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HAVOK Analysis of a Droplet Number Signal at Subatmospheric Conditions

Elab Report
from
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I assure to have written the work independently and to have used only the given sources and tools. The passages that have been adopted either literally or in terms of content are marked as such. The statutes of the Karlsruhe Institute of Technology (KIT) to ensure good scientific practice have been observed by me in their current version.

Karlsruhe, September 25, 2020

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Abbreviations

Abbreviations	Description
HAVOK	Hankel alternative view of Koopman
SINDy	Sparse Identification of Nonlinear Dynamics
SVD	Singular Value Decomposition

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1 Introduction

Machine Learning becomes increasingly popular, even in the field of fluid dynamics. It is still unclear how big the potential of these methods is, to gain deeper understanding of far complex dynamical systems. The large availability of data and simultaneous absence of appropriate models to describe a dynamic system has necessitated the need to look for data driven approaches to reconstruct a mathematical description. In this work, effort is made to use the HAVOK analysis to understand how a state space model of a dynamic system can be reproduced using raw data. First, the HAVOK analysis (Steven L. Brunton (2017)) is applied to a sine wave model for verification and then to analyze a droplet number signal of an atomizer utilized in a combustion chamber of an aero-engine at sub-atmospheric conditions.

2 Methodology

2.1 HAVOK

This method has been developed at University of Washington by Brunton et. al. (Steven L. Brunton (2017)). This work presents a data-driven approximation of deterministic chaos as an intermittently forced linear system. The method uses the eigen-time-delay coordinates as the training and validation data obtained by applying a singular value decomposition (SVD) to the Hankel matrix and Sparse Identification of Nonlinear Dynamics (SINDy) is applied consecutively to generate a state space model of the chaotic dynamical system. The state space model can be simulated using a validation data corresponding to a time signal $x(t)$.

2.2 Hankel Matrix

Hankel matrix is a square matrix constructed using the time shifted copies of measurement data as shown below. A matrix **V** of eigen-time-delay coordinates is obtained when SVD is applied on the Hankel matrix resulting in $H = U \sum V^T$. The number of rows q of the Hankel matrix is a deterministic parameter in the methodology. Details about the further implementation can be found in the supplementary of (Steven L. Brunton (2017)).

$$\mathbf{H} = \begin{bmatrix} y(t_1) & y(t_2) & \cdots & y(t_p) \\ y(t_2) & y(t_3) & \cdots & y(t_p+1) \\ y(t_2) & y(t_3) & \cdots & y(t_p+1) \\ \vdots & \vdots & \ddots & \vdots \\ y(t_q) & y(t_q+1) & \cdots & y(t_m) \end{bmatrix}$$

2.3 SINDy

SINDy is a regression method which is used to extract governing equations from a given data set. According to this method, most physical systems have only a few relevant basis function which determine the dynamical evolution, making the governing equations sparse in a high-dimensional nonlinear function space (Brunton et al. (2016)).

3 Results and Discussion

3.1 Sine Wave

To verify the HAVOK methodology, it is applied on a generic sine signal. Here, different frequencies are used to generate the training time series data from the sine wave equation. The results are displayed in Figure 3.1. The parameters for the modeling are given in Table 3.1 and the source code can be found in the gitlab repo.

The attractor (2D phase space plot) for the sine signal is well known to be an ellipse. An embedded attractor is obtained by eigen-time-delay coordinates and is reconstructed with the SINDy method. The color coding in the figures indicates the evolution in the time starting with the blue and closing with the red.

The results depend on the following degrees of freedom, as depicted in Figure 3.1

- No. of rows q considered for the Hankel matrix
- The number of rows r from the eigen-time-delay coordinates considered for the appropriate linear state space model
- The considered basis function on the SINDy method

We can obviously see that the attractor of the sine signal is reproduced correctly as an ellipse. However, we additionally observe that the models are not equally smooth for all frequencies. As the frequency increase, the attractor gets smoother. In general we can state that the more data we have the more accurate model we achieve. The chosen model number r plays an important role. Here we choose $r = 4$ because it is the highest order model that reconstructs attractor dynamics, retains neutral stability and has a meaningful sparse model structure. This is purely an artifact and can be achieved only with varying combinations of the above mentioned parameters (Steven L. Brunton (2017)).

Table 3.1: Sine Wave Model Parameters

Parameter	**Value**
Equation	xdat_x =sin(n*pi*t)
q	200
r	4
Polyorder	2

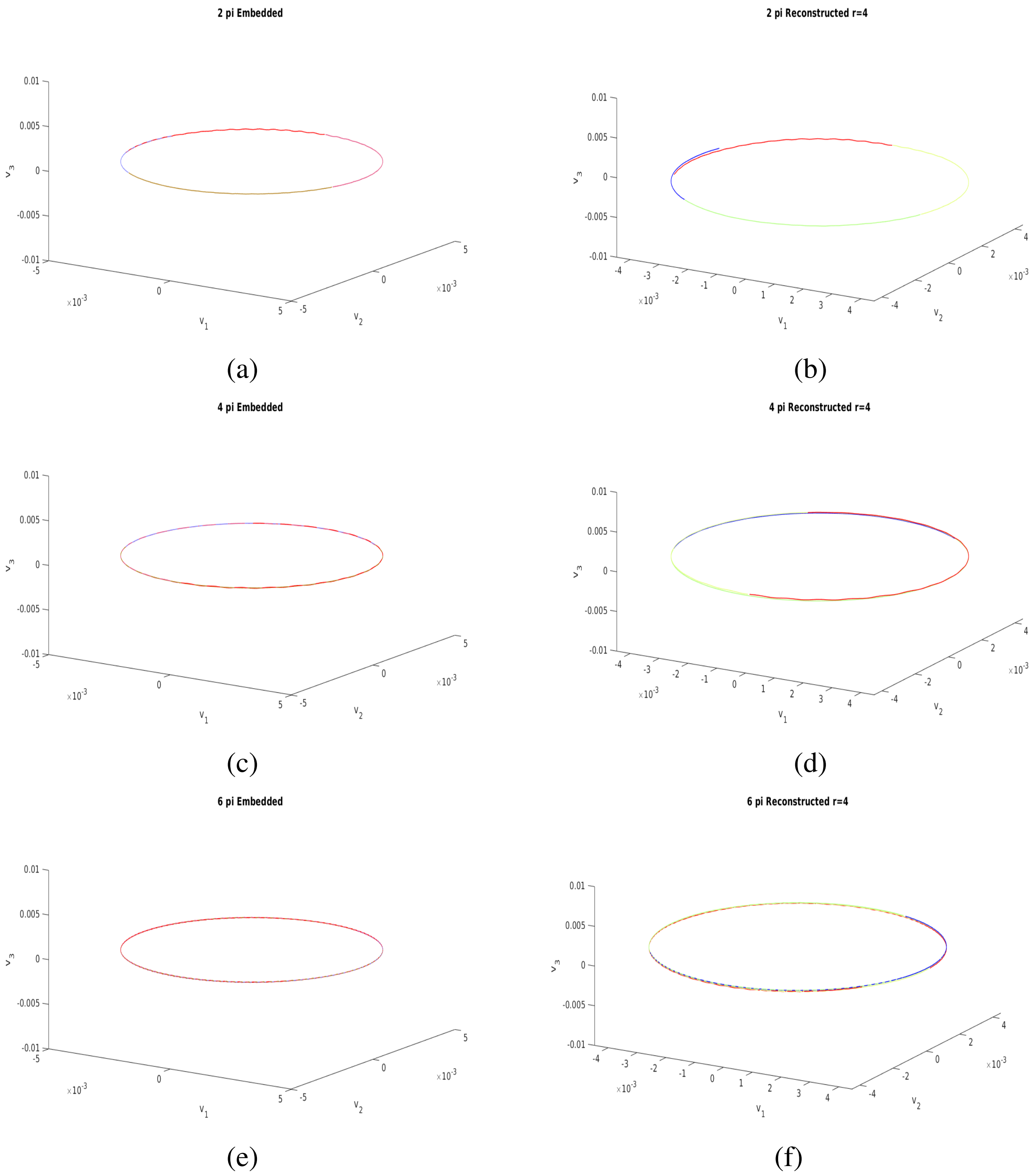


Figure 3.1: Embedded and reconstructed attractors of the sine signal

3.2 Modeling of a Droplet Number Signal

The time series considered for this section is obtained from the simulation of atomizer utilized for aero-engines. Since the chaotic behavior of the atomization is not understood yet, the HAVOK analysis is a potential tool to explore the dynamics.

The study is conducted choosing the parameters of varying degrees of freedom. The simulated data is cubical interpolated to achieve reasonable amount of data (Appendix 2). The results are displayed in the figures below. The source code and the data file can be found on the gitlab repo.

From the results we observe that, even though they seem to be promising, the reconstructed attractor is very diverse for every chosen parameter. Given that there can be very wide range of combination of the parameters and ambiguity in the results, it is hard to draw a final conclusion. However, we can see that the embedded attractor constructed by means of the eigen-time-delay coordinates is almost similar, independent of the chosen parameter, which can be helpful in understanding the behavior of the system.

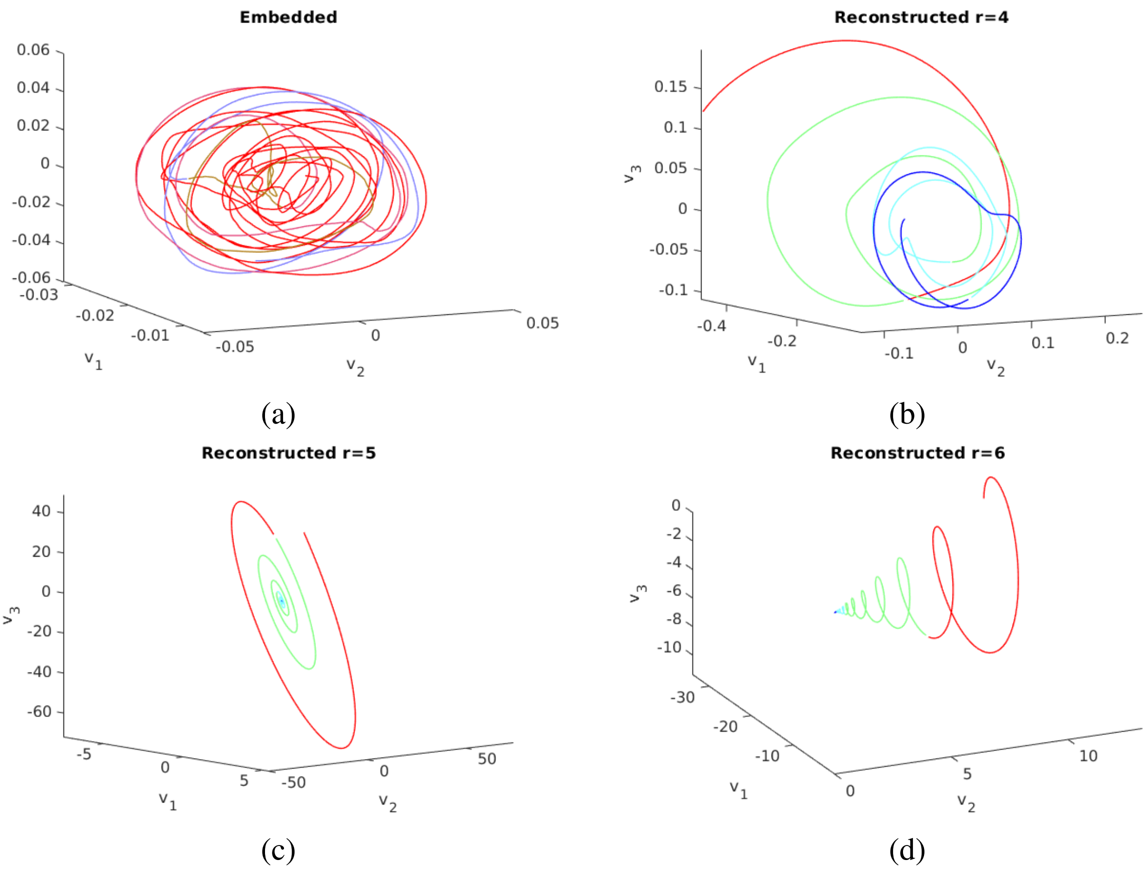


Figure 3.2: Droplet Number Model with q=100, polyorder = 2

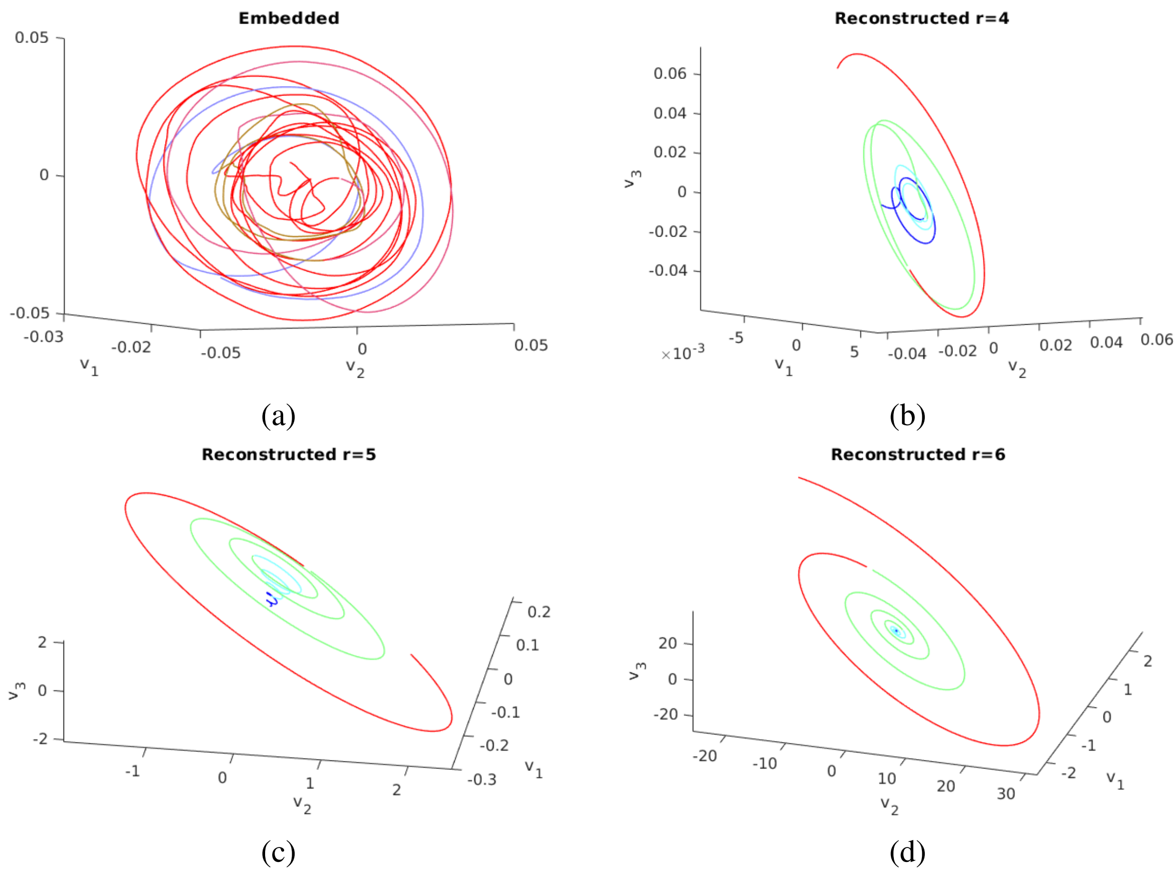


Figure 3.3: Droplet Number Model with q=200, polyorder = 2

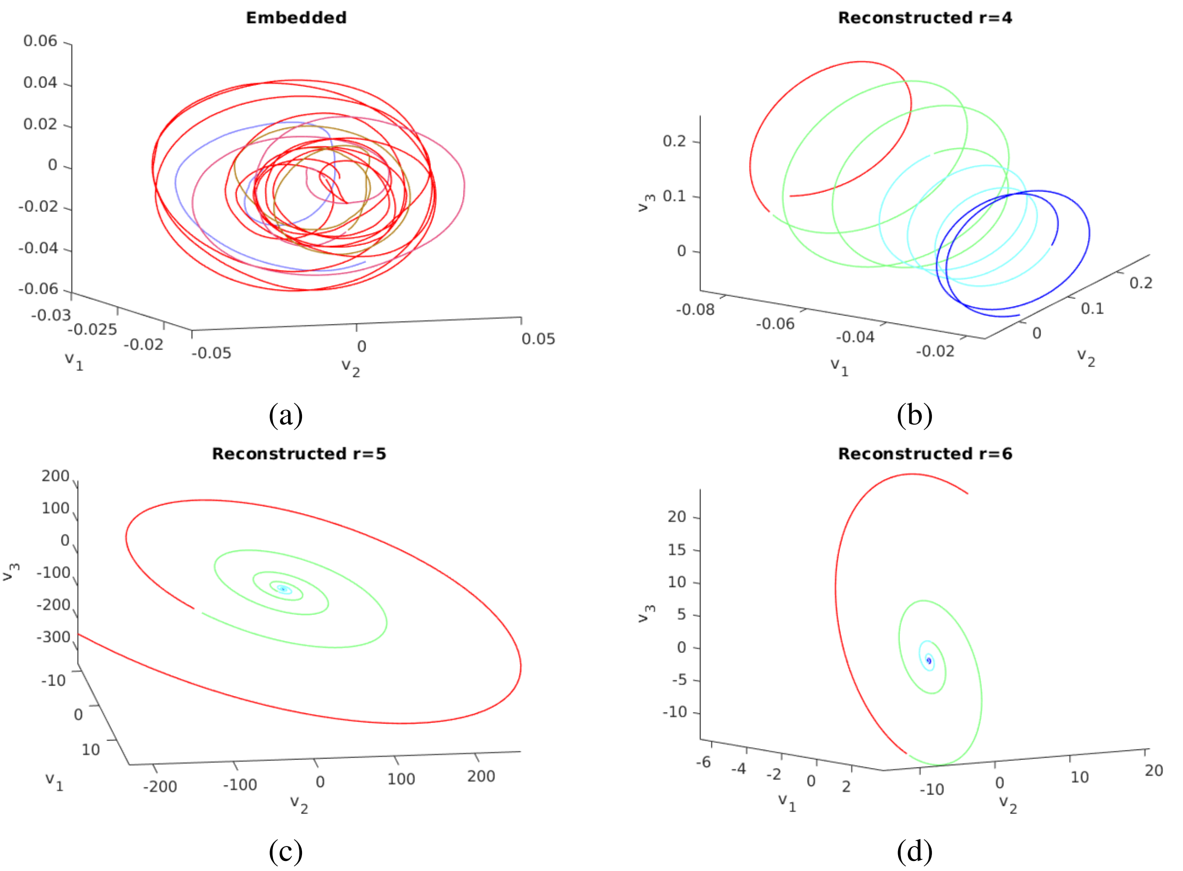


Figure 3.4: Droplet Number Model with q=300, polyorder = 2

4 Summary and future work

As we see from the above discussion there is a potential in this approach to explore chaotic nature of atmoization. The HAVOK tools used in this work are just the tip of iceberg of the ongoing research. Few other sophisticated tools have been introduced by the researchers like

- PySINDy which is a python module which generates the model using the input data and parameters.
- Autoencoder SINDy which is an upgraded SINDy algorithm coupled with deep learning model.

Bibliography

Brunton, Steven L., Proctor, Joshua L. und Kutz, J. Nathan (2016): *Discovering governing equations from data by sparse identification of nonlinear dynamical systems*. Proceedings of the National Academy of Sciences, Bd. 113, S. 3932–3937. ISSN 0027-8424.

Steven L. Brunton, Bingni W. Brunton, Joshua L. Proctor Eurika Kaiser J. Nathan Kutz (2017): *Chaos as an intermittently forced linear system*.

Appendix

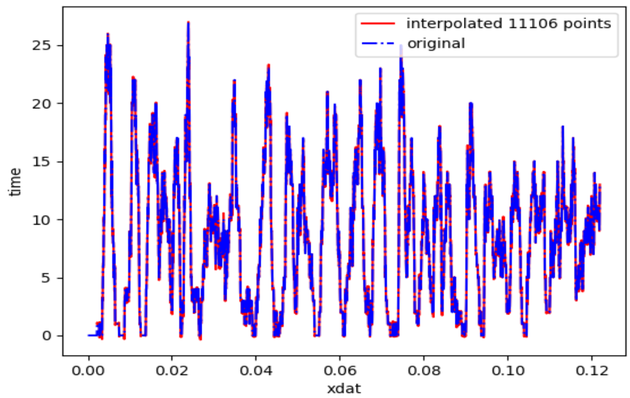
A.1 Appendix 1

```
function Xi = sparsifyDynamics(Theta,dXdt,lambda,n)
% Copyright 2015, All Rights Reserved
% Code by Steven L. Brunton
% For Paper, "Discovering Governing Equations from Data:
%        Sparse Identification of Nonlinear Dynamical Systems"
% by S. L. Brunton, J. L. Proctor, and J. N. Kutz

% compute Sparse regression: sequential least squares
Xi = Theta\dXdt;  % initial guess: Least-squares

% lambda is our sparsification knob.
for k=1:10
    smallinds = (abs(Xi)<lambda);    % find small coefficients
    Xi(smallinds)=0;                 % and threshold
    for ind = 1:n                    % n is state dimension
        biginds = ~smallinds(:,ind);
        % Regress dynamics onto remaining terms to find sparse Xi
        Xi(biginds,ind) = Theta(:,biginds)\dXdt(:,ind);
    end
end
```

A.2 Appendix 2



(a) Interpolated Time Series Data