AKTU DSTL

DISCRETE MATHEMATICS

ONE SHOT REVISION

UNIT-I

SET THEORY& RELATIONS,
POSET & LATTICES

GATEWAY CLASSES

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Lec-1

Today's Target

- What is set?
- Standard sets
- Number system

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP
4	Unit wise set of PYQs
5	Course will be completed by ‘Last week of Nov 2023’
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SET

SET: A Set is a Well-defined collection of objects

Well-defined collection: Collection does not change person to person

Not Well-defined collection: Collection can be change person to person

Example-1 The collection of all odd natural number. less than 10

Abhishek - 1, 3, 5, 7, 9
Sachin - 1, 3, 5, 7, 9
Wilson - 1, 3, 5, 7, 9

So, this collection is well defined and therefore, it is a set

Example-2 The collection of top 3 batsmen in the World

Abhishek - Sachin, Rahul Dravid, Ponting
Sachin - Sachin, Dhoni, Smith
Wilson - Virat, Dhoni, Devilliers

So, this collection is not well defined and therefore, it is not a set.

NOTE- Every set is a collection but every collection is not a set.

Example- Group of good cricket players is a collection but it is not a set.

$$A = \{a, e, i, o, u\}$$

Name of Set

Elements or member or objects of set

Sets are usually denoted by capital letters, A, B, C, X, Y, Z etc.

The elements of a set are represented by small letters a, b, c, y, z etc.

NOTE

(i) a belongs to A

$$\boxed{a \in A}$$

(ii) b does not belong to A

$$\boxed{b \notin A}$$

Where ε = epsilon

Some Standard Sets

N: The set of all natural numbers

W : Set of whole numbers

Z or I : The set of all integers

Q: The set of all rational numbers

R: The set of real numbers

C: The set of all complex number

I^+ or Z^+: The set of positive integers

Q^+: The set of positive rational numbers.

R^+: The set of positive real numbers.

$N = \{1, 2, 3, 4 \ldots\}$

$W = \{0, 1, 2, 3 \ldots\}$

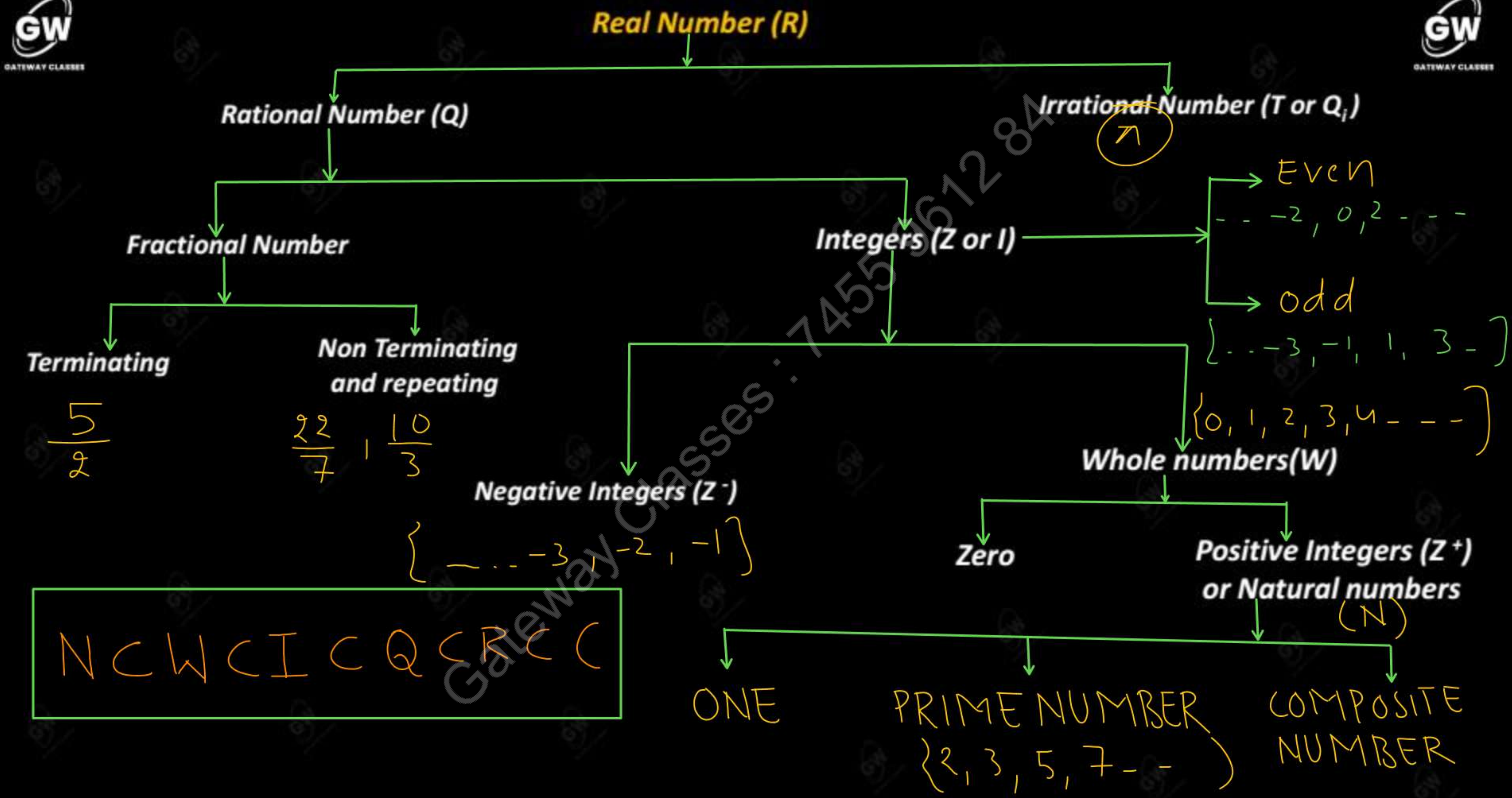
$I = \{\ldots -3, -2, -1, 0, 1, 2 \ldots\}$

COMPLEX NUMBER (C)

Real Number

Imaginary Number

$\sqrt{-1}, \sqrt{-2}, \sqrt{-3} \ldots$


Real Number (R)
Rational Number (Q)
Irrational Number (T or Q_i)
π
Fractional Number
Integers (Z or I)
Even
- - -2, 0, 2 - - -
odd
{- - -3, -1, 1, 3 - -}
Terminating
$\frac{5}{2}$
Non Terminating and repeating
$\frac{22}{7}$, $\frac{10}{3}$
{0, 1, 2, 3, 4 - - -}
Whole numbers(W)
Negative Integers (Z^-)
{- - - -3, -2, -1}
Zero
Positive Integers (Z^+) or Natural numbers (N)
NCWCICQCRCC
ONE
PRIME NUMBER {2, 3, 5, 7 - - -}
COMPOSITE NUMBER

Rational Numbers: - These are the real numbers which can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Example-

Irrational Numbers: - (Non-terminating and non repeating decimal) : These are the real numbers which can not be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$.

Example- $\pi, \sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{6}$ - - - -

Non Positive Integers : $0, -1, -2, -3$ - - - -

Non Negative Integers : $0, 1, 2, 3$ - - -

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Thank You

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) $\boxed{\text{Set Theory}}$ & Relations

(ii) POSET & Lattices

Lec-2

Today's Target

- *Representation of Set*
- Types of sets

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Representation of Set

Roster Form
Or
Tabular Form

Set – builder Form
or
Rule Method

Roster Form or Tabular Form

In roster form, all the elements are being separated by commas and are enclosed with in curly brackets { }

Examples

(1) The set of all natural numbers which divide 42

$N = \{1, 2, 3, 6, 7, 14, 21, 42\}$

(2) The set of all prime number less than 20

$A = \{2, 3, 5, 7, 11, 13, 17, 19\}$

(3) The set of odd natural number .

$N = \{1, 3, 5, 7 \ldots\}$

(4) The set of Binary digits.

$A = \{0, 1\}$

Note: (i) All infinite sets can not be written in roster form

Example: The set of real number can not be written in roster form.

(ii) In roster form the order in Which the elements of a set are written is not important .

Example: $A = \{1, 2, 3\}$ $B = \{2, 1, 3\}$ Here A and B are same set

(iii) In roster form, an element is not generally repeated, i.e all elements are taken as distinct

Example: $A = \{1, 2, 3\}$

$B = \{1, 2, 3, 2, 1\}$ Here set A and set B are same

Set – Builder Form

In Set – builder form, all the elements of a set possess a single common property which is not possessed by any element out side the set .

such that

$$A = \{x : p(x)\}$$

$N = \{1, 2, 3 - - -\}$

Where $P(x)$ is the property by which $x \in A$

Note:- A vertical bar is also used in place of colon (:)

Examples

(i) The set of all natural number less than 10 .

$N = \{x : x \in N \text{ and } x < 10\}$

(ii) The set of Real number .

$R = \{x : x \text{ is a real number}\}$

(iii) The set of natural number .

$N = \{x : x \text{ is a natural number}\}$

Types of Set

1. Empty Set / Null Set/ Void Set

- A Set having no element in it.
- It is denoted by { } or ϕ

Examples

① $A = \{x : x \in N, 4 < x < 5\} = \phi$

② $A = \{x : x^2 = 9, x \in \text{Even number}\} = \phi$

2. Singleton Set

A Set containing exactly one element is called a singleton set.

Examples

① $A = \{5\}$

② $B = \{x : x^2 = 4, x \in N\} = \{2\}$

Cardinal Number of a Set

OR

Cardinality of a set

The number of distinct elements containing in any set A is called cardinal number of set A

It is denoted by n (A) or |A|

Examples

(1) $A = \{2, 4, 6, 8\}$

$n(A) = 4$

(2) $A = \{\phi\}$

$n(A) = 1$

$B = \{1, 2, 3, 2, 4, 1\}$

$n(B) = 4$

Finite and infinite Set

3. Finite Set - *A Set is said to be finite if it contains finite number of elements .*

Examples ① $A = \{1, 2, 3\}$

② The set students in a class

4. Infinite Set - *A Set is said to be infinite if it contains infinite number of elements .*

Examples

(a) The Set of natural number

$N = \{1, 2, 3, 4 \ldots\}$

(b) The Set of integers

$Z = \{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$

5. Equal Set : Two Sets A and B are said to be equal, if they have exactly the same elements and we Write

$$A = B$$

Examples

$A = \{1, 4, 6, 7\}$

$B = \{4, 1, 6, 7\}$

6. Equivalent Set : Two finite Sets A and B are said to be equivalent if

$$\boxed{n(A) = n(B)}$$

Examples

$A = \{2, 4, 6, 8\}$

$B = \{a, b, c, d\}$

Subset and Super Set

7. Subset : a set A is said to be a subset of set B, if every element of A is also an element of B

$$A \subseteq B$$

Examples $A = \{2, 4, 6\}$

$B = \{1, 2, 3, 4, 5, 6\}$

$A \subseteq B$

Note: (i) If A is not a subset of B $\quad A \nsubseteq B$

(ii) Null set is the subset of all set $\quad \phi \subseteq A$

(iii) Every set is a subset of itself $\quad A \subseteq A$

The total number of subset of a finite set containing n elements = 2^n

Q.1 Write down all possible subsets of

(i) $A = \{a, b\}$

(ii) $B = \{1, 2, 3\}$

(i) $A = \{a, b\}$

Subsets of A

$\{a\}, \{b\}, \{\ \}, \{a, b\}$

(ii) $B = \{1, 2, 3\}$

Subsets of B

$\{\ \}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}$

8. Super Set : *If $A \subseteq B$, the B is called a superset of A and we write*

$$B \supseteq A$$

Examples

$A = \{a, b, c\}$

$B = \{a, b, c, d\}$

$A \subseteq B$

$\boxed{B \supseteq A}$

9. Proper subset: If $A \subseteq B$ and $A \neq B$, then A is called a proper subset of B and we write

$$A \subset B$$

Examples $A = \{1, 2\}$

$B = \{1, 2, 3\}$

Note (1) $\boxed{N \subset W \subset Z \subset Q \subset R}$

(2) Set A is not proper subset of A

The total number of proper subset of a finite set containing n elements $= 2^n - 1$

10. Power Set

- The collection of all subsets of a set A is called the power set of A
- It is denoted by P(A)

$m = 3$

In P(A) , every element is a set

If $n(A) = m \quad then \quad n[P(A)] = 2^m$

Q.1. Write down the power set of A = {1, 2, 3}

$$P(A) = \{ \{\ \}, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\} \}$$

$$n[P(A)] = 8$$

11. Universal Set:- Collection of all elements under consideration is called universal set

Example

$A = \{1, 2, 3\}$

$B = \{2, 3, 4, 5\}$

$C = \{4, 5, 6, 7\}$

$U = \{1, 2, 3, 4, 5, 6, 7\}$

Interval as subsets of R: If a, b are real numbers such that $a < b$

(1) Closed interval

$$[a,\ b] = \{x : x \in R \ \ and \ \ a \le x \le b\}$$

(2) Open interval

$$(a, b) =]\,a, b\,[= \{x : x \in R \ \ and \ \ a < x < b\}$$

(3) Semi-open or semi closed interval

(i) $[a,\ b) = \{x : x \in R \ \ and \ \ a \le x < b\}$

(ii) $(a,\ b] = \{x : x \in R \ \ and \ \ a < x \le b\}$

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DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Lec- 3

Today's Target

Operations on Sets

[redacted]

Course Details(Paid) : All Subjects

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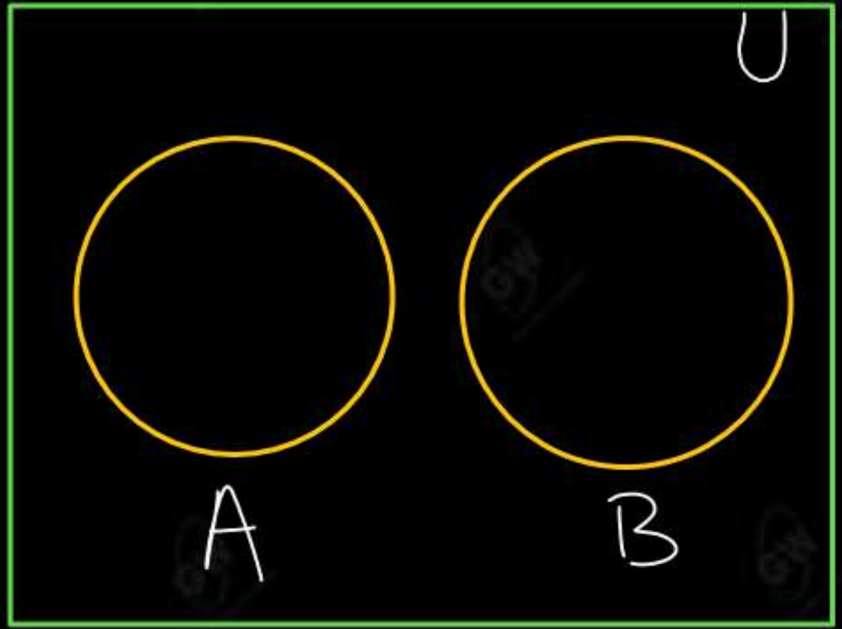
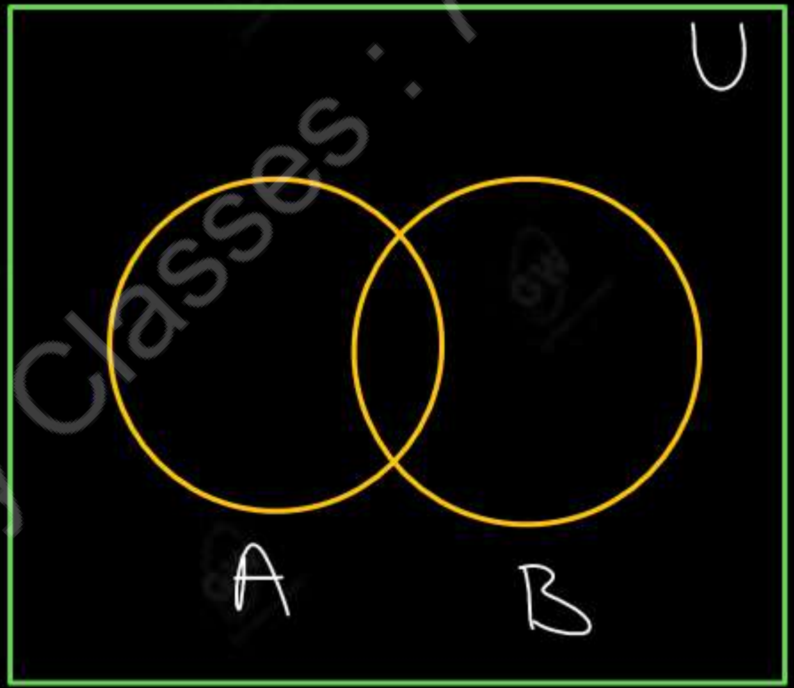
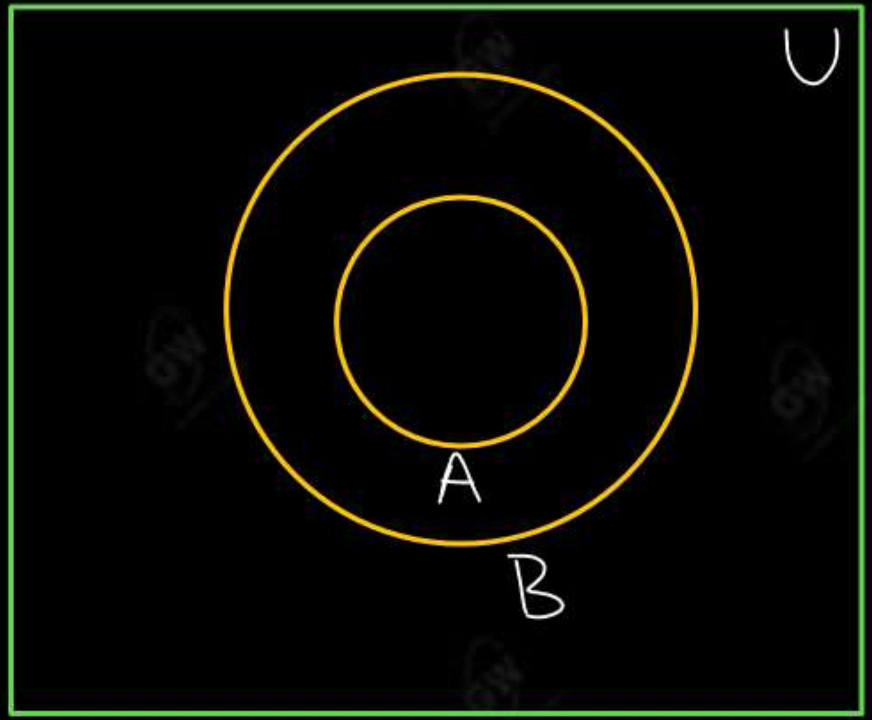
Venn Diagram

- *Most of the relationships between sets can be represented by means of diagram which are known as Venn Diagram*
- *The universal set is represented by the interior of a rectangle and its subsets are represented by circular areas drawn within the rectangle.*





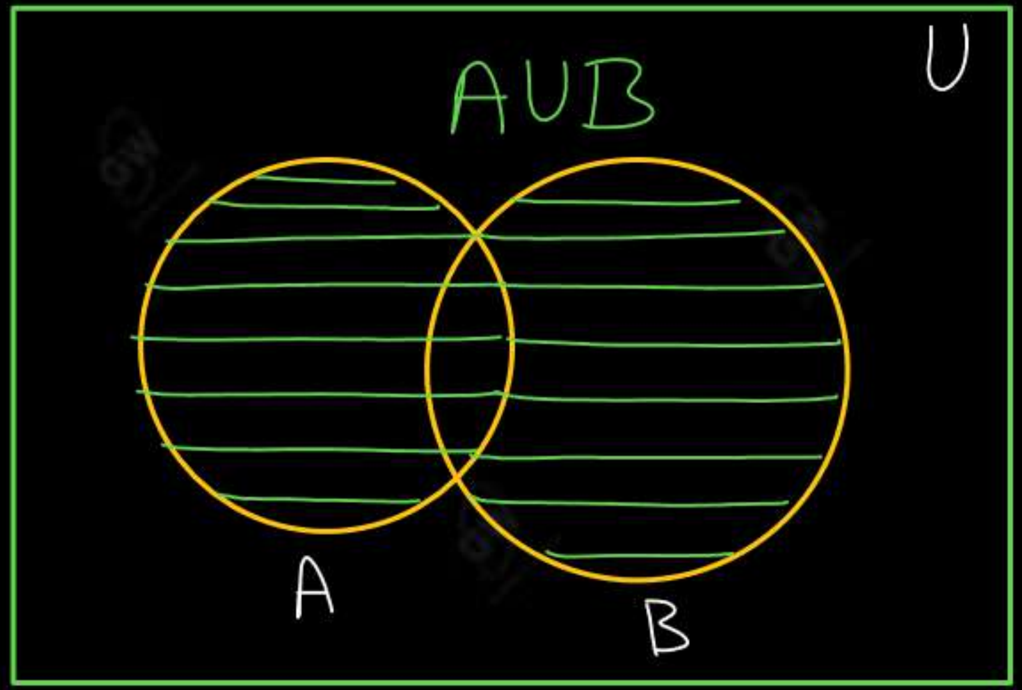


Operations on Sets

1. Union of Set:- *The union of two sets A and B is the set of all elements which are either in A or in B or in both A and B.*

It is denoted by $A \cup B$

$$\boxed{A \cup B = \{x : x \in A \text{ or } x \in B\}}$$



Note:

(1) The common elements being taken only once

(2) $x \in A \cup B \Rightarrow$ $x \in A$ or $x \in B$

(3) $x \notin A \cup B \Rightarrow$ $x \notin A$ and $x \notin B$

Example: Let $A = \{1, 2, 3, 4\}$ and $B = \{2, 4, 6, 8\}$ Find $A \cup B$.

$A \cup B = \{1, 2, 3, 4, 6, 8\}$

Example: If $A = \{x : x \text{ is a prime number less than } 15\}$

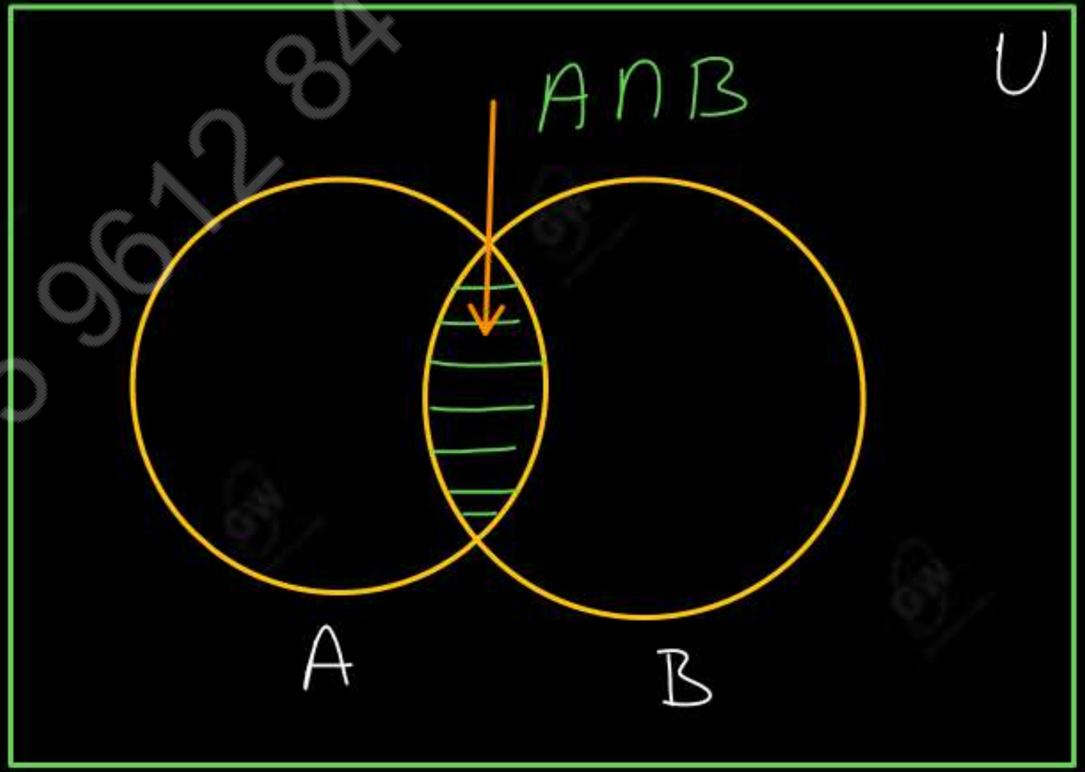
and $B = \{x : x \in N, x \text{ is a factor of } 12\}$ Find $A \cup B$.

$A = \{2, 3, 5, 7, 11, 13\}$ and $B = \{1, 2, 3, 4, 6, 12\}$

$A \cup B = \{1, 2, 3, 5, 6, 7, 11, 12, 13\}$

2. Intersection of sets:- *The intersection of two sets A and B is the set of all elements which are common to both A and B.*

- *It is denoted by* $A \cap B$
- $\boxed{A \cap B = \{x : x \in A \text{ and } x \in B\}}$



Note: (1) $x \in A \cap B \Rightarrow$ $x \in A$ and $x \in B$

(2) $x \notin A \cap B \Rightarrow$ $x \notin A$ or $x \notin B$

Example :- *If* $A = \{2, 4, 6, 8, 10\}$ *and* $B = \{1, 2, 3, 4\}$ *Find* $A \cap B$.

$A \cap B = \{2, 4\}$

3. Disjoint Sets:- Two sets A and B are said to be disjoint if

$$\boxed{A \cap B = \emptyset}$$

Null set

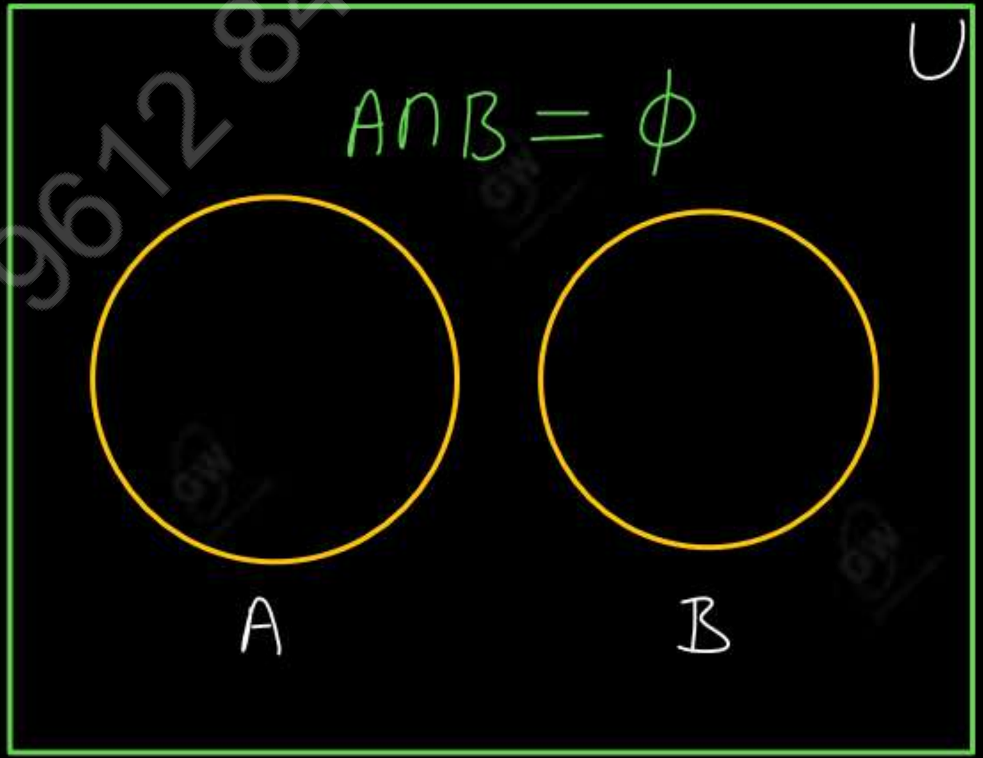
Example :-

If $A = \{2, 4, 6, 8, 10\}$

and $B = \{1, 3, 5, 7, 9\}$

$$\boxed{A \cap B = \phi}$$

Here, A and B are disjoint sets

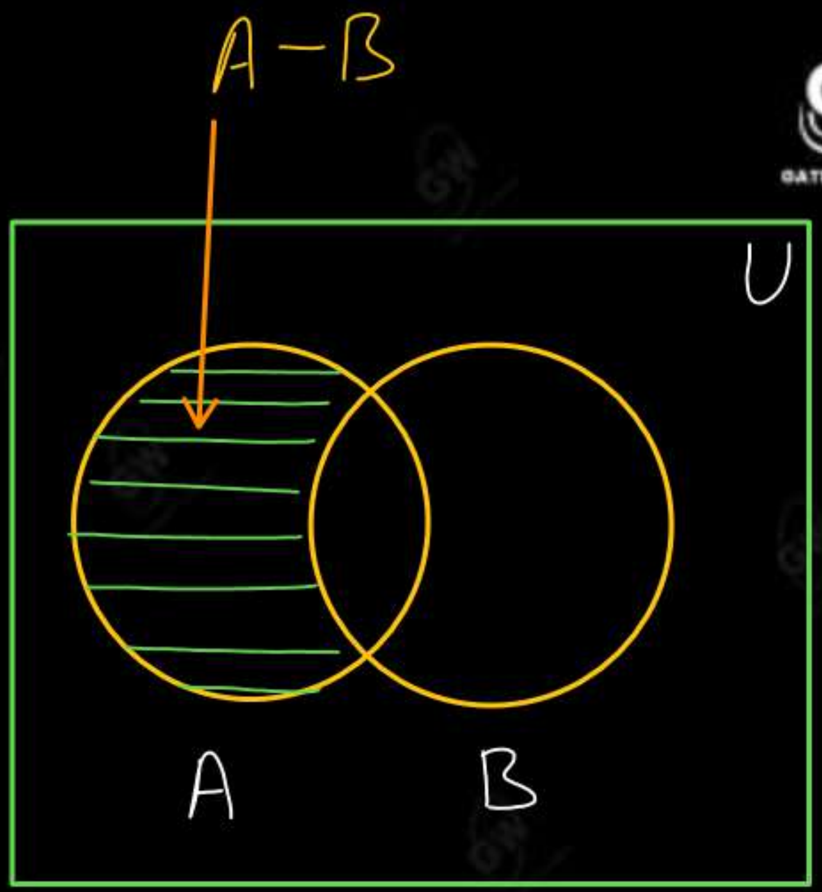


4. Difference of Sets : For any two sets A and B

(i) A - B =

It contains all the elements of set A which are not present in set B.

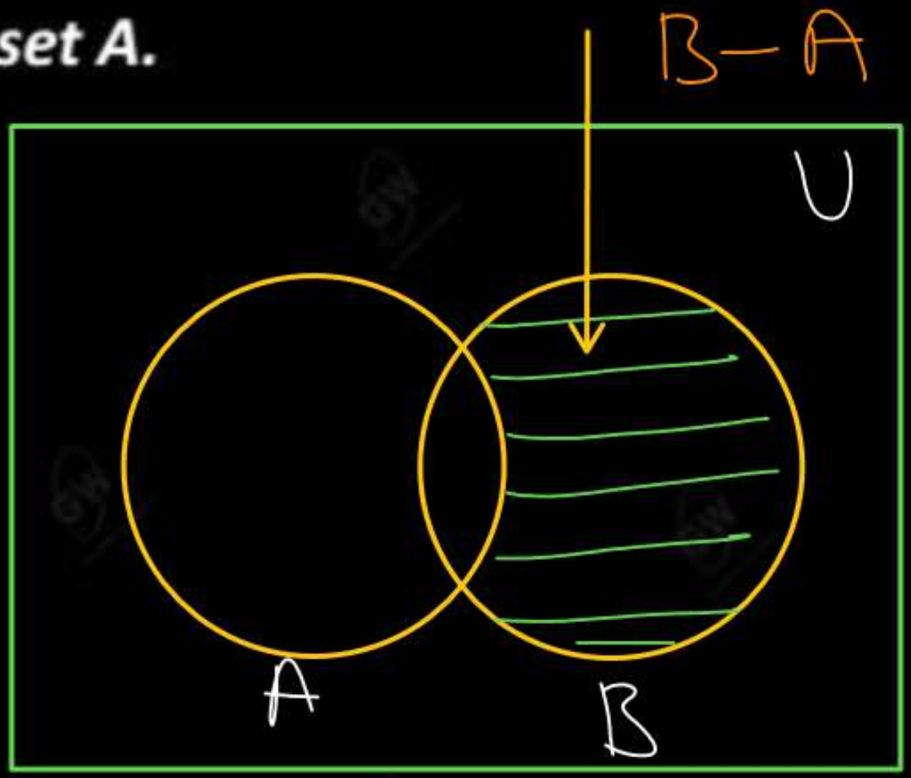
$$A - B = \{x : x \in A \text{ and } x \notin B\}$$



(ii) B - A

It contains all the elements of set B which are not present in set A.

$$B - A = \{x : x \in B \text{ and } x \notin A\}$$



Example:- If $A = \{3, 5, 6, 8, 9\}$ and $B = \{5, 8\}$ Find $A - B$

$$A - B = \{3, 6, 9\}$$

Example :- If $A = \{x : x \in N, x \text{ is a factor of } 12\}$ and

$B = \{x : x \in N, x \text{ is a factor of } 15\}$ then Find (i) $A-B$ (ii) $B-A$.

$A = \{1, 2, 3, 4, 6, 12\}$ and $B = \{1, 3, 5, 15\}$

$$A - B = \{2, 6, 12\}$$

$$B - A = \{5, 15\}$$

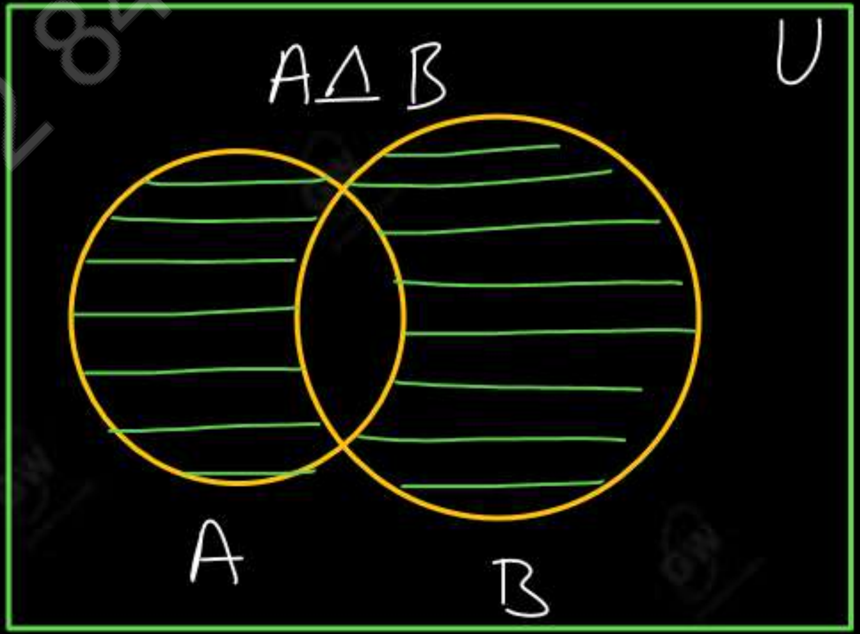
(5) Symmetric difference of sets

The symmetric difference of any two sets A and B is denoted by $A \triangle B$ and defined as

$$A \triangle B = (A - B) \cup (B - A)$$

OR

$$A \triangle B = (A \cup B) - (B \cap A)$$



Example:- If $A = \{1, 2, 3, 6\}$ and $B = \{3, 6, 9, 12\}$, then find $A \triangle B$.

$A - B = \{1, 2\}$

$B - A = \{9, 12\}$

$A \triangle B = \{1, 2, 9, 12\}$

$A \cup B = \{1, 2, 3, 6, 9, 12\}$

$A \cap B = \{3, 6\}$

$A \triangle B = \{1, 2, 9, 12\}$

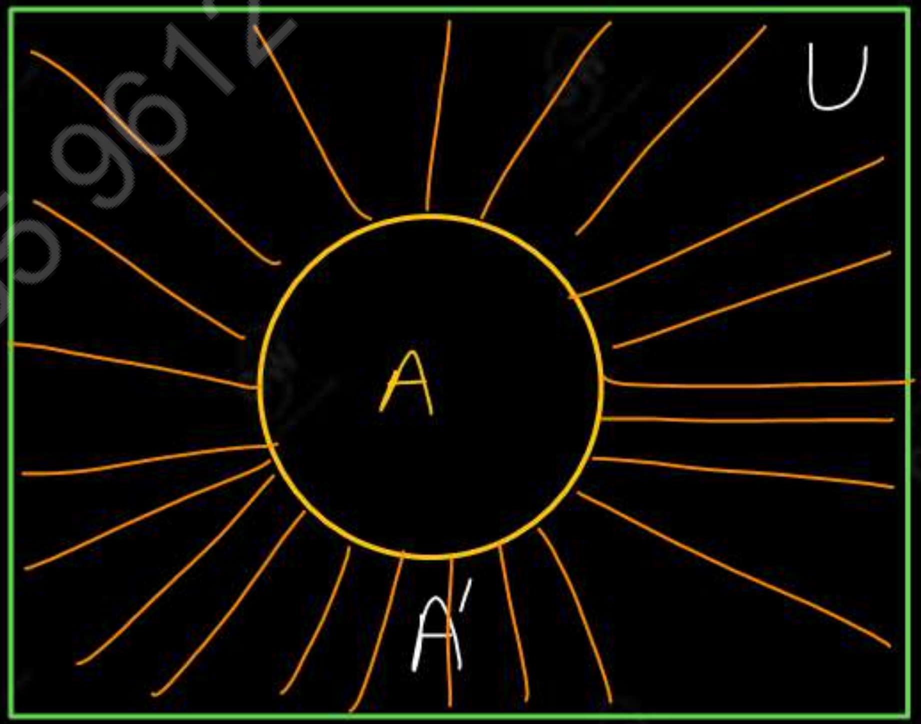
(6) Complement of set :- Let ∪ be the universal set and A be any subset of ∪.

The complement of A, denoted by A' or A^c is the set of elements which belongs to ∪ but which does not belongs to A.

$$A' = \cup - A$$

$$\boxed{A' = \{x : x \in \cup \text{ and } x \notin A\}}$$

$$x \in A^c \Rightarrow x \notin A$$



Note:- $A \cap A^c = \emptyset$

$A \cup A^c = \cup$

$(A^c)^c = A$

Example:- If $\cup = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ *and*

$A = \{1, 3, 5, 7, 9, 12\}$

Find

(i) A^c

(ii) $A \cap A^c$

(iii) $A \cup A^c$

(iv) $(A^c)^c$

(i) $A^c = \{2, 4, 6, 8, 10, 11\}$

(ii) $A \cap A^c = \phi$

(iii) $A \cup A^c = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

(iv) $(A^c)^c = ?$

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UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Lec-4

Today's Target

- *Algebra of sets*

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Operations on Sets

Revise

1. **Union of Set:**

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

Note:

(a) $x \in A \cup B \Rightarrow x \in A$ or $x \in B$

(b) $x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$

2. **Intersection of sets:**

$A \cap B = \{x : x \in A \text{ and } x \in B\}$

Note:

(a) $x \in A \cap B \Rightarrow x \in A$ and $x \in B$

(b) $x \notin A \cap B \Rightarrow x \notin A$ or $x \notin B$

3. Difference of Sets : For any two sets A and B

$A - B = \{x : x \in A \text{ and } x \notin B\}$

$B - A = \{x : x \in B \text{ and } x \notin A\}$

$x \in (A-B) = x \in A$ and $x \notin B$

$x \in (B-A) = x \in B$ and $x \notin A$

4. Symmetric difference of sets

$A \triangle B = (A - B) \cup (B - A)$

OR

$A \triangle B = (A \cup B) - (B \cap A)$

A' or A^c

5. Complement of set :-

$A' = \cup - A$

$A' = \{x : x \in \cup \text{ and } x \notin A\}$

$x \in A^c \Rightarrow x \notin A$

$x \in A \Rightarrow x \notin A^c$

Note:- $A \cap A^c = \emptyset$

$A \cup A^c = \cup$

$(A^c)^c = A$

Algebra of sets

1. Idempotent Laws

(a) $A \cup A = A$

(b) $A \cap A = A$

2. Commutative Laws

(a) $A \cup B = B \cup A$

(b) $A \cap B = B \cap A$

3. Associative Laws

(a) $(A \cup B) \cup C = A \cup (B \cup C)$

(b) $(A \cap B) \cap C = A \cap (B \cap C)$

4. Distributive Laws

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. DeMorgan's Laws

(a) $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$

6. Identity Laws

(a) $A \cup \phi = A$

(b) $A \cap U = A$

(c) $A \cup U = U$

(d) $A \cap \phi = \phi$

$A = \{1, 2, 3\}$

$B = \{1, 2, 3\}$

$A \subseteq B$

$B \subseteq A$

$\boxed{A = B}$

7. Involution Laws

$(A')' = A$

8. Complement Laws

(a) $A \cup A' = U$

(b) $A \cap A' = \phi$

(a) $U' = \phi$

(b) $\phi' = U$

9. Absorption Laws

(a) $A \cup (A \cap B) = A$

(b) $A \cap (A \cup B) = A$

Methods

1. By definition
2. By venn diagram ×
3. For any two sets A and B

$A \subseteq B$ — (1)

$B \subseteq A$ — (2) $\Rightarrow A = B$

1. *Idempotent Laws* : For any set A, prove that

(a) $A \cup A = A$

Proof :

Let $x \in A$

$\Rightarrow x \in A$ or $x \in A$

$\Rightarrow x \in A \cup A$

$\Rightarrow A \subseteq A \cup A$ — ①

Conversely if $x \in A \cup A$

$\Rightarrow x \in A$ or $x \in A$

$\Rightarrow x \in A$

$\Rightarrow A \cup A \subseteq A$ — ②

From ① and ②

$\boxed{A \cup A = A}$ Hence, proved

2nd Method

$A \cup A = \{x : x \in A \text{ or } x \in A\}$

$= \{x : x \in A\}$

$\boxed{A \cup A = A}$ Hence proved

(b) $A \cap A = A$

Proof:

Let $x \in A$

$\Rightarrow x \in A$ and $x \in A$

$\Rightarrow x \in A \cap A$

$\Rightarrow A \subseteq A \cap A$ —— 1

Conversely if $x \in A \cap A$

$\Rightarrow x \in A$ and $x \in A$

$\Rightarrow x \in A$

$\Rightarrow A \cap A \subseteq A$ —— 2

From 1 and 2

$$\boxed{A \cap A = A}$$

2nd Method

$A \cap A = \{x : x \in A \text{ and } x \in A\}$

$= \{x : x \in A\}$

$$\boxed{A \cap A = A}$$

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2. Commutative Laws: For any two sets A and B, Prove that

(a) $A \cup B = B \cup A$

Proof

Let $x \in A \cup B$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in B \cup A$

$\Rightarrow A \cup B \subseteq B \cup A$ — (1)

Conversely, if

$x \in B \cup A$

$\Rightarrow x \in B$ or $x \in A$

$\Rightarrow x \in A$ or $x \in B$

$\Rightarrow x \in A \cup B$

$\Rightarrow B \cup A \subseteq A \cup B$ — (2)

From (1) and (2)

$$\boxed{A \cup B = B \cup A}$$

2nd Method

$A \cup B = \{x : x \in A \text{ or } x \in B\}$

$= \{x : x \in B \text{ or } x \in A\}$

$= B \cup A$

$$\boxed{A \cup B = B \cup A}$$

GW GATEWAY CLASSES

(b) $A \cap B = B \cap A$

Proof

Let $x \in A \cap B$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in B \cap A$

$\Rightarrow A \cap B \subseteq B \cap A$ — (1)

Conversely, if

$x \in B \cap A$

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in A$ and $x \in B$

$\Rightarrow x \in A \cap B$

$\Rightarrow B \cap A \subseteq A \cap B$ — (2)

From (1) and (2)

$$\boxed{A \cap B = B \cap A}$$

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3. Associative Laws: *For any three sets A, B, C, prove that*

(a) $(A \cup B) \cup C = A \cup (B \cup C)$

Proof

Let $x \in (A \cup B) \cup C$

$\Rightarrow x \in (A \cup B)$ or $x \in C$

$\Rightarrow (x \in A$ or $x \in B)$ or $x \in C$

$\Rightarrow x \in A$ or $(x \in B$ or $x \in C)$

$\Rightarrow x \in A$ or $x \in (B \cup C)$

$\Rightarrow x \in A \cup (B \cup C)$ — ①

$\Rightarrow (A \cup B) \cup C \subseteq A \cup (B \cup C)$ — ①

Similarly

$A \cup (B \cup C) \subseteq (A \cup B) \cup C$ — ②

From ① and ②

$$\boxed{(A \cup B) \cup C = A \cup (B \cup C)}$$

2nd Method

$(A \cup B) \cup C = \{x : x \in (A \cup B) \text{ or } x \in C\}$

$= \{x : (x \in A \text{ or } x \in B) \text{ or } x \in C\}$

$= \{x : x \in A \text{ or } (x \in B \text{ or } x \in C)\}$

$$(A \cup B) \cup C = \{x : x \in A \text{ or } (x \in B \text{ or } x \in C\}$$

$$= \{x : x \in A \text{ or } x \in (B \cup C)\}$$

$$= \{x : x \in A \cup (B \cup C)\}$$

$$= A \cup (B \cup C)$$

$$\boxed{(A \cup B) \cup C = A \cup (B \cup C)}$$

(b) $(A \cap B) \cap C = A \cap (B \cap C)$

4. Distributive Laws: For any three sets A, B, C, prove that

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Proof

Let $x \in A \cup (B \cap C)$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A$ or $(x \in B$ and $x \in C)$

$\Rightarrow x \in A$ or $x \in B$ and $x \in A$ or $x \in C$

$\Rightarrow x \in (A \cup B)$ and $x \in (A \cup C)$

$\Rightarrow x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ — (1)

conversely if

$x \in (A \cup B) \cap (A \cup C)$

$\Rightarrow (x \in A$ or $x \in B)$ and $(x \in A$ or $x \in C)$

$\Rightarrow x \in A$ or $(x \in B$ and $x \in C)$

$\Rightarrow x \in A$ or $x \in (B \cap C)$

$\Rightarrow x \in A \cup (B \cap C)$

$(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$ — (2)

From (1) and (2)

$$\boxed{A \cup (B \cap C) = (A \cup B) \cap (A \cup C)}$$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. DE Morgan's Laws: *For any two sets A and B, Prove that*

(a) $(A \cup B)' = A' \cap B'$

Proof:

Let $x \in (A \cup B)'$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in A' \cap B'$

$\Rightarrow (A \cup B)' \subseteq A' \cap B'$ —— (1)

conversely if

$x \in A' \cap B'$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \in (A \cup B)'$

$\Rightarrow A' \cap B' \subseteq (A \cup B)'$ —— (2)

From (1) and (2)

$$\boxed{(A \cup B)' = A' \cap B'}$$

(b) $(A \cap B)' = A' \cup B'$

Proof

$$A' \cup B' = \{x : x \in A' \text{ or } x \in B'\}$$

$$= \{x : x \notin A \text{ and } x \notin B\}$$

$$= \{x : x \notin (A \cap B)\}$$

$$= \{x : x \in (A \cap B)'\}$$

$$= (A \cap B)'$$

$$A' \cup B' = (A \cap B)'$$

$$\boxed{(A \cap B)' = A' \cup B'}$$

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Thank You

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Lec-5

Today's Target

- *Question Based on Set theory*
- *PYQ*
- *DPP*

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP
4	Unit wise set of PYQs
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Algebra of sets

General Identities

1. Idempotent Laws

(a) $A \cup A = A$

(b) $A \cap A = A$

2. Commutative Laws

(a) $A \cup B = B \cup A$

(b) $A \cap B = B \cap A$

3. Associative Laws

(a) $(A \cup B) \cup C = A \cup (B \cup C)$

(b) $(A \cap B) \cap C = A \cap (B \cap C)$

4. Distributive Laws

(a) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

5. DeMorgan's Laws

(a) $(A \cup B)' = A' \cap B'$

(b) $(A \cap B)' = A' \cup B'$

6. Identity Laws

(a) $A \cup \phi = A$

(b) $A \cap U = A$

(c) $A \cup U = U$

(d) $A \cap \phi = \phi$

7. Involution Laws

$(A')' = A$

8. Complement Laws

(a) $A \cup A' = U$

(b) $A \cap A' = \phi$

(a) $U' = \phi$

(b) $\phi' = U$

9. Absorption Laws

(a) $A \cup (A \cap B) = A$

(b) $A \cap (A \cup B) = A$

Q. 1 Let $A = \{a, b\}$ then calculate $A \cup P(A)$, where $P(A)$ is a power set of A.

$A = \{a, b\}$

subset of A

$\phi, \{a\}, \{b\}, \{a, b\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

$$\boxed{A \cup P(A) = \{a, b, \phi, \{a\}, \{b\}, \{a, b\}\}}$$

Q.2 For any set A and B , show that $A - B = A \cap B^c$. V. Imp

To Prove

$A - B = A \cap B^c$

Let $x \in (A - B)$

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in A$ and $x \in B^c$

$\Rightarrow x \in A \cap B^c$

$\Rightarrow A - B \subseteq A \cap B^c$ — (1)

Conversely if

$x \in A \cap B^c$

$\Rightarrow x \in A$ and $x \in B^c$

$\Rightarrow x \in A$ and $x \notin B$

$\Rightarrow x \in (A - B)$

$\Rightarrow A \cap B^c \subseteq A - B$ — (2)

From (1) and (2)

$$\boxed{A - B = A \cap B^c}$$

Q.3 Show that for any two sets A and B : $A - (A \cap B) = A - B$ AKTU-2021-2022

To Prove

$A - (A \cap B) = A - B$

LHS

$A - (A \cap B)$

$= A \cap (A \cap B)^c \quad \{A - B = A \cap B^c\}$

$= A \cap (A^c \cup B^c)$

$= (A \cap A^c) \cup (A \cap B^c)$ (By Distributive law)

$= \phi \cup (A \cap B^c)$

$= A \cap B^c$

$= A - B \quad \{\because A - B = A \cap B^c\}$

$\boxed{LHS = RHS}$

Hence proved

Q.4 If A, B, C are three sets then show that $A \cup (B - A) = A \cup B$

To Prove

$$A \cup (B - A) = A \cup B$$

LHS

$$A \cup (B - A)$$

$$= A \cup (B \cap A') \quad \{\because A - B = A \cap B'\}$$

$$= (A \cup B) \cap (A \cup A')$$

$$= (A \cup B) \cap U$$

$$= A \cup B$$

$$\boxed{LHS = RHS}$$

Hence proved

Q. 5 If A, B, C are three sets then prove that $A-(B\cup C)=\overline{(A-B)}\cap\overline{(A-C)}$

To Prove

$A-(B\cup C)=(A-B)\cap(A-C)$

LHS

$A-(B\cup C)$

$=A\cap(B\cup C)'$ $\{\because A-B=A\cap B'\}$

$=A\cap(B'\cap C')$ $\{$By Demorgan's law$\}$

$=A\cap A\cap(B'\cap C')$ $\{$By idempotent law$\}$

$=(A\cap B')\cap(A\cap C')$

$=(A-B)\cap(A-C)$ $\{\because A-B=A\cap B'\}$

$\boxed{LHS=RHS}$

Hence proved

Q. 6 *If A, B, C are three sets then show that* $A - (B \cap C) = (A - B) \cup (A - C)$.

AKTU-2021-2022

To Prove

$A - (B \cap C) = (A - B) \cup (A - C)$

LHS

$A - (B \cap C)$

$= A \cap (B \cap C)'$ $\{A - B = A \cap B'\}$

$= A \cap (B' \cup C')$ {By Demorgan's law}

$= (A \cap B') \cup (A \cap C')$ {By Distributive law}

$= (A - B) \cup (A - C)$ $\{\because A - B = A \cap B'\}$

$\boxed{LHS = RHS}$

Hence proved

Q.7 If A, B, C are three sets then show that $(A \cap B) \cup (A - B) = A$

To Prove

$(A \cap B) \cup (A - B) = A$

LHS

$(A \cap B) \cup (A - B)$

$= (A \cap B) \cup (A \cap B')$ $\{\because A - B = A \cap B'\}$

$= A \cap (B \cup B')$ {By Distributive law}

$= A \cap U$ $\{\because A \cup A' = U\}$

$= A \cap U$

$= A$

$\boxed{LHS = RHS}$

Hence proved

Q.8 Prove that $A \subseteq B$, then $B' \subseteq A'$

Given

$A \subseteq B$

If $x \in A \implies x \in B$

$x \notin B \implies x \notin A$

$x \in B^c \implies x \in A^c$

$\implies \boxed{B^c \subseteq A^c}$ Hence proved

DSTL (by Gulshan Sir)

DPP-1 to 5

Topic : SET Theory

Q .1 *State De Morgan's law and Absorption law* AKTU-2021-22

Q .2 Prove for any two sets A and B $(A \cup B)' = A' \cap B'$ AKTU-2014-15

Q .3 $Let\ A = \{a, \phi\}\ then\ calculate\ A \cup P(A), where\ P(A) is\ a\ power\ set\ of\ A$ AKTU-2022-23

Q .4 *List the subsets of the set* $\mathrm{A} = \{0, 1, 2\}$

Q .5 *List the subset of the set* $\mathrm{B} = \{1, \{2, 3\}\}$

Q .6 *If* $A = \{1, 2, 3, 4\}, B = \{2, 3, 4, 5\}$ *and* $C = \{3, 4, 5, 6\}$, *find* $A - B,\ B - C,\ A - (B - C),\ (A - B) - C$

Q .7 *If* $A = \{1, 2, 3, 4\},\ B = \{2, 4, 6, 8\}$ *and* $C = \{3, 4, 5, 6\}$ *find* *(i)* $(A \cap B) \cap C$ *(ii)* $A \cap (B \cap C)$

Q .8 *Prove that* $A \cap (B - C) = (A \cap B) - (A \cap C)$

Q .9 *Prove that* $A \cap (B - C) = (A \cap B) - (A \cap C)$

Q .10 *Prove that* $A - (B - C) = (A - B) \cup (A \cap C)$

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) Set Theory & $\boxed{\text{Relations}}$

(ii) POSET & Lattices

Lec-6

Today's Target

- *Ordered Pair*
- *Cartesian product of two sets*
- *Relation or Binary Relation, Relation on a set*
- *Domain and Range of relation*
- *Operation on relation*

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP
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Ordered Pair:- An ordered pair consist of two elements grouped together in a particular order

Example :- $(1, 4)$

In the ordered pair (a, b)

(i) a is called first element

(ii) b is called second element

Universal set

Cartesian product of two sets :- The set of all possible ordered pairs (a, b), where $a \in A$ and $b \in B$ is called the Cartesian product of set A and B and is denoted by $A \times B$

usually $\boxed{A \times B \neq B \times A}$

Note :- If $n(A) = m$ and $n(B) = n$ then

$$n(A \times B) = mn$$

Example :- *If* $A = \{1, 2\}$ *and* $B = \{a, b, c\}$ *then find*

(i) $A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$ $\boxed{n(A \times B) = 6}$

(ii) $B \times A = \{(a, 1) (a, 2) (b, 1) (b, 2) (c, 1) (c, 2)\}$ $\boxed{n(B \times A) = 6}$

(iii) $A \times A = \{(1, 1) (1, 2) (2, 1) (2, 2)\}$ $\boxed{n(A \times A) = 4}$

(iv) $B \times B = \{(a, a) (a, b) (a, c) (b, a) (b, b) (b, c) (c, a) (c, b) (c, c)\}$

$\boxed{n(B \times B) = 9}$

Relation or Binary relation :- Let A and B be two non-empty sets, then any subset R of the Cartesian product $A \times B$ is called a relation from set A to set B

$$\boxed{R \subseteq A \times B}$$

$$R = \{(x,\ y) : x \in A, y \in B \ \text{ and } \ x\,R\,y\}$$

Note :-

(i) If $(x,\ y) \in R$ then we say that 'x is related to y' and we write $x\,R\,y$

(ii) If $(x,\ y) \notin R$ then we say that 'x is not related to y' and we write $x \not{R}\, y$

(iii) If $n(A) = m$ and $n(B) = n$, then $n(A \times B) = mn$

$$\boxed{\text{Number of relations from } A \text{ to } B = \text{Total number of subsets of } A \times B = 2^{mn}}$$

Example : *If* $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ *and* $B = \{1, 2, 3, 4, 5\}$ *and* $R = \{(a, b): a \in A, b \in B \text{ and } a = 2b\}$ *then find* R

Given

$a = 2b$

$R = \{(2, 1), (4, 2), (6, 3), (8, 4)\}$

Relation on a set:- A Relation R from set A to the set A itself is called a relation on set A

$$R \subseteq A \times A$$

$$\boxed{R = \{(x,\ y) : x \in A, y \in B \ \ and \ \ x\, R\, y\}}$$

Example :- If $A = \{1, 2, 3\}$. Find relation on A defined by " is less than equal to "

$A = \{1, 2, 3\}$, $A = \{1, 2, 3\}$

$A \times A = \{(1,1)(1,2)(1,3)(2,1),(2,2),(2,3)(3,1)(3,2)(3,3)\}$

$1 = 1 \implies (1,1) \in R$
$1 < 2 \implies (1,2) \in R$
$1 < 3 \implies (1,3) \in R$
$2 \nleq 1 \implies (2,1) \notin R$

$2 = 2 \implies (2,2) \in R$
$2 < 3 \implies (2,3) \in R$
$3 \nleq 1 \implies (3,1) \notin R$
$3 \nleq 2 \implies (3,2) \notin R$

$3 = 3 \implies (3,3) \in R$

$$R = \{(1,1)(1,2)(1,3)(2,2)(2,3)(3,3)\}$$

Domain and Range of Relation :-

Let $R = \{(x,\ y) \colon x \in A,\ \ y \in B\ \ and\ \ x\,R\,y\}$

Domain :- The set of all first elements of the ordered pairs belonging to R is called the domain of R. it is denoted by Dom(R) or d(R)

$d(R) = \{x : x \in A \text{ and } (x, y) \in R\}$

Range :- The set of all second elements of the ordered pairs belonging to R is called the range of R.

it is denoted by Ran(R) or r(R)

$r(R) = \{y : y \in B \text{ and } (x, y) \in R\}$

Example :- Let $A = \{2, 4, 6\}$ and $B = \{1, 4, 5, 6\}$ then find out the relation from A to B defined by "is less than or equal to". Find out the domain and range of the relation

$A \times B = \{(2,1)\ (2,4)\ (2,5)\ (2,6)\ (4,1)\ (4,4)\ (4,5)\ (4,6)\ (6,1)\ (6,4)\ (6,5)\ (6,6)\}$

Relation from A to B "is less than or equal to"

$2 \nleq 1 \Rightarrow (2,1) \notin R$

$2 < 4 \Rightarrow (2,4) \in R$

$2 < 5 \Rightarrow (2,5) \in R$

$2 < 6 \Rightarrow (2,6) \in R$

$4 \nleq 1 \Rightarrow (4,1) \notin R$

$4 = 4 \Rightarrow (4,4) \in R$

$4 < 5 \Rightarrow (4,5) \in R$

$4 < 6 \Rightarrow (4,6) \in R$

$6 \nleq 1 \Rightarrow (6,1) \notin R$

$6 \nleq 4 \Rightarrow (6,4) \notin R$

$6 \nleq 5 \Rightarrow (6,5) \notin R$

$6 = 6 \Rightarrow (6,6) \in R$

$\boxed{R = \{(2,4)\ (2,5)\ (2,6)\ (4,4)\ (4,5)\ (4,6)\ (6,6)\}}$

$\boxed{d(R) = \{2, 4, 6\}}$

$\boxed{r(R) = \{4, 5, 6\}}$

Operation on relation

(a) Complement of Relation

(b) Inverse relation

(5) Intersection and Union of Relation

Complement of a Relation

Consider a relation R from set the set A to B. The complement of relation R denoted by $\overline{\mathrm{R}}$ or R' is a relation from A to B such that

$$\boxed{\bar{R} = \{(a,b) : (a,b) \notin R\}}$$

Example : *Let R be a relation from X to Y, where $X = \{1, 2, 3\}$ and $Y = \{8, 9\}$ and $R = \{(1, 8), (2, 8)(1, 9), (3, 9)\}$ find the complement of relation R*

$R = \{(1,8), (2,8) (1,9) (3,9)\}$

$X \times Y = \{\underset{\times}{(1,8)} \underset{\times}{(1,9)} \underset{\times}{(2,8)} (2,9) (3,8) \underset{\times}{(3,9)}\}$ Universal set

$$\boxed{\bar{R} = \{(2,9) (3,8)\}}$$

Inverse Relation

Let R be relation from a set A to B. The inverse of relation R denoted by R^{-1} is a relation from B to A such that

$$\boxed{R^{-1} = \{(b,a) : (a,b) \in R\}}$$

Thus , to find R^{-1} we write in reverse order all ordered pairs belonging to R.

Example: Find the R^{-1} to the relation R on A defined "$x + y$" divisible by 2.

For $A = \{1, 2, 3, 4, 6\}$.

$R = \{(1,1)\ (1,3)\ (2,2)\ (2,4)\ (2,6)\ (3,1)\ (3,3)\ (4,2)$
$(4,4)\ (4,6)\ (6,2)\ (6,4)\ (6,6)\}$

$R^{-1} = \{(1,1)\ (3,1)\ (2,2)\ (4,2)\ (6,2)\ (1,3)\ (3,3)\ (2,4)\ (4,4)\ (6,4)\ (2,6)$
$(4,6)\ (6,6)$

Intersection and Union of Relation

If R and S are the two relations then intersection of R and S denoted by $R \cap S$ and the union of R and S denoted by $R \cup S$ are two new relation that can be formed from R and S.

Thus $R \cup S = \{(x, y): xRy \text{ or } xSy\}$

$R \cap S = \{(x, y): xRy \text{ and } xSy\}$

Example : Let $R = \{(1,1), (2, 2), (3, 3), (4, 4), (3, 4), (4, 3)\}$

and $S = \{(1,1), (2, 2), (3, 3), (4, 4), (1, 2), (2, 1)\}$. Find $R \cup S$ and $R \cap S$

$R \cup S = \{(1,1)(2,2)(3,3)(4,4)(3,4)(4,3)(1,2)(2,1)$

$R \cap S = \{(1,1)(2,2)(3,3)(4,4)$

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Thank You

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Lec-7

Today's Target

- *Properties of Relation*

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
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Link in Description

Properties of Relation (Types of Relation)

1. **Void Relation:-** A Relation R in a set A is called a void relation or an empty relation, if no element of A is related to any element of A and we denote such a relation by ϕ

Thus

$$\boxed{R = \phi}$$

where $\phi \subseteq A \times A$

Example :- Consider a relation R on the set $A = \{1, 2, 3, 4\}$ and defined by $R = \{(a, b): a - b = 12\}$

$R = \{ \ \} = \phi$

2. Universal Relation:- *A Relation R in a set A is called a universal relation, if each element of A is related to every element of A*

Thus

$$\boxed{R = A \times A}$$

where $A \times A \subseteq A \times A$

Example :- *Consider a relation R on the set* $A = \{2, 3, 4\}$ *and* $R = \{(a, b) : a + b < 9\}$

$A \times A = \{(2,2)(2,3)(2,4)(3,2)(3,3)(3,4)(4,2)(4,3)(4,4)\}$

$R = \{(2,2)(2,3)(2,4)(3,2)(3,3)(3,4)(4,2)(4,3)(4,4)\}$

3. Identify Relation:- *A Relation R in a set A is called an identify relation, if every element of A is related to it self only .*

Example :- *Consider the relation R on set* $A = \{1, 2, 3, 4\}$ *and* $R = \{(a, b) : a \div b = 1\}$

$$R = \{(1,1)(2,2)(3,3)(4,4)\}$$

4. Reflexive Relation:- A relation R on set A is reflexive if every element of A is related to itself.

Imp

$$\boxed{\text{Thus } R \text{ is Reflexive} \Leftrightarrow (a,a) \in R \ for\ all\ \ a \in A}$$

aRa for all a ∈ A

Example : Consider the relation R on the set $A = \{1,2,3,4\}$ and $R = \{(a,b): a,b \in A,\ (a \div b) \in Integer\}$

$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (2,1)\ (3,1)\ (4,1)\ (4,2)\}$

Note :- Every identify relation is reflexive but every reflexive relation is not identify relation.

5. Irreflexive Relation:- A Relation R on Set A is irreflexive if

$(a, a) \notin R$ for all $a \in A$

Example:- Let $A = \{1, 2\}$ and $R = \{(1, 2), (2, 1)\}$

$a \not{R} a \ \forall \ a \in A$

6. Non-reflexive Relation:- A Relation R on a Set A is non-reflexive if R is neither reflexive nor irreflexive.

Example:- Let $A = \{1, 2\}$ and $R = \{(1, 2), (2, 1)(1, 1)\}$

7. Symmetric Relation:- Imp A relation R on Set A is symmetric relation if a is related to b then b is also related to a.

$$\boxed{\text{Thus R is symmetric if } (a,b) \in R \Longrightarrow (b, a) \in R \ \ for\ all\ \ a,\ b \in A}$$

$aRb \Rightarrow bRa\ \forall\ a, b \in A$

Example 8 :-Consider the relation R_1 and R_2 on set $A = \{2, 4, 5, 6\}$.

$$R_1 = \{(2,4), (4,2), (4,5), (5,4), (6,6)\} \text{ and } R_2 = \{(2,4), (2,6), (6,2), (5,4), (4,5)\}$$

For R_1

$(2,4) \in R_1 \Longrightarrow (4,2) \in R_1$

$(4,5) \in R_1 \Longrightarrow (5,4) \in R_1$

$(6,6) \in R_1 \Longrightarrow (6,6) \in R_1$

Hence R_1 is symmetric

For R_2

$(2,4) \in R_2 \Longrightarrow (4,2) \notin R_2$

Hence R_2 is not symmetric

8. Asymmetric Relation:- A Relation R on set A is asymmetric if $(a, b) \in R$ then $(b, a) \notin R$ for $a \neq b$.

Example:- Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (1, 1), (2, 3), (3, 1)\}$ is a asymmetric relation.

$(1,2) \in R \Rightarrow (2,1) \notin R$

$(2,3) \in R \Rightarrow (3,2) \notin R$

$(3,1) \in R \Rightarrow (1,3) \notin R$

Hence R is Asymmetric Relation

9. Antisymmetric Relation:- A Relation R is said to be antisymmetric if $(a, b) \in R$ and $(b, a) \in R$ for $a = b$

Example:- The relation $R = \{(1, 1), (3, 3)\}$ is Antisymmetric Relation on $A = \{1, 2, 3\}$

$A \times A = \{(1,1)(1,2)(1,3)(2,1)(2,2)(2,3)(3,1)(3,2)(3,3)\}$

$R = \{(1,1)(3,3)\}$

10. Transitive Relation:- The Relation R on set A is transitive relation if a is related to b, b is related to c then a is also related to c

Imp

$$\boxed{\text{Thus R is transitive if } (a,b) \in R \ \ and \ \ (b,\ c) \in R \Rightarrow (a,\ c) \in R \ \ forall\ \ a,\ b, c \in A}$$

Thus R is transitive if aRb and bRc $\Rightarrow$ aRc $\forall$ a, b, c $\in$ A

Example:- Consider a relation R on set $A = \{1, 2, 3, 4\}$ and $R = \{(a, b): a < b\}$

$A \times A = \{(1,1)(1,2)(1,3)(1,4)(2,1)(2,2)(2,3)(2,4)(3,1)(3,2)(3,3)(3,4)$
$(4,1)(4,2)(4,3)(4,4)\}$

$R = \{(1,2)(1,3)(1,4)(2,3)(2,4)(3,4)\}$

$(1,2) \in R$ and $(2,3) \in R \Rightarrow (1,3) \in R$

$(1,2) \in R$ and $(2,4) \in R \Rightarrow (1,4) \in R$

$(1,3) \in R$ and $(3,4) \in R \Rightarrow (1,4) \in R$

$(2,3) \in R$ and $(3,4) \in R \Rightarrow (2,4) \in R$

Hence R is transitive

V.imp

7 Marks

Equivalence Relation:- A Relation R on set A is said to be an equivalence relation if and only if it is Reflexive, Symmetric and transitive simultaneously.

NOTE: A relation R in a set A is called

(i) Reflexive if $(a, a) \in R \quad for\ all\ \ a \in A$

(ii) Symmetric if $(a, b) \in R \Rightarrow (b,\ a) \in R \quad fo\ all \quad a,\ b \in A$

(iii) Transitive if $(a, b) \in R \ \ and \ \ (b,\ c) \in R \Rightarrow (a,\ c) \in R \quad for\ all \quad a,\ b, c \in A$

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Thank You

DISCRETE STRUCTURES & THEORY OF LOGICS
(Discrete Mathematics)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Lec-8

Today's Target

- *Equivalence relation*
- *PYQ*
- *DPP*

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP
4	Unit wise set of PYQs

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Link in Description

Equality of Relation
Or
Equivalence Relation

Let A be a non-empty set and R be a relation defined on set A, then R is said to be equivalence relation if it is

(i) Reflexive: $(a, a) \in R \ \forall \ a \in A$

$$\boxed{aRa \ \forall \ a \in A}$$

For all

(ii) Symmetric: If $(a, b) \in R \Rightarrow (b, a) \in R \ \forall \ a, b \in A$

$$\boxed{aRb \Rightarrow bRa \ \forall \ a, b \in A}$$

(iii) Transitive: If $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \ \forall \ a, b, c \in A$

$$\boxed{aRb \text{ and } bRc \Rightarrow aRc \ \forall \ a, b, c \in A}$$

Equivalence class

Let R be an equivalence relation on a non-empty set A. The equivalence class of an element $a \in A$ is

the set of elements of A which are related to 'a'

It is denoted by [a] or $\overline{a}$

$$[a] = equivalence\ class\ of\ a$$

$$[a] = \{x : x \in A, x\ R\ a\}$$

Q.1. Let $A = \{a, b, c\}$ and let $\boxed{R = \{(a,a),(b,b),(c,c),(a,b),\ (b,a)\}}$ Where R is clearly an equivalence relation. Find the equivalence classes of the elements of A.

Equivalence classes

$[a] = \{a, b\}$

$[b] = \{a, b\}$

$[c] = \{c\}$

Q.2. *Let* $A = \{0, 1, 2, 3, 4\}$. *Show that the relation* $R = \{(0,0), (0,4), (1,1), (1,3), (2,2), (3,1), (3,3), (4,0), (4,4)\}$ *is an equivalence relation. Also find the distinct equivalence Classes of R.*

$A = \{0, 1, 2, 3, 4\}$

$R = \{(0,0), (1,1), (2,2), (3,3), (4,4), (0,4), (1,3), (3,1), (4,0)\}$

Reflexive:

since $(0,0) \in R$
$(1,1) \in R$
$(2,2) \in R$
$(3,3) \in R$
$(4,4) \in R$

$\Rightarrow (a, a) \in R \ \forall a \in A$

$\Rightarrow a R a \ \forall a \in A$

So, R is Reflexive

Symmetric

since $(0,4) \in R \Rightarrow (4,0) \in R$
$(1,3) \in R \Rightarrow (3,1) \in R$
$(3,1) \in R \Rightarrow (1,3) \in R$
$(4,0) \in R \Rightarrow (0,4) \in R$

$\Rightarrow (a,b) \in R \Rightarrow (b,a) \ \forall \ a,b \in A$

$\Rightarrow aRb \Rightarrow bRa \ \forall \ a,b \in A$

so, R is symmetric

Transitive:

$(0,4) \in R$ and $(4,0) \in R \Rightarrow (0,0) \in R$

$(0,4) \in R$ and $(4,4) \in R \Rightarrow (0,4) \in R$

$(1,3) \in R$ and $(3,1) \in R \Rightarrow (1,1) \in R$

$(1,3) \in R$ and $(3,3) \in R \Rightarrow (1,3) \in R$

$(4,0) \in R$ and $(4,4) \in R \Rightarrow (4,4) \in R$

$\Rightarrow (a,b) \in R$ and $(b,c) \in R \Rightarrow$

$(a,c) \in R \ \forall \ a,b,c \in A$

$\Rightarrow aRb$ and $bRc \Rightarrow aRc \ \forall \ a,b,c \in A$

so, R is transitive

Equivalence classes

$[0] = \{0,4\}$

$[1] = \{1,3\}$

$[2] = \{2\}$

$[3] = \{1,3\}$

$[4] = \{0,4\}$

Q.3 Let $X = \{1, 2, 3, \ldots\ldots 7\}$ and $R = \{(x, y) : x - y$ is divisible by $3\}$. Show that R is an equivalence relation.

R is an equivalence Relation if

(i) Reflexive: For any $x \in X$

$x - x = 0$, which is divisible by 3

$\Rightarrow (x, x) \in R \;\forall\; x \in X$

So, R is Reflexive

(ii) Symmetric: Let $x, y \in X$, such that $(x, y) \in R$

$\Rightarrow x - y$ is divisible by 3

$\Rightarrow x - y = 3n \quad , n \in I$

$\Rightarrow -(x - y) = -3n$

$\Rightarrow y - x = -3n$

$\Rightarrow (y, x) \in R$

So, R is symmetric

Transitive :

Let $x, y, z \in X$ such that

$(x, y) \in R$ and $(y, z) \in R$

$\Rightarrow (x-y)$ is divisible by 3 and $(y-z)$ is divisible by 3

$\Rightarrow x-y = 3n_1$ and $y-z = 3n_2$

$\Rightarrow x-\cancel{y}+\cancel{y}-z = 3n_1+3n_2$

$\Rightarrow x-z = 3(n_1+n_2)$

$\Rightarrow (x, z) \in R$

So, R is transitive

Hence

R is an equivalence Relation

Q4: Let $A = \{1,2,3,4,6,7,8,9\}$ and let R be the relation on $A \times A$ defined by $(a,b)\ R\ (c,d)$ if $a+d = b+c$ for all $(a,b),\ (c,d) \in A \times A$. Prove that

(a) R is an equivalence relation .

(b) Find $[(2,5)]$.

R is an equivalence Relation if

(1) Reflexive: For any $(a,a) \in A \times A$

$(a,a)\ R\ (a,a)$ if $a+a = a+a$ (true)

So, R is Reflexive

Symmetric:

Let $(a,b), (c,d) \in A \times A$ such that $(a,b)\ R\ (c,d)$

$\Rightarrow a+d = b+c$

$\Rightarrow b+c = a+d$

$\Rightarrow c+b = d+a$

$\Rightarrow (c,d)\ R\ (a,b)$

So, R is symmetric

Transitive:

Let $(a,b), (c,d), (e,f) \in A \times A$ such that

$(a,b) R (c,d)$ and $(c,d) R (e,f)$

$\Rightarrow a+d = b+c$ and $c+f = d+e$

$\Rightarrow a+\cancel{d}+\cancel{c}+f = b+\cancel{c}+\cancel{d}+e$

$\Rightarrow a+f = b+e$

$\Rightarrow (a,b) R (e,f)$

$(a,b) R (c,d)$

$a+d = b+c$

So, R is transitive

Hence

R is an equivalence Relation

Equivalence class

$$\boxed{[(2,5)] = \{(1,4)(2,5)(3,6)(4,7)(5,8)(6,9)\}}$$

Q.5: Let $A = R \times R$ (R be a set of Real numbers and define the following relation on A $(a,b)\ R\ (c,d) \Leftrightarrow a^2+b^2=c^2+d^2$. Verify that R is an equivalence relation

R is an equivalence Relation if

(I) Reflexive : For any $(a,a) \in A$

$(a,a) R (a,a)$ if $a^2+a^2 = a^2+a^2$ (True)

so, R is Reflexive

(II) Symmetric

Let $(a,b), (c,d) \in A$ such that

$(a,b) R (c,d)$

$\Rightarrow a^2+b^2 = c^2+d^2$

$\Rightarrow c^2+d^2 = a^2+b^2$

$\Rightarrow (c,d) R (a,b)$

so, R is symmetric

(III) Transitive :

Let $(a,b) (c,d) (e,f) \in A$

such that

$(a,b)\,R\,(c,d)$ and $(c,d)\,R\,(e,f)$

$\Rightarrow a^2+b^2=c^2+d^2$ and $c^2+d^2=e^2+f^2$

$\Rightarrow a^2+b^2+\cancel{c^2}+\cancel{d^2}=\cancel{c^2}+\cancel{d^2}+e^2+f^2$

$\Rightarrow a^2+b^2=e^2+f^2$

$\Rightarrow (a,b)\,R\,(e,f)$

So, R is transitive

Hence, R is an equivalence Relation

NOTE: Two integers a and b are said to be congruence modulo m if a-b is divisible by m and we write

$a \equiv b \pmod{m}$

Thus, $a \equiv b \pmod{m} \iff (a-b)$ is divisible by m

Q.6: Let A be the set of all integers and a relation R is defined as

$\boxed{R = \{(x, y) : x \equiv y \pmod{m},\, m\ divide\ (x - y)\}}$ where m is a positive integer.

Prove that R is an equivalence relation.

R is an equivalence Relation if

(I) Reflexive: For any $x \in A$

$x - x = 0$ which is divisible by m

$\Rightarrow x \equiv x \pmod{m}$

$\Rightarrow xRx \ \forall x \in A$

So, R is Reflexive

(II) Symmetric:

Let $x, y \in A$ such that

$(x, y) \in R$

$\Rightarrow x - y$ is divisible by m

$\Rightarrow x \equiv y \pmod{m}$

$\Rightarrow x-y = mn \qquad n \in I$

$\Rightarrow -(x-y) = -mn$

$\Rightarrow (y-x) = -mn$

$\Rightarrow (y,x) \in R$

$\Rightarrow y \equiv x \pmod m$

So, R is symmetric

Transitive

Let $x, y, z \in A$ such that

$(x,y) \in R$ and $(y,z) \in R$

$(x-y)$ is divisible by m and $(y-z)$ is divisible by m

$x \equiv y \pmod m$ and $y \equiv z \pmod m$

$\Rightarrow x-y = mn_1$ and $y-z = mn_2$

$\Rightarrow x-\cancel{y}+\cancel{y}-z = mn_1+mn_2$

$\Rightarrow x-z = m(n_1+n_2)$

$\Rightarrow (x-z)$ is divisible by m

$\Rightarrow x \equiv z \pmod{m}$

$\Rightarrow (x, z) \in R$

So, R is transitive

Hence

R is an equivalence Relation

DPP- 6 to 8

Topic : Equivalence Relation

Q.1. *Let* $N = \{1, 2, 3, ----- -\}$ and a relation is defined in N X N as follows: (a, b) is related to (c, d) iff $\mathrm{ad} = \mathrm{bc}$. *Then show that whether R is an equivalence relation or not.*

Q.2. *Let R be a binary relation defined as* $R = \{(a, b) \in R^2 : (a - b) \leq 3\}$

Determine whether R is an equivalence relation.

Q.3. *Let S be the set of all points in a plane. Let R be a relation such that for any two points a and b* $(a,\ b) \in R$ *is not an equivalence relation.*

Thank You

DISCRETE STRUCTURES & THEORY OF LOGICS
(Discrete Mathematics)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Lec-9

Today's Target

- *Important Theorem on Equivalence relation*
- *Composition of relation*
- *PYQ*
- *DPP*

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
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Equivalence Relation

Let A be a non-empty set and R be a relation defined on set A, then R is said to be equivalence relation if it is

(i) Reflexive: $(a, a) \in R \ \forall \ a \in A$

i.e $aRa \ \forall \ a \in A$

(ii) Symmetric: $(a, b) \in R \Rightarrow (b, a) \in R \ \forall \ a, b \in A$

i.e $aRb \Rightarrow bRa \ \forall \ a, b \in A$

(iii) Transitive: $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \ \forall \ a, b, c \in A$

i.e aRb and $bRc \Rightarrow aRc \ \forall \ a, b, c \in A$

Theorem-1

If R is an equivalence relation on A, then prove that R^{-1} is also an equivalence relation on A .

Given

since R is an equivalence relation

(i) R is Reflexive

(ii) R is symmetric

(iii) R is transitive

To Prove

R^{-1} is also an equivalence relation

Proof :

(i) Reflexive : For any $x \in A$

$(x, x) \in R$ $\{\because R \text{ is reflexive}\}$

$\Rightarrow (x, x) \in R^{-1} \ \forall \ x \in A$

So, R^{-1} is reflexive

(II) Symmetric:

Let $x, y \in A$ such that

$(x, y) \in R \Rightarrow (y, x) \in R$

$\{\because R$ is symmetric$\}$

$\Rightarrow (y, x) \in R^{-1} \Rightarrow (x, y) \in R^{-1}$

$\Rightarrow (x, y) \in R^{-1} \Rightarrow (y, x) \in R^{-1}$

So, R^{-1} symmetric

Transitive: Let $x, y, z \in A$ such that

$(x, y) \in R$ and $(y, z) \in R \Rightarrow (x, z) \in R$

$\{\because R$ is transitive$\}$

$(y, x) \in R^{-1}$ and $(z, y) \in R^{-1} \Rightarrow (z, x) \in R^{-1}$

$(z, y) \in R^{-1}$ and $(y, x) \in R^{-1} \Rightarrow (z, x) \in R^{-1}$

So, R^{-1} is transitive Relation

Hence

R^{-1} is also an equivalence Relation

Theorem-2

If R_1 and R_2 are two equivalence relations on set A, then prove that $R_1 \cap R_2$ is also an equivalence relation.

Given

R_1 and R_2 are equivalence relation

(i) R_1 and R_2 are reflexive

(ii) R_1 and R_2 are symmetric

(iii) R_1 and R_2 are transitive

To Prove

$R_1 \cap R_2$ is also an equivalence Relation

Proof

(i) Reflexive For any $x \in A$

$(x, x) \in R_1$ $\{\because R_1$ is Reflexive$\}$

$(x, x) \in R_2$ $\{\because R_2$ " $\}$

$\Rightarrow (x, x) \in R_1 \cap R_2 \ \forall x \in A$

So, R is reflexive

(ii) Symmetric: Let $x, y \in A$ such that

$(x, y) \in R_1 \Rightarrow (y, x) \in R_1$ {R is symmetric}

$(x, y) \in R_2 \Rightarrow (y, x) \in R_2$ { " " }

$\Rightarrow (x, y) \in R_1 \cap R_2 \Rightarrow (y, x) \in R_1 \cap R_2 \ \forall \ x, y \in A$

So, $R_1 \cap R_2$ is symmetric

(iii) Transitive:

Let $x, y, z \in A$ such that

$(x, y) \in R_1$ and $(y, z) \in R_1 \Rightarrow (x, z) \in R_1$

$(x, y) \in R_2$ and $(y, z) \in R_2 \Rightarrow (x, z) \in R_2$

{$\because$ R_1 and R_2 are transitive}

$\Rightarrow (x, y) \in R_1 \cap R_2$ and $(y, z) \in R_1 \cap R_2$

$\Rightarrow (x, z) \in R_1 \cap R_2$

So, $R_1 \cap R_2$ is transitive

Hence, $R_1 \cap R_2$ is also an equivalence relation

Theorem-3

If R_1 and R_2 are two equivalence relations on set A, then prove that $R_1 \cup R_2$ is not necessarily an equivalence relation.

Example: Let $A = \{1, 2, 3\}$ and Equivalence relation. $R_1 = \{(1,1)(2,2)(3,\ 3)\}, (1,2), (2,1)$ and $R_2 = \{(1,1)(2,2)(3,\ 3)(2,3), (3,2)\}$, Show that $R_1 \cup R_2$ is not an equivalence relation.

Proof: $R_1 \cup R_2 = \{(1,1)(2,2)(3,3)(1,2)(2,1)(2,3)(3,2)\}$

since $(3,2) \in R_1 \cup R_2$ and $(2,1) \in (R_1 \cup R_2)$

But $(3,1) \notin R_1 \cup R_2$

So, $R_1 \cup R_2$ is not an equivalence Relation

Composition of Relations
or
Composite Relation

Let A, B and C be three non-empty sets.

Let R is a relation from A to B   i.e $R \subseteq A \times B$

and S is a relation from B to C.   i.e $S \subseteq B \times C$

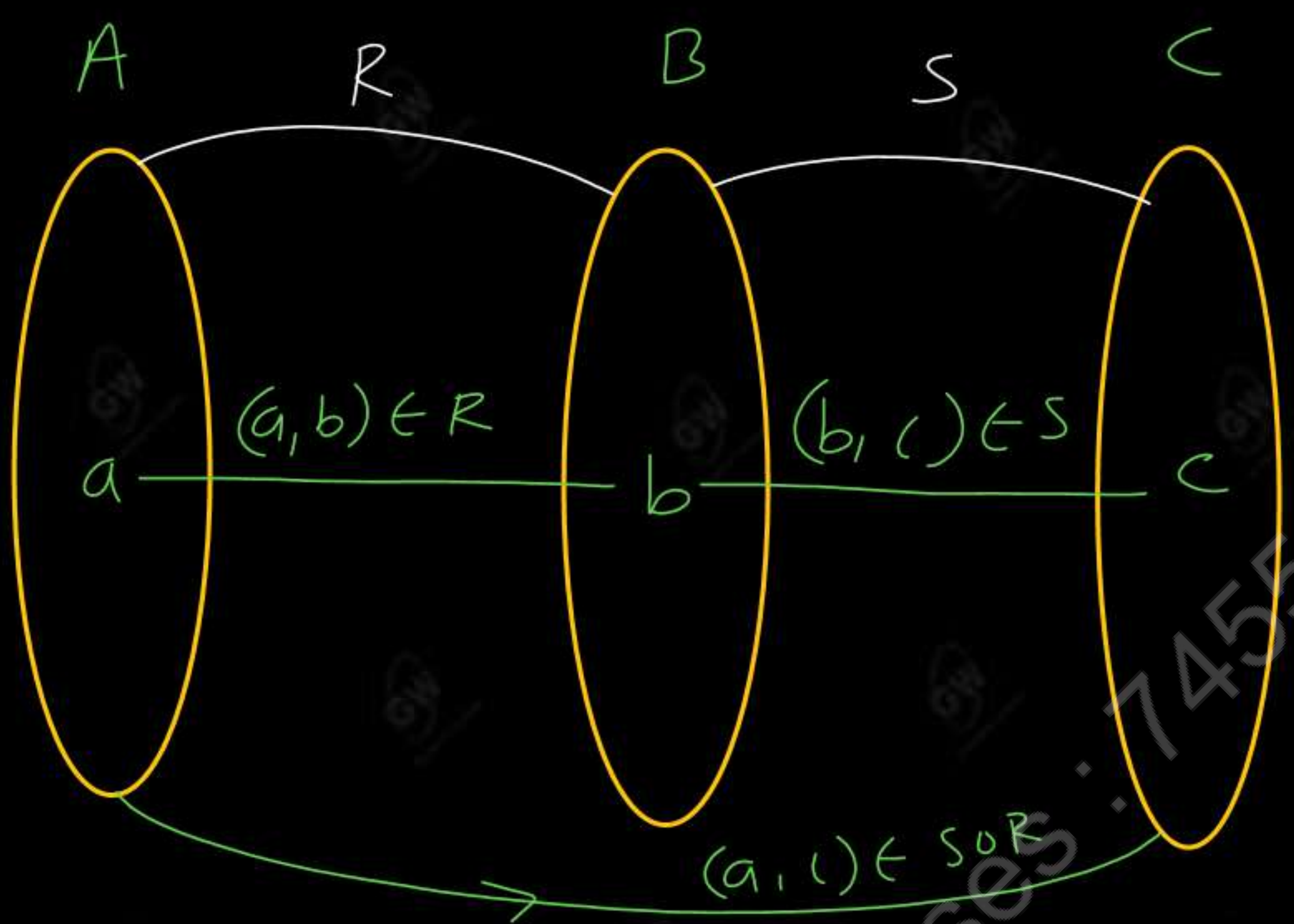
Then the composition of Relation of R and S denoted by SoR, is a relation from A to C and and defined as

$$\boxed{SoR = \{(a,c) \in A \times C : \text{for some } b \in B \text{ such that } (a,b) \in R \text{ and } (b,c) \in S\}}$$

Where $a \in A,\ b \in B,\ c \in C$

Hence we can say

$$\boxed{(a,b) \in R,\ (b,c) \in S \Rightarrow (a,c) \in SoR}$$



Note: (i) If R and S are relation from A to B and C to D, then SoR is not defined unless B=C

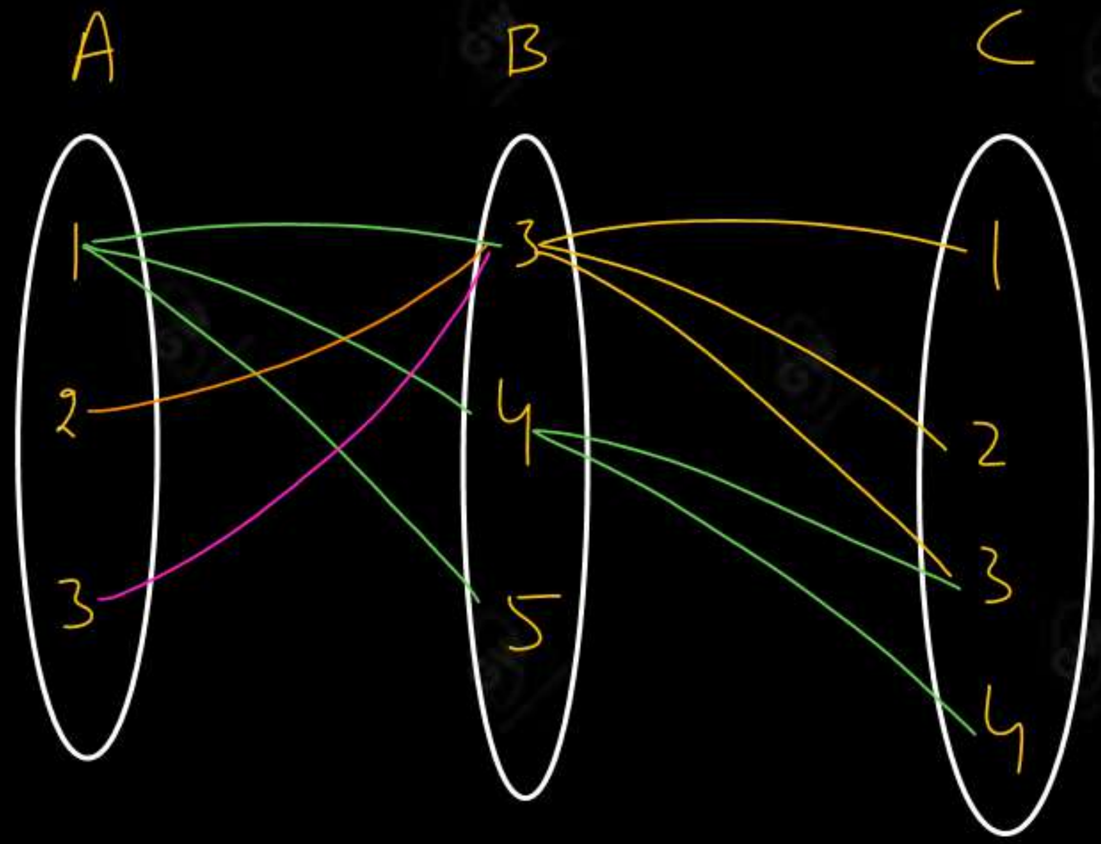
(ii) The composition of R with R : $RoR = R^2$

(iii) The composition of R^2 with R : $R^2oR = R^3 = RoRoR$

(iv) The composition of R with SoR : $Ro\,(SoR)$

Q.1. Let $A = \{1, 2, 3\}$ $B = \{3, 4, 5\}$, $C = \{1, 2, 3, 4\}$ and Let $R = \{(1,3)(1,4)(1,5)(2,3)(3,3)\}$

$S = \{(3,1)(3,2)(3,3)(4,3)(4,4)\}$. compute $S \circ R$, $R \circ S$



$S \circ R = \{(1,1)(1,2)(1,3)(1,4)(2,1)$
$(2,2)(2,3)(3,1)(3,2)(3,3)\}$

$R = \{(1,3)(1,4)(1,5)(2,3)(3,3)\}$

$S = \{(3,1)(3,2)(3,3)(4,3)(4,4)\}$

$S \circ R = \{(1,1)(1,2)(1,3)(1,4)(2,1)(2,2)$
$(2,3)(3,1)(3,2)(3,3)\}$

Q.2 Let $R = \{(1,2)(3,4)(2,\ 2)\}$ and $S = \{(4,2)(2,5)(3,\ 1)(1,3)\}$

Find (i) SoR (ii) RoS (iii) RoR (iv) $RoRoR$ (v) $Ro\,(SoR)$ (vi) $(RoS)oR$

(i) SOR

$R = \{(1,2)(3,4)(2,2)\}$

$S = \{(4,2)(2,5)(3,1)(1,3)\}$

$SOR = \{(1,5)(3,2)(2,5)\}$

✓✓

(ii) ROS

$S = \{(4,2)(2,5)(3,1)(1,3)\}$

$R = \{(1,2)(3,4)(2,2)\}$

$ROS = \{(4,2)(3,2)(1,4)\}$

✓✓

(III) $R \circ R$

$R = \{(1,2)(3,4)(2,2)\}$

$R = \{(1,2)(3,4)(2,2)\}$

$R \circ R = \{(1,2)(2,2)\}$

$R^2 = \{(1,2)(2,2)\}$

(IV) $R \circ R \circ R = R^2 \circ R$

$R = \{(1,2)(3,4)(2,2)\}$

$R^2 = \{(1,2)(2,2)\}$

$R \circ R \circ R = \{(1,2)(2,2)\}$

$R^3 = \{(1,2)(2,2)\}$

(V) $R \circ (S \circ R)$

$S \circ R = \{ \quad \}$

$R = \{ \quad \}$

(VI) $(R \circ S) \circ R$

$R = \{ \quad \}$

$R \circ S = \{ \quad \}$

Theorem:- *Let R be a relation from the set A to the Set B and S be a relation from the Set B to Set C, then*

$$(SOR)^{-1} = R^{-1}\, O\, S^{-1}$$ ***(Reversal law in composite relation)***

Proof

Let $(c, a) \in (SOR)^{-1}$

$\Rightarrow (a, c) \in SOR$

$\Rightarrow$ For $b \in B$ such that

$\Rightarrow (a, b) \in R$ and $(b, c) \in S$

$\Rightarrow (b, a) \in R^{-1}$ and $(c, b) \in S^{-1}$

$\Rightarrow (c, b) \in S^{-1}$ and $(b, a) \in R^{-1}$

$\Rightarrow (c, a) \in R^{-1} o S^{-1}$

Hence

$$\boxed{(SOR)^{-1} = R^{-1} o S^{-1}}$$

Hence proved

Q.3. Let $A = \{a,\ b\}$

$$R = \{a,\ a\}(b, a)(b, b)\}$$

$$S = \{a,\ b\}(b, a)(b, b)\}$$

Then verify $(SOR)^{-1} = R^{-1}\, O\, S^{-1}$

LHS

$$SOR = \{(a,b)(b,b)(b,a)\}$$

$$(SOR)^{-1} = \{(b,a)(b,b)(a,b)\} \quad (1)$$

RHS

$$S^{-1} = \{(b,a)(a,b)(b,b)\}$$

$$R^{-1} = \{(a,a)(a,b)(b,b)\}$$

$$R^{-1} o S^{-1} = \{(b,a)(b,b)(a,b)\} \quad (2)$$

From (1) and (2)

$$\boxed{(SOR)^{-1} = R^{-1} o S^{-1}}$$

DPP- 9

Topic : Composition of relation

Q.1. Let $R = \{(1,1)(2,1)(3,\ 2),(4,3)\}$, compute R^2, R^3, R^4 .

Q.2. Let $A = \{\ 1,2,3\}\ \ B = \{p, q, r\}$ and $C = \{x, y, z\}$

and let $R = \{(1,p)(1,r)(2,q)(3,q)\}$

and $S = \{(p,y)(q,x)(r,z)\}$ compute SoR

Thank You

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Lec-10

Today's Target

- ***Total Number of relation***
- **Partial Order Relation**

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP
4	Unit wise set of PYQs

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$A \times A = n^2$

Total Number of relations

Let A be a set with n elements i.e n(A) = n, then the total number of

(i) Relation $= 2^{n^2}$

(ii) Reflexive relation $= 2^{n^2-n}$ or $2^{n(n-1)}$

(iii) Irreflexive relation $= 2^{n^2-n}$ or $2^{n(n-1)}$

(iii) Symmetric relation $= 2^{\frac{n(n+1)}{2}}$

(iv) Asymmetric relation $= 3^{\frac{n(n-1)}{2}}$

(v) Anti – symmetric relation $= 2^n \times 3^{\frac{n(n-1)}{2}}$

(vi) Reflexive and symmetric $= 2^{\frac{n^2-n}{2}}$

$A = \{1, 2, 3\}$

$$A \times A = \{(1,1)\ (1,2)\ (1,3)$$
$$(2,1)\ (2,2)\ (2,3)$$
$$(3,1)\ (3,2)\ (3,3)\}$$

$n^2 - n$

PARTIAL ORDER RELATION

A relation R on set A is called partial order relation If it is

(i) Reflexive : $(a, a) \in R \ \forall \ a \in A$

(ii) Anti symmetric : $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b \ \forall \ a, b \in A$

(iii) Transitive : $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \ \forall \ a, b, c \in A$

Q.1. Identify the partial order relation from the following Relation on a set $A = \{1, 2, 3, 4\}$

X (i) $R_1 = \phi$

X (ii) $R_2 = A \times A$

Reflexive ✓
Anti-symmetric X $(2,1) \in R$ and $(1,2)$ but $2 \neq 1$
Transitive ✓

✓ (iii) $R_3 = \{(1,1)(2,2)(3,3)(4,4)(1,2)(2,3)(1,3)\}$

R ✓
A ✓
T ✓

✓ (iv) $R_4 = \{(1,1)(2,2)(3,3)(4,4)\}$

R ✓
A ✓
T ✓

X (v) $R_5 = \{(1,1)(2,2)(3,3)(4,4)(1,2)(2,1)\}$

R ✓
A X

Q.2. Let N be the set of natural numbers, prove that the relation of " less than or equal to" is a partial order relation on N .

$R = \{(a,b) : a \le b \text{ and } a, b \in N\}$

R is a partial order relation if

(I) Reflexive : For Any $a \in N$

since $a \le a$ (True)

$(a,a) \in R \ \forall a \in N$

So, R is a reflexive Relation

(II) Anti-Symmetric

Let $a, b \in N$ such that

$(a,b) \in R$ and $(b,a) \in R$

$\Rightarrow a \le b$ and $b \le a$

$\Rightarrow a = b$

So, R is Anti-symmetric Relation

(iii) Transitive: Let $a, b, c \in N$ such that

$(a, b) \in R$ and $(b, c) \in R$

$\Rightarrow a \leq b$ and $b \leq c$

$\Rightarrow a \leq c$

$\Rightarrow (a, c) \in R \;\forall\; a, b, c \in N$

$\Rightarrow$ So, it Transitive

Hence, given relation is a partial order relation

Q.3. Which of the following is not a partial order relation ?

(i) $R_1 = \{(a,b) : a, b \in z, a < b\}$

(ii) $R_2 = \{(a,b) : a, b \in z, a \leq b\}$

(iii) $R_3 = \{(a,b) : a, b \in z, \frac{b}{a} \leq z\}$

(i) $R_1 = \{(a,b) : a, b \in Z, a < b\}$

Reflexive: For any $a \in Z$

$a < a$ (False)

$\Rightarrow (a,a) \notin R$

$\Rightarrow$ so, R is not reflexive

Hence, it is not a partial order relation

Discrete Mathematics by Gulshan Sir

DPP- 10

Topic : Partial Order Relation

Q.1. Let $A = \{1, 2, 3\}$. Show that $R = \{(1,1)(2,2)(3,3)(1,2)(2,3)(1,3)\}$ is a partial order relation.

Q.2. Show that the relation $R = \{(x, y): x \geq y\}$ where x , y ∈ z is a partial order relation

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DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Lec-11

Today's Target

- *Digraphs of Relation*
- Matrix Representation of Relation

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP
4	Unit wise set of PYQs

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Digraphs of Relation

OR

Directed Graph of Relation

If A is finite set and R is a relation on A, then we can also represent R pictorially as

(i) Draw a small circle for each element of A and label the circle with corresponding elements of A, these circles are called nodes or vertices

(ii) Draw an arrow, called an edge , from vertex

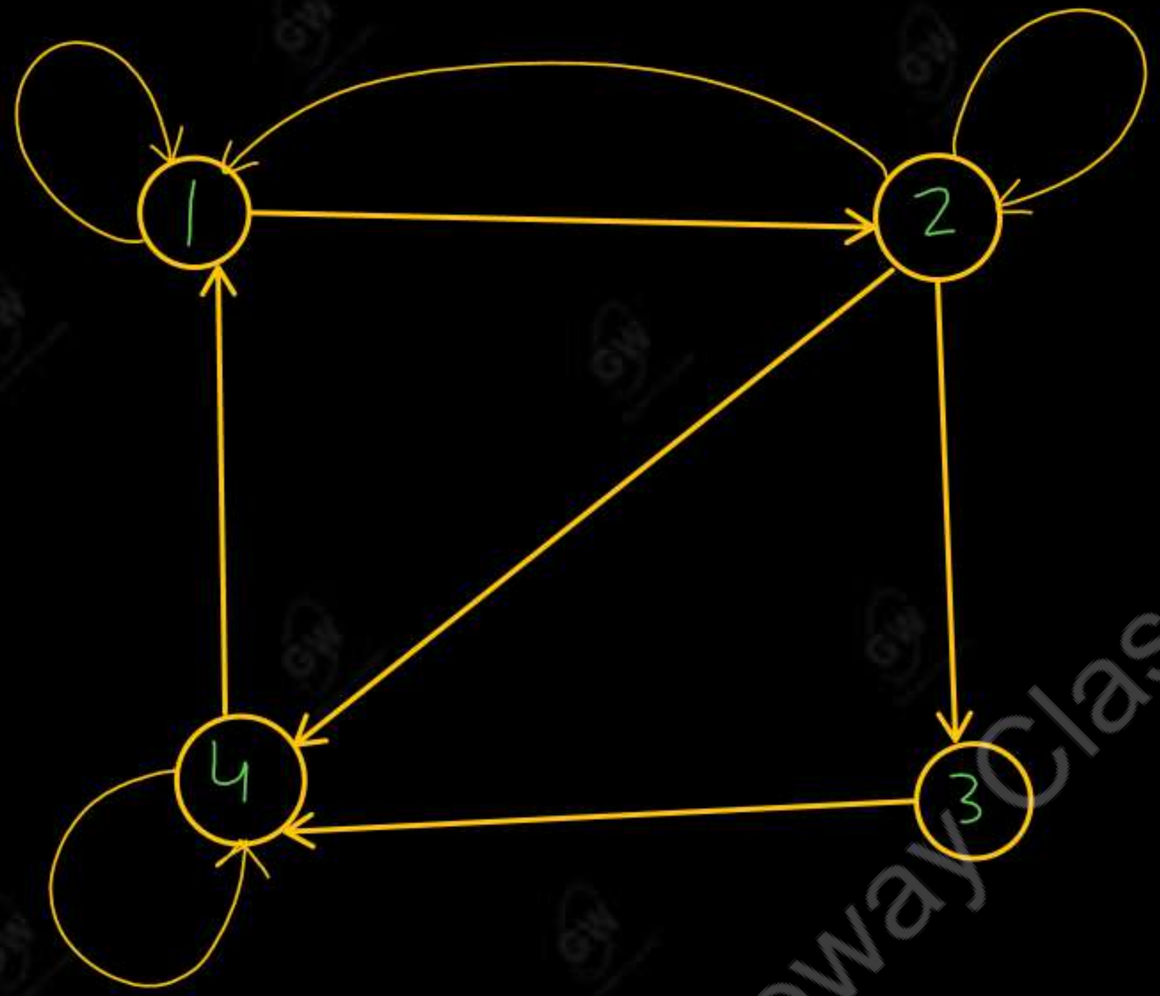
$$a_i \text{ to } a_j \Leftrightarrow a_i \, R \, a_j$$

The resulting pictorial representation of R is called a Directed graph or Digraph of R.

Q.1- Let $A = \{1, 2, 3, 4, \}$ $R = \{(1,1), (1,2), (2,1), (2,2), (2,3), (2,4), (3,4), (4,1), (4,4)\}$

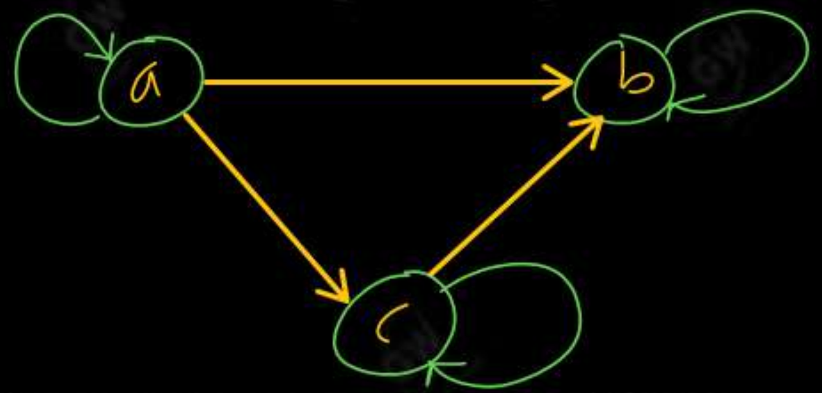
Construct the digraph of R

solution

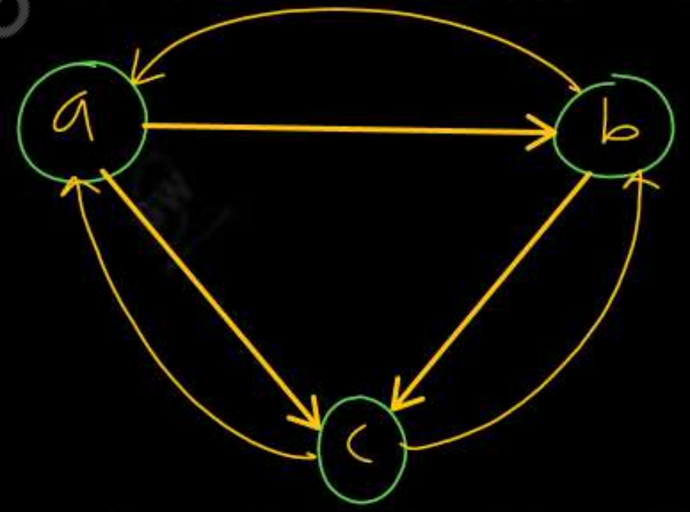


It is digraph of the given Relation

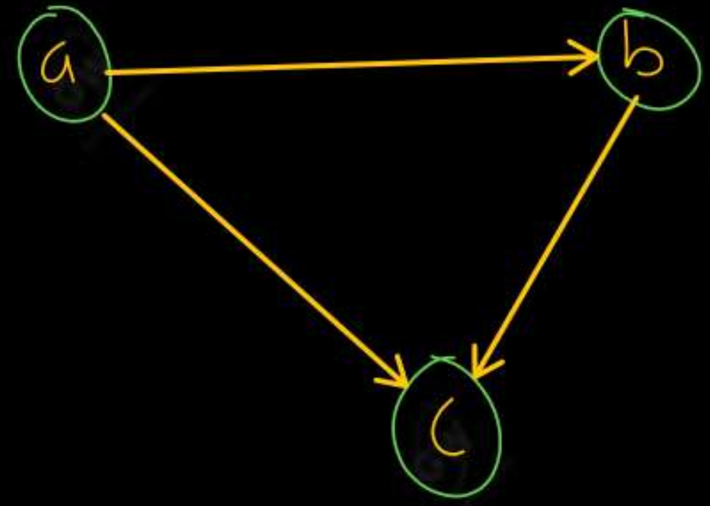
(1) **Reflexive -** A Relation R is reflexive if there is a loop at every Vertex of the directed graph



(2) **Symmetric -** A Relation R is symmetric, if for every edge between distinct vertices in its digraph there is an edge in the opposite direction.

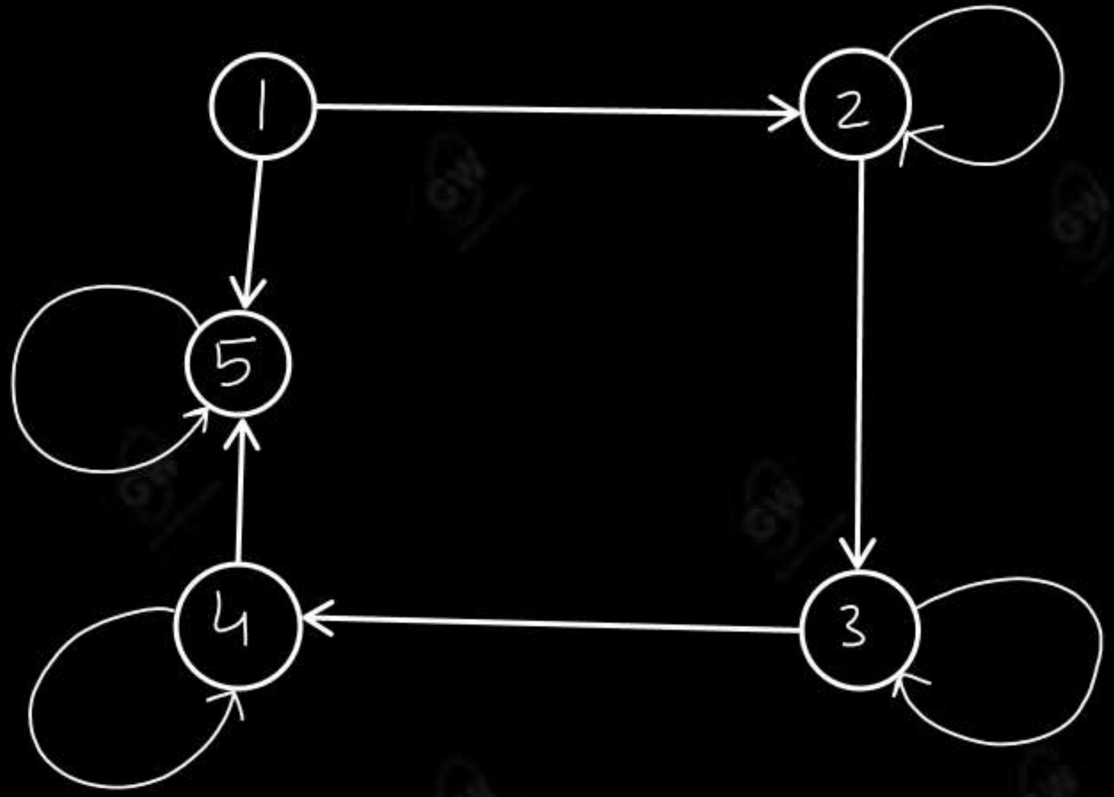


(3) **Transitive -** A Relation is transitive if there is an edge from vertex a to vertex b and from vertex b to vertex c, then there is an edge from vertex a to vertex c.



Q.2- Find the relation by the following figures

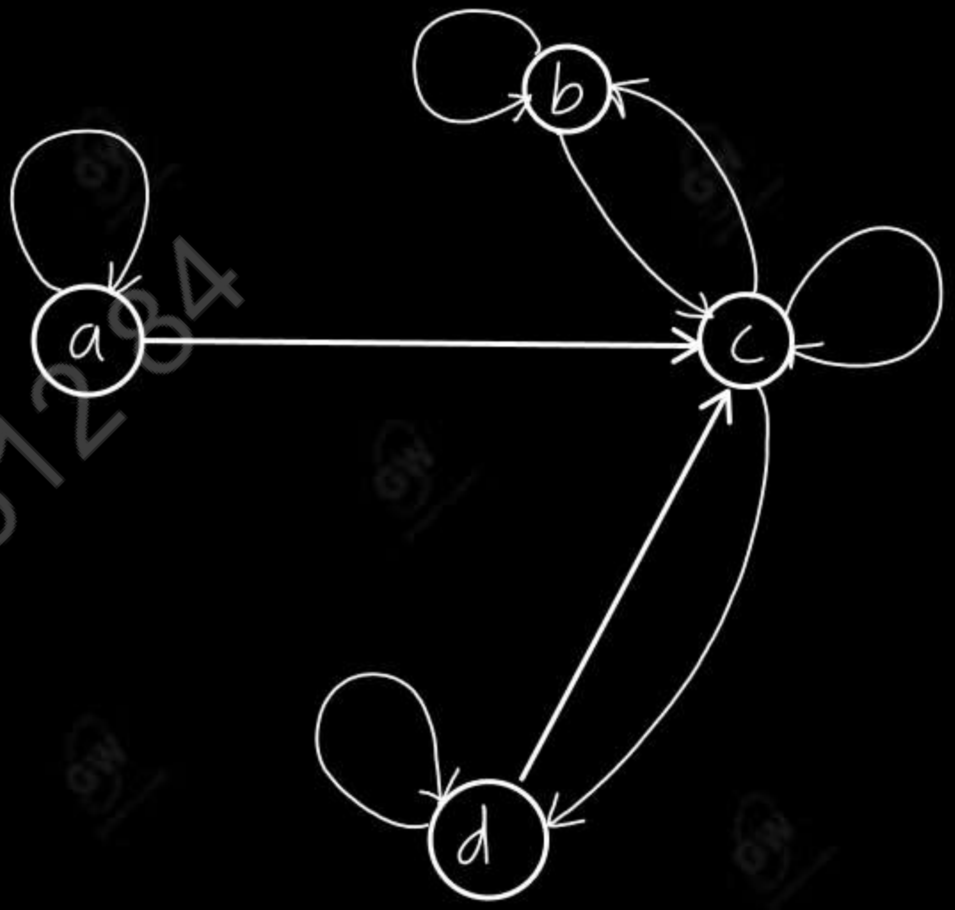
(i)



Solution

$R = \{(1,2), (2,2), (2,3), (3,3), (3,4), (4,4), (4,5) (5,5) (1,5)\}$

(ii)



solution

$R = \{(a,a), (a,c), (c,c), (c,b) (b,c) (b,b) (c,d) (d,c) (d,d)\}$

Matrix Representation of Relation

Let $\boxed{A = \{a_1, a_2, a_3 \text{ ---- } -a_m\}}$ and $\boxed{b = \{b_1, b_2, b_3 \text{ ---- } -b_n\}}$ are finite sets containing m and n elements respectively and let R be a relation from A to B

Then R can be represented by the m x n matrix

$$M_R = [m_{ij}]$$

Where

$$\boxed{m_{ij} = \begin{cases} 1 & if \ (a_i, b_j) \in R \\ 0 & if \ (a_i, b_j) \notin R \end{cases}}$$

The matrix M_R is called matrix of Relation R

Q.3- *Let R be the relation from the set* $A = \{1, 3, 4, \}$ *on itself and defined by*

$R = \{(1,1),\ (1,3), (3,1)\ (3,3), (4,4)\}$ *then find relation matrix*

Given $A = \{1, 3, 4\}$

$R = \{(1,1)(1,3)(3,1)(3,3)(4,4)\}$

$$M_R = \begin{array}{c c} & \begin{array}{ccc} 1 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Q.4- Let $A = \{a, b, c, d\}$ and let $M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ Find R

Solution

$A = \{a, b, c, d\}$

$$M_R = \begin{array}{c|c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$R = \{(a,a)(a,b)(b,c)(b,d)(c,c)$
$(c,d)(d,a)(d,c)\}$

Q.5- Let $A = \{1, 2, 3, 4\}$ and let R be a relation on A whose matrix is $M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ show that R is transitive.

Sol

$A = \{1, 2, 3, 4\}$

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 1 & 1 & 1 & 1 \\ 4 & 0 & 1 & 0 & 0 \end{array}$$

First Method

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (3,1)\ (3,2)\ (3,3)\ (3,4)\ (4,2)\}$

Second Method

R is transitive if

$$\boxed{M_R^2 + M_R = M_R}$$

$$M_R^2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$M_R^2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_R^2 + M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

$$M_R^2 + M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \end{bmatrix} = M_R$$

Since

$$\boxed{M_R^2 + M_R = M_R}$$

Hence, R is a transitive Relation

Q.6- Let $P = \{2, 3, 4, 5\}$. Consider the relation R and S on P defined by

$$R = \{(2,2), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5), (5,4\} \text{ and}$$

$$S = \{(2,3), (2,5), (3,4), (3,5), (4,2), (4,3), (4,5), (5,2), (5,5\}$$

Find the matrices of the above relations

Use matrices to find the composition relation of R and S

$$M_R = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$M_S = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$SOR = M_R \times M_S$

DPP- 11

Topic : Matrix Representation of Relation

Q.1. Let $A = \{a_1, a_2, a_3\}$ and $B = \{b_1, b_2, b_3, b_4\}$ which ordered pairs are in the relation R represented by the matrix

$$M_R = \begin{array}{c} \\ a_1 \\ a_2 \\ a_3 \end{array} \begin{array}{c} \begin{array}{cccc} b_1 & b_2 & b_3 & b_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}.$$

Q.2. Let $A = \{1, 2, 3, 4, 8\}$, $B = \{1, 4, 6, 9\}$. Let a R b iff a divides b. Find the relation matrix

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DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Lec-12

Today's Target

- *Closure of Relation*
- Warshall's Algorithm
- Univ PYQs
- DPPs

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP
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Closure of Relation

Let R be a relation on a set A. R may or may not have some property P, such as reflexivity, symmetry or transitivity. If we add some pairs then we have the desired property, the smallest relation S on A that contains R and posses the desired property P is called Closure of relation R with respect to that property.

$$\boxed{R \subseteq S}$$

Set A
↓
Relation R (does not satisfy some property p)
↓
Add some order pairs
↓
New Relation S
where s is called closure of Relation

Reflexive Property | Symmetric property | Transitive property

Closer of Relation (S)

Reflexive closure R^{r} | **Symmetric closure** R^{s} | **Transitive closure** R^{+}

(i) Reflexive closure

The reflexive closure R^{r} of a relation R is the *smallest* reflexive relation that contains R as a subset.

Reflexive closure is obtained by taking union of R and I_A.

$$\boxed{R^{r} = R \cup I_A}$$

where $I_A = \{(a, a) : a \in A\}$

= Identity Relation

$A = \{1, 2, 3\}$

$I_A = \{(1,1)(2,2)(3,3)\}$

Q.1 Let $A = \{1, 2, 3, 4\}$ *and* $R = \{(1,1)(1,2)(2,3)\}$ be the relation on A. Find the reflexive closure of R.

Given

$A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,2), (2,3)\}$

$I_A = \{(1,1), (2,2), (3,3), (4,4)\}$

$R^r = R \cup I_A$

$$\boxed{R^r = \{(1,1), (2,2), (3,3), (4,4), (1,2), (2,3)\}}$$

(ii) Symmetric closure

The Symmetric closure R^S is the smallest symmetric relation that contains R as a subset.

The symmetric closure is obtained by taking the union of R and R^{-1}

$$\boxed{R^S = R \cup R^{-1}}$$

$$R^{-1} = \{(b,a) : (a,b) \in R\}$$

$$R = \{(1,2)(1,3)\}$$

$$R^{-1} = \{(2,1)(3,1)\}$$

Q.2 If $R = \{(1,2)(4,3)(2,2)(2,1)(3,1)\}$ be a relation on A= $\{1,2,3,4\}$. Find the symmetric closure.

Given $A = \{1,2,3,4\}$

$R = \{(1,2),(4,3),(2,2),(2,1),(3,1)\}$

$R^{-1} = \{(2,1),(3,4),(2,2),(1,2),(1,3)\}$

$R^{s} = R \cup R^{-1}$

$$\boxed{R^{s} = \{(1,2)(4,3)(2,2)(2,1)(3,1)(3,4)(1,3)\}}$$

Transitive closure

The relation obtained by adding the least number of ordered pairs to ensure transitivity is called the transitive closure of the relation.

The transitive closure of relation, R is denoted by R^+

Consider a relation, R on Set $A = \{a_1, a_2, a_3 \dots \dots \dots \dots a_m\}$

Then transitive closure of R

$$\boxed{R^+ = R \cup R^2 \cup R^3 \dots\dots\dots \cup R^m}$$

$R^2 = R \circ R$

$R^3 = R^2 \circ R$

$R^4 = R^3 \circ R$

$R^5 = R^4 \circ R$

Q.3 Let $R=\{(1,2)(2,3)(3,1)\}$ defined on $A=\{1,2,3\}$ find the transitive closure of R .

Given

$A = \{1,2,3\}$

$R = \{(1,2)(2,3)(3,1)\}$

$R^{+} = R \cup R^2 \cup R^3$

$R^2 = R \circ R$

$R^2 = \{(1,2)(2,3)(3,1)\} \circ \{(1,2)(2,3)(3,1)\}$

$R^2 = \{(1,3),(2,1),(3,2)\}$

$R^3 = R^2 \circ R$

$R^3 = \{(1,2)(2,3)(3,1)\} \circ \{(1,3)(2,1)(3,2)\}$

$R^3 = \{(1,1)(2,2)(3,3)\}$

$$R^+ = \{(1,2)(2,3)(3,1)\} \cup \{(1,3),(2,1),(3,2)\} \cup \{(1,1),(2,2),(3,3)\}$$

$$\boxed{R^+ = \{(1,2)(2,3)(3,1)(1,3)(2,1)(3,2)(1,1)(2,2)(3,3)\}}$$

Warshall's Algorithm to find transitive Closure

A more efficient method to calculate transitive closure of relation

Step-1: Let n be the number of elements in a given Set A

To find transitive closure of Relation R on A, maximum n warshall sets can be find.

$w_0, w_1, w_2 \dots\dots\dots\dots w_n$ $(Here\ w_0 = M_R)$

Step-2: Procedure to calculate w_k from w_{K-1}

(i) Copy I from w_{K-1} in w_k with same position

(ii) In K^{th} row and K^{th} column of w_{K-1}, see K^{th} column and check the position of 1 and put below C

See K^{th} row and check the position of 1 and put below R

(iii) Mark entries in w_k as 1 for (C, R) of w_k if there are not already 1.

Step-3: Stop when w_n is obtained and it the required transitive closure.

Q.4 Let $A = \{1,2,3,4\}$ *and* $R = \{(1,4)(2,1)(2,3)(3,1)(3,4)(4,3)\}$ be a relation on A.

Find transitive closure using Warshall's Algorithm.

Given

$A = \{1, 2, 3, 4\}$

$R = \{(1,4)(2,1)(2,3)(3,1)(3,4)(4,3)\}$

$n(A) = 4$

$$M_R = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$W_0 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

From First colum and First Row

$$W_1 = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

C $\{2,3\}$ R $\{4\}$

$C \times R = \{(2,4)(3,4)\}$

From second column and second Row

$$W_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

C: $\{ \}$ R: $\{1, 3, 4\}$

$C \times R = \{ \}$

From Third column and third Row

$$W_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 0 & 1 \\ 4 & 1 & 0 & 1 & 1 \end{array}$$

C: $\{2, 4\}$ R: $\{1, 4\}$

$C \times R = \{(2,1)(2,4)(4,1)(4,4)\}$

From Fourth column and Fourth Row

$$W_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 0 & 1 & 1 \\ 2 & 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 1 & 1 \\ 4 & 1 & 0 & 1 & 1 \end{array}$$

C: $\{1, 2, 3, 4\}$ R: $\{1, 3, 4\}$

$C \times R = \{(1,1)(1,3)(1,4)(2,1)(2,3)(2,4)(3,1)(3,3)(3,4)(4,1)(4,3)(4,4)\}$

Transitive closure

$$\boxed{R^+ = \{(1,1)(1,3)(1,4)(2,1)(2,3)(2,4)(3,1)(3,3)(3,4)(4,1)(4,3)\}}$$

DPP- 12

Topic : *Closure of Relation and* Warshall's Algorithm

Q.1. *Let* $A = \{1, 2, 3, 4\}$ *and* $R = \{(1,2)(2,3)(3,4)\}$ *be a relation on set A . Find the transitive closure.*

Q.2. Let $A = \{4, 6, 8, 10\}$ *and* $R = \{(4,4)(4,10)(6,6)(6,8)(8,10)\}$ is a relation on set A. Determine the transitive closure of R using Warshall's Algorithm.

Q.3. Let $R = \{(1,2)(2,3)(3,1)\}$ defined on $A = \{1, 2, 3\}$ find the transitive closure of R using Warshall's Algorithm.

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DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Today's Target

- Partial order relation
- Partial order set or POSET
- Graphical representation of POSET - Hasse Diagram.
- Hasse Diagram from Directed graph.
- Easiest way to draw Hasse diagram

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A relation R on set A is called partial order relation if it is

(i) Reflexive $(a, a) \in R \ \forall \ a \in A$

(ii) Anti-symmetric $(a, b) \in R$ and $(b, a) \in R \Rightarrow a = b \ \forall \ a, b \in A$

(iii) Transitive $(a, b) \in R$ and $(b, c) \in R \Rightarrow (a, c) \in R \ \forall \ a, b, c \in A$

Partial order set or POSET

If A is any non-empty set and R is a partial order relation on set A, then the order pair (A, R) is called Partial order set or POSET

POSET is denoted by $\boxed{(A, R) \text{ Or } (A, \leq)}$

Hasse Diagram

Relation

A partial order ≤ on a set A can be represented by means of a diagram known as Hasse Diagram of (A, ≤)

i.e Hasse diagram is a graphical representation of POSET

Hasse Diagram from directed graph or digraph

Step 1:- Start with directed Graph

Step 2:- Remove self loop

Reason : Self loops are quite obvious and they must be present for all veriticesbecause R is reflexive, so we can remove them to simply diagram.

Step 3:- Remove all transitive edges.

Reason: Transitive edges are quite obvious and they must be present because R is transitive. So, we can remove all transitive edges to Simply diagram.

Step 4:- Remove circle by dot

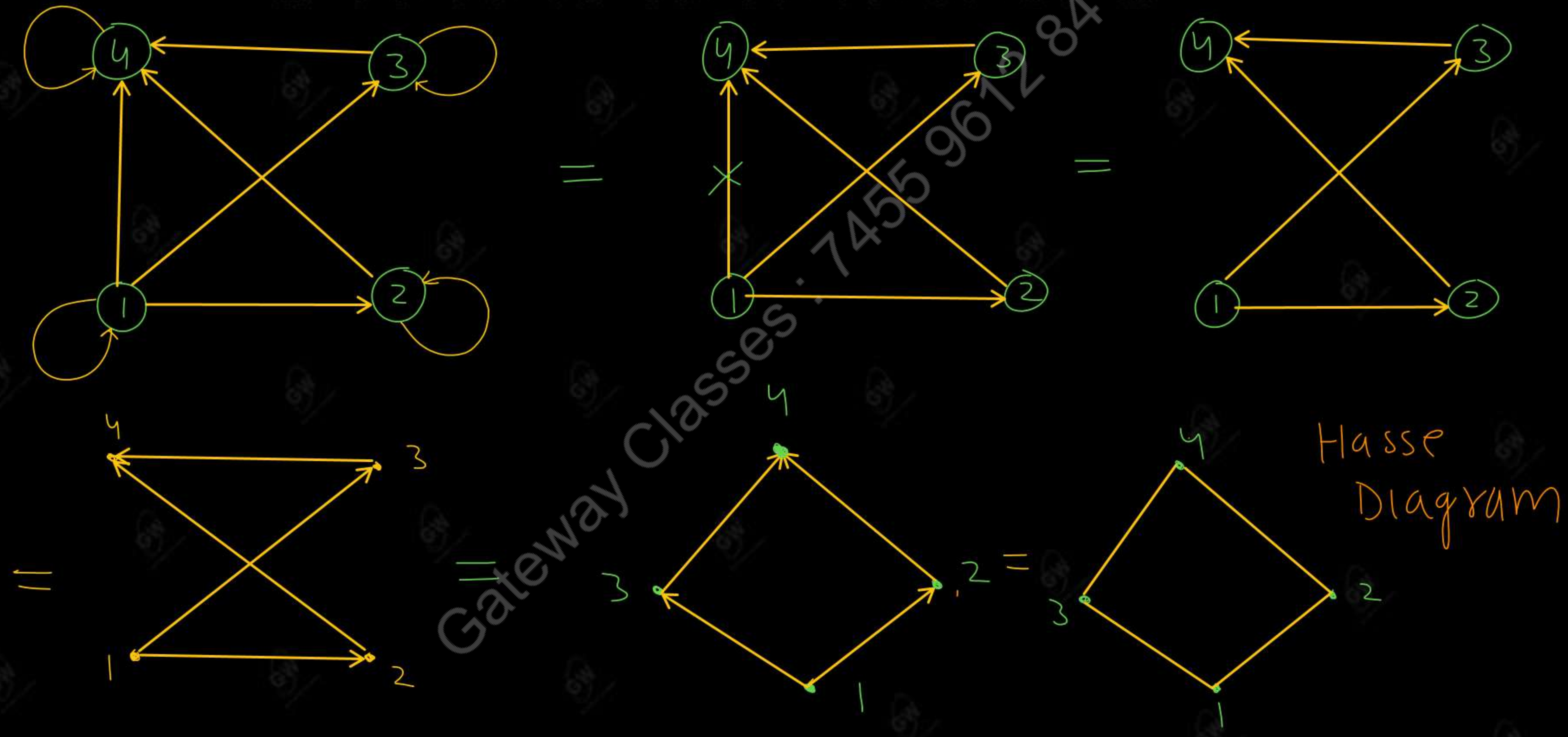
Step-5: Arrange each edge so that its initial vertex is below its terminal vertex i.e. all edges are pointing upwards.

Step-6: Remove all the arrows the directed graph.

Reason: since all edges are pointing upwards, so there is no need to show the directions in the graph.

Q.1 *Determine the Hasse diagram of the relation R on set* $A = \{1, 2, 3, 4\}$ (A, R)

$$R = \{(1,1), (1,2), (2,2), (2,4), (1,3), (3,3), (3,4), (1,4), (4,4)\}$$

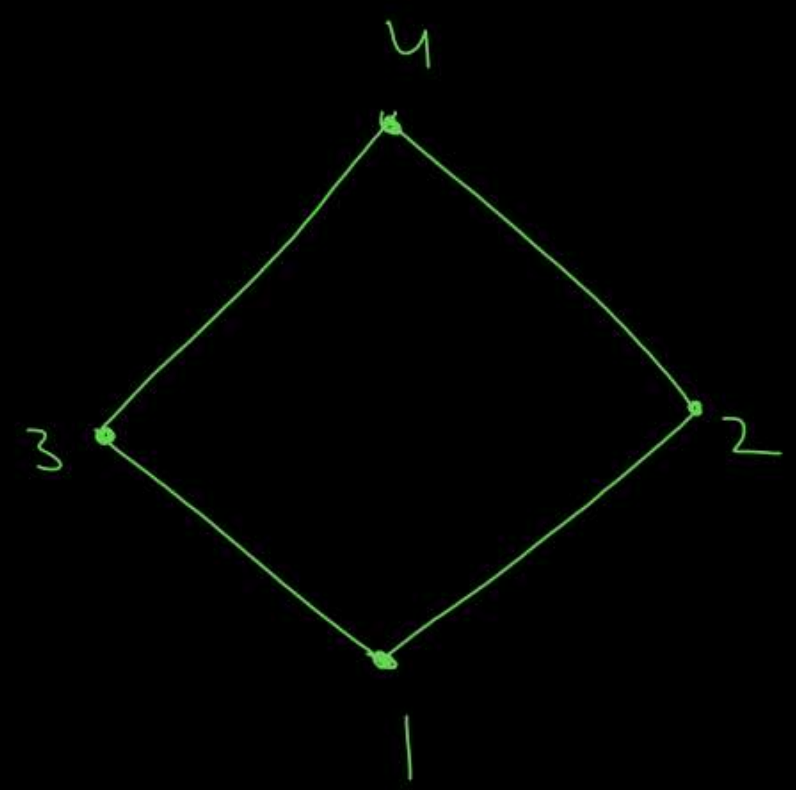


Easy Method

$$R = \{(1,1)\,(1,2)\,(2,2)\,(2,4)\,(1,3)\,(3,3)\,(3,4)\,(1,4)\,(4,4)\}$$

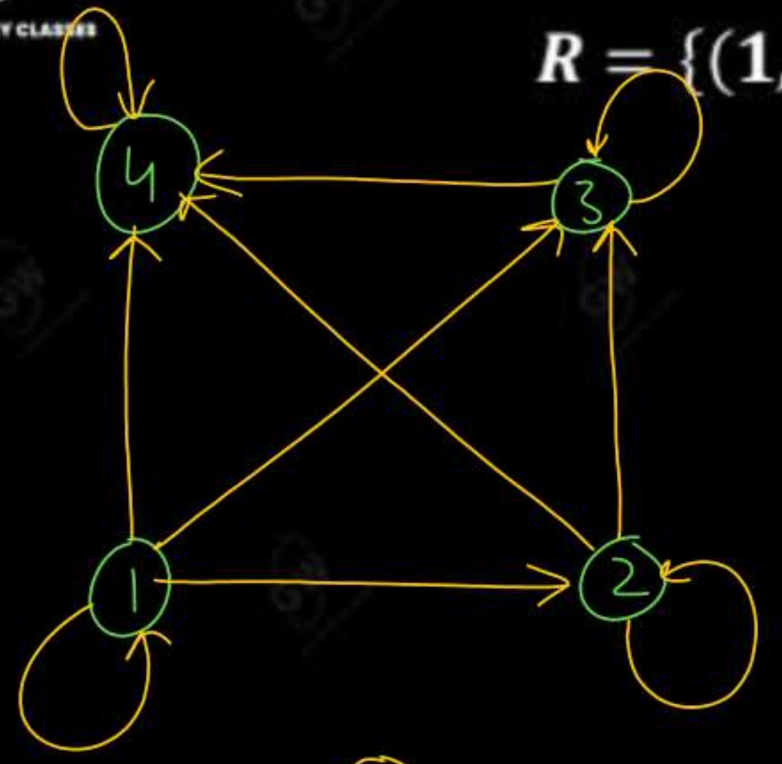
$$R = \{(1,2)\,(2,4)\,(1,3)\,(3,4)\,\cancel{(1,4)}\}$$

$$R = \{(1,2)\,(2,4)\,(1,3)\,(3,4)\}$$



Q.2 Determine the Hasse diagram of the relation R on set $A = \{1, 2, 3, 4\}$ (A, R)

$$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$$



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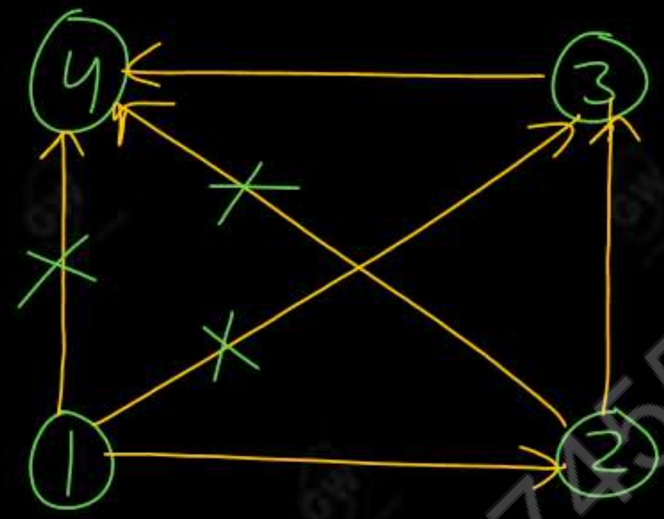

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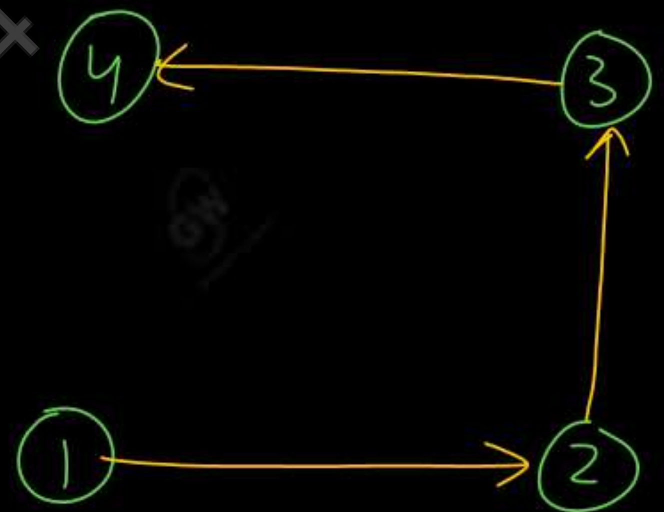

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Hasse Diagram

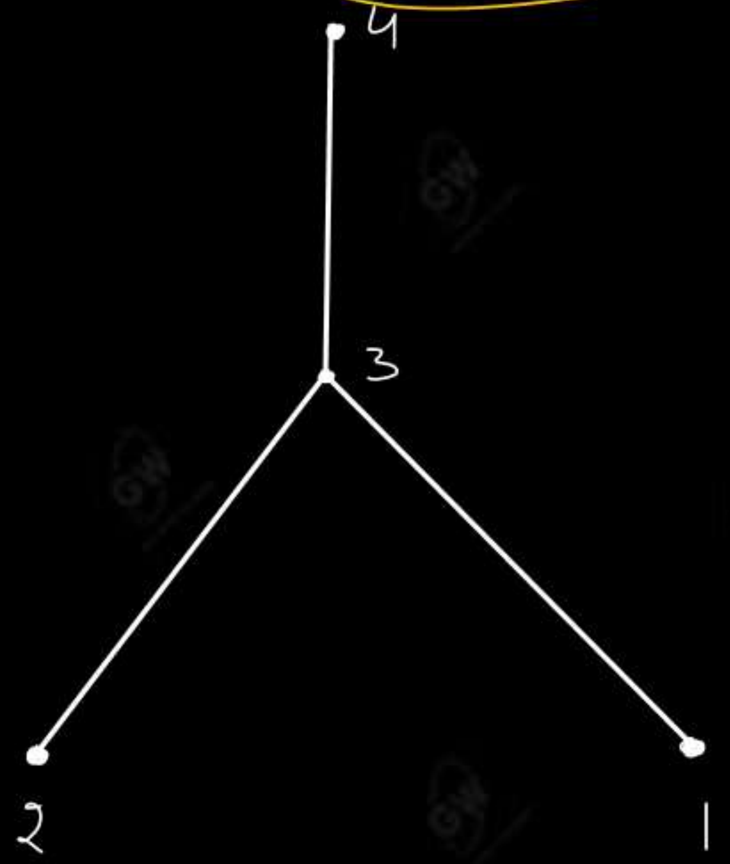
Easy Method

$$R = \{(1,2)\,\overset{\times}{(1,3)}\,\overset{\times}{(1,4)}\,(2,3)\,\overset{\times}{(2,4)}\,(3,4)\}$$

$$R = \{(1,2)\,(2,3)\,(3,4)\}$$

Hasse Diagram

Q.3- Describe the order pairs in the relation determined by the Hasse diagram of a POSET $(A, \leq)$ on the set $A = \{1, 2, 3, 4\}$



solution

From Hasse Diagram

$(1,3)\ (2,3)\ (3,4)$

Reflexive order pair

$(1,1)\ (2,2)\ (3,3)\ (4,4)$

Transitive Pair

$(2,4)\ (1,4)$

$$\boxed{R = \{(1,1)(2,2)(3,3)(4,4)(1,3)(2,3)(3,4)(2,4)(1,4)\}}$$

Easiest Way to draw the Hasse diagram

Q.4 Draw the Hasse diagram for the following POSET

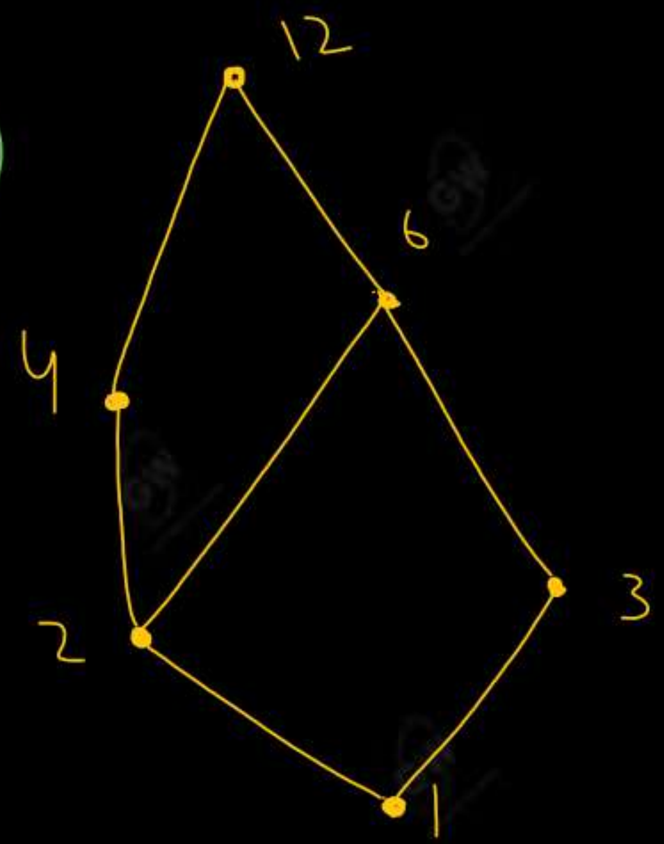
(i) $(A, \leq)$ where $A = \{1, 2, 3, 4, 5, 6\}$

(ii) $(A, |)$ where $A = \{1, 2, 3, 4, 6, 12\}$

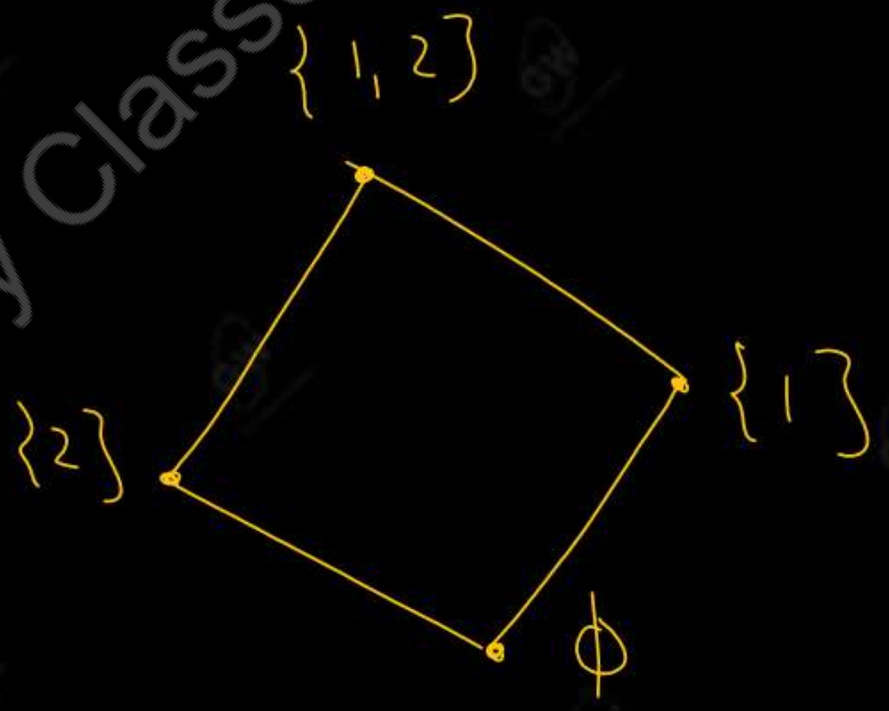
(iii) $(A, \subseteq)$ where $A = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$

(i)

(ii)



(iii)



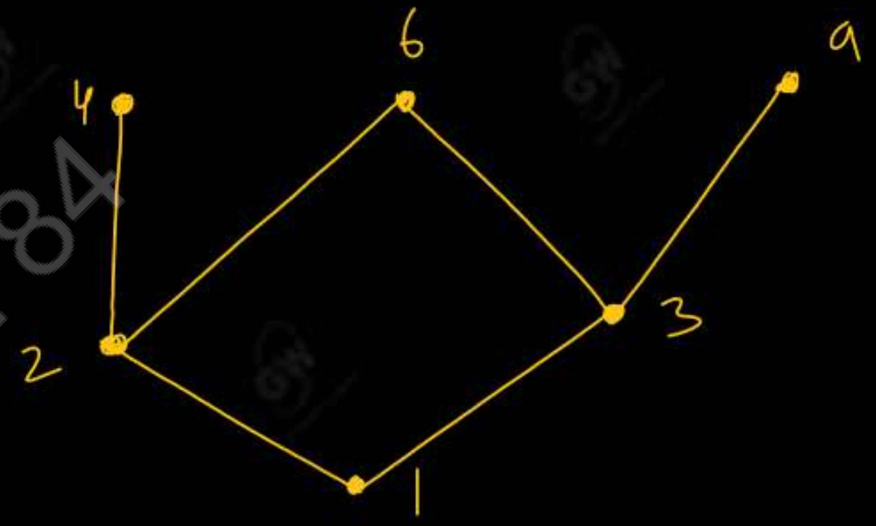
(iv) $(A, |)$ where $A = \{1, 2, 3, 4, 6, 9\}$

(v) $(A, |)$ where $A = \{2, 3, 5, 30, 60, 120, 180, 360\}$

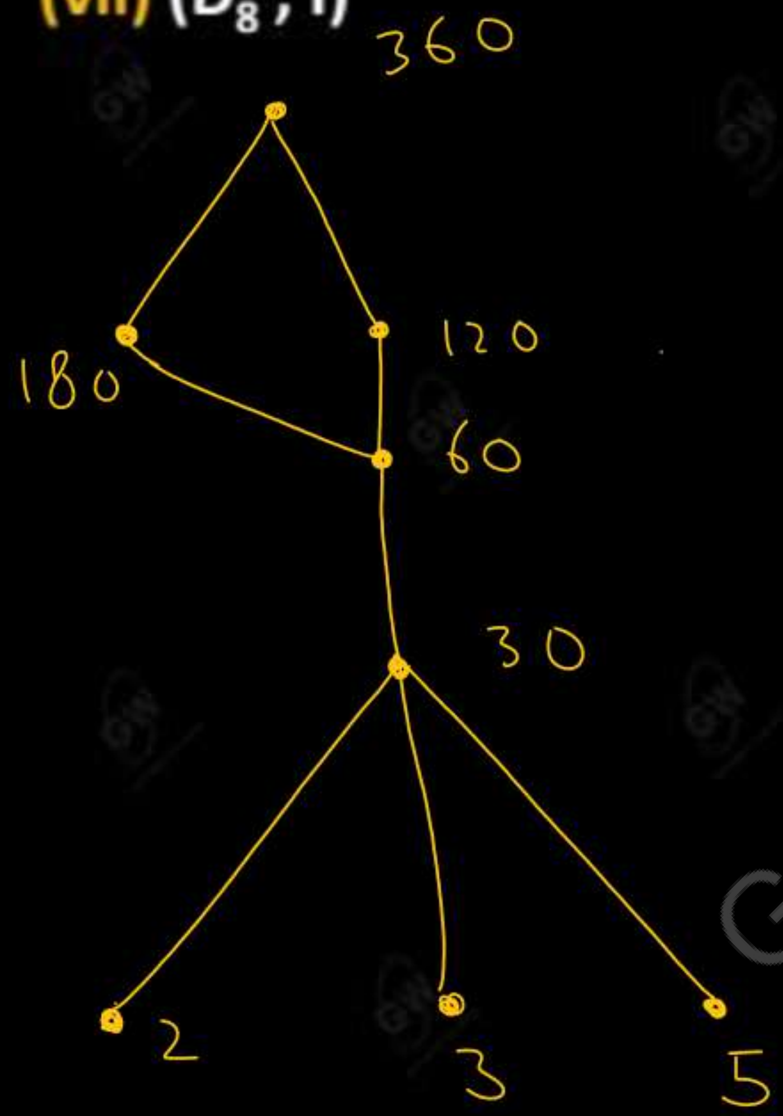
(vi) $(A, |)$ where $A = \{1, 2, 3, 5, 6, 10, 15, 30\}$

(vii) $(D_8, |)$

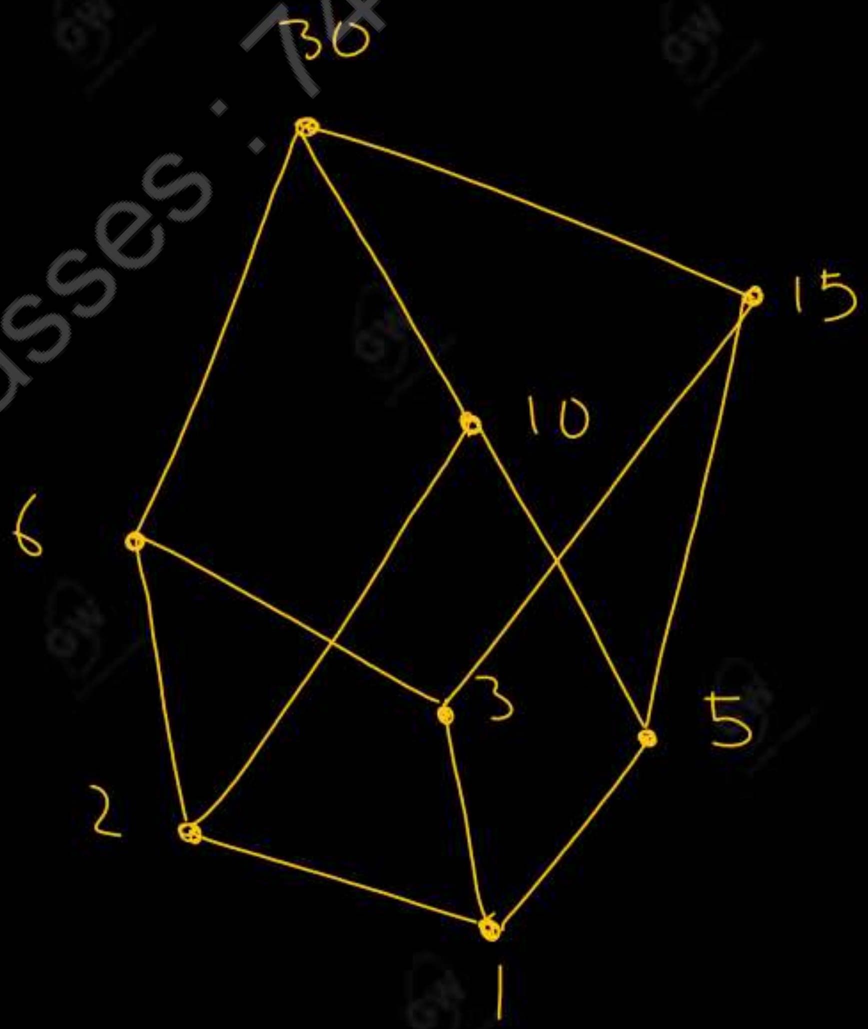
(IV)



(V)



(VI)



(VII) $(D_8, |)$

$A = \{1, 2, 4, 8\}$

GW

Q.5- Let $X = \{1, 2, 3, 4, 5, 6\}$, then I is a partial order relation on X. Draw the Hasse diagram of (X, I)

Hasse Diagram

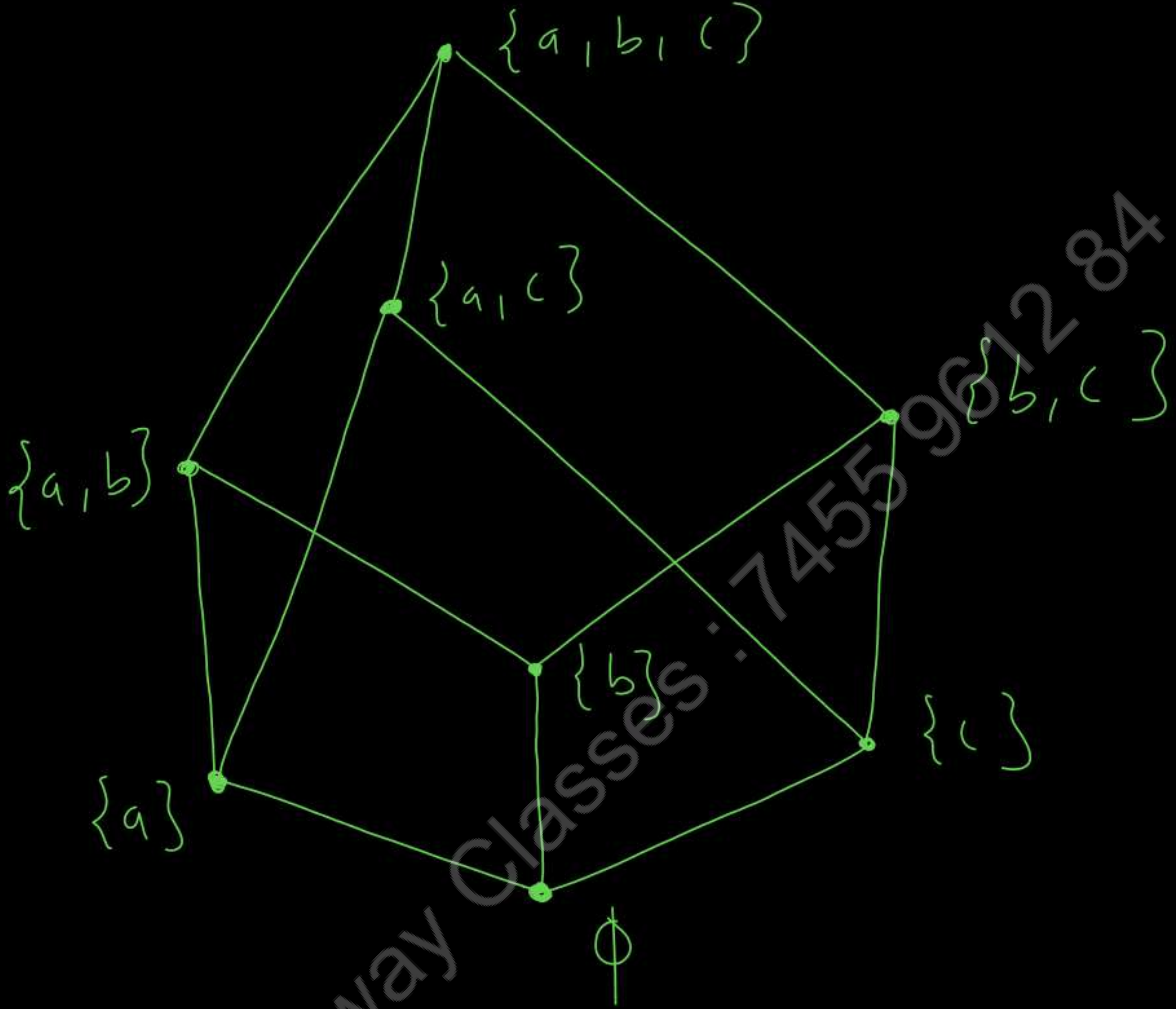
Q.6- Draw the Hasse diagram for the POSET $(P(s), \subseteq)$ where $P(s)$ is the power set on $A = \{a, b, c\}$

$A = \{a, b, c\}$

Subset of A

$\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}$

$P(S) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$


{a, b, c}
{a, c}
{b, c}
{a, b}
{b}
{c}
{a}
ϕ

Q.7- Let $X = \{1, 3, 6, 12, 24, 36\}$ and the relation $\leq$ be such that $x \leq y$ if 'x *divides* y'

Relation

$x R y$

Draw the Hasse diagram.

DPP- 13

Topic : Hasse's diagram

Q.1. Draw the Hasse's diagram of the POSET (L, ⊆) Where

$L = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$ where the sets are given by

$S_0 = \{a, b, c, d, e, f\}$, $S_1 = \{a, b, c, d, e\}$, $S_2 = \{a, b, c, e, f\}$, $S_3 = \{a, b, c, e\}$, $S_4 = \{a, b, c\}$, $S_5 = \{a, b\}$,

$S_6 = \{a, c\}$, $S_7 = \{a\}$

Q.2. Let $A = \{1, 2, 3, 4, 6, 8, 9, 12, 18, 24\}$ be ordered by the relation 'a divides b' find the Hasse diagram.

Q.3 *Draw the Hasse diagram of the relation S defined as "divides" on set B where*

$$B = \{2, 3, 4, 6, 12, 36, 48\}$$

Q.4 *Draw Hasse diagram for the following relations on set*

$A = \{1, 2, 3, 4, 12\}$

$R = \{(1,1), (2,2), (3,3), (4,4), (12,12), (1,2), (4,12), (1,3), (1,4), (1,12), (2,4), (2,12), (3,12)\}$

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Thank You

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Today's Target

- Maximal and Minimal elements in POSET
- Maximum and Minimum elements in POSET
- Upper Bound and lower Bound in Hasse diagram
- Least upper Bound and Greatest Lower Bound in Hasse diagram
- PYQ
- DPP

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP
4	Unit wise set of PYQs

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Link in Description

Maximal and Minimal elements in POSET

Maximal element : *If in a POSET, an element is not related to any other element, then it is called maximal elements*

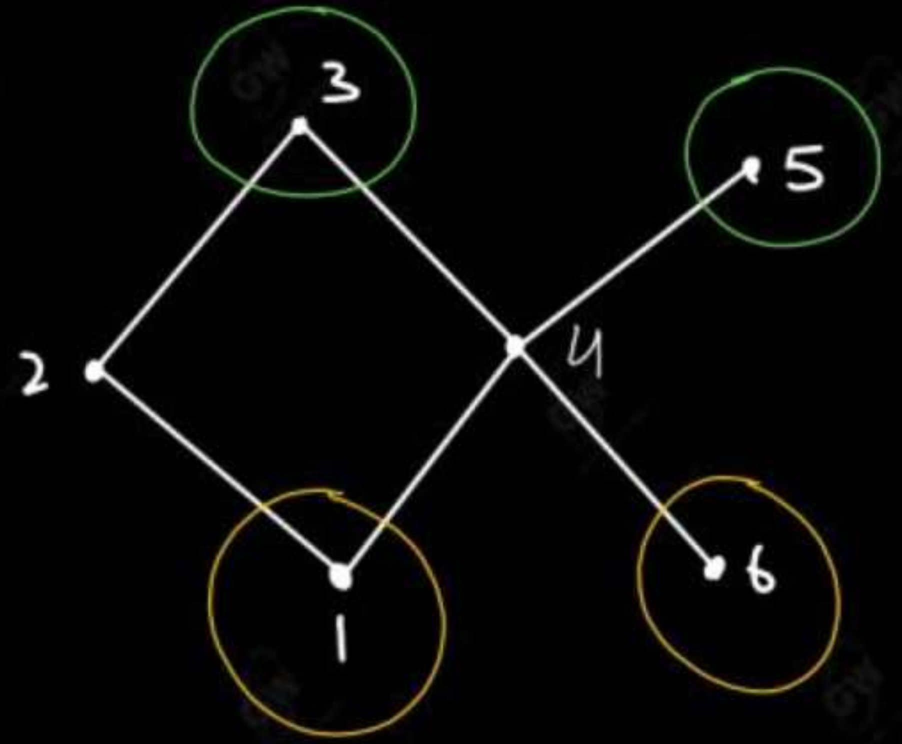
Green circle

Minimal elements : *If in a POSET, no element is related to any other element, then it is called minimal elements*

Yellow circle

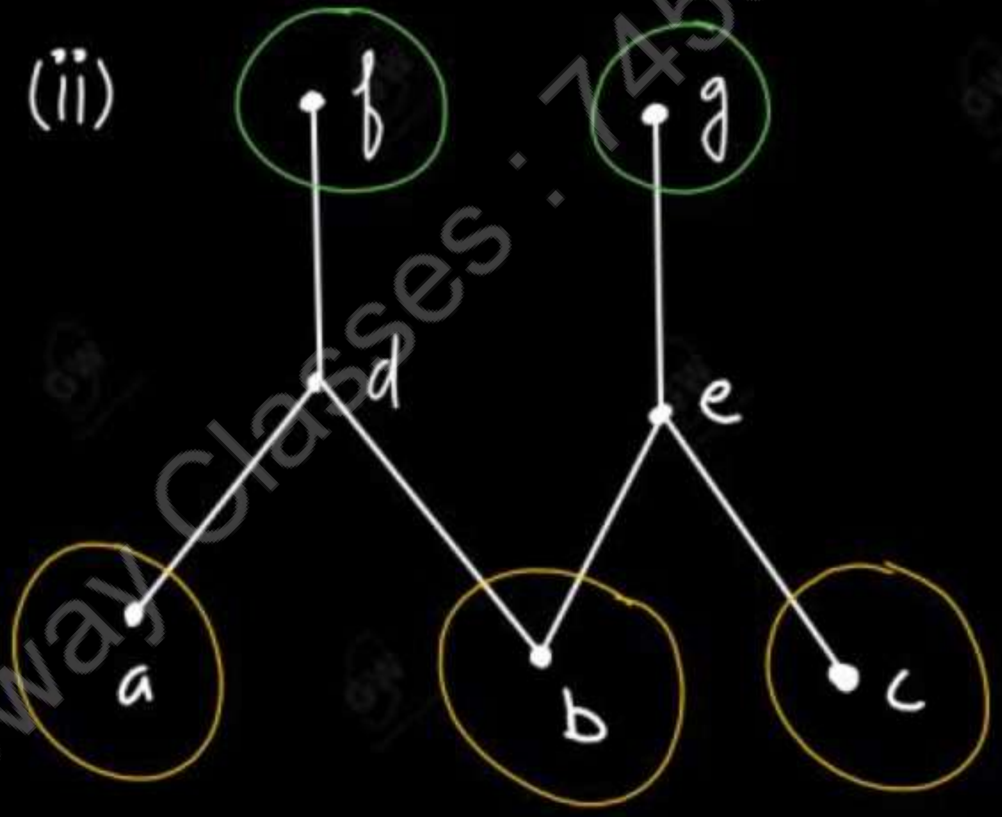
Q.1: *Find all the maximal and minimal elements of the POSET whose Hasse diagrams are given below.*

(i)

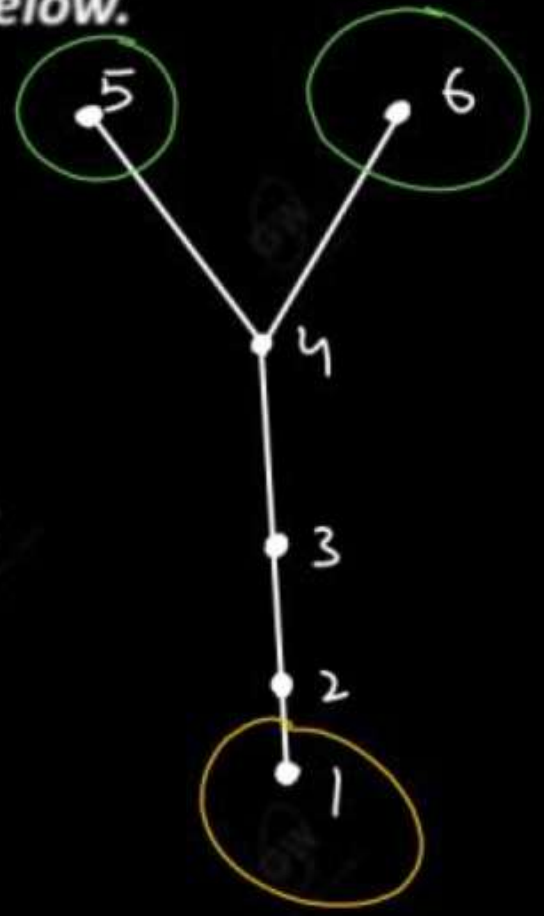


Maximal elements = 3, 5

(ii)

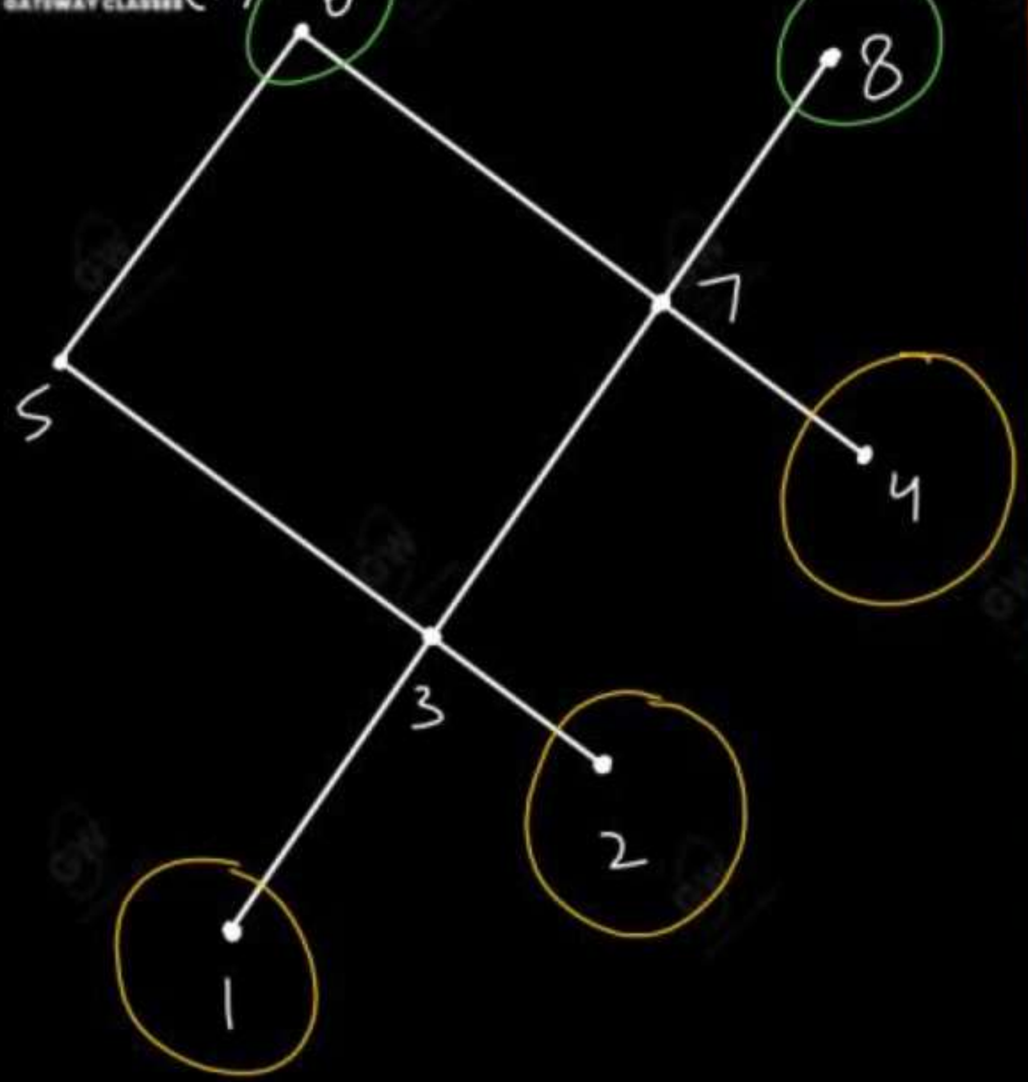


(iii)

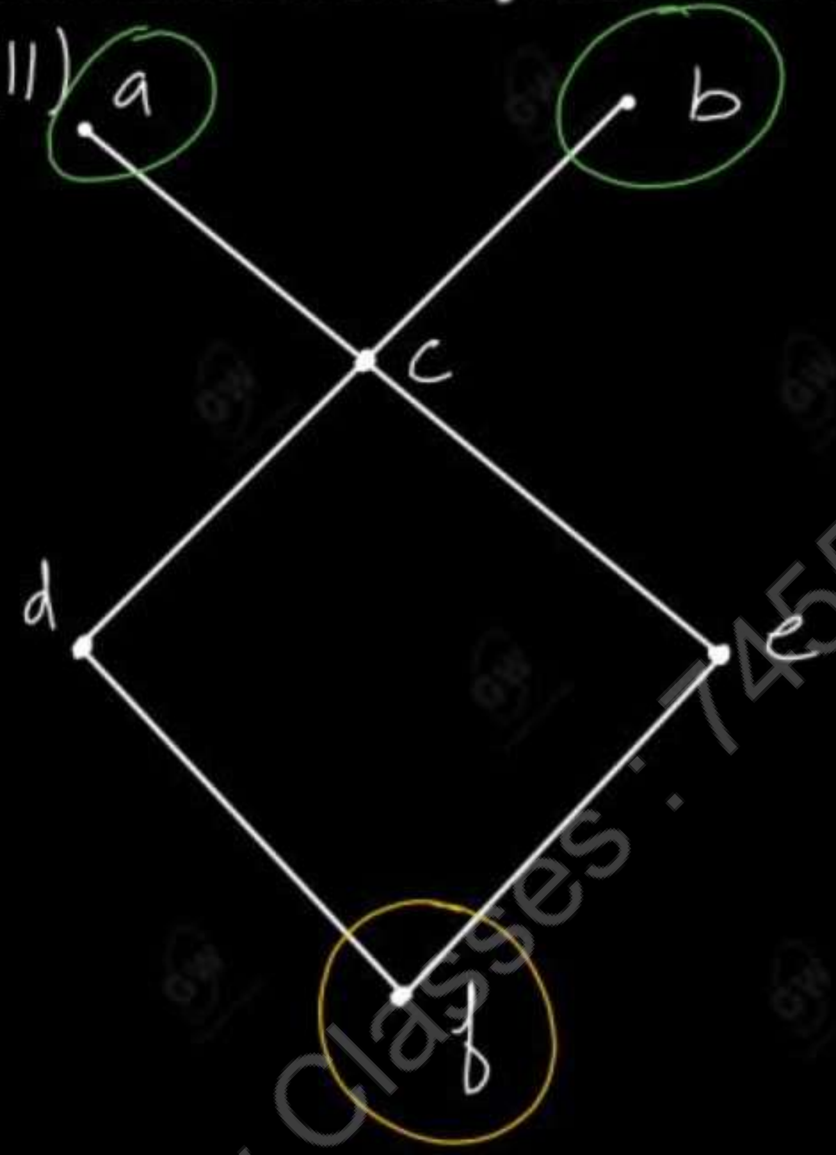


Q.2: Find all the maximal and minimal elements of the POSET whose Hasse diagrams are given below.

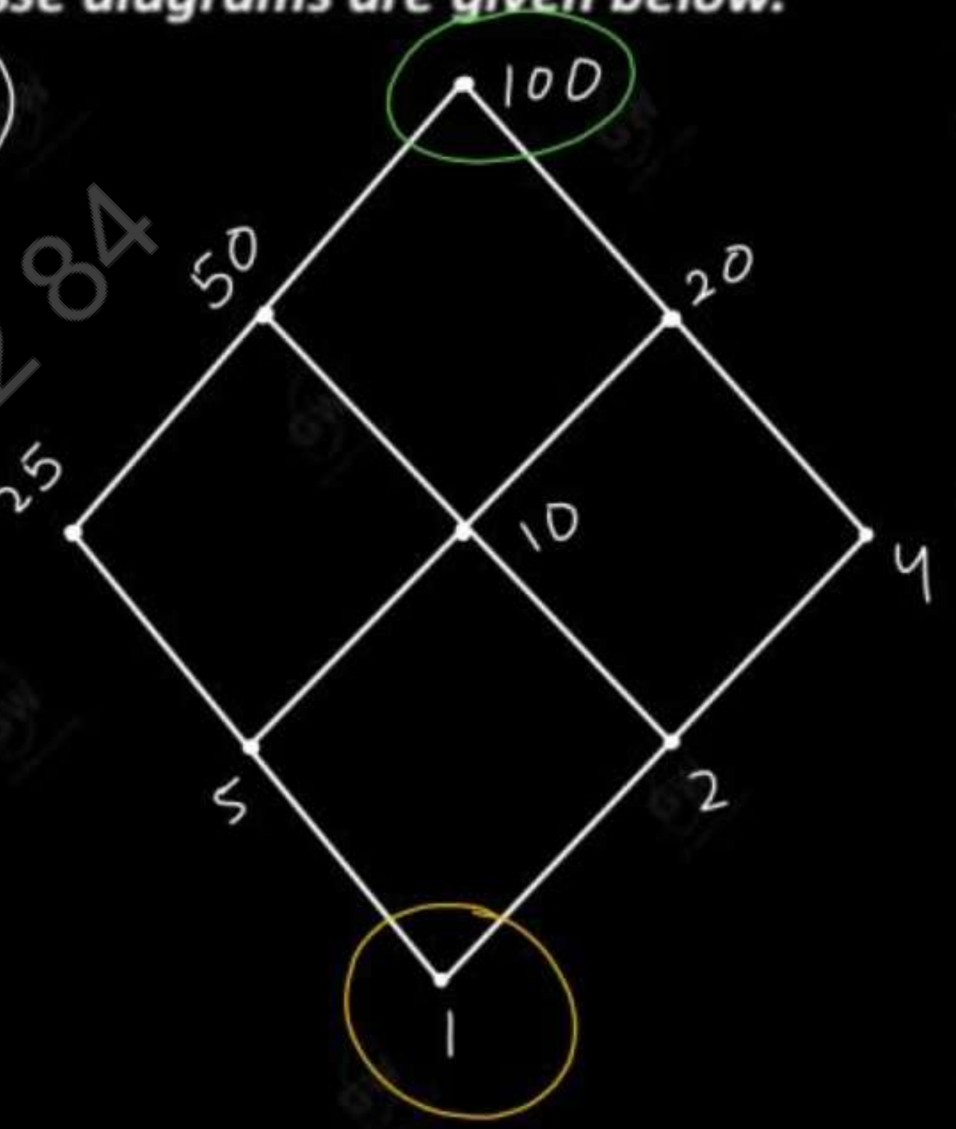
(i)



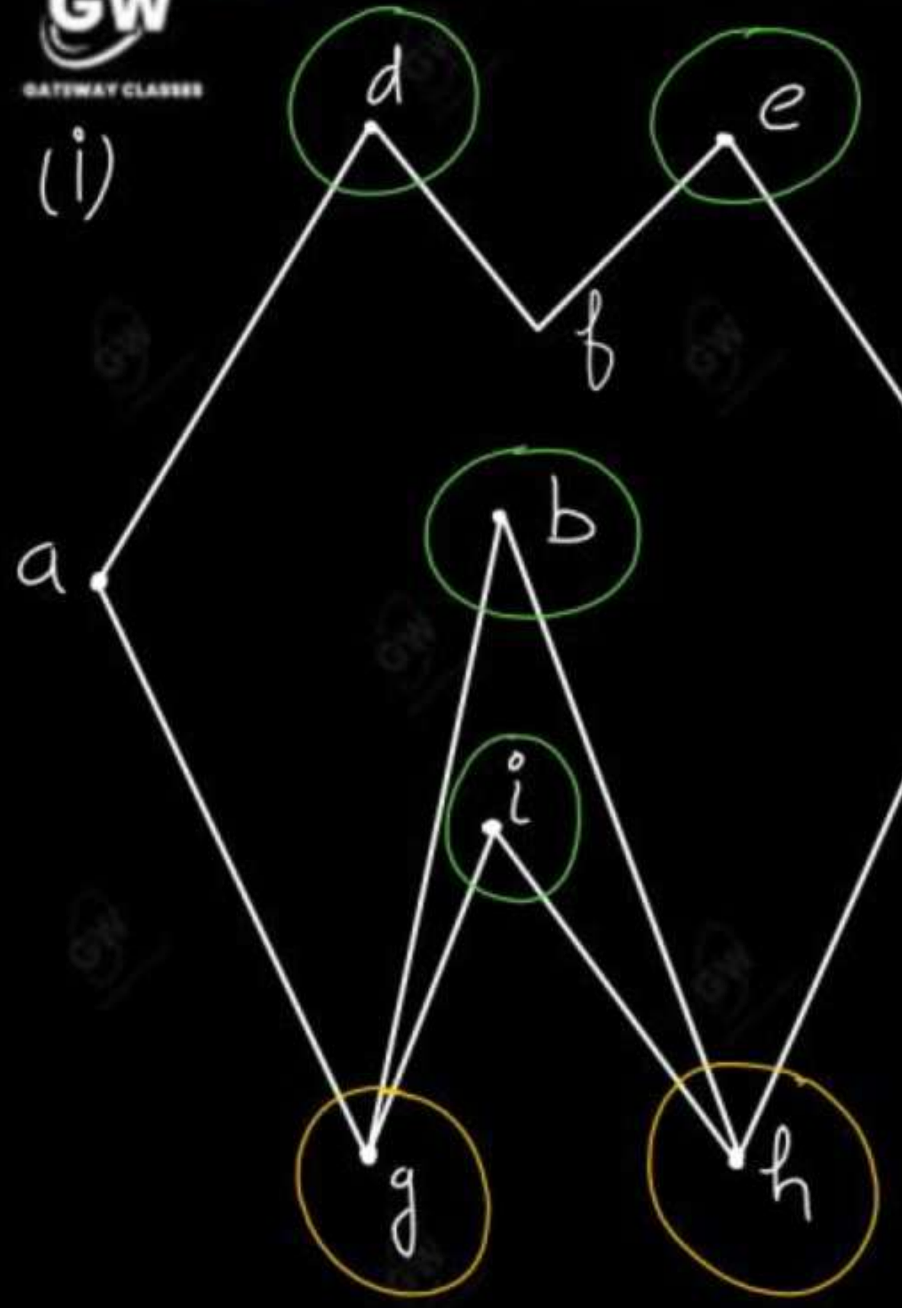
(II)

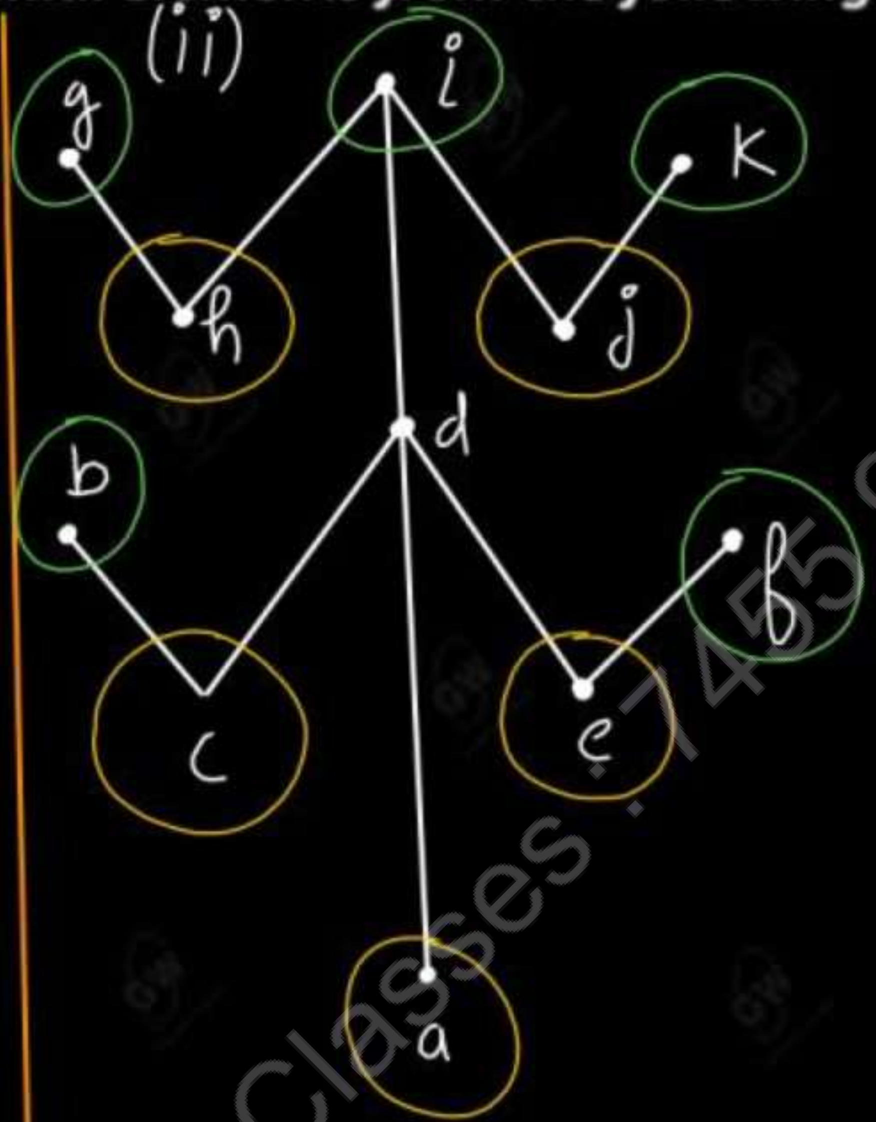


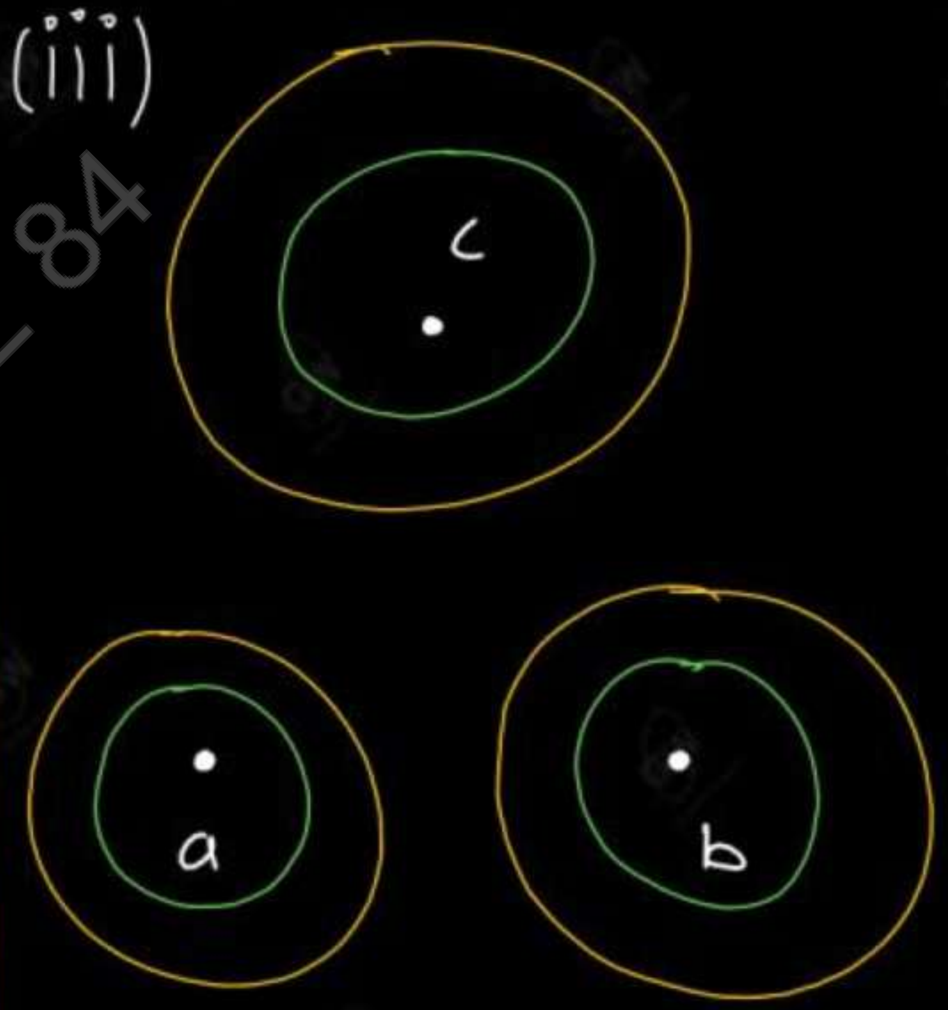
(III)



Q.3:- Find all the maximal and minimal elements from the following Hasse diagrams.







Maximal = a, b, c

Minimal = a, b, c

Maximum and Minimum element

Maximum element / Greatest element: *If it is maximal and every element is related to it*

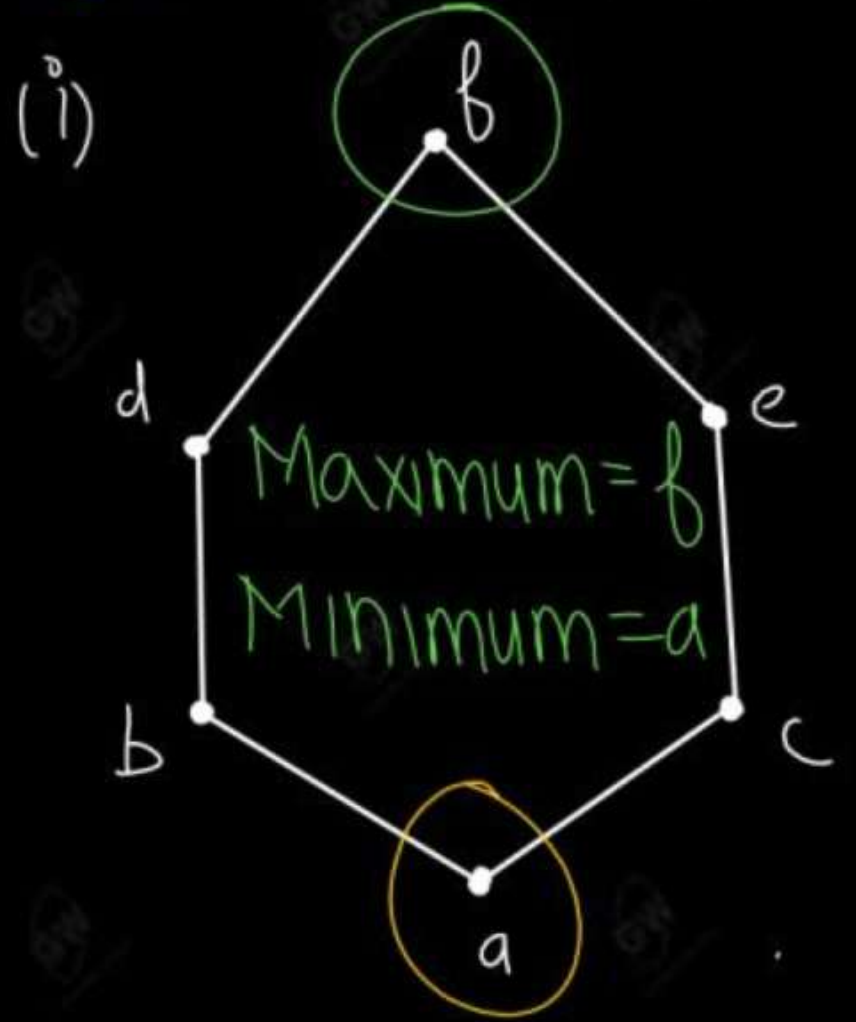
Minimum elements / Least element : *If it is minimal and it is related to every element.*

Maximum = Null

Minimum = Null

Q.4 : *Find the maximum and minimum elements from the following Hasse diagrams*

(i)



Maximum = f

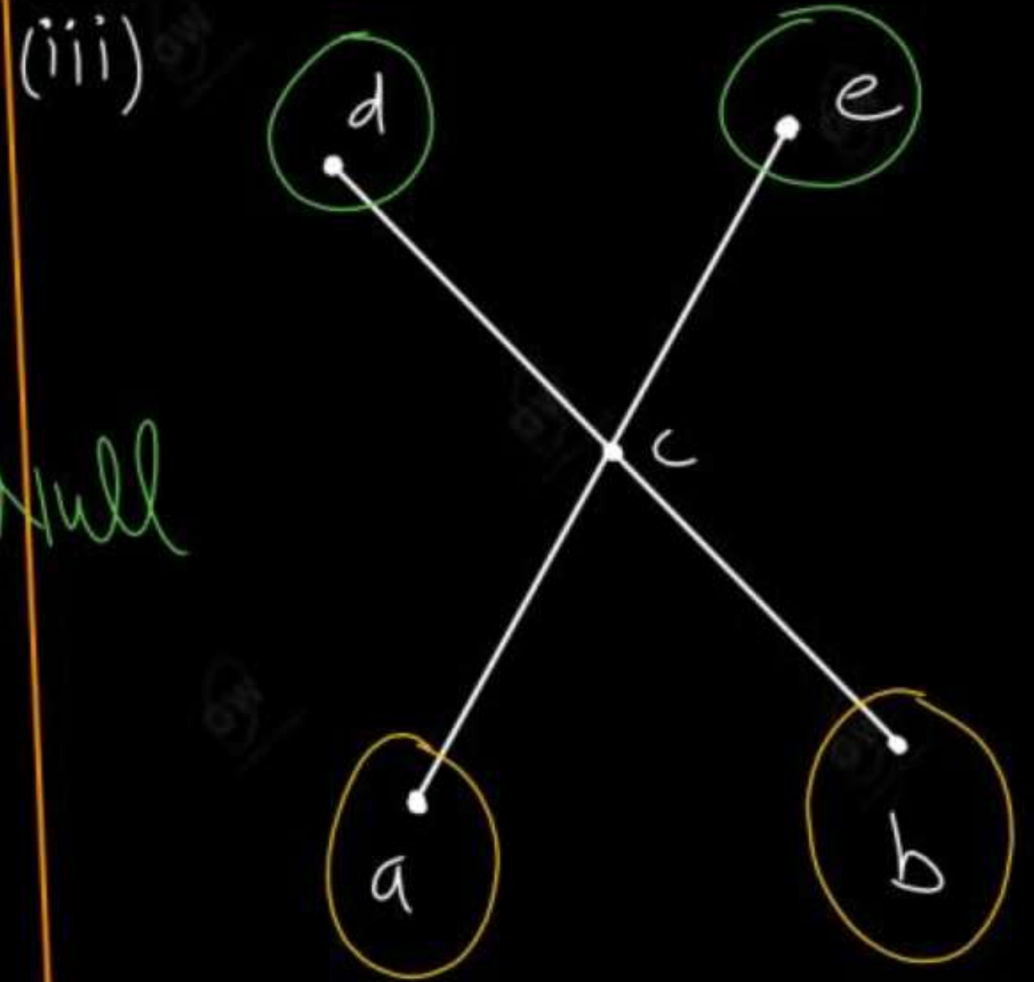
Minimum = a

(ii)

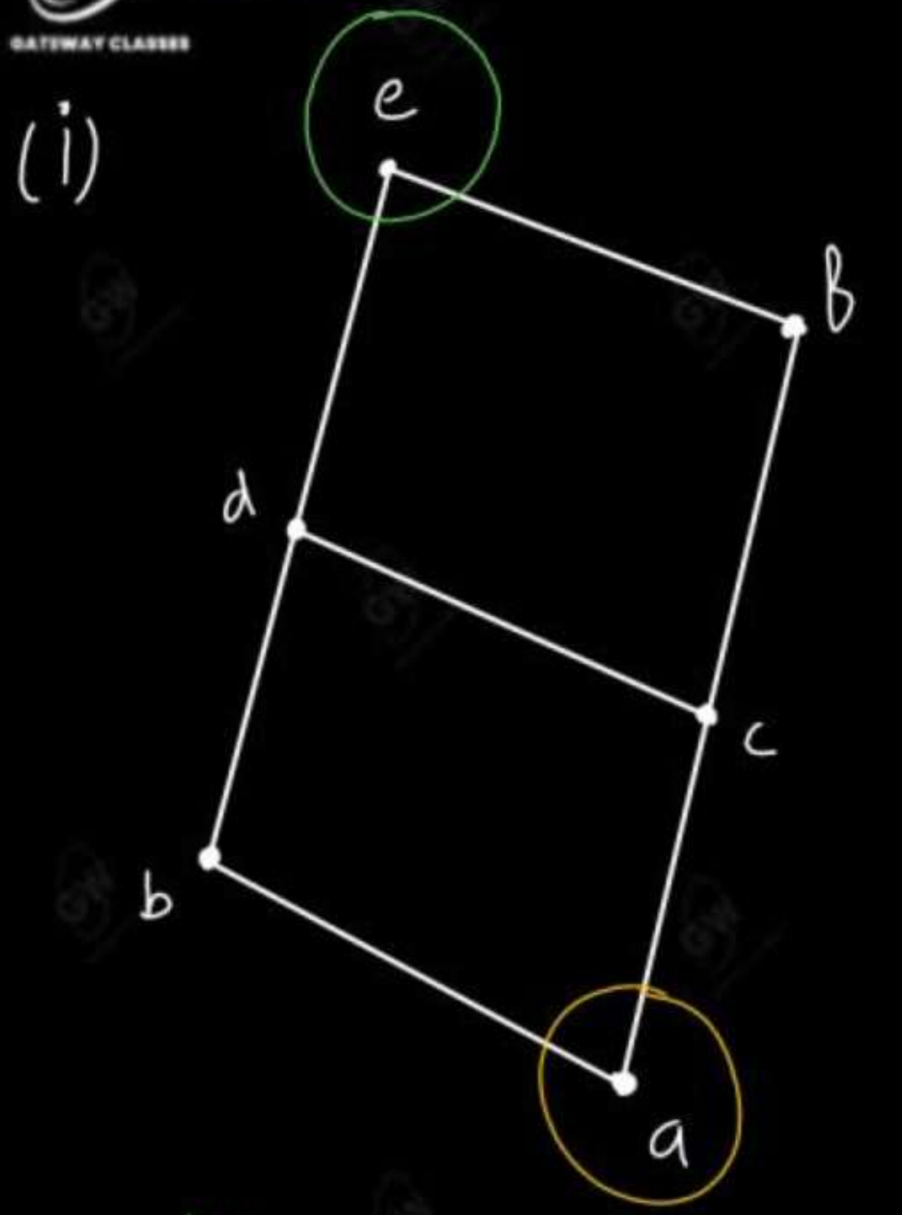
Maximum = d

Minimum = Null

(iii)

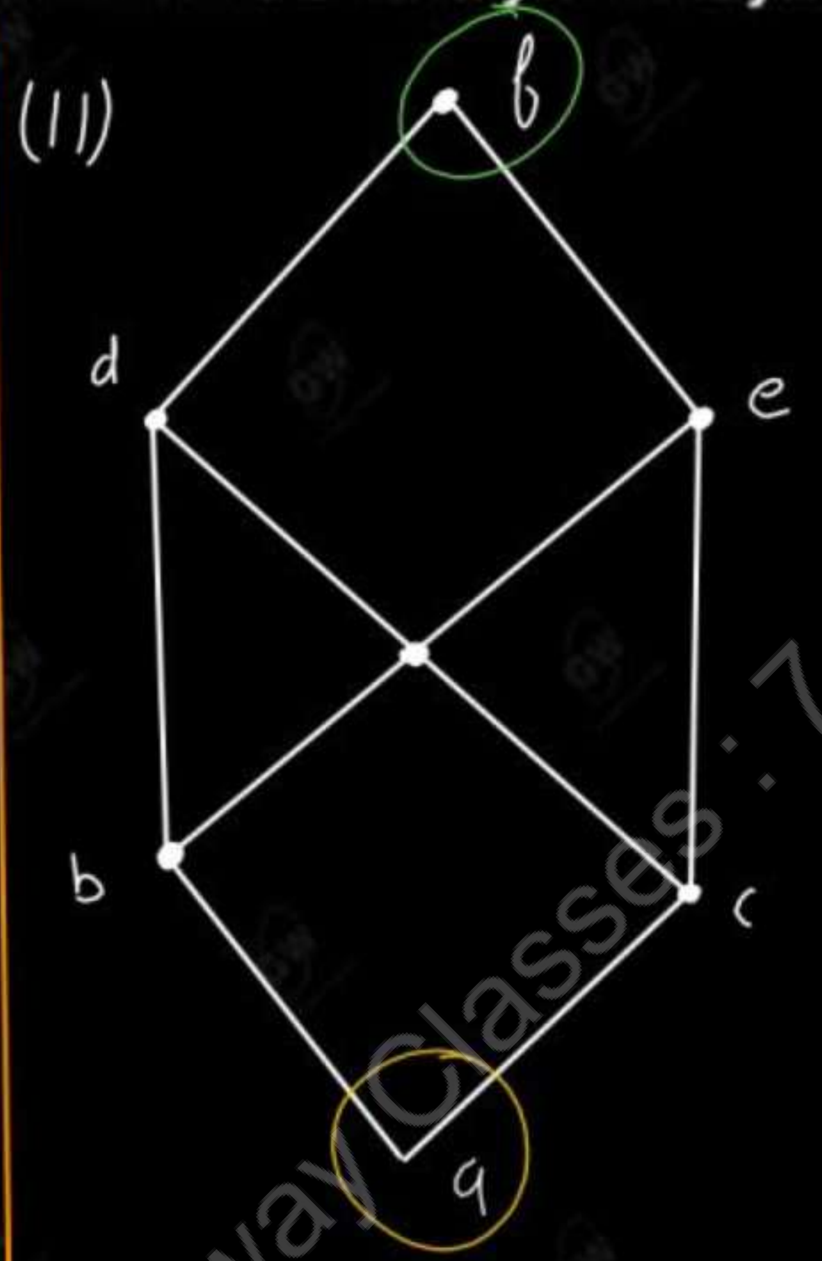


Q.5 : Find the maximum and minimum elements from the following Hasse diagrams



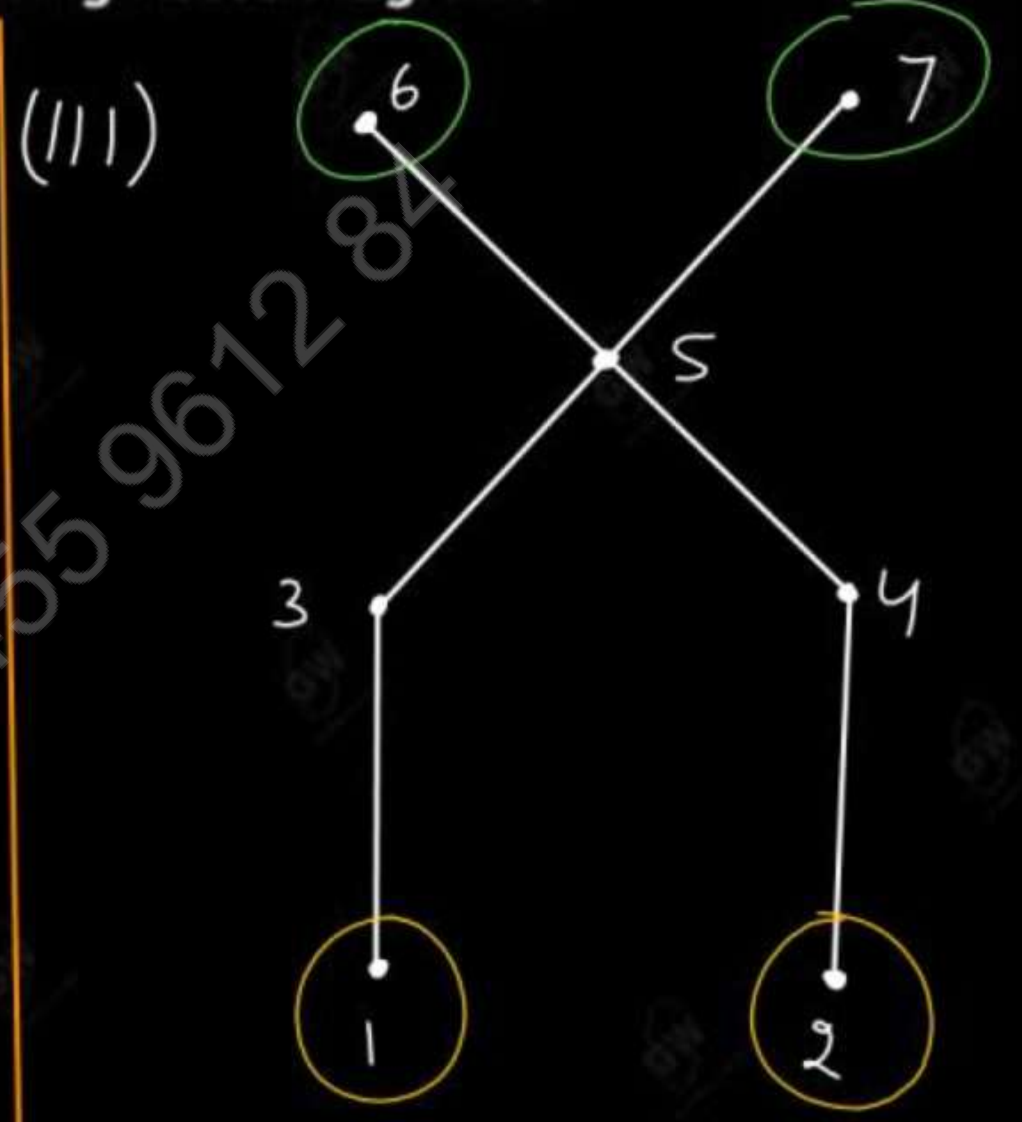
Maximum = e

Minimum = a



Maximum = m

Minimum = a

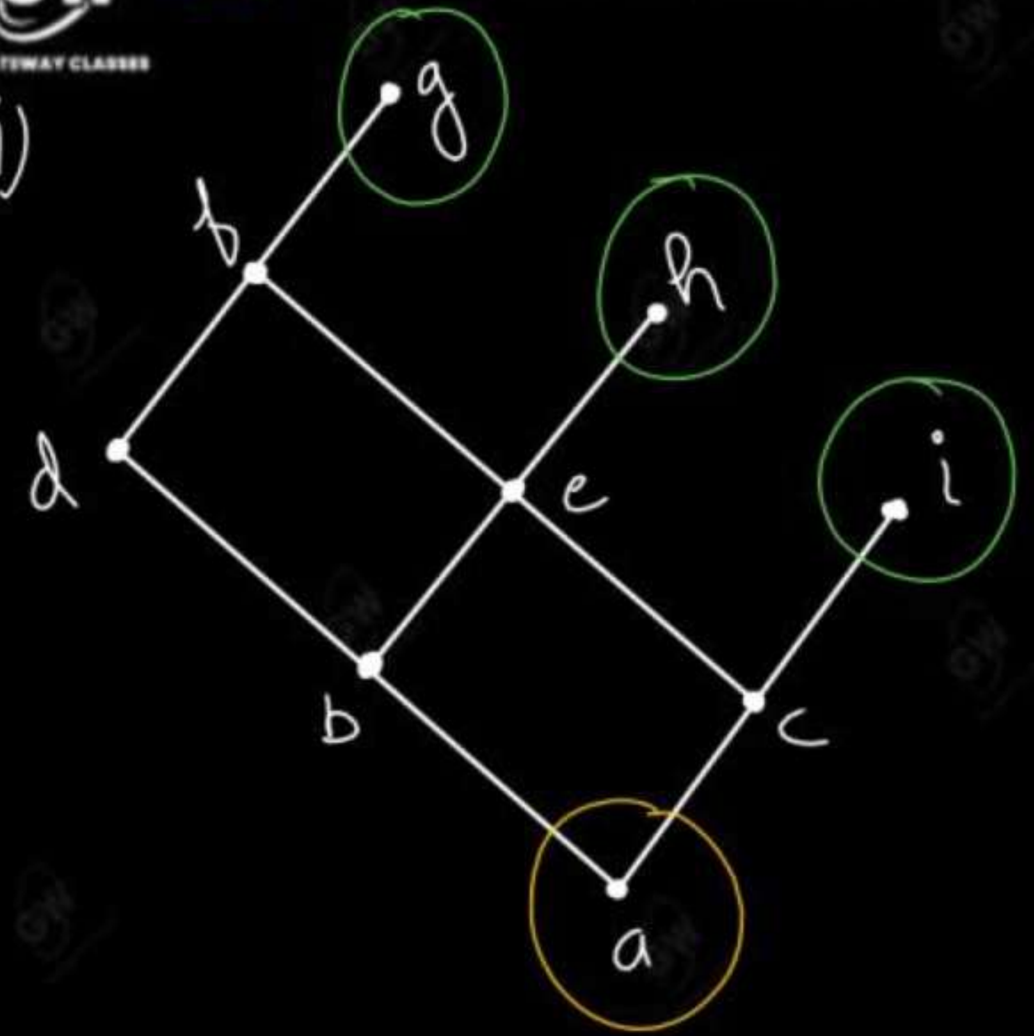


Maximum = X

Minimum = X

Q.6 : Find the Greatest and Least elements of the following Hasse diagrams.

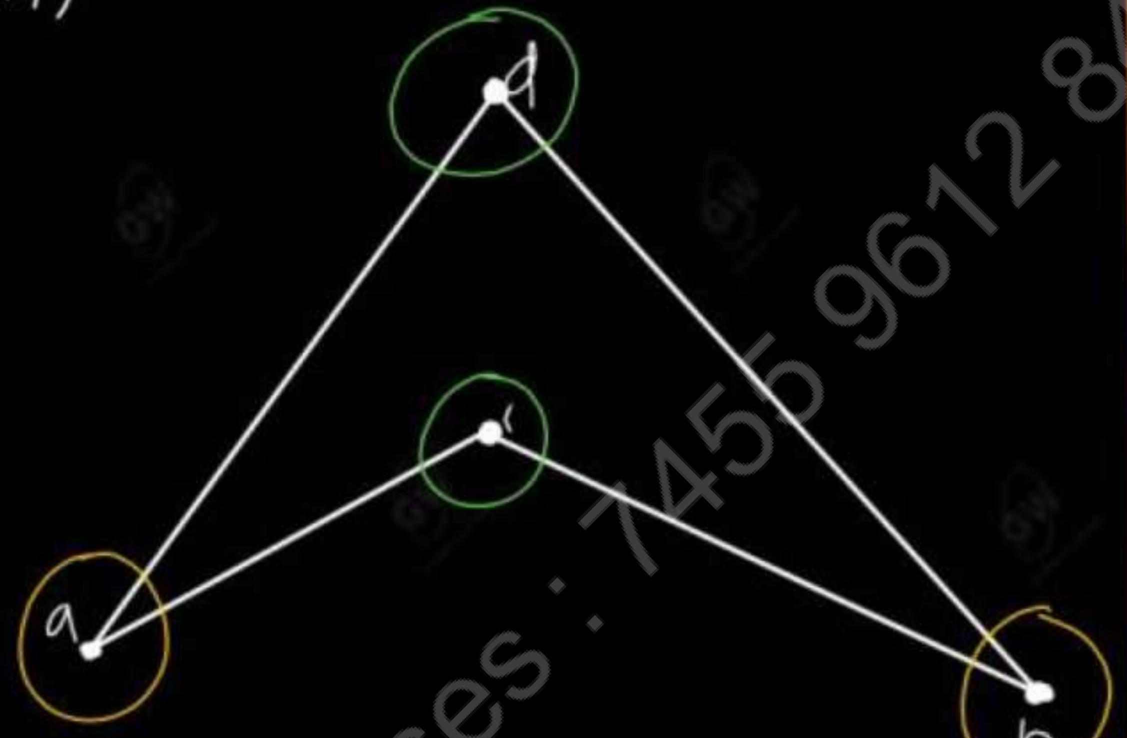
(i)



Maximum = X

Minimum = a

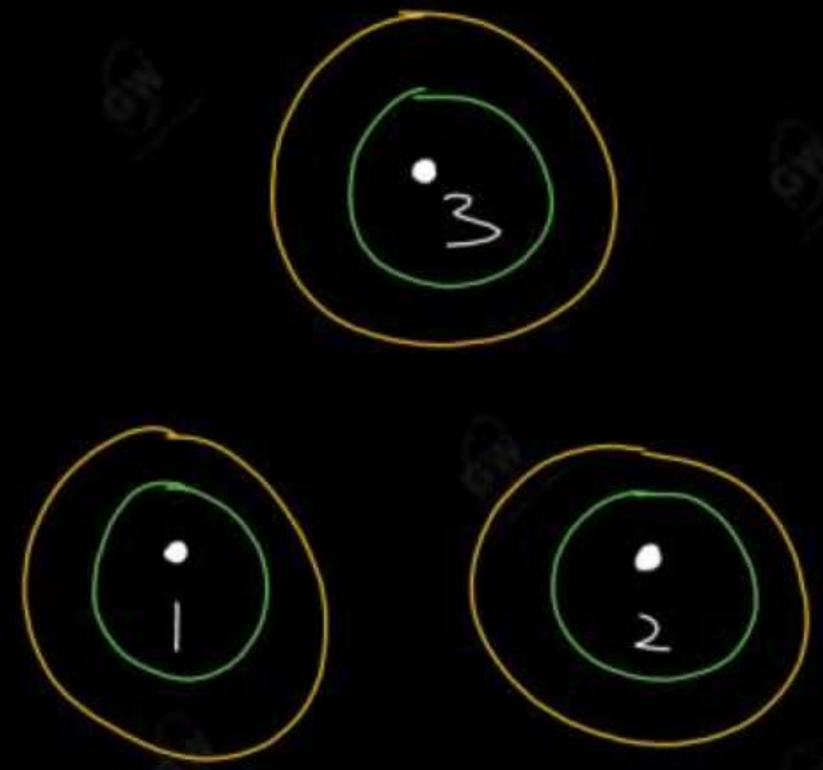
(ii)



Maximum = X

Minimum = X

(iii)



Maximum = X

Minimum = X

$B \subseteq A$

Upper Bound

Let $(A, \leq)$ be POSET and let B be a subset of A. An element $x \in A$ is called an upper bound of B if x succeeds every element of B. i. e

$$\text{i.e } a \leq x \quad \forall \quad a \in B$$

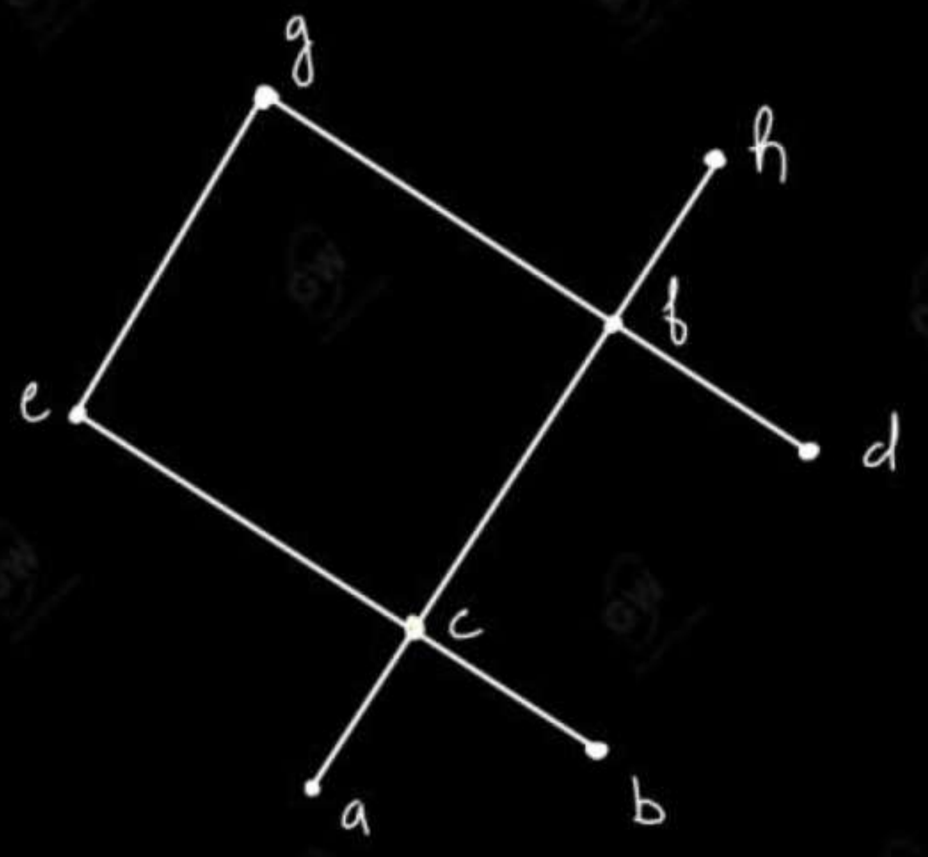
Lower Bound

Let $(A, \leq)$ be a POSET and let B be a subset of A. An element $x \in A$ is called a Lower bound of B if x preceeds every element of B.

$$\text{i.e } x \leq a \quad \forall \quad a \in B$$

Q.7 Find the upper Bound and Lower Bound of set B from the following

(i)



$B = \{e, c\}$

$LB = a, b, c$

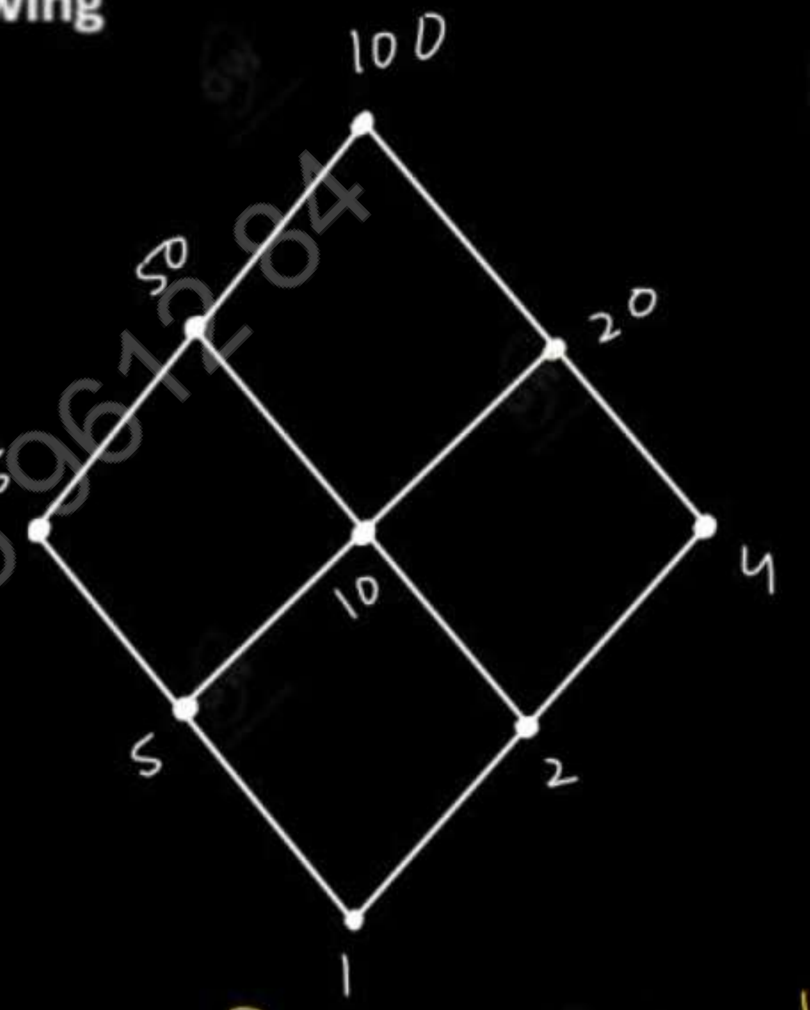
$UB = g$

$B = \{c, f, d\}$

$LB = \phi$

$UB = f, g, h$

(ii)



$B = \{5, 10\}$

$LB = 1, 5$

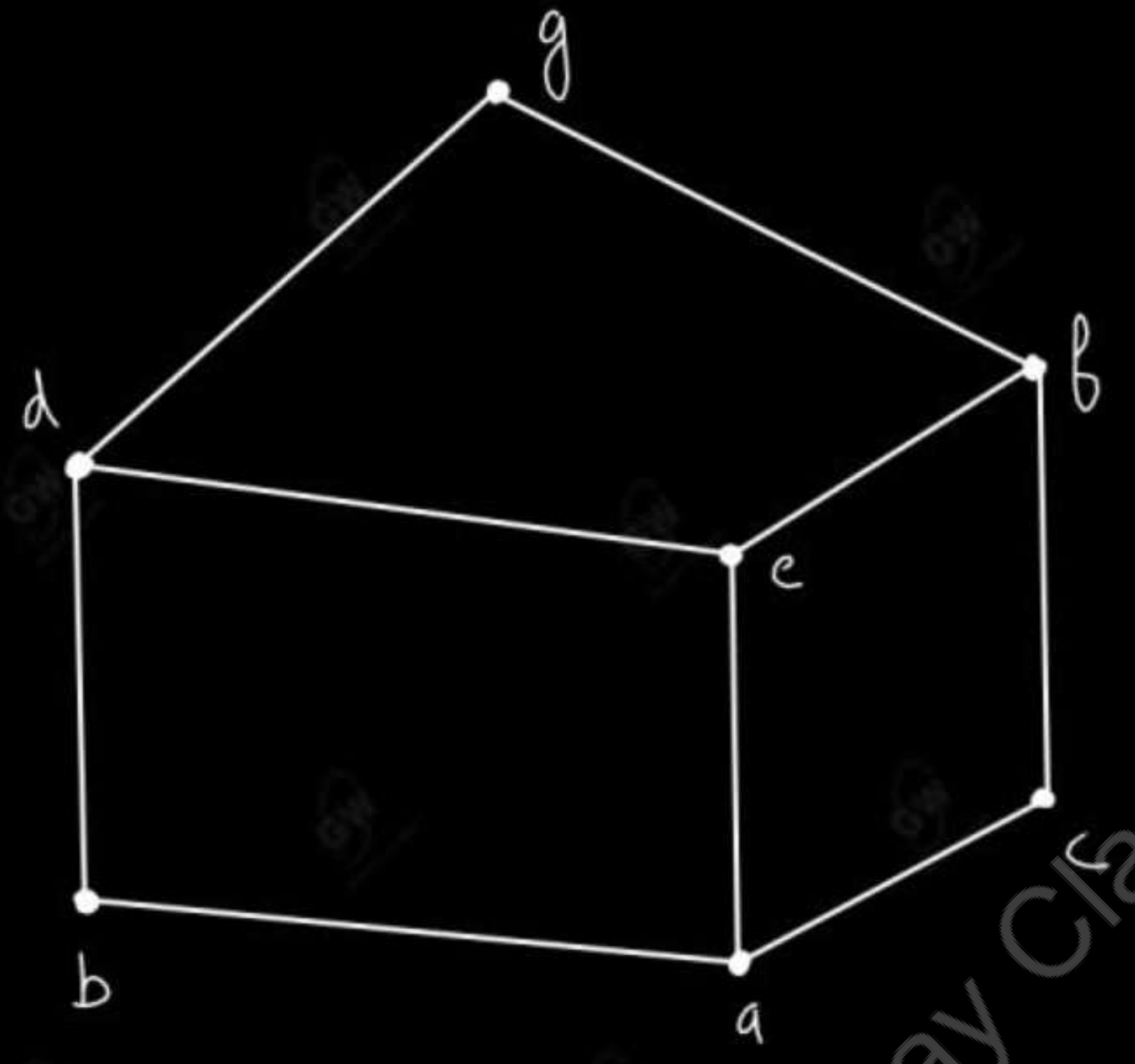
$UB = 10, 20, 50, 100$

$B = \{5, 10, 2, 4\}$

$LB = 1$

$UB = 20, 100$

Q.8 Find the upper Bound and Lower Bound of set B from the following



$B = \{d, g\}$

$\boxed{LB = b, a, \cancel{g}\ e, d}$

$UB = g$

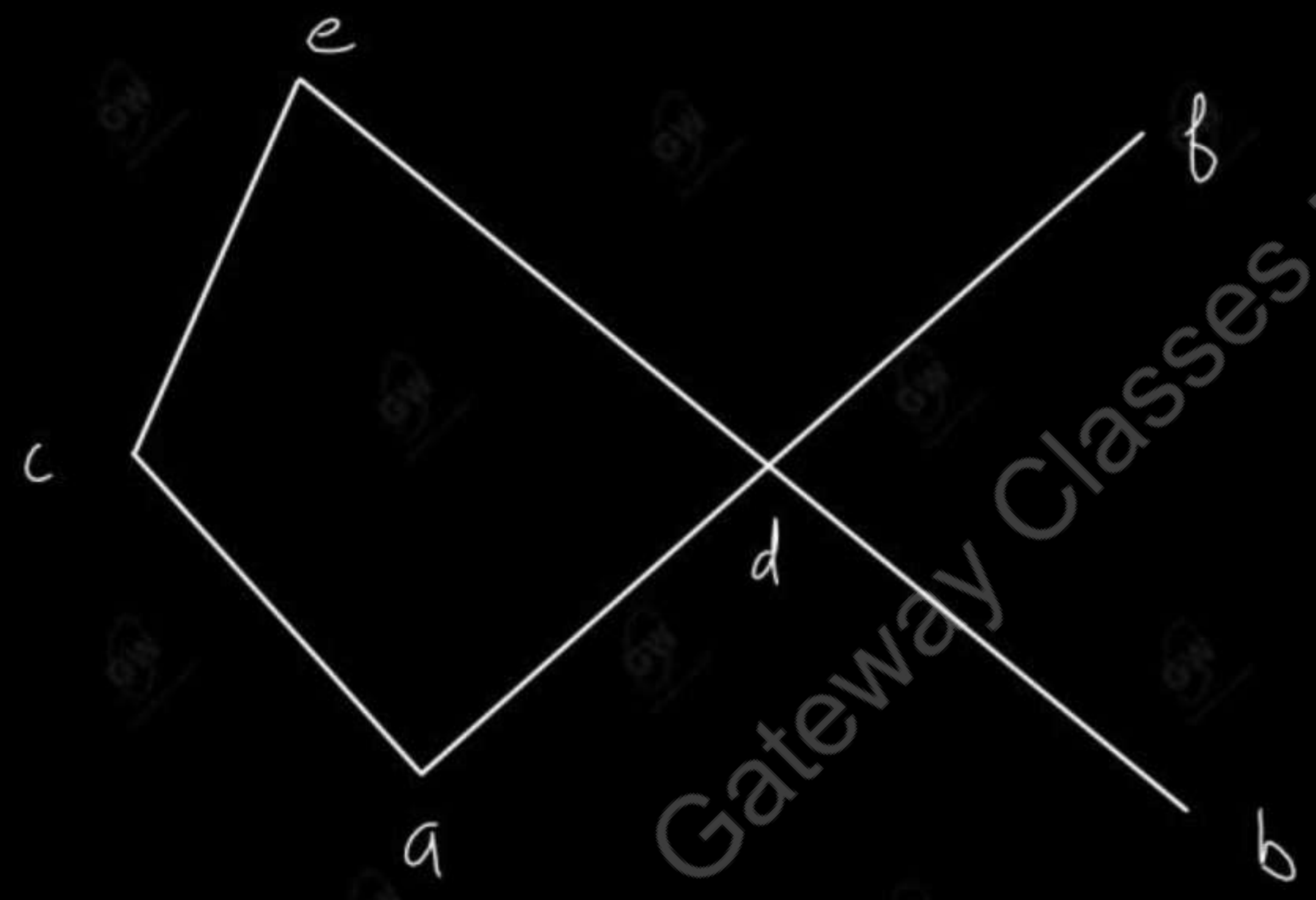
$B = \{e, f\}$

$LB = a, e$

$UB = g, f$

1. **Least Upper Bound / LUB/ Supremum/ Join/($\vee$ or $\cup$)** : Least (minimum) element in upper Bound
2. **Greatest Lower Bound/GLB/Infimum/Meet /($\wedge$ or $\cap$)** : Greatest (maximum) element in lower Bound

Q.9 Find the Least upper Bound and Greatest Lower Bound of set B from the following



$B = \{c, d\}$

UB = e

LUB = e

LB = a

GLB = a

$B = \{a, b\}$

UB = d, e, f

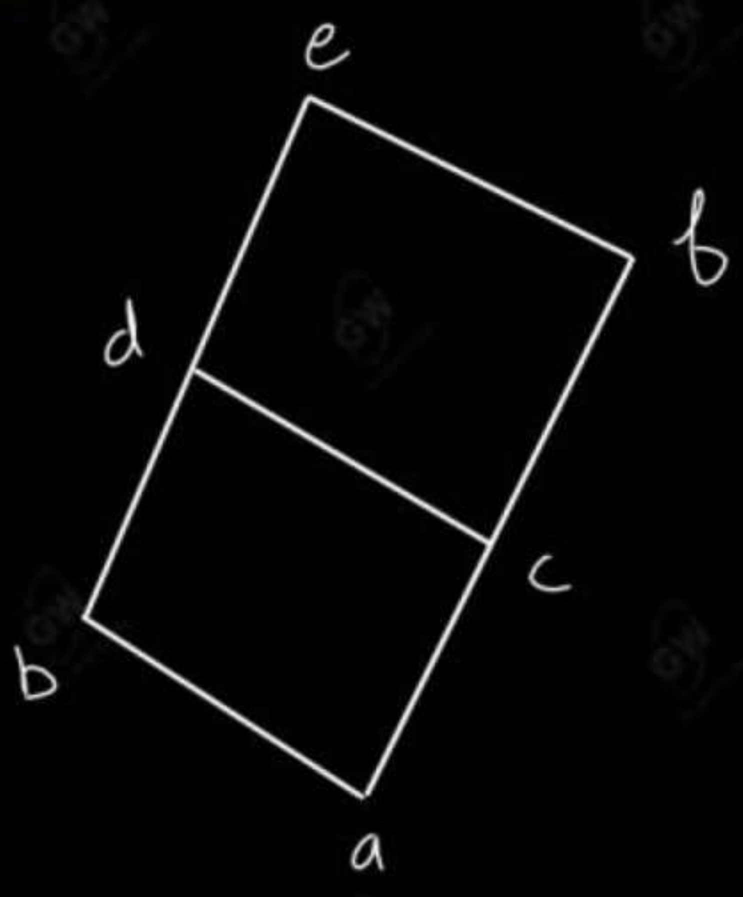
LUB = d

LB = ϕ

GLB = ϕ

Q.10 Find the Least upper Bound and Greatest Lower Bound of set B from the following

(i)



$B = \{a, c\}$

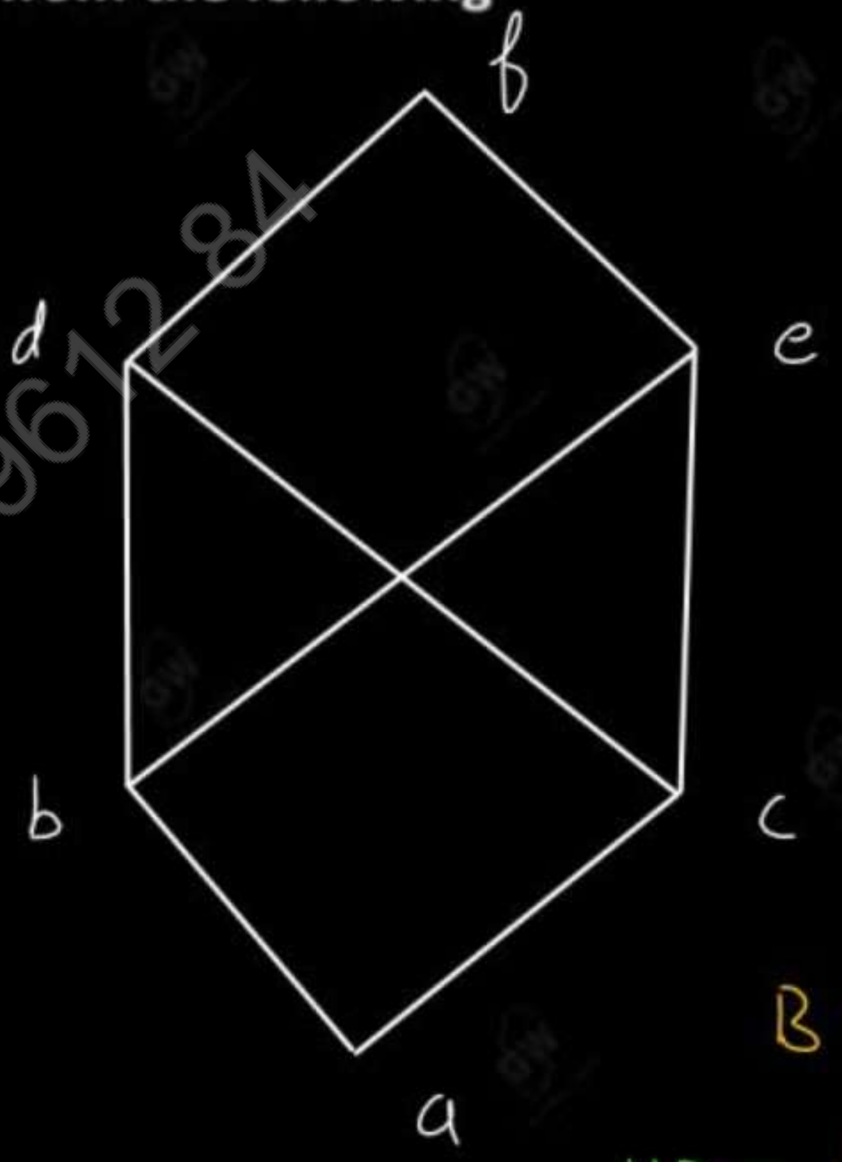
$UB = c, f, d, e$

$LUB = c$

$LB = a$

$GLB = a$

(ii)



$B = \{d, e\}$

$UB = f$

$LUB = f$

$LB = a, b, c$

$GLB = \phi$

$B = \{b, c\}$

$UB = d, e, f$

$LUB = \phi$

$LB = a$

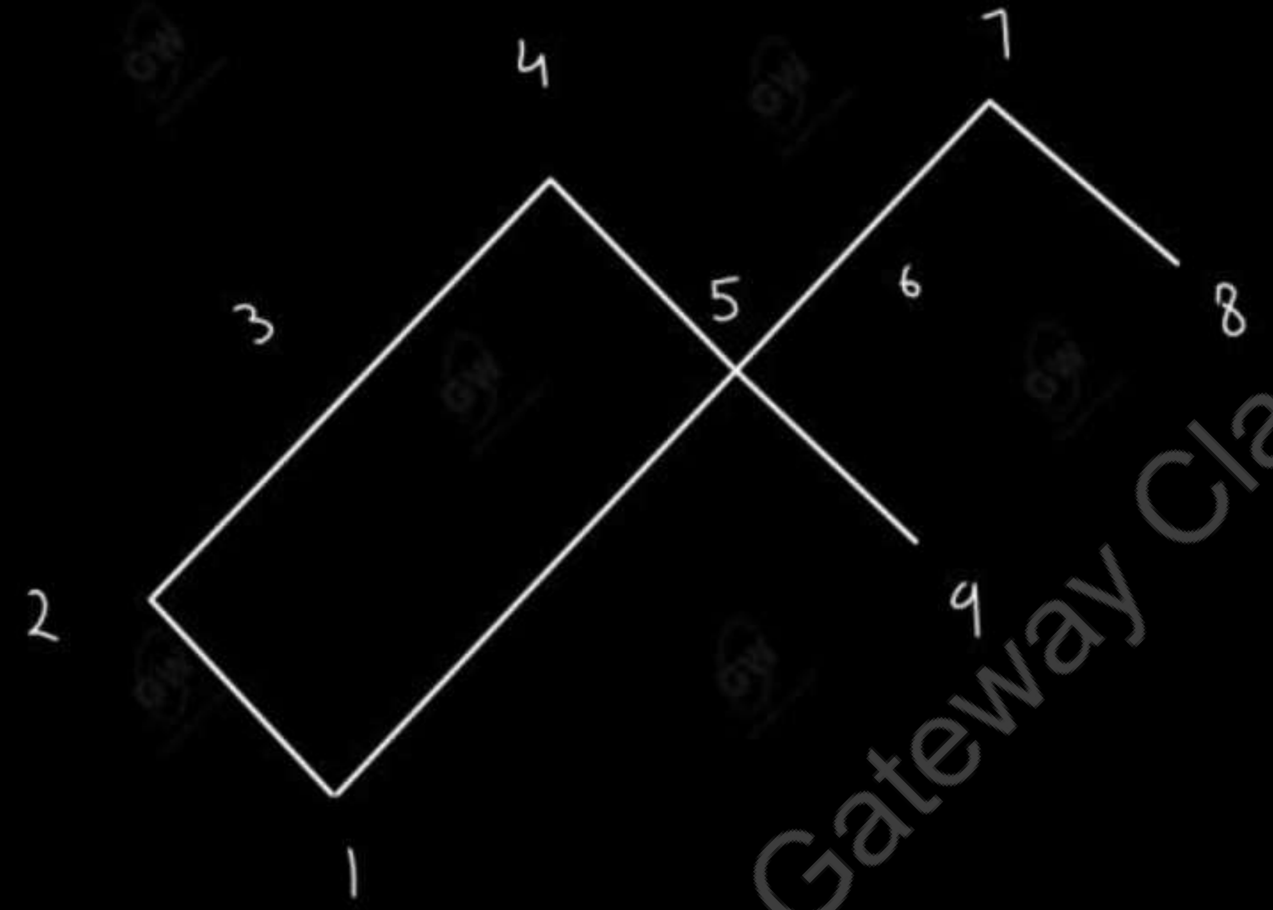
$GLB = a$

DPP- 10

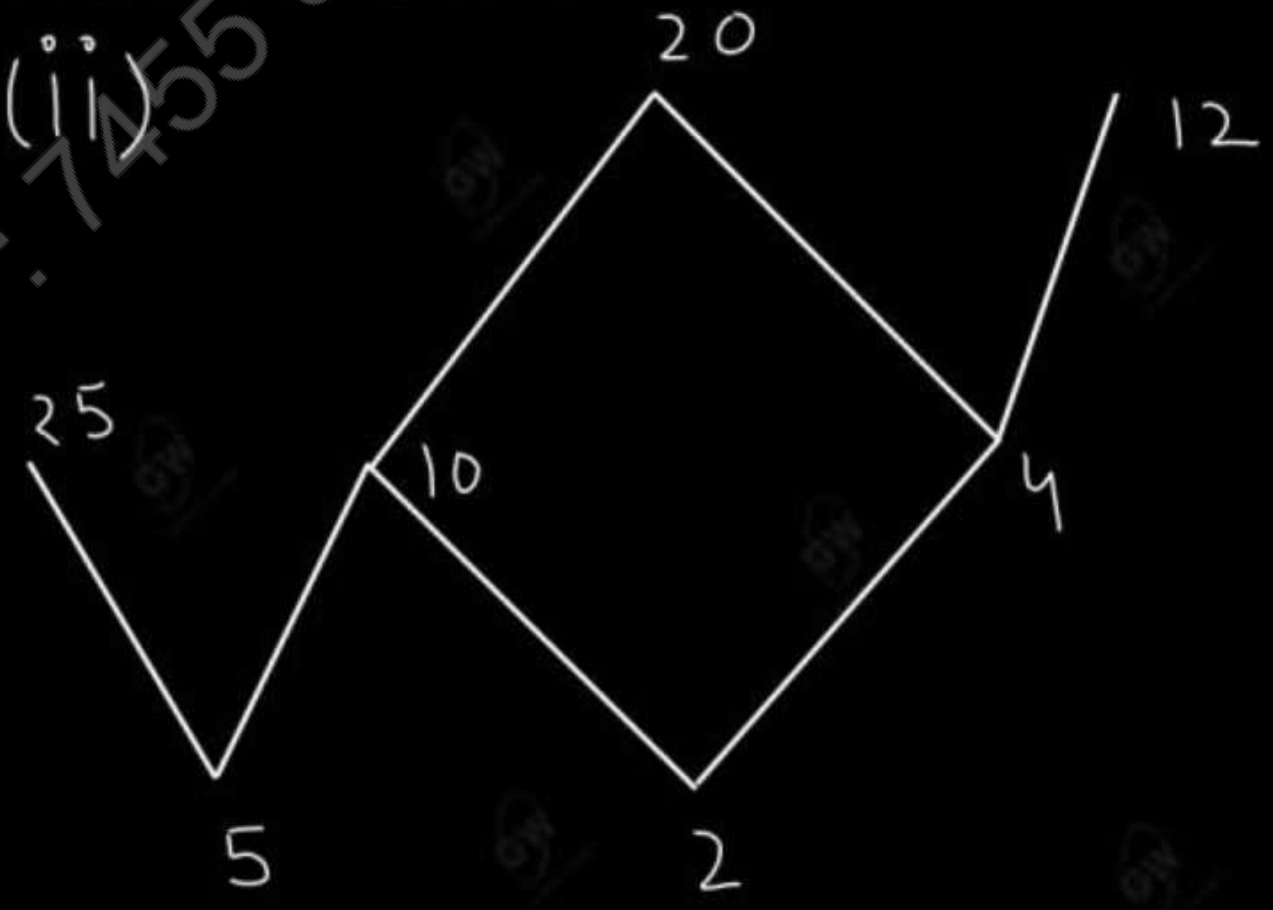
Topic : **Partial Order Relation**

Q.1. Find all the maximal and minimal elements from the following Hasse diagrams.

(i)



(ii)



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DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattice

Today's Target

- ***LATTICE***
- ***Properties of Lattice***
- ***Distributive Lattice***
- ***Bounded Lattice***
- ***Complemented lattice***
- PYQ
- DPP

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP
4	Unit wise set of PYQs

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Link in Description

Join Semi Lattice

A POSET $(L, \leq)$ is a join semi lattice if $\forall\, a, b \in L$, $a \vee b$ exist in L.

$a \vee b$

Note: $a \vee b = lub\,\{a, b\} = sup\,\{a, b\}$

$a \vee b = a \text{ join } b$

lub – least upper bound & sup – supremum

OR

In a POSET if join exist for every pair of elements, then POSET is called Join Semi Lattice

Meet Semi Lattice

A POSET $(L, \leq)$ is a meet semi lattice if $\forall\, a, b \in L$, $a \wedge b$ exist in L.

$a \wedge b$

Note: $a \wedge b = glb\,\{a, b\} = inf\,\{a, b\}$

$a \wedge b = a \text{ meet } b$

glb – greatest lower bound & inf – infimum

OR

In a POSET if meet exist for every pair of elements, then POSET is called Join Semi Lattice

Lattice

A POSET $(L, \leq)$ is a lattice if $\forall a, b \in L$, $a \vee b$ and $a \wedge b$ exist in L.

Where $a \vee b = lub\{a, b\} = sup\{a, b\}$

$a \wedge b = glb\{a, b\} = inf\{a, b\}$

Note: $a \vee b = a \text{ join } b$ and $a \wedge b = a \text{ meet } b$

lub – least upper bound & sup – supremum

glb – greatest lower bound & inf – infimum

OR

In a POSET if join and meet exist for every pair of elements, then POSET is called Lattice

OR

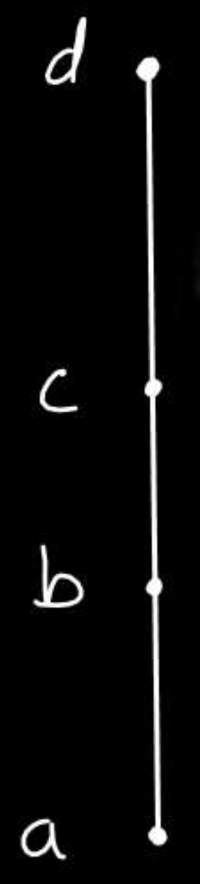
A POSET $(L, \leq)$ is a lattice if it is :

(i) Join Semi Lattice

(ii) Meet Semi Lattice

Q.1. ***Determine whether the following Hasse diagram represent lattice or not***

(i)



lub table

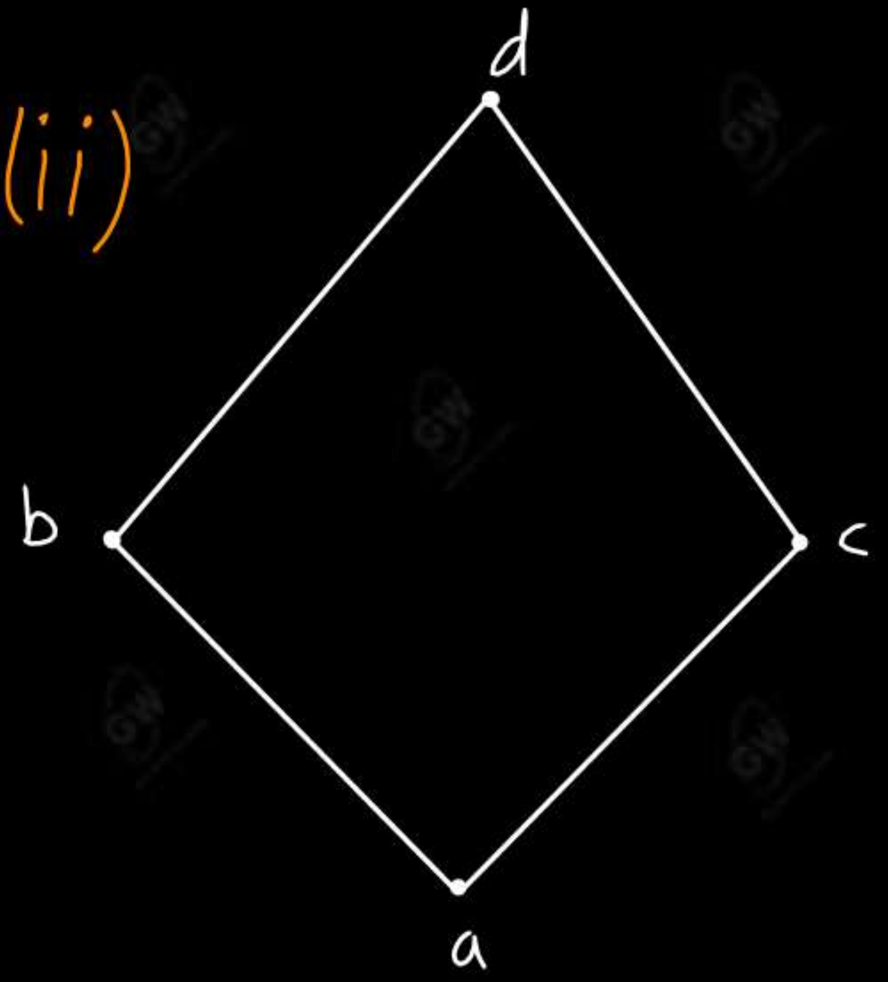
$\vee$	a	b	c	d
a	a	b	c	d
b	b	b	c	d
c	c	c	c	d
d	d	d	d	d

glb table

$\wedge$	a	b	c	d
a	a	a	a	a
b	a	b	b	b
c	a	b	c	c
d	a	b	c	d

It represent a lattice

(ii)



lub table

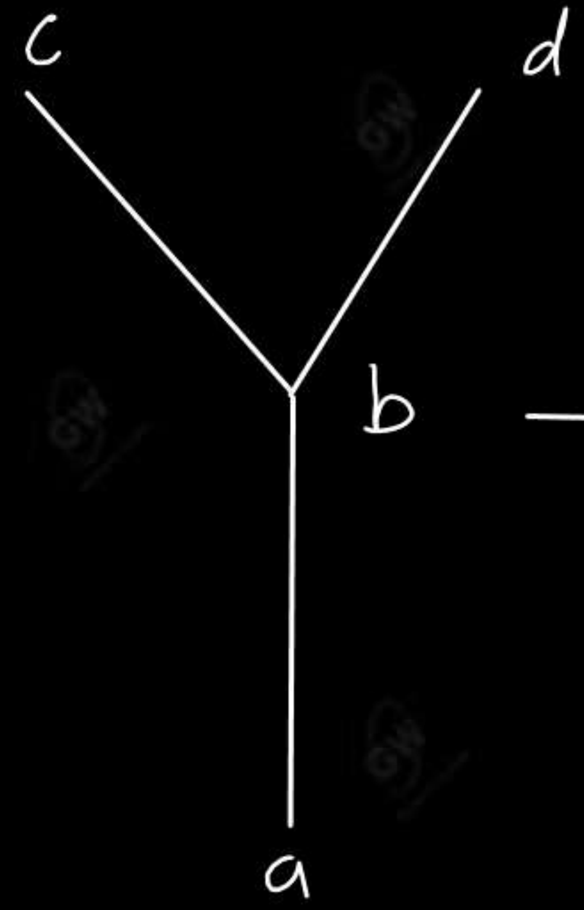
∨	a	b	c	d
a	a	b	c	d
b	b	b	d	d
c	c	d	c	d
d	d	d	d	d

glb table

∧	a	b	c	d
a	a	a	a	a
b	a	b	a	b
c	a	a	c	c
d	a	b	c	d

It represent a lattice

(iii)



lub table

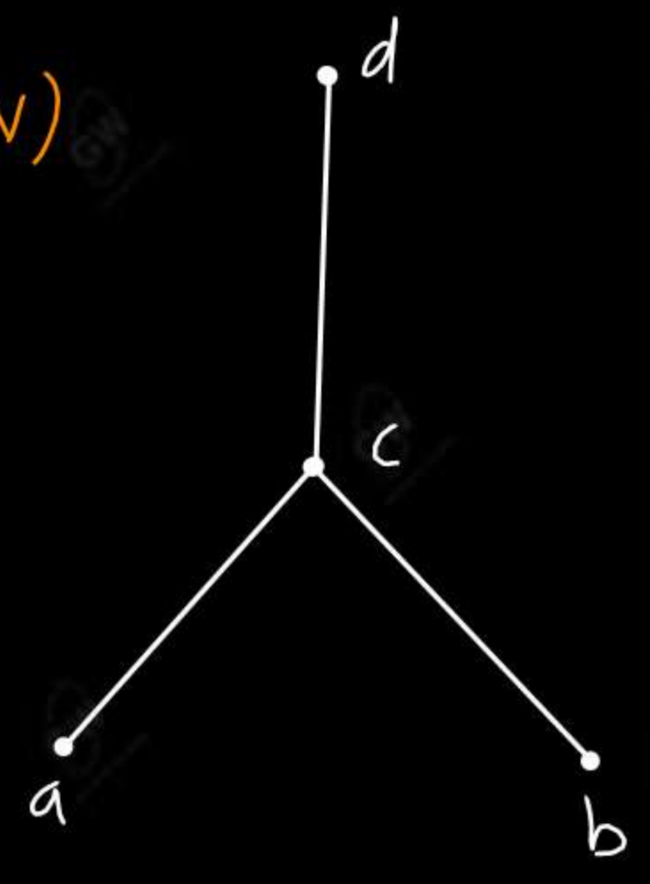
$\vee$	a	b	c	d
a	a	b	c	d
b	b	b	c	d
c	c	c	c	—
d	d	d	—	d

glb table

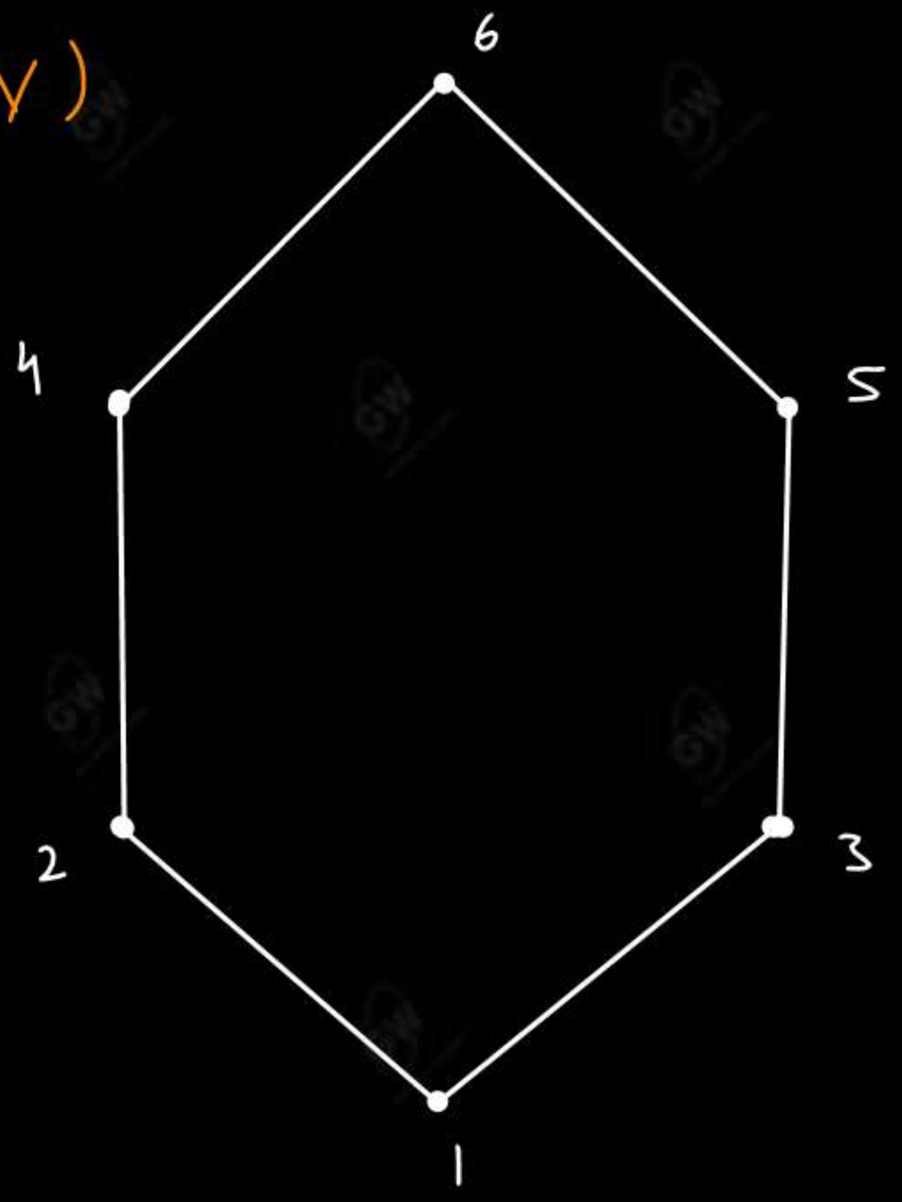
$\wedge$	a	b	c	d
a	a	a	a	a
b	a	b	b	b
c	a	b	c	b
d	a	b	b	d

It does not represent lattice

(IV)



(V)



lub table

V	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	6	4	6	6
3	3	6	3	6	5	6
4	4	4	6	4	6	6
5	5	6	5	6	5	6
6	6	6	6	6	6	6

glb table = ?

It represent a lattice

Properties of Lattice

Let (L, ≤) be a Lattice, then the following results holds

(i) Idempotent law : $\forall \ a \in L$

(i) $a \vee a = a$

(ii) $a \wedge a = a$

(ii) Commutative law :- $\forall a, \ b \in L$

(i) $a \vee b = b \vee a$

(ii) $a \wedge b = b \wedge a$

(iii) Associative law :- $\forall a, \ b, c \in L$

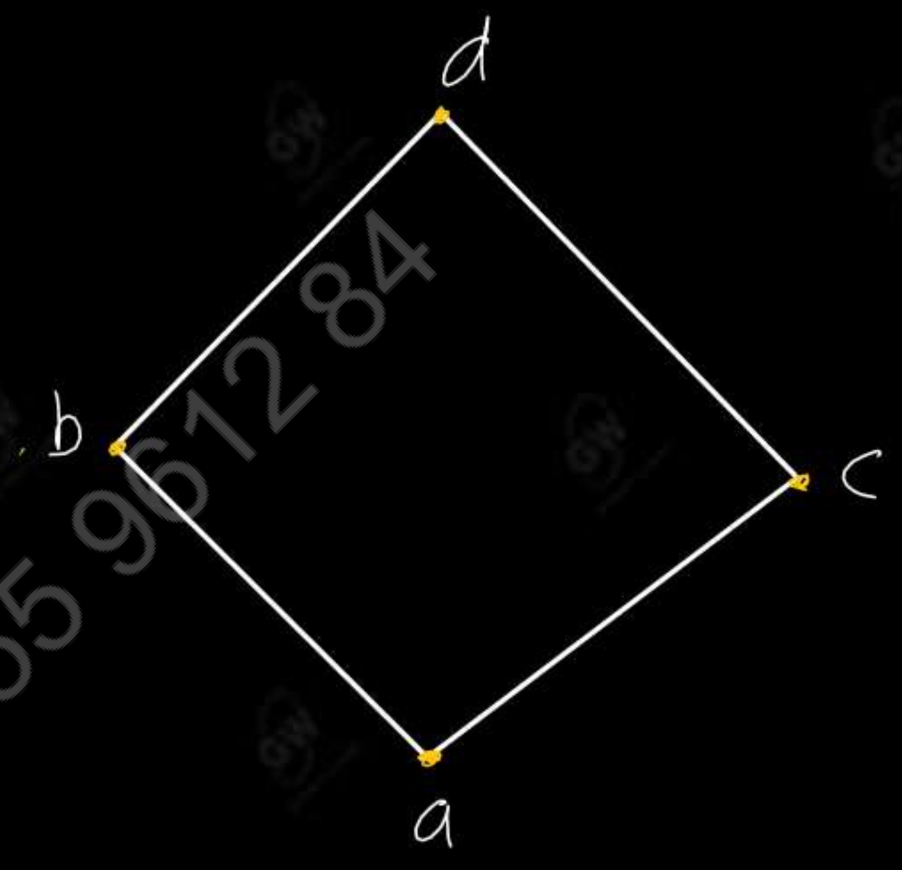
(i) $a \vee (b \vee c) = (a \vee b) \vee c$

(ii) $a \wedge (b \wedge c) = (a \wedge b) \wedge c$

(iv) Absorption law :- for all $a, \ b \in L$

(i) $a \vee (a \wedge b) = a$

(ii) $a \wedge (a \vee b) = a$



Distributive Lattice

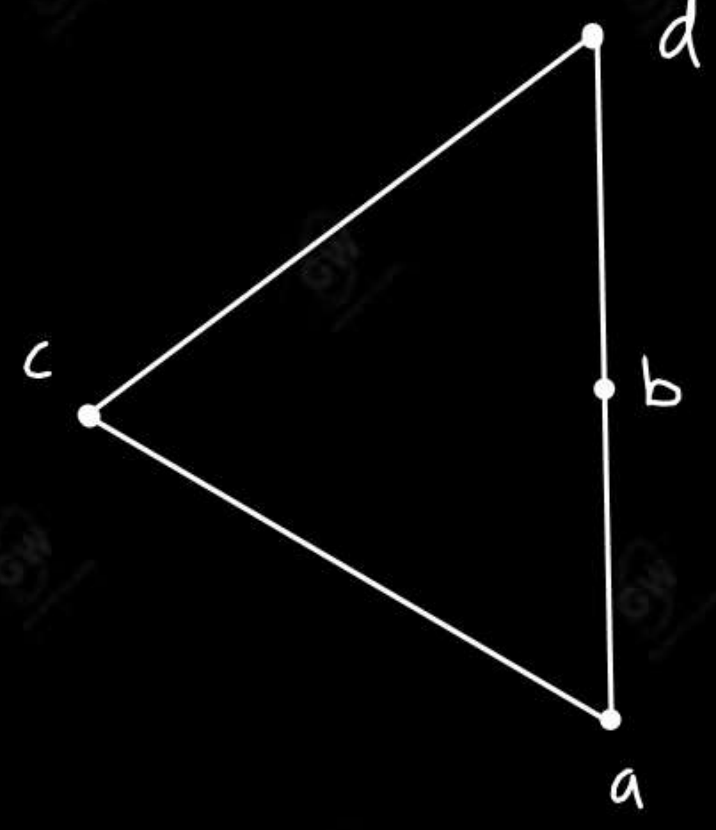
A lattice (L, ≤) is called distributive lattice if $\forall\, a,\ b, c\ \in L$ *, it satisfies the following properties*

(i) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

(ii) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

- *If the lattice L does not satisfy the above properties, the it is called a non-distributive lattice*
- *If the lattice L satisfy the above properties, the it is distributive lattice*

Q.2. ***Show that the given lattice is distributive***



Given lattice is distributive If it satisfy the following properties

(i) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

$a \vee a = b \wedge c$

$a = a$

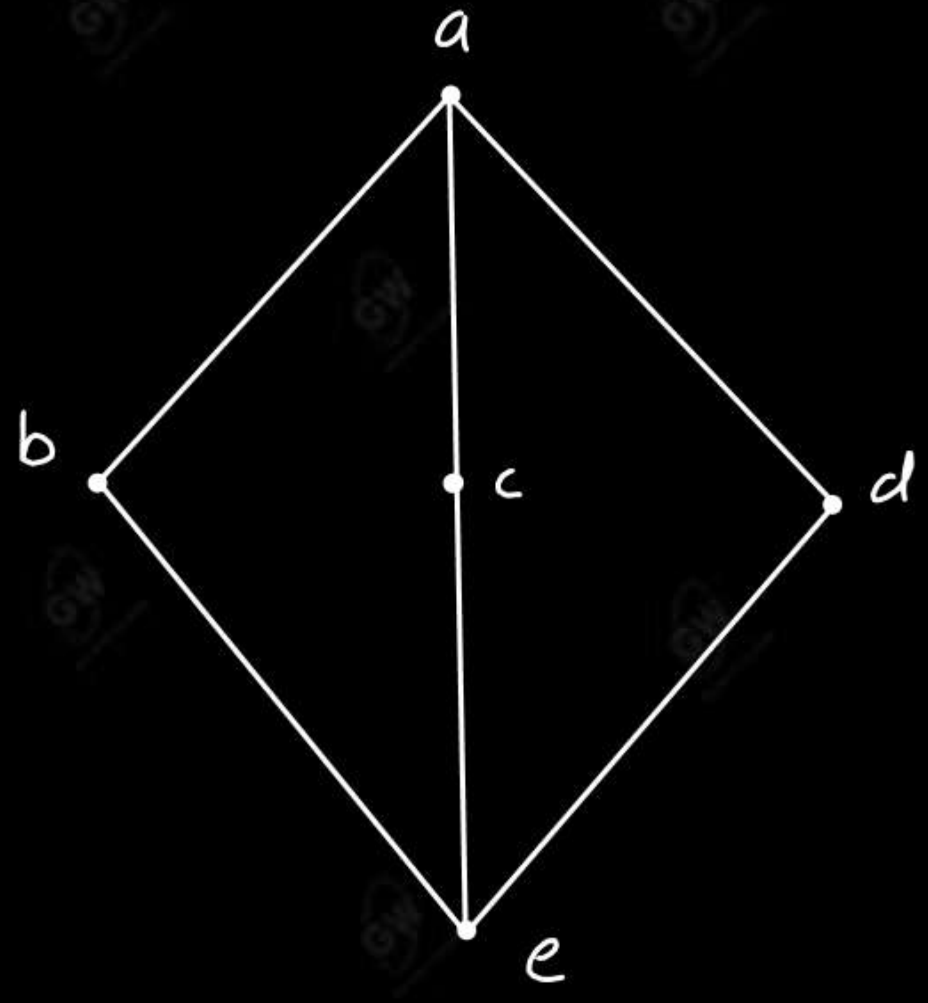
(ii) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

$a \wedge d = a \vee a$

$a = a$

Hence given lattice is distributive

Q.3. Show that the given lattice is not distributive



For b, c, d

(i) $b \vee (c \wedge d) = (b \vee c) \wedge (b \vee d)$

$b \vee e = a \wedge a$

$b \neq a$

LHS $\neq$ RHS

Hence given lattice is not distributive

Theorem :

Let $a,\ b,\ c \in L$ where $(L, \leq)$ is a distributive lattice then

$$a \vee b = a \vee c \text{ and } a \wedge b = a \wedge c \Rightarrow b = c$$

Given

$(L, \leq)$ is a distributive lattice

Where

$a \vee b = a \vee c$

$a \wedge b = a \wedge c$

To Prove

$b = c$

Proof

$b = b \vee (b \wedge a)$ By Aborption law

$= b \vee (a \wedge b)$ By commutative law

$= b \vee (a \wedge c)$ Given

$= (b \vee a) \wedge (b \vee c)$ By Distributive law

$= (a \vee b) \wedge (c \vee b)$ By commutative law

$= (a \vee c) \wedge (c \vee b)$ Given

$= (c \vee a) \wedge (c \vee b)$ By commutative law

$= c \vee (a \wedge b)$ By distributive law

$= c \vee (a \wedge c)$ Given

$= c \vee (c \wedge a)$ By commutative law

$= c$ By Absorption law

$\therefore \boxed{b = c}$ Hence proved

Bounded Lattice :

A lattice (L, ≤) is called a bounded lattice if it has a greatest element 1 and a least element 0.

Properties of Bounded Lattice :

If (L, ≤) is a bounded lattice then for any element $a \in L$ we have

(i) $a \vee 1 = 1$

(ii) $a \wedge 1 = a$

(iii) $a \vee 0 = a$

(iv) $a \wedge 0 = 0$

Note:- Every finite lattice is bounded

$A \cup U = U$

$A \cap U = A$

$A \cup \phi = A$

$A \cap \phi = \phi$

Infinite Lattice

Complement of an element in a lattice :

Let $(L, \leq)$ is a bounded lattice with least element 0 and greatest element 1, then $a' \in L$ is called a complement of $a \in L$ if

(i) $a \vee a' = 1$

(ii) $a \wedge a' = 0$

$$\therefore a' = a$$

Note:- (1) It is not necessary that every element $a \in L$ has a complement

(2) An element $a \in L$ have more than one complement

(3) $1' = 0$ and $0' = 1$

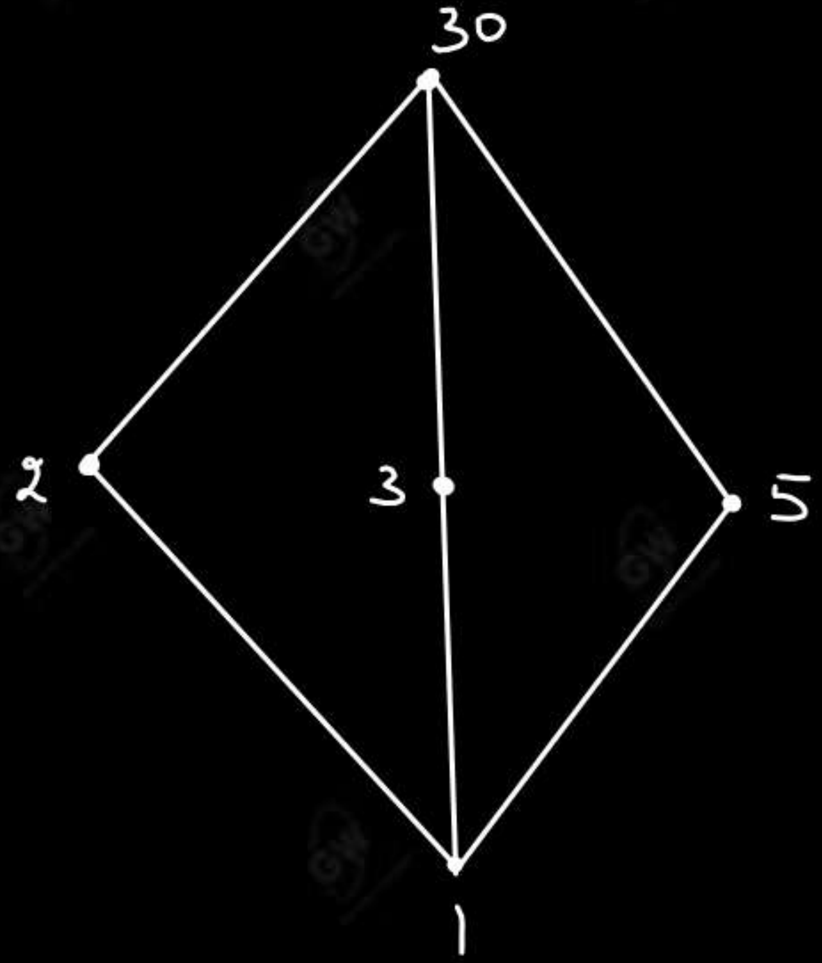
Complemented lattice :

A Lattice $(L, \leq)$ is called a complemented lattice if

(i) if L is bounded

(ii) Every element $a \in L$ has a complement

Q.4. Let $A = \{1, 2, 3, 5, 30\}$ and $a \le b$ iff a divided b. Then find the complement of 2.



(i) $2 \vee 3 = 30$ $\quad 2 \wedge 3 = 1$

(ii) $2 \vee 5 = 30$ $\quad 2 \wedge 5 = 1$

Hence

3 and 5 are the complement of 2

$2' = 3$

$2' = 5$

Theorem:

If a and b are the elements in a bounded distributive lattice L and if 'a' has complement a' then

(i) $(a')' = a$

(ii) $(a \vee b)' = a' \wedge b'$ DE Morgan's law

(iii) $(a \wedge b)' = a' \vee b'$ DE Morgan's law

(iv) $a \vee (a' \wedge b) = a \vee b$

(v) $a \wedge (a' \vee b) = a \wedge b$

a a'

$\boxed{a \vee a' = 1}$

(ii) complement of $a \vee b = a' \wedge b'$

By property of complement

$(a \vee b) \vee (a' \wedge b') = 1$ and $(a \vee b) \wedge (a' \wedge b') = 0$

$= [(a \vee b) \vee a'] \wedge [(a \vee b) \vee b']$ By Distributive law

$= [(b \vee a) \vee a'] \wedge [a \vee (b \vee b')]$ By commutative law

$= [b \vee (a \vee a')] \wedge [a \vee 1]$

$= (b \vee 1) \wedge (a \vee 1)$

$= 1 \wedge 1$

$= 1$

and

$(a \vee b) \wedge (a' \wedge b') = 0$

$= (a \vee b) \wedge (a' \wedge b')$

$= (a' \wedge b') \wedge (a \vee b)$ By commutative law

$= [(a' \wedge b') \wedge a] \vee [(a' \wedge b') \wedge b]$ By Distributive law

$= [(b' \wedge a') \wedge a] \vee [a' \wedge (b' \wedge b)]$ By commutative law

$= [b' \wedge (a' \wedge a)] \vee [a' \wedge 0]$

$= (b' \wedge 0) \vee (a' \wedge 0)$

$= 0 \vee 0$

$= 0$

LHS = RHS

Hence

$(a \vee b)' = a' \wedge b'$

(iv) $a \vee (a' \wedge b) = a \vee b$

LHS

$= a \vee (a' \wedge b)$

$= (a \vee a') \wedge (a \vee b)$ By Distributive law

$= 1 \wedge (a \vee b)$

$= a \vee b$

$=$ RHS

Hence proved

(v) $a \wedge (a' \vee b) = a \wedge b$

LHS

$= a \wedge (a' \vee b)$

$= (a \wedge a') \vee (a \wedge b)$ By Distributive law

$= 0 \vee (a \wedge b)$

$= a \wedge b$

$=$ RHS

Hence proved

THEOREM

Show that in a distributive Lattice, if an element has a complement then this complement is unique.

OR

Prove that for a bounded lattice L, the complements are unique if they exist.

Let $(L, \leq)$ be a bounded distributive lattice

Let $a \in L$ having two complements b and c

then show that

$$b = c$$

Since b and c be the complement of a then

$a \vee b = 1 \qquad a \wedge b = 0$

$a \vee c = 1 \qquad a \wedge c = 0$

Proof

$b = b \wedge 1$

$b = b \wedge (a \vee c)$ Given

$b = (b \wedge a) \vee (b \wedge c)$ Distributive law

$b = (a \wedge b) \vee (b \wedge c)$ commutative law

$b = 0 \vee (b \wedge c)$ Given

$b = (a \wedge c) \vee (b \wedge c)$

$b = (a \vee b) \wedge c$

$b = 1 \wedge c$

$\boxed{b = c}$

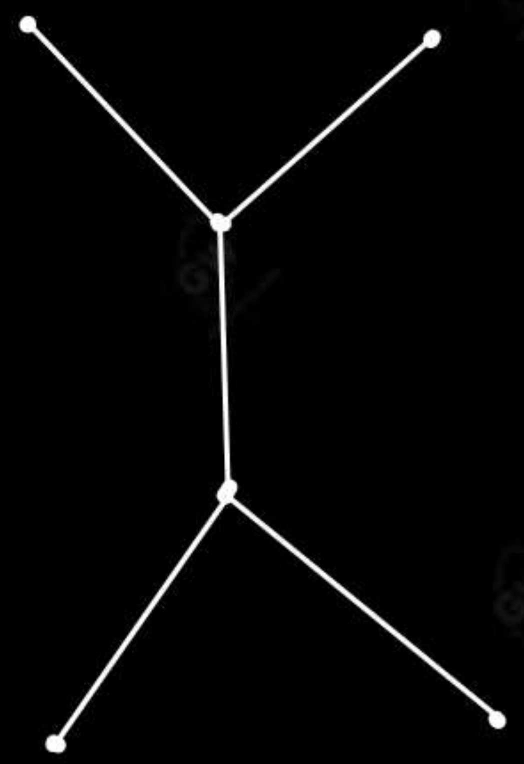
Hence proved

DPP- 10

Topic : *LATTICE*

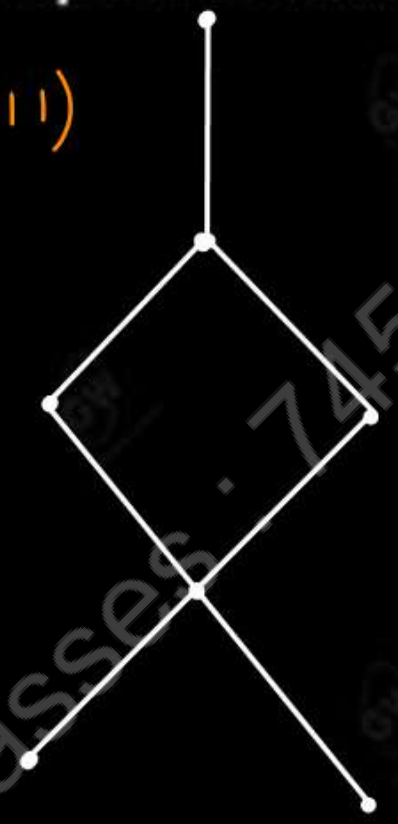
Q.1. Determine whether the following Hasse diagram represent lattice or not

(I)

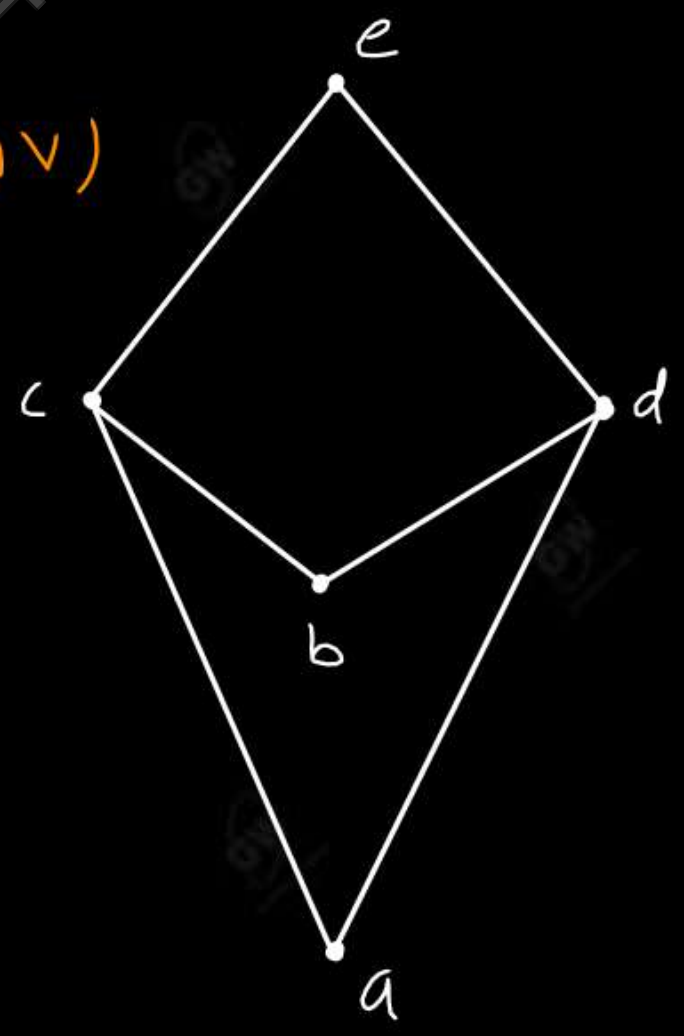


(II)

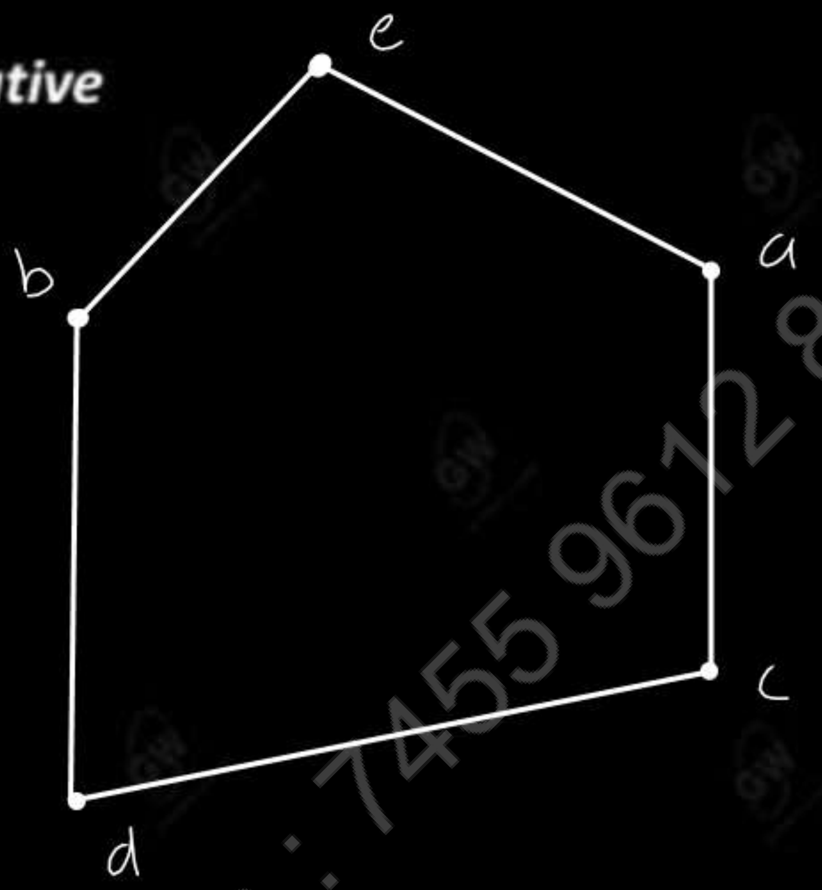
(III)



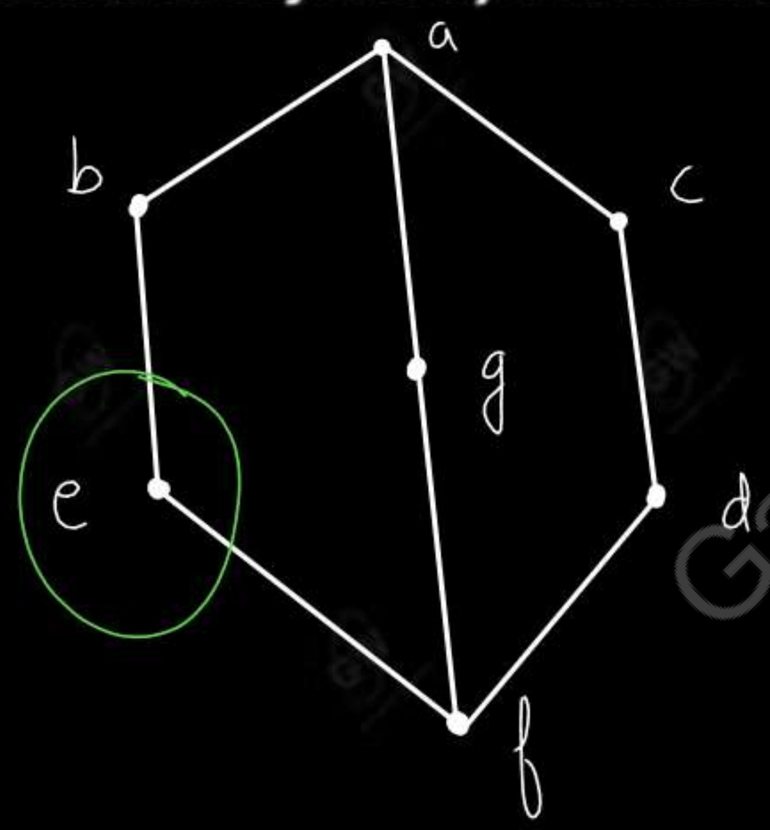
(IV)



Q.2. ***Show that the given lattice is not distributive***



Q.3. ***In the Lattice defined by the Hasse Diagram. How many complements does the element 'e' have ?***



Thank You

DISCRETE STRUCTURES & THEORY OF LOGICS (BCS303)

UNIT -1 : (i) Set Theory & Relations

(ii) POSET & Lattices

Today's Target

- ***Modular LATTICE***
- ***Complete Lattice***
- ***Sub Lattice***
- ***Isomorphic Lattice***
- PYQ
- DPP

Course Details(Paid) : All Subjects

1	Recorded Video Lectures (100 % Syllabus Coverage)
2	Pdf Notes
3	Lecture wise DPP
4	Unit wise set of PYQs

Paid Courses are available in Gateway Classes Application

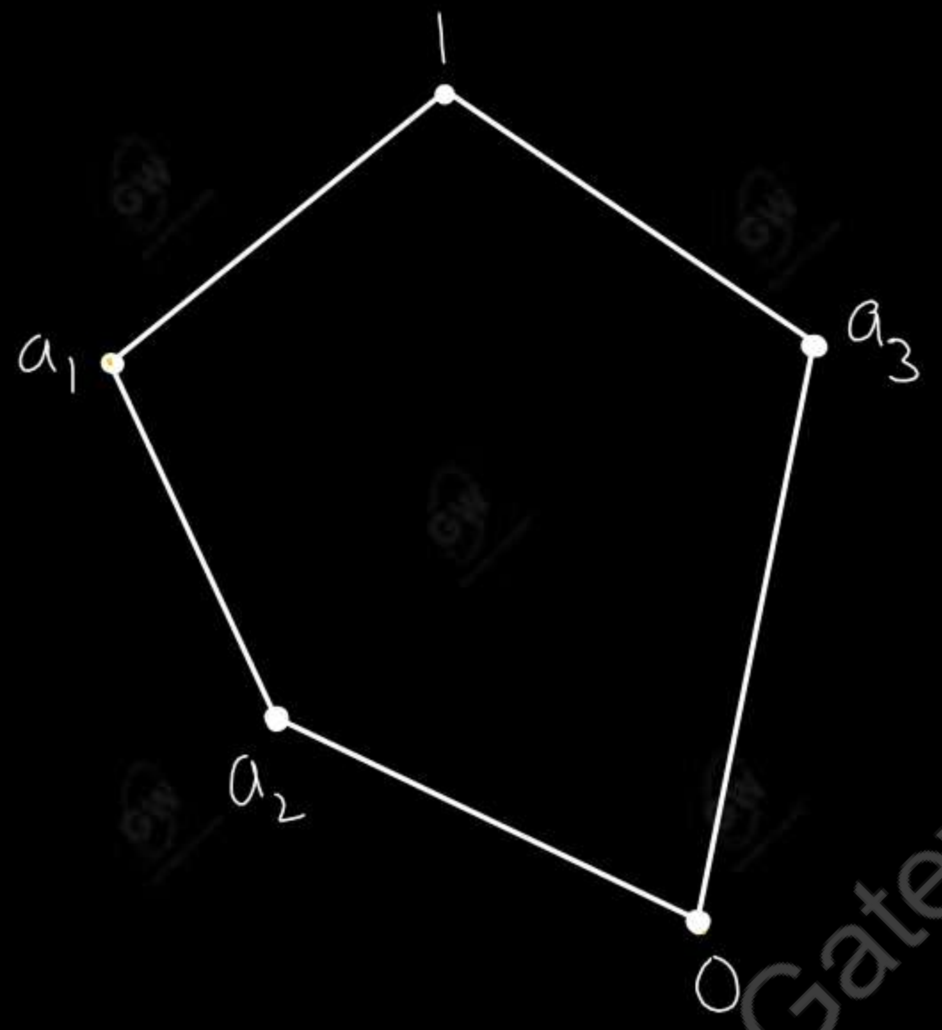
Link in Description

Modular Lattice

A lattice (L, ≤) is said to be modular lattice if ∀ a, b, c ∈ L

$$\boxed{a \vee (b \wedge c) = (a \vee b) \wedge c} \text{ whenever } a \le c$$

Q.1. Show that that pentagonal lattice given below is not modular



consider a, b, c in such a way that : $a \le c$

✓ $a = a_2$

$b = a_3$

✓ $c = a_1$

Now

$a \vee (b \wedge c) = a_2 \vee (a_3 \wedge a_1)$

$= a_2 \vee 0$

$= a_2$

$$(a \vee b) \wedge c = (a_2 \vee a_3) \wedge a_1$$

$$= 1 \wedge a_1$$

$$= a_1$$

$$\therefore \boxed{A \vee (b \wedge c) \neq (a \vee b) \wedge c}$$

Hence, given lattice is not a modular lattice

Theorem

Prove that every distributive lattice is modular.

Let $(L, \leq)$ be a distributive lattice and $a, b, c \in L$ such that

$a \leq c$ then

$$a \vee c = c \quad \text{—— (1)}$$

By Distributive law

$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$$

By using (1)

$$\boxed{a \vee (b \wedge c) = (a \vee b) \wedge c}$$

Hence, every distributive lattice is modular

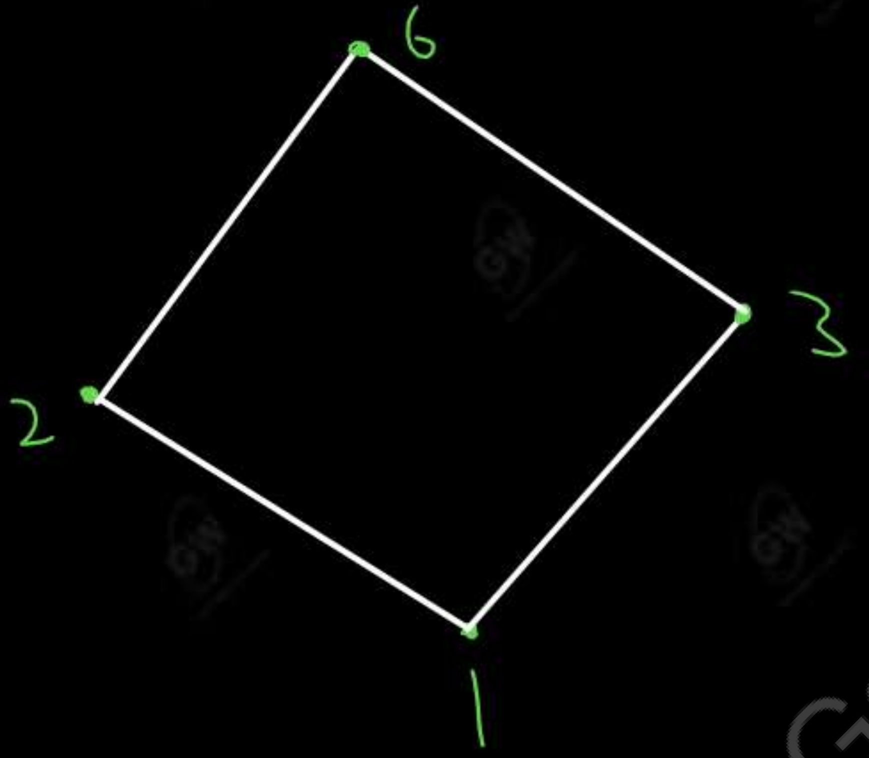
<u>Note</u> - Modular lattice may or may not a distributive lattice

Complete lattice :-

A Lattice $(L, \leq)$ is complete, if every non-empty subset of L has a least upper bound (sup) and greatest lower bound (inf) in L.

Example : Show that $(D_6, |)$ is a complete lattice

$D_6 = \{1, 2, 3, 6\}$



Subsets

$A_1 = \{1, 2\}$

$A_2 = \{1, 2, 6\}$

$A_3 = \{2, 6\}$

$A_4 = \{3, 6\}$

Hence, the given lattice is a complete lattice

$D_n - Lattice$

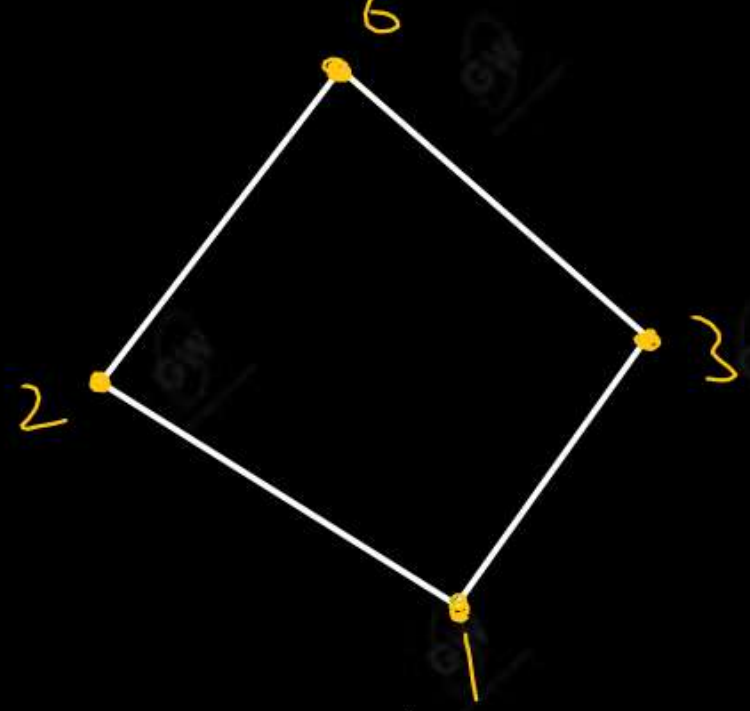
Let n be a positive integer and let D_n be the set of all +ve divisors on n

Then D_n is a lattice under the relation of Divisibility

It is denoted by (D_n, I)

Q.1. Prove that $(D_6, \ I)$ is a lattice .

$D_6 = \{1, 2, 3, 6\}$



lub table

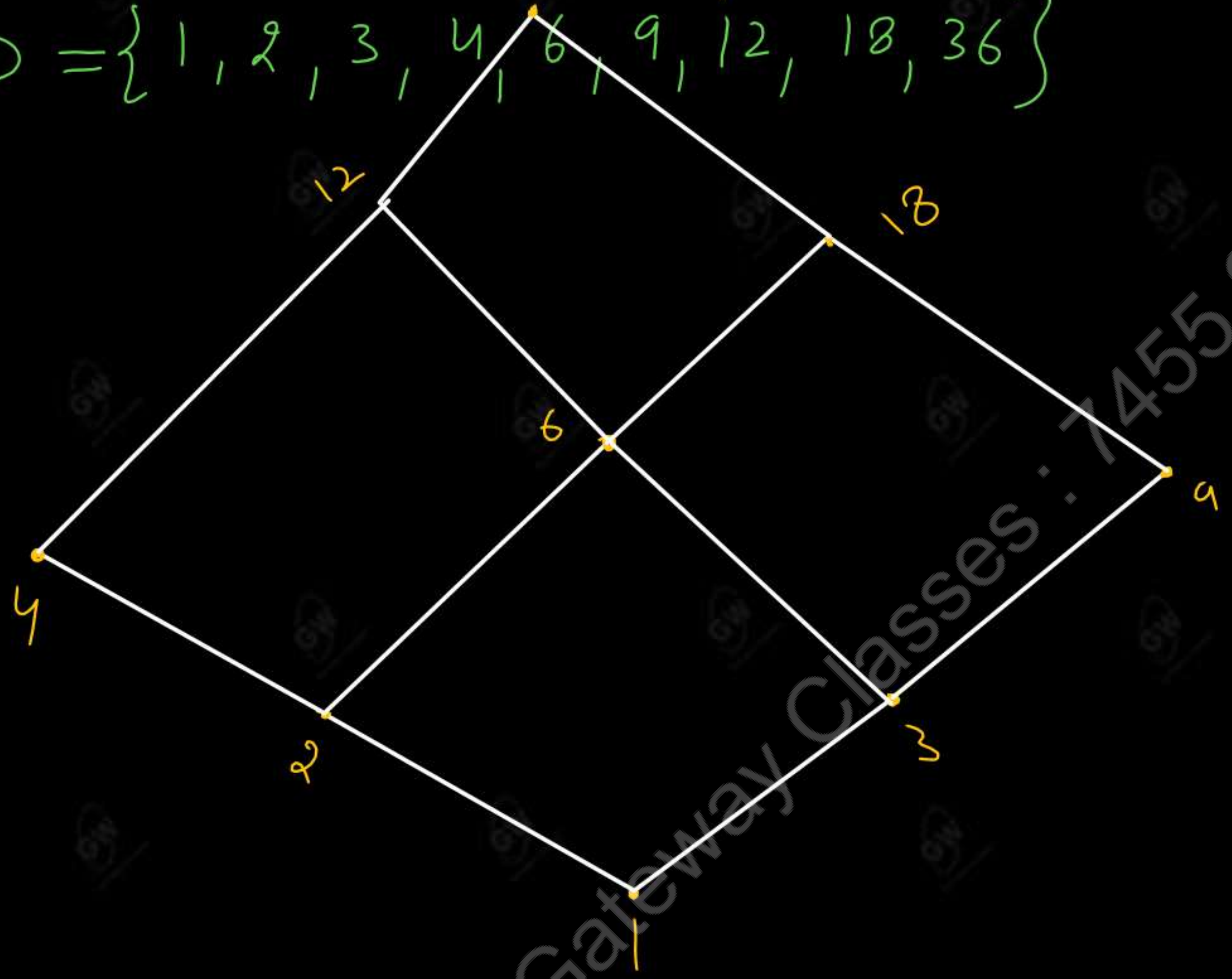
$\vee$	1	2	3	6
1	1	2	3	6
2	2	2	6	6
3	3	6	3	6
6	6	6	6	6

glb table

$\wedge$	1	2	3	6
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
6	1	2	3	6

Q.2: Prove that $(D_{36}, |)$ is a lattice .

$D = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$



lub table

$\vee$	1	2	3	4	6	9	12	18	36
1									
2									
3									
4									
6									
9									
12									
18									
36									

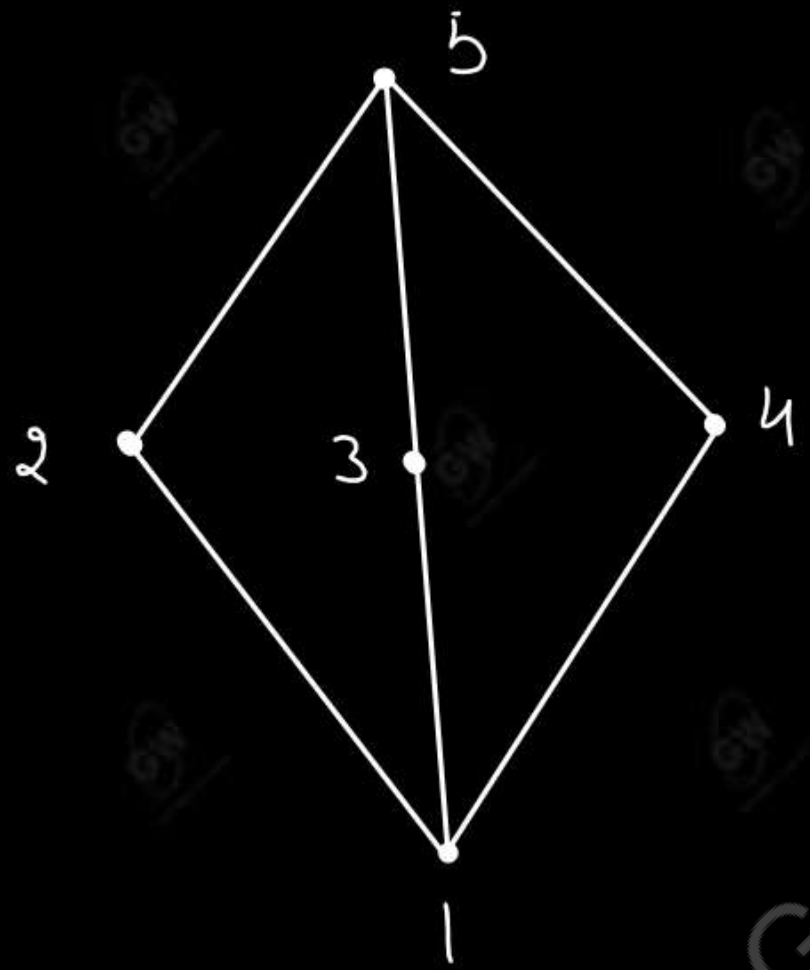
glb table

$\vee$	

Sublattice :-

A non- empty subset M of Lattice (L, ≤) is said to be a sub-lattice of L if M is closed with respect to join (∨) and meet (∧) i.e $a, b \in M \Rightarrow a \vee b \in M$ and $a \wedge b \in M$

Q.3. Consider the lattice $L = \{1, 2, 3, 4, 5\}$ given below Determine all sub lattices with three or more elements.



$L_1 = \{1, 2, 5\}$

$L_2 = \{1, 3, 5\}$

$L_3 = \{1, 4, 5\}$

$L_4 = \{1, 2, 5, 3\}$

$L_5 = \{1, 4, 5, 3\}$

$L_6 = \{1, 2, 5, 4\}$

Isomorphic Lattice

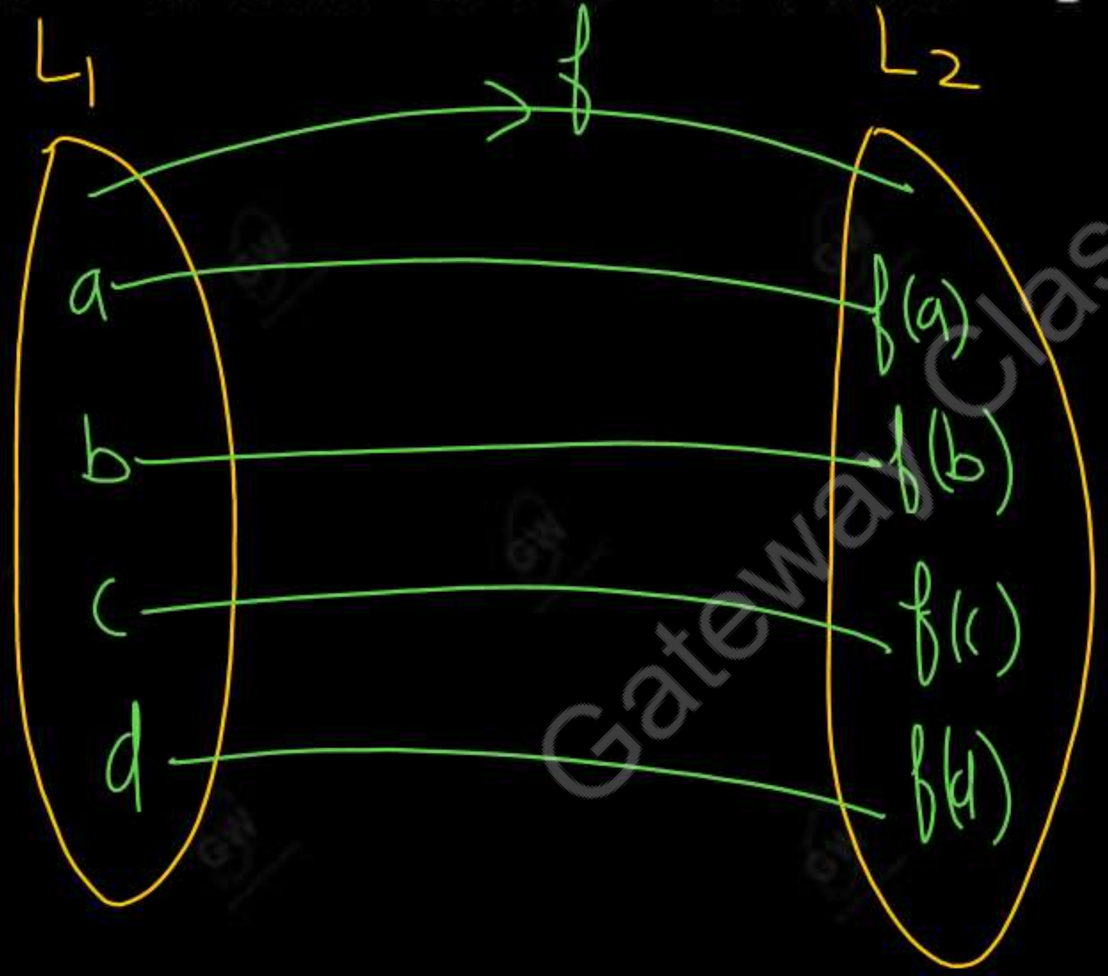
$f: L_1 \longrightarrow L_2$

Two lattice L_1 and L_2 are isomorphic if

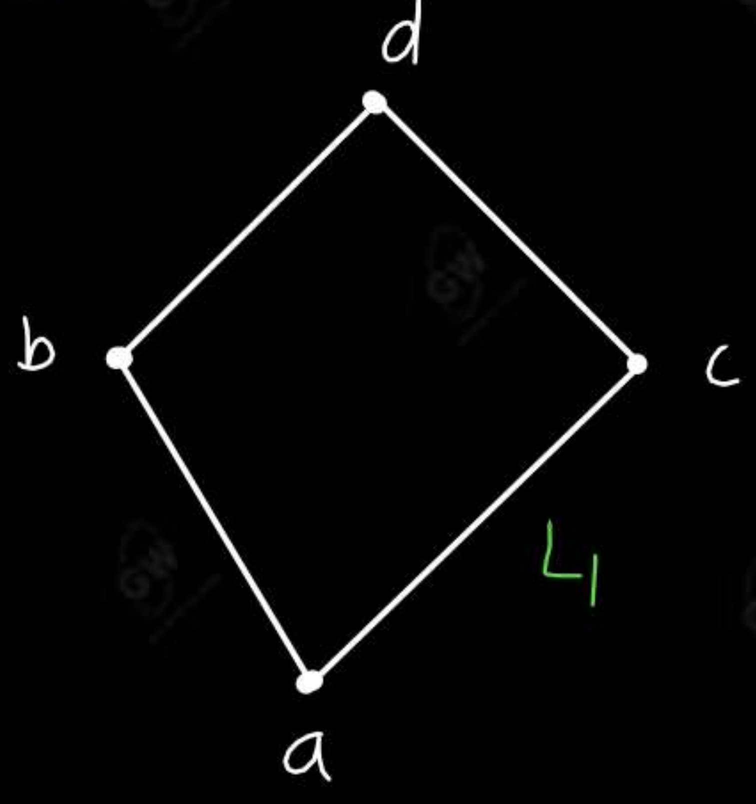
(i) There exist one-to-one correspondence between vertices and edges (Bijection)

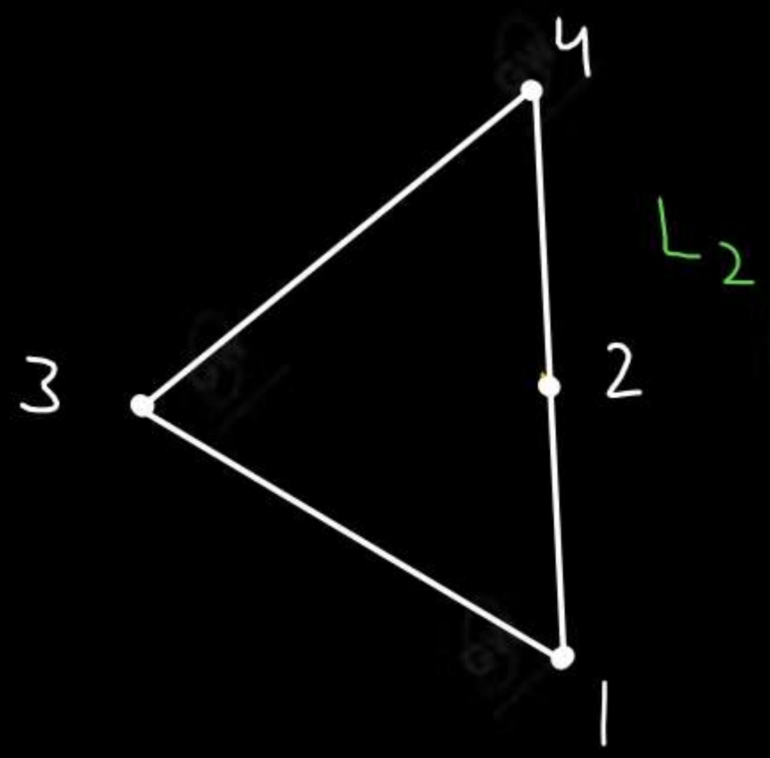
(ii) $f(a \vee b) = f(a) \vee f(b) \quad \forall\, a, b \in L_1$

$f(a \wedge b) = f(a) \wedge f(b) \quad \forall\, f(a), f(b) \in L_2$

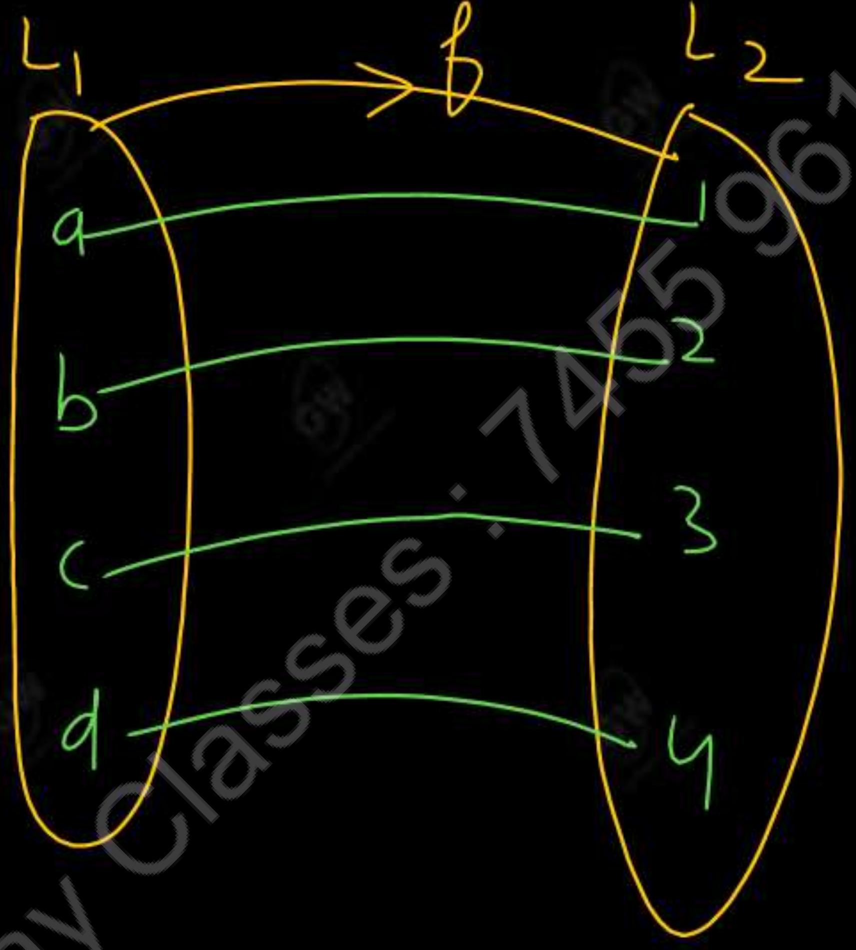


Q.4. Determine whether the lattice given below are isomorphic or not.





$f : L_1 \longrightarrow L_2$



such that

$f(a) = 1$

$f(b) = 2$

$f(c) = 3$

$f(d) = 4$

$f(a \vee b) = f(b) = 2$ — ①

$f(a) \vee f(b) = 1 \vee 2 = 2$ — ②

From ① and ②

$$\boxed{f(a \vee b) = f(a) \vee f(b)}$$

$f(a \wedge b) = f(a) = 1$ — ③

$f(a) \wedge f(b) = 1 \wedge 2 = 1$ — ④

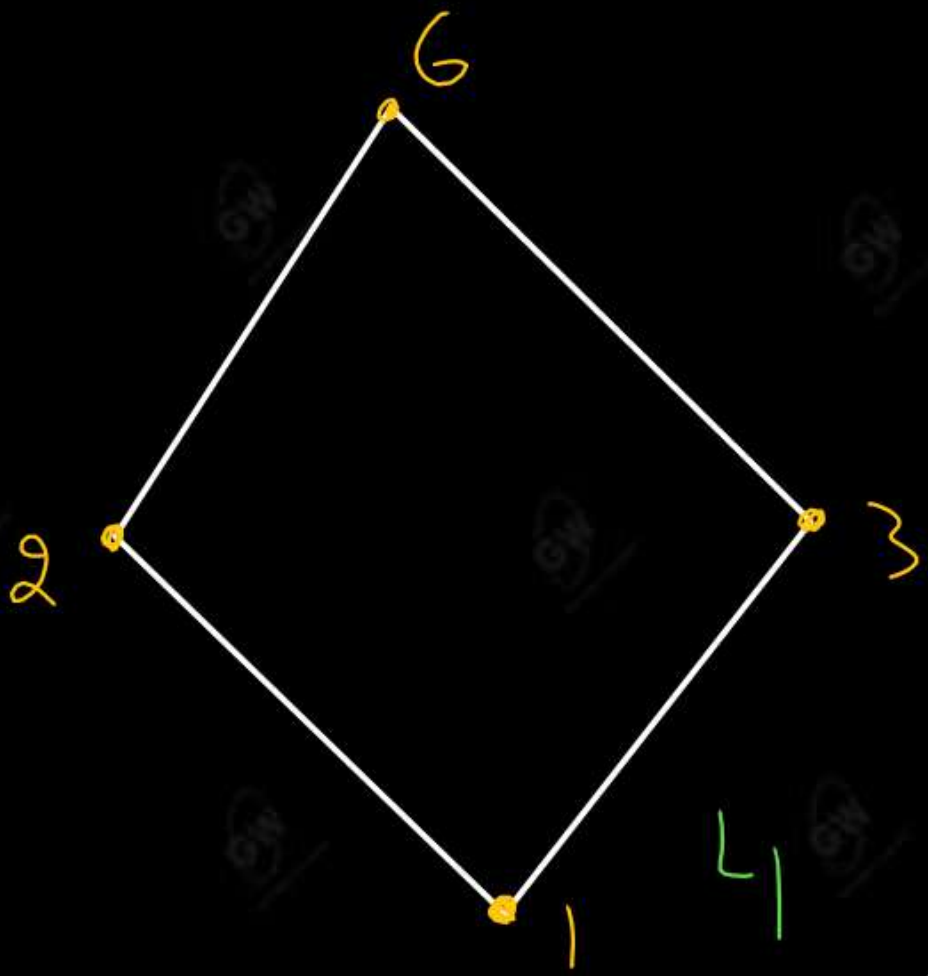
From ③ and ④

$$\boxed{f(a \wedge b) = f(a) \wedge f(b)}$$

Hence L_1 and L_2 are isomorphic lattices

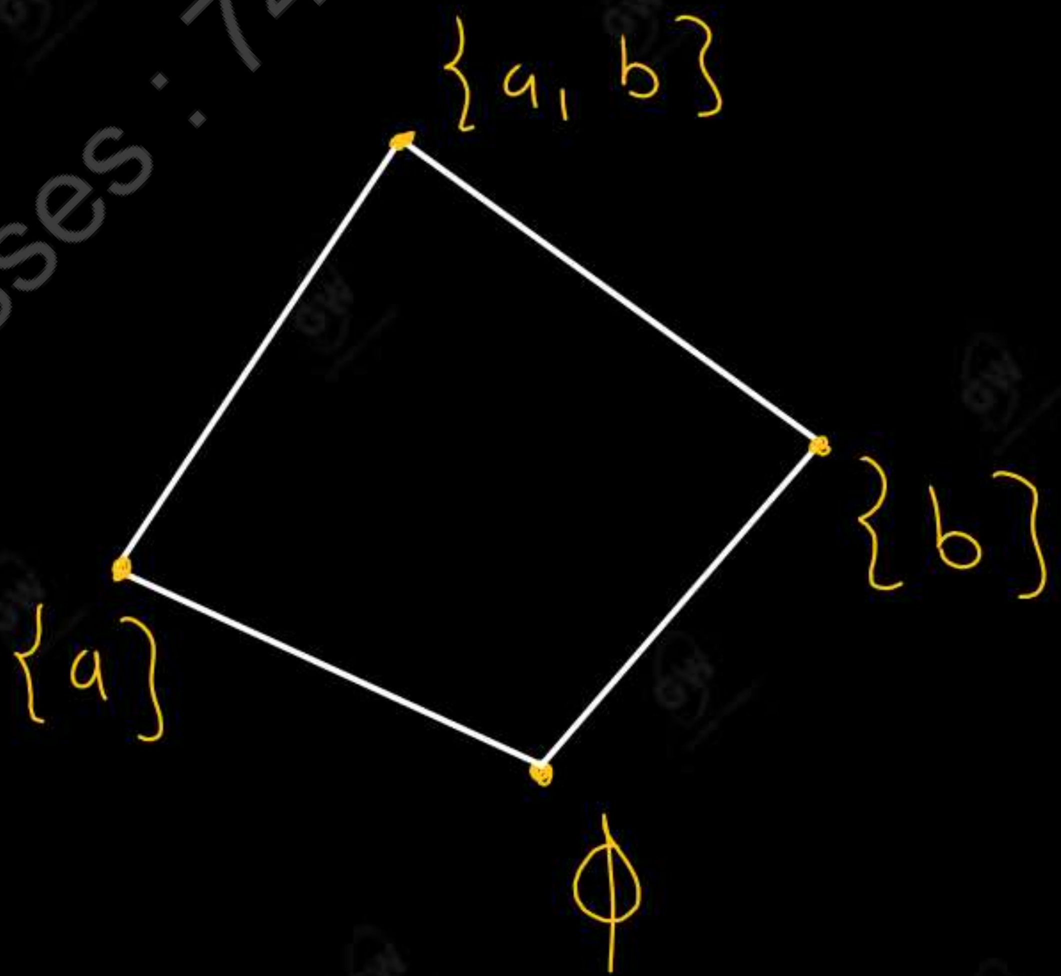
Q.5. Let L_1 be the lattice defined as D_6 and L_2 be the lattice $(P(S), \subseteq)$, where P(S) is the power set defined on set S = {a, b} justify that the two lattices are isomorphic.

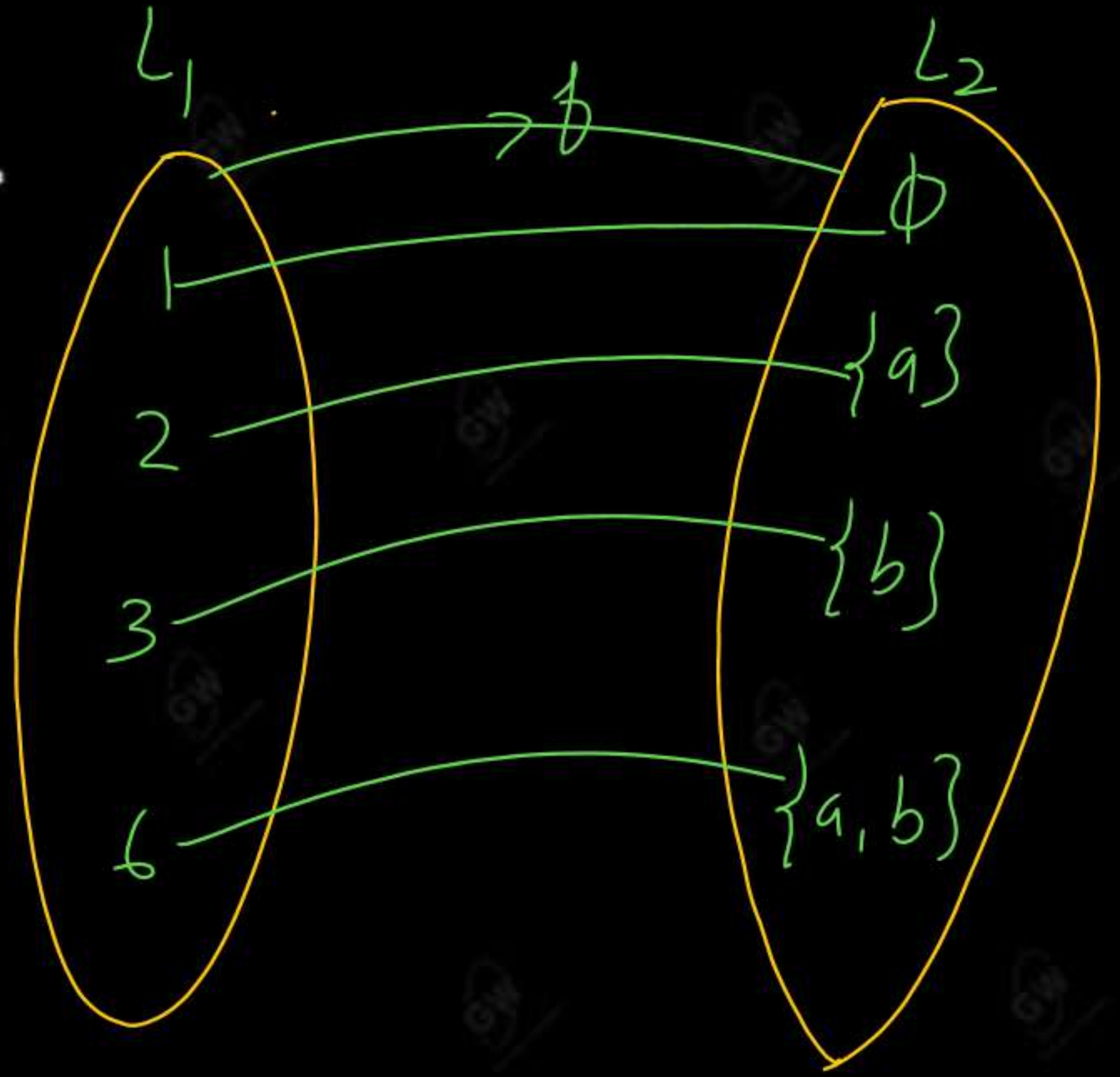
$D_6 = \{1, 2, 3, 6\}$



$L_2 = (P(S), \subseteq)$

$P(S) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$





$$f(1 \vee 2) = f(2) = \{a\}$$

$$f(1) \vee f(2) = \phi \vee \{a\} = \{a\}$$

$$\therefore \boxed{f(1 \vee 2) = f(1) \vee f(2)}$$

$$f(1 \wedge 2) = f(1) = \phi$$

$$f(1) \wedge f(2) = \phi \wedge \{a\} = \phi$$

$$\boxed{f(1 \wedge 2) = f(1) \wedge f(2)}$$

DPP- 16

Topic : *LATTICE*

Q.1. Prove that $(D_{20}, 1)$ *is a lattice*

Q.2. Prove that $(D_{30}, 1)$ *is a lattice*

Q.3. Determine all the sub lattices of D_{30} *that contain four elements.*

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Thank You