rains then the crops will grow .

$q \to p$: If the crops grow then there has been rain.

$\sim p \to \sim q$: If it does not rain then the crop will not grow.

$\sim q \to \sim p$: If the crops do not grow then there has been no rain rains.

(5) Bi-conditional Statement ($\leftrightarrow$ *or* $\Leftrightarrow$)

If p *and* q are statements, then the compound statement 'p if and only if q', denoted by $p \leftrightarrow q$ is called a bi-conditional statement .

The compound statement $p \leftrightarrow q$ is true when both components have save values.

T T → T

F F → T

Truth Table

p	q	$p \longleftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Example :

$(i)\ p : Sales\ of\ houses\ fall$

$q : Interes\ rate\ rises$

$p \leftrightarrow q :$ Sales of houses fall if and only if interest rate rises .

$(ii)\ p : 7 > 5$

$q : 7 - 5 > 0$

$p \leftrightarrow q : 7 > 5\ iff\ 7 - 5 > 0$

SUMMARY

Negation (NOT)	$\sim$	**T** $\longrightarrow$ **F** **F** $\longrightarrow$ **T**		
Conjunction (AND)	$\wedge$	T T $\longrightarrow$ T	Otherwise	F
Disjunction (OR)	$\vee$	F F $\longrightarrow$ F	Otherwise	T
Conditional (IF....THEN)	$\longrightarrow$ $\Rightarrow$	T F $\longrightarrow$ F	Otherwise	T
Bi-conditional (IFF)	$\leftrightarrow$ $\Leftrightarrow$	T T $\longrightarrow$ T F F $\longrightarrow$ T	Otherwise	F

Q.1 :- Construct a truth table for the following statement $(p \vee q \Rightarrow r) \Leftrightarrow \big((p \Rightarrow r) \vee (q \Rightarrow r)\big) = A$ **(2015, 13)**

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p	q	r	$p \vee q$	$(p \vee q \Rightarrow r)$	$p \Rightarrow r$	$q \Rightarrow r$	$(p \Rightarrow r) \vee (q \Rightarrow r)$	A
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	T	F
F	T	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T	F
F	F	T	F	T	T	T	T	T
F	F	F	F	T	T	T	T	T

Q.2 :- Find the truth table for the statement $(p \Leftrightarrow q \wedge r) \Rightarrow (\sim r \Rightarrow \sim p) = A$ (2006, 13)

p	q	r	$\sim p$	$\sim r$	$q \wedge r$	$(p \Leftrightarrow q \wedge r)$	$(\sim r \Rightarrow \sim p)$	A
T	T	T	F	F	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	F	F	F	F	T	T
T	F	F	F	T	F	F	F	T
F	T	T	T	F	T	F	T	T
F	T	F	T	T	F	T	T	T
F	F	T	T	F	F	T	T	T
F	F	F	T	T	F	T	T	T

Q.3 :- Given that the value of $p \leftrightarrow q$ is true. Can you determine the value of $\sim p \vee (p \leftrightarrow q)$? (2007)

p	q	$p \leftrightarrow q$ ✓	$\sim p$ ✓	$\sim p \vee (p \leftrightarrow q)$
T	T	$\boxed{T}$	F	$\boxed{T}$
T	F	F	F	F
F	T	F	T	T
F	F	$\boxed{T}$	T	$\boxed{T}$

When $p \leftrightarrow q$ is true then $\sim p \vee (p \leftrightarrow q)$ is true

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Q.4 :- Given that the value of $p \longrightarrow q$ is false, determine the value of $(\sim p \vee \sim q) \Rightarrow q = A$ (2009)

p	q	$p \to q$	$\sim p$	$\sim q$	$(\sim p \vee \sim q)$	A
T	T	T	F	F	F	T
T	F	$\boxed{F}$	F	T	T	$\boxed{F}$
F	T	T	T	F	T	T
F	F	T	T	T	T	F

Value of $(\sim p \vee \sim q) \Rightarrow q$ is False

Discrete Mathematics by Gulshan Sir

DPP- 3

Topic : Truth table for compound statement

Q.1. Construct a truth table for the following statement : $(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$ [AKTU]

Q.2 Prepare a table for the following statement : $(p \Leftrightarrow q) \wedge (r \vee q)$

Q.3 Prepare a table for the following statement : $\{(p \vee q) \wedge r\} \Rightarrow q)$ [AKTU]

Q.4 Construct the truth tables for the following : $(\sim p \longrightarrow r) \longrightarrow ((p \longrightarrow q) \longrightarrow (p \longrightarrow r))$

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)
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UNIT -3 : *THEORY OF LOGICS*

Lecture - 04

Today's Target

- Tautology
- Contradiction
- Contingency
- Satisfiability
- PYQ
- DPP

SUMMARY

Negation (NOT)	$\sim$	T F		F T		
Conjunction (AND)	$\wedge$	T	T	T	Otherwise	F
Disjunction (OR)	$\vee$	F	F	F	Otherwise	T
Conditional (IF....THEN)	$\longrightarrow$ or $\Rightarrow$	T	F	F	Otherwise	T
Bi-conditional (IFF)	$\leftrightarrow$ or $\Leftrightarrow$	T F	T F	T T	Otherwise	F

Tautology:

A compound Statement (or proposition) which contain only T for all cases in the last column of its truth table is called a Tautology.

A tautology is also called Logically valid or Logically true statement

Contradiction:

A compound Statement (or proposition) which contain only F for all cases in the last column of its truth table is called a contradiction.

A contradiction is also called Logically invalid or Logically false statement

Contingency:

A Statement (or proposition) that is neither a Tautology nor a contradiction is called a contingency.

A contingency is also called Logically valid or Logically true statement

Note - Last Colum contain both T and F

Satisfiability

A compound statement is satisfiability if there is at least one true result in its truth table

Statement

- Tautology (Always true)
- Contingency (Either true or False)
- Contradiction (Always False)

Tautology, Contingency → Satisfiable → Logically valid

Contradiction → Unsatisfiable → Logically in valid

Q.1 Show that the given statement is a tautology: $((P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R)) \rightarrow R$

Let $(P \vee Q) \wedge (P \rightarrow R) = A$ and $((P \vee Q) \wedge (P \rightarrow R) \wedge (Q \rightarrow R)) = B$

P	Q	R	$P \vee Q$	$P \rightarrow R$	A	$Q \rightarrow R$	B	$B \rightarrow R$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	F	F	T	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	F	F	T
F	F	T	F	T	F	T	F	T
F	F	F	F	T	F	T	F	T

Since, all entries in the last column are T, then the given statement is a Tautology

Q.2 Prove that the statement $(p \vee q) \wedge (\sim p) \wedge (\sim q)$ is a contradiction.

Let $(p \vee q) \wedge (\sim p) = A$

p	q	$\sim p$	$\sim q$	$p \vee q$	A	$A \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	T	F
F	F	T	T	F	F	F

Since, all the entries in the last column of truth table are F only

$\Rightarrow$ The given statement is a contradiction

Q.3 Prove that the statement $(p \Rightarrow \sim q) \Leftrightarrow (q \Rightarrow p)$ is a contingency

p	q	$\sim q$	$p \Rightarrow \sim q$	$q \Rightarrow p$	$(p \Rightarrow \sim q) \Leftrightarrow (q \Rightarrow p)$
T	T	F	F	T	F
T	F	T	T	T	T
F	T	F	T	F	F
F	F	T	T	T	T

The given statement is neither a Tautology nor a contradiction

$\Rightarrow$ It is a contingency

Q.4 Show that the given formula is a tautology:

Let $\sim p \wedge (\sim q \vee \sim r) = A$

$$\{(p \vee q) \wedge \sim(\sim p \wedge (\sim q \vee \sim r))\} \vee (\sim p \wedge \sim q) \vee (\sim p \wedge \sim r)$$ **(UPTU-2007)**

Let $\{(p \vee q) \wedge \sim A\} = B$, Let $(\sim p \wedge \sim q) = C$ Let $(\sim p \wedge \sim r) = D$

p	q	r	$\sim p$	$\sim q$	$\sim r$	$p \vee q$	$\sim q \vee \sim r$	A	$\sim A$	B	C	$B \vee C$	D	$B \vee C \vee D$
T	T	T	F	F	F	T	F	F	T	T	F	T	F	T
T	T	F	F	F	T	T	T	F	T	T	F	T	F	T
T	F	T	F	T	F	T	T	F	T	T	F	T	F	T
T	F	F	F	T	T	T	T	F	T	T	F	T	F	T
F	T	T	T	F	F	T	F	F	T	T	F	T	F	T
F	T	F	T	F	T	T	T	T	F	F	F	F	T	T
F	F	T	T	T	F	F	T	T	F	F	T	T	F	T
F	F	F	T	T	T	F	T	T	F	F	T	T	T	T

Q.5 Prove that the given statement is a tautology: $(p \Rightarrow q) \vee r \Leftrightarrow [(p \vee r) \Rightarrow (q \vee r)]$

Let $(p \Rightarrow q) \vee r = A$ Let $(p \vee r) \Rightarrow (q \vee r) = B$

OR

Prove that the truth value of the following are independent of their components : $(p \Rightarrow q) \vee r \Leftrightarrow [(p \vee r) \Rightarrow (q \vee r)]$

p	q	r	$p \Rightarrow q$	A	$p \vee r$	$q \vee r$	B	$A \Leftrightarrow B$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	F	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	F	F	T	T

$\Rightarrow$ Tautology

Hence, the truth values of the given statement are independent of their components

DPP- 4

Topic : Truth table for compound statement

Q.1 Prove that the given statement is a tautology: $((p \longrightarrow q) \wedge (q \longrightarrow r)) \longrightarrow (p \longrightarrow r)$

OR

Prove that the truth value of the following are independent of their components : $((p \longrightarrow q) \wedge (q \longrightarrow r) \longrightarrow (p \longrightarrow r)$

Q.2 *Prove that* $(p \Leftrightarrow q) \wedge (q \Leftrightarrow r) \Rightarrow (p \Leftrightarrow r)$ *is a tautology.* (UPTU-2013)

Q.3 Prove that $[(p \wedge r) \vee (q \wedge \sim r)] \Leftrightarrow [(\sim p \wedge r) \vee (\sim q \wedge \sim r)]$ is a contradiction.

Q.4 Determine whether the following statements are tautology, contradiction or statisfiable:

(i) $(p \longrightarrow q) \wedge (q \longrightarrow r) \wedge (p \wedge \sim r)$ (UPTU-2016)

(ii) $(p \vee \sim q) \wedge (\sim p \wedge \sim q) \vee q$ (UPTU-2017)

(iii) $[p \wedge (p \longrightarrow q)] \longrightarrow \sim q$

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

DISCREE MATHEMATICS

UNIT -3 : *THEORY OF LOGICS*

Lecture - 05

Today's Target

- **Logical Equivalence**
- PYQ
- DPP

SUMMARY

Negation (NOT)	$\sim$	T F		F T		
Conjunction (AND)	$\wedge$	T	T	T	Otherwise	F
Disjunction (OR)	$\vee$	F	F	F	Otherwise	T
Conditional (IF....THEN)	$\rightarrow$ or $\Rightarrow$	T	F	F	Otherwise	T
Bi-conditional (IFF)	$\leftrightarrow$ or $\Leftrightarrow$	T F	T F	T T	Otherwise	F

Kinds of conditional

If $p \to q$ is a conditional statement or Direct implication.

(1) *Converse Implication* : $q \longrightarrow p$

(2) *Inverse Implication* : $\sim p \longrightarrow \sim q$

(3) *Contrapositive Implication* : $\sim q \longrightarrow \sim p$

Logical Equivalence

Two statement (or proposition) are called logically equivalent if the truth values of both the statements (or proposition) are always identical.

If two statement $P and\ Q$ are logically equivalent, then these are represented by

$$\boxed{P \equiv Q}$$

Note: The necessary and sufficient condition for two statements P and Q to be logically equivalent is that

$P \Leftrightarrow Q$ is a Tautology.

Q.1 Show that $\sim(p \Rightarrow q) \equiv \{p \wedge (\sim q)\}$

Let $P = \sim(p \Rightarrow q)$

$Q = p \wedge (\sim q)$

To Prove

$P \equiv Q$

p	q	$\sim q$	$p \Rightarrow q$	P	Q
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

since truth value of P and Q are identical

$\Rightarrow P \equiv Q$

Q.2 Show that the statement $p \rightarrow (q \vee r)$ *and* $[(p \rightarrow q) \vee (p \rightarrow r)]$ *are equivalent.*

GW GATEWAY CLASSES

Let $P = p \rightarrow (q \vee r)$ and $Q = [(p \rightarrow q) \vee (p \rightarrow r)]$

To Prove

$P \equiv Q$

p	q	r	$q \vee r$	P	$p \rightarrow q$	$p \rightarrow r$	Q	$P \Leftrightarrow Q$
T	T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T	T
T	F	T	T	T	F	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T

since $P \Leftrightarrow Q$ is a Tautology

$\Rightarrow P \equiv Q$

Hence proved

Q.3 If p & q are two statements, show that the implication $p \Rightarrow q$ and its contra-positive $(\sim q) \Rightarrow (\sim p)$ are logically equivalent. **[U.P.T.U 2015]**

To Prove

$$p \Rightarrow q \equiv (\sim q) \Rightarrow (\sim p)$$

p	q	$\sim p$	$\sim q$	$p \Rightarrow q$	$(\sim q) \Rightarrow (\sim p)$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

Since, truth values of both statement are idendical

$\therefore$ $\boxed{p \Rightarrow q \equiv (\sim q) \Rightarrow (\sim p)}$

DPP- 1

Topic : Truth table for compound statement

Q.1. Show that $(p \vee q) \Longrightarrow (p \wedge q) \equiv p \Leftrightarrow q$ (UPTU-2019)

Q.2 Show that $q \Rightarrow p$ converse of $p \Rightarrow q$ and its inverse $(\sim p) \Rightarrow (\sim q)$ are logically equivalent.

UNIT -3 : *THEORY OF LOGICS*

Lecture - 06

Today's Target

- Laws of Proposition / Algebra of Proposition
- PYQ
- DPP

SUMMARY

Negation (NOT)	$\sim$	**T** **F**		**F** **T**		
Conjunction (AND)	$\wedge$	T	T	T	Otherwise	F
Disjunction (OR)	$\vee$	F	F	F	Otherwise	T
Conditional (IF....THEN)	$\longrightarrow$ or $\Rightarrow$	T	F	F	Otherwise	T
Bi-conditional (IFF)	$\longleftrightarrow$ or $\Leftrightarrow$	T F	T F	T T	Otherwise	F

Tautology:

A compound Statement (or proposition) which contain only T for all cases in the last column of its truth table is called a Tautology.

A tautology is also called Logically valid or Logically true statement

Contradiction:

A compound Statement (or proposition) which contain only F for all cases in the last column of its truth table is called a contradiction.

A contradiction is also called Logically invalid or Logically false statement

Logical Equivalence

Two statement (or proposition) are called logically equivalent if the truth values of both the statements (or proposition) are always identical.

If two statement $P and\ Q$ are logically equivalent, then these are represented by

$$\boxed{P \equiv Q}$$

P ≡ Q

Note: The necessary and sufficient condition for two statements P and Q to be logically equivalent is that

$P \Leftrightarrow Q$ is a Tautology.

Laws of Proposition / Algebra of Proposition

1. Idempotent Law
2. Commutative Law
3. Associative Law
4. Distributive Law
5. De-Morgan's Law
6. Identity Law
7. Complement Law
8. Absorption Law

$A \cap A = A$

$A \cup A = A$

1. Idempotent Law

$(i)\ p \wedge p = p$

p	p	$p \wedge p$
T	T	T
F	F	F

Since column under p and $p \wedge p$ are identical

$\therefore \boxed{p \wedge p = p}$

$(ii)\ p \vee p = p$

p	p	$p \vee p$
T	T	T
F	F	F

2. Commutative Law

(i) $p \wedge q = q \wedge p$

$A \cap B = B \cap A$

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

since column under $p \wedge q$ and $q \wedge p$ are identical

(ii) $p \vee q = q \vee p$

$A \cup B = B \cup A$

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

3. Associative Law

$(A \cup B) \cup C = A \cup (B \cup C)$

(2005 & 2010)

(i) $(p \vee q) \vee r = p \vee (q \vee r)$

p	q	r	$(p \vee q)$	$(q \vee r)$	$(p \vee q) \vee r$ ✓	$p \vee (q \vee r)$ ✓
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

(ii) $(p \wedge q) \wedge r = p \wedge (q \wedge r)$

4. Distributive Law

$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(i) $p \vee (q \wedge r) = (p \vee q) \wedge (p \vee r)$

p	q	r	$(q \wedge r)$	$(p \vee q)$	$(p \vee r)$	$p \vee (q \wedge r)$	$(p \vee q) \wedge (p \vee r)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

(ii) $p \wedge (q \vee r) = (p \wedge q) \vee (P \wedge r)$

(5) - De-Morgan's Law

$(A \cap B)^c = A^c \cup B^c$

(i) $\sim(p \wedge q) = (\sim p) \vee (\sim q)$

p	q	$(p \wedge q)$	$\sim p$	$\sim q$	$\sim(p \wedge q)$	$(\sim p) \vee (\sim q)$
T	T	T	F	F	F	F
T	F	F	F	T	T	T
F	T	F	T	F	T	T
F	F	F	T	T	T	T

(ii) $\sim(p \vee q) = (\sim p) \wedge (\sim q)$

$(A \cup B)^c = A^c \cap B^c$

6. Identity Law (i) $A \cap U = A$ (ii) $A \cup \phi = A$

(i) The identity element for conjunction is Tautology (t)

$$p \wedge t = t \wedge p = p$$

p	t	$p \wedge t$	$t \wedge p$
T	T	T	T
F	T	F	F

(ii) The identity element for disjunction is Contradiction (f)

$$p \vee f = f \vee p = p$$

p	f	$p \vee f$	$f \vee p$
T	F	T	T
F	F	F	F

$A \cup A^c = U$ $\qquad$ $A \cap A^c = \phi$

7. Complement Law

For every statement p there exist its negation$(\sim p)$ such that

(i) $p \vee (\sim p) = t$ $\qquad$ (ii) $p \wedge (\sim p) = f$

p	$\sim p$	$p \vee (\sim p)$	t
T	F	T	T
F	T	T	T

p	$\sim p$	$p \wedge (\sim p)$	f
T	F	F	F
F	T	F	F

8. Absorption Law

$A \cup (A \cap B) = A$

(i) $p \vee (p \wedge q) = p$

p	q	$p \wedge q$	$p \vee (p \wedge q)$
T	T	T	T
T	F	F	T
F	T	F	F
F	F	F	F

(ii) $p \wedge (p \vee q) = p$

DPP- 6

Topic : Laws of Proposition / Algebra of Proposition

Q.1 Explain all the laws of Proposition

1. Idempotent Law
2. Commutative Law
3. Associative Law
4. Distributive Law
5. De-Morgan's Law
6. Identity Law
7. Complement Law
8. Absorption Law

UNIT -3 : *THEORY OF LOGICS*

Lecture - 07

Today's Target

- **Theory of inference** (Part-1)
- PYQ
- DPP

SUMMARY

Negation (NOT)	$\sim$	**T** **F**		**F** **T**		
Conjunction (AND)	$\wedge$	T	T	T	Otherwise	F
Disjunction (OR)	$\vee$	F	F	F	Otherwise	T
Conditional (IF....THEN)	$\longrightarrow$ or $\Rightarrow$	T	F	F	Otherwise	T
Bi-conditional (IFF)	$\longleftrightarrow$ or $\Leftrightarrow$	T F	T F	T T	Otherwise	F

Kinds of conditional

If $p \to q$ is a conditional statement or Direct implication.

(1) *Converse Implication* : $q \to p$

(2) *Inverse Implication* : $\sim p \to \sim q$

(3) *Contrapositive Implication* : $\sim q \to \sim p$

Laws of Proposition

1. Idempotent Law

(i) $p \wedge p = p$

(ii) $p \vee p = p$

2. Commutative Law

(i) $p \wedge q = q \wedge p$

(ii) $p \vee q = q \vee p$

5. De-Morgan's Law

(i) $\sim (p \wedge q) = (\sim p) \vee (\sim q)$

(ii) $\sim (p \vee q) = (\sim p) \wedge (\sim q)$

Argument

- An argument is a process by which a conclusion is drawn from a given set of porpositions
- the given set of porpositions are called premises.
- The final proposition derived from given proposition is called conclusion.

An Argument which yield a conclusion c f~~o~~rom the premises $p_1, p_2, p_3 \dots \dots, p_n$ denoted by

$$\boxed{p_1, p_2, p_3 \dots \dots \dots, p_n \vdash C}$$

Valid Argument

- An argument $p_1, p_2, \dots, p_n \vdash C$ is said to be Valid argument if the conclusion is true whenever all the premisesare true.
- An argument $p_1, p_2, p_3 \dots \dots \dots, p_n \vdash C$ is said to be Valid if and only if the statement $\boxed{(p_1 \wedge p_2 \wedge \cdots \wedge p_n) \rightarrow C}$ is tautology.
- Any conclusion which is arrived by following rules of inferences is called a valid conclusion and the argument is called Valid argument.

Representation of an Argument

An argument $p_1, p_2, \ldots, p_n \vdash p$ *is written as*

$$\left.\begin{array}{c} p_1 \\ p_2 \\ \vdots \\ p_n \end{array}\right\} \text{premises}$$
$$\overline{\qquad\qquad\qquad\qquad}$$
$$c \ (\text{conclusion})$$

In the above representation premises are listed above the horizontal line and the conclusion below the horizontal line.

Method to determine whether a given argument is valid or not

(i) By truth table

(ii) By Rules of inference

T F → F

- The Rules of inference are criteria for determining the validity of an argument.
- *Any conclusion which is arrived by following rules of inferences is called a valid conclusion and the argument is called Valid argument.*

1. Addition

(i) $\dfrac{p}{\therefore p \vee q}$ (ii) $\dfrac{q}{\therefore p \vee q}$

p	q	$p \vee q$	$p \to (p \vee q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

2. Simplification

(i) $\dfrac{p \wedge q}{p}$ (ii) $\dfrac{p \wedge q}{q}$

3. Conjunction

$$\begin{array}{c} p \\ \underline{q} \\ \therefore p \wedge q \end{array}$$

4. Modus ponens

Rule of Detachment

$$\begin{array}{c} p \to q \\ \underline{p} \\ \therefore q \end{array}$$

p	q	$p \to q$	$(p \to q) \wedge q$	$[(p \to q) \wedge q] \to q$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	F	T

5. Modus tollens

$$\begin{array}{c} p \to q \\ \underline{\sim q} \\ \sim p \end{array}$$

6. Hypothetical syllogism

$$\begin{array}{l} p \rightarrow q \\ \underline{q \rightarrow r} \\ \therefore p \rightarrow r \end{array}$$

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$p \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$ (A)	$A \rightarrow p \rightarrow r$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

7. Disjunction syllogism

$$\begin{array}{c} p \vee q \\ \underline{\sim p} \\ q \end{array}$$

8. Absorption

$$\begin{array}{c} \underline{p \rightarrow q} \\ p \rightarrow (p \wedge q) \end{array}$$

9. *Constructive dilemma*

$$\frac{\begin{array}{c}(p \to q) \wedge (r \to s)\\ p \vee r\end{array}}{q \vee s}$$

10. *Destructive dilemma*

$$\frac{\begin{array}{c}(p \to q) \wedge (r \to s)\\ \sim q \vee \sim s\end{array}}{\sim p \vee \sim r}$$

Q.1 Show that the following argument is valid :

$$\begin{array}{c} p \vee q \\ \sim p \\ \hline q \end{array}$$

By Truth table

Above Argument is valid if
$[(p \vee q) \wedge \sim p] \longrightarrow q$ is a Tautology

p	q	$\sim p$	$p \vee q$	$(p \vee q) \wedge \sim p$	$[(p \vee q) \wedge \sim p] \rightarrow q$
T	T	F	T	F	T
T	F	F	T	F	T
F	T	T	T	T	T
F	F	T	F	F	T
	C	P	P		

Hence given Argument is valid

By Rules of inferences

$$\begin{array}{c} p \vee q \\ \sim p \\ \hline q \end{array}$$

Above Argument is valid by Disjunction Syllogism

Q.2 ***Show that the argument*** $\overset{1}{p}, \overset{2}{p \to q}, \overset{3}{q \to r} \vdash r$ ***is valid:***

By Truth table → Above Argument is valid if

$[p \wedge (p \to q) \wedge (q \to r)] \longrightarrow r$ is tautology

p	q	r	$p \to q$	$q \to r$			
T	T	T	T	T			
T	T	F	T	F			
T	F	T	F	T			
T	F	F	F	T			
F	T	T	T	T			
F	T	F	T	F			
F	F	T	T	T			
F	F	F	T	T			

By the rules of inferences

(1) $p \to q$

p

---

q (By Modus ponens

(2) $q \longrightarrow r$

q

---

r (By Modus ponen)

Hence given Argument is valid

GW Q.3 Test the validity of the following argument: If a man is a bachelor, he is worried, If a man is a worried, he dies young. Therefore Bachelors die young

$$\begin{array}{l} p \to q \\ q \to r \\ \hline p \to r \end{array}$$

Let

p = Man is a bachelor

q = He is worried

r = He dies young

$$\begin{array}{c} p \longrightarrow q \\ q \longrightarrow r \\ \hline p \longrightarrow r \end{array}$$

Above Statement is valid By Hypothetical Syllogism

By Truth table

p	q	r	$p \to q$ (P)	$q \to r$ (P)	$p \to r$ (C)	$(p\to q)\wedge(q\to r)$ (A)	$A \to (p \to r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	F	T
T	F	F	F	T	F	F	T
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	T	T

Q.4 Prove that the argument $\overset{1}{p \to q},\ \overset{2}{q \to r},\ \overset{3}{r \to s},\ \overset{4}{\sim s},\ \overset{5}{p \vee t} \overset{c}{\vdash t}$ **is valid without using truth table.**

Solution

1.	$p \to q$	*Premise (Given)*
2.	$q \to r$	*Premise (Given)*
3.	$r \to s$	*Premise (Given)*
~~4.~~	$\cancel{p \to r}$	~~*Hypothetical syllogism using 1 and 2*~~
~~5.~~	$\cancel{p \to s}$	~~*Hypothetical syllogism using 3 and 4*~~
4.	$\sim s$	*Premise (Given)*
~~7.~~	$\cancel{\sim p}$	~~*Modus tollens using 5 and 6*~~
5.	$p \vee t$	*Premise (Given)*
~~9.~~	$\cancel{t}$	~~*disjunctive syllogism using 7 and 8*~~

(1) $\dfrac{\begin{array}{c} p \to q \\ q \to r \end{array}}{p \to r}$ (By Hypothetical syllogism)

(2) $\dfrac{\begin{array}{c} p \to r \\ r \to s \end{array}}{p \to s}$ (")

(3) $\dfrac{\begin{array}{c} p \to s \\ \sim s \end{array}}{\sim p}$ (By Modus tollens)

(4) $p \vee t$

$\sim p$

---

t (By Disjunction Syllogism

Hence, given Argument is valid

Q.5 Show that s *is a valid conclusion from the premises* $p \to q$, $p \to r$, $\sim(q \land r)$ *and* $s \lor p$.

Solution

1.	$p \to q$	*Premise (Given)*
2.	$p \to r$	*Premise (Given)*
3.	$(p \to q) \land (p \to r)$	*Conjunction using 1 and 2*
4.	$\sim(q \land r)$	*Premise (Given)*
5.	$\sim q \lor \sim r$	*Demorgan's law using 4*
6.	$\sim p \lor \sim p$	*Destructive dilema using 3 and 5*
7.	$\sim p$	*Idempotent law using 6*
8.	$s \lor p$	*Premise (Given)*
9.	s	*Disjunctive syllogism using 7 and 8*

Q.6 ***Prove the validity of the following argument "If I get the job and work hard, then I will get promoted. If I get promoted, then I will be happy. I will not be happy. therefore, either I will not get the job or I will not work hard"***

Solution: Let

p = I get the job

q = Work hard

r = I will get promoted

s = I will be happy

$$(p\wedge q)\rightarrow r$$
$$r\rightarrow s$$
$$\sim s$$
$$\sim p\vee\sim q$$

1. $(p \wedge q) \rightarrow r$ — *Premise (Given)*
2. $r \rightarrow s$ — *Premise (Given)*
3. ~~$(p \wedge q) \rightarrow \cancel{r}\ s$ — *Hypothetical syllogism using 1 and 2*~~
4. $\sim s$ — *Premise (Given)*
5. $\sim (p \wedge q)$ — *Modus tollens using 3 and 4*
6. $\sim p \vee \sim q$ — *Demorgan's law using 5 (conclusion)*

Hence the argument is valid

$$(p\wedge q)\rightarrow r$$
$$r\rightarrow s$$
$$(p\wedge q)\rightarrow s$$

(2) $(p\wedge q)\rightarrow s$

$\sim s$

$\sim(p\wedge q)$

(3) $\sim(p\wedge q)$

$\sim p\vee\sim q$

Hence given argument is valid

DPP- 7

Topic : Theory of inference (Part -1)

Q.1 *Show that the argument* $p,\ p \to q \vdash q$ *is valid*

Q.2 *Test the validity of the following argument: "If it rains then it will be cold, If it is cold then I shall stay at home. Since it rains therefore I shall stay at home".*

Q.3 *Test the validity of the argument:*

If two side of a triangle are equal, then the opposite angles are equal.

Two side of a triangle are not equal.

The opposite angles are not equal

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

DISCREE MATHEMATICS

UNIT -3 : *THEORY OF LOGICS*

Lecture - 08

Today's Target

- **Theory of inference** (Part-2)
- PYQ
- DPP

Rules of inference

- The Rules of inference are criteria for determining the validity of an argument.
- *Any conclusion which is arrived by following rules of inferences is called a valid conclusion and the argument is called Valid argument.*

1. *Addition*

(i) $\dfrac{p}{\therefore p \vee q}$

(ii) $\dfrac{q}{\therefore p \vee q}$

2. *Simplification*

(i) $\dfrac{p \wedge q}{p}$

(ii) $\dfrac{p \wedge q}{q}$

3. Conjunction

$$\begin{array}{c} p \\ \underline{q} \\ \therefore p \wedge q \end{array}$$

4. Modus ponens

$$\begin{array}{c} p \rightarrow q \\ \underline{\quad q \quad} \\ \therefore q \end{array}$$

5. Modus tollens

$$\begin{array}{c} p \rightarrow q \\ \underline{\quad \sim q \quad} \\ \sim q \end{array}$$

6. *Hypothetical syllogism*

$$\begin{array}{c} p \to q \\ \underline{q \to r} \\ \therefore p \to r \end{array}$$

7. *Disjunction syllogism*

$$\begin{array}{c} p \vee q \\ \underline{\sim p} \\ q \end{array}$$

8. *Absorption*

$$\begin{array}{c} \underline{p \to q} \\ p \to (p \wedge q) \end{array}$$

9. *Constructive dilemma*

$$\frac{\begin{array}{c}(p \to q) \land (r \to s) \\ p \lor r\end{array}}{q \lor s}$$

10. *Destructive dilemma*

$$\frac{\begin{array}{c}(p \to q) \land (r \to s) \\ \sim p \lor \sim s\end{array}}{\sim p \lor \sim r}$$

Q.1 *Test the validity of following argument. If I will select in IAS examination, then I will not be able to go to London. Since, I am going to London, I will not select in IAS examination*

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Let

p : I will select in IAS Examination

q : I am going to london.

1. $p \longrightarrow \sim q$
2. q

$\sim p$

By Truth table – First Method

p	q	$\sim p$	$\sim q$	$p \rightarrow \sim q$
T	T	F	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Whenever premises are conclusion also true

Hence, Argument is valid

By Truth table - Second Method

Above Argument is valid if

$[(p \to \sim q) \wedge q] \longrightarrow \sim p$ is a Tautology

p	q	$\sim p$	$\sim q$	$p \to \sim q$	$(p \to \sim q) \wedge q$	$[(p \to \sim q) \wedge q] \to \sim p$
T	T	F	F	F	F	T
T	F	F	T	T	F	T
F	T	T	F	T	T	T
F	F	T	T	T	F	T

Hence, given Argument is valid

By Rules of inferences

$$\begin{array}{l} p \to \sim q \\ q \\ \hline \sim p \\ \hline \end{array}$$

(By Modus tollens)

Hence, Above Argument is a valid argument

Q.2 Consider the following argument and determine whether it is valid.

Either *I will get good marks or I will not graduate. If I did not graduate I will go to Canada. I get good marks. Thus, I would not go to Canada.*

Let

p : I will get good marks

q : I will graduate

r : I will to to canada

1. $p \vee \sim q$
2. $\sim q \longrightarrow r$
3. p

$\sim r$

p	q	r	$\sim q$	$\sim r$	$p \vee \sim q$	$\sim q \to r$
T	T	T	F	F	T	T
T	T	F	F	T	T	T
T	F	T	T	F	T	T
T	F	F	T	T	T	F
F	T	T	F	F	F	T
F	T	F	F	T	F	T
F	F	T	T	F	T	T
F	F	F	T	T	T	F

Whenever primes are true, conclusion are not true. Hence Arument is not valid

Q.3 Check the validity of the following arguments. Show with the use of symbolic notation. $x^2 = y^2$ only if $x = y$

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$$\frac{x = y}{x^2 = y^2}$$

Let $p : x^2 = y^2$

$q : x = y$

1. $p \rightarrow q$
2. q

$\therefore \; p$

Above Argument is valid if $[(p \rightarrow q) \wedge q] \rightarrow p$ is a tautology

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	$[(p \rightarrow q) \wedge q] \rightarrow p$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	T

$[(p \rightarrow q) \wedge q] \rightarrow p$ is not a Tautology

Hence, it is not a Tautology

DPP- 8

Topic : Theory of inference (Part -2)

Q.1. *"If the labour market is perfect then the wages of all persons in a particular employment will be equal. But it is always the case that wages for such persons are not equal therefore the balour market is not perfect". Test the validity of the argument.* AKTU

Q.2 *Prove the validity of the following argument "if the races are fixed so the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases, then the police be happy. The police force is never happy. Therefore, the races are not fixed.* AKTU

Q.3 *Use rules of inference to Justify that the three hypotheses (i)"If it does not rain or if it is not fogy, then the sailing race will be held and the lifesaving demonstrationwill go on." (ii) If the sailing race is held, then the trophy will be awarded. (iii) "The trophy was not awarded." imply the conclusion* (iv) *It rained."* AKTU

Q. 4 *Prove the validity of the followng argument.*
If Mary runs for office, She will be elected. If Mary attends the meeting, she will run for office. Either Mary will attend the meeting or she will go to India. But Mary cannot go to India. AKTU

Q.5 *Test the validity of the argument:*

"If Ashok wins then Ram will be happy. If Kamal wins Raju will be happy. Either Ashol will win or Kamal will win. However if Ashok win, Raju will not be happy and if Kamal wins Ram will not be happy. So Ram will be happy if and only if Raju is not happy".

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